TO: **Professor Andreas Linninger / Grant Hartung**

FROM: **Kehinde Abioye**

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SUBJECT: **Visualization in MATLAB**

The purpose of this assignment is to use the equation Ax=b to solve for unknown variables (or intersections) of lines or planes in multidimensional coordinate systems and plot those lines in MATLAB.

Introduction

Part#1: Lines

The variable x in the equation Ax=b, is the linearly independent set of equations 1 and 2. By using Gaussian elimination (Appendix A) the matrix created for the equations can be reduced to find a solution for y and use the solution for y in the first equation to solve for x. There are also MATLAB functions that can be used to make this process easier.

$$2x + y = 2; \quad \text{Eq 1}$$

$$x + 3y = 1; \quad \text{Eq 2}$$

Part#2: Planes

The variable x in the equation Ax=b, is the intersection of the equations 3, 4 and 5. By using Gaussian elimination (Appendix A) the matrix created for the equations can be reduced to find a solution for z resulting in finding y with reduced equation 4 and x with the values found for z and y. A grid of all possible solutions for a given interval is created for each equation by using for loops. The for loops run through every possible point in an interval and runs in through equation 3, 4, and 5. The result is a plane for each equation. The solution is where all planes meet.

$$x + y + z = 2; \quad \text{Eq 3}$$

$$2x + 3y + z = 1; \quad \text{Eq 4}$$

$$3x + 2y + z = 3; \quad \text{Eq 5}$$

Part#3: 3D Points

The observed points were plotted in an X-Y-Z coordinate system. The points from the table were put in matrix form. A scatter plot can be created for each X-Y-Z point by using the X, Y, and Z vectors from the matrix.

	X	**Y**	**Z**
POINT 1	10	2	5
POINT 2	8	1	4
POINT 3	6	9	3
POINT 4	3	8	6
POINT 5	7	9	8
POINT 6	8	7	7
POINT 7	9	4	9
POINT 8	5	5	5
POINT 9	2	3	2
POINT 10	1	2	1

Table 1: Points observed in X-Y-Z coordinate system

Part#4: Plotting the Energy of a System

Vector b can be subtracted on both sides of the equation Ax=b to produce the residual error equation, r=Ax-b (eq. 6). Solution vector x will give r = null vector while all other possible points will give a number. The error measures the accuracy of your solution. The residual error can be produced into a surface of using the phi equation, P $(x) = r^T r = (Ax - b)^T (Ax - b)$ (eq. 7). Equation 7 gives the length of the residual vector. The longer the distance the more error. The solution x looks as if it resides in the center of the surface which would be when r equals a null vector.

$$r = Ax - b; \text{ Eq 6}$$

$$P\,(x) = r^T r = (Ax - b)^T (Ax - b); \text{ Eq 7}$$

Methods

Part#1: Lines

Equation 1 and 2 were rearranged in the form Ax=b. The unknown variables in vector x were solved using MATLAB and manual Gaussian elimination. Equations 1 and 2 were plotted while varying x using the “plot” function.

Part#2: Planes

Equation 3, 4, and 5 were rearranged in the form Ax=b. The unknown variables in vector x were solved using MATLAB and manual Gaussian elimination. Variables x and y were given intervals using “linspace.” Equations 3,4 and 5 were plotted using the “surf” function and the intersecting vector x was plotted using the “scatter3” function.

Part#3: 3D Points

The points from Table 1 were formed in a 10x3 matrix in MATLAB. The x, y, and z values were assigned to their respective values and were inputted in the “scatter3” function to create a 3D scatter plot.

Part#4: Plotting the Energy of a System

Equations 1 and 2 from part #1 were rearranged into residual form r=Ax-b. The residual error was calculated on the interval ($-10 \leq x \leq 10$, $-10 \leq y \leq 10$) using the “linspace” function, for loops, and the residual error equations. The residual surface was calculated by inputting the residual error equation into equation 7. The surfaces were plotted using the “surf” function and the solution from part #1 was plotted using the “scatter” function.

Results

Part#1: Lines

Equations 1 and 2 were plotted by setting them equal to y and vector x was found and plotted on the same graph. Figure 1 shows the solution to the linearly independent set where the two lines meet and is represented by an orange circle.

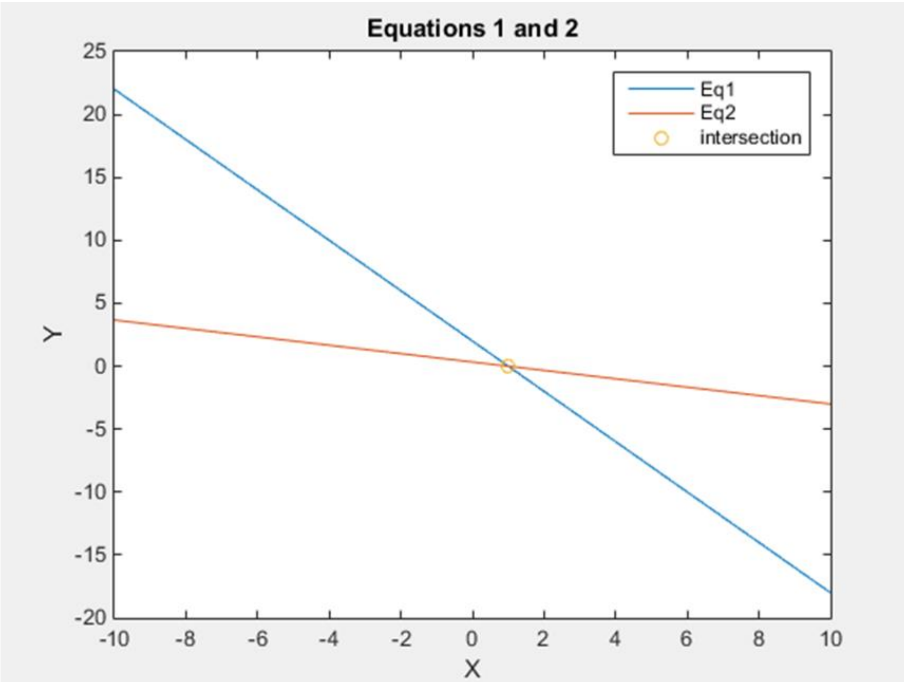

Figure 1: Linear algebraic equations 1 (2x + y = 2) and equation 2 (x + 3y = 1) with solution x on X-Y coordinate system

The solution can be checked by using the matrix equation, Ax=b as shown in equations 8 and 9. When rearranged, the equation Ax-b=0 is produced. In equation 6, 0 is a null vector and is represented by r.

$$\begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} - \begin{bmatrix} 2 \\ 1 \end{bmatrix} = 0; \quad \text{Eq. 8}$$

$$\begin{bmatrix} 2 * 1 + 1 * 0 - 2 \\ 1 * 1 + 3 * 0 - 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \quad \text{Eq. 9}$$

Since r equals a null vector, the solution is correct.

Part#2: Planes

Equations 3, 4, and 5 were set equal to z. Three grids of possible solutions were created for Equations 3, 4, and 5 using for loops and plotted on a three-dimensional plane shown in figure 2. Vector x was found and plotted on the same graph and is represented by a ble circle.

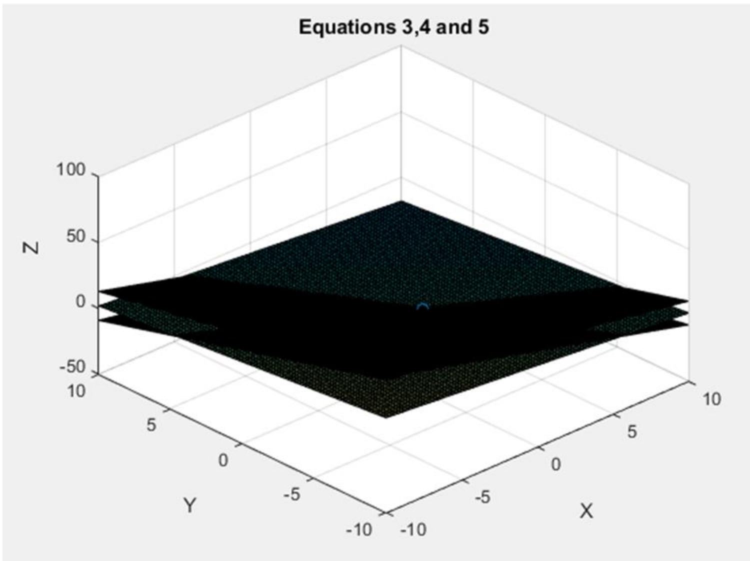

Figure 2: Linear algebraic equations 3 (x + y + z = 2), equation 4 (2x + 3y + z = 1) and equation 2 (3x + 2y + z = 3) with solution x on X-Y coordinate system

The solution can be checked by using the matrix equation, Ax=b. When rearranged, the equation Ax-b=0 is produced, which is also the residual error equation r=Ax-b, where r is a null vector. This is shown in equations 10 and 11.

$$\begin{bmatrix}1 & 1 & 1\\ 2 & 3 & 1\\ 3 & 2 & 1\end{bmatrix}\begin{bmatrix}1\\ -1\\ 2\end{bmatrix}-\begin{bmatrix}2\\ 1\\ 3\end{bmatrix}=0; \quad \text{Eq. 10}$$

$$\begin{bmatrix}1*1+1*-1+1*2-2\\ 2*1+3*-1+1*2-1\\ 3*1+2*-1+1*2-3\end{bmatrix}=\begin{bmatrix}0\\ 0\\ 0\end{bmatrix}; \quad \text{Eq. 11}$$

Since r equals a null vector, the solution is correct

Part#3: 3D Points

A three-dimensional scatter plot was created using values of x, y, and z.

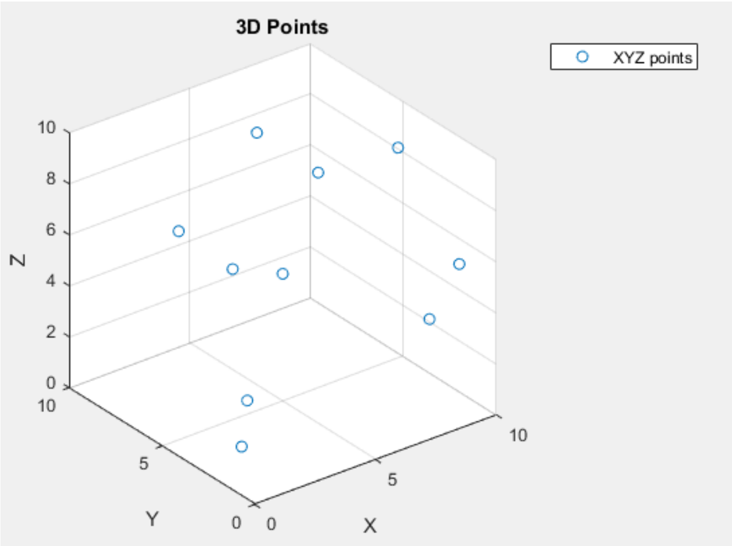

Figure 3: A scatter plot of observed points in the X-Y-Z coordinate system.

Part#4: Plotting the energy of a system

Equations 1 and 2 were set equal to y. A grid of all possible solutions was created with the residual error and surface equations using for loops and plotted on a three-dimensional plane. The solution from case#1 was plotted on the same graph.

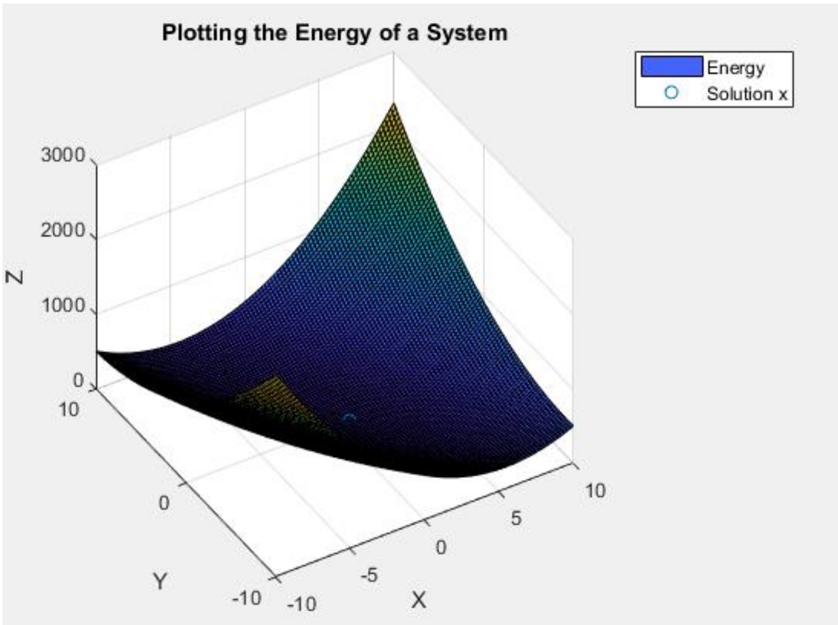

Figure 4: Three-dimensional graph of the surface of residual error of equations 1 and 2 along with the solution from case#1

Conclusion

All cases produced a null vector when plugged into the residual error function, which validates the solutions. The residual surface produced an image that showed the further away from the solution vector x, the longer the distance. The outer surfaces of the graph curved upward which would have a larger magnitude.

Discussion

I have learned that the matrix equation can be applied to solving for unknown variables of a matrix. I also learned the residual error equation can be derived from the matrix equation and be applied to check the accuracy of the solution x of the matrix equation.

Appendix A

```
clc
clear all
% 1. Lines
% Visualize the two lines in MATLAB. Find the intersection of the two
lines
% by manual Gauss elimination and visualize the solution along the two
lines.
% Find the intersection programmatically using Ax = b to validate your
answer.
% 2x + y = 2 eq 1
% x + 3y = 1 eq 2
C=[2 1; 1 3]
b = [2;1]
A=[C b]
A(2,:)=(2*(A(2,:)))-A(1,:)
A(1,:)=(A(1,:))/2
%5y=0
%y=0/5
%y=0
%x+0.5y=1
%x+0.5(0)=1
%x+0=1
%x=1
x=[1;0]
xy=C\b
r=(C*x)-b
%r is a null vector so soln is correct

%Plot
x=linspace(-10,10);
y1=(-2*x)+2
y2=(-x+1)/3
figure;
plot(x,y1,x,y2)
hold on
scatter(1,0)
title('Equations 1 and 2')
xlabel('X')
ylabel('Y')
legend('Eq1','Eq2','intersection')


% 2. Planes
% Visualize the three planes in MATLAB. Find the intersection of the
three
% planes by manual Gauss elimination and indicate the solution with the
three
% planes. Find the intersection programmatically using Ax = b to
validate your answer.
% x + y + z = 2 eq 3
% 2x + 3y + z = 1 eq 4
% 3x + 2y + z = 3 eq 5
C2=[1 1 1;2 3 1;3 2 1]
b2=[2;1;3]
```

```
A2=[C2 b2]
A2(2,:)=((A2(2,:))/2)-A2(1,:)
A2(3,:)=A2(3,:)-(3*(A2(1,:)))
A2(3,:)=A2(3,:)+(2*(A2(2,:)))
%-3z=-6
%z=2
%0.5y-0.5(2)=-1.5
%0.5y-1=-1.5
%0.5y=-0.5
%y=-1
%x-1+2=2
%x=1
xyz=C2\b2
x2=[1;-1;2]
r2=(C2*x2)-b2
%r is a null vector so soln is correct

%Plot
x=linspace(-10,10);
y=x;
%[X,Y] = meshgrid(x,y);
for i=1:length(x)
    for j=1:length(y)
        z1(i,j) =-x(i)-y(j)+2;
        z2(i,j)=(-2*(x(i)))-(3*(y(j)))+1;
        z3(i,j)=(-3*(x(i)))-(2*(y(j)))+3;
    end
end
figure;
surf(x,y,z1)
hold on
surf(x,y,z2)
hold on
surf(x,y,z3)
hold on
scatter3(1,-1,2)
title('Equations 3,4 and 5')
xlabel('X')
ylabel('Y')
zlabel('Z')


% 3. 3D Points
% The following 10x3 matrix represents ten experimentally observed points in
an
% X-Y-Z coordinate system. Visualize the experimental points in MATLAB

data=[10 2 5;8 1 4;6 9 3;3 8 6;7 9 8;8 7 7;9 4 9;5 5 5;2 3 2;1 2 1]
x1=data(:,1);
y1=data(:,2);
z1=data(:,3);
scatter3(x1,y1,z1)
title('3D Points')
xlabel('X')
ylabel('Y')
zlabel('Z')
```

```
legend('XYZ points')


% 4. Plotting the Energy of a system
% To further practice MATLAB plotting functions, compute the residual error
(Eq 6) over
% a range (-10<x<10, -10<y<10) and plot the resulting residual error surface
in 3-D
%
% where Ax=b is the linear algebraic representation of Eq 1 and Eq 2. Here,
rearrange
% Eq 1 and Eq 2 to be written in in residual form (Eq. 6) and evaluate the
residual surface
% using Eq 7. Also plot the solution found in Part 1 and discuss where the
point is located
% on the 3D plane.
%Plot
x=linspace(-10,10);
y=x;
%[X,Y] = meshgrid(x,y);
for i=1:length(x)
    for j=1:length(y)
        r=([2 1; 1 3]*[x(i);y(j)])-[2;1];
        p(i,j)=r'*r;
    end
end


figure;
surf(x,y,p)
hold on
scatter(1,0)
xlabel('X')
ylabel('Y')
zlabel('Z')
title('Plotting the Energy of a System')
legend('Energy','Solution x')
```