TO: Professor Andreas Linninger / Grant Hartung

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SUBJECT: Eigenvalues and eigenvectors

The purpose of this assignment is to practice calculating the eigenvalues and eigenvectors by hand and in MATLAB for eigen decomposition.

Introduction

Eigenvalues are used to uncouple coupled systems to obtain a solution more easily. This assignment focuses on solving for these systems by solving for eigenvalues and eigenvectors by hand. When the values are found, they are verified in MATLAB. The solutions are then plotted in MATLAB.

Methods

Part Aa:

The eigenvalues and vectors are calculated 6 equations by hand and verified using MATLAB. The "eig" function was used in MATLAB to calculate the eigenvalues. The eigenvalue decomposition of matrix A was proven by using equation (1).

$$A = M\Lambda M^{-1} \qquad (1)$$

The two eigenvectors create the modal matrix.

Part Ab

Y(t) was calculated using the equation 2 and 3 below at y(t=0):

$$z_0 = M^{-1} * y_0 \qquad (2)$$

$$Y(t) = Me^{\Lambda t}z_0 \qquad (3)$$

Part Ac:

The solution was verified using equation 4:

$$\dot{Y}=AY \qquad (4)$$

Part Ad:

Y(t) was plotted in MATLAB using the equations from part b.

Part B:

The same was done for equations 7, 8, and 9 as in Part A, but with 3x3 matrices.

Part C:

The same was done for equations 10 and 11 as in Part A, but in state space form with 4x4 matrices.

Results

Part Ad:

Eigenvectors and eigen values were found by hand. Y(t) was plotted for the first 6 equations below:

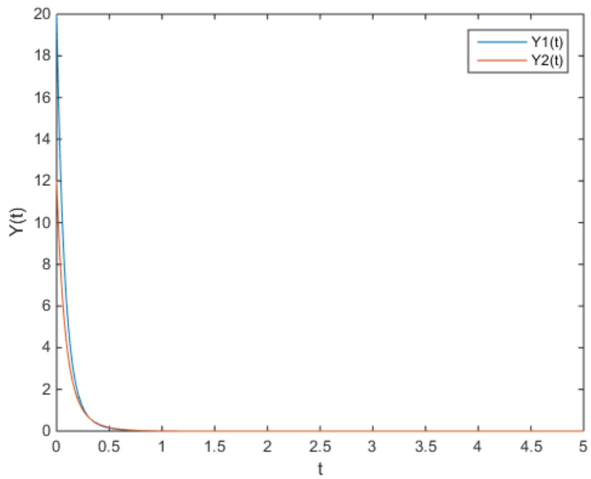

Figure 1 Y(t) for equation 1

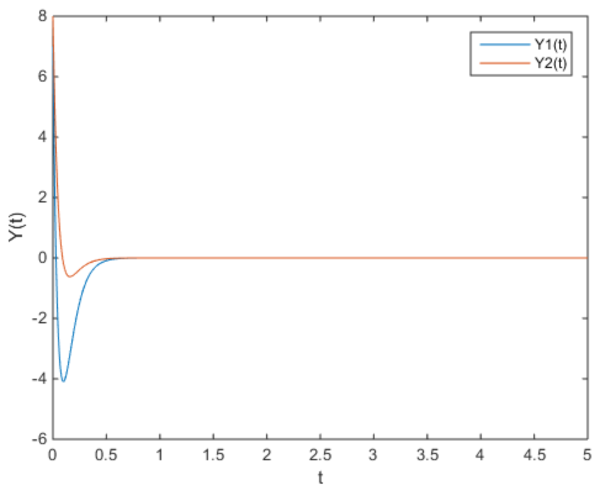

Figure 2 Y(t) for equation 2

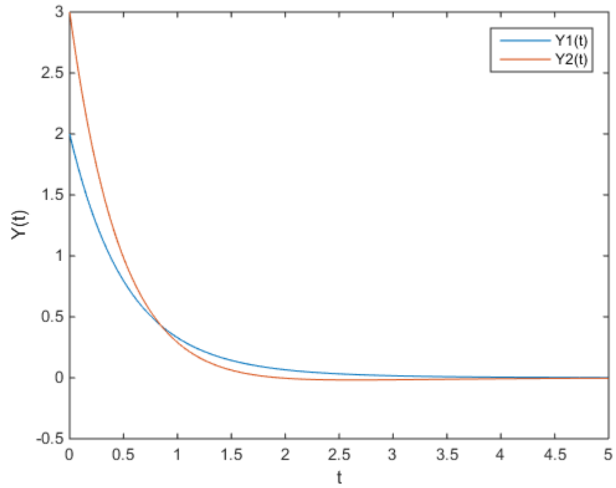

Figure 3 Y(t) for equation 3

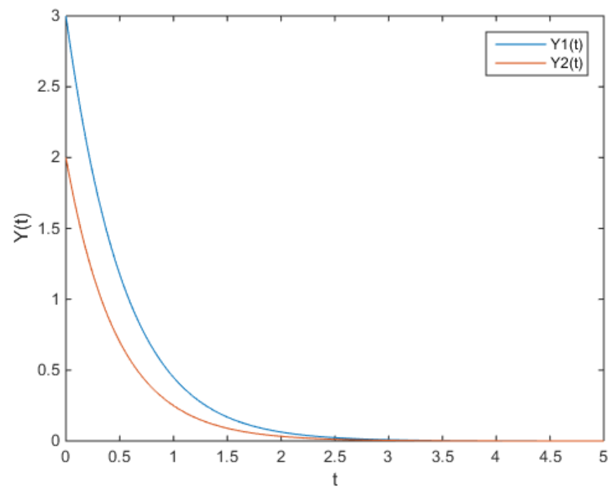

Figure 4 Y(t) for equation 4

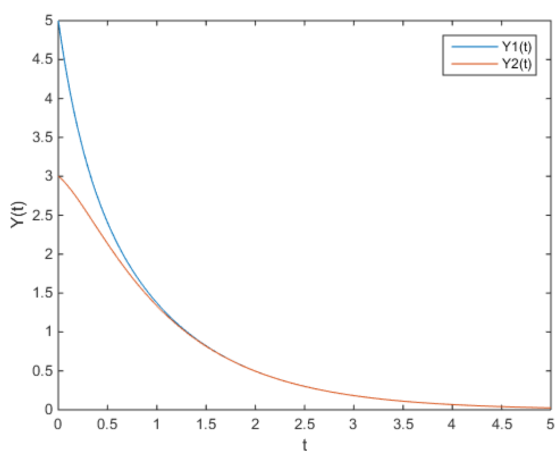

Figure 5 Y(t) for equation 5

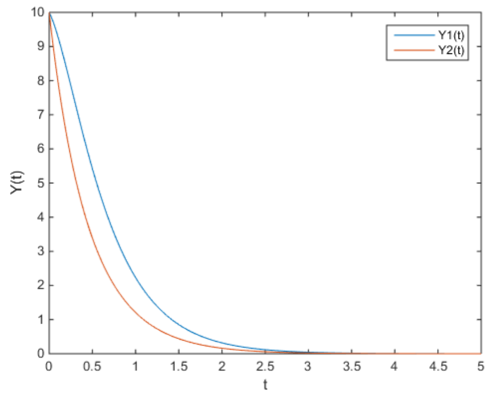

Figure 6 Y(t) for equation 6

Conclusion

Repetitively solving for eigenvalues by hand has been effective in learning eigenvalue decomposition. Coupled systems require all the equations to solve for all variables. By uncoupling the equations, you can get a solution for each equation. For instance, the ways cells communicate can be a coupled system.

Discussion

The use of eigenvalues and eigenvectors help solve for coupled systems which is important when solving for biological systems.

Appendix A

```
clc
clear all
close all

t = 0:.01:5;

%Equation 1
A1 = [-58/3 32/3;-20/3 -2/3];
eig1=eig(A1)
%z0=(M^-1)*y0
%y(t)=(Me^?t)*z0
y01=[20;12];
M1=[1 1;1/2 5/4];
z01=M1\y01
Av=M1\M1*eig1
y1 = (exp(-14*t)*z01(1,:))+(exp(-6*t)*z01(2,:));
y2 = ((1/2)*exp(-14*t)*z01(1,:))+((5/4)*exp(-6*t)*z01(2,:));
plot(t,y1,t,y2)
xlabel('t')
ylabel('Y(t)')
legend('Y1(t)','Y2(t)');

%Equation 2
A2 = [2 -45;6 -31];
eig2=eig(A2)
y02=[7;8];
M2=[1 1;1/3 2/5];
z02=M2\y02
Av=M2\M2*eig2
y1 = (exp(-13*t)*z02(1,:))+(exp(-16*t)*z02(2,:));
y2 = ((1/3)*exp(-13*t)*z02(1,:))+((2/5)*exp(-16*t)*z02(2,:));
figure;
plot(t,y1,t,y2)
xlabel('t')
ylabel('Y(t)')
legend('Y1(t)','Y2(t)');

%Equation 3
A3 = [-3/2 -1/4;-1 -3/2];
eig3=eig(A3)
y03=[2;3];
M3=[1 1;2 -2];
z03=M3\y03
Av=M3\M3*eig3
y1 = (exp(-2*t)*z03(1,:))+(exp(-t)*z03(2,:));
y2 = (2*exp(-2*t)*z03(1,:))+(-2*exp(-t)*z03(2,:));
figure;
plot(t,y1,t,y2)
xlabel('t')
ylabel('Y(t)')
legend('Y1(t)','Y2(t)');

%Equation 4
A4 = [-5/2 1;1/4 -5/2];
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```
eig4=eig(A4)
y04=[3;2];
M4=[1 1;-1/2 1/2];
z04=M4\y04
Av=M4\M4*eig4
y1 = (exp(-3*t)*z04(1,:))+(exp(-2*t)*z04(2,:));
y2 = ((-1/2)*exp(-3*t)*z04(1,:))+(1/2*exp(-2*t)*z04(2,:));
figure;
plot(t,y1,t,y2)
xlabel('t')
ylabel('Y(t)')
legend('Y1(t)','Y2(t)');

%Equation 5
A5 = [-3 2;1 -2];
eig5=eig(A5)
y05=[5;3];
M5=[1 1;-1/2 1];
z05=M5\y05
Av=M5\M5*eig5
y1 = (exp(-4*t)*z05(1,:))+(exp(-t)*z05(2,:));
y2 = ((-1/2)*exp(-4*t)*z05(1,:))+(exp(-t)*z05(2,:));
figure;
plot(t,y1,t,y2)
xlabel('t')
ylabel('Y(t)')
legend('Y1(t)','Y2(t)');

%Equation 6
A6 = [-7/2 3;1/4 -5/2];
eig6=eig(A6)
y06=[10;10];
M6=[1 1;-1/6 1/2];
z06=M6\y06
Av=M6\M6*eig6
y1 = (exp(-4*t)*z06(1,:))+(exp(-2*t)*z06(2,:));
y2 = ((-1/6)*exp(-4*t)*z06(1,:))+((1/2)*exp(-2*t)*z06(2,:));
figure;
plot(t,y1,t,y2)
xlabel('t')
ylabel('Y(t)')
legend('Y1(t)','Y2(t)');

%Equation 7
A7=[-1 -3 1;4 -8 1;3 -3 -2];
eig7=eig(A7)

%Equation 8
A8 = [-15/4 15/2 -1;1/8 -1/4 -1/2;5/4 3/2 -3];
eig8=eig(A8)

%Equation 9
A9 = [-11/4 -1 5/4;-1/8 -9/2 11/8;-3/4 -5 5/4];
eig9=eig(A9)
```

```
%Equation 10
A10 = [-11 0 1 2;-9 -2 3 0;-21 0 -1 6;-15 0 3 0];
eig10=eig(A10)

%Equation 11
A11 = [0 -4 -1 3;-30 34 -13 -21;24 -30 2 18;-66 82 -23 -51];
eig11=eig(A11)
```