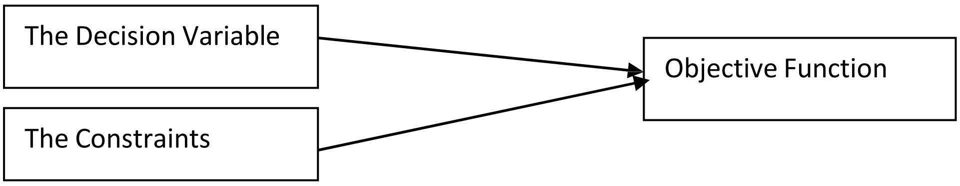
INTRODUCTION TO LINEAR PROGRAMMING

Linear Programming is a problem solving approach that has been developed to help managers to make decisions. Linear Programming is a mathematical technique for determining the optimum allocation of resources and obtaining a particular objective when there are alternative uses of the resources, money, manpower, material, machine and other facilities.

FORMULATION OF THE LINEAR PROGRAMMING MODEL

Three components are:

- The decision variable
- The environment (uncontrollable) parameters
- The result (dependent) variable



Linear Programming Model is composed of the same components

TERMINOLOGY USED IN LINEAR PROGRAMMING PROBLEM

Components of LP Problem: Every LPP is composed of

a. Decision Variable

b. Objective Function,

c. Constraints.

Optimization: Linear Programming attempts to either maximize or minimize the values of the objective function.

Profit of Cost Coefficient: The coefficient of the variable in the objective function express the rate at which the value of the objective function increases or decreases by including in the solution one unit of each of the decision variable.

Constraints: The maximization (or minimization) is performed subject to a set of constraints. Therefore LP can be defined as a constrained optimization problem. They reflect the limitations of the resources.

Input-Output coefficients: The coefficient of constraint variables is called the Input- Output Coefficients. They indicate the rate at which a given resource is unitized or depleted. They appear on the left side of the constraints.

Capacities: The capacities or availability of the various resources are given on the right hand side of the constraints.

THE MATHEMATICAL EXPRESSION OF THE LP MODEL

The general LP Model can be expressed in mathematical terms as shown below:

Let

O_{ij} = Input-Output Coefficient

C_j = Cost (Profit) Coefficient

b_i = Capacities (Right Hand Side)

X_j = Decision Variables

Find a vector (x_1, x_2, x_3 ....x_n) that minimise or maximise a linear objective function F(x)

where $F(x) = c_1x_1 + c_2x_2 + \ldots\ldots\ldots\ldots + c_nx_n$

subject to linear constraints

$a_1x_1 + a_2x_2 + \ldots\ldots\ldots\ldots + a_nx_n = b_2$

$a_1x_1 + a_2x_2 + \ldots\ldots\ldots\ldots + a_nx_n \leq b_2$

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$am_1x_1 + am_2x_2 + \ldots\ldots\ldots\ldots + am_nx_n \leq b_2$

and non-negativity constraints

$x_1 \geq 0, x_2 \geq 0, \ldots\ldots\ldots\ldots, x_n \geq 0$

FORMULATION OF LPP STEPS

- Identify decision variables
- Write objective function
- Formulate constraints

EXAMPLE 1. (PRODUCTION ALLOCATION PROBLEM)

A firm produces three products. These products are processed on three different machines. The time required manufacturing one unit of each of the three products and the daily capacity of the three machines are given in the table below:

Machine	Time per unit (Minutes)			Machine Capacity (minutes/day)
	Product 1	Product 2	Product 3	
M1	2	3	2	440
M2	4	-	3	470
M3	2	5	-	430

It is required to determine the daily number of units to be manufactured for each product. The profit per unit for product 1, 2 and 3 is Rs. 4, Rs.3 and Rs.6 respectively. It is assumed that all the amounts produced are consumed in the market. Formulate the mathematical (L.P.) model that will maximize the daily profit.

Formulation of Linear Programming Model

Step 1

From the study of the situation find the key-decision to be made. In the given situation key decision is to decide the extent of products 1, 2 and 3, as the extents are permitted to vary.

Step 2

Assume symbols for variable quantities noticed in step 1. Let the extents (amounts) of products 1, 2 and 3 manufactured daily be x1, x2 and x3 units respectively.

Step 3

Express the feasible alternatives mathematically in terms of variable. Feasible alternatives are those which are physically, economically and financially possible. In the given situation feasible alternatives are sets of values of x1, x2 and x3 units respectively.

where $x1, x2 \text{ and } x3 \geq 0$.

Since negative production has no meaning and is not feasible.

Step 4

Mention the objective function quantitatively and express it as a linear function of variables. In the present situation, objective is to maximize the profit.

i.e., $Z = 4x1 + 3x2 + 6x3$

Step 5

Put into words the influencing factors or constraints. These occur generally because of constraints on availability (resources) or requirements (demands). Express these constraints also as linear equations/inequalities in terms of variables.

Here, constraints are on the machine capacities and can be mathematically expressed as

$2x1 + 3x2 + 2x3 \leq 440$

$4x1 + 0x2 + 3x3 \leq 470$

$2x1 + 5x2 + 0x3 \leq 430$

EXAMPLE 2: PRODUCT MIX PROBLEM

A factory manufactures two products A and B. To manufacture one unit of A, 1.5 machine hours and 2.5 labor hours are required. To manufacture product B, 2.5 machine hours and 1.5 labor hours are required. In a month, 300 machine hours and 240 labor hours are available. Profit per unit for A is Rs. 50 and for B is Rs. 40. Formulate as LPP.

Solution:

Products	Resource/unit	
	Machine	Labour
A	1.5	2.5
B	2.5	1.5
Availability	300 hrs	240 hrs

There will be two constraints. One for machine hours availability and for labor hours availability.

Decision variables

X1 = Number of units of A manufactured per month. X2 = Number of units of B manufactured per month.

The objective function:

Max Z = 50x1+ 40x2

Subjective Constraints For machine hours

1.5x1+ 2.5x2 ≤ 300

For labor hours

2.5x1+ 1.5x2 ≤ 240

Non negativity

x1, x2 ≥0

EXAMPLE: 3

A company produces three products A, B, C.

For manufacturing three raw materials P, Q and R are used. Profit per unit

A - Rs. 5, B - Rs. 3, C - Rs. 4

Resource requirements/unit

Raw Material Product	P	Q	R
A	-	20	50
B	20	30	-
C	30	20	40

Maximum raw material availability:

P - 80 units; Q - 100 units; R - 150 units. Formulate LPP.

Solution:

Decision variables:

x1 = Number of units of A

x2 = Number of units of B

x3 = Number of units of C

Objective Function

Since Profit per unit is given, objective function is maximization

Max Z = 5x1+ 3x2 + 4x3

Constraints:

For P: For Q:

For R: $0x_1 + 20x_2 + 30x_3 \leq 80$

$20x_1 + 30x_2 + 20x_3 \leq 100$

$50x_1 + 0x_2 + 40x_3 \leq 150$

$x_1, x_2, x_3 >= 0$

EXAMPLE 4: PORTFOLIO SELECTION (INVESTMENT DECISIONS)

An investor is considering investing in two securities 'A' and 'B'. The risk and return associated with these securities is different.

Security 'A' gives a return of 9% and has a risk factor of 5 on a scale of zero to 10. Security 'B' gives return of 15% but has risk factor of 8.

Total amount to be invested is Rs. 5, 00, 000/- Total minimum returns on the investment should be 12%. Maximum combined risk should not be more than 6. Formulate as LPP. Solution:

Decision Variables:

x1 = Amount invested in Security A

x2 = Amount invested in Security B

The objective is to maximize the return on total investment.

$\therefore$ Max Z = 0.09 X1 + 0.015 X2 ((% = 0.09, 15% = 0.15)

Constraints:

Related to Total Investment:

X1 + X2 = 5, 00, 000

Related to Risk:

5X1 + 8X2 = (6 X 5, 00, 000)

5X1 + 8X2 = 30, 00, 000

Related to Returns:

0.09X1 + 0.15X2 = (0.12 X 5, 00, 000)

$\therefore$0.09X1 + 0.15X2 = 60, 000

Non-negativity X1, X2 $\geq$ 0

EXAMPLE 5: MEDIA SELECTION

An advertising agency is planning to launch an ad campaign. Media under consideration are T.V., Radio & Newspaper. Each medium has different reach potential and different cost.

Minimum 10, 000, 000 households are to be reached through T.V. Expenditure on newspapers should not be more than Rs. 10, 00, 000. Total advertising budget is Rs. 20 million.

Following data is available:

Medium	Cost per Unit (Rs.)	Reach per unit (No. of households)
Television	2, 00, 000	20, 00, 000
Radio	80, 000	10, 00, 000
Newspaper	40, 000	2, 00, 000

Solution:

Decision Variables:

x1 = Number of units of T.V. ads,

x2 = Number of units of Radio ads,

x3 = Number of units of Newspaper ads.

Objective function: (Maximize reach)

Max. Z = 20, 00, 000 x1 + 10, 00, 000 x2 + 2, 00, 000x3

Subject to constraints:

20, 00, 000 x1 ≥ 10, 000, 000 (for T.V.)

40, 000 x3 ≤ 10, 00, 000(for Newspaper)

2, 00, 000x1 + 80, 000x2 + 40, 000x3 ≤ 20, 000, 000 (Ad. budget)

x1, x2, x3 ≥ 0

∴ Simplifying constraints:

for T.V.	2 x1	≥ 10	∴ x1 ≥ 5
for Newspaper	4 x3	≤ 100	∴ x3 ≤ 25
Ad. Budget			

20 x1 + 8 x2 + 4 x3 ≤ 2000

5 x1 + 2x2 + x3 ≤ 500

x1, x2, x3 ≥ 0

EXAMPLE 6: DIET PROBLEM

Vitamins B1 and B2 are found in two foods F1 and F2. 1 unit of F1 contains 3 units of B1 and 4 units of B2. 1 unit of F2 contains 5 units of B1 and 3 units of B2 respectively.

Minimum daily prescribed consumption of B1 & B2 is 50 and 60 units respectively. Cost per unit of F1 & F2 is Rs. 6 & Rs. 3 respectively.

Formulate as LPP.

Solution:

Vitamins	Foods		Minimum Consumption
	F1	F2	
B1	3	5	30
B2	5	7	40

Decision Variables:

x1 = No. of units of F1 per day.

x2 = No. of units of F2 per day.

Objective function:

Min. Z = 100 x1 + 150 x2

Subject to constraints:

$3x1 + 5x2 \geq 30$ (for N1)

$5x1 + 7x2 \geq 40$ (for N2)

$x1, x2 \geq 0$

EXAMPLE 7: BLENDING PROBLEM

A manager at an oil company wants to find optimal mix of two blending processes. Formulate LPP.

Data:

Process	Input (Crude Oil)		Output (Gasoline)	
	Grade A	Grade B	X	Y
P1	6	4	6	9
P2	5	6	5	5

Profit per operation:

Process 1 (P1) = Rs. 4, 000

Process 2 (P2) = Rs. 5, 000

Maximum availability of crude oil:

Grade A = 500 units

Grade B = 400 units

Minimum Demand for Gasoline:

X = 300 units

Y = 200 units

Solution:

Decision Variables:

x1 = No. of operations of P1

x2 = No. of operations of P2

Objective Function:

Max. $Z = 4000\, x1 + 5000\, x2$

Subjective to constraints:

$6x1 + 5x2 \leq 500$

$4x1 + 6x2 \leq 400$

$6x1 + 5x2 \geq 300$

$9x1 + 5x2 \geq 200$

$x1, x2 \geq 0$

EXAMPLE 8: FARM PLANNING

A farmer has 200 acres of land. He produces three products X, Y & Z. Average yield per acre for X, Y & Z is 4000, 6000 and 2000 kg. Selling price of X, Y & Z is Rs. 2, 1.5 & 4 per kg respectively. Each product needs fertilizers. Cost of fertilizer is Rs. 1 per kg. Per acre need for fertilizer for X, Y & Z is 200, 200 & 100 kg respectively. Labor requirements for X, Y & Z are 10, 12 & 10 man hours per acre. Cost of labor is Rs. 40 per man hour. Maximum availability of labor is 20, 000 man hours.

Formulate as LPP to maximize profit.

Solution:

Decision variables:

The production/yield of three products X, Y & Z is given as per acre. Hence,

$x1$ = No. of acres allocated to X

$x2$ = No. of acres allocated to Y

$x3$ = No. of acres allocated to Z

Objective Function:

Profit = Revenue - Cost

Profit = Revenue - (Fertilizer Cost + Labor Cost)

Product	X	Y	Z
Revenue	2 (4000) x1	1.5 (6000) x2	4 (2000) x3
Fertilizer Cost	1 (200) x1	1 (200) x2	1 (100) x3
Labor Cost	40 (10) x1	40 (12) x2	40 (10) x3
Profit	7400 x1	8320 x2	7500 x3

∴ Objective function

Max. = 7400 x1 + 8320 x2 + 7500 x3

Subject to constraints:

$x1 + x2 + x3 = 200$ (Total Land)

$10 x1 + 12 x2 + 10 x3 \leq 20,000$ (Max Man hours)

$x1, x2, x3 \geq 0$

MERITS OF LPP

- Helps management to make efficient use of resources.
- Provides quality in decision making.
- Excellent tools for adjusting to meet changing demands.
- Fast determination of the solution if a computer is used.
- Finds solution to problems with a very large or infinite number of possible solutions.