ELSEVIER

Journal of Economic Behavior & Organization
Vol. 34 (1998) 477–488

JOURNAL OF
Economic Behavior
& Organization

Spatial competition with intermediated matching

Chris van Raalte[a], Harry Webers[b,*]

[a]*Department of Econometrics, Tilburg University, P.O.Box 90153, NL-5000 LE Tilburg, The Netherlands*
[b]*Policy Research Corporation NV, Jan Moorkensstraat 68, B-2600 Antwerp, Belgium*

Received 21 August 1995; received in revised form 30 January 1997

Abstract

This paper studies a two-sided market where the one type of agents needs the service of a middleman or matchmaker in order to be matched with the other type. The matchmakers compete for agents of both types by means of commission fees. In addition to the fee, the agents also lose a certain transaction cost in case of a match, which may differ among agents. Furthermore, agents have a limited willingness to pay for the service provided by the matchmakers. In certain cases the matchmakers are willing to subsidize one side of the market. Then one of the types of agents is free-riding. © 1998 Elsevier Science B.V.

JEL classification: C72; L11; L13

Keywords: Matching; Middlemen; Competition in fees

1. Introduction

In this paper, we study a bilateral matching market with intermediation. In a bilateral matching market, there are two types of agents, each agent seeking to be matched with one agent of the other type. Examples are the markets for marriage, employment and housing. Often, the two types of market participants are not able to interact directly, and need the help of an intermediary. A considerable part of the literature on bilateral matching is concerned with the trade-off between direct interaction ('search' models) and intermediated interaction. Two typical kinds of externalities are incurred with direct interaction. First, there is the uncertainty that an agent will meet another agent, and second, uncertainty is involved with the matching process itself. Yavas (1994, 1996)

* Corresponding author. E-mail: PolicyResearch@innet.be

PII S0167-2681(97)00080-2

introduces a middleman who internalizes these externalities. A similar approach is taken by Gehrig (1993), who considers competition between two middlemen. The models of Yavas (1994, 1996) and Gehrig (1993) all consider middlemen as matchmakers. A matchmaker's job is to establish matches between market parties without being involved into the interaction between matched agents. This is the case for marketmakers, who buy commodities from sellers and resell them to buyers. Gehrig (1993), Rubinstein and Wolinsky (1987) and Bhattacharya and Yavas (1993) analyze this type of middleman. See also Bhattacharya and Hagerty (1987). Yavas (1993) compares the performance of marketmakers and matchmakers in different kind of markets.

A different type of middlemen is considered by Hänchen and Von Ungern-Sternberg (1985). These middlemen inform consumers about commodity prices and qualities. Biglaiser (1992) and Biglaiser and Friedman (1994) consider middlemen who are able to differentiate better than other buyers the quality of commodities, and so to profitably establish a reputation for offering high quality goods. On middlemen in housing markets, see for example, Carroll (1989).

In our model, we consider intermediation by matchmakers. We analyze a variant of the Salop (1979) model of spatial competition where two matchmakers compete in commission fees in a one-stage game. In our model there are two types of agents, distributed uniformly over a circular city. The types have different densities. The two matchmakers are located symmetrically on the circle, that is, at maximal distance from each other[1].

Every agent going to a matchmaker pays a commission fee, and loses a certain relational cost[2], provided he is matched. The relational costs are incurred in the process of matching, and are directly proportional to the distance along the circle to a matchmaker. Positive relational costs make agents heterogeneous, by which the model becomes more general. The case of zero relational costs is analyzed as a limit case; this case represents 'pure' competition in commission fees. Furthermore, each agent has a reservation price, indicating how much he is willing to spend, in terms of the fee plus the relational cost, in order to be matched. The reservation price may differ between the two types. The valuation for matches is assumed to be homogeneous among agents and matchmakers.

With respect to the agents, we make the assumption that each of them goes to the matchmaker whose sum of fee and relational cost is the smaller. Also, the matchmakers expect the agents to behave that way. We do not incorporate more sophisticated expectations of the firms with respect to agents' behavior, or of the agents with respect to the other agents[3]. We think there is no strong reason to incorporate such expectations, since neither the firms nor the agents possess any a priori information, based on which

[1] Kats (1995) shows that symmetric location of firms along the circle is an equilibrium outcome in the traditional circular model (i.e. the model with (essentially) one market side). Although this location equilibrium is not unique in case of linear transportation cost, it is unique in case of quadratic transportation cost. Therefore we think it is reasonable to assume that firms are located symmetrically.

[2] Relational costs could be interpreted as agent-specific transaction costs, as suggested by one of the referees. Van Raalte (1996) incorporates relational costs in models of costly intermediation in commodity markets.

[3] If they would do so, the risk of not being matched would enter the agents' utility function. In some sense, this risk could be related to the risk associated with the timely delivery of products (see Espinosa, 1992).

they could form any reasonable expectations about the other agents' behavior. Since we focus on the competition in commission fees, we choose not to complicate the model unnecessarily. The behavioral assumptions made with respect to the agents are consistent with strategic behavior. Given that the matchmakers set equilibrium fees, the agents cannot do better than indeed go to the 'cheapest' matchmaker.

The price game played by the matchmakers is analyzed for the case of unequal densities of agents. The symmetric, equal density, case is non-generic and therefore omitted in this paper[4]. As is shown in Webers (1996, Chapter 6, pp. 87–111), the symmetric and asymmetric cases yield structurally different equilibrium sets, where the symmetric case is rather complex. We show that, generically, there exists a unique Nash equilibrium. In equilibrium, the fees are such that every matchmaker has an equal share of type 1 and 2 agents[5].

Among the equilibria, we find equilibria in which one type is charged a zero fee. The matchmakers even desire to subsidize this type, that is, charge a negative fee. In the local 'monopoly' case, where reservation prices are so low that firms do not compete for 'indifferent agents,' this kind of equilibrium occurs if the difference in reservation prices is sufficiently large. In that case, the type with the lower reservation price is 'free-riding.' This phenomenon arises from the fact that the profit of a matchmaker is determined by the minimum of his shares of type 1 and type 2. To increase his profit, he needs to increase his shares of both types. In order to achieve this, the type with the low reservation price should be charged a relatively low fee, or even a zero fee. In that case, the 'low' type is needed only to attract the 'high' type, from which positive fees are collected.

This free-riding phenomenon is also found outside the local monopoly region. For the case with unequal densities, equilibria may exist where the short side of the market is served for free, under competition. The reason is that the matchmakers compete for an indifferent agent only on the short side. The fee for the agents on the long side is chosen to adjust the share of these agents to the share of short-side agents. A real-life example of a market with an advantageous position of the agents on the short side is the housing market. In the US, for example, often the real-estate brokers only charge a fee from the sellers, since they form the long side of the market. Another example are dating agencies, where, usually, females are attracted by 'subsidizing' them.

The remainder of the paper is organized as follows. In Section 2, the model is formulated, and the different types of competition are introduced. Section 3 analyzes the symmetric equilibria of the price game. Section 4 concludes, among other things by giving some insight into the symmetric case of equal densities.

---

[4] Would matchmakers each organize a competitive market for the commodity, in the equal density case a general equilibrium would exist, by equality of demand and supply. In the unequal density case we can only have partial equilibrium, namely, in commission fees. Van Raalte (1996) considers costly, local competitive markets organized by middlemen.

[5] Shares could be interpreted as demand and supply in certain markets. It is often assumed in the literature, that either supply is not binding or the demand functions are exogenous. In our model, the demand functions are endogenous. The model can be seen as a 'strategic market coverage' type of model. Strategic market coverage through advertising was considered by Boyer and Moreaux (1993).

2. The model

There is a continuum of agents distributed uniformly along a circle with perimeter one. Agents of type $\theta \in \{1,2\}$ are distributed with density α^θ along this circle. We consider the generic asymmetric case $\alpha^1 < \alpha^2$; the non-generic symmetric case $\alpha^1 = \alpha^2$ is not analyzed here, as it complicates the analysis considerably. Readers interested in the equal density case are referred to Webers (1996, Chapter 6, pp. 87–111).

The agents are on a bilateral matching market. This means that an agent of type 1 desires to be matched with an agent of type 2, and vice versa. In order to be matched, the agents need a third, intermediating, party. There are two of such intermediaries, indexed $i \in \{1,2\}$, further referred to as firms. Both firms are located diametrically opposite to each other along the circle. Both types of agents face identical linear relational costs[6] with unit cost $t>0$. Furthermore, agents of type θ have a reservation price $r^\theta>0$ for the relational cost and fees charged by any of the two firms, meaning that they want to pay up to an amount r^θ for the firms' services. The reservation prices are assumed to be given exogenously. Firm i charges a fee $p_i^\theta \in [0,r^\theta]$ from agents of type θ for providing the service. Non-negative fees imply that firms are not to able to provide subsidies[7]. Let $p_i \in [0,r^1] \times [0,r^2]$ denote the tuple of fees $\langle p_i^1, p_i^2 \rangle$ for firm i. It may happen that the fee and the relational costs are so high that the reservation price of some type cannot cover these.

Definition 1. *For firm i,* **the potential market area** *of agents of type θ at fee p_i^θ, denoted by $M_i^\theta(p_i^\theta)$, is the set of agents of type θ for which the sum of the relational cost and the fee p_i^θ charged by firm i does not exceed the reservation price r^θ.*

More formally, $M_1^\theta(p_1^\theta) = \{x \in [0,1] \mid p_1^\theta + tx \le r^\theta \text{ or } p_1^\theta + t(1-x) \le r^\theta\}$ and $M_2^\theta(p_2^\theta) = \{x \in [0,1] \mid p_2^\theta + t(\frac{1}{2}-x) \le r^\theta \text{ or } p_2^\theta + t(x-\frac{1}{2}) \le r^\theta\}$ for type θ. Note that for each firm both potential market areas form an interval. The notion of potential market areas is used to describe the structure of competition among the two firms[8] We distinguish between three different situations.

Definition 2. *At given fees, there is* **strong competition** *if the potential market areas for the firms have a nonempty intersection for both types of agents. There is* **weak competition** *if the potential market areas for the firms have a nonempty intersection for one of the two types of agents and for the other type the intersection is either a point or empty. There is* **no competition** *if the potential market areas of the firms have an intersection which is either a point or empty for both types of agents.*

---

[6] That is, we assume a uniform distribution on relational costs to intermediaries. Of course, any distribution on relational costs could be imagined. As we choose not to complicate the model without gaining significant insight, we impose a uniform distribution.

[7] Webers (1996, Chapter 6, pp. 87–111) analyzes the effect of subsidies, that is, negative fees.

[8] The notion of potential market area is a generalization of the one formulated by Gabszewicz and Thisse (1986) which holds for fees equal to zero.

It is easy to verify that for both firms the potential market areas of agents of type θ have a nonempty intersection in case $\frac{p_1^\theta+p_2^\theta}{2}+\frac{t}{4}\leq r^\theta$ and have an intersection which is either a point or empty in case $\frac{p_1^\theta+p_2^\theta}{2}+\frac{t}{4}\geq r^\theta$.

For firm i, the size of its potential market area of agents of type θ at fee p_i^θ is the length of the interval of agents of type θ for which the sum of the relational cost to firm i and the fee p_i^θ of firm i does not exceed the reservation price r^θ, times the density α^θ of type θ agents.

In case of no competition, the *demand* of firm i is equal to

$$\min\left\{\frac{2\alpha^1}{t}(r^1-p_i^1),\frac{2\alpha^2}{t}(r^2-p_i^2)\right\}.$$

In case of weak competition for, say, type 1 agents, the demand of firm i is equal to

$$\min\left\{\alpha^1\left(\frac{1}{2}+\frac{p_j^1-p_i^1}{t}\right),\frac{2\alpha^2}{t}(r^2-p_i^2)\right\},\ \text{where } j\neq i\in\{1,2\}.$$

The demand of a firm is determined by the set of agents that is actually served by that firm, expressed by the minimum condition. Firm i's profit is equal to the sum of both types' fees times demand, where production costs are assumed to be normalized to zero.

We look for a Nash equilibrium for the game in which the firms simultaneously choose fees, or prices, so as to maximize their profits. Since firms are located symmetrically it makes sense to look for an equilibrium in which both firms choose the same prices.

3. Equilibrium price configurations

As a first, straightforward, result we derive that for each firm, the shares of both types of agents must be equal in equilibrium. For, suppose some firm's share of type θ agents is higher than its share of the other type of agents. Then, increasing the fee for the type θ agents increases profits as the share of the other type of agents does not change.

Proposition 1. *In any Nash equilibrium, firm $i\in\{1,2\}$ has equal shares of type 1 and type 2 agents.*

It is easily derived that the situation of strong competition can never occur in any symmetric equilibrium. Namely, equality of shares can then be satisfied only in case $\alpha_1=\alpha_2$.

Essentially, we can distinguish two types of symmetric equilibria. First, equilibria in which both types are charged a strictly positive fee. Second, equilibria in which one type of agents is charged a zero fee, that is, one type is served for free. More specifically, three different regions can be distinguished in which one type of agents is served for free: two types inhibit a regime of no competition, and another type a regime of weak competition.

Theorem 1. *There are regions of reservation prices for which the corresponding equilibrium fee is zero for at least one type of agents, while there is no competition.*

In the no-competition regime, where neither of the firms competes for an indifferent agent because of relatively low reservation prices, the type with the low reservation price is served for free. Actually, firms desire to subsidize the type with the low reservation price. The type of agents with the high reservation price is able to pay a higher fee than the other type of agents. Competition drives fees down to zero for the type of agents with the low reservation price and only the type with the high reservation price brings in a positive amount of money. The market externality associated with bilateral matching implies that an equal share of the type of agents with the low reservation price has to be attracted. Due to their relatively low willingness to pay, the firms have to charge these agents a low fee in order to attract this type of agents. Nevertheless, the negative effect on profits of the zero fee is more than compensated by the positive effect of the types with the low reservation price.

Theorem 2. *There are regions of reservation prices for which the corresponding equilibrium fee is zero for at least one type of agents, while there is weak competition.*

In the other region in which one type of agents is served for free, we have a regime of weak competition. There, both firms compete for the indifferent consumers of type 1. The type of agent with the low density is served for free, whereas the other type is charged a relatively high fee. Due to the fact that the fees for one of the types of agents are zero, no firm can gain by charging a lower fee for the other type of agents, because the market size cannot be enlarged.

In total, six equilibrium price regions can be distinguished. In three of them, one of the types is served for free, as explained above. In the other three regions, both types pay a positive fee. Typically, the symmetric price equilibrium is unique.

Theorem 3. *The price game has a symmetric Nash equilibrium in fees, which is generically unique.*

Generic uniqueness of equilibrium is caused by the density asymmetry between market sides. Were densities equal, uniqueness is no longer guaranteed. Of course, for the regime of no competition, the analysis is not changed. For the regimes of weak and strong competition, however, several continua of equilibria are found, as derived in Chapter 6 of Webers (1996, pp. 87–111).

In Table 1, the different equilibria are given. The different areas are illustrated graphically in Fig. 1.

It is easily checked that the corresponding fees and profits change continuously in and between the areas, except between the areas II^a and IV^a where $r^2 = \frac{\alpha^1 t}{4\alpha^2}$ and $r^1 > \frac{(\alpha^1+3\alpha^2)t}{4\alpha^2}$. There exists a continuum of equilibria at the intersection of areas II^a and IV^a for $r^1 > \frac{(\alpha^1+3\alpha^2)t}{4\alpha^2}$. Equilibrium fees are $\langle \varphi, 0 \rangle$ and profits are $\frac{\alpha^1}{2}\varphi$ with $\varphi \in [\frac{(\alpha^1+2\alpha^2)t}{4\alpha^2}, r^1 - \frac{t}{4}]$. In equilibrium the buyers are served for free and the per firm market size equals $\frac{\alpha^1}{2}$.

3.1. Areas I, II^a, II^b: No competition

In the areas I, II^a, and II^b, the reservation price of at least one of the types of agents is so low that both firms establish 'local monopolies.' In area I, the differences between the

Table 1
Equilibrium fees and profits

Area	Fees	Profits
I	$\langle\frac{(2\alpha^1+\alpha^2)r^1-\alpha^2r^2}{2(\alpha^1+\alpha^2)}, \frac{(\alpha^1+2\alpha^2)r^2-\alpha^1r^1}{2(\alpha^1+\alpha^2)}\rangle$	$\frac{\alpha^1\alpha^2}{2(\alpha^1+\alpha^2)t}(r^1+r^2)^2$
II^a	$\langle r^1-\frac{\alpha^2}{\alpha^1}r^2, 0\rangle$	$2\frac{\alpha^2}{t}r^2(r^1-\frac{\alpha^2}{\alpha^1}r^2)$
II^b	$\langle 0, r^2-\frac{\alpha^1}{\alpha^2}r^1\rangle$	$2\frac{\alpha^1}{t}r^1(r^2-\frac{\alpha^1}{\alpha^2}r^1)$
III	$\langle r^1-\frac{t}{4}, r^2-\frac{\alpha^1t}{4\alpha^2}\rangle$	$\frac{\alpha^1}{2}\left(r^1+r^2-\frac{(\alpha^1+\alpha^2)t}{4\alpha^2}\right)$
IV^a	$\langle\frac{(\alpha^1+\alpha^2)t}{2\alpha^2}-r^2, r^2-\frac{\alpha^1t}{4\alpha^2}\rangle$	$\frac{\alpha^1(\alpha^1+2\alpha^2)t}{8\alpha^2}$
IV^b	$\langle 0, r^2-\frac{\alpha^1t}{4\alpha^2}\rangle$	$\frac{\alpha^1}{2}\left(r^2-\frac{\alpha^1t}{4\alpha^2}\right)$

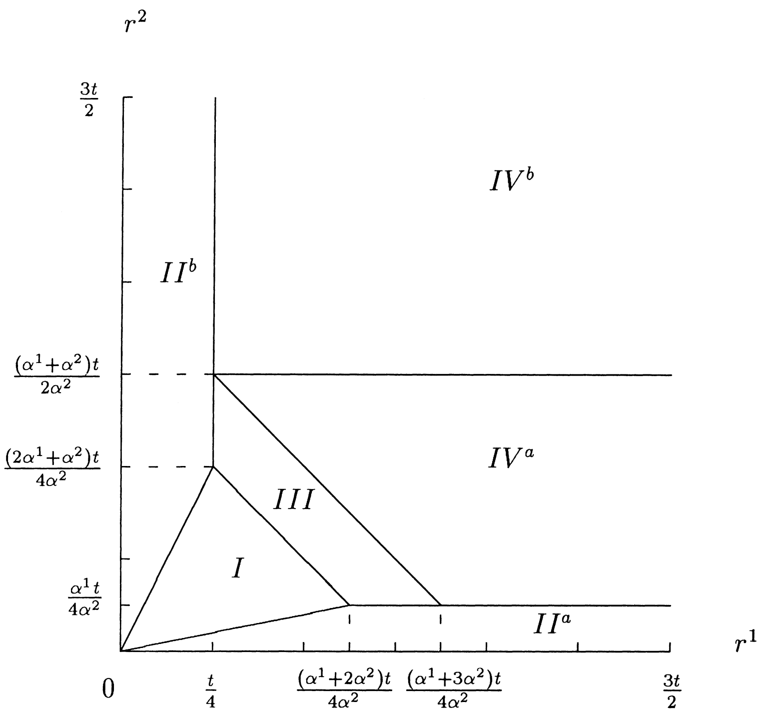

Fig. 1. The different regions of fees.

reservation prices of the sellers and buyers are sufficiently low to obtain an equilibrium with both fees positive. In equilibrium, the fees are such that the agents with the higher reservation price also pay a higher fee. In areas II^a and II^b, the fee for one type is zero.

3.2. Areas III, IV^a,IV^b: Weak competition

In area *III*, the type 1 agents located at a distance $\frac{1}{4}$ from the firms have a zero surplus. A fraction $\alpha^2-\alpha^1$ of the type 2 agents is not served. Firms do not try to capture these agents, since it is optimal to have the shares of type 1 and type 2 agents to be equal.

In areas IV^a and IV^b, the symmetric treatment between the two types of agents disappears. Now, the reservation prices are so high, that the type 1 agents located at a distance $\frac{1}{4}$ from both firms claim a positive surplus. These agents form the short side of the market and are able to take advantage of their position in the market. The negative effect of charging lower fees is more than compensated by the positive effect of attracting more agents of type 1.

The advantageous market position of type 1 agents in case of high reservation prices is exercised maximally in area IV^b. Similar to the area II^b, the firms desire to subsidize the type 1 agents. This results in these agents being served for free. The profits in IV^b increase in the reservation price of the type 2 agents, with no upper bound. Since competition on the long side of the market never occurs in equilibrium, the type 2 agents can be charged a very high price compared to their reservation price.

For the special case $r^1=r^2=r$ equilibrium fees as a function of the reservation price r are drawn in Fig. 2. We see that type 1 agents are charged a lower fee than type 2 agents for all values of the reservation price. Furthermore, agents of type 1 are served for free for relatively high values of the reservation price.

A limit result for the case with zero transportation costs is stated. In this limit case, we have 'pure' Bertrand competition between firms. Then, at least one type is served for free in any Nash equilibrium.

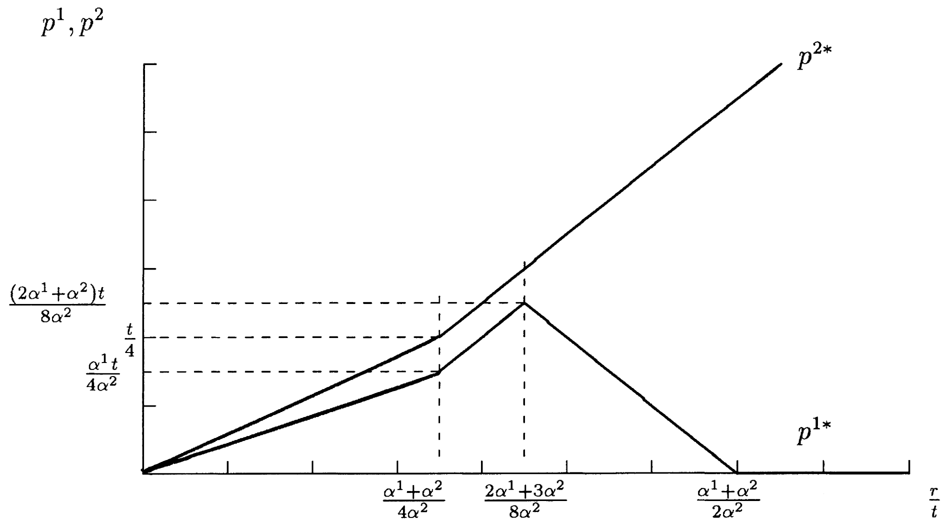

Fig. 2. Equilibrium fees in case $r^1=r^2$.

Proposition 2. *In the limit case t=0, at least one type is served for free in equilibrium. Thereby,* $p_1^* = p_2^* = \langle p, 0\rangle$ with $p \in [0,r^1]$ if $r^1 \geq 0$ and $r^2=0$, and $p_1^* = p_2^* = \langle 0, r^2\rangle$ if $r^1 \geq 0$ and $r^2>0$.

For the special case where both types of agents have equal reservation prices, we find that the short side of the market has an advantage over the long side of the market in the sense that type 1 agents pay a lower fee than type 2 agents. This reflects the market externality inherent in bilateral matching.

Proposition 3. *Suppose* $r^1 = r^2 = r$. *Type 1 agents are served at a lower fee than type 2 agents in equilibrium. If r is relatively large, type 1 agents are served for free.*

4. Conclusion

We have analyzed the competition in commission fees between two matchmakers operating on a bilateral matching market. The symmetric Nash equilibria of the price game between matchmakers were derived. We find that one type of agent may be served for free. The market position of this type is sufficiently strong to induce matchmakers to 'subsidize' it. This 'free-riding' is a result of the externality associated with bilateral matching markets, namely, that the profit of matchmakers is determined by the minimum of the shares of both types.

The model has a Salop (1979)-type circular structure. For two firms, the circle with the firms located diametrically opposite to each other could be replaced by a line segment with the firms located on its far edges. However, if a third firm were added, a line segment poses problems with respect to the existence of price equilibria. A circular model does not pose existence problems, as symmetry is preserved when firms are added. In order to be able to easily extend the model to more than two firms, the circular model is preferred.

The analysis pertained to the asymmetric case, both types having unequal densities. The symmetric case is ignored in this paper, as its analysis is rather complicated while being non-generic. In Webers (1996), the symmetric case is analyzed. The equilibrium sets corresponding to the symmetric and asymmetric case turn out to be structurally different. While in the case of unequal densities there exists a generically unique equilibrium, we find in the case of equal densities also several continua of equilibria. Then, namely, the matchmakers may compete for both types of agents (strong competition). If densities are unequal, only competition for one type occurs, namely the type with the lowest density. These types entirely determine the fee for the type with high density, thereby resolving the indeterminacy. This means a considerable indeterminacy for the agents[9].

[9] We refer to Young (1993) in this respect. Young analyzes the evolution of bargaining over some amount of money between two agents of different types. Young finds several splits between types as equilibrium outcomes. If, however, some agents change type sometimes only, the equal split remains as an equilibrium. In our model, adding or deleting an arbitrarily small group of agents eliminates the continua of equilibria.

Acknowledgements

The authors would like to thank Eric van Damme, Dave Furth, Rob Gilles, Hans Haller, Pieter Ruys, Dolf Talman, and two anonymous referees for valuable comments. Also, we acknowledge comments by the participants of the International Symposium on Economic Modelling, 19–21 July 1995, Bologna, Italy, the 22nd Annual Conference of E.A.R.I.E., 3–6 September 1995, Juan les Pins, France, and the 11th World Congress of the International Economic Association, 18–22 December 1995, Tunis, Tunisia.

Appendix

We provide the derivations of the equilibria mentioned in Theorems 1, 2, 3. The derivations of Propositions 2 and 3 are mentioned briefly.

Proof of Theorem 1.

In the region of prices where there is no competition firm $i \neq j \in \{1,2\}$ chooses prices p_i^1 and p_i^2 that maximize

$$(p_i^1 + p_i^2)\min\left\{\frac{2\alpha^1}{t}(r^1 - p_i^1), \frac{2\alpha^2}{t}(r^2 - p_i^2)\right\} \tag{1}$$

subject to the price constraints

$$\begin{aligned} r^1 &\leq \tfrac{p_1^1+p_2^1}{2} + \tfrac{t}{4}, \quad 0 \leq p_i^1 \leq r^1 \\ r^2 &\leq \tfrac{p_1^2+p_2^2}{2} + \tfrac{t}{4}, \quad 0 \leq p_i^2 \leq r^2. \end{aligned}$$

Because demand and supply must be equal in equilibrium, we can substitute $p_i^2 = r^2 - \frac{\alpha^1}{\alpha^2}R^1 + \frac{\alpha^1}{\alpha^2}p_j^1$ into maximization problem (1) for firm i. Note that one of the constraints becomes redundant. If we denote the vector of Lagrange multipliers by $\lambda_i \in \mathbb{R}_+^5$, the corresponding Lagrangian maximized by firm i reads $\mathscr{L}_i(p_i^1, \lambda_i) = (\frac{\alpha^1+\alpha^2}{\alpha^2}p_i^1 + r^2 - \frac{\alpha^1}{\alpha^2}r^1)(2(r^1 - p_i^1)) - \lambda_{i1}(2r^1 - p_1^1 - p_2^1 - \frac{t}{2}) - \lambda_{i2}(\frac{\alpha^1}{\alpha^2}R^1 + r^2 - \frac{\alpha^1}{\alpha^2}p_i^1 - p_j^2 - \frac{t}{2}) - \lambda_{i3}(-p_i^1) - \lambda_{i4}(p_i^1 - r^1) - \lambda_{i5}(r^1 - \frac{\alpha^2}{\alpha^1}R^2 - p_i^1)$ with $j \neq i$.

From the first-order conditions for profit maximization one derives a candidate equilibrium price $p_i^* = \langle p^{1*}, p^{2*}\rangle$ for firm i. Using symmetry, we get the following candidate equilibria with one type being charged a zero fee:

$$p_1^* = p_2^* = \begin{cases} \langle 0, r^2 - \frac{\alpha^1}{\alpha^2}r^1\rangle & \text{if } r^1 \leq \frac{\alpha^2}{2\alpha^1+\alpha^2}r^2, r^1 \leq \frac{t}{4} \\ \langle r^1 - \frac{\alpha^2}{\alpha^1}r^2, 0\rangle & \text{if } r^2 \leq \frac{\alpha^1}{\alpha^1+2\alpha^2}r^1, r^2 \leq \frac{\alpha^1 t}{4\alpha^2} \end{cases}$$

For the limit case t=0, we find $p_1^* = p_2^* = \langle 0, r^2\rangle$ if r^1=0, $p_1^* = p_2^* = \langle r^1, 0\rangle$ if r^2=0.

We examine the candidate equilibria for deviation. All solutions have the property that deviation by setting a higher price for either type decreases profits. The more interesting situation is deviation by setting a lower price for type 1, which of course cannot occur in case the other firm charges fees $\langle 0, r^2 - \frac{\alpha^1}{\alpha^2}r^1\rangle$. If the other firm charges $\langle r^1 - \frac{\alpha^2}{\alpha^1}R^2, 0\rangle$, deviation by setting a lower price for the sellers decreases profits, because demand cannot increase.

Proof of Theorem 2.

In the region of prices where there is weak competition and the firms compete for the type $k\neq l \in \{1,2\}$ agents, firm $i \neq j \in\{1,2\}$ chooses prices p_i^1 and p_i^2 that maximize

$$(p_i^1 + p_i^2)\min\left\{\frac{\alpha^k}{t}(p_j^k - p_i^k + \frac{t}{2}), \frac{2\alpha^l}{t}(r_l - p_i^l)\right\} \tag{2}$$

subject to price constraints

$$\begin{array}{ll} r_k \geq \frac{p_1^k+p_2^k}{2} + \frac{t}{4}, & 0 \leq p_i^k \leq r_k \\ r_l \leq \frac{p_1^l+p_2^l}{2} + \frac{t}{4}, & 0 \leq p_i^l \leq r_l. \end{array}$$

Because demand and supply have to be equal in equilibrium, we can substitute $r^2 - \frac{\alpha^1}{2\alpha^2}(p_j^1 - p_i^1 + \frac{t}{2})$ for p_i^2 into maximization problem (2) for $j\neq i \in\{1,2\}$. If we denote the vector of Lagrange multipliers by $\lambda_i \in \mathbb{R}_+^6$, the corresponding Lagrangian for firm i, reads $\mathscr{L}_i(p_i^1, \lambda_i) = (\frac{\alpha^1+2\alpha^2}{2\alpha^2}p_i^1 - \frac{\alpha^1}{2\alpha^2}p_j^1 + r^2 - \frac{\alpha^1 t}{4\alpha^2})\ (p_j^1 - p_i^1 + \frac{t}{2}) - \lambda_{i1}(p_1^1 + p_2^1 + \frac{t}{2} - 2r^1)$ $-\lambda_{i2}(2r^2 - t + \frac{\alpha^1 t}{2\alpha^2} - 2p_j^2 + \frac{\alpha^1}{\alpha^2}(p_j^1 - p_i^1)) - \lambda_{i3}(-p_i^1) - \lambda_{i4}(p_i^1 - r^1) - \lambda_{i5}(p_j^1 + \frac{t}{2} - \frac{2\alpha^2}{\alpha^1}r^2$ $-p_i^1) - \lambda_{i6}(p_i^1 - p_j^1 - \frac{t}{2})$.

Due to symmetry the first-order conditions are solved by $p_i^* = \langle p^{1*}, p^{2*}\rangle$. Solving these equations we get a candidate equilibrium with type 1 charged a zero fee, namely

$$p_1^* = p_2^* = \langle 0, R^2 - \frac{\alpha^1 t}{4\alpha^2}\rangle$$

For the limit case t=0, we find $p_1^* = p_2^* = \langle 0, r^2\rangle$.

We have to check whether or not this solution can be improved upon. As argued in the proof of Theorem 1, such improvement is not possible.

Proof of Theorem 3.

In the proofs of Theorems 1 and 2, we derived the equilibria in which one type is served for free. Besides these equilibria, the first-order conditions for maximization problems (1) and (2) also yield the candidate equilibria for which both types pay a strictly positive fee. First, for the case of no competition, these are the following.

$$\begin{cases} \langle \frac{(2\alpha^1+\alpha^2)r^1-\alpha^2 r^2}{2(\alpha^1+\alpha^2)}, \frac{(\alpha^1+2\alpha^2)r^2-\alpha^1 r^1}{2(\alpha^1+\alpha^2)}\rangle & \text{if } r^1 + r^2 \leq \frac{(\alpha^1+\alpha^2)t}{2\alpha^2}, \\ & \frac{\alpha^2}{2\alpha^1+\alpha^2}r^2 \leq r^1 \leq \frac{\alpha^1+2\alpha^2}{\alpha^1}r^2 \\ \langle r^1 - \frac{t}{4}, r^2 - \frac{\alpha^1 t}{4\alpha^2}\rangle & \text{if } r^1 + r^2 \geq \frac{(\alpha^1+\alpha^2)t}{2\alpha^2}, r^1 \geq \frac{t}{4}, \\ & r^2 \geq \frac{\alpha^1 t}{4\alpha^2}. \end{cases}$$

For the limit case t=0, we find $p_1^* = p_2^* = \langle r^1, r^2\rangle$.

We check whether or not these candidate equilibria can be improved upon by deviation. If the other firm charges $\langle r^1 - \frac{t}{4}, r^2 - \frac{\alpha^1 t}{4\alpha^2}\rangle$, deviating by setting a lower price for the sellers decreases profits as long as $r^1 + r^2 \leq \frac{(2\alpha^1+3\alpha^2)t}{4\alpha^2}$. Finally, if the other firm charges $\langle\frac{(2\alpha^1+\alpha^2)r^1-\alpha^2 r^2}{2(\alpha^1+\alpha^2)}, \frac{(\alpha^1+2\alpha^2)r^2-\alpha^1 r^1}{2(\alpha^1+\alpha^2)}\rangle$, deviating by setting a lower price for the sellers decreases profits. For the solution $p_1^* = p_2^* = \langle r^1 - \frac{t}{4}, r^2 - \frac{\alpha^1 t}{4\alpha^2}\rangle$ we thus have to impose the additional requirement that $r^1 + r^2 \leq \frac{(2\alpha^1+3\alpha^2)t}{4\alpha^2}$.

Second, for the case of weak competition, the candidate equilibria are

$$\begin{cases} \langle\frac{(\alpha^1+\alpha^2)t}{2\alpha^2} - r^2, r^2 - \frac{\alpha^1 t}{4\alpha^2}\rangle & \text{if } r^1 + r^2 \geq \frac{(2\alpha^1+3\alpha^2)t}{4\alpha^2}, \frac{\alpha^1 t}{4\alpha^2} \leq r^2 \leq \frac{(\alpha^1+\alpha^2)t}{2\alpha^2} \\ \langle r^1 - \frac{t}{4}, r^2 - \frac{\alpha^1 t}{4\alpha^2}\rangle & \text{if } r^1 + r^2 \leq \frac{(2\alpha^1+3\alpha^2)t}{4\alpha^2}, r^1 \geq \frac{t}{4}, r^2 \geq \frac{\alpha^1 t}{4\alpha^2}. \end{cases}$$

As in the no-competition case, we have to impose the additional requirement that $r^1 + r^2 \geq \frac{(\alpha^1+\alpha^2)t}{2\alpha^2}$ for the solution $\langle r^1 - \frac{t}{4}, r^2 - \frac{\alpha^1 t}{4\alpha^2}\rangle$.

References

Bhattacharya, U., Hagerty, K., 1987. Dealerships, trading externalities, and general equilibrium. In: Prescott and Wallace (Eds.), Contractual Arrangements for Intertemporal Trade. University of Minnesota Press, Minneapolis, MN.

Bhattacharya, U., and Yavas, A., 1993. In search of the right middleman, Economics Letters 42, 341–347.

Biglaiser, G., 1992. Middlemen as experts, Rand Journal of Economics 24, 212–224.

Biglaiser, G., and Friedman, J.W., 1994. Middlemen as guarantors of quality, International Journal of Industrial Organization 12, 509–531.

Boyer, M., and Moreaux, M., 1993. Strategic market coverage in spatial competition, International Journal of Industrial Organization 11, 299–326.

Carroll, W., 1989. Fixed-percentage commissions and moral hazard in residential real estate brokerage, Journal of Real Estate Finance and Economics 2, 349–365.

Espinosa, M.P., 1992. On the efficiency of location decisions under discriminatory pricing, International Journal of Industrial Organization 10, 273–296.

Gehrig, T., 1993. Intermediation in search markets, Journal of Economics and Management Strategy 2, 97–120.

Gabszewicz, J.J., and Thisse, J.-F., 1986. Spatial competition and the location of firms, Fundamentals of Pure and Applied Economics 5, 1–71.

Hänchen, T., Von Ungern-Sternberg, T., 1985. Information costs, intermediation and equilibrium price, Economica 52, 407–419.

Kats, A., 1995. More on hotelling's stability in competition, International Journal of Industrial Organization 13, 89–93.

Raalte, C. van, 1996. Market Formation and Market Selection, Center for Economic Research Dissertation. Thesis Publishers, Amsterdam.

Rubinstein, A., Wolinsky, A., 1987. Middlemen, Quarterly Journal of Economics, 581–593.

Salop, S.C., 1979. Monopolistic competition with outside goods, Bell Journal of Economics 10, 141–156.

Webers, H.M., 1996. Competition in Spatial Location Models, Center for Economic Research Dissertation. Thesis Publishers, Amsterdam.

Yavas, A., 1993. Market makers versus match makers, Journal of Financial Intermediation 2, 33–58.

Yavas, A., 1994. Middlemen in bilateral search markets, Journal of Labor Economics 12, 406–429.

Yavas, A., 1996. Search and Trading in Intermediated Markets, Journal of Economics and Management Strategy 2, 97–120.

Young, H.P., 1993. An evolutionary model of bargaining, Journal of Economic Theory 59, 145–168.