The Informational Role of Warranties and Private Disclosure about Product Quality

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1. INTRODUCTION

A fundamental role of competition is to facilitate the allocation of resources. This is achieved by prices which, to some extent, reflect and transmit the underlying worth of resources. The ability of prices to reflect and transmit information derives from the attempt of economic agents to buy or sell based on their information. Competition among all those who want to buy wheat because they think that wheat will be scarce tomorrow drives up the price of wheat. Hence their information can be transmitted by the price system to those who store wheat but do not have direct access to information about next period's wheat demand. This mechanism works because there is some future state of nature which will lead to prices which reward those who buy or sell today.[1]

Unfortunately, there are situations where no such prices exist. An important case involves information about product quality. Sellers may know the quality of the item they sell but it may be in their interest to withhold that information. If there is no way for buyers to learn about the sellers' quality, then this will force all items to sell at the same price. If there is no way sellers of good-quality items can distinguish themselves from sellers of low-quality items, then the low-quality sellers will find it in their interest to hide their quality. This has been called the "lemons problem."[2]

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[1] Sanford J. Grossman, Existence of Futures Markets, Noisy Rational Expectations, and Informational Externalities, 44 Rev. Econ. Stud. 431 (1977).

[2] George A. Akerlof, The Market for "Lemons"; Qualitative Uncertainty and the Market Mechanism, 84 Q. J. Econ. 488 (1970), and Charles Wilson, The Nature of Equilibrium in Markets with Adverse Selection, Bell J. Econ. 11(2) p. 108, (1980).

In this paper, we will consider cases where good sellers have an incentive to distinguish themselves from bad sellers. Of particular interest is the case where there is a single seller. Consumer information is often quite poor about those products which are new. It is just these products where (temporary) monopoly is likely to be found. Thus, we will be concerned with cases where a monopolist could have an incentive to reveal his quality even when it is low. In order for this incentive to exist, there must be some event which occurs after the sale which will reward sellers as a function of their true quality. We consider two cases.

In our first case, the seller can make statements about the product's quality which are *ex post* verifiable. For example, a diamond seller can disclose the weight of a diamond he is selling. He can give the buyer a warranty which states the weight of the diamond. That is, in Section 2 we will consider situations where sellers can make any disclosure about their product's quality and give a complete warranty which guarantees that the disclosure is true (for example, the diamond seller gives the buyer a written statement guaranteeing that the diamond can be returned if an objective party finds that its weight is less than specified).

The second case which we consider is where statements about product quality are too costly either to communicate or verify *ex post*. For this reason, the statements cannot be guaranteed. However, we assume that there is some characteristic which is observable *ex post*. For example, the quality of an automobile's construction is difficult to describe or verify *ex post*. However, it may be easy to verify *ex post* whether the auto "breaks down." If it is the case that low-quality items have a higher probability of breakdown than do high-quality, then warranties which guarantee against (the *ex post* cheaply observable) breakdown can substitute for guarantees regarding (the *ex post* very costly to observe) quality.

This paper is primarily concerned with situations where consumers have had no experience and will have no further experience with the monopolist. This is the case where the monopolist will have the greatest incentive to mislead. Our basic result is, however, that the monopolist will *not* be able to mislead rational consumers about the quality of his product. In Section 2, we assume that the monopolist has the ability to make *ex post* verifiable statements about his quality. We show that consumers with rational expectations will assume that the monopolist is of the worst possible quality consistent with his disclosure when he makes less than a full disclosure. The monopolist, realizing this, decides to make a full disclosure. This result generalizes the result in Grossman and Hart.[3]

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[3] Sanford J. Grossman & Oliver Hart, Disclosure Laws and Takeover Bids, 35 J. Finance 323 (1980).

In Section 3, where verifiable disclosure is assumed to be impossible, the monopolist can offer warranties. It is assumed that the warranty is conditional on an event, the probability of which depends *only* on the seller's quality. Further, the consumer is assumed to be risk averse. In this case, we show that if a seller offers less than a full warranty, consumers with rational expectations will conclude that he is trying to mislead them about the product's quality. Each consumer knows that it is Pareto optimal for the seller to sell the item at the consumer's reservation price with a complete warranty. Hence a seller would only offer less than a complete warranty if it would make the seller better off than the complete warranty contract. But then it would have to make the consumer worse off than the complete warranty contract. But this would give the consumer less than his reservation price, so he does not make the purchase.

The above arguments are based upon adverse selection against sellers which make consumers bear the risk associated with not knowing the quality of their product. This is to be contrasted with signaling arguments. Spence has presented a model with many different sellers who have marginal costs of production that depend on their quality.[4] In his model (where consumers differ in risk aversion), there is an equilibrium distribution of qualities and warranties outstanding at a given time. Further, high-quality firms offer a larger warranty than do low-quality firms, thus signaling their quality. This result is very different from mine. It requires that consumers have information about the statistical relationship between warranties and quality. He further requires that there are many competing sellers extant at a given moment. I have presented a much weaker equilibrium concept which is appropriate for markets with a *single* seller and many buyers. Further, because there is only a single seller, I am able to be more precise in defining consumers' conjectures about the seller's quality than is normally done in a signaling model. In a signaling model, consumers can only get information from the equilibrium contract schedule, while in my model it is the consumers' conjectures about the monopolist's quality *out* of equilibrium which forces the monopolist to choose a particular equilibrium. In my model, the monopolist is unable to mislead consumers because of the rational conjectures consumers have regarding what his quality must be if he deviates from a full-information Pareto optimal contract. In the equilibrium I present, consumers are *unable* to determine the seller's quality by inverting the equilibrium contract. This is what distinguishes the result and model presented here from Spence and other signaling models.

[4] Michael Spence, Consumer Misperceptions, Product Failure, and Producer Liability, 44 Rev. Econ. Stud. 561 (1977).

2. A MODEL OF PRIVATE DISCLOSURE

In this section we assume that it is possible for a seller to make *ex post* verifiable disclosures. For example, a diamond seller can specify the weight of the diamond; a doctor can specify the medical school he graduated from, his class standing, the number of malpractice suits he is engaged in, and so forth. In this section, we will be concerned with disclosures that have negligible *ex post* verification cost and also negligible communications cost. For example, it would be very costly for a doctor to explain to a patient, in detail, his contribution to the study of ulcers. This might involve imparting four years' worth of medical school training to the patient. Yet, if the patient must choose between two doctors, this information gross of acquisition costs is very valuable (while net of acquisition costs it is of no value). Note further than the doctor may desire to substitute low communications cost information such as, ''I am the best ulcer doctor in the world,'' but such a statement is not easily subject to verification (primarily because it is not sufficiently detailed—what does ''best'' mean?).

In this section we will not model a consumer's decision about verifying *ex post* the truth of the statement made by the seller. Rather, for simplicity, we will limit attention to situations where all of the seller's statements are costlessly verifiable *ex post*. A clear example is where the seller is selling boxes of oranges. If the seller states that there are ten oranges in a box, then this becomes verifiable *ex post* for free. It is important that the information be publically verifiable for the purpose of this section. In particular, if the seller states that a product will ''make the buyer happy,'' then this fact is not open to easy third-party verification. When the seller states that the diamond weighs one ounce, this is very cheap to verify *ex post*.

There has been much recent interest in laws which require sellers to make particular disclosures. This is to be distinguished from antifraud laws which make it illegal for a seller to lie.[5] It is sometimes argued that there are disclosures that are of negligible cost but are not made by sellers in an attempt to mislead buyers. Hence a law is needed which requires such disclosures. This section focuses on cases of costless disclosure to derive insight about the issue of what a firm would voluntarily disclose.

We restrict attention to disclosures which are truthful. (For example, a

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[5] The best examples of positive disclosure laws occur in the buying and selling of securities. See Grossman & Hart, *supra* note 3, for an analysis of tender offers, when the offerer is required to state his purpose for buying shares in a company. See also Stephen Ross, Disclosure Regulation in Financial Markets, in Issues in Financial Regulations (Franklin Edwards ed. 1978), for an analysis of the incentive role of positive disclosure.

seller who says nothing is making a truthful disclosure. A seller with a diamond which weighs one ounce who states that it weighs at least one-half ounce is making a truthful disclosure, while if the same seller said that his diamond weighs two ounces it would not be truthful.) We consider only truthful disclosures for two reasons. First, we are interested in analyzing the benefits of a positive disclosure law which are above and beyond those provided by a law against lying. Second, if there are zero *ex post* verification costs, sellers would warranty their disclosures. Any seller who did not warranty his disclosure would immediately be assumed to be lying; that is, saying nothing. As can easily be seen from the analysis to be given below, unwarranteed disclosures could easily be incorporated.

In situations where there is objective, costless *ex post* verification and no communications costs, we can most clearly elucidate the role of *positive disclosure*. In situations where sellers do not lie, the only issue is how much of the truth they will decide to tell. In particular, if a seller has a bad product, will he say nothing, leading consumers to believe his product is of average quality? Will adverse selection by the low-quality sellers drive out the high-quality sellers? If the market is competitive, then this will clearly not be the case. That is, if there is free entry into the sellers' activity, then good sellers will make disclosures to distinguish themselves from bad sellers. If any good seller should be lumped with the bad sellers due to nondisclosure, then the good seller could costlessly disclose his quality and be distinguished, getting a higher price.

The case of free entry is reasonably obvious. However, in many important cases involving consumer uncertainty, free entry may be an inappropriate assumption. Consumer information is relatively poor about new products. There may only be one firm selling a new product because of patent protection or because that firm is a particularly rapid innovator. Schumpeter has argued that an extremely important role of the competitive system is in encouraging innovation via the temporary monopoly power won by the fastest innovator.[6] Thus it is important to ask whether a monopolist would find it in his interest to make a full disclosure. It is remarkable that if the monopolist has customers with rational expectations, then it will be in his interest to make a complete disclosure. It will be shown that adverse selection works *against* a monopolist who makes less than a full disclosure. The idea of the analysis can be seen from a simple example. Suppose the monopolist is selling boxes of apples. He can label the boxes with an exact number of apples, but if he does then his must be the true amount under the above "no lying" assumptions. However, he could also put no label as to the quantity or he could state,

[6] Joseph Schumpeter, Capitalism, Socialism, and Democracy (3d ed. 1950).

"There are at least three apples in the box." Suppose that from the size of the box consumers can tell that the box holds between zero and 100 apples, and they also know that the seller knows how many apples are in the box. Suppose the seller says nothing about the number in the box. Then a consumer could rationally conclude that the box contains no apples, for if there were, say, three apples in the box, the seller could have said, "There are at least three apples in the box." Similarly, suppose a seller makes the statement, "There are at least six apples in the box." This must mean that there are *exactly* six apples in the box, for if there were really seven, then the seller could have made more profit by saying that there are at least seven apples in the box. This is because the expected number of apples in the box under the latter statement is higher than the former, so consumers will be willing to pay more. Thus there is a kind of adverse selection against a seller who does not make a full disclosure, even though he is the only seller. Consumers rationally expect a seller's quality to be the poorest possible consistent with his disclosure. The seller, knowing that consumers will only offer to pay the lowest amount consistent with his disclosure, finds it optimal to disclose the highest possible quality consistent with the truth; that is, he discloses the truth when he knows it. The remainder of this section is devoted to providing a formal model of the above.

Let q be a vector of characteristics which gives a consumer utility. A product of unknown quality can be modeled as a product where the consumer does not know which particular vector of characteristics the commodity contains (for example, see Leland for an analysis which uses characteristics to model product quality.)[7] Let Q represent the set of all possible vectors of characteristics. Assume that if a consumer has income I, then his willingness to pay for a particular known vector q is given by the p such that

$$U(q, I - p) = \bar{u}, \tag{1}$$

where $\bar{u}$ is the utility he can get by not consuming the commodity. We will sometimes call p "the consumer's reservation price." We will take $\bar{u}$ as an exogenous description of the best alternative available opportunity for the consumer. We assume that the consumer is only interested in zero or one unit and that all consumers are identical. Assume that U is increasing in I.

This paper is concerned with situations where sellers know more than buyers about the quality of a product. We represent the buyers' knowledge about the product by specifying a probability distribution on Q. For example, if there are only two possible qualities, q_1 and q_2, and γ is the

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[7] Hayne E. Leland, Quality Choice and Competition, 67 Am. Econ. Rev. 127 (1977).

probability that the consumer places on q_1, then the consumer's expected utility is $V(\gamma, p) \equiv \gamma U(q_1, I - p) + (1 - \gamma)U(q_2, I - p)$. In general, there may be many possible vectors of characteristics $\{q_1, q_2, q_3, \ldots\} \equiv Q$, where the consumer assesses γ_i to be the probability that q_i is the true quality. If we let $\gamma = (\gamma_1, \gamma_2, \ldots)$ be his probability assessment, then his expected utility is

$$V(\gamma, p) = \sum_i \gamma_i U(q_i, I - p). \tag{2}$$

Hence, if the buyer has beliefs γ, his willingness to pay will be $p(\gamma)$, which is the p such that

$$V(\gamma, p) = \bar{u}. \tag{3}$$

The seller wants to maximize the price he gets for the item. We assume that the seller has no long-term relationship with the buyer or with other buyers that a given buyer may communicate with. (I make this assumption to create the strongest case for nondisclosure.) The seller can choose to make a disclosure about q. We model this disclosure by assuming that the seller picks a set of q's denoted by D and states: ''My q is an element of D (denoted by $q \in D$).'' For example, if the seller states $D = \{q_1, q_2\}$, then this means that his quality is either q_1 or q_2. Note that the larger is the set D which a seller reports, the less a consumer can infer about his true quality. A seller could state $q \in Q$ and this is equivalent to making no disclosure, for then he is merely stating that his quality is any possible quality. Let $D(q)$ be the disclosure made by the seller if his true quality is q. Then it must be the case that

$$q \in D(q). \tag{4}$$

This is because we have assumed lying is impossible.

Equilibrium is characterized by the function $D(q)$, which describes the optimal disclosure set for a seller when his quality is q. There are two ways by which consumers can infer q from $D(q)$. First, the seller may reveal everything so $D(q) \equiv \{q\}$.[8] Second, the consumer may be able to reason that only a particular quality seller, say q_i, would find it profit maximizing to make a disclosure $D(q_i)$. That is, the function $D(q)$ may be invertible. For example, suppose that the consumer convinces himself that only a seller of type q_i would make a disclosure like $D_i = \{q_i, q_{i+1}, q_{i+2}, \ldots\}$ (that is, the seller states, ''My quality is at least q_i''); then in this case the quality of the seller can be inferred from D, even though D is not a one-element set.

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[8] $\{q\}$ is the set with a single element, namely q.

In order to define a seller's profit-maximizing disclosure, it is necessary to decide how much a consumer is willing to pay for the item, given that a particular disclosure D is made. Let p_i denote the consumer's willingness to pay for an item of sure quality q_i; that is, p_i solves

$$U(q_i, I - p_i) = \bar{u}. \tag{5}$$

It is convenient to label the qualities in order of consumers' willingness to pay, so that

$$p_1 < p_2 < p_3 < \ldots . \tag{6}$$

(There is no loss generality in assuming that all the prices are different. For if $p_1 = p_2$, then at this price the consumer is indifferent between q_1 and q_2, so it does not matter to him which is the truth.)

If the seller discloses a set, say $D = \{q_3, q_6, q_7\}$, then the consumer knows that the true quality is q_3, q_6 or q_7, but he may know more. Note that p_7 is larger than p_3 or p_6. The consumer reasons that if the seller's quality was really q_7, then the seller would have disclosed $D = \{q_7\}$ rather than $D = \{q_3, q_6, q_7\}$, since in that case the seller would have received more money by shifting the consumer's beliefs toward the better-quality item. Continuing this argument, the consumer concludes that the quality must be q_3.

To make the above idea precise, let $\gamma_i(D)$ denote the consumer's beliefs about the probability that the item has quality q_i after he observes a disclosure D. Let $\gamma(D) \equiv \{\gamma_1(D), \gamma_2(D), \ldots\}$. Note that, given the consumer's beliefs $\gamma(D)$, we can use (3) to find his willingness to pay $p[\gamma(D)]$. We now give necessary conditions for a particular $\gamma(D)$ to be a rational expectations inference function. Appendix A gives a rigorous definition of $\gamma(D)$ and proves that in equilibrium the seller's disclosure reveals his quality. In the remainder of this section, we give the basic idea.

The seller's disclosure must maximize his profit. Therefore it must be the case that the price he receives when he makes the disclosure, D, $p[\gamma(D)]$, must be as large as the price he would get if he made any other disclosure. One disclosure which is always open to a seller is to disclose his exact quality and receive p_i when he is of quality q_i. Hence, for $D(q)$ to be an optimal disclosure for a seller of quality q, when buyers have an inference function $\gamma(D)$, it must be the case that

$$p\{\gamma[D(q_i)]\} \geq p_i \qquad \text{for all qualities } i. \tag{7}$$

The right-hand side of (7) is the price the seller would receive if he reveals his true quality.

We now show that this implies that $p\{\gamma[D(q_i)]\} = p_i$, and that $D(q)$ reveals q. It must be the case that

$$V\big(\gamma[D(q_i)], p\{\gamma[D(q_i)]\}\big) = \bar{u} \qquad (8)$$

for the consumer to buy the item and for the seller to have maximized profits. Note that if $D(q)$ reveals q, then (7) must hold as an equality. So suppose $D(q)$ does not reveal q. Then it must be the case that the seller would make the same disclosure for two different q's, say q_i and q_j, with $p_i > p_j$. That is, (8) holds for the common disclosure $D = D(q_i) = D(q_j)$. (For if there was a distinct disclosure set for each distinct q, then the particular disclosure set would reveal q; see the example in the paragraph after eq. (4).) Further, (8) states that the consumer's *average* utility is $\bar{u}$, but this means that either (a) the consumer gets $\bar{u}$ whether the quality is q_i or q_j; that is, $U\big(q_i, I - p\{\gamma[D(q_i)]\}\big) = U\big(q_j, I - p\{\gamma[D(q_i)]\}\big) = \bar{u}$, or (b):

$$U\big(q_i, I - p\{\gamma[D(q_i)]\}\big) > \bar{u} > U\big(q_j, I - p\{\gamma[D(q_i)]\}\big). \qquad (9)$$

But (a) is impossible by the convention that different qualities are associated with different willingnesses to pay, see (6). Hence (b) must hold. But (9) states that the consumer is doing strictly better than $\bar{u}$ in the event of high quality q_i. This means that the seller is getting a lower price than he would if he revealed exactly that $q = q_i$ when his quality is q_i; that is, $p\{\gamma[D(q_i)]\} < p_i$. This contradicts (7).

We have shown that an equilibrium inference function $\gamma[D(q)]$ must reveal q to the consumer. The essence of the argument is the assumption that the seller could have always disclosed the exact quality. The only reason for the seller not to do so is that he could make higher profit by reporting a larger set. However, the only way he can get a higher profit is by making the same disclosure both when he has high quality and when he has low quality. But this is not optimal. When the seller has high quality, he should certainly reveal exactly what his quality is. Hence consumers know immediately that he has low quality when the seller does not make a full disclosure.

It is essential to note that we *do not* assume that consumers have had a lot of experience with the seller, and are thus able to learn the relationship between the actual disclosure $D(q)$ made by a seller of type q and his true quality q. Instead, we argue that, for example, if a seller says nothing about his quality the consumers infer that he is of the lowest possible quality, because *only* a seller of the lowest quality would find it profit maximizing to say nothing. A seller whose actual quality is q_2 (just above the lowest quality) would have been better off saying, "My quality is at

least q_2." With lying impossible, every rational consumer will be willing to pay more to a seller who says his q is at least q_2 than they will to a seller who says nothing. Hence only the worst possible seller would say nothing. A similar argument shows that only a seller of quality q_i would find it profit maximizing to state: "My quality is at least q_i."[9]

3. Warranties and Indirect Disclosure

The previous section considered commodities where disclosure about quality involves (a) negligible communication cost and (b) negligible cost of making ex post verifiable statements. As noted earlier, there exist situations where the seller has some information about a product, the disclosure of which would be costly. (For example, a used-car salesman can make statements about the kind of driver who previously owned the car, but the making of verifiable statements might be costly, as would an objective inspection of the car.) In this section, we consider the extreme case where the cost to the seller of making relevant statements or of the buyer determining the quality before purchase is larger than the difference in value between the best and worst possible commodity. We maintain the assumption that the seller knows the quality of the commodity, and this quality is exogenous. As before, we consider a situation where buyers have no experience with sellers and will have no future relationship with them.

Under the above assumptions, all sellers will be judged to be identical. Each seller, whether his commodity is best or worst, will receive the same price. There are two situations to consider. If there are no warranties or any other device other than price to signal quality, then we will have the usual "lemons" problem. There will be adverse selection against high-quality sellers. Each high-quality seller will want to be distinguished from those of average quality, but in this case there is no way for him to do so.

In some situations, sellers can attempt to distinguish their quality even if disclosures are very costly. This can be done with warranties. It seems intuitively plausible that a seller of a high-quality item can offer a better warranty than can be offered by a low-quality seller. Of course, for this to make sense, it is necessary that something be ex post objectively observable by buyers and sellers. For example, a doctor can warranty a patient against the recurrence of an illness, a manufacturer can offer a warranty against breakdown, a lawyer can take a fee contingent on success, etc. In

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[9] See Paul Milgrom, Good News and Bad News: Representation Theorems and Applications, Discussion Paper 407R Northwestern School of Management, October (1980), for a game-theoretic version of the same argument, and possible generalizations.

each of these cases, it is observable *ex post* when the patient gets sick, the car breaks, or the legal battle is lost. In many cases such events will be much easier to verify *ex post* than will be the *ex ante* statements a seller makes about his quality, and will have lower communication cost. For example, the lawyer could try to tell the customer about all his previous cases, why losing particular cases was not his fault, compare his record with the records of other lawyers, and so on. Such statements are far more complicated and costly to the buyer and seller than a statement like "Pay me $1,000 if I lose and $10,000 if I win." Sometimes the information is impossible to convey in an *ex post* verifiable way. A doctor may know that he is the best doctor in existence, but there is no way (at a reasonable cost) that he can prove this to a prospective patient. In situations in which a seller's information cannot be conveyed to a buyer, the seller's warranty can, in effect, transmit that information to the buyer. There is a sense in which the degree of warranty can be a sufficient statistic for the seller's information.

To develop the warranty idea, we consider a special case of product quality. In particular, for simplicity assume there are two states of nature: a "good" state and a "bad" state. Let b_1 be the benefit the consumer gets in the good state and b_2 his benefit in the bad state, where $b_1 > b_2$. Suppose that the only possible difference between products is the probability that the good state will arise. (For example, the only difference among cars is the probability of breakdown or among lawyers the probability that the case is won.) A warranty gives the consumer some payment w in state 2 only, while in the good state the consumer gets no payment from the seller. For notational simplicity, we consider a two-period model with a zero interest rate between the periods. In the initial period, a contract is signed and the consumer pays p dollars, while in the second period if state 2 arises the firm pays the consumer w, and in state 1 it pays nothing to the consumer. *We will assume that there are no moral hazards on the consumer's part, so that the consumer cannot affect the probability of the states, and that the seller knows the buyer's benefits b_1 and b_2.* In this case, it does not matter whether a warranty involves a dollar payment or some repairs, or a combination of both.

We consider first what would happen if the buyer knows the quality of the seller's item. Let π be the probability of the good state and let the seller charge a price p with warranty w. If the seller sells many units of the product, his revenue per unit, by the law of large numbers, will be nonstochastic. His revenue per unit is

$$R(\pi, p, w) \equiv \pi p + (1 - \pi)(p - w), \tag{10}$$

since he gets p no matter what the state is, but must pay out w in the state

which is bad for the consumer. If the consumer thinks the probability of the good state is π^e, then his expected utility is

$$V(\pi^e, p, w) = \pi^e u(b_1 - p) + (1 - \pi^e)u(b_2 - p + w); \qquad (11)$$

b_i is the dollar value of the commodity in state i and $u(\cdot)$ is the consumer's von Neumann-Morgenstern utility function.

A Pareto optimal contract would involve choosing p, w to maximize $R(\pi, p, w)$ subject to the constraint that $V(\pi^e, p, w) \geqslant \lambda$, where λ is a number which determines the division of consumer surplus between the seller and buyer. Let $\bar{u}$ be the best utility level the consumer could attain elsewhere. Then monopoly is the Pareto optimal contract with $\lambda = \bar{u}$. If many firms with the same π competed and $\pi^e = \pi$, then the competitive equilibrium would involve a choice of p, w to maximize $V(\pi, p, w)$ subject to $R(\pi, p, w) \geqslant 0$ (where we take production as having already occurred and no more production is possible; more will be said about this assumption below). Clearly, for any λ, if the consumer is strictly risk averse ($u'' < 0$), then a Pareto optimal contract p^0, w^0 involves equalizing the consumer's net income in both states; that is, $b_1 - p^0 = b_2 - p^0 + w^0$, equivalently

$$b_1 - b_2 = w^0. \qquad (12)$$

In the case of a monopolist, p^0 would be chosen so that

$$V(\pi^e, p^0, w^0) = u(b_1 - p^0) = \bar{u}, \qquad (13)$$

while perfect competition among sellers with an identical π will drive p^0 down to where $R(\pi, p^0, w^0) = 0$; that is, where

$$p^0 = (1 - \pi)w^0 = (1 - \pi)(b_1 - b_2). \qquad (14)$$

We are interested in situations where consumers do not know π. To facilitate the analysis, consider Figure 1, which is drawn for two types of firms; the firm with the highest π denoted by π_h, and the firm with the lowest π denoted by π_l. For each firm, the zero revenue line is drawn, and labeled $R(\pi, p, w) = 0$. The consumer's indifference curve, which is tangent to each revenue line, is also drawn, and labeled $V(\pi, p, w)$. The slope of the consumer's indifference curve is

$$\left.\frac{dw}{dp}\right|_{V=\text{constant}} = \frac{1}{1 - \pi}\left\{1 + \pi\left[\frac{u'(b_1 - p)}{u'(b_2 - p + w)} - 1\right]\right\}, \qquad (15)$$

while the slope of the firm's iso-revenue line is

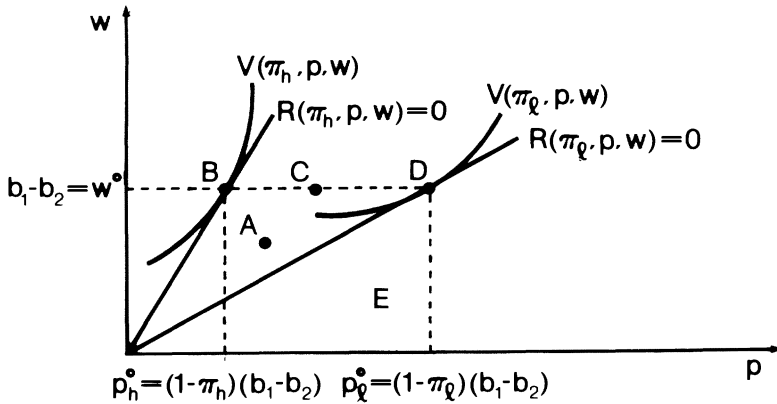$$\left.\frac{dw}{dp}\right|_{R=\text{constant}} = \frac{1}{1 - \pi}. \qquad (16)$$

FIGURE 1

When the firm and the consumer have the same beliefs about π, a tangency occurs at the full insurance point $w = b_1 - b_2$. Note that firm revenue is higher below and to the right of the iso-revenue line, while the consumer's utility is higher to the left and above his indifference curve.

It is easy to dispose of the competitive cases when the consumer does not know a particular firm's π but where the consumer knows the distribution of π's across firms. (By competitive, we mean that there are more potential products than buyers, and sellers do not collude.) First, suppose all firms have the same π; for example, π_h. Suppose one firm offers a contract like the point A in the figure. Then another firm could offer a slightly higher warranty, and this would be purchased since the consumer would know that they are of equal quality. The only undominated contract is at the tangency point B.

Another competitive case which is easy to analyse is where there are, say, two types in the population π_l and π_h, but consumers cannot identify which firm is of which quality. There are two subcases to consider, depending on which type is the marginal firm.[10] First suppose that the number of identical consumers, say n, is such that the high-quality firms can satisfy all of demand. In this case, the equilibrium contract will be B in

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[10] Note that in this model there is no production. Further, to insure that firms supply competitively, we assume that all firms together can more than supply the n identical consumers, each of whom wants at most one unit. For this reason, if consumers observe π, equilibrium will be characterized by a zero net price: $R(\pi^*, p, w^0) = 0$ for the marginal quality π^*. The only issue is which firms will sell their products. If we assumed that there was a production cost c, then competitive equilibrium with full information would be characterized by $R(\pi^*, p, w^0) = c$ for the marginal quality π^* (as long as this contract gives consumers at least $\bar{u}$).

Figure 1. For if a contract like A is offered, then it will be dominated by a full warranty contract on the line BC. Note that consumers do not have to make inferences about the quality of items sold with full warranty contracts. Hence a consumer will always prefer a contract like B to a contract like A or C. The other competitive subcase is when the lower-quality firms are the marginal quality; that is, the high-quality firms cannot satisfy demand, but all the potential firms together more than satisfy demand. In this case, the equilibrium contract will be D. (Note that because D involves full insurance, a consumer does not care about π.) To see this, note that the only contract firms that would prefer to offer to D would be to the right of the $R(\pi_l, p, w) = 0$ line. Suppose the best firm, π_h, switches from D to a point in the region labeled E. There will be points in E which are better for a π_h firm than is D. However, there is no point in region E which can be better for both the π_h firm and for the consumer than is D. This is because D is a Pareto optimal point; it involves full insurance. Hence a consumer offered a contract in E would know that it was worse than D if it knew that a good firm offered it. If a consumer would not buy the best firm's offer of something in E if he could get D, he surely would not buy any (unidentified) firm's offer of something in E when he can get D. All other points are also clearly worse than D since they involve higher prices and less warranty than D.

The previous results are based on the idea that when the consumer is offered contracts involving a complete warranty (that is, where $w = b_1 - b_2$), then he does not care about quality. If there are a sufficient number of firms, then competition among firms will drive out contracts which do not offer a full warranty. As we noted earlier, consumers will be least informed about those products which are new, and it is exactly the case of new products where monopoly is most likely to be found. Thus it is extremely important to see whether the results of this section generalize to the case of monopoly. In particular, will a monopolist of very low quality be treated as if he is of average quality by a consumer? Without disclosure or warranties, then, the answer is yes. However, we will extend the results of the last section to show that there will be adverse selection against a monopolist who does not offer a full warranty, and thus even monopoly will be characterized by the same type of contracts which would arise as in the case where consumers know the monopolist's quality exactly.

Consider Figure 2. We have drawn two indifference curves for the consumer. The steep curve is the (p, w) combination, which leaves the consumer indifferent when he knows the item is of high quality π_h. The less steep curve corresponds to an item of low-quality π_l. Both indifference curves are drawn to give the consumer a level of utility $\bar{u}$, which is
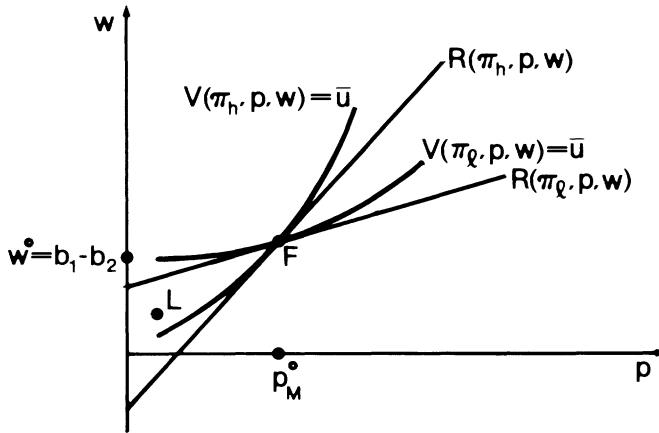
FIGURE 2

the best he can do if he does not purchase the monopolist's product. Figure 2 also contains two iso-revenue lines, each line tangent to its respective indifference curve. As we noted earlier, tangency occurs at the full insurance point. In Figure 2, p_M^0 is the price such that $u(b_1 - p_M^0) = \bar{u}$. Both indifference curves and both iso-revenue lines go through the point (p_M^0, w^0), which is labeled point F in Figure 2. Thus, for example, $R(\pi_l, p, w)$ is the line with slope $(1 - \pi_l)^{-1}$ through the point (p_M^0, w^0). The point F is the contract which would arise if a monopolist of either quality maximized his revenue subject to the constraint that *a consumer knew his quality* and that the consumer must get an expected utility of $\bar{u}$.

We will now use an argument similar to that of the last section to show that F is also the outcome even if the consumer does not know π. An optimal contract for a monopolist of type π is a pair (p, w) which maximizes his revenue subject to the constraint that a rational consumer is willing to buy it. In order to describe the set of contracts that a consumer is willing to purchase, it is necessary to describe what a consumer expects a firm's quality to be as a function of the contract offered; that is, to define $\pi^e(p, w)$. First, note that if a firm offers a contract with a complete warranty, then it does not matter what $\pi^e(\cdot)$ is; the consumer will purchase the contract if and only if $p \leqslant p_M^0$. However, suppose a consumer sees a contract like L in Figure 2. (In Figure 2, L is a point (w, p) just above and to the left of the indifference curve $V(\pi_h, p, w) = \bar{u}$.) Should the consumer buy it? Note that it is below his $\bar{u}$ indifference curve if it is the low quality, while it is above his $\bar{u}$ indifference curve if it is the high quality (that is, he would not buy it if he knew it was of low quality). Note further that a low-quality firm will make more profit offering L than it will

offering F. Hence, in order to see whether F is an equilibrium, it is necessary to see whether the consumer would be willing to buy L.

It is rational for the consumer to reason as follows. Suppose the firm offering L was a high-quality firm; then, from the figure, the point L gives a high-quality firm strictly less revenue than the point F. A high-quality firm knows that if it offered F then the consumer would buy it, since F involves complete insurance and thus the consumer knows he will get $\bar{u}$. Hence why would a high-quality firm offer a contract like L rather than F? It would not. Thus if a firm offers L it must be of lower quality.

If there were exactly two qualities possible, the consumer would infer from L that the firm is of quality π_l and would not purchase the contract since it puts him on a π_l indifference curve below $\bar{u}$. However, there may be other possible qualities. Consider an iso-revenue line through L and F. That is, if $L \equiv (p_L, w_L)$, the iso-revenue line is the set of p, w such that

$$\frac{p - p_L}{w - w_L} = \frac{p - p_M^0}{w - w^0}.$$

Such a line would have slope $(w^0 - w_L) \div (p_M^0 - p_L)$. That slope can be used to define the quality π_L at which a firm would get the same revenue offering (p_L, w_L) and (p_M^0, w^0). All firms with quality below π_L will get higher revenue offering L than F. This is illustrated in Figure 3. Hence the consumer knows that if the firm offers L its quality is below π_L. Consider the consumer's $\bar{u}$ indifference curve for a product of known quality π_L. That indifference curve must be tangent to the revenue line LF at the
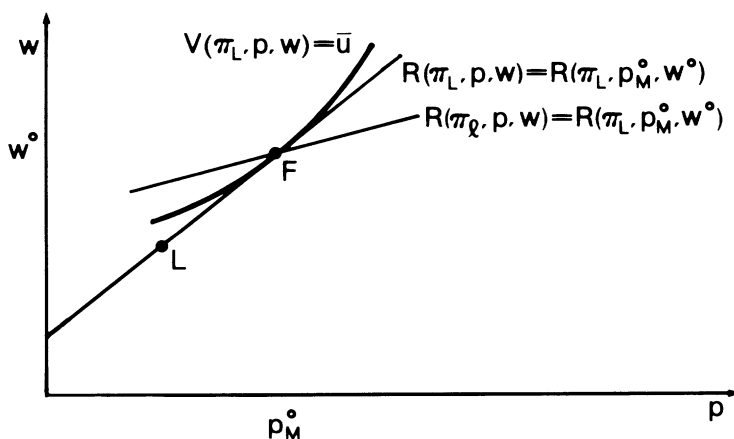

FIGURE 3

point F. Therefore L is below the indifference curve $V(\pi_L, p, w) = \bar{u}$. It follows immediately that L is also below all $\bar{u}$ indifference curves for qualities less than π_L, since these curves are less steep. Hence the consumer knows for sure that if he purchases L and L was profit maximizing for the firm offering it, then he will get less than $\bar{u}$. Hence he does not buy it.

The above argument holds for any contract L which is not on the $R(\pi_h, p, w)$ iso-revenue curve in Figure 2. Therefore the only candidates for equilibrium are points on that iso-revenue line. Consider a firm with quality less than π_h. Such a firm would get strictly higher revenue at F than any other point on the $R(\pi_h, p, w)$ line, since its iso-revenue line is flatter than is $R(\pi_h, p, w)$. Hence any firm of quality less than π_h will offer F. Consider a firm of quality π_h. That firm is indifferent about anything on $R(\pi_h, p, w)$, so F is an optimal policy for it. However, if the firm should choose anything but F on its iso-revenue line, the consumer would immediately know that its quality is π_h and thus that the consumer's expected utility is less than $\bar{u}$. Hence F is strictly optimal for even the highest quality firm.

We have shown that the full insurance point will be the monopoly solution even if consumers do not know product quality. All other contracts can be eliminated because consumers would infer that they are being offered only by firms that are offering qualities insufficient for the consumers to attain a utility level of $\bar{u}$. Such policies would only be offered by low-quality firms who wanted to be thought of as high-quality firms. One way to think of our argument is that the consumer knows that the point F is Pareto optimal. Hence he knows that any other point can only make the firm better off if it makes the consumer worse off than F. Since the consumer can do as well as F elsewhere (he can get $\bar{u}$ by not buying the product), he knows that he should not buy the product at a contract which gives the firm higher revenue than F gives the firm.[11]

Appendix B gives a formal definition of equilibrium and a proof of the above statements.

4. EXTENSIONS AND CONCLUSIONS

We have analyzed the question of whether a monopolist can mislead consumers about his quality. Our basic principle can be described as

[11] Joseph Stiglitz in Monopoly, Non-Linear Pricing and Imperfect Information, Rev. Econ. Stud., vol. LXLIV, no. 138 pp. 407–32, has analyzed markets where a monopolist selling insurance chooses contracts which attempt both to price discriminate and screen out bad risks. In his model, all customers know the qualities of the monopolist, but the monopolist does not know the qualities of the buyers. This is the reverse case from what I have considered. The basic equilibrium concept is very similar, however.

follows: Consider the best contract that the monopolist could offer in a world where the consumers know the monopolist's quality ("the known-quality contract"). This contract extracts all the consumer surplus from the consumer and makes the monopolist as much profit as he can get. Consumers are left indifferent between buying and not buying the item. In a world where there is incomplete information, a monopolist with low quality has the potential of doing even better. If he can mislead consumers into thinking he is of high quality, then he can make the consumers worse off from buying his product instead of their next best alternative. In Section 3 we showed that this is impossible if consumers can make "rational inferences." If the "known-quality" contract does not depend on the monopolist's quality (as it does not when it involves a complete warranty), then the consumer knows that anything which makes the monopolist better off than the full information contract must make him worse off, so he does not purchase anything else.

To prove the above result, we assumed that there are no moral hazards on the part of consumers; that is, the firm solely determines the probability of breakdown of the item. It is clear that our result will still hold when there are consumer moral hazards of the following type. Suppose the consumer can affect the probability of breakdown, so that even if the firm and the consumer knew the quality of the item, full insurance would not be Pareto optimal. However, suppose the known-quality contract which is best for the monopolist provides a level of insurance which is independent of his quality. In this case, the previous argument will go through unchanged and the "unknown-quality" contract will be the same as the "known-quality" contract. Similarly, if there are production costs which depend on quality, then the "known-quality" contract will still be independent of the quality for all produced qualities (there will be a full warranty and a price set to extract all of consumer surplus). Thus, in this case, our result will be true.

There is a problem in generalizing the results of this paper to situations involving many types of consumers. As Salop has shown, if consumers who have different willingness to pay for an item also have different risk aversions, then random prices can help the monopolist sort consumers by willingness to pay.[12] This increases the surplus he can extract. In our model, the monopolist, by making less than a full disclosure, or offering less than a full warranty, makes consumers bear some excess risk. Thus he may be able to increase his profit via this sorting mechanism. However, it is probably the case that there are better random devices (like

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[12] Steven C. Salop, The Noisy Monopolist: Imperfect Information, Price Dispersion, and Price Discrimination, 44 Rev. Econ. Stud. 393 (1977).

random price reductions) which the monopolist can use for sorting high elasticity from low elasticity of demand consumers than incomplete disclosure or incomplete warranties.

We have tried to give some examples of situations in which firms will have an incentive to communicate their quality. Warranties seem like an incredibly useful device for getting around asymmetric information about product quality. There are many products sold with warranties, but I find it surprising that they are not used even more often. The reader might think that the answer lies with moral hazard. Yet there are many risks which are insured by insurance companies but not by sellers. A person can purchase health insurance but not usually from his doctor. I can buy theft insurance, but I cannot purchase it from the seller of burglar-alarm systems. It is very important that the insurance be sold by the commodity seller so that the terms of the insurance vary by seller. This would not matter if the insurance company knew the quality of sellers and sold insurance for products sold by different sellers at different prices. However, this also seems rare.

This paper has been concerned with showing that when firms have tools available which they could use to convey information they will do so. It is not in a monopolist's interest to withhold information about product quality. If information transmittal or warranties are costless, then there is no role for government intervention to encourage disclosure. Thus, the argument that there should be a positive disclosure law or government-mandated warranties cannot be justified on the grounds that these tools have negligible social and private cost, and high benefits through giving consumers more information about product quality or less risk about product quality. One might conclude that a positive disclosure law does no harm as well. Unfortunately, disclosure laws are often very broad. Securities law requires the issuer of a new stock to disclose all facts which are *material* to a purchaser.[13] This requirement may have disadvantages relative to what would arise if there is no positive disclosure law. After the purchase bad events do occur which were not perfectly predictable.

[13] It is no accident that positive disclosure laws are very broad. If there were a few very specific pieces of information relevant to a buyer, then the theorem of this paper would apply: a buyer would simply ask the seller about these pieces of information. A positive disclosure law appears to have benefits when the characteristics of product quality are so vague that a consumer literally has no idea of what to ask. For example, there are so many possible defects in a house that the information costs of disclosing that each possible characteristic works well is much higher than having a seller make a disclosure only in the event that he is aware that there is a specific defect. This is presumably what a positive disclosure law is trying to remedy. By requiring the seller to disclose any information *material* to a buyer, the law attempts to reduce disclosure costs by, for example, requiring the seller to disclose only defects in his product. However, for reasons described below in the text, these laws can actually raise disclosure costs.

Buyers can always bring suit claiming that a material fact was not disclosed regarding the possibility of the bad event. The buyer can then attempt to search the seller's records for evidence. Since this can make the seller bear costs, the seller, anticipating this, discloses an enormous amount of information in the first place. Some of this information may also surely be irrelevant but could be costly to disclose. The seller, by making excessive disclosures, makes the buyer bear more costs in trying to interpret the disclosure. This can convert a situation which involves costless disclosure of truly material facts into a situation where both the buyer and seller must bear costs. An important negative consequence of this is that disclosures may no longer reveal the quality of the seller because they have become so noisy. Thus in the case where disclosure of the (truly) material facts are costless, we are better off without a positive disclosure law.

It would be useful to see how far this policy conclusion can be extended to cases where disclosure or warranties are costly. I disclose that the voluntary disclosure theorem will not be true when disclosure is costly. Further, there are important externalities involved when search or disclosure is costly. Thus the reader should view this policy conclusion with extreme caution.

APPENDIX A

FORMAL DEFINITION OF DISCLOSURE EQUILIBRIUM

We define jointly the equilibrium disclosure function $D(q)$ and the equilibrium inference function $\gamma(D)$. For those to form an equilibrium, it must be the case that

$$\text{for each } q, p\{\gamma[D(q)]\} \geq p[\gamma(D')] \quad \text{for all sets } D' \text{ with } q \in D'; \quad \text{(A1)}$$

$$\gamma_i(D) = \text{Prob (seller } i \text{ would find it optimal to make a disclosure } D). \quad \text{(A2)}$$

Condition (A1) just states that a seller of type q must find it optimal to disclose $D(q)$. Recall that costs are zero, so the seller's profit is just the price he receives for the item. The next condition is quite vague. If this was a screening model, then buyers would try to invert $D(q)$ to discover q. That is, if from experience buyers knew the joint distribution of q and $D(q)$, they could learn something about q from a disclosure D by finding the set of q's such that $D(q) = D$.

However, the function $\gamma(D)$ must be well defined for *all D*, since the monopolist can choose any D (as long as he does not lie). For example, if there are just three possible qualities q_1, q_2, q_3, then in equilibrium there would be at most three disclosure sets $D_1 \equiv D(q_1)$, $D_2 \equiv D(q_2)$, $D_3 \equiv D(q_3)$. But a monopolist with quality q_1 can choose *any* D. In order to show that D_1 is profit maximizing, we must define his profit for all D, not only D_1, D_2, and D_3. For this reason we have had to go beyond signalling to define a rational inference function. Thus (A2) states that the consumer knows that D was optimal for the seller.

With this in mind, we make (A2) more precise. First, given a disclosure set D and a disclosure function $\gamma(\cdot)$, define a set of qualities (that is, the integer labels of the qualities) $J(D; \gamma)$ as follows: $J(D; \gamma)$ contains all integers l such that

$$q_l \in D; \tag{A3}$$

$$\text{if } q_l \in D', \text{ then } p[\gamma(D)] \geq p[\gamma(D')]. \tag{A4}$$

We further require that if D is a one-element set, say $D = \{q_l\}$, then $J(D; \gamma) = \{l\}$:

$$J(\{l\}, \gamma) = \{l\}. \tag{A5}$$

Conditions (A3) and (A4) state that $J(D; \gamma)$ is the set of indexes of qualities q_l with the property that a seller of quality q_l would find D to be his optimal feasible disclosure (recall that lying is impossible, so that a disclosure D' is feasible for a seller of type q_l if and only if $q_l \in D'$). If $\bar{\gamma}_i$ is the buyer's prior probability that the seller is of type i, then (A2) can be written as

$$\gamma_i(D) = \begin{cases} \bar{\gamma}_i \div \displaystyle\sum_{l \in J(D; \gamma)} \bar{\gamma}_l & \text{if } i \in J(D; \gamma) \\ 0 & \text{if } i \notin J(D; \gamma). \end{cases} \tag{A6}$$

Note that (A6) gives a condition that $\gamma_i(D)$ must satisfy; it is a functional equation since $\gamma(\cdot)$ appears on both sides of the equation. Any $\gamma(D)$ which satisfies (A6) for all D is called a *rational inference function*.

The first result is the one given in the text:

Theorem 1. If $\gamma(D)$ is a rational inference function and $D(q)$ is the best disclosure for a seller of type q (in the sense of [A1]), then

$$p\{\gamma[D(q_i)]\} = p_i \quad \text{for all } i.$$

Proof: If $D(q)$ is an optimal disclosure, it must be at least as good as a complete disclosure. Hence

$$p\{\gamma[D(q_i)]\} \geq p_i \quad \text{for all } i. \tag{A7}$$

Suppose (A7) holds with strict inequality for some i. Now simply follow the argument in the text of Section 2. That is, (8) must hold and this contradicts (9). Q.E.D.

It is easy to see that a rational inference function exists. For example, define

$$\gamma_i^e(D) = \begin{cases} 0 & \text{if } q_i \text{ is not the lowest quality in } D \\ 1 & \text{if } q_i \text{ is the lowest quality in } D \end{cases} \tag{A8}$$

To see that (A5) holds, all we must do is show that $J(D; \gamma^e)$ is the index of the lowest quality in D. To see this, let q_i be the lowest quality in D. Suppose a seller has a higher quality, say k; that is, $p_k > p_i$. Then a seller with quality k would get a higher price by announcing $D' = \{q_k\}$ rather than D under the function γ^e in (A8). This shows that only seller i would find it optimal to choose the set D. Thus $J(D; \gamma^e) = \{i\}$ when q_i is the worst quality in D.

APPENDIX B

THE MONOPOLIST'S OPTIMAL WARRANTY UNDER ASYMMETRIC INFORMATION

We want to define an equilibrium policy for each possible product quality π. That is, we want to define a function $[p(\pi), w(\pi)]$ which gives the policy which a firm of type π would offer. Further, it is necessary to define an inference function $\pi^e(p, w)$ which states what a consumer thinks π is when he observes a particular contract. Given a contract function $[p(\pi), w(\pi)]$, we can define a policy set $\delta = \{(p, w) \mid [p(\pi), w(\pi)] = (p, w) \text{ for some } \pi\}$. Note that δ need not be a very large set. In particular, $\pi^e(p, w)$ must be defined for all p, w even those not in δ. Assume that consumers know the distribution of possible π's. Let $G(\pi)$ be the cumulative distribution function of qualities. If consumers know that the equilibrium (as yet undefined) is $p(\cdot), w(\cdot)$, then they can compute the set δ. If they observe a $(p, w) \in \delta$ they can make an inference about π as follows. They can compute the conditional expectation of π given that $\pi \in \{\pi \mid [p(\pi), w(\pi)] = p, w\} \equiv M_e(p, w)$. However, if the consumer observes a $(p, w) \notin \delta$, then $M_e(p, w)$ is empty so another definition must be given. When the consumer observes (p, w), he knows that p, w must be the best policy for the firm in the class of all feasible policies. Given a $\pi^e(\cdot)$ function, a policy is feasible if $V[\pi^e(p, w), p, w] \geq \bar{u}$. Hence a firm of type π will offer a policy (p, w) if and only if

$$R(\pi, p, w) \geq R(\pi, \hat{p}, \hat{w}) \text{ for all } (\hat{p}, \hat{w}) \text{ such that } V[\pi^e(\hat{p}, \hat{w}), \hat{p}, \hat{w}] \geq \bar{u}. \quad \text{(B1)}$$

Hence, given $\pi^e(\cdot)$, the consumer knows that if $(p, w) \notin \delta$ is observed, then π must satisfy the above inequality. Let the set of π's which satisfy the above inequality be given by $M(p, w)$. Note that the inequality must also be true if $(p, w) \in \delta$. Hence $\pi^e(p, w)$ is a rational inference function if, for all p, w:

$$\pi^e(p, w) = \begin{cases} E\{\pi \mid [\pi \in M(p, w)] \text{ and } [\pi \in M_e(p, w)]\} & \text{if } (p, w) \in \delta \\ E\{\pi \mid [\pi \in M(p, w)]\} & \text{if } (p, w) \notin \delta. \end{cases}$$

Note that (p, w) will be in the equilibrium policy set if and only if (A1) is satisfied. Hence we define $\pi^e(p, w)$ to be a rational inference function if

$$\pi^e(p, w) = E\{\pi \mid [\pi \in M(p, w)]\}, \quad \text{for all } (p, w). \quad \text{(B2)}$$

From (B1), $M(p, w)$ depends on the function $\pi^e(\cdot)$ so that (B2) is a functional equation in the mapping $\pi^e(\cdot)$. Define $p(\pi), w(\pi)$ to be an equilibrium policy when, for each π, (B1) holds at $[p(\pi), w(\pi)] = (p, w)$; that is, p, w maximizes $R(\pi, p, w)$ subject to $V[\pi^e(p, w), p, w] \geq \bar{u}$.

Before proving the existence of an equilibrium policy function and inference function, we show that for any equilibrium inference function there will be only one policy function; namely, full insurance.

Theorem 2. Assume that $u(\cdot)$ is strictly concave and $G(\cdot)$ is such that $\pi = 0$ and $\pi = 1$ have zero probability. If equilibrium policy and inference functions $\pi^e(\cdot)$ exist, then $p(\pi) \equiv p_M^0$ and $w(\pi) = w^0$, where those numbers are given in the text by $u(b_1 - p_M^0) \equiv \bar{u}$ and $w^0 \equiv b_1 - b_2$.

Proof: Suppose $(p, w) \neq (p_M^0, w^0)$, we derive a contradiction. If (p, w) maximizes $R(\pi, p, w)$ so that $V(\pi^e, p, w) \geq \bar{u}$, then it must be the case that

$$R(\pi, p, w) \geq R(\pi, p_M^0, w^0), \quad \text{(B3)}$$

since p_M^0, w^0 gives the consumer $\bar{u}$ irrespective of π^e. It must also be the case that

$$E\{[\pi u(b_1 - p) + (1 - \pi)u(b_2 - p + w)] \mid \pi \in M(p, w)\} \geq \bar{u}. \qquad (B4)$$

Note that since $u(\cdot)$ is strictly concave for each π, there is no $(p, w) \neq (p_M^0, w^0)$ such that (B3) holds and $\pi u(b_1 - p) + (1 - \pi)u(b_2 - p + w) \geq \bar{u}$, since p_M^0, w^0 is Pareto optimal. Hence for all π such that (B3) holds, $\pi u(b_1 - p) + (1 - \pi)u(b_2 - p + w) < \bar{u}$. If $\pi \in M(p, w)$, then (B3) holds. Hence (B4) is impossible. Q.E.D.

It is easy to show that a rational inference function exists.

Theorem 3. Define $\pi(p, w) \equiv E[\pi \mid R(\pi, p, w) \geq R(\pi, p_M^0, w^0)]$, then $\pi(p, w)$ is a rational inference function.

Proof: It must be shown that (B2) holds. We need only show that if $R(\pi, p, w) \geq R(\pi, p_M^0, w^0)$, then $R(\pi, p, w) \geq R(\pi, \hat{p}, \hat{w})$ for all $(\hat{p}, \hat{w})$ satisfying $V[\pi(\hat{p}, \hat{w}), \hat{p}, \hat{w}] \geq \bar{u}$. Equivalently, we must show that there is no $(\hat{p}, \hat{w})$ such that $R(\pi, \hat{p}, \hat{w}) > R(\pi, p_M^0, w^0)$ and $V[\pi(\hat{p}, \hat{w}), \hat{p}, \hat{w}] \geq \bar{u}$. As in the proof of theorem 2, if there was such a $\hat{p}$, $\hat{w}$ then $E[\pi u(b_1 - \hat{p}) + (1 - \pi)u(b_2 - \hat{p} + \hat{w}) \mid R(\pi, \hat{p}, \hat{w}) \geq R(\pi, p_M^0, w^0)] \geq \bar{u}$, but this implies that there exists some π such that $\pi u(b_1 - \hat{p}) + (1 - \pi)u(b_2 - \hat{p} + \hat{w}) \geq \bar{u}$ and $R(\pi, \hat{p}, \hat{w}) > R(\pi, p_M^0, w^0)$, which contradicts the Pareto optimality of p_M^0, w^0. Q.E.D.