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To the memory of my parents

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Chapter 1
Introduction

1.1 Economic Analysis of Law

In the subdiscipline of law and economics, laws, legal rules, regulations, procedures, doctrines, etc., are analysed by using what is usually called the economic method. One of the central elements of the economic method is that individuals are assumed to be rational, i.e. purposive. It is also generally assumed that individuals are self-regarding. The notions of rationality and self-regardingness are independent of each other; rationality per se does not imply self-regardingness and neither does the implication hold the other way round. One can be purposive without being solely concerned with one's own welfare; and one can be quite self-centric without exhibiting a high degree of rationality. Rationality postulate consists of assuming that individuals have well-defined preferences over the set of alternatives or outcomes and that they act in accordance with their preferences. The postulate of self-regardingness of individuals consists of assuming that in judging different alternatives individuals take account of only those aspects of the alternatives that concern them. For instance, in the theory of consumer behaviour, it is assumed that the consumer can rank all possible consumption bundles and that in any given situation he[1] will choose a bundle best among all feasible bundles. In evaluating different allocations of goods, a consumer is assumed to take into account only what he would obtain under them and not what others would obtain.

Actions of rational individuals in the furtherance of their goals depend on the institutional structure within the framework of which they are constrained to act. Given the institutional structure, the totality of actions taken by all the individuals determines the social outcome. The outcomes generated by an institution can be

[1]Throughout this work, unless used for specific persons, 'he' will stand for 'he or she', 'him' for 'him or her', 'his' for 'his or her', and 'himself' for 'himself or herself'.

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analysed from the perspective of different characteristics which the outcomes may or may not possess. In the economic analysis of law, the outcomes are almost invariably analysed from the perspective of efficiency.

A method, which assumes that individuals are rational (purposive); determines the actions of individuals under the assumption that they will opt for optimal actions given the institutional structure; determines the social outcome generated by the totality of actions undertaken by individuals; and analyses the characteristics of outcomes thus generated; by its very nature is so general that in principle it can be applied to any human institution.[2]

Analysis of institutions in general, and legal institutions in particular, can be done from a normative perspective as well as from a positive perspective. One way to analyse an institution normatively is to ask what kind of outcomes could be expected under it as a result of actions undertaken by rational individuals. In particular, one could analyse the outcomes from the perspective of those values which are considered important and determine whether the outcomes generated by the institution in question satisfy those values. Alternatively, one could start with a given set of values and explore what kind of institutional structure would be required if the given values are to be realized. In the economic analysis of law, both these approaches have been taken. A positive analysis of an institution normally attempts to explain why the institution exists in the way it does. In terms of comprehending an institution in its entirety, the best possible scenario would be to discover an idea in terms of which the whole of the institution in all its complexity could be explained or understood. The economic analysis of law, both positive and normative, has been done essentially from the perspective of efficiency. Modern beginnings of economic analysis of law are generally attributed to two papers: 'The Problem of Social Cost' by Ronald H. Coase in 1960 and 'Some Thoughts on Risk-Distribution and the Law of Torts' by Guido Calabresi in 1961. The vast amount of law and economics literature which has come into existence since then covers almost all areas of law. Among the various areas of law, the tort law possibly has received the maximum attention. The pioneering contributions by Coase and Calabresi also dealt with tort law. This text will be almost exclusively concerned with liability rules of tort law.[3]

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[2]Becker (1976, p. 8) states the generality of the economic method in these words:

> Indeed, I have come to the position that the economic approach is a comprehensive one that is applicable to all human behavior, be it behavior involving money prices or imputed shadow prices, repeated or infrequent decisions, large or minor decisions, emotional or mechanical ends.

[3]On economic analysis of law in general and the use of economic method for analysing law in particular, see, among others, Baker (1975, 1980), Posner (1975, 1987), Becker (1976), Veljanovski (1980, 1990), Burrows and Veljanovski (1981), Symposium (on Law and Economics) (1985), Cooter (1985, 1991), Landes and Posner (1987), Shavell (1987, 2004), Polinsky (1989), Barnes and Stout (1992a,b), Miceli (1996, 2004), Parisi and Posner (1997), Posner (2007), and Cooter and Ulen (2011). Landes and Posner (1987) and Shavell (1987) are among the most important texts on economic analysis of tort law.

1.2 Efficiency Criteria

There are several different notions of efficiency which are employed in economics. The most important of these is that of Pareto-optimality. Pareto-optimality is defined in terms of Pareto-superiority. An alternative is defined to be Pareto-superior to another alternative if and only if (iff) every individual in the society considers the former to be at least as good as the latter and at least one individual considers the former to be better than the latter. An alternative is defined to be Pareto-optimal or Pareto-efficient iff there is no feasible alternative which is Pareto-superior to it. That is to say, an alternative is Pareto-efficient iff it is not possible to make some individual better off without making anyone worse off. The attractiveness of the idea of Pareto-efficiency stems from the value judgment of the Pareto-criterion. According to the Pareto-criterion, an alternative is to be regarded as socially better than another alternative if the former is Pareto-superior to the latter. If one accepts the value judgment of the Pareto-criterion, it follows that if an alternative is not Pareto-efficient then there exists another alternative which is socially better than it. Consequently, if the social choice is to be rational, it must be from among efficient alternatives only. The desirability of institutions which have the property of always giving rise to efficient outcomes, to be called efficient institutions, also stems from this consideration. Some care, however, is required in interpreting the significance of an outcome being efficient or of an institution which invariably gives rise to efficient outcomes. Acceptance of Pareto-criterion does not imply that every efficient alternative is socially better than every inefficient alternative, not even that every efficient alternative is socially at least as good as every inefficient alternative. Therefore, simply on the basis of Pareto-criterion, one cannot conclude that an efficient institution is to be preferred over an inefficient institution.

Another important efficiency notion is based on the Kaldor criterion, also known as Kaldor-Hicks criterion. Under the Kaldor criterion a change is considered better if gainers can compensate the losers and still be better off. Actual compensation need not be paid. Intuitively, it seems that if the gainers can compensate the losers and still be better off, then the total wealth after the change would be larger than before the change. The underlying, though often implicit, premise in using this criterion is that the questions of total wealth and its distribution can somehow be separated. The main drawback of the Kaldor criterion is that of the two social states it is possible that a move from the first to the second is such that the gainers can compensate the losers and still be better off; and also that if there is a move from the second to the first then once again the gainers can compensate the losers and still be better off. The paradox associated with the Kaldor criterion is indicative of the difficulty of separating the notion of aggregate wealth from its distribution. Notwithstanding the difficulties associated with the Kaldor-Hicks criterion, its use in the law and economics literature is widespread.[4]

[4]For Pareto-criterion, Pareto-optimality, and Kaldor, Hicks, and Scitovsky compensation principles, see Kaldor (1939), Hicks (1939), Scitovsky (1941), Arrow (1951, 1963), Sen (1970),

1.3 The Coase Theorem

One of the most important results of economic theory relates to the Pareto-efficiency of the competitive equilibrium. Under ideal conditions, which include price-taking behaviour of agents and absence of externalities, the institution of market has the property of resulting in a Pareto-optimal allocation of resources. When these ideal conditions are not there, the link between market and Pareto-optimality is ruptured. Inefficient allocation of resources under the market mechanism, because of non-satisfaction of one or more of the ideal conditions under which market necessarily gives rise to efficient outcomes, is construed as a market failure. When there are externalities there is no longer any guarantee that the allocation arrived at under the market mechanism would be Pareto-efficient. When an activity undertaken by a person results in harm to another person, but this harm is not a cost to the person undertaking the activity, a negative externality is said to exist.[5] Externalities create a divergence between private and social costs. Consider a pollutant-emitting factory harming those living in its neighbourhood. Suppose the factory is operating under competitive conditions, so that the price of the good which is being produced by the factory is equal to its marginal cost. If the factory reduces its output by one unit, the net private loss to the factory owner would be zero. What would be lost because of decrease in revenue would be exactly matched by the gain because of reduction in costs. However, the net gain to the society would be positive as the harm due to the marginal unit would be eliminated. It is because of this divergence that socially inappropriate decisions would be taken by the factory owner. The socially optimal quantity of good is that amount at which price is equal to the sum of marginal harm and private marginal cost. On the other hand the actual amount which would be produced would be that level of output at which the price would be equal to the private marginal cost. Thus, from a social point of view, there would be overproduction of the good manufacturing of which is pollution generating.

What is required for efficient allocation of resources is that the divergence between private costs and social costs be eliminated. One way to bring about alignment of private and social costs is to use Pigovian taxes and subsidies to correct problems of negative and positive externalities, respectively. If the factory owner is made to pay taxes equal to the harm inflicted on the people living in the neighbourhood, then while deciding on how much output to produce, the factory owner is going to take into consideration not only the costs of producing the good but also the harm which would take place as he would have to pay taxes equal to the harm. Thus, while deciding on the amount to be produced, he would be taking into

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Chipman (1987), and Feldman and Serrano (2006). On wealth maximization see Calabresi (1980, 1991), Symposium (on Efficiency as a Legal Concern) (1980), Dworkin (1980), Veljanovski (1981), and Posner (1985), among others. In Chap. 2 of this text, efficiency notions and related issues are discussed in detail.

[5] Similarly, when an activity undertaken by a person results in benefit to another person, but this benefit is not a gain to the person undertaking the activity, a positive externality is said to exist.

consideration the full social costs of his decision and therefore would arrive at the socially appropriate amount. Pigovian taxes and subsidies therefore constitute one method of solving the externality problem.

Coase in his celebrated paper ‘The Problem of Social Cost’ took an entirely different approach. Consider a negative externality such that individual A’s activity harms individual B. According to Coase if property rights are well defined and the individuals can negotiate costlessly, then an efficient outcome would be reached regardless of whether individual A is liable for the harm or not. Assume that A’s activity if undertaken would inflict harm on B of 20; and that in the absence of A’s activity B’s net gain from his activity is 100. If A is liable for the harm inflicted on B, then in deciding whether to undertake the activity in question, A is going to compare his gains from the undertaking of the activity with the costs of the activity inclusive of harm to B as being liable the harm to B is a cost to him; and he will undertake the activity only if the gains are at least as large as costs. The decision arrived at by him would be socially optimal as there is no divergence between private and social gains and costs. For instance, if the excess of A’s gain over costs exclusive of harm to B is 10, then A would decide not to undertake the activity.

Next suppose that A is not liable to B for the harm from his activity. One might think that if the excess of A’s gain over costs exclusive of harm to B is 10, then A will decide to undertake the activity as he does not have to pay for the harm to B. Coase argues that it would not be correct to think along these lines. In fact, according to Coase, A’s decision would be the same as before. The net gain to A from the activity is 10; and net loss to B from the activity is 20. There is clearly scope for a mutually beneficial agreement between A and B. Assuming that transaction costs are zero, if A is offered a sum greater than 10 and less than 20 by B for agreeing not to undertake the harmful activity, then both parties would benefit. Thus, it makes no difference whether A is liable for the harm to B or not. In either case A will undertake the activity if social gains exceed social costs; and will not undertake the activity if social costs exceed social gains.

The logic of examples considered by Coase is generally stated as a theorem. One way to state the Coase theorem is to assert that if transaction costs are zero, then the allocation of resources will be efficient regardless of liability assignments. In the examples which Coase had considered in his paper, the allocation of resources in fact was invariant with respect to liability assignments, as indeed was the case with the example considered above. As a result the first set of statements of the Coase theorem asserted both efficiency and invariance of allocation of resources with respect to different liability assignments. The invalidity of the invariance claim was soon realized. When the injurer is liable for the harm to the victim, the distribution of wealth would be in favour of the victim; and when the harm has to be borne by the victim himself, the wealth distribution would be in favour of the injurer. Consequently, if in the context of some activity the liability regime is changed, then one set of people would become wealthier and another set of people would become less wealthy. This in general would have differing implications for demands of different goods and in general would result in a different allocation of resources.

There is an important implication of the logic of the Coase theorem regarding when, what the law is, is relevant from efficiency perspective. What the Coase theorem tells us is that in a world of zero transaction costs, efficient allocation of resources would be achieved regardless of the liability assignments. From the kind of examples and considerations which were used to assert the Coase theorem, it follows that if the transaction costs are prohibitively high then the efficiency or otherwise of allocation of resources would depend on the liability assignments. In the example which was considered above, it is clear that if transaction costs are so high as to preclude private bargaining then A would not undertake the activity in case he is liable for the harm to B. On the other hand, if he is not liable then he would undertake the activity as it would benefit him to the tune of 10 to do so. When transaction costs were zero, the payment from B made the gain from not undertaking the activity at least equal to 10, which now is ruled out because of the prohibitively high transaction costs. When transaction costs are high, in the example considered here, imposing the liability on the injurer results in greater wealth than imposing it on the victim. This is because we assumed that the net social gain from the injurer's activity is negative $(10 - 20 = -10)$. When transaction costs are zero the liability law is irrelevant from the perspective of efficiency; but not if transaction costs are positive.[6]

1.4 Liability Rules

One of the fundamental problems of tort law is how to apportion harms which ensue interactions between injurers and victims. A rule which apportions the harm between the injurer and the victim is called a liability rule. Strict liability and no liability constitute two polar cases. Under the rule of strict liability, the injurer is made liable regardless of any other consideration. On the other hand, under no liability the liability invariably falls on the victim. When one person's activity results in harm for another person, it would generally be the case that the probability and extent of harm can be reduced by care taken by one or both parties. A rule for assigning liabilities may take into account the actual care levels of the parties in relation to the care levels which might be legally prescribed, called the due care levels. If a party's actual care level is greater than or equal to the due care level then the party is called nonnegligent; and called negligent otherwise. Most of the liability rules which are used in practice make liability assignments dependent on negligence or otherwise of one or both parties. At times, the degree of negligence is also taken

[6]For a lucid exposition of the Coase theorem, see Demsetz (1972). There is a vast literature on the interpretation, validity, and applicability of the Coase theorem and related issues. See, among others, Nutter (1968), Arrow (1969), Mishan (1971), Baumol (1972), Regan (1972), Inada and Kuga (1973), Samuels (1974), Baker (1975), Cheung (1978, 1980), Cooter (1982), Barzel (1989), Eggertsson (1990), Allen (1991), and Medema (1994). For surveys of literature on the Coase theorem, see Medema and Zerbe (2000), and Parisi (2005).

into account. The rules which take into account negligence or otherwise of one or both parties include rule of negligence, negligence with the defence of contributory negligence, and strict liability with the defence of contributory negligence. The comparative negligence takes into account degrees of negligence as well.

A liability rule determines the proportions in which the two parties are to bear the loss in case of accident. A very general way to define the notion of a liability rule is to make it a function of nonnegligence proportions of the parties. Given any nonnegligence proportions of the two parties, a liability rule uniquely determines the proportions in which accident loss is to be borne by the parties. Let f be a liability rule, p and q the nonnegligence proportions of the victim and the injurer, respectively, and x and y the proportions of accident loss to be borne by the victim and the injurer, respectively, where $p, q \in [0, \infty),\ \ x, y \in [0, 1],\ \ x + y = 1.$[7] More formally, a liability rule is a function from the Cartesian product of the set of nonnegative numbers with itself to the Cartesian product of the closed unit interval with itself, i.e. from $[0, \infty)^2$ to $[0, 1]^2$. In any given accident context, if the due care for a party is positive, then the nonnegligence proportion of the party is equal to the actual care level divided by the due care level; and if the due care is zero then the nonnegligence proportion is taken to be 1 regardless of the actual level of care taken by the party. If the nonnegligence proportion of a party is less than 1, then the party is called negligent; if equal to 1, then exactly nonnegligent; and if greater than 1, then overly nonnegligent.

One of the most important concerns of the economic analysis of tort law has been with the question of efficiency or otherwise of different liability rules. A liability rule is efficient if it invariably induces the parties to behave in ways which are social costs minimizing. The first formal analysis of important liability rules from the perspective of social costs minimization was done by Brown (1973). For most of the liability rules which are commonly used, the question of efficiency was settled by Brown's contribution. In the context of his model, Brown showed that the rules of negligence, negligence with the defence of contributory negligence, and strict liability with the defence of contributory negligence all have the property of invariably inducing both the victim and the injurer to take levels of care which are appropriate from the perspective of minimization of total social costs (TSC). He also showed that the rules of strict liability and no liability do not possess this property.[8]

[7]Let $a < b$. We use the standard notation to denote:
$\{x \mid a \leq x \leq b\}$ by $[a, b]$, $\{x \mid a \leq x < b\}$ by $[a, b)$, $\{x \mid a < x \leq b\}$ by $(a, b]$, $\{x \mid a < x < b\}$ by (a, b), $\{x \mid x \geq a\}$ by $[a, \infty)$, $\{x \mid x > a\}$ by (a, ∞), $\{x \mid x \leq a\}$ by $(-\infty, a]$, and $\{x \mid x < a\}$ by $(-\infty, a)$.

[8]The literature on the efficiency of liability rules is quite vast. See in addition to the pioneering contribution by Brown (1973), Posner (1972, 1973), Epstein (1973), Rabin (1976, 1981), Schwartz (1978, 1981), Rizzo (1980), Shavell (1980, 1987, 2007), Assaf (1984), Haddock and Curran (1985), Cooter and Ulen (1986), Craswell and Calfee (1986), Rea (1987), Rubinfeld (1987), Hylton (1990), Orr (1991), Symposium (on The Economics of Liability) (1991), Gilles (1992), Chung (1993), Goldberg (1994), Endres and Querner (1995), Wittman et al. (1997), Feldman and Frost (1998), Jain and Singh (2002), Bar-Gill and Ben-Shahar (2003), Kim (2004), Dari-Mattiacci

While in practice only a small number of liability rules are used, in theory there is an infinite number of them. An analysis of the totality of all liability rules shows that a liability rule is efficient iff it satisfies two conditions, namely, the requirement of non-reward for over-nonnegligence (RNO) and the condition of negligence liability (NL).[9] The requirement of non-reward for over-nonnegligence essentially requires that if one party is exactly nonnegligent, then the other party must not benefit by moving from a position of exact nonnegligence to over-nonnegligence. More precisely, what is required is that if one party is exactly nonnegligent, then the liability share of the other party when he is over-nonnegligent is greater than or equal to his liability share when he is exactly nonnegligent. The condition of negligence liability requires that if one party is exactly nonnegligent and the other party is negligent, then the entire loss in case of accident must be borne by the negligent party. Thus, it is possible to have a liability rule which is efficient and punishes individuals for excessive care although none of the liability rules employed in practice do so. If one does not make a distinction between the due care and more than the due care, as indeed is the case as a general rule, then RNO is automatically satisfied; and consequently it follows that the efficient liability rules then will be characterized by condition NL alone. If one does not make a distinction between the due care and more than the due care, one can define nonnegligence proportion of a party to be 1 if the care level is greater than or equal to the due care and equal to the actual care level divided by the due care in case of actual care level being less than the due care. The notion of a liability rule then can be defined as a function from the Cartesian product of the closed unit interval with itself to the Cartesian product of the closed unit interval with itself. Thus, if the notion of a liability rule is defined as a function from $[0,1]^2$ to $[0,1]^2$, then negligence liability condition is both necessary and sufficient for efficiency. The negligence liability condition has a straightforward interpretation. By making the negligent party fully liable for the entire accident loss when the other party is nonnegligent, a liability rule satisfying the negligence liability condition makes each of the two parties internalize the entire accident loss and thereby induces each of them to take socially appropriate decisions. As the rules of negligence, negligence with the defence of contributory negligence, comparative negligence, and strict liability with the defence of contributory negligence, all satisfy the negligence liability condition, their efficiency follows as a corollary of the general characterization theorem. As strict liability and no liability violate the negligence liability condition, it follows that they are inefficient. If one considers only those cases where optimal care by the victim is zero then the victim can never be negligent, and consequently the negligence liability condition reduces to requiring that whenever the injurer is negligent he must be fully liable for the entire accident loss. As under strict liability the injurer is fully liable in all cases, the requirement imposed by the negligence liability condition is fulfilled, and therefore it follows that strict liability is efficient under the unilateral care case.

(2005), Dari-Mattiacci and De Geest (2005), Dari-Mattiacci and Parisi (2005), Singh (2007a,b), and Jain (2010a). The list is by no means exhaustive.

[9]The formal statement of the characterization theorem and its proof are given in Chap. 3.

Although as a general rule no distinction is made in tort law between the due care and more than the due care, the practice of making no such distinction does not seem to have a basis in the efficiency requirement. From the necessity and sufficiency of the conjunction of RNO and NL for efficiency, it follows that there are efficient rules which make a distinction between the due care and more than the due care as well as those which do not; and also there are inefficient rules which make a distinction between the due care and more than the due care and which do not. Because of complete logical independence between efficiency on the one hand and making no distinction between the due care and more than the due care on the other, it follows that efficiency alone cannot be an explanation of this tort law feature. The feature must, at least partly, have a non-efficiency explanation. It can, however, be shown that every monotonic liability rule which is efficient must be such that it makes no distinction between the due care and more than the due care. A liability rule is monotonic iff it is such that if a party's nonnegligence proportion goes up, then his liability share must not increase, given that there is no change in the other party's nonnegligence proportion. Monotonicity can be regarded as a formalization of an aspect of fairness. Because monotonicity, by itself, like efficiency, is unrelated to the no distinction between the due care and more than the due care feature, in view of the result linking efficient monotonic liability rules with no distinction between the due care and more than the due care feature, it seems appropriate to claim that the no distinction between the due care and more than the due care feature is grounded partly in efficiency and partly in fairness.

The framework within which the characterization of efficient liability rules as those satisfying the negligence liability condition is obtained is similar to the framework employed by Brown (1973), although somewhat more general. Two important elements of this framework, to be referred to as the standard tort model, are: (i) The activity levels of both the parties are fixed. This assumption is crucial as there is no liability rule which is efficient if both activity and care levels can be varied.[10] (ii) The legally specified due care levels are appropriate from the perspective of minimization of TSC. Efficient outcomes are not guaranteed under any liability rule if the legally specified due care levels are different from the ones which minimize social costs. Chapter 3 contains the basic framework within which the liability rules are analysed in this text.

1.5 Decoupled Liability

A feature of every liability rule is that the payment to the victim always equals the liability of the injurer. Liability is said to be 'coupled' if the liability imposed on the injurer equals payment to the victim; and to be 'decoupled' if the two amounts are unequal. Thus, liability is coupled under every liability rule. An example of a rule with decoupled liability is provided by the rule under which the injurer pays

[10] Shavell (1980).

tax equal to the harm and the victim bears his loss. Unlike the case of liability rules, where the proportions of the loss that the two parties bear sum to 1, in the case of the rule under which the injurer pays tax equal to the harm and the victim bears his loss, the proportions of the loss that the two parties bear sum to 2. In general one can consider rules where the proportions of the loss that the two parties bear sum to any nonnegative number. For liability rules, i.e. the rules where the proportions of the loss that the two parties bear sum to 1, we know that a necessary and sufficient condition for efficiency is that the condition of negligence liability holds. The question arises as to what the corresponding result is if one considers not only liability rules but all rules where the sum of the proportions of the loss that the two parties bear could be any nonnegative number. Chapter 4 deals with this question. It turns out that decoupled liability is inconsistent with efficiency.[11] In other words, even if we consider the totality of all rules where the sum of the proportions of the loss that the two parties bear could be any nonnegative number, rather than only liability rules, the set of efficient rules remains the same. A corollary of this general result showing that decoupled liability is inconsistent with efficiency is that the rule under which the injurer pays tax equal to the harm and the victim bears his loss is not an efficient rule.

It was noted earlier that the reason why liability rules satisfying the negligence liability condition and only liability rules satisfying the negligence liability condition are efficient is that they make both the parties internalize the entire harm due to interaction between the parties. The impossibility result with decoupled liability points to another requirement for efficiency. Efficiency also depends on whether the proposed rule for solving the externality problem has the closure property with respect to the parties involved in the interaction giving rise to the externality. When we consider an externality resulting from an interaction between two parties, the apportioning of the loss resulting from the interaction between them only keeps the externality closed with respect to the parties whose interaction is the cause of it. On the other hand, if the loss apportionment involves not only the interacting parties but the government or some other third party also, then the externality is not closed with respect to the parties involved in the interaction giving rise to it. It is the involvement of the third parties which distorts incentives for taking care. Decoupled liability is inconsistent with efficiency because decoupling of liability necessarily results in non-closure of the externality with respect to parties involved in the interaction giving rise to it.

1.6 Two Notions of Negligence

As mentioned earlier, in the law and economics literature, the notion of negligence is defined in terms of shortfall from the due care level. If an individual's care is less than the due care, then the individual is adjudged as negligent; and if his care

[11] Jain (2012).

is greater than or equal to the due care, he is adjudged to be nonnegligent. This notion of negligence has been criticized on the ground that the way courts construe negligence is quite different from this way of defining negligence.[12] According to this view whether courts judge a party to be negligent depends on whether the existence of an untaken precaution can be shown such that if it had been taken, the decrease in expected harm would have been greater than the increase in care cost. That is to say, a party is negligent iff there exists a cost-justified precaution which was not taken; and nonnegligent iff there does not exist any cost-justified untaken precaution.[13]

When negligence is defined as shortfall from the due care, a liability rule is efficient iff it satisfies the condition of negligence liability. However, it turns out that if negligence is determined on the basis of existence or otherwise of cost-justified untaken precautions, then there is no liability rule which is efficient.[14] Earlier we had seen two different sources of inefficiency, namely, failure by at least one of the two parties to internalize the entire harm due to externality and the non-closure of the externality with respect to the parties involved in the interaction giving rise to the externality. The impossibility theorem with negligence defined in terms of cost-justified untaken precautions reveals a third potential source of inefficiency. When negligence is defined in terms of shortfall from the due care, whether one is negligent or not depends only on one's own care level and not on the care level of the other party. In contrast, if negligence is defined as existence of a cost-justified untaken precaution, then whether one is negligent or not depends both on one's own care level and the care level of the other party. Suppose TSC are minimized at a unique configuration of care levels of the two parties, and suppose the victim is taking care at the TSC-minimizing level. If the injurer also takes care at the TSC-minimizing level, then for the victim there would not exist any cost-justified untaken precaution, and consequently he would be adjudged nonnegligent. If, however, the cares by the two parties are substitutes for each other, then the possibility exists that if the injurer's care is less than the TSC-minimizing level, there might exist a cost-justified untaken precaution for the victim. Thus, the injurer by taking a less than the TSC-minimizing level of care can convert the status of the victim from nonnegligent to negligent even though the victim is taking care at the TSC-minimizing level. It is because of the possibility of this kind of strategic manipulation that there is no liability rule which can invariably give rise to efficient outcomes when negligence is defined in terms of cost-justified untaken precautions.

In view of this a strong case can be made out for defining negligence as shortfall from the due care level. However, if the assumptions of the standard tort model do not hold, then it is not possible to claim superiority for defining negligence as shortfall from the due care level over defining negligence as existence of a

[12] See Grady (1983, 1984, 1989) on this.

[13] Chapter 5 is concerned with this way of defining negligence and its implications for the efficiency of liability rules.

[14] Jain (2006).

cost-justified untaken precaution. The costs of determining the socially optimal due care levels by courts can be quite high and may more than offset the efficiency gains. There is an additional point in favour of defining negligence in terms of cost-justified untaken precautions. This approach is more in harmony with the adversarial system than is the case with the due care level approach. In the due care level approach, the courts must necessarily play an active role to gather and process information for calculating the due care levels correctly. Under the untaken precaution approach, it is for the party which wants to establish the negligence of the other party to show the existence of a cost-justified precaution which the opposite party could have taken but did not.

1.7 Incremental Liability Rules

Under the negligence rule, as it is usually defined, the entire accident loss falls on the injurer if the injurer is negligent; and the entire loss is borne by the victim if the injurer is nonnegligent. There is, however, another version of negligence rule, to be called incremental negligence rule, under which the injurer, if nonnegligent, bears no liability; and, if negligent, bears liability equal to the harm which can be ascribed to his negligence. Consider a simple example of negative externality in which harm occurs with certainty. The quantum of harm, however, depends on the care levels of the two parties. Suppose further that the configuration of care levels at which the social costs are minimized is unique, and let the injurer's due care be fixed at the social costs minimizing level. Suppose, given that the victim is taking the social costs minimizing level of care, if the injurer takes the due care, the harm is 10; and if he takes no care, then the harm is 30. Then it is clear that of the total harm of 30, only harm of 20 can be attributed to the negligence of the injurer as the remaining harm of 10 would have taken place even if the injurer had not been negligent. Under the standard negligence rule, the injurer would be liable for the entire harm of 30 when he does not take care. Under the incremental version of the negligence rule, however, he would be liable for only 20 of the total harm of 30 when he does not take care.

Incremental negligence rule is a member of a large class of rules which can be termed as the class of incremental liability rules. A standard liability rule determines the proportions, in which the two parties are required to bear the loss in case of occurrence of accident, on the basis of the nonnegligence proportions of the parties involved in the interaction. On the other hand, an incremental liability rule is a rule which, as a function of the nonnegligence proportions of the two parties, specifies (i) which of the two parties, the victim or the injurer, is to be the non-residual liability holder; and (ii) the proportion of the incremental loss, which can be ascribed to the negligence of the non-residual liability holder, to be borne by the non-residual party. Under the incremental negligence rule, the injurer is the non-residual liability holder for all configurations of proportions of nonnegligence of the two parties; and whenever the injurer is negligent, he is liable for the entire loss which is due to his negligence.

For the standard liability rules, negligence liability is both necessary and sufficient for efficiency. The corresponding result for incremental liability rules is: Let, when both parties are nonnegligent, the party which is the residual liability holder be designated as r and the party which is the non-residual liability holder as nr. An incremental liability rule is efficient iff its structure is such that: (i) If party r is negligent and party nr is nonnegligent, then party r must remain the residual liability holder. (ii) If party nr is negligent and party r is nonnegligent, then party nr must either become the residual liability holder or liability of nr must be equal to the entire incremental loss which can be ascribed to the negligence of nr.[15] In the case of incremental negligence rule, the residual liability holder is always the victim. Consequently the requirement for efficiency simply becomes: If the injurer is negligent and the victim is nonnegligent, then the injurer's liability must be equal to the entire incremental loss which can be ascribed to the negligence of the injurer. Such indeed being the case under the incremental negligence rule, it follows that the rule is efficient.[16]

For standard liability rules the efficiency requirement does not impose any restrictions on liability assignments when both parties are nonnegligent or both parties are negligent. For incremental liability rules also the efficiency requirement does not impose any restrictions on liability assignments when both parties are negligent. However, when both parties are nonnegligent, from the very definition of an incremental liability rule it follows that the entire loss must be borne by one of the two parties alone.

Negligence liability condition is symmetric with respect to the two parties. The conditions characterizing efficient incremental liability rules, on the other hand, are asymmetric with respect to the parties. This is unsurprising in view of incremental liability rules treating the parties asymmetrically. This asymmetry can be of considerable use. If one wants to view the interactions involving negative externalities asymmetrically as being caused by injurers and consequently wishes to make injurers liable on considerations of justice and fairness, and at the same time does not wish to forego efficiency, then by making the victims the non-residual liability holders, one can have efficiency without significant sacrifice of justice and fairness.

[15] The necessary and sufficient conditions for incremental liability rules were first obtained in Jain (2009a). Incremental liability rules from efficiency perspective are discussed in Chap. 6 of the text.

[16] The efficiency of the incremental version of the negligence rule for the unilateral case was established in Kahan (1989). A proof of the efficiency of the incremental negligence rule for the bilateral case is provided in Jain (2010a). Chapter 7 of this text contains a detailed efficiency analysis of both the versions of the negligence rule, for the unilateral as well as the bilateral case. The analysis is carried out with negligence defined as shortfall from due care as well as with negligence defined as existence of a cost-justified untaken precaution.

1.8 The Idea of Negligence and Strategic Manipulation

So far three sources of inefficiency have been noted: the failure of at least one party to internalize the entire harm from the negative externality; the non-closure of the externality with respect to the parties involved in the interaction giving rise to the externality; and strategic manipulation. The last one occurring if negligence is defined as existence of a cost-justified untaken precaution. There is yet another kind of strategic manipulation which impedes efficiency. Unlike the former kind of strategic manipulation to which only one way of defining negligence is subject to, the latter kind of strategic manipulation afflicts both ways of defining negligence.[17]

The logic behind the idea of individualized negligence determination is that those who are in a position to undertake reduction of expected loss at a lower cost than the others are the ones who should be taking greater care compared to those who can do so only at a higher cost. Therefore, if one party's, say the victim's, costs of taking care remain the same but the injurer's costs of taking care vary, one would expect the care burden on the injurer tending to increase as his costs of taking various levels of care decrease. The goal of minimizing social costs requires that the burden of care on more capable persons be greater than the burden on the less capable. This creates perverse incentives for the individuals. If an individual can successfully hide his true capability, then the due care level that would be set for him would be lower than what it would be if his true capability were known. Successful non-revelation of true capability has twofold undesirable effect on social costs. The social costs would tend to increase because of the due care being set at a lower level than required for social costs minimization. Also, in general there would be wasteful expending of resources for the purpose of hiding one's true capabilities. Furthermore, individualized negligence determination implies that in some cases cost-effective measures for reducing costs of care would not be undertaken. If the net gain from a care effectiveness increasing investment is less than the loss because of enhanced level of the due care resulting from the increased effectiveness of care, then such an investment would not be undertaken.

If the individualized negligence determination is given up and a uniform due care level is adopted, then the wastage of resources on account of efforts for the purpose of misrepresenting one's capabilities would get eliminated. Also, cost-effective measures for increasing effectiveness of care would be undertaken. The social costs, however, would not be at a minimum as abandoning individualized negligence determination means that in some cases the due care would be greater than and in some cases less than that required for minimizing social costs.

Although having a uniform due care for everyone eliminates at an individual level any incentive to manipulate strategically, the incentive for strategic manipulation is not eliminated at the collective level. If uniform negligence determination is to serve the purpose of reducing social costs, then the uniform negligence determination has

[17] Jain (2010b).

to be based on the general level of capability of the set of injurers or the set of victims as the case may be. But then the set of individuals in question would have the same perverse incentives to manipulate strategically as individuals had in the case of individualized negligence determination.[18]

1.9 Loss Decomposition and Efficiency

The negligence liability condition which characterizes efficiency for liability rules has an all-or-none character. When one party is negligent and the other nonnegligent, the entire loss must be borne by the negligent party if efficiency is to obtain. In view of this it seems that if the choice of a liability rule is to be from the set of efficient liability rules, then the non-efficiency considerations, including those of fairness and restitutive justice, cannot possibly play any role in assigning liability in cases where one party is negligent and the other is not. Other considerations at best can have a role only in situations when either both parties are negligent or both are nonnegligent. If liabilities of the parties are to be specified as proportions of total accident loss, as is done in the context of liability rules, then what has been said about efficiency considerations precluding other considerations altogether in cases where one party is negligent and the other nonnegligent is indeed correct. But if one is willing to go outside the framework of liability rules, then it turns out that the scope for non-efficiency considerations is much greater than what would appear to be the case at first glance. In providing, from the perspective of efficiency, correct incentives to the parties, it turns out that a part of accident loss does not play any role and can therefore be apportioned between the two parties independently of their care levels.

Let the loss which takes place in case of occurrence of accident when both parties are taking due care be called optimal loss. Let θ be a nonnegative number. Let excess loss be defined as excess of total accident loss over θ times the adjusted optimal loss (adjusted to take into account differing probabilities of accident with different care levels) if total accident loss is greater than θ times the adjusted optimal loss; otherwise as zero. And let the specified loss be the difference between total accident loss and excess loss.[19] A liability rule apportions the accident loss between the

[18]The matters pertaining to strategic manipulation are discussed in greater detail in Chap. 7.

[19]Let the loss in case of accident when the victim's care is c and the injurer's care is d be given by $H(c, d)$. Let $\pi(c, d)$ denote the probability of accident when the care levels of the victim and the injurer are c and d, respectively. Let social costs be minimized when victim's care level is c^* and injurer's care level is d^*. Let c^* and d^* be the due care levels for the victim and the injurer, respectively. Then optimal loss is given by $H(c^*, d^*)$ and adjusted optimal loss by $H(c^*, d^*)\frac{\pi(c^*,d^*)}{\pi(c,d)}$. If $H(c, d)$ is greater than $\theta H(c^*, d^*)\frac{\pi(c^*,d^*)}{\pi(c,d)}$, then excess loss is given by $H(c, d) - \theta H(c^*, d^*)\frac{\pi(c^*,d^*)}{\pi(c,d)}$ and specified loss by $\theta H(c^*, d^*)\frac{\pi(c^*,d^*)}{\pi(c,d)}$. If $H(c, d)$ is less than or equal to $\theta H(c^*, d^*)\frac{\pi(c^*,d^*)}{\pi(c,d)}$, then excess loss is 0 and specified loss is $H(c, d)$.

parties as a function of the parties' proportions of nonnegligence. Corresponding to any liability rule, one can define a two-parameter (λ, θ), $\lambda \in [0, 1]$, $\theta \geq 0$, family of rules in the following way: (i) The excess loss is to be apportioned between the two parties as specified by the liability rule in question as a function of proportions of nonnegligence of the parties. (ii) The specified loss is to be assigned between the two parties in fixed proportions $(\lambda, 1 - \lambda)$. This more general notion of a rule would be called a (λ, θ)-decomposed liability rule. In other words, while a liability rule apportions the entire accident loss on the basis of which parties are negligent and to what extent; a decomposed liability rule does so only for one part of the accident loss (excess loss), the other part (specified loss) being divided up between the two parties in fixed proportions. It should be noted that if $\theta = 0$, then the notion of a decomposed liability rule coincides with that of a liability rule.

It can be shown that if $0 \leq \theta \leq 1$, then a decomposed liability rule is efficient iff the corresponding liability rule is efficient. If $\theta > 1$, then it turns out that no decomposed liability rule can be efficient.[20] In other words, efficiency properties remain unaffected provided the quantum of loss that is assigned independently of care levels does not exceed the adjusted optimal loss. Regardless of whether a decomposed liability rule corresponds to an efficient liability rule or an inefficient liability rule, if $\theta > 1$ then the decomposed liability rule would be inefficient. In view of these results, it is clear that it is the amount of loss that is in excess of the adjusted optimal loss which constitutes the irreducible minimum which must be assigned to the negligent party to ensure efficiency, in cases where one party is negligent and the other is not. Thus, if actual loss is greater than the adjusted optimal loss, then it is the adjusted optimal loss part of the total loss which has no bearing on efficiency; and if actual loss is less than or equal to the adjusted optimal loss, then there is no part of the total loss which has any bearing on efficiency. From this it is clear that the requirements imposed by efficiency considerations can be quite mild depending on the context.[21]

1.10 Multiparty Interactions

The usual liability rules are defined for one victim and one injurer. The notion of a liability rule when there are one victim and multiple injurers can be defined along the same lines as that of a liability rule with one victim and one injurer. A liability rule for one victim and n injurers is a function which for every configuration of nonnegligence proportions of $n + 1$ parties assigns the proportions in which the loss in case of accident is to be borne by the parties. Thus, while a usual liability rule is a function from $[0, 1]^2$ to $[0, 1]^2$, a liability rule defined for one victim and n injurers is a function from $[0, 1]^{n+1}$ to $[0, 1]^{n+1}$. In Chap. 9 the totality of all

[20] See Jain and Kundu (2004).

[21] Chapter 8 analyses the class of decomposed liability rules corresponding to the negligence rule.

liability rules defined for one victim and multiple injurers is considered; and a sufficient condition for efficiency is derived. It is shown that a condition called the collective negligence liability is sufficient to ensure efficiency of a liability rule defined for one victim and multiple injurers. A liability rule defined for one victim and multiple injurers satisfies the condition of collective negligence liability iff its structure is such that whenever some individuals are negligent, no nonnegligent individual bears any loss in case of occurrence of accident. For the case of one victim and one injurer, the condition of collective negligence liability reduces to that of negligence liability. Thus, the condition of collective negligence liability can be viewed as a generalization of the condition of negligence liability when there are one victim and multiple injurers. The question whether this condition is necessary for efficiency is an open question. In Jain and Kundu (2006), it has been shown that collective negligence liability is both necessary and sufficient for efficiency for the class of simple liability rules defined for one victim and multiple injurers. Simple liability rules do not differentiate between no care and care levels which are below the due care, that is to say, treat all nonnegligence proportions belonging to $[0, 1)$ the same. In view of the necessity and sufficiency of collective negligence liability for efficiency of simple liability rules defined for one victim and multiple injurers, it follows that for the entire class of liability rules defined for one victim and multiple injurers there are only two possibilities. These possibilities are: (i) the condition of collective negligence liability is both necessary and sufficient for efficiency for any liability rule defined for one victim and multiple injurers, and (ii) for the class of liability rules defined for one victim and multiple injurers as a whole, there does not exist any condition which is both necessary and sufficient for efficiency. It is an open question as to which of these two possibilities in fact holds.[22]

A multiple-victim-one-injurer liability rule is a rule which specifies the proportions in which each victim's loss is to be divided between the victim in question and the injurer in case of occurrence of accident as a function of nonnegligence proportions of the individuals. Unlike the case of one victim and multiple injurers, in the case of multiple victims and one injurer, it turns out that there is no liability rule which is efficient for all applications.[23] The reason for this difference lies in the fact that for efficiency it is required that all parties involved in the interaction internalize the totality of harm resulting from the interaction. This is possible when there are multiple injurers and one victim, but not when there are multiple victims and one injurer. Regardless of how much care is taken by a victim, he can at most be made to bear his own loss in entirety, but not any part of loss incurred by another victim. There are contexts in which the loss that a particular victim suffers depends not only on the care taken by himself and the injurer but also on the care levels

[22]The first result with one victim and multiple injurers is due to Landes and Posner (1980). They showed the efficiency of the rule of negligence with one victim and multiple injurers. Multiple-tortfeasor context was also analysed in Tietenberg (1989), Kornhauser and Revesz (1989), and Miceli and Segerson (1991).

[23]Jain (2009b).

of other victims. In such situations, given that the victims can at most be made to bear their own losses in entirety, there is no way that a victim could be made to internalize the loss incurred by another victim. Consequently, unlike the case of one victim and multiple injurers, in the case of one injurer and multiple victims, one gets an impossibility theorem. If one considers only those applications where expected loss of a victim depends only on his own care level and the care level of the injurer, but not on the care levels of other victims, then a sufficient condition for a multiple-victim-one-injurer liability rule to be efficient is that its structure be such that: (i) whenever the injurer is negligent and a particular victim is nonnegligent, the entire loss incurred by that victim must be borne by the injurer; and (ii) whenever a particular victim is negligent and the injurer is nonnegligent, the entire loss incurred by that victim must be borne by the victim himself. Like the condition of collective negligence liability, the above condition, called $(n,1)$-negligence liability, also reduces to that of negligence liability for the case of one victim and one injurer. Thus, the condition of $(n,1)$-negligence liability can also be thought of as a generalization of negligence liability condition. For an important subclass of multiple-victim-one-injurer liability rules, characterized by the condition that the proportions in which the loss incurred by a particular victim is to be borne by the injurer and that victim depend only on the nonnegligence proportions of the injurer and that victim, the condition of $(n,1)$-negligence liability is both necessary and sufficient for efficiency with respect to the subclass of applications where expected loss of a victim depends only on his own care level and the care level of the injurer, but not on the care levels of other victims.[24]

In view of the fact that there is no liability rule defined for multiple victims and one injurer which is efficient for all applications, it follows that there does not exist any liability rule defined for multiple injurers and multiple victims which is efficient for all applications.

1.11 Legal Institutions and Values

In the context of efficiency analysis of laws, two important questions arise. The first question relates to whether efficiency provides a unified explanation for common law; the question has arisen in view of the connection between common law and efficiency which has been revealed by economic analysis. While some scholars contend that efficiency provides a unified explanation for the whole of common law, some others favour a more moderate view of regarding efficiency as only one of the explanatory values, but an important one.[25] Two sets of arguments can be advanced in favour of the moderate viewpoint. First, not every feature of common law can be explained solely in terms of efficiency. For instance, the tort law feature of making

[24] Chapter 9 contains a detailed treatment of issues relating to multiparty interactions.

[25] See Calabresi (1980), Posner (1981, 1987), and Landes and Posner (1987) among others.

no distinction between the due care and more than the due care, the mention of which was made earlier, cannot be explained solely in terms of efficiency, an explanation solely in terms of efficiency being impossible because of the complete logical independence between efficiency and the no-distinction feature in the context of liability rules. Second, in order to explain adoption of some efficient rules and non-adoption of other efficient rules, criteria other than efficiency are required as a matter of necessity. The second question relates to the relationship between efficiency and values like justice which are commonly associated with legal institutions. Efficiency is an important value, but so are some other values like justice and protection of basic rights. There are contexts when efficiency is not nonconflictive with values regarded as important. Selection of efficient laws in all cases can consequently have implications detrimental to other values in such contexts.

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Chapter 2
Efficiency Criteria

In economics several different, though related, notions of efficiency are employed. The most important notion of efficiency is that of Pareto-optimality, which is based on the Pareto-criterion. A social alternative x is defined to be Pareto-superior to another social alternative y iff every individual in the society considers x to be at least as good as y and at least one individual considers x to be better than y. According to the Pareto-criterion, alternative x is regarded socially better than alternative y if x is Pareto-superior to y. A social alternative is defined to be Pareto-optimal or Pareto-efficient iff there is no feasible alternative which is Pareto-superior to it. That is to say, an alternative is Pareto-efficient iff it is not possible to make some individual better off without making anyone worse off.

From the definition it is clear that the notion of Pareto-efficiency is defined with respect to a particular set of social alternatives and a particular set of individuals. If the set of social alternatives or the set of individuals changes, then the set of Pareto-efficient alternatives would also in general change. In particular, if the set of social alternatives contracts, then an alternative which earlier was Pareto-inefficient might become Pareto-efficient; and if the set of social alternatives expands, then an alternative which earlier was Pareto-efficient might become Pareto-inefficient.[1] If the set of individuals expands or contracts, then an alternative which earlier was Pareto-inefficient might become Pareto-efficient; and an alternative which earlier was Pareto-efficient might become Pareto-inefficient.

Most people find the value judgment of the Pareto-criterion to be rather compelling. It should, however, be noted that the Pareto-criterion can easily conflict with some important value judgments. Suppose under state x a small number of individuals are better off compared to state y and the remaining individuals are equally well off under the two states. While according to the Pareto-criterion state

[1] The alternatives which are Pareto-inefficient before the expansion of the set of alternatives must of course remain so after the expansion as well; and the alternatives which are Pareto-efficient before the contraction must continue to remain so after the contraction.

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x would be judged to be socially better than state y, it is possible that state y might be judged to be less inegalitarian than state x. In which case the Pareto-criterion would be in conflict with the value judgment which declares a state to be better than another if the former is more egalitarian than the latter.

If one accepts the value judgment of the Pareto-criterion, then it follows that the choice of the society or the collective should be from the set of Pareto-efficient alternatives. Choice of a Pareto-inefficient alternative would imply selection of an inferior alternative and rejection of a superior alternative as Pareto-inefficiency of an alternative implies the existence of a Pareto-superior alternative and therefore a socially better alternative in the set of feasible alternatives. Thus, for the purpose of making social choices, only the sets of Pareto-efficient alternatives need be considered if the collective subscribes to the Pareto-criterion. It is, however, rather important to note that the acceptance of the Pareto-criterion does not imply that any arbitrary Pareto-efficient alternative is better than any arbitrary Pareto-inefficient alternative. Pareto-inefficiency of an alternative merely entails that there exists another alternative which is Pareto-superior to it and therefore socially better in view of the Pareto-criterion. Under the Pareto-criterion only some pairs of alternatives can be compared. It is perfectly possible that a particular Pareto-inefficient alternative may be noncomparable with a particular Pareto-efficient alternative in terms of the Pareto-criterion but better in terms of another evaluative criterion. An alternative can be Pareto-efficient and at the same time socially highly undesirable. Pareto-efficiency is discussed formally and more fully in the first section of this chapter.

One difficulty with the Pareto-criterion is that its scope is rather limited. If there is a person in the collective who prefers alternative x to alternative y and another person who prefers y to x, then under the Pareto-criterion x and y cannot be compared regardless of the preferences of the remaining individuals. Thus, in general, only a small proportion of all pairs of alternatives would be comparable under the Pareto-criterion. Because of this one would also generally expect the sets of Pareto-efficient alternatives to be large. In many contexts, therefore, the notion of efficiency that is used is that which is based on the Kaldor compensation principle rather than that based on the Pareto-criterion. An alternative x is defined to be Kaldor-superior to another alternative y iff if there is a move from y to x then the gainers can compensate the losers and still be better off. Actual compensation, however, need not be paid. Under the Kaldor criterion alternative x is regarded as socially better than alternative y if x is Kaldor-superior to y. Intuitively, it seems that if the gainers can compensate the losers and still be better off, then the total wealth after the change must be larger than before the change. It should be noted that if actual compensation is not paid then there is no consensual basis to the change. The underlying, though often implicit, premise in using this notion of efficiency is that the questions of total wealth and its distribution can somehow be separated. The main drawback of the Kaldor compensation principle lies in the fact that the Kaldor-superior relation fails to be asymmetric. It is possible that of the two social alternatives x and y, each is Kaldor-superior to the other. That is to say, examples can be constructed such that if there is a move from alternative y to alternative x, the gainers can compensate the losers and still be better off; and also that if there is a move from alternative x to alternative y, the gainers can compensate the losers and still be better off. One way to resolve this difficulty is to modify the compensation principle along the lines suggested by Scitovsky. Under the Kaldor compensation principle, one infers 'social alternative x is better than social alternative y' from the statement 'if there is a move from social alternative y to social alternative x, then the gainers can compensate the losers and still be better off'. Under the Scitovsky criterion, one infers 'social alternative x is better than social alternative y' from 'if there is a move from social alternative y to social alternative x, then the gainers can compensate the losers and still be better off; and if there is a move from social alternative x to social alternative y, then it is not the case that the gainers can compensate the losers and still be better off'. Unlike the Kaldor-superior relation, the Scitovsky-superior relation is asymmetric; and therefore it can never happen that of the two alternatives x and y, each is Scitovsky-superior to the other. The social 'at least as good as' relation generated by the Scitovsky criterion, however, fails to satisfy transitivity.[2] The logical difficulties associated with Kaldor and Scitovsky compensation principles are particular instances of the Arrow paradox relating to aggregation of individual preferences into social preferences. Another way to look at these difficulties is to see them as manifestations of the fact that the notion of wealth in general cannot be separated from its distribution. The second section of the chapter discusses the compensation principles in greater detail.[3]

2.1 Pareto-Efficiency

Let $N = \{1, \ldots, n\}, n \geq 2$, denote the set of individuals comprising the society; and let S be the nonempty set of social alternatives. Let $R_i, i \in N$, denote the 'at least as good as' binary relation of individual i on S. Thus, $xR_iy, x, y \in S$, will stand for 'individual i considers x to be at least as good as y'. 'At least as good as' binary relation of the society will be denoted by R. The asymmetric parts of binary relations R_i, R, etc., will be denoted by P_i, P, etc., respectively; and the symmetric parts by I_i, I, etc., respectively.[4] Each $R_i, i \in N$, will be assumed to be an ordering, i.e. a reflexive, connected, and transitive binary relation.[5]

[2]See Arrow (1963).

[3]Some important references for Pareto-optimality, compensation principles, and wealth maximization are Arrow (1951, 1963), Sen (1970), Symposium (on Efficiency as a Legal Concern) (1980), Posner (1985), and Feldman and Serrano (2006).

[4]Let R be a binary relation on a set S. The asymmetric part P and the symmetric part I of R are defined by: $(\forall x, y \in S)[(xPy \leftrightarrow xRy \wedge \sim yRx) \wedge (xIy \leftrightarrow xRy \wedge yRx)]$. Thus, if R stands for 'at least as good as', then P and I stand for 'preferred to' and 'indifferent to'.

[5]A binary relation R over a set S is (i) reflexive iff $(\forall x \in S)(xRx)$, (ii) connected iff $(\forall x, y \in S)[x \neq y \rightarrow xRy \vee yRx]$, (iii) transitive iff $(\forall x, y, z \in S)[xRy \wedge yRz \rightarrow xRz]$, and (iv) an ordering iff it is reflexive, connected, and transitive.

Let Q be the unanimity relation defined by: $(\forall x, y \in S)[xQy \leftrightarrow (\forall i \in N)(xR_iy)]$. Thus, xQy iff everyone in the society considers x to be at least as good as y. An alternative $x \in S$ is defined to be Pareto-superior to another alternative $y \in S$ iff $[xQy \wedge \sim yQx]$, i.e. iff $[(\forall i \in N)(xR_iy) \wedge (\exists i \in N)(xP_iy)]$. That is to say, $x \in S$ is Pareto-superior to $y \in S$ iff everyone considers x to be at least as good as y and at least one person considers x to be better than y. An alternative $x \in S$ is defined to be Pareto-indifferent to another alternative $y \in S$ iff $[xQy \wedge yQx]$, i.e. iff $(\forall i \in N)(xI_iy)$. That is to say, $x \in S$ is Pareto-indifferent to $y \in S$ iff everyone considers x to be as good as y. From the definition of unanimity relation, it follows that it is a reflexive and transitive binary relation on S.

Proposition 2.1. *Let N and S be the set of individuals and the set of alternatives, respectively. If every individual belonging to N has an ordering on S, then unanimity relation is reflexive and transitive.*

Proof. As by definition $xQx \leftrightarrow (\forall i \in N)(xR_ix)$, the reflexivity of Q follows from the reflexivity of individual R_i's.
Let $xQy \wedge yQz$.
$xQy \to (\forall i \in N)(xR_iy)$.
$yQz \to (\forall i \in N)(yR_iz)$.
$(\forall i \in N)(xR_iy) \wedge (\forall i \in N)(yR_iz) \to (\forall i \in N)(xR_iz),$ as each R_i is transitive.
$(\forall i \in N)(xR_iz) \to xQz$.
Thus, Q is transitive. □

From the transitivity of the unanimity relation Q, it follows that 'Pareto-superior' relation and 'Pareto-indifferent' relation are transitive.[6] As 'Pareto-superior' relation is the asymmetric part of the unanimity relation, it follows that it is irreflexive.[7] 'Pareto-indifference' relation is reflexive as Q is reflexive.

An alternative $x \in S$ is defined to be Pareto-optimal or Pareto-efficient iff $\sim (\exists y \in S)[(\forall i \in N)(yR_ix) \wedge (\exists i \in N)(yP_ix)]$. That is to say, an alternative $x \in S$ is Pareto-efficient iff there is no alternative which is Pareto-superior to it. An alternative is Pareto-inoptimal or Pareto-inefficient iff it is not Pareto-optimal.

The following example illustrates the definitions of Pareto-superiority and Pareto-optimality.

Example 2.1. Let $S = \{x, y, z, w\}$ and $N = \{1, 2, 3, 4\}$.
Let the preference orderings of individuals in N be as follows:
$R_1 = xyzw$
$R_2 = w(yz)x$
$R_3 = (xy)zw$
$R_4 = (xyz)w$

[6]If a binary relation is transitive, then its asymmetric and symmetric parts are transitive. See Sen (1970).

[7]A binary relation R over a set S is irreflexive iff $(\forall x \in S)(\sim xRx)$.

(Notation: Alternatives within the parentheses are indifferent to each other. If an alternative is left of another, then the former is preferred to the latter. Thus, individual 3 is indifferent between x and y, prefers both x and y to both z and w, and prefers z to w.)

Checking pairwise we find: (i) As individual 1 prefers x to y and individual 2 prefers y to x, it follows that neither x is Pareto-superior to y nor y is Pareto-superior to x. (ii) As individual 1 prefers x to z and individual 2 prefers z to x, it follows that neither x is Pareto-superior to z nor z is Pareto-superior to x. (iii) As individual 1 prefers x to w and individual 2 prefers w to x, it follows that neither x is Pareto-superior to w nor w is Pareto-superior to x. (iv) Individuals 1 and 3 prefer y to z and individuals 2 and 4 are indifferent between y to z; therefore it follows that y is Pareto-superior to z. (v) As individual 1 prefers y to w and individual 2 prefers w to y, it follows that neither y is Pareto-superior to w nor w is Pareto-superior to y. (vi) As individual 1 prefers z to w and individual 2 prefers w to z, it follows that neither z is Pareto-superior to w nor w is Pareto-superior to z.

Thus, from (i) to (vi) we have: (a) There is no alternative which is Pareto-superior to x. (b) There is no alternative which is Pareto-superior to y. (c) There is an alternative which is Pareto-superior to z. (d) There is no alternative which is Pareto-superior to w. Consequently, alternatives x, y, w are Pareto-optimal and alternative z is not Pareto-optimal. $\Diamond$

Remark 2.1. The notion of Pareto-optimality or Pareto-efficiency is defined relative to particular sets of individuals and alternatives. If these sets change, then in general the set of Pareto-optimal alternatives would also change even when there is no change in the preference orderings of individuals. Consider a given N and preferences of individuals in N. If the set of alternatives S expands and becomes $S' \supset S$, then it is possible for an alternative that was Pareto-optimal before the expansion to become Pareto-inoptimal after the expansion. Such would be the case if one of the alternatives in $S' - S$ happens to be Pareto-superior to the alternative in question. An alternative that was Pareto-inoptimal before the expansion cannot of course become Pareto-optimal. If an alternative is inoptimal with respect to S, then there must exist an alternative belonging to S which is Pareto-superior to it. As $S' \supset S$, there would exist a Pareto-superior alternative after the expansion as well. If the set of alternatives S contracts and becomes $S'' \subset S$, then it is possible for an alternative that was Pareto-inoptimal before the contraction to become Pareto-optimal after the contraction. Such would be the case if alternatives Pareto-superior to the alternative in question do not belong to S''. An alternative that was Pareto-optimal before the contraction would of course remain Pareto-optimal even after the contraction. Non-existence of a Pareto-superior alternative in S implies non-existence of a Pareto-superior alternative in S''. When N expands or contracts, both kinds of changes are possible. A previously Pareto-optimal alternative might become Pareto-inoptimal, or a previously Pareto-inoptimal alternative might become Pareto-optimal. The following examples illustrate these points. $\Diamond$

Example 2.2. Let $S = \{x, y, z, w, t, v\}$ and $N = \{1, 2, 3\}$.
Let the preference orderings of individuals in N be as follows:
$R_1 = vxyzwt$
$R_2 = vxzwyt$
$R_3 = xwyztv$

As every individual prefers x to all of y, z, w, t, it follows that none of y, z, w, t is Pareto-optimal. As individual 3 prefers x to v, it follows that no alternative in S is Pareto-superior to x, and consequently x is Pareto-optimal. Individual 1 prefers v to any other alternative in S; therefore it follows that no alternative in S is Pareto-superior to v implying that it is Pareto-optimal.

Now, consider a contraction of the set of alternatives to $S' = \{y, z, w, t, v\}$. As no alternative belonging to S' is Pareto-superior to any alternative belonging to $S' - \{t\}$, it follows that all of y, z, w, v are Pareto-optimal. As every individual prefers all of y, z, w to t, t is Pareto-inoptimal.

In this example when the set of alternatives expands from S' to S, (i) alternatives y, z, w Pareto-optimal with respect to S' become Pareto-inoptimal with respect to S, and (ii) t, Pareto-inoptimal alternative with respect to S', is Pareto-inoptimal with respect to S as well. When the set of alternatives contracts from S to S', (i) alternatives y, z, w Pareto-inoptimal with respect to S become Pareto-optimal with respect to S', and (ii) v, Pareto-optimal alternative with respect to S, is Pareto-optimal with respect to S' as well. $\Diamond$

Example 2.3. Let $S = \{x, y, z\}$ and $N = \{1, 2, 3\}$.
Let the preference orderings of individuals in N be as follows:
$R_1 = xyz$
$R_2 = yzx$
$R_3 = (xy)z$

As every individual prefers y to z, it follows that z is not Pareto-optimal. As individual 1 prefers x to both y and z, it follows that no alternative in S is Pareto-superior to x, and consequently x is Pareto-optimal. Individual 2 prefers y to any other alternative in S; therefore it follows that no alternative in S is Pareto-superior to y implying that it is Pareto-optimal.

Now, consider an expansion of the set of individuals to $N' = \{1, 2, 3, 4\}$ with $R_4 = zyx$. Individual 4 prefers z to any other alternative in S; therefore it follows that no alternative in S is Pareto-superior to z implying that it is Pareto-optimal. Thus, all alternatives are Pareto-optimal with respect to S and N'.

In this example when the set of individuals expands from N to N', alternative z Pareto-inoptimal with respect to N becomes Pareto-optimal with respect to N'. When the set of individuals contracts from N' to N, alternative z Pareto-optimal with respect to N' becomes Pareto-inoptimal with respect to N. $\Diamond$

Example 2.4. Let $S = \{x, y, z\}$ and $N = \{1, 2, 3\}$.
Let the preference orderings of individuals in N be as follows:
$R_1 = (xy)z$
$R_2 = z(yx)$
$R_3 = (xy)z$

As individual 2 prefers z to both x and y, it follows that no alternative in S is Pareto-superior to z, and consequently z is Pareto-optimal. Every individual is indifferent between x and y, and individual 1 prefers both x and y to z; therefore there is no alternative in S which is Pareto-superior to x or y. Thus, all alternatives in S are Pareto-optimal.

Now, consider an expansion of the set of individuals to $N' = \{1, 2, 3, 4\}$ with $R_4 = zyx$. Individual 4 prefers y to x and individuals 1,2,3 are indifferent between x and y, implying y is Pareto-superior to x. Consequently x is not Pareto-optimal with respect to S and N'.

In this example when the set of individuals expands from N to N', alternative x Pareto-optimal with respect to N becomes Pareto-inoptimal with respect to N'. When the set of individuals contracts from N' to N, alternative x Pareto-inoptimal with respect to N' becomes Pareto-optimal with respect to N. $\Diamond$

These examples make it clear that while discussing Pareto-efficiency or otherwise of alternatives, it is crucial that the set of individuals and the set of alternatives remain fixed throughout the discourse to avoid the possibility of erroneous inferences.

2.1.1 *Paretian Value Judgment*

The Pareto-criterion is a value judgment according to which, if alternative x is Pareto-superior to y then x is socially better than y, and if alternative x is Pareto-indifferent to y then x is socially as good as y. Being based on unanimity, Pareto-criterion is generally regarded as noncontroversial.[8] Because the unanimity relation Q is reflexive and transitive, it follows that the social 'at least as good as' relation induced by the Pareto-criterion is reflexive and transitive, but in general not complete. Two alternatives can be compared in terms of the Pareto-criterion if one of them is Pareto-superior to the other or if the two are Pareto-indifferent to each other; but not otherwise. In particular, if some individual prefers alternative x to alternative y and some other individual prefers y to x, then in terms of the Pareto-criterion x and y cannot be compared. Pareto-criterion implies that if an alternative

[8] It is rather important to note that merely from the fact of x being Pareto-superior to y, one cannot infer that x is socially better than y. The persuasive nature of a normative criterion cannot make a normative inference from a purely factual premise possible.

is Pareto-inefficient, then there exists an alternative which is socially better than the alternative in question. From this it follows that if the social choice has to be a best alternative,[9] i.e. an alternative at least as good as every feasible alternative, then it must be from among the subset of Pareto-efficient alternatives.

While from the Pareto-criterion it can be inferred that for every Pareto-inefficient alternative there exists a socially better alternative, one cannot infer that every Pareto-efficient alternative is better than every Pareto-inefficient alternative. In Example 2.1 we found that alternatives x, y, w are Pareto-efficient and alternative z is Pareto-inefficient. z, however, is Pareto-incomparable with x and w. So, although z is Pareto-inefficient and x and w Pareto-efficient, one cannot infer that x and w are better than z. From this it follows that a movement from a Pareto-inefficient state to a Pareto-efficient state does not imply that the society has moved to a socially better state than before.

Because the Pareto-criterion is based on unanimity, in general there will be few pairs of alternatives which would be comparable. There are two ways by which greater comparability could be attained. If one is essentially unwilling to make use of any value judgment other than the Pareto-criterion, then one way to make all pairs of alternatives comparable is to declare all Pareto-incomparable alternatives to be socially indifferent to each other. If the value judgment of the Pareto-criterion is supplemented by the value judgment that all Pareto-incomparable alternatives are equally good, then it follows that all Pareto-efficient alternatives are equally good; and every Pareto-efficient alternative is at least as good as any Pareto-inefficient alternative. Thus, all Pareto-efficient alternatives are best and none of Pareto-inefficient alternatives is best. For instance, in the case of Example 2.1, by the use of the value judgment of the Pareto-criterion supplemented by the value judgment that all Pareto-incomparable alternatives are equally good, we obtain that alternatives x, y, w are equally good; y is socially better than z; and z is socially indifferent to x and w. When Pareto-criterion is supplemented by the value judgment that all Pareto-incomparable alternatives are equally good, the social 'at least as good as' relation is no longer transitive as is clear from Example 2.1.[10]

If for comparing alternatives, which are incomparable in terms of the Pareto-criterion, one makes use of other value judgments, then there is no longer any guarantee that every Pareto-efficient alternative will be socially at least as good as every Pareto-inefficient alternative. The example given below illustrates the point.

Example 2.5. Let $N = \{1, 2, 3, 4\}$ and $S = \left\{(x_1, x_2, x_3, x_4) \mid \sum_{i \in N} x_i \leq 4\right\}$, where x_i denotes the amount of money held by individual i.
The preference orderings of individuals in N are given by:
$(\forall i \in N)\,[(x_1, x_2, x_3, x_4)\, R_i\, (x_1', x_2', x_3', x_4') \leftrightarrow x_i \geq x_i']$.
Let $a = (4, 0, 0, 0)$ and $b = (1, 1, 1, 0.9999)$. a is Pareto-efficient and b is Pareto-inefficient. Under the Pareto-criterion the two alternatives are not comparable. If one

[9] Alternative $x \in S$ is best in S according to binary relation R iff $(\forall y \in S)(xRy)$.

[10] The 'socially better than' relation, however, is transitive. See Sen (1970).

uses some criterion like justice, fairness, or equality to compare a and b, then b would be regarded as socially better than a. Thus, the use of a criterion to compare Pareto-incomparable alternatives can lead to a Pareto-inefficient alternative being regarded as socially better than a Pareto-efficient alternative. ◊

Some laws, rules, and institutions have the property that when purposive and self-regarding individuals act within their framework, the outcomes are Pareto-efficient. As a shorthand, we can term such laws, rules, and institutions themselves as Pareto-efficient. It is generally taken for granted that Pareto-efficient laws, rules, and institutions are to be preferred over those which do not have the property of invariably giving rise to Pareto-efficient outcomes. Simply on the basis of the Paretian value judgment, such a conclusion is not warranted. This is irrespective of whether Paretian value judgment is given primacy over all other value judgments or not. As we saw above, what is required to reach the conclusion that Pareto-efficient laws, rules, and institutions are to be preferred over the Pareto-inefficient ones is to have only the Paretian value judgment supplemented by the value judgment that all Pareto-incomparable alternatives are socially indifferent, to the exclusion of other value judgments.

2.2 The Kaldor Criterion and Wealth Maximization

Comparing of alternative policies in terms of the Pareto-criterion for any real society is a non-starter. As almost all policies tend to benefit some and harm some others, comparison in terms of Pareto-criterion is not possible. To surmount this difficulty Kaldor (1939) proposed that in order to determine whether a proposed change is desirable, one should find out whether those benefiting from the change are in a position to compensate those who would be losing out from the change and still be better off compared to the pre-change situation. If the gainers can compensate the losers and still be better off than before then the change is preferable; otherwise not. If gainers can compensate the losers and still be better off, then intuitively it seems that the change is wealth increasing, regardless of whether compensation is paid or not. If compensation is paid the post-compensation state is clearly Pareto-superior to the pre-change state as no one is worse off than before and the gainers are better off. But the crucial point is that even if compensation is not paid, from the fact that the gainers can compensate the losers and still be better off, it appears that there is now greater wealth than before.

Let $S(x)$ denote the set of social states accessible from social state x through compensations. Social state x is defined to be Kaldor-superior to social state y iff there exists some $z \in S(x)$ which is Pareto-superior to y. The value judgment of the Kaldor criterion declares x to be socially better than y if x is Kaldor-superior to y. On the basis of the existence of z, which can be reached through compensations from x, and under which everyone is at least as well off as under y and some people are better off, the Kaldor criterion declares x itself to be socially better than y.

The value judgment underlying the Kaldor criterion, however, is self-inconsistent. It is possible for x to be Kaldor-superior to y and y to be Kaldor-superior to x.[11] As it is not possible to have both x to be socially better than y and y to be socially better than x, the value judgment which declares an alternative to be socially better than another if the former is Kaldor-superior to the latter is incapable of being held in all cases. The example below illustrates the inconsistency of the value judgment of Kaldor-superiority implying social strict preference.

Example 2.6. Let $N = \{1, 2\}$. Let $((a_1, b_1); (a_2, b_2))$ denote the allocation in which individual 1 has a_1 amount of good 1 and b_1 amount of good 2, and individual 2 has a_2 amount of good 1 and b_2 amount of good 2. Let $S = \{x, y, z, w\}$, where:
$x = ((9, 1); (1, 4))$
$y = ((4, 1); (1, 9))$
$z = ((4, 3); (1, 7))$
$w = ((7, 1); (3, 4))$.
The preference orderings of individuals in N are given by:
$R_1 = zxwy$
$R_2 = wyzx$

x is Kaldor-superior to y as the gainer if there is a move from y to x, individual 1, can more than compensate individual 2 by giving up 2 units of good 1 to individual 2 and still be better off. Allocation w is reached from x when individual 1 transfers 2 units of good 1 to individual 2. w is preferred to y by both the individuals.

y is Kaldor-superior to x as the gainer if there is a move from x to y, individual 2, can more than compensate individual 1 by giving up 2 units of good 2 to individual 1 and still be better off. Allocation z is reached from y when individual 2 transfers 2 units of good 2 to individual 1. z is preferred to x by both the individuals. $\diamondsuit$

One way out of the inconsistency, following the Scitovsky suggestion, is to regard of the two social states x and y, x to be socially better than y if x is Kaldor-superior to y and y is not Kaldor-superior to x; y to be socially better than x if y is Kaldor-superior to x and x is not Kaldor-superior to y; and x to be socially indifferent to y if it is the case that both x is Kaldor-superior to y and y is Kaldor-superior to x or if it is the case that neither x is Kaldor-superior to y nor y is Kaldor-superior to x. It can be shown that the social indifference relation generated by this modified compensation principle is not transitive.[12]

In connection with the use of compensation principles for evaluating social states, two important points need to be noted. First, when one state is declared socially better than another state, it ought to mean that taking into consideration all relevant things from an overall viewpoint, the former is a better state for the society than the latter. In judging social states both aggregate wealth and its distribution

[11] That it is possible to have both x to be Kaldor-superior to y and y to be Kaldor-superior to x was first pointed out by Scitovsky (1941).

[12] Arrow (1951).

matter. Thus, comparing social states solely on the basis of aggregate wealth, to the exclusion of its distribution, constitutes a questionable method. Second, even if one is solely interested in aggregate wealth, and not its distribution, there is no way that the notions of magnitude of wealth and its distribution can be separated except in trivial cases.[13] The logical difficulties associated with compensation criteria are a reflection of the general impossibility of defining aggregate wealth independent of its distribution. The use of Kaldor-Hicks criterion in the law and economics literature is quite common, notwithstanding the difficulties discussed above.

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[13]The difficulties associated with the compensation principles have been elucidated by Arrow (1963, pp. 39–40) as follows:

> A matter which immediately springs to mind is the desirability of the goal which Kaldor and Hicks set for themselves, that of separating the production aspects of a desired change in social state from the distribution aspects. Any given choice is made on the basis of both considerations; even if a clear-cut meaning were given to an ordering of social states in terms of production, it would not be in the least obvious what use that ordering would be in relation to the desired ordering of social states in terms of all relevant factors, including distributional elements as well as production.
>
> But a deeper objection is that, in a world of more than one commodity, there is no unequivocal meaning to comparing total production in any two social states save in terms of some standard of value which makes the different commodities commensurable; and usually such a standard of value must depend on the distribution of income. In other words, there is no meaning to total output independent of distribution.

Chapter 3
The Structure of Efficient Liability Rules

One of the central problems of tort law is how to apportion the accident loss between the injurer and the victim and on what basis. Under the rule of strict liability the entire accident loss is borne by the injurer; and under the rule of no liability by the victim. In most accident contexts, the probability of accident as well as the quantum of loss in case of accident depend on the levels of care that the parties involved in the interaction took for the prevention of harm. The level of care which is deemed appropriate for a party is termed as the due care for that party. If a party's level of care is less than the due care level, the party in question can be regarded as at fault. One basis for apportionment of accident loss thus can be whether and to what extent the parties involved in the interaction were at fault in the sense of not having taken care at least equal to the due care level. A party is called negligent if its level of care is less than the due care; and nonnegligent otherwise.

Among the liability rules which are used in law, there are several rules which determine the liability shares of the two parties on the basis of whether the injurer's or the victim's care level was less than the due care level, and sometimes taking the extent of shortfall from the due care as well. The rules of negligence, negligence with the defence of contributory negligence, strict liability with the defence of contributory negligence, and comparative negligence are some of the most important liability rules in this category of rules. Under the negligence rule, the entire accident loss is borne by the injurer if he is negligent; and by the victim otherwise. Under the negligence with the defence of contributory negligence rule, the entire accident loss is borne by the injurer if he is negligent and the victim is nonnegligent; and by the victim otherwise. Under the rule of comparative negligence, if the injurer is negligent and the victim is nonnegligent then the entire accident loss is borne by the injurer; if both the injurer and the victim are negligent then the accident loss is apportioned between the two parties on the basis of their extents of negligence; and if the injurer is nonnegligent then the entire accident loss is borne by the victim. Under the rule of strict liability with the defence of contributory negligence,

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Table 3.1 Negligence rule

	Injurer negligent	Injurer nonnegligent
Victim negligent	Injurer bears entire loss	Victim bears entire loss
Victim nonnegligent	Injurer bears entire loss	Victim bears entire loss

Table 3.2 Negligence rule with the defence of contributory negligence

	Injurer negligent	Injurer nonnegligent
Victim negligent	Victim bears entire loss	Victim bears entire loss
Victim nonnegligent	Injurer bears entire loss	Victim bears entire loss

Table 3.3 Comparative negligence rule

	Injurer negligent	Injurer nonnegligent
Victim negligent	Loss apportioned between victim and injurer	Victim bears entire loss
Victim nonnegligent	Injurer bears entire loss	Victim bears entire loss

Table 3.4 Strict liability with the defence of contributory negligence

	Injurer negligent	Injurer nonnegligent
Victim negligent	Victim bears entire loss	Victim bears entire loss
Victim nonnegligent	Injurer bears entire loss	Injurer bears entire loss

Table 3.5 Expected loss

	Injurer does not take care	Injurer takes care
Victim does not take care	10	8
Victim takes care	8	5

the entire accident loss is borne by the victim if he is negligent; and by the injurer otherwise. In tabular form these rules can be depicted as given in Tables 3.1–3.4.

One important problem in the context of harmful interactions is to analyse how different liability rules affect the incentives of the parties to the interaction regarding taking of care and thereby affect the costs of interaction. If one ignores administrative and litigation costs, then the costs of interaction consist of cost of care by the victim, cost of care by the injurer, and the expected loss. In general, these costs will be different under different rules. Consider a simple interaction context such that taking care by either party costs 1. If neither party takes care, loss of 10 will fall on the victim; if one of the two parties takes care and the other does not, then the quantum of loss will be 8; and if both parties take care, then loss of 5 will occur. The Table 3.5 depicts expected loss for different care-configurations given above.

Costs of interaction therefore would be as given in Table 3.6.

Thus costs of interaction or social costs are minimized when both parties take care. If liability law is the rule of strict liability, then the victim will not take care.

Table 3.6 Costs of interaction

	Injurer does not take care	Injurer takes care
Victim does not take care	10	9
Victim takes care	9	7

If victim does not take care, then his costs will be zero as he will be fully compensated for his loss (which will be 10 if the injurer does not take care and 8 in case he does). On the other hand, if he takes care, his costs will be 1, as while he will be fully compensated for his loss (which will be 8 if the injurer does not take care and 5 in case he does), he will have to bear the cost of care himself. Consequently a rational victim will not take care. Knowing that the victim will not take care, the injurer will decide to take care as his costs will be 9 if he takes care (liability of 8 plus care cost of 1) and 10 if he does not (liability of 10). Thus the outcome under the strict liability rule will consist of injurer taking care and victim not taking care resulting in total costs of 9. A similar argument establishes that under the rule of no liability, the victim will take care but not the injurer, and total costs of interaction will be 9.

Unlike the rules of strict liability and no liability, the negligence rule will result in both parties taking care. If the injurer takes care, his costs will be only 1, the cost of care. On the other hand, if he does not take care, his costs will be 8 or 10 depending on whether the victim takes care or not. Therefore the injurer will take care. Knowing that the injurer will take care, the victim will decide to take care as well as his costs will be 6 if he takes care (loss of 5 plus care cost of 1) and 8 if he does not (loss of 8). Thus the outcome under the negligence rule will consist of both parties taking care resulting in total costs of 7. Analogous arguments show that under each of the rules of negligence with the defence of contributory negligence, comparative negligence, and the strict liability with the defence of contributory negligence, both parties will take care and costs of interaction will be at a minimum.

A liability rule is termed efficient if it has the property of invariably inducing the parties to take levels of care which will result in minimization of social costs. The first formal analysis of liability rules from the efficiency perspective was put forward by Brown (1973). He showed, among others, that the rule of strict liability with the defence of contributory negligence and the negligence rule induce both victims and injurers to take optimal levels of care. There is a vast literature on the efficiency of liability rules.[1] Regarding the rules generally employed in law, the main conclusion that has emerged from this literature is that while various negligence rules as well as the rule of strict liability with the defence of contributory negligence are efficient, the rules of no liability and strict liability are not.

In the literature pertaining to efficiency of liability rules, the problem has generally been considered within the framework of accidents resulting from interaction of two parties. It is assumed that, in case of occurrence of an accident, the entire loss falls, to begin with, on the victim. The probability of accident and the amount

[1] See, among others, the references cited in Footnote 8 of Chap. 1.

of loss in case of occurrence of accident are assumed to depend on the levels of care taken by the two parties. Both parties are assumed to be risk-neutral. Total costs of interaction (social costs) consist of costs of care taken by the two parties and expected accident loss. In the literature, it is generally assumed that costs of care and expected accident loss functions are such that there is a unique configuration of care levels of the two parties at which total social costs (TSC) attain their minimum. Depending on the liability rule, there could be legally binding due care levels for both parties, for only the injurer, for only the victim, or for neither party. It is assumed that in case there are legally binding due care levels for both parties, the configuration of the legally binding due care levels is identical with the unique TSC-minimizing configuration of care levels; that in case there is legally binding due care level for only the injurer, it is identical with the care level of the injurer which figures in the unique TSC-minimizing configuration of care levels; and that in case there is legally binding due care level for only the victim, it is identical with the care level of the victim which figures in the unique TSC-minimizing configuration of care levels.

In the context of liability rules, there are two conceptually distinct ways in which the notions of negligence and nonnegligence figure. One can in the first instance ask whether the behaviour of the parties is appropriate from the point of view of the objective of minimizing TSC. This is a meaningful question and can be asked regardless of the legal position regarding what constitutes negligent behaviour for each of the two parties. The technical notion of negligence or nonnegligence characterizes the parties' behaviour in terms relevant from efficiency point of view. The legal notions of negligence and nonnegligence, on the other hand, are relevant for determining the actual behaviour of the parties. It is the conjuncture of these two notions of negligence and nonnegligence which is crucial for the efficiency question.

A liability rule determines the proportions in which the two parties are to bear the loss in case of accident as a function of whether and by how much the parties' levels of care were below the due care levels. A liability rule is efficient if it induces both parties to take the TSC-minimizing care levels. In the presence of the assumption that there is a unique configuration of care levels which is TSC minimizing, the question of efficiency of a liability rule reduces to the question of whether the configuration of TSC-minimizing care levels of the two parties constitutes a unique Nash equilibrium or not.

This chapter is concerned with the derivation of conditions which are necessary and sufficient for a liability rule to be efficient. The framework within which the efficiency question is analysed here is somewhat more general than the framework described above, which can be termed as the standard tort model. In particular, no assumptions are made on the costs of care and expected loss functions, apart from postulating that they are such that the minimum of TSC exists and that a higher level of care never results in higher probability of accident or greater accident loss. The possibility that there could be more than one configuration of care levels at which TSC are minimized is not ruled out.

As mentioned above, a party is called nonnegligent if its care level is at least equal to the due care level; otherwise it is called negligent. In particular, it makes no difference whether a party's care level is equal to the due care or more than the due care. In this chapter, however, we differentiate between taking the due care and taking more than the due care. A party is called negligent if its care level is less than the due care; exactly nonnegligent if its care level is equal to the due care; and over-nonnegligent if its care level is greater than the due care. A liability rule is defined as a function which for any nonnegligence proportions of the two parties specifies how the loss is to be apportioned between the two parties in case of accident.

An analysis of the totality of all liability rules shows that the subclass of efficient liability rules is characterized by the conjunction of two conditions, namely the requirement of non-reward for over-nonnegligence (RNO) and the condition of negligence liability (NL). The requirement of non-reward for over-nonnegligence essentially requires that if one party is exactly nonnegligent, then the other party must not benefit by moving from a position of exact nonnegligence to over-nonnegligence. More precisely, what is required is that if one party is exactly nonnegligent, then the liability share of the other party when he is over-nonnegligent is greater than or equal to his liability share when he is exactly nonnegligent. The condition of negligence liability requires that if one party is exactly nonnegligent and the other party is negligent, then the entire loss in case of accident must be borne by the negligent party. Thus it is possible to have a liability rule which is efficient and punishes individuals for excessive care although none of the liability rules employed in practice do so. If one does not make a distinction between the due care and more than the due care, as indeed is the case as a general rule, then RNO is automatically satisfied; and consequently it follows that the efficient liability rules are characterized by condition NL alone.

Although as a general rule no distinction is made in tort law between the due care and more than the due care, the practice of making no such distinction does not seem to have a basis in the efficiency requirement.[2] From the necessity and sufficiency of the conjunction of RNO and NL for efficiency, it follows that there are efficient rules which make a distinction between the due care and more than the due care as well as those which do not; and also there are inefficient rules which make a distinction between the due care and more than the due care and which do not. Because of complete logical independence between efficiency on the one hand and making no distinction between the due care and more than the due care, it follows that efficiency alone cannot be an explanation of this tort law feature. The feature must, at least partly, have a non-efficiency explanation. An attempt is made here to provide an explanation, although a tentative one, of the no-distinction feature. For this purpose, a condition called monotonicity is introduced, which requires that if a party's nonnegligence proportion goes up, then his liability share must not increase, given that there is no change in the other party's nonnegligence proportion. As taking

[2]Because the usual definition of a liability rule itself incorporates the tort law feature of making no distinction between the due care and more than the due care, this feature is not subjected to scrutiny in the course of analysing liability rules. In particular, the question whether this feature of tort law has anything to do with efficiency can be asked only by defining the notion of a liability rule in a more general way than the standard way of defining it.

greater care never results in greater expected accident loss, fairness would seem to require that a party taking greater care should not be penalized. Therefore it can be argued that monotonicity condition is a formalization of at least one important aspect of fairness. It is shown here that every monotonic liability rule which is efficient must be such that it makes no distinction between the due care and more than the due care. Because monotonicity, by itself, like efficiency, is unrelated to the no-distinction feature, in view of the result linking efficient monotonic liability rules with no-distinction feature, it seems appropriate to claim that the no-distinction feature is grounded partly in efficiency and partly in fairness.

The chapter is divided into three sections. Section 3.1 lays down the framework of analysis. This framework will be used throughout this text. Section 3.2 discusses the theorem stating that a liability rule is efficient iff it satisfies the requirement of non-reward for over-nonnegligence and the condition of negligence liability. The proof of the characterization theorem is given in the appendix to the chapter. The last section contains a discussion of the interrelationships among efficiency, monotonicity, and the no distinction between due care and more than due care feature.

3.1 The Framework

In this chapter, as well as in most of the chapters that follow, we consider accidents resulting from interaction of two parties. The parties will be assumed to be strangers to each other.[3] It will be assumed that, to begin with, the entire accident loss falls on one party to be called the victim; the other party would be referred to as the injurer. We denote by $c \geq 0$ the cost of care taken by the victim; and by $d \geq 0$ the cost of care taken by the injurer. We assume that c and d are strictly increasing functions of levels of care of the two parties. This of course implies that c and d themselves can be taken as indices of levels of care of the victim and the injurer, respectively.

Let

$C = \{c \mid c$ is the cost of some feasible level of care which can be taken by the victim$\}$

and

$D = \{d \mid d$ is the cost of some feasible level of care which can be taken by the injurer$\}$.

We assume $0 \in C \wedge 0 \in D$. (A1)

$c = 0$ will be identified as no care by the victim; and $d = 0$ as no care by the injurer. Assumption (A1) merely says that taking no care is always a feasible option for both parties.

[3]This assumption ensures that the transaction costs of negotiating an agreement are prohibitively high.

Let π denote the probability of occurrence of accident and $H \geq 0$ the loss in case of occurrence of accident. Both π and H will be assumed to be functions of c and d; $\pi = \pi(c, d), H = H(c, d)$. Let $L = \pi H$. L is thus the expected loss due to accident.
We assume:

$$(\forall c, c' \in C)\,(\forall d, d' \in D)\,[[c > c' \;\rightarrow\; \pi(c,d) \leq \pi\,(c', d)] \;\wedge [d > d' \rightarrow\; \pi(c,d) \leq \pi\,(c, d')]] \tag{A2}$$

and

$$(\forall c, c' \in C)\,(\forall d, d' \in D)\,[[c > c' \;\rightarrow\; H(c,d) \leq H\,(c', d)] \wedge [d > d' \;\rightarrow\; H(c,d) \leq H\,(c, d')]]\,. \tag{A3}$$

In other words, it is assumed that a larger expenditure on care by either party, given the expenditure on care by the other party, does not result in higher probability of occurrence of accident or in larger accident loss.
From (A2) and (A3) it follows that:
$(\forall c, c' \in C)\,(\forall d, d' \in D)\,[[c > c' \;\rightarrow\; L(c,d) \leq L\,(c', d)] \wedge [d > d' \;\rightarrow\; L(c,d) \leq L\,(c, d')]]\,.$
In other words, a larger expenditure on care by either party, given the expenditure on care by the other party, results in lesser or equal expected accident loss.[4]

Total social costs (TSC) are defined to be the sum of cost of care by the victim, cost of care by the injurer, and the expected loss due to accident; $\text{TSC} = c + d + L(c, d)$. Let $M = \{(c', d') \in C \times D \mid c' + d' + L\,(c', d')$ is minimum of $\{c + d + L(c, d) \mid c \in C \;\wedge\; d \in D\}\}$. Thus M is the set of all costs of care configurations (c', d') which are TSC minimizing. It will be assumed that:

$$C, D, \pi \text{, and } H \text{ are such that } M \text{ is nonempty.} \tag{A4}$$

In order to characterize a party's level of care as negligent or otherwise, a reference point (the due care level) for the party needs to be specified. Let c^* and d^*, where $(c^*, d^*) \in M$, denote the due care levels of the victim and the injurer, respectively. We define nonnegligence functions p and q as follows:

$p : C \mapsto [0, \infty)$ such that
$$p(c) = \frac{c}{c^*} \text{ if } c^* > 0;$$
$$= 1 \;\text{ if } c^* = 0$$

---

[4] L, in general, is not strictly decreasing in c and d. L may become zero for sufficiently high values of c and d; then increasing them beyond these levels will not bring about any further decrease in L. In some cases of complementarities between cares of the two parties, L may be such that at a particular combination of care levels of the two parties a reduction in L can be brought about only if both care levels are increased.

$q : D \mapsto [0, \infty)$ such that:

$$q(d) = \frac{d}{d^*} \text{ if } d^* > 0;$$
$$= 1 \text{ if } d^* = 0.$$

Remark 3.1. Instead of defining $p(c) = 1$ for all $c \in C$ when $c^* = 0$, one could also define it in any other way subject to the following two restrictions without affecting any of the results:

(i) $p(c^*) = 1$
(ii) p is an increasing function of c, i.e. $(\forall c, c' \in C)\, [c > c' \;\rightarrow\; p(c) \geq p(c')]$.

Analogous remarks apply for function q when $d^* = 0$. ◊

p and q would be interpreted as proportions of nonnegligence of the victim and the injurer, respectively. The victim would be called negligent if $p < 1$, exactly nonnegligent if $p = 1$, and over-nonnegligent if $p > 1$. Similarly, the injurer would be called negligent if $q < 1$, exactly nonnegligent if $q = 1$, and over-nonnegligent if $q > 1$.

In case there is a legally binding due care level for the victim, it would be taken to be identical with c^* figuring in the definition of function p; and in case there is a legally binding due care level for the injurer, it would be taken to be identical with d^* figuring in the definition of function q. Knowledge of C, D, π, H, and the legally specified due care levels will be assumed to be part of common knowledge.

Remark 3.2. Liability rules can be divided into four mutually exclusive and exhaustive subclasses: (i) the rules with legally binding due care levels for both the victim and the injurer, (ii) the rules with legally binding due care level for only the injurer, (iii) the rules with legally binding due care level for only the victim, and (iv) the rules with legally binding due care levels for neither party. The rule of negligence with the defence of contributory negligence, the rule of negligence, the rule of strict liability with the defence of contributory negligence, and the rule of strict liability provide examples, respectively, of these four categories of liability rules. If the liability rule belongs to subclass (i), c^* and d^* used in the definitions of functions p and q will be taken to be identical with the legally binding due care levels for the victim and the injurer, respectively. If the liability rule belongs to subclass (ii), d^* used in the definition of function q will be taken to be identical with the legally binding due care level for the injurer. c^* used in the definition of function p can be any element of $\{c \in C \mid (c, d^*) \in M\}$. $\{c \in C \mid (c, d^*) \in M\}$ may contain more than one element. If the liability rule belongs to subclass (iii), c^* used in the definition of function p will be taken to be identical with the legally binding due care level for the victim. d^* used in the definition of function q can be any element of $\{d \in D \mid (c^*, d) \in M\}$. $\{d \in D \mid (c^*, d) \in M\}$ may contain more than one element. If the liability rule belongs to subclass (iv), then any element of M can be used for the purpose of defining functions p and q. Thus in all cases c^* would denote the legally binding due care level for the victim whenever the idea of legally binding due care level for the victim is applicable; and d^* would denote the legally binding due care level for the injurer whenever the idea of legally binding due care level for the injurer is applicable. Thus implicitly it is being assumed that the legally binding due care levels are set appropriately from the point of view of minimizing TSC.[5] $\Diamond$

We now proceed to define formally the notion of a liability rule. The way the notion of a liability rule is defined here is more general than the one employed in the law and economics literature. From a technical point of view, it is desirable to define the notion of a liability rule independently of its applications. The context in which a liability rule can be applied is completely specified if in addition to $C, D, \pi,$ and H, we also specify the configuration of due care levels $(c^*, d^*) \in M$. The set of all applications $< C, D, \pi, H, (c^*, d^*) \in M >$ satisfying (A1)–(A4) will be denoted by $\mathscr{A}$.[6]

Formally, a liability rule is a function f from $[0, \infty)^2$ to $[0, 1]^2$, $f : [0, \infty)^2 \mapsto [0, 1]^2$, such that: $f(p, q) = (x, y)$, where $x + y = 1$.

Thus, a liability rule is a rule which specifies the proportions in which the two parties are to bear the loss in case of occurrence of accident as a function of proportions of nonnegligence of the two parties.

Remark 3.3. In the law and economics literature, the notion of a liability rule is usually defined as a function which for any levels of care of the two parties and their due care levels assigns the liability shares of the parties. In view of the fact that the costs of care are usually taken to be indices of levels of care, a liability rule then becomes a function of costs of care of the two parties and the costs of their due care levels. That is to say, in terms of the notation of this book, a liability rule is a function which for any c, d, c^*, d^* determines the liability shares of the two parties.

In the absence of further restrictions, this way of defining liability rules results in the notion of a liability rule becoming ambiguous. In the absence of any restrictions, there is no guarantee that one would have $f(mc, md, mc^*, md^*) = f(c, d, c^*, d^*)$, for $m > 0$. Let u and u' be two different monetary units such that $mu = u', m > 0$. If $f(mc, md, mc^*, md^*) \neq f(c, d, c^*, d^*)$, then the liability shares determined by the rule would be different depending on whether the problem is described in terms of monetary unit u or monetary unit u'.

Thus it is clear that for the notion of a liability rule to be unambiguous, it is necessary that it must satisfy the requirement:

$$(\forall m > 0) \left[f\left(mc, md, mc^*, md^*\right) = f\left(c, d, c^*, d^*\right) \right]. \tag{R1}$$

In fact, for the definition of a liability rule to be unambiguous, a requirement more stringent than (R1) is needed. The requirement is:

$$(\forall m, n > 0) \left[f\left(mc, nd, mc^*, nd^*\right) = f\left(c, d, c^*, d^*\right) \right]. \tag{R2}$$

[5]This is a standard assumption and is crucial for the results on the efficiency of liability rules.

[6]In this and subsequent chapters, it will be assumed that the activity levels of both the parties are fixed. Shavell (1980) has shown that there does not exist any liability rule which is efficient if both activity and care levels can be varied.

The more stringent requirement is needed as the same underlying problem can be described by using different monetary units for the two parties. Accepting (R2) is essentially equivalent to viewing the notion of a liability rule as a function of proportions $\frac{c}{c^*}$ and $\frac{d}{d^*}$, $c^* \neq 0, d^* \neq 0$. $\lozenge$

Let f be a liability rule. Then, for any proportions of nonnegligence (p, q) by the two parties, f assigns the proportions (x, y) in which the loss is to be borne by the parties in case of occurrence of accident. An application of the liability rule consists of specification of C, D, π, H, and $(c^*, d^*) \in M$. Once C, D, π, H, and $(c^*, d^*) \in M$ have been specified, for any configuration of costs of care (c, d) by the two parties, proportions of nonnegligence p and q are uniquely determined. The liability rule then uniquely determines the liability proportions (x, y) corresponding to (p, q).

Let f be a liability rule. Consider a particular application $< C, D, \pi, H, (c^*, d^*) \in M >$ of the rule. If accident takes place and loss of $H(c, d)$ materializes, then $xH(c, d)$ will be borne by the victim and $yH(c, d)$ by the injurer, where $(x, y) = f(p, q) = f[p(c), q(d)]$. As, to begin with, in case of occurrence of accident, the entire loss falls upon the victim, $yH(c, d)$ represents the liability payment by the injurer to the victim. The expected costs of the victim and the injurer, to be denoted by EC_1 and EC_2, respectively, therefore are $c + xL(c, d)$ and $d + yL(c, d)$, respectively. Both parties are assumed to prefer smaller expected costs to larger expected costs and be indifferent between alternatives with equal expected costs.

Let $f : [0, \infty)^2 \mapsto [0, 1]^2$ be a liability rule. f is defined to be efficient for a given application $< C, D, \pi, H, (c^*, d^*) \in M >$ belonging to $\mathscr{A}$ iff $(\forall(\overline{c}, \overline{d}) \in C \times D)[(\overline{c}, \overline{d})$ is a Nash equilibrium $\rightarrow (\overline{c}, \overline{d}) \in M]$ and $(\exists(\overline{c}, \overline{d}) \in C \times D)[(\overline{c}, \overline{d})$ is a Nash equilibrium]. In other words, a liability rule is efficient for a particular application satisfying (A1)–(A4) iff (i) every $(\overline{c}, \overline{d}) \in C \times D$ which is a Nash equilibrium is TSC minimizing, and (ii) there exists at least one $(\overline{c}, \overline{d}) \in C \times D$ which is a Nash equilibrium. A liability rule is defined to be efficient with respect to a class of applications iff it is efficient for every application belonging to that class.

Remark 3.4. When one considers the class of all liability rules, with respect to Nash equilibria, all possibilities are open as the following examples show. $\lozenge$

Example 3.1. Consider the liability rule f which requires equal sharing of losses regardless of care levels of the two parties, i.e., the rule given by:
$(\forall p \in [0, \infty))(\forall q \in [0, \infty)) \left[f(p, q) = \left(\frac{1}{2}, \frac{1}{2}\right)\right]$.
Consider the application given below.
$C = \{0, 1\}, D = \{0, 1\}$, and $L(c, d)$ for $(c, d) \in C \times D$ is given by:

$L(c, d)$

	$d = 0$	$d = 1$
$c = 0$	10.0	8.2
$c = 1$	8.2	5.0

$(1,1)$ is the unique TSC-minimizing configuration of care levels.
Let $(c^*, d^*) = (1,1)$.
$(EC_1(c,d), EC_2(c,d))$ for $(c,d) \in C \times D$ is given by:

$(EC_1(c,d), EC_2(c,d))$

	$d=0$	$d=1$
$c=0$	$(\underline{5.0}, \underline{5.0})$	$(4.1, 5.1)$
$c=1$	$(5.1, 4.1)$	$(\underline{3.5}, \underline{3.5})$

Here both $(0,0)$ and $(1,1)$ are Nash equilibria. While $(1,1)$ is TSC minimizing, $(0,0)$ is not, and thus the rule is inefficient for the given application. $\Diamond$

Example 3.2. Consider the rule of no liability, i.e., the rule f given by: $(\forall p \in [0,\infty))(\forall q \in [0,\infty))[f(p,q) = (1,0)]$.
Consider the application given below.
$C = \{0,1\}, D = \{0,1\}$, and $L(c,d)$ for $(c,d) \in C \times D$ is given by:

$L(c,d)$

	$d=0$	$d=1$
$c=0$	10	5
$c=1$	5	0

$(1,1)$ is the unique TSC-minimizing configuration of care levels.
Let $(c^*, d^*) = (1,1)$.
$(EC_1(c,d), EC_2(c,d))$ for $(c,d) \in C \times D$ is given by:

$(EC_1(c,d), EC_2(c,d))$

	$d=0$	$d=1$
$c=0$	$(10, \underline{0})$	$(5, 1)$
$c=1$	$(\underline{6}, \underline{0})$	$(\underline{1}, 1)$

Here $(1,0)$ is the unique Nash equilibrium. As $(1,0)$ is not TSC minimizing, the rule is inefficient for the given application. $\Diamond$

Example 3.3. Consider the negligence rule f given by:
$(\forall p \in [0,\infty))(\forall q \in [0,1))[f(p,q) = (0,1)] \wedge (\forall p \in [0,\infty))(\forall q \in [1,\infty))$ $[f(p,q) = (1,0)]$.
Consider the application given below.
$C = \{0,1\}, D = \{0,1\}$, and $L(c,d)$ for $(c,d) \in C \times D$ is given by:

$L(c,d)$

	$d=0$	$d=1$
$c=0$	10	7
$c=1$	6	0

$(1,1)$ is the unique TSC-minimizing configuration of care levels.
Let $(c^*, d^*) = (1,1)$.
$(EC_1(c,d), EC_2(c,d))$ for $(c,d) \in C \times D$ is given by:

$$(EC_1(c,d), EC_2(c,d))$$

	$d=0$	$d=1$
$c=0$	$(\underline{0}, 10)$	$(7, \underline{1})$
$c=1$	$(1, 6)$	$(\underline{1}, \underline{1})$

Here $(1,1)$ is the unique Nash equilibrium. Thus the rule is efficient for the given application. $\lozenge$

Example 3.4. Consider the rule f given by:
$(\forall p \in [0,1))(\forall q \in [0,1)) \left[f(p,q) = \left(\frac{1}{2}, \frac{1}{2}\right)\right] \wedge (\forall p \in [0,1))(\forall q \in [1,\infty)) \left[f(p,q) = \left(\frac{3}{17}, \frac{14}{17}\right)\right] \wedge (\forall p \in [1,\infty))(\forall q \in [0,1)) \left[f(p,q) = \left(\frac{2}{17}, \frac{15}{17}\right)\right] \wedge (\forall p \in [1,\infty))(\forall q \in [1,\infty)) \left[f(p,q) = \left(\frac{2}{7}, \frac{5}{7}\right)\right]$.
Consider the application given below.
$C = \{0,1\}, D = \{0,1\}$, and $L(c,d)$ for $(c,d) \in C \times D$ is given by:

$$L(c,d)$$

	$d=0$	$d=1$
$c=0$	10.0	8.5
$c=1$	8.5	7.0

$(1,1)$ is the unique TSC-minimizing configuration of care levels.
Let $(c^*, d^*) = (1,1)$.
$(EC_1(c,d), EC_2(c,d))$ for $(c,d) \in C \times D$ is given by:

$$(EC_1(c,d), EC_2(c,d))$$

	$d=0$	$d=1$
$c=0$	$(5, \underline{5})$	$(\underline{1.5}, 8)$
$c=1$	$(\underline{2}, 7.5)$	$(3, \underline{6})$

Here there is no $(c,d) \in C \times D$ which is a Nash equilibrium. Thus the rule is inefficient for the given application. $\lozenge$

Throughout this book we denote $f(1,1)$ by (x^*, y^*), i.e., we write $x(1,1)$ as x^* and $y(1,1)$ as y^*. We also denote $\pi(c^*, d^*), H(c^*, d^*)$, and $L(c^*, d^*)$ by π^*, H^*, and L^*, respectively.

3.2 The Characterization of Efficient Liability Rules

First, we define two conditions on liability rules.

Condition of Negligence Liability (NL): A liability rule f satisfies the condition of negligence liability iff $[[\forall p \in [0, 1)][f(p, 1) = (1, 0)] \ \wedge \ [\forall q \in [0, 1)][f(1, q) = (0, 1)]]$.

In other words, a liability rule satisfies the condition of negligence liability iff its structure is such that (i) whenever the injurer is exactly nonnegligent and the victim is negligent, the entire loss in case of occurrence of accident is borne by the victim, and (ii) whenever the victim is exactly nonnegligent and the injurer is negligent, the entire loss in case of occurrence of accident is borne by the injurer.

Requirement of Non-Reward for Over-Nonnegligence (RNO): A liability rule f satisfies the requirement of non-reward for over-nonnegligence iff $[[\forall p \in (1, \infty)][x(p, 1) \geq x(1, 1)] \ \wedge \ [\forall q \in (1, \infty)][y(1, q) \geq y(1, 1)]]$.

That is to say, a liability rule satisfies the requirement of non-reward for over-nonnegligence iff its structure is such that (i) given that the injurer is exactly nonnegligent, the liability share of the victim when he is over-nonnegligent is greater than or equal to his liability share when he is exactly nonnegligent, and (ii) given that the victim is exactly nonnegligent, the liability share of the injurer when he is over-nonnegligent is greater than or equal to his liability share when he is exactly nonnegligent.

The efficient liability rules are characterized by the conjunction of the two conditions defined above. The following is the formal statement of the theorem.

Theorem 3.1. *A liability rule $f : [0, \infty)^2 \mapsto [0, 1]^2$ is efficient with respect to $\mathscr{A}$ iff it satisfies the requirement of non-reward for over-nonnegligence and the condition of negligence liability.*

The characterization theorem is proved via four propositions whose statements and proofs are given in the appendix at the end of the chapter. Here we discuss the logic of the propositions and the characterization theorem informally. Proposition 3.1 establishes that if a liability rule satisfies both RNO and NL, then regardless of which application belonging to $\mathscr{A}$ is considered, $(c^*, d^*) \in M$ constitutes a Nash equilibrium. An intuitive explanation of Proposition 3.1 is as follows. Suppose the injurer is using d^*. Consider a change by the victim from c^* to some $c > c^*$. Now, the reduction in expected loss $L(c^*, d^*) - L(c, d^*)$ cannot exceed the increase in cost of care $c - c^*$; otherwise (c^*, d^*) could not have been a TSC-minimizing configuration. By RNO, the shift from c^* to c, $c > c^*$, cannot result in a decrease in the liability share x of the victim. Consequently, a change from c^* to c, $c > c^*$, cannot be advantageous for the victim. Next consider a change from c^* to some $c < c^*$. The increase in expected loss $L(c, d^*) - L(c^*, d^*)$ must be at least as large as the decrease in cost of care $c^* - c$; otherwise (c^*, d^*) could not have been TSC minimizing. By condition NL, the entire increase in expected loss must be borne by the victim. Consequently a shift from c^* to c, $c < c^*$, can never be advantageous. Thus, given the choice of d^* by the injurer, c^* is best for the victim. By an analogous argument, one shows that, given the choice of c^* by the victim, d^* is best for the injurer.

Proposition 3.2 establishes that regardless of which application belonging to $\mathscr{A}$ of a liability rule satisfying RNO and NL is considered, all $(\overline{c}, \overline{d}) \in C \times D$ which are Nash equilibria are TSC minimizing. The argument underlying the proof of the proposition can be put as follows. Let $(\overline{c}, \overline{d})$ be a Nash equilibrium. First, we note that if $\overline{d} < d^*$, then $x[p(c^*), q(\overline{d})] = 0$ by condition NL; and if $\overline{d} > d^*$, then $x[p(c^*), q(\overline{d})] \leq x^*$ by condition RNO. Furthermore if $\overline{d} > d^*$, then $L[c^*, \overline{d}] \leq L[c^*, d^*]$. Therefore regardless of the value of d, we have $x[p(c^*), q(\overline{d})]L[c^*, \overline{d}] \leq x^* L[c^*, d^*]$. As $(\overline{c}, \overline{d})$ is a Nash equilibrium, it must be the case that $EC_1(\overline{c}, \overline{d}) \leq EC_1(c^*, \overline{d})$. Consequently we have $EC_1(\overline{c}, \overline{d}) \leq c^* + x[p(c^*), q(\overline{d})]L[c^*, \overline{d}] \leq c^* + x^* L[c^*, d^*]$. Thus, a consequence of conditions NL and RNO and the hypothesis that $(\overline{c}, \overline{d})$ is a Nash equilibrium is that the expected costs of the victim at (c^*, d^*) must be at least as large as expected costs at $(\overline{c}, \overline{d})$. By an analogous argument, it follows that the expected costs of the injurer at (c^*, d^*) must be at least as large as expected costs at $(\overline{c}, \overline{d})$. As TSC are sum of expected costs of the two parties, it immediately follows that $(\overline{c}, \overline{d})$ is TSC minimizing.

Propositions 3.1 and 3.2 together thus show that a liability rule satisfying RNO and NL is efficient for every application belonging to $\mathscr{A}$ and therefore establish the sufficiency of RNO and NL for efficiency with respect to $\mathscr{A}$. Propositions 3.1 and 3.2 together, in fact, establish more than the sufficiency part. To establish sufficiency part, we need, for every application belonging to $\mathscr{A}$, only (i) all $(\overline{c}, \overline{d}) \in C \times D$ which are Nash equilibria to be TSC minimizing and (ii) the existence of a $(\overline{c}, \overline{d}) \in C \times D$ which is a Nash equilibrium. Rather than merely showing the existence of a $(\overline{c}, \overline{d}) \in C \times D$ which is a Nash equilibrium, Proposition 3.1 establishes a much stronger result, namely that the configuration of due care levels is always a Nash equilibrium.

Proposition 3.3 establishes that RNO is a necessary condition for efficiency with respect to $\mathscr{A}$ of any liability rule. That is to say, if a liability rule violates RNO, then the rule must be inefficient for some application belonging to $\mathscr{A}$. Suppose the liability rule f is such that when one party is exactly nonnegligent (say the victim) and the other party (the injurer) is over-nonnegligent, the liability assignments are such that the over-nonnegligent party's (the injurer's) liability share is less than its liability share when both parties are exactly nonnegligent, i.e. for some $q' > 1, f(1, q') = (x_{q'}, y_{q'})$, where $y_{q'} < y^*$. Consider an application of f belonging to $\mathscr{A}$ such that (i) TSC-minimizing configuration is unique and both care levels figuring in it are positive; (ii) when both parties take optimal care, then the expected loss is positive; and (iii) $q'd^*$ is an element of D. Given that the victim is using c^*, if we consider a shift by the injurer from d^* to $q'd^*$, then the decrease in expected loss $L(c^*, d^*) - L(c^*, q'd^*)$ must be less than the increase in the cost of care $(q' - 1)d^*$. Although the entire increase in the cost of care is borne by the injurer, he can still be better off as he is now liable for a smaller share of the loss. In other words, one can always find an application such that, in addition to (i)–(iii), $y^*L(c^*, d^*) - y_{q'}L(c^*, q'd^*) > (q' - 1)d^*$. This establishes the existence of an application for which the unique TSC-minimizing configuration is not a Nash equilibrium and consequently the existence of an application for which the rule is inefficient.

Proposition 3.4 establishes that NL is a necessary condition for efficiency with respect to $\mathscr{A}$ of any liability rule. That is to say, if a liability rule violates condition NL, then the rule must be inefficient for some application belonging to $\mathscr{A}$. Suppose the liability rule f is such that when one party is exactly nonnegligent (say the victim) and the other party (the injurer) is negligent, the liability assignments are such that the negligent party (the injurer) is not required to bear the entire loss, i.e., for some $q' < 1, f(1, q') = (x_{q'}, y_{q'})$, where $y_{q'} < 1$. Consider an application of f belonging to $\mathscr{A}$ such that (i) TSC-minimizing configuration is unique and both care levels figuring in it are positive; (ii) if both parties take optimal care, then the expected loss is zero; and (iii) $q'd^*$ is an element of D. Given that the victim is using c^*, if we consider a shift by the injurer from d^* to $q'd^*$, then the increase in expected loss $L(c^*, q'd^*) - L(c^*, d^*) = L(c^*, q'd^*)$ must be greater than the reduction in the cost of care $(1 - q')d^*$. As $y_{q'} < 1$, the injurer bears only a part of the increase in expected loss. On the other hand, the entire decrease in cost of care accrues to the injurer. Therefore, it follows that one can always find an application such that, in addition to (i)–(iii), $L(c^*, q'd^*) > (1 - q')d^* > y_{q'}L(c^*, q'd^*)$. This establishes the existence of an application for which the unique TSC-minimizing configuration is not a Nash equilibrium and consequently the existence of an application for which the rule is inefficient.

Thus Propositions 3.3 and 3.4 together establish the necessity of conjunction of RNO and NL for efficiency with respect to $\mathscr{A}$. The theorem stating that the efficient liability rules are characterized by the conjunction of RNO and NL therefore follows immediately from Propositions 3.1 to 3.4.

A remarkable aspect of the characterizing conditions should be noted. Both RNO and NL put restrictions on the assignment of liability shares when one party is exactly nonnegligent and the other party is not. Neither of the two conditions constrains in any way if neither party is exactly nonnegligent or if both parties are exactly nonnegligent. In particular if one party is negligent and the other party is over-nonnegligent, then neither condition constrains liability assignments in any way whatsoever. Consequently it follows that it is possible for a liability rule to be efficient and at the same time exhibit rather perverse features. Consider the following example.

Example 3.5. Let the liability rule $f : [0,\infty)^2 \mapsto [0,1]^2$ be defined by:

$$
\begin{aligned}
f(p,q) &= \left(\frac{1}{2},\frac{1}{2}\right) \text{ if } p < 1 \wedge q < 1;\\
&= \ (1,0) \quad \text{if } p < 1 \wedge q = 1;\\
&= \ (0,1) \quad \text{if } p < 1 \wedge q > 1;\\
&= \ (0,1) \quad \text{if } p = 1 \wedge q < 1;\\
&= \left(\frac{1}{2},\frac{1}{2}\right) \text{ if } p = 1 \wedge q = 1;\\
&= \left(\frac{1}{2},\frac{1}{2}\right) \text{ if } p = 1 \wedge q > 1;\\
&= \ (1,0) \quad \text{if } p > 1 \wedge q < 1;\\
&= \left(\frac{1}{2},\frac{1}{2}\right) \text{ if } p > 1 \wedge q = 1;\\
&= \left(\frac{1}{2},\frac{1}{2}\right) \text{ if } p > 1 \wedge q > 1.
\end{aligned}
$$

This liability rule is efficient as it satisfies both RNO and NL. The perverse feature of the rule lies in the fact that when one party is negligent and the other is over-nonnegligent, the entire liability falls on the over-nonnegligent party. $\diamondsuit$

In the law and economics literature relating to liability rules, it is generally assumed that costs of care and expected loss functions are such that there is a unique (c,d) configuration at which TSC are minimized, i.e. M consists of a single element. Let $\mathscr{A}' \subset \mathscr{A}$ be the set of applications that satisfy (A1)–(A4) and are such that M is a singleton. The next theorem establishes that the characterization of efficient liability rules remains unchanged even if efficiency is considered with respect to $\mathscr{A}'$ rather than $\mathscr{A}$.

Theorem 3.2. *A liability rule $f : [0,\infty)^2 \mapsto [0,1]^2$ is efficient with respect to $\mathscr{A}'$ iff it satisfies the requirement of non-reward for over-nonnegligence and the condition of negligence liability.*

Proof. As $\mathscr{A}' \subset \mathscr{A}$, sufficiency of conjunction of NL and RNO follows from Theorem 3.1. The necessity of RNO follows from the proof of Proposition 3.3 in view of the fact that the application considered there, in addition to satisfying (A1)–(A4), is such that M is a singleton. The necessity of NL follows from the proof of Proposition 3.4 as the application considered there also, in addition to satisfying (A1)–(A4), is such that M is a singleton. $\square$

Remark 3.5. Let $\mathscr{B} \subset \mathscr{A}$ and $\mathscr{B}' \subset \mathscr{A}$ be two sets of applications such that $\mathscr{B}' \subset \mathscr{B}$. Suppose some condition α is a sufficient condition for efficiency with respect to $\mathscr{B}$. This means that if a liability rule satisfies α, then for every application belonging to $\mathscr{B}$, the outcome that would result under it would be efficient. Given $\mathscr{B}' \subset \mathscr{B}$, it then follows that if a liability rule satisfies α, then efficiency would obtain for every application belonging to $\mathscr{B}'$. Thus whenever a condition is sufficient for efficiency with respect to a particular set of applications $\mathscr{B}$, it would also constitute a sufficient condition for efficiency with respect to any nonempty set of applications which is a subset of $\mathscr{B}$. If α is a necessary condition for efficiency with respect to $\mathscr{B}$, then if a liability rule violates α, there must exist an application belonging to $\mathscr{B}$ for which the rule in question would not give rise to an efficient outcome. But it is possible that although the rule does not give rise to efficient outcomes for every application belonging to $\mathscr{B}$, it gives rise to an efficient outcome for every application belonging to $\mathscr{B}'$. Thus a condition necessary for efficiency with respect to a set of applications $\mathscr{B}$ need not be necessary for efficiency with respect to a set of applications that is a subset of $\mathscr{B}$. From these considerations it follows that Theorem 3.2 is independent of Theorem 3.1. Furthermore, although the conjunction of RNO and NL is both necessary and sufficient with respect to $\mathscr{A}$ as well as with respect to that subset of $\mathscr{A}$ wherein M is always a singleton, for particular subsets of applications, conditions weaker than the conjunction of RNO and NL may emerge as necessary and sufficient for efficiency. $\Diamond$

3.2.1 Unilateral Care

As a liability rule is inefficient with respect to a class of applications iff it is inefficient for at least one application belonging to that class, it follows that a rule which is inefficient with respect to a class of applications may be efficient with respect to a proper subclass of that class. Let
$\mathscr{A}_1^0 = \{< C, D, \pi, H, (c^*, d^*) \in M > \in \mathscr{A} \mid c^* = 0\}$;
$\mathscr{A}_2^0 = \{< C, D, \pi, H, (c^*, d^*) \in M > \in \mathscr{A} \mid d^* = 0\}$.
In other words, $\mathscr{A}_1^0 \subset \mathscr{A}$ is that subclass of applications belonging to $\mathscr{A}$ such that $c^* = 0$; and $\mathscr{A}_2^0 \subset \mathscr{A}$ is that subclass of applications belonging to $\mathscr{A}$ such that $d^* = 0$. Thus, $\mathscr{A}_1^0$ is the set of applications where only care by injurer is required for minimization of social costs; and $\mathscr{A}_2^0$ is the set of applications where only care by victim is required for minimization of social costs.

If $c^* = 0$, then we have $(\forall c \in C)[p(c) = 1]$. Consequently, $[\forall p \in [0,1)][f(p,1) = (1,0)]$ part of the NL condition and $[\forall p \in (1,\infty)][x(p,1) \geq x(1,1)]$ part of the RNO condition are trivially satisfied.[7] Therefore the set of liability rules efficient with respect to $\mathscr{A}_1^0$ is given by the conjunction of $[\forall q \in [0,1)][f(1,q) = (0,1)]$ and $[\forall q \in (1,\infty)][y(1,q) \geq y(1,1)]$. Similarly, if $d^* = 0$, then we have $(\forall d \in D)[q(d) = 1]$; and consequently $[\forall q \in [0,1)][f(1,q) = (0,1)]$ part of NL and $[\forall q \in (1,\infty)][y(1,q) \geq y(1,1)]$ part of RNO are trivially satisfied. Therefore the set of liability rules efficient with respect to $\mathscr{A}_2^0$ is given by the

[7] A conditional is true if its antecedent is false.

conjunction of $[\forall p \in [0,1)][f(p,1) = (1,0)]$ and $[\forall p \in (1,\infty)][x(p,1) \geq x(1,1)]$. From the above it is also clear that every liability rule is efficient with respect to $\mathscr{A}_1^0 \cap \mathscr{A}_2^0$.

3.3 The No Distinction Between the Due Care and More Than the Due Care Feature, Monotonicity, and Efficiency

One of the basic features of tort law is that for the purpose of assigning liability shares, as a general rule, no distinction is made between the due care and more than the due care. There are two different ways in which this characteristic feature of tort law can be incorporated in the analysis of liability rules. One can either treat the no-distinction feature as a condition on liability rules or the feature can be incorporated in the definition of a liability rule itself. The no-distinction feature of tort law as a condition on liability rules can be stated as follows.

The no distinction between the due care and more than the due care requirement (NDMR): A liability rule f satisfies the no distinction between the due care and more than the due care requirement iff $[\forall p, q \in [0,\infty)][[p \geq 1 \rightarrow f(p,q) = f(1,q)] \;\wedge\; [q \geq 1 \rightarrow f(p,q) = f(p,1)]]$.

It is immediate that any liability rule which satisfies NDMR would also satisfy RNO. Therefore, it follows, in view of the theorem that the efficient liability rules are characterized by the conjunction of NL and RNO, that for the subclass of liability rules satisfying NDMR, a necessary and sufficient condition for efficiency is that NL holds. We formally state the result as a theorem.

Theorem 3.3. *Let liability rule $f : [0,\infty)^2 \mapsto [0,1]^2$ belong to the subclass of liability rules satisfying the no distinction between the due care and more than the due care requirement. Then, a necessary and sufficient condition for f to be efficient with respect to $\mathscr{A}$ is that it satisfy the condition of negligence liability.*

If the no distinction between the due care and more than the due care requirement is to be incorporated in the definition of a liability rule itself, then the most appropriate way to do so seems to be to define a liability rule as a function from $[0,1]^2$ to $[0,1]^2$, rather than from $[0,\infty)^2$ to $[0,1]^2$, along with the required changes in the definitions of functions p and q. Under this procedure for incorporating the no distinction between the due care and more than the due care requirement, a liability rule f would be defined by:

$f : [0,1]^2 \mapsto [0,1]^2, \text{ where } f(p,q) = (x,y),\; x + y = 1.$

An application of f would consist of specification of $C, D, \pi, H, (c^*, d^*) \in M$ satisfying (A1)–(A4), along with functions p and q defined as follows[8]:

[8]If $c < c^*$, then we must have $c^* > 0$ as $(\forall c \in C)(c \geq 0)$; and if $d < d^*$, then we must have $d^* > 0$ as $(\forall d \in D)(d \geq 0)$.

$p : C \mapsto [0,1]$ such that:

$$
\begin{aligned}
p(c) &= \frac{c}{c^*} \text{ if } c < c^*; \\
&= 1 \quad \text{if } c \geq c^*
\end{aligned}
$$

$q : D \mapsto [0,1]$ such that:

$$
\begin{aligned}
q(d) &= \frac{d}{d^*} \text{ if } d < d^*; \\
&= 1 \quad \text{if } d \geq d^*.
\end{aligned}
$$

RNO, of course, is inapplicable in this framework. Within this framework, the efficient liability rules are characterized by condition NL.[9]

Theorem 3.4. *Let $f : [0,1]^2 \mapsto [0,1]^2$ be a liability rule. Then, a necessary and sufficient condition for f to be efficient with respect to $\mathscr{A}$ is that it satisfy the condition of negligence liability.*

The following important corollary follows immediately from Theorem 3.4.

Corollary 3.1. *(i) The rules of negligence, negligence with the defence of contributory negligence, comparative negligence, and strict liability with the defence of contributory negligence are efficient with respect to $\mathscr{A}$.*
(ii) The rules of no liability and strict liability are not efficient with respect to $\mathscr{A}$.

Remark 3.6. As is the case with liability rules with domain $[0,\infty)^2$, in the case of liability rules with domain $[0,1]^2$ also, the set of efficient rules with respect to $\mathscr{A}'$ is the same as with respect to $\mathscr{A}$. ♢

Remark 3.7. From the discussion of Sect. 3.2 on unilateral care, it follows that when the notion of a liability rule is defined as a function from $[0,1]^2$ to $[0,1]^2$, then a liability rule is efficient for every application belonging to $\mathscr{A}_1^0$ iff $[\forall q \in [0,1)][f(1,q) = (0,1)]$ holds; and a liability rule is efficient for every application belonging to $\mathscr{A}_2^0$ iff $[\forall p \in [0,1)][f(p,1) = (1,0)]$ holds. From this it follows that strict liability is efficient with respect to $\mathscr{A}_1^0$; and no liability is efficient with respect to $\mathscr{A}_2^0$. ♢

The no distinction between the due care and more than the due care requirement is logically completely independent of efficiency. There are both efficient and inefficient liability rules satisfying NDMR as well as violating it. Therefore, it follows that this very important feature of tort law cannot possibly have an explanation solely rooted in the normative criterion of economic efficiency. In what follows we argue that this feature of tort law is based partly on fairness considerations and partly on efficiency considerations. In order to argue this, we first introduce the property of monotonicity.

As taking greater care never results in greater expected accident losses, fairness would seem to require that greater levels of care should not be associated with

[9] The result that efficient liability rules in this framework are characterized by condition NL is established in Jain and Singh (2002).

greater liability shares. In other words, the use of liability rules which perversely associate larger liability shares for larger proportions of nonnegligence might be thought inappropriate on grounds of fairness. A liability rule which does not perversely associate larger liability shares with larger proportions of nonnegligence would be called a monotonic liability rule. Intuitively, the monotonicity requirement seems to be quite compelling on considerations of fairness and justice. It is therefore not surprising that all liability rules used in practice satisfy the monotonicity requirement. Formally, the monotonicity condition is defined as follows:

Monotonicity (M): A liability rule f satisfies the condition of monotonicity iff $[\forall p, p', q, q' \in [0,\infty)][[p \geq p' \rightarrow x(p,q) \leq x(p',q)] \ \wedge \ [q \geq q' \rightarrow y(p,q) \leq y(p,q')]]$.

If one considers only the subclass of monotonic liability rules, then, in view of necessity and sufficiency of conjunction of NL and RNO for efficiency with respect to $\mathscr{A}$, it follows that all efficient monotonic liability rules satisfy the no-distinction requirement of tort law as is shown in the following theorem.

Theorem 3.5. *Let liability rule $f : [0,\infty)^2 \mapsto [0,1]^2$ belong to the subclass of liability rules satisfying the condition of monotonicity. Then, if f is efficient for every application belonging to $\mathscr{A}$, then it satisfies the no distinction between the due care and more than the due care requirement.*

Proof. Let f be an efficient monotonic liability rule. Therefore, f satisfies RNO and NL by Theorem 3.1.

$$p \geq 1 \rightarrow x(p,q) \leq x(1,q), \text{ by condition M} \tag{3.1}$$

$$q \geq 1 \rightarrow x(1,q) \leq x(1,1), \text{ by RNO} \tag{3.2}$$

$$q \geq 1 \rightarrow x(p,q) \geq x(p,1), \text{ by M} \tag{3.3}$$

$$p \geq 1 \rightarrow x(p,1) \geq x(1,1), \text{ by RNO} \tag{3.4}$$

From (3.1) and (3.2) we conclude:

$$p \geq 1 \wedge q \geq 1 \rightarrow x(p,q) \leq x(1,1) \tag{3.5}$$

From (3.3) and (3.4) we conclude:

$$p \geq 1 \wedge q \geq 1 \rightarrow x(p,q) \geq x(1,1) \tag{3.6}$$

(3.5) and (3.6) imply:

$$p \geq 1 \wedge q \geq 1 \rightarrow x(p,q) = x(1,1) \tag{3.7}$$

In view of (3.1), (3.2), and (3.7), it follows that:

$$p \geq 1 \wedge q \geq 1 \rightarrow x(p, q) = x(1, q) \tag{3.8}$$

Now,

$$q < 1 \rightarrow x(1, q) = 0, \text{ by NL} \tag{3.9}$$

(3.1) and (3.9) imply:

$$p \geq 1 \wedge q < 1 \rightarrow x(p, q) = x(1, q) \tag{3.10}$$

(3.8) and (3.10) imply:

$$p \geq 1 \rightarrow x(p, q) = x(1, q) \tag{3.11}$$

By an analogous argument, it follows that

$$q \geq 1 \rightarrow x(p, q) = x(p, 1) \tag{3.12}$$

(3.11) and (3.12) establish the theorem. □

The monotonicity condition, like the NDMR condition, is logically completely independent of efficiency. From the necessity and sufficiency of conjunction of NL and RNO for efficiency with respect to $\mathscr{A}$, it follows that the class of efficient liability rules includes both monotonic and non-monotonic rules and so does the class of inefficient rules. Monotonicity by itself is unrelated to NDMR as well, as there are monotonic rules both satisfying and violating NDMR and non-monotonic rules both satisfying and violating NDMR. The NDMR, however, seems to be, in view of Theorem 3.5, partly grounded in fairness and partly in efficiency.

Most liability rules used in practice do not distinguish among levels of care which are less than the due care. A notable exception is the rule of comparative negligence. The requirement that no distinction be made among different levels of care as long as they are all less than the due care can be formalized as follows.

The requirement of no distinction among levels of care which are less than the due care (RNDL): A liability rule f satisfies the requirement of no distinction among levels of care which are less than the due care iff $[\forall p, q \in [0, \infty)][[p < 1 \rightarrow f(p, q) = f(0, q)] \wedge [q < 1 \rightarrow f(p, q) = f(p, 0)]]$.

It is easy to see that this condition is not only unrelated to efficiency and monotonicity (which is being interpreted as a formalization of an aspect of fairness) taken singly but also when they are considered jointly. Thus, this requirement, although analogous to NDMR, does not seem to be based on any value of a compelling nature.

The structure of liability rules which satisfy both NDMR and RNDL is extremely simple as the liability assignments depend only on whether and which parties are negligent. If both the conditions are incorporated in the definition of a liability rule itself, then a liability rule f becomes a function from $\{0,1\}^2$ to $[0,1]^2$; $f : \{0,1\}^2 \mapsto [0,1]^2$, where $f(p,q) = (x,y)$, $x + y = 1$. An application of f consists of specification of $C, D, \pi, H, (c^*, d^*) \in M$ satisfying (A1)–(A4) along with functions p and q defined as follows:

$p : C \mapsto \{0,1\}$ such that:
$$p(c) = 0 \text{ if } c < c^*;$$
$$= 1 \text{ if } c \geq c^*$$

$q : D \mapsto \{0,1\}$ such that:
$$q(d) = 0 \text{ if } d < d^*;$$
$$= 1 \text{ if } d \geq d^*.$$

In view of the results obtained here, it seems fairly clear that as far as the question of how best the notion of a liability rule can be formalized is concerned, there are essentially two alternatives depending on whether one incorporates the tort law feature of making no distinction between the due care and more than the due care into the definition of a liability rule or as a condition. The formalization, which makes a liability rule a function from the Cartesian product of the set of nonnegative real numbers with itself to the Cartesian product of the closed unit interval with itself, has the advantage of being more general. The results of this chapter, however, indicate that the framework incorporating the tort law feature of making no distinction between the due care and more than the due care into the definition of a liability rule, notwithstanding the fact that it is less general, offers certain advantages. Even if efficiency is accorded primacy, it is reasonable to suppose that once the efficiency criterion is satisfied, one may want to fulfil other desirable criteria, particularly criteria grounded in fairness. It is in this context that non-monotonic liability rules are unlikely to be acceptable. As Theorem 3.5 shows, all efficient monotonic liability rules satisfy the no distinction between the due care and more than the due care requirement. Consequently by restricting oneself to the less general framework in which the no distinction between the due care and more than the due care requirement is incorporated in the definition of a liability rule itself, one is not excluding any monotonic efficient liability rule from the analysis. This seems to suggest that the more general framework is perhaps unnecessary. In any case, incorporation of the no distinction between the due care and more than the due care feature into the analysis is in conformity with the tort law as it has evolved. In the remainder of the book, liability rules will be viewed as functions from the Cartesian product of the closed unit interval with itself to the Cartesian product of the closed unit interval with itself.

Appendix

Proposition 3.1. *If a liability rule* $f : [0,\infty)^2 \mapsto [0,1]^2$ *satisfies conditions NL and RNO, then for any application* $< C, D, \pi, H, (c^*, d^*) \in M >$ *belonging to* $\mathscr{A}$*,* (c^*, d^*) *is a Nash equilibrium.*

Proof. Let liability rule f satisfy conditions NL and RNO. Take any application $< C, D, \pi, H, (c^*, d^*) \in M >$ belonging to $\mathscr{A}$.
Suppose (c^*, d^*) is not a Nash equilibrium. This implies:

$$(\exists c' \in C)[c' + x[p(c'), q(d^*)]L(c', d^*) < c^* + x^* L^*] \ \vee \ (\exists d' \in D)[d' + y[p(c^*), q(d')]L(c^*, d') < d^* + y^* L^*]. \quad (3.13)$$

Suppose $(\exists c' \in C)[c' + x[p(c'), q(d^*)]L(c', d^*) < c^* + x^* L^*]$ holds. (3.14)

$$c' < c^* \wedge (3.14) \rightarrow c' + L(c', d^*) < c^* + x^* L^*, \text{ as } x[p(c'), q(d^*)] = 1 \text{ by condition NL}$$
$$\rightarrow c' + L(c', d^*) < c^* + L^*, \text{ as } x^* \in [0,1] \text{ and } L^* \geq 0$$
$$\rightarrow c' + d^* + L(c', d^*) < c^* + d^* + L^*$$
$$\rightarrow \mathrm{TSC}(c', d^*) < \mathrm{TSC}(c^*, d^*).$$

This is a contradiction as TSC are minimum at (c^*, d^*). Therefore we conclude:

$c' < c^* \rightarrow$ (3.14) cannot hold. (3.15)

For $c' > c^*$, we have the following:
If $c^* = 0$, then $x[p(c'), q(d^*)] = x(1,1) = x^*$;
If $c^* > 0$, then $x[p(c'), q(d^*)] \geq x^*$, by condition RNO.
Therefore,
$c' > c^* \rightarrow x[p(c'), q(d^*)] \geq x^*$.
Consequently,

$$c' > c^* \wedge (3.14) \rightarrow c' + x^* L(c', d^*) < c^* + x^* L^*$$
$$\rightarrow (1 - x^*)c' + x^*[c' + d^* + L(c', d^*)] < (1 - x^*)c^* + x^*[c^* + d^* + L^*]$$
$$\rightarrow (1 - x^*)c' < (1 - x^*)c^*, \text{ as } \mathrm{TSC}(c', d^*) \geq \mathrm{TSC}(c^*, d^*). \quad (3.16)$$

$$(1 - x^*) > 0 \wedge (3.16) \rightarrow c' < c^*, \text{ which contradicts the hypothesis that } c' > c^*. \quad (3.17)$$

$$(1 - x^*) = 0 \ \wedge \ (3.16) \rightarrow 0 < 0, \text{ a contradiction.} \tag{3.18}$$

(3.17) and (3.18) establish that (3.16) cannot hold. Therefore it follows that:

$$c' > c^* \rightarrow (3.14) \text{ cannot hold.} \tag{3.19}$$

(3.15) and (3.19) establish that (3.14) cannot hold. By an analogous argument, one can show that $(\exists d' \in D)[d' + y[p(c^*), q(d')]L(c^*, d') < d^* + y^* L^*]$ cannot hold.

This establishes that (c^*, d^*) is a Nash equilibrium. □

Proposition 3.2. *If a liability rule* $f : [0, \infty)^2 \mapsto [0, 1]^2$ *satisfies conditions NL and RNO, then for every application* $< C, D, \pi, H, (c^*, d^*) \in M >$ *belonging to* $\mathscr{A}$*:* $(\forall(\overline{c}, \overline{d}) \in C \times D)[(\overline{c}, \overline{d})$ *is a Nash equilibrium* $\rightarrow (\overline{c}, \overline{d}) \in M]$.

Proof. Let liability rule f satisfy conditions NL and RNO. Take any application $< C, D, \pi, H, (c^*, d^*) \in M >$ belonging to $\mathscr{A}$.
Let $(\overline{c}, \overline{d})$ be a Nash equilibrium. $(\overline{c}, \overline{d})$ being a Nash equilibrium implies:

$$(\forall c \in C)[\overline{c} + x[p(\overline{c}), q(\overline{d})]L(\overline{c}, \overline{d}) \leq c + x[p(c), q(\overline{d})]L(c, \overline{d})] \tag{3.20}$$

and

$$(\forall d \in D)[\overline{d} + y[p(\overline{c}), q(\overline{d})]L(\overline{c}, \overline{d}) \leq d + y[p(\overline{c}), q(d)]L(\overline{c}, d)] \tag{3.21}$$

(3.20) and (3.21) imply, respectively:

$$\overline{c} + x[p(\overline{c}), q(\overline{d})]L(\overline{c}, \overline{d}) \leq c^* + x[p(c^*), q(\overline{d})]L(c^*, \overline{d}) \tag{3.22}$$

$$\overline{d} + y[p(\overline{c}), q(\overline{d})]L(\overline{c}, \overline{d}) \leq d^* + y[p(\overline{c}), q(d^*)]L(\overline{c}, d^*) \tag{3.23}$$

Adding inequalities (3.22) and (3.23) we obtain:

$$\overline{c} + \overline{d} + L(\overline{c}, \overline{d}) \leq c^* + d^* + x[p(c^*), q(\overline{d})]L(c^*, \overline{d}) + y[p(\overline{c}), q(d^*)]L(\overline{c}, d^*). \tag{3.24}$$

By the definitions of functions p and q and condition RNO:

If $\overline{c} \geq c^*$, then $y[p(\overline{c}), q(d^*)] \leq y^*$; and
If $\overline{d} \geq d^*$, then $x[p(c^*), q(\overline{d})] \leq x^*$.

By condition NL:

If $\overline{c} < c^*$, then $y[p(\overline{c}), q(d^*)] = 0$; and
If $\overline{d} < d^*$, then $x[p(c^*), q(\overline{d})] = 0$.

Also, by (A2) and (A3),

If $\overline{c} \geq c^*$, then $L(\overline{c}, d^*) \leq L^*$; and
If $\overline{d} \geq d^*$, then $L(c^*, \overline{d}) \leq L^*$.

In view of the above,

$$\overline{c} \geq c^* \wedge \overline{d} \geq d^* \wedge (3.24) \rightarrow \overline{c} + \overline{d} + L(\overline{c}, \overline{d}) \leq c^* + d^* + x^* L(c^*, \overline{d}) + y^* L(\overline{c}, d^*)$$

$$\rightarrow \overline{c} + \overline{d} + L(\overline{c}, \overline{d}) \leq c^* + d^* + x^* L^* + y^* L^* = c^* + d^* + L^*; \tag{3.25}$$

$$\overline{c} < c^* \wedge \overline{d} \geq d^* \wedge (3.24) \rightarrow \overline{c} + \overline{d} + L(\overline{c}, \overline{d}) \leq c^* + d^* + x[p(c^*), q(\overline{d})] L(c^*, \overline{d})$$

$$\rightarrow \overline{c} + \overline{d} + L(\overline{c}, \overline{d}) \leq c^* + d^* + L(c^*, \overline{d})$$

$$\rightarrow \overline{c} + \overline{d} + L(\overline{c}, \overline{d}) \leq c^* + d^* + L^*; \tag{3.26}$$

$$\overline{c} \geq c^* \wedge \overline{d} < d^* \wedge (3.24) \rightarrow \overline{c} + \overline{d} + L(\overline{c}, \overline{d}) \leq c^* + d^* + y[p(\overline{c}), q(d^*)] L(\overline{c}, d^*)$$

$$\rightarrow \overline{c} + \overline{d} + L(\overline{c}, \overline{d}) \leq c^* + d^* + L(\overline{c}, d^*)$$

$$\rightarrow \overline{c} + \overline{d} + L(\overline{c}, \overline{d}) \leq c^* + d^* + L^*; \tag{3.27}$$

$$\overline{c} < c^* \wedge \overline{d} < d^* \wedge (3.24) \rightarrow \overline{c} + \overline{d} + L(\overline{c}, \overline{d}) \leq c^* + d^*$$

$$\rightarrow \overline{c} + \overline{d} + L(\overline{c}, \overline{d}) \leq c^* + d^* + L^*. \tag{3.28}$$

(3.25)–(3.28) establish that:
(3.24) $\rightarrow \mathrm{TSC}(\overline{c}, \overline{d}) \leq \mathrm{TSC}(c^*, d^*)$.
As $\mathrm{TSC}(c^*, d^*)$ is minimum of TSC, it follows that we must have $\mathrm{TSC}(\overline{c}, \overline{d}) = \mathrm{TSC}(c^*, d^*)$.

This establishes that $(\overline{c}, \overline{d}) \in M$. □

Proposition 3.3. *If a liability rule $f : [0, \infty)^2 \mapsto [0, 1]^2$ is efficient for every application $< C, D, \pi, H, (c^*, d^*) \in M >$ belonging to $\mathscr{A}$, then it satisfies condition RNO.*

Proof. Let liability rule f violate condition RNO. Then:
$[\exists p \in (1, \infty)][x(p, 1) < x(1, 1)] \ \vee \ [\exists q \in (1, \infty)][y(1, q) < y(1, 1)]$.
Suppose $[\exists q \in (1, \infty)][y(1, q) < y(1, 1)]$ holds. Let $y(1, q') < y(1, 1), q' > 1$.
Let t be a positive number. Choose u such that:
$u > \frac{(q'-1)t}{y(1,1)-y(1,q')}$.
It should be noted that $u > 0$.
Choose a positive number μ such that $\mu < (q' - 1)t$.

Let s, ϵ, and δ be any positive numbers. Let C and D be specified as follows:
$C = \{0, s\}, D = \{0, t, q't\}$.
Let $L(c, d)$ for $(c, d) \in C \times D$ be as follows:

$L(c, d)$

	$d = 0$	$d = t$	$d = q't$
$c = 0$	$s + \epsilon + t + \delta + u$	$s + \epsilon + u$	$s + \epsilon + u - (q' - 1)t + \mu$
$c = s$	$t + \delta + u$	u	$u - (q' - 1)t + \mu$

$\epsilon > 0, \delta > 0$, and $0 < \mu < (q' - 1)t$ imply that (s, t) is the unique TSC-minimizing configuration, i.e. $M = \{(s, t)\}$.
Let $(c^*, d^*) = (s, t)$.
Now,
$EC_2(s, t)$

$= t + y(1, 1)L(s, t)$

$= t + y(1, 1)u$

$EC_2(s, q't)$

$= q't + y(1, q')L(s, q't)$

$= q't + y(1, q')[u - (q' - 1)t + \mu]$

$EC_2(s, t) - EC_2(s, q't)$

$= t + y(1, 1)u - q't - y(1, q')[u - (q' - 1)t + \mu]$

$= -(q' - 1)t + [y(1, 1) - y(1, q')]u + y(1, q')[(q' - 1)t - \mu]$

As $y(1, q')[(q' - 1)t - \mu] \geq 0$, we conclude:

$EC_2(s, t) - EC_2(s, q't) \geq -(q' - 1)t + [y(1, 1) - y(1, q')]u.$

As $u > \frac{(q'-1)t}{y(1,1)-y(1,q')}$, it follows that:
$EC_2(s, t) - EC_2(s, q't) > 0$.
This implies that the unique TSC-minimizing configuration (s, t) is not a Nash equilibrium. Consequently f is not efficient.

If $[\exists p \in (1, \infty)][x(p, 1) < x(1, 1)]$ holds, then by an analogous argument one can show that f is not efficient.

This establishes the proposition. □

Proposition 3.4. *If a liability rule* $f : [0, \infty)^2 \mapsto [0, 1]^2$ *is efficient for every application* $< C, D, \pi, H, (c^*, d^*) \in M >$ *belonging to* $\mathscr{A}$, *then it satisfies condition NL.*

Proof. Let liability rule f violate condition NL. Then:
$[\exists p \in [0,1)][f(p,1) \neq (1,0) \ \vee \ [\exists q \in [0,1)][f(1,q) \neq (0,1)]$.
Suppose $[\exists q \in [0,1)][f(1,q) \neq (0,1)]$ holds. Let $y(1,q') < 1, q' < 1$.
Let $t > 0$. Choose r such that $0 \leq y(1,q')t < r < t$. Let $v = \frac{r}{1-q'}$. Let $u > 0$ and $\epsilon > 0$.
Let $C = \{0,u\}, D = \{0,q'v,v\}$.
Let $L(c,d)$ for $(c,d) \in C \times D$ be specified as follows[10]:

$L(c,d)$

	$d = 0$	$d = q'v$	$d = v$
$c = 0$	$u + \epsilon + t + q'v$	$u + \epsilon + t$	$u + \epsilon$
$c = u$	$t + q'v$	t	0

$\epsilon > 0$ and $t > r = (1-q')v$ imply that $M = \{(u,v)\}$.
Let $(c^*, d^*) = (u,v)$.

Now,
$EC_2(u,v) = v$

$EC_2(u,q'v)$

$= q'v + y(1,q')L(u,q'v)$

$= q'v + y(1,q')t$

$EC_2(u,v) - EC_2(u,q'v)$

$= v - q'v - y(1,q')t$

$= (1-q')v - y(1,q')t$

$= r - y(1,q')t$

$> 0.$

This implies that the unique TSC-minimizing configuration (u,v) is not a Nash equilibrium. f is therefore not efficient.

If $[\exists p \in [0,1)][f(p,1) \neq (1,0)]$ holds, then by an analogous argument it can be shown that f is not efficient.

This establishes the proposition. □

Proof of Theorem 3.1. Let liability rule f satisfy the requirement of non-reward for over-nonnegligence and the condition of negligence liability. Then by Propositions 3.1 and 3.2, f is efficient for every application $< C, D, \pi, H, (c^*, d^*) \in M >$ belonging to $\mathscr{A}$. Propositions 3.3 and 3.4 establish that if f is efficient for every

[10] L has been specified in such a way that no inconsistency would arise even if $q' = 0$.

application $< C, D, \pi, H, (c^*, d^*) \in M >$ belonging to $\mathscr{A}$, then it satisfies the requirement of non-reward for over-nonnegligence and the condition of negligence liability. $\square$

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Chapter 4
Decoupled Liability and Efficiency

Liability is said to be 'coupled' if the liability imposed on the injurer equals payment to the victim and to be 'decoupled' if the two amounts are unequal. Liability is coupled under every liability rule, as a liability rule apportions the accident loss between the injurer and the victim and consequently liability imposed on the injurer necessarily equals the payment to the victim. An example of a rule with decoupled liability is provided by the rule under which the injurer pays tax equal to the harm and the victim bears his loss. This chapter examines the relationship between the coupled liability feature of tort law and efficiency within a framework essentially that of Chap. 3. It is rigorously established here that decoupled liability is inconsistent with efficiency.

In order to discuss decoupled liability, we introduce the notion of a hybrid liability rule, to be written as h-liability rule in abbreviated form. Like a liability rule, a h-liability rule also determines the proportions in which the two parties are to bear the loss in case of accident as a function of whether and by what proportions the parties' levels of care were below the due care levels. But unlike the case of a liability rule, there is no requirement that the liability shares of the two parties must sum to one. The sum of the liability shares of the two parties can be any nonnegative number. If the sum is less than one, then part of the loss falls on a third party; if the sum is greater than one, then there is a net transfer to a third party. If the third party is the government, then the cases of less than one and greater than one correspond to the government subsidy and tax, respectively. For the rule under which the injurer pays tax equal to harm and the victim bears his loss, we have the sum of the liability shares as two. A liability rule is a special case of a h-liability rule. The definition of a h-liability rule reduces to that of a liability rule when the sum of the liability shares equals 1.

As in the case of liability rules, a h-liability rule is efficient if and only if it invariably induces both parties to take total social cost (TSC)-minimizing care levels. The main theorem of this chapter shows that no h-liability rule with decoupled liability can be such that it will invariably induce both parties to take

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TSC-minimizing care levels. In other words, in the context of the standard tort model, decoupled liability is inconsistent with efficiency. As a corollary it follows that the rule under which the injurer pays tax equal to harm and the victim bears his loss is inefficient.[1]

There is a simple and intuitively clear way to understand this result. In the context of externalities, it is generally thought that if every party to the interaction can be made to internalize the harm, then the outcome will be efficient; inefficiencies result because of failure on the part of one or the other party to internalize the harm. The reason why rules like strict liability or no liability do not in general lead to efficient outcomes when care is bilateral is that under these rules only one of the two parties to the interaction is induced to internalize the harm. What the result of this chapter establishing the necessity of coupled liability for efficiency, with its corollary of inefficiency of the rule of injurer paying tax equal to harm and victim bearing his loss, shows is that while internalization of harm by both the parties is required if each of them is to take socially efficient level of care, this requirement by itself is not sufficient. There is another important requirement, namely, whether the proposed rule for solving the externality problem has the closure property with respect to the parties involved in the interaction giving rise to the externality. When we consider an externality resulting from an interaction between two parties, the apportioning of the loss resulting from the interaction between them only keeps the externality closed with respect to the parties whose interaction is the cause of it. On the other hand, if the loss apportionment involves not only the interacting parties but also the government or some other third party, then the externality is not closed with respect to the parties involved in the interaction giving rise to it. It is the involvement of the third parties which distorts incentives for taking care. Decoupled liability is inconsistent with efficiency because decoupling of liability necessarily results in non-closure of the externality with respect to parties involved in the interaction giving rise to it. When we have coupled liability, i.e. use a liability rule, then violation of negligence liability condition is inconsistent with efficiency as violation of negligence liability implies that one or the other party fails to internalize the harm. Thus efficiency within the framework of the standard model obtains only when we have liability rules satisfying the condition of negligence liability.

The main policy conclusion that emerges from the analysis of decoupled liability is that other things being equal tort law (assuming of course that a liability rule satisfying the condition of negligence liability is used) method for solving externality problem is superior to the taxation approach. If in a particular context government intervention in the form of taxes is advocated for the solution of a negative externality problem, it needs to be justified in terms of reduction

[1]This rule is generally regarded as efficient. According to Shavell (2007, p. 147) under this rule efficiency would obtain even when both care and activity levels can be varied: 'However, fully optimal behavior can readily be induced with tools other than liability rules. For example, if injurers have to pay the state for harm caused and victims bear their own losses, both victims and injurers will choose levels of care and of activity optimally'. See also Faure (2009, p. 22). As shown in this chapter, the rule is not efficient even in the case of fixed activity levels.

of litigation and other costs over and above the additional costs resulting from inefficiencies on account of the use of a decoupled liability system.

This chapter is divided into three sections.[2] The first section discusses and formalizes the notion of a hybrid liability rule, a generalization of the concept of a liability rule. The second section discusses the logic of the impossibility theorem. Although no decoupled h-liability rules are efficient, some of them exhibit the interesting property of always yielding the configuration of due care levels as a Nash equilibrium. The characterization of all hybrid liability rules exhibiting this property is discussed in the last section. All formal statements and proofs have been relegated to the appendix to this chapter.

4.1 Decoupled Liability

For enabling a systematic analysis of decoupled liability, the notion of a liability rule needs to be generalized. For this purpose we introduce the notion of a hybrid liability rule. Let $\overline{s} \geq 0$ be a given constant. Formally, a hybrid liability rule (h-liability rule) is a function f from $[0,1]^2$ to $[0,\infty)^2$, $f : [0,1]^2 \mapsto [0,\infty)^2$, such that $f(p,q) = (x,y)$, where $x + y = \overline{s}$. Thus, a h-liability rule is a rule which specifies the multiples of loss the two parties are to bear in case of occurrence of accident as a function of proportions of nonnegligence of the two parties. If $\overline{s} = 1$, then the definition of a h-liability rule reduces to that of a liability rule.

If $\overline{s} \neq 1$, then in case of harm, it must necessarily be the case that the net payment by the injurer is unequal to the net payment to the victim, as $yH \neq (1-x)H$. $yH - (1-x)H$ represents the net payment to the government as tax. Thus the liability is decoupled iff $\overline{s} \neq 1$; and coupled iff $\overline{s} = 1$.

As in the case of a liability rule, we define a h-liability rule $f : [0,1]^2 \mapsto [0,\infty)^2$ to be efficient for a given application $< C, D, \pi, H, (c^*, d^*) \in M > \in \mathscr{A}$ iff $(\forall (\overline{c}, \overline{d}) \in C \times D)[(\overline{c}, \overline{d})$ is a Nash equilibrium $\rightarrow (\overline{c}, \overline{d}) \in M]$ and $(\exists (\overline{c}, \overline{d}) \in C \times D)[(\overline{c}, \overline{d})$ is a Nash equilibrium]; and to be efficient with respect to a class of applications iff it is efficient for every application belonging to that class.

Below we consider the example of 'injurer pays tax equal to harm and victim bears his loss' rule, one of the most important decoupled liability rules:

Example 4.1. Let h-liability rule $f : [0,1]^2 \mapsto [0,\infty)^2$ be defined by:
$(\forall p, q \in [0,1])[f(p,q) = (1,1)]$.
Under f, regardless of the nonnegligence proportions, the victim bears his loss and the injurer pays tax equal to the loss.
We consider the application specified below.
Let C and D be given by:
$C = \{0,1,2\}, D = \{0,1,2\}$.

[2] This chapter relies on Jain (2012).

For $(c, d) \in C \times D$, let $L(c, d)$ be as given in the following array:

$L(c, d)$

	$d = 0$	$d = 1$	$d = 2$
$c = 0$	10	6	5
$c = 1$	6	2	1
$c = 2$	5	1	0

We have $M = \{(1, 1), (1, 2), (2, 1), (2, 2)\}$.
Let $(c^*, d^*) = (1, 1)$.
Under the h-liability rule f, the following array gives (EC_1, EC_2) for $(c, d) \in C \times D$:

(EC_1, EC_2)

	$d = 0$	$d = 1$	$d = 2$
$c = 0$	$(10, 10)$	$(6, \underline{7})$	$(5, \underline{7})$
$c = 1$	$(\underline{7}, 6)$	$(\underline{3}, \underline{3})$	$(\underline{2}, \underline{3})$
$c = 2$	$(\underline{7}, 5)$	$(\underline{3}, \underline{2})$	$(\underline{2}, \underline{2})$

$(c, d) \in C \times D$ which are Nash equilibria are $(1, 1)$, $(1, 2)$, $(2, 1)$, $(2, 2)$. As all of these belong to M, it follows that f is efficient for the application under consideration. $\Diamond$

4.2 Efficiency of h-Liability Rules

Under the rule according to which the victim bears his loss and the injurer pays tax equal to the harm inflicted on the victim, both parties fully internalize the harm. For the application of Example 4.1, this rule would give rise to an efficient outcome. The rule, however, does not always lead to efficient outcomes, as the following example shows.

Example 4.2. Consider the following application of the h-liability rule defined by: $(\forall p, q \in [0, 1])[f(p, q) = (1, 1)]$.
$C = D = \{0, 1\}$.
For $(c, d) \in C \times D$, let $L(c, d)$ be as given in the following array:

$L(c, d)$

	$d = 0$	$d = 1$
$c = 0$	100	90.5
$c = 1$	90	89.9

We have $M = \{(1, 0)\}$. Let $(c^*, d^*) = (1, 0)$.

We obtain $(EC_1(c,d), EC_2(c,d))$ for $(c,d) \in C \times D$ as given in the following array:

$(EC_1(c,d), EC_2(c,d))$

	$d=0$	$d=1$
$c=0$	$(100, 100)$	$(\underline{90.5}, \underline{91.5})$
$c=1$	$(\underline{91}, \underline{90})$	$(90.9, 90.9)$

Thus both $(1,0)$ and $(0,1)$ are Nash equilibria.
As $(0,1) \notin M$, it follows that the rule is inefficient for this application. $\Diamond$

The reason why this rule does not invariably result in efficient outcomes in spite of both the parties being forced to internalize the entire loss is due to the fact that the liability under the rule is decoupled. In Theorem 4.1, it is shown that if a h-liability rule is efficient for every application belonging to $\mathscr{A}$, then the liability must be coupled. In other words, a necessary condition for a h-liability rule to be efficient for every application belonging to $\mathscr{A}$ is that the liability be coupled. An equivalent statement would be that a necessary condition for a h-liability rule to be efficient for every application belonging to $\mathscr{A}$ is that it be a liability rule.

The proof of necessity of coupled liability for a h-liability rule to be efficient for every application belonging to $\mathscr{A}$ is given in the appendix to this chapter. Here we discuss the arguments which establish the result informally. To begin with, it is shown in Lemma 4.1 that a necessary condition for a h-liability rule f to be efficient with respect to $\mathscr{A}$ is that the structure of f be such that when one party is negligent and the other is nonnegligent, the liability of the negligent party must be at least equal to the total loss, i.e. the liability share of the negligent party must be greater than or equal to 1. Suppose f is such that when one party is nonnegligent, say the victim, with $p = 1$, and the other party, the injurer, is negligent with $q = q_0 < 1$, the liability assignments are such that $y(1, q_0) < 1$. Consider an application of f belonging to $\mathscr{A}$ such that (i) TSC-minimizing configuration is unique, $M = \{(c_0, d_0)\}$, $(c_0, d_0) = (c^*, d^*)$; (ii) both c^* and d^* are positive, and if both parties take optimal care, then the expected loss is zero; and (iii) $q_0 d^*$ is an element of D. Given that the victim is using c^*, if we consider a shift by the injurer from d^* to $q_0 d^*$, then the increase in expected loss $L(c^*, q_0 d^*) - L(c^*, d^*) = L(c^*, q_0 d^*)$ ($L(c^*, d^*)$ being 0) must be greater than the reduction in cost of care $(1 - q_0)d^*$ as TSC-minimizing configuration is unique. The injurer, however, bears only a part of the increase in expected loss as $y(1, q_0) < 1$. On the other hand, the entire decrease in cost of care accrues to the injurer. Therefore, it follows that one can always find an application such that, in addition to (i)–(iii), $L(c^*, q_0 d^*) > (1 - q_0)d^* > y(1, q_0)L(c^*, q_0 d^*)$. This establishes the existence of an application for which the unique TSC-minimizing configuration is not a Nash equilibrium and consequently the existence of an application for which the rule is inefficient. This establishes the necessity of the requirement that when one party is negligent and the other is nonnegligent, the liability share of the negligent party must be greater than or equal to 1 for efficiency with respect to $\mathscr{A}$. The requirement in turn implies that no h-liability rule with $\overline{s} < 1$ can be efficient with respect to $\mathscr{A}$ (Corollary 4.1).

The next step in the argument (Lemma 4.3) shows that if f is efficient with respect to $\mathscr{A}$, then its structure must be such that when both parties take at least due care, neither party's liability share exceeds 1. Suppose h-liability rule f is such that $x(1,1) = x^* > 1$. Consider an application of f belonging to $\mathscr{A}$ such that (i) TSC-minimizing configuration is unique, $M = \{(c_0, d_0)\}$, $(c_0, d_0) = (c^*, d^*)$, (ii) $L(c^*, d^*) > 0$, and (iii) C contains $c^* + \epsilon$, $\epsilon > 0$. Given that the injurer is using d^*, if we consider a shift by the victim from c^* to $c^* + \epsilon$, then the decrease in expected loss $L(c^*, d^*) - L(c^* + \epsilon, d^*)$ must be less than the increase in cost of care ϵ, as TSC-minimizing configuration is unique. The entire increase in cost of care falls on the victim; but the reduction in the amount of expected loss borne by the victim is greater than the reduction in expected loss $L(c^*, d^*) - L(c^* + \epsilon, d^*)$ as $x^* > 1$. Therefore, it follows that one can always find an application such that, in addition to (i)–(iii), $x^*[L(c^*, d^*) - L(c^* + \epsilon, d^*)] > \epsilon > L(c^*, d^*) - L(c^* + \epsilon, d^*)$. This establishes the existence of an application for which the unique TSC-minimizing configuration is not a Nash equilibrium and consequently the existence of an application for which the rule is inefficient. This establishes the necessity of the requirement that when both parties are nonnegligent, neither party's liability share must exceed 1 for efficiency with respect to $\mathscr{A}$. The requirement in turn implies that no h-liability rule with $\overline{s} > 2$ can be efficient with respect to $\mathscr{A}$ (Corollary 4.3).

Thus the set of h-liability rules which are efficient with respect to $\mathscr{A}$ is contained in the set of h-liability rules for which $1 \leq \overline{s} \leq 2$. The next step in the argument shows that if f is such that $1 < \overline{s} \leq 2$, then the rule cannot be efficient with respect to $\mathscr{A}$. It is shown that if $1 < \overline{s} \leq 2$, then one can find an application belonging to $\mathscr{A}$ in which a configuration where both parties are taking more than due care, though not TSC minimizing, is a Nash equilibrium under f. The proof of this part is rather complex. Intuitively, the reason why one can always find such an application is as follows. $\overline{s} > 1$ implies that reduction in the expected liability which takes place for both the parties together is more than the reduction in the total expected loss. It is because of this that a configuration which does not minimize TSC and in which both parties are taking more than due care can emerge a Nash equilibrium under f.

This completes the argument establishing the impossibility of having both decoupled liability and efficiency with respect to $\mathscr{A}$; as liability is decoupled iff $\overline{s} \neq 1$, and coupled iff $\overline{s} = 1$. In view of the impossibility theorem, it follows that the set of h-liability rules efficient with respect to $\mathscr{A}$ is identical with the set of liability rules efficient with respect to $\mathscr{A}$. As a liability rule is efficient with respect to $\mathscr{A}$ iff it satisfies the condition of negligence liability, we conclude that a h-liability rule is efficient for every application belonging to $\mathscr{A}$ iff it satisfies the condition of negligence liability.

4.3 Characterization of Quasi-efficient h-Liability Rules

Although the rule under which the victim bears his loss and the injurer pays tax equal to the loss inflicted on the victim considered in Examples 4.1 and 4.2, like any other rule with decoupled liability, is not efficient with respect to $\mathscr{A}$, it does have an interesting property. Under this rule the configuration of due care levels $(c^*, d^*) \in M$ is always a Nash equilibrium. Let us define a h-liability rule to be quasi-efficient with respect to an application $< C, D, \pi, H, (c^*, d^*) \in M >$ iff $(c^*, d^*) \in M$ is a Nash equilibrium. A h-liability rule is defined to be quasi-efficient with respect to a class of applications iff it is quasi-efficient for every application belonging to that class. A rule being quasi-efficient with respect to an application of course is no guarantee that the outcome corresponding to that application would be efficient. But, unlike the cases where all equilibria are non-TSC minimizing, the possibility of an efficient outcome is not ruled out altogether either. Thus the class of h-liability rules which are quasi-efficient with respect to $\mathscr{A}$ is of some interest. Theorem 4.3 given in the appendix at the end of this the chapter completely characterizes the set of h-liability rules which are quasi-efficient with respect to $\mathscr{A}$. It is shown that a h-liability rule f is quasi-efficient with respect to $\mathscr{A}$ iff it satisfies the following two properties: (i) when one party is negligent and the other is nonnegligent, the liability share of the negligent party must be greater than or equal to 1, and (ii) when both parties take at least the due care, neither party's liability share exceeds 1. It should be noted that these two conditions together, as we have seen earlier, imply that $1 \leq \overline{s} \leq 2$.

While discussing the logic of the impossibility theorem in the previous section, it was noted that if (i) is violated or (ii) is violated by f, then one can find an application belonging to $\mathscr{A}$ in which $(c^*, d^*) \in M$ is not a Nash equilibrium. Thus both (i) and (ii) are necessary for quasi-efficiency with respect to $\mathscr{A}$. An intuitive explanation of sufficiency of (i) and (ii) for quasi-efficiency with respect to $\mathscr{A}$ is as follows. Suppose the injurer is using d^*. Consider a change by the victim from c^* to some $c > c^*$. Now, the reduction in expected loss $L(c^*, d^*) - L(c, d^*)$ cannot exceed the increase in cost of care $c - c^*$; otherwise (c^*, d^*) could not have been a TSC-minimizing configuration. As $x^* \leq 1$ by condition (ii), it follows that the reduction in the victim's liability $x^*[L(c^*, d^*) - L(c, d^*)]$ must be less than or equal to the increase in victim's expenditure on care $c - c^*$. Thus, a change from c^* to c, $c > c^*$, cannot be advantageous for the victim. Next consider a change from c^* to some $c < c^*$. The increase in expected loss $L(c, d^*) - L(c^*, d^*)$ must be at least as large as the decrease in cost of care $c^* - c$; otherwise (c^*, d^*) could not have been TSC minimizing. By conditions (i) and (ii), the increase in victim's liability must be at least equal to the increase in expected loss. Consequently a shift from c^* to c, $c < c^*$, can never be advantageous for the victim. Thus, given the choice of d^* by the injurer, c^* is best for the victim. By an analogous argument one shows that, given the choice of c^* by the victim, d^* is best for the injurer.

Concluding Remarks

For a proper perspective on the result showing inefficiency of every h-liability rule in which liability is decoupled, two important points need to be noted. If it is the case that in a particular context not all possible applications can arise, only applications belonging to some specific subclass can arise, then what matters is whether the rule under consideration gives rise to efficient outcomes for applications belonging to this specific subclass, and not whether it gives rise to efficient outcomes for all possible applications. As an inefficient rule with respect to a class of applications can very well be efficient with respect to a subclass of the class of applications with respect to which it is inefficient, therefore, although every decoupled h-liability rule is inefficient, a particular decoupled h-liability rule may very well be efficient with respect to some specific class of applications which may be found relevant in a given context. Thus, depending on the context, use of decoupled h-liability rules is possible without sacrificing efficiency.

A major limitation of the standard tort model is its neglect of administrative and litigation costs. If in addition to care costs and expected accident loss, administrative and litigation costs are also taken into account, then there are no rules which are invariably efficient. In a particular context some rules might perform better than others; it is perfectly possible for a rule with decoupled liability to perform better than a rule with coupled liability. The contribution by Polinsky and Che (1991) illustrates this point. In Polinsky-Che model, the amount of care that the injurer takes depends on the probability of a suit being filed on the one hand and his liability in case of occurrence of accident and filing of the suit on the other. It is assumed that injurer will take greater care with higher liability or higher probability of suit. Victim's decision to file a suit depends on the amount which will be awarded to him in case of a trial. The higher the award to the victim, the greater will be his incentive to litigate. Now, first consider any particular level of injurer's liability when liability is coupled. The level of injurer's liability (equal to victim's award) will determine the incentive of the victim to sue and the incentive of the injurer to take care. These in turn will determine the social costs, assumed to be the sum of the injurer's cost of care, the victim's expected harm, and the parties' expected litigation costs. Next, consider decoupling liability, starting from any particular level of coupled liability. If the amount paid by the injurer is increased, then he will increase his care; and if the amount of award to the victim is lowered, thereby reducing his incentive to sue, then the injurer's incentive to take care will be lowered. Thus, by simultaneously raising the injurer's liability and lowering the victim's award, the probability

(continued)

of suit can be lowered without affecting the injurer's incentive to take care. If injurer's care remains the same, then the expected accident loss remains unaffected, but because of a lower incentive to sue, litigation costs become lower. Consequently, it follows that given any level of coupled liability, there exists a system of decoupled liability where the social costs are lower. Given the logic of the model, it is clear that in the optimal system of decoupled liability, the defendant's payment will be as high as possible. However, in Polinsky and Che model, the optimal award to the plaintiff may be less than or greater than the optimal payment by the injurer.

There is, however, a potential problem, as noted by Polinsky and Che, if award to the victim is greater than the injurer's liability as it may create an incentive for individuals to falsely claim that an accident has occurred, when in fact no accident has occurred, to obtain the government subsidy equal to the difference between the award to the victim and the liability imposed on the injurer. This is a particular instance of the general point made above that decoupling because of its disruption of the closure of the externality with respect to the parties involved in the interaction giving rise to it tends to distort incentives which can lead to inefficiencies.

The notion of a hybrid liability rule is more general than that of a liability rule. An even more general concept for assigning liability than that of hybrid liability rules can be defined as follows. Let $p, q \in [0, 1], x, y \in [0, \infty)$. Let a function be called a liability assignment rule if it assigns liability multiples for any combination of nonnegligence proportions. Thus a liability assignment rule f is a function from $[0, 1]^2$ to $[0, \infty)^2$, $f : [0, 1]^2 \mapsto [0, \infty)^2$. For any combination of proportions of nonnegligence (p, q), f assigns the liability multiples (x, y). When liability assignment is done in accordance with a liability rule, then the sum of x and y must necessarily be 1. When liability assignment is done according to a hybrid liability rule, then the sum of x and y can be any nonnegative number, but the sum must be the same for all $(p, q) \in [0, 1]^2$. Under a liability assignment rule, the sum of x and y can be different for different combinations of (p, q). It is immediate that the notions of liability rules and hybrid liability rules are special cases of the more general notion of liability assignment rules. Although unlikely to be easily solvable, it would clearly be interesting to find necessary and sufficient conditions for a liability assignment rule to be efficient. Of particular interest is the question whether there exists a liability assignment rule exhibiting decoupled liability for some combinations of (p, q) which is efficient.

Appendix

Lemma 4.1. *Let f be a h-liability rule; $f : [0,1]^2 \mapsto [0,\infty)^2$. If f is efficient for every application belonging to $\mathscr{A}$, then $[\forall p, q \in [0,1)][x(p,1) \geq 1 \wedge y(1,q) \geq 1]$.*

Proof. Let f be a h-liability rule; $f : [0,1]^2 \mapsto [0,\infty)^2$. Suppose $[\exists q \in [0,1)][y(1,q) < 1]$.
Let $y(1,q_0) < 1$.
Let $t > 0$. Choose r such that $0 \leq y(1,q_0)t < r < t$. Let $d_0 = \frac{r}{1-q_0}$. Let $c_0 > 0$ and $\epsilon > 0$.
Let C and D be specified as follows:
$C = \{0, c_0\}, D = \{0, q_0 d_0, d_0\}$.
For $(c,d) \in C \times D$, let $L(c,d)$ be as given in the following array[3]:

$L(c,d)$

	$d = 0$	$d = q_0 d_0$	$d = d_0$
$c = 0$	$c_0 + \epsilon + t + q_0 d_0$	$c_0 + \epsilon + t$	$c_0 + \epsilon$
$c = c_0$	$t + q_0 d_0$	t	0

$\epsilon > 0$ and $t > r = (1 - q_0)d_0$ imply that $M = \{(c_0, d_0)\}$.
Let $(c^*, d^*) = (c_0, d_0)$.
Now,
$EC_2(c_0, d_0) = d_0$

$EC_2(c_0, q_0 d_0)$

$= q_0 d_0 + y(1,q_0)L(c_0, q_0 d_0)$

$= q_0 d_0 + y(1,q_0)t$

$EC_2(c_0, d_0) - EC_2(c_0, q_0 d_0)$

$= d_0 - q_0 d_0 - y(1,q_0)t$

$= (1 - q_0)d_0 - y(1,q_0)t$

$= r - y(1,q_0)t$

> 0.

This implies that the unique TSC-minimizing configuration (c_0, d_0) is not a Nash equilibrium. f is therefore not efficient for the application under consideration.

If $[\exists p \in [0,1)][x(p,1) < 1]$ holds, then by an analogous argument one can demonstrate the existence of an application belonging to $\mathscr{A}$ for which f is not efficient.

The proposition is therefore established. □

The following corollary follows immediately from Lemma 4.1.

[3] L has been specified in such a way that no inconsistency would arise even if $q_0 = 0$.

Corollary 4.1. *Let f be a h-liability rule; $f : [0,1]^2 \mapsto [0,\infty)^2$. If f is efficient for every application belonging to $\mathscr{A}$, then $\overline{s} \geq 1$.*

For every h-liability rule such that $[\exists p \in [0,1)][x(p,1) < 1] \vee [\exists q \in [0,1)][y(1,q) < 1]$, the proof of Lemma 4.1 shows the existence of an application belonging to $\mathscr{A}$ for which the configuration of costs of due care levels $(c^*, d^*) \in M$ is not a Nash equilibrium. Thus from the proof of Lemma 4.1, it follows that the following lemma holds.

Lemma 4.2. *Let f be a h-liability rule; $f : [0,1]^2 \mapsto [0,\infty)^2$. If f is such that for every application belonging to $\mathscr{A}$ the configuration of costs of due care levels $(c^*, d^*) \in M$ is a Nash equilibrium, then $[\forall p, q \in [0,1)][x(p,1) \geq 1 \wedge y(1,q) \geq 1]$.*

The following corollary follows immediately from Lemma 4.2.

Corollary 4.2. *Let f be a h-liability rule; $f : [0,1]^2 \mapsto [0,\infty)^2$. If f is such that for every application belonging to $\mathscr{A}$ the configuration of costs of due care levels $(c^*, d^*) \in M$ is a Nash equilibrium, then $\overline{s} \geq 1$.*

Lemma 4.3. *Let f be a h-liability rule; $f : [0,1]^2 \mapsto [0,\infty)^2$. If f is efficient for every application belonging to $\mathscr{A}$, then $[x^* \leq 1 \wedge y^* \leq 1]$.*

Proof. Suppose $x^* > 1$.
Choose $0 < \frac{1}{x^*} < t < 1$.
Choose $c_0, d_0, \epsilon, \delta_1, \delta_2 > 0$.
Let C and D be specified as follows:
$C = \{0, c_0, c_0 + \epsilon\}, D = \{0, d_0\}$.
For $(c, d) \in C \times D$, let $L(c, d)$ be as given in the following array:

$L(c, d)$

	$d = 0$	$d = d_0$
$c = 0$	$c_0 + \delta_1 + d_0 + \delta_2 + t\epsilon$	$c_0 + \delta_1 + t\epsilon$
$c = c_0$	$d_0 + \delta_2 + t\epsilon$	$t\epsilon$
$c = c_0 + \epsilon$	$d_0 + \delta_2$	0

$\delta_1 > 0$, $\delta_2 > 0$, and $0 < t < 1$ imply that (c_0, d_0) is the unique TSC-minimizing configuration, i.e. $M = \{(c_0, d_0)\}$.
Let $(c^*, d^*) = (c_0, d_0)$.
Now,

$EC_1(c_0, d_0) = c_0 + x^* t\epsilon$

$EC_1(c_0 + \epsilon, d_0) = c_0 + \epsilon$

$EC_1(c_0, d_0) - EC_1(c_0 + \epsilon, d_0) = c_0 + x^* t\epsilon - c_0 - \epsilon$

$= \epsilon[x^* t - 1]$

$> 0.$

Thus the unique TSC-minimizing configuration of costs of care is not a Nash equilibrium, establishing that the rule is not efficient for the application in question.

If $y^* > 1$, an analogous argument shows that there exists an application belonging to $\mathscr{A}$ for which the rule is not efficient.

The lemma is therefore established. □

The following corollary follows immediately from Lemma 4.3.

Corollary 4.3. *Let f be a h-liability rule; $f : [0,1]^2 \mapsto [0,\infty)^2$. If f is efficient for every application belonging to $\mathscr{A}$, then $\overline{s} \leq 2$.*

For every h-liability rule such that $[x^* > 1 \vee y^* > 1]$, the proof of Lemma 4.3 shows the existence of an application belonging to $\mathscr{A}$ for which the configuration of costs of due care levels $(c^*, d^*) \in M$ is not a Nash equilibrium. Thus from the proof of Lemma 4.3, it follows that the following lemma holds.

Lemma 4.4. *Let f be a h-liability rule; $f : [0,1]^2 \mapsto [0,\infty)^2$. If f is such that for every application belonging to $\mathscr{A}$, the configuration of costs of due care levels $(c^*, d^*) \in M$ is a Nash equilibrium, then $[x^* \leq 1 \wedge y^* \leq 1]$.*

The following corollary follows immediately from Lemma 4.4.

Corollary 4.4. *Let f be a h-liability rule; $f : [0,1]^2 \mapsto [0,\infty)^2$. If f is such that for every application belonging to $\mathscr{A}$, the configuration of costs of due care levels $(c^*, d^*) \in M$ is a Nash equilibrium, then $\overline{s} \leq 2$.*

Theorem 4.1. *Let f be a h-liability rule; $f : [0,1]^2 \mapsto [0,\infty)^2$. If f is efficient for every application belonging to $\mathscr{A}$, then $\overline{s} = 1$.*

Proof. Let h-liability rule $f : [0,1]^2 \mapsto [0,\infty)^2$ be efficient for every application belonging to $\mathscr{A}$. Then from Corollaries 4.1 and 4.3, we have $1 \leq \overline{s} \leq 2$.
Suppose $1 < \overline{s} \leq 2$.
We have $x^* + y^* > 1$ as $\overline{s} > 1$.
Therefore, we obtain: $[1 \geq x^* > 0 \wedge 1 \geq y^* > 0]$, in view of Lemma 4.3.
Let:

(i) $c_0, d_0, \epsilon_1, \epsilon_2 > 0$
(ii) $\beta > 0$
(iii) $\frac{x^*}{x^*+y^*}\beta < \alpha_1 < \theta_1 < x^*\beta$
(iv) $\frac{y^*}{x^*+y^*}\beta < \alpha_2 < \theta_2 < y^*\beta$.

Let C and D be specified as follows:
$C = \{0, c_0, c_0 + \theta_1\}, D = \{0, d_0, d_0 + \theta_2\}$.

For $(c,d) \in C \times D$, let $L(c,d)$ be as given in the following array:

$L(c,d)$

	$d=0$	$d=d_0$	$d=d_0+\theta_2$
$c=0$	$c_0+\epsilon_1+d_0+\epsilon_2+\theta_1+\theta_2$	$c_0+\epsilon_1+\theta_1+\theta_2$	$c_0+\epsilon_1+\theta_1+\alpha_2$
$c=c_0$	$d_0+\epsilon_2+\theta_1+\theta_2$	$\theta_1+\theta_2$	$\theta_1+\alpha_2$
$c=c_0+\theta_1$	$d_0+\epsilon_2+\alpha_1+\theta_2$	$\alpha_1+\theta_2$	$\alpha_1+\alpha_2-\beta$

$\epsilon_1 > 0$, $\epsilon_2 > 0$, $\alpha_1 > 0$, $\alpha_2 > 0$ and $\alpha_1+\alpha_2-\beta > 0$ imply[4] that $M = \{(c_0, d_0)\}$.
Let $(c^*, d^*) = (c_0, d_0)$.
Now,

$$EC_1(c_0+\theta_1, d_0+\theta_2) = c_0+\theta_1+x^*(\alpha_1+\alpha_2-\beta) \tag{4.1}$$

$$EC_1(c_0, d_0+\theta_2) = c_0+x^*(\theta_1+\alpha_2) \tag{4.2}$$

$$EC_1(0, d_0+\theta_2) = x(0,1)(c_0+\epsilon_1+\theta_1+\alpha_2) \tag{4.3}$$

$$EC_2(c_0+\theta_1, d_0+\theta_2) = d_0+\theta_2+y^*(\alpha_1+\alpha_2-\beta) \tag{4.4}$$

$$EC_2(c_0+\theta_1, d_0) = d_0+y^*(\alpha_1+\theta_2) \tag{4.5}$$

$$EC_2(c_0+\theta_1, 0) = y(1,0)(d_0+\epsilon_2+\alpha_1+\theta_2) \tag{4.6}$$

$$\begin{aligned}
&EC_1(c_0+\theta_1, d_0+\theta_2) - EC_1(c_0, d_0+\theta_2) = [c_0+\theta_1+x^*(\alpha_1+\alpha_2-\beta)] \\
&- [c_0+x^*(\theta_1+\alpha_2)] \\
&= (1-x^*)\theta_1 + x^*\alpha_1 - x^*\beta \\
&< (1-x^*)\theta_1 + x^*\theta_1 - x^*\beta, \text{ as } \theta_1 > \alpha_1 > 0 \text{ and } x^* > 0 \\
&= \theta_1 - x^*\beta \\
&< 0
\end{aligned} \tag{4.7}$$

$$\begin{aligned}
&EC_1(c_0+\theta_1, d_0+\theta_2) - EC_1(0, d_0+\theta_2) \\
&= [c_0+\theta_1+x^*(\alpha_1+\alpha_2-\beta)] - x(0,1)[c_0+\epsilon_1+\theta_1+\alpha_2] \\
&\le [c_0+\theta_1+x^*(\alpha_1+\alpha_2-\beta)] - [c_0+\epsilon_1+\theta_1+\alpha_2], \text{ as } x(0,1) \ge 1 \text{ by Lemma 4.1} \\
&= x^*(\alpha_1-\beta) - (1-x^*)\alpha_2 - \epsilon_1 \\
&< 0
\end{aligned} \tag{4.8}$$

[4] Adding the inequalities $\frac{x^*}{x^*+y^*}\beta < \alpha_1$ and $\frac{y^*}{x^*+y^*}\beta < \alpha_2$, we obtain $\alpha_1+\alpha_2-\beta > 0$.

$$
\begin{aligned}
&EC_2(c_0+\theta_1, d_0+\theta_2) - EC_2(c_0+\theta_1, d_0) = [d_0+\theta_2+y^*(\alpha_1+\alpha_2-\beta)] \\
&-[d_0+y^*(\alpha_1+\theta_2)] \\
&= (1-y^*)\theta_2 + y^*\alpha_2 - y^*\beta \\
&< (1-y^*)\theta_2 + y^*\theta_2 - y^*\beta, \text{ as } \theta_2 > \alpha_2 > 0 \text{ and } y^* > 0 \\
&= \theta_2 - y^*\beta \\
&< 0
\end{aligned} \tag{4.9}
$$

$$
\begin{aligned}
&EC_2(c_0+\theta_1, d_0+\theta_2) - EC_2(c_0+\theta_1, 0) \\
&= [d_0+\theta_2+y^*(\alpha_1+\alpha_2-\beta)] - y(1,0)[d_0+\epsilon_2+\alpha_1+\theta_2] \\
&\leq [d_0+\theta_2+y^*(\alpha_1+\alpha_2-\beta)] - [d_0+\epsilon_2+\alpha_1+\theta_2], \text{ as } y(1,0) \geq 1 \text{ by Lemma 4.1} \\
&= y^*(\alpha_2-\beta) - (1-y^*)\alpha_1 - \epsilon_2 \\
&< 0
\end{aligned} \tag{4.10}
$$

(4.7)–(4.10) establish that $(c_0+\theta_1, d_0+\theta_2)$ is a Nash equilibrium. But $(c_0+\theta_1, d_0+\theta_2) \notin M$, which implies that f is not efficient for the application under consideration.

This contradiction establishes the theorem. □

As a h-liability rule $f : [0,1]^2 \mapsto [0,\infty)^2$ exhibits decoupled liability iff $\overline{s} \neq 1$, Theorem 4.1 can also be stated as follows.

Theorem 4.2. *If a h-liability rule $f : [0,1]^2 \mapsto [0,\infty)^2$ exhibits decoupled liability, then it is not the case that f is efficient for every application belonging to $\mathscr{A}$.*

Theorem 4.3. *Let f be a h-liability rule; $f : [0,1]^2 \mapsto [0,\infty)^2$. f has the property that for every application belonging to $\mathscr{A}$ the configuration of costs of due care levels $(c^*, d^*) \in M$ is a Nash equilibrium iff it satisfies the following two conditions:*

(i) $[\forall p, q \in [0,1)][x(p,1) \geq 1 \wedge y(1,q) \geq 1]$
(ii) $x^* \leq 1 \wedge y^* \leq 1$.

Proof. Suppose f has the property that for every application belonging to $\mathscr{A}$ the configuration of costs of due care levels $(c^*, d^*) \in M$ is a Nash equilibrium. Then (i) holds by Lemma 4.2 and (ii) holds by Lemma 4.4.

Next, assume that f satisfies conditions (i) and (ii). Take any application $< C, D, \pi, H, (c^*, d^*) \in M >$ belonging to $\mathscr{A}$. Suppose (c^*, d^*) is not a Nash equilibrium. This implies:

$$
\begin{aligned}
&(\exists c' \in C)[c' + x[p(c'), q(d^*)]L(c', d^*) < c^* + x^*L^*] \ \vee \ (\exists d' \in D) \\
&[d' + y[p(c^*), q(d')]L(c^*, d') < d^* + y^*L^*].
\end{aligned} \tag{4.11}
$$

$$\text{Suppose } (\exists c' \in C)[c' + x[p(c'), q(d^*)]L(c', d^*) < c^* + x^* L^*] \text{ holds.} \tag{4.12}$$

$c' < c^* \wedge (4.12) \to c' + L(c', d^*) < c^* + x^* L^*, \text{ as } x[p(c'), q(d^*)]$

≥ 1 by condition (i)

$\to c' + L(c', d^*) < c^* + L^*, \text{ as } x^* \leq 1$ by condition (ii)

$\to c' + d^* + L(c', d^*) < c^* + d^* + L^*$

$\to \text{TSC}(c', d^*) < \text{TSC}(c^*, d^*).$

This is a contradiction as TSC are minimum at (c^*, d^*). Therefore we conclude:

$$c' < c^* \to (4.12) \text{ cannot hold.} \tag{4.13}$$

For $c' > c^*$, we have: $x[p(c'), q(d^*)] = x(1, 1) = x^*$.
Consequently,

$c' > c^* \wedge (4.12) \to c' + x^* L(c', d^*) < c^* + x^* L^*$

$\to (1 - x^*)c' + x^*[c' + d^* + L(c', d^*)] < (1 - x^*)c^* + x^*[c^* + d^* + L^*]$

$$\to (1 - x^*)c' < (1 - x^*)c^*, \text{ as } \text{TSC}(c', d^*) \geq \text{TSC}(c^*, d^*). \tag{4.14}$$

$$(1 - x^*) > 0 \wedge (4.14) \to c' < c^*, \text{ which contradicts the hypothesis that } c' > c^*. \tag{4.15}$$

$$(1 - x^*) = 0 \wedge (4.14) \to 0 < 0, \text{ a contradiction.} \tag{4.16}$$

(4.15) and (4.16) establish that (4.14) cannot hold. Therefore it follows that:

$$c' > c^* \to (4.12) \text{ cannot hold.} \tag{4.17}$$

(4.13) and (4.17) establish that (4.12) cannot hold.

By an analogous argument one can show that $(\exists d' \in D)[d' + y[p(c^*), q(d')] L(c^*, d') < d^* + y^* L^*]$ cannot hold.

This establishes the theorem. □

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Chapter 5
Negligence as Failure to Take Some Cost-Justified Precaution

In Chap. 3 the notion of negligence was defined in terms of shortfall from the due care level. If an individual's care is less than the due care, then the individual is adjudged as negligent; and if his care is greater than or equal to the due care, he is adjudged to be nonnegligent. In the mainstream of law and economics, this is the way the notion of negligence is defined. This way of conceptualizing the idea of negligence, however, is not entirely uncontroversial. In contradistinction to the mainstream view, it has been argued that the courts do not determine negligence as visualized in the law and economics literature. Rather, whether a party is adjudged to be negligent or not depends on whether the opposite party is able to show the existence of some cost-justified precaution which could have been taken but was not taken. That is to say, a party is deemed to be negligent iff it can be shown that the party could have averted some harm by taking care which would have cost less than the loss due to harm.[1] This chapter is concerned with deriving the implications of this way of defining negligence for the efficiency of liability rules. The main result of this chapter shows that if negligence is determined on the basis of cost-justified untaken precautions, then there is no liability rule which is efficient.

This chapter is divided into two sections. In the first section the idea of negligence as existence of a cost-justified untaken precaution is formalized. The second section discusses the impossibility theorem which states that if negligence is defined as the existence of a cost-justified untaken precaution, then there is no liability rule which is efficient. The proof of the impossibility theorem is given in the appendix at the end of this chapter.[2]

[1] This view has been most consistently, and cogently, articulated by Grady (1983, 1984, 1989).

[2] This chapter draws on Jain (2006).

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5.1 Negligence as Failure to Take Some Cost-Justified Precaution

In what follows, we formalize the notion of negligence as failure to take some cost-justified precaution.

Corresponding to each $(c, d) \in C \times D$, we define:

$$C^u(c, d) = \{c^u \in C \mid c^u > c \wedge L(c, d) - L(c^u, d) > c^u - c\}$$
$$D^u(c, d) = \{d^u \in D \mid d^u > d \wedge L(c, d) - L(c, d^u) > d^u - d\}.$$

Thus, $C^u(c, d)$ is the set of all cost-justified care levels at (c, d) which the victim could have taken; and $D^u(c, d)$ is the set of all cost-justified care levels at (c, d) which the injurer could have taken.

We define:

$$\begin{aligned} \hat{c}(c, d) &= \sup C^u(c, d) && \text{if } C^u(c, d) \neq \emptyset \\ &= c && \text{if } C^u(c, d) = \emptyset \end{aligned}$$

and

$$\begin{aligned} \hat{d}(c, d) &= \sup D^u(c, d) && \text{if } D^u(c, d) \neq \emptyset \\ &= d && \text{if } D^u(c, d) = \emptyset \end{aligned}$$

From the definition of $\hat{c}$, it is clear that $c < \hat{c}$ iff at (c, d) there exists a cost-justified care level which the victim could have taken but did not; and $c = \hat{c}$ iff at (c, d) there does not exist a cost-justified care level which the victim could have taken. Analogously, $d < \hat{d}$ iff at (c, d) there exists a cost-justified care level which the injurer could have taken but did not; and $d = \hat{d}$ iff at (c, d) there does not exist a cost-justified care level which the injurer could have taken.

Given C, D, π, and H, we define nonnegligence functions p and q as follows[3]:

$$\begin{aligned} p : C \times D \mapsto [0, 1] \text{ by} : p(c, d) &= \tfrac{c}{\hat{c}(c,d)} && \text{if } \hat{c}(c, d) \neq 0 \\ &= 1 && \text{if } \hat{c}(c, d) = 0 \\ q : C \times D \mapsto [0, 1] \text{ by} : q(c, d) &= \tfrac{d}{\hat{d}(c,d)} && \text{if } \hat{d}(c, d) \neq 0 \\ &= 1 && \text{if } \hat{d}(c, d) = 0. \end{aligned}$$

Remark 5.1. To define proportions of nonnegligence the way they have been defined here seems to be an appropriate way to do so in a framework where

[3] It should be noted that $\hat{c}(c, d) = 0$ iff $C^u(c, d) = \emptyset$ and $c = 0$; and $\hat{d}(c, d) = 0$ iff $D^u(c, d) = \emptyset$ and $d = 0$.

negligence is determined on the basis of existence of some cost-justified untaken precaution. It should, however, be noted that there are other plausible ways of defining proportions of nonnegligence. For the impossibility result of this chapter, the particular way in which proportions of nonnegligence have been defined here is not crucial. $\Diamond$

Let f be a liability rule. When negligence is defined as failure to take some cost-justified precaution, then an application of f consists of specification of C, D, π, and H satisfying (A1)–(A4). The class of all applications $< C, D, \pi, H >$ satisfying (A1)–(A4) will be denoted by $\mathscr{A}^u$.

Remark 5.2. It should be noted that the notion of an application of a liability rule crucially depends on how negligence is defined. When negligence is defined as failure to take at least the due care, then an application consists of specification of $< C, D, \pi, H, (c^*, d^*) \in M >$ satisfying assumptions (A1)–(A4). Nonnegligence proportions p and q depend on all of $C, D, \pi, H, (c^*, d^*) \in M$. In contrast, when negligence is defined as failure to take some cost-justified precaution, nonnegligence proportions p and q depend only on C, D, π, H. Given C, D, π, H, if there is a unique total social cost (TSC)-minimizing configuration of care levels, then there will be only one application belonging to $\mathscr{A}$ corresponding to C, D, π, H; but in case of multiple TSC-minimizing configurations of care levels, the number of applications belonging to $\mathscr{A}$ corresponding to C, D, π, H will be equal to the number of elements in M. However, corresponding to given C, D, π, H, there will be only one application belonging to $\mathscr{A}^u$ regardless of the number of TSC-minimizing configurations of care levels. $\Diamond$

The following example illustrates some of the definitions introduced above.

Example 5.1. Let liability rule f be the rule of strict liability with the defence of contributory negligence defined by: $(\forall p \in [0,1))(\forall q \in [0,1])[f(p,q) = (1,0)] \wedge (\forall q \in [0,1])[f(1,q) = (0,1)]$.
Consider the following application of f:
$C = D = \{0,1,2\}$.
For $(c,d) \in C \times D$, let $L(c,d)$ be as given in the following array:

$L(c,d)$

	$d=0$	$d=1$	$d=2$
$c=0$	10.0	7.0	5.5
$c=1$	7.0	4.0	2.5
$c=2$	5.5	2.5	1.0

Here $(2,2)$ is the unique TSC-minimizing configuration of costs of care.

For $(c, d) \in C \times D$, we have $C^u(c, d)$ as given in the following array:

$C^u(c, d)$

	$d = 0$	$d = 1$	$d = 2$
$c = 0$	$\{1, 2\}$	$\{1, 2\}$	$\{1, 2\}$
$c = 1$	$\{2\}$	$\{2\}$	$\{2\}$
$c = 2$	$\emptyset$	$\emptyset$	$\emptyset$

And the following array gives $D^u(c, d)$ for $(c, d) \in C \times D$:

$D^u(c, d)$

	$d = 0$	$d = 1$	$d = 2$
$c = 0$	$\{1, 2\}$	$\{2\}$	$\emptyset$
$c = 1$	$\{1, 2\}$	$\{2\}$	$\emptyset$
$c = 2$	$\{1, 2\}$	$\{2\}$	$\emptyset$

Therefore, we obtain:
$(\forall(c, d) \in C \times D)[\hat{c}(c, d) = 2 \ \wedge \ \hat{d}(c, d) = 2]$.

The following array gives, for $(c, d) \in C \times D$, $(EC_1(c, d), EC_2(c, d))$:

$(EC_1(c, d), EC_2(c, d))$

	$d = 0$	$d = 1$	$d = 2$
$c = 0$	$(10, \underline{0})$	$(7, 1)$	$(5.5, 2)$
$c = 1$	$(8, \underline{0})$	$(5, 1)$	$(3.5, 2)$
$c = 2$	$(\underline{2}, 5.5)$	$(\underline{2}, 3.5)$	$(\underline{2}, \underline{3})$

Therefore $(2, 2)$ is the only $(c, d) \in C \times D$ that is a Nash equilibrium. Thus f is efficient for this application. $\Diamond$

Remark 5.3. The notion of negligence as shortfall from due care with due care being defined appropriately from the perspective of minimization of social costs (negligence-dc) and the notion of negligence as existence of a cost-justified untaken precaution (negligence-up) are logically completely independent of each other as can be seen from the two examples which follow:

Example 5.2. Let $C = D = \{0, 1, 2\}$.
For $(c, d) \in C \times D$, let $L(c, d)$ be as given in the following array:

$L(c, d)$

	$d = 0$	$d = 1$	$d = 2$
$c = 0$	10.0	8.5	7.4
$c = 1$	8.5	7.0	6.1
$c = 2$	7.4	6.1	5.3

Total social costs $\mathrm{TSC}(c, d)$, $(c, d) \in C \times D$, are as given in the following array:

$\mathrm{TSC}(c, d)$

	$d = 0$	$d = 1$	$d = 2$
$c = 0$	10.0	9.5	9.4
$c = 1$	9.5	9.0	9.1
$c = 2$	9.4	9.1	9.3

Thus $(1, 1)$ is the unique $\mathrm{TSC}(c, d)$-minimizing configuration of costs of care. If due care level for the injurer (d^*) has to be set appropriately from the perspective of minimization of social costs, then it has to be $d^* = 1$.

Now, consider the situation when both the victim and the injurer are taking 0 care. Under the notion of negligence-dc, the injurer is negligent because $0 < d^*$. The injurer is negligent under the notion of negligence-up as well because if the injurer takes care equal to 1, expected loss will decrease by 1.5 and the cost of care will increase by 1; thus $d = 1$ is a cost-justified untaken precaution.

Next, consider the situation when the victim is taking 0 care and the injurer is taking care = 1. Under the notion of negligence-dc, the injurer is nonnegligent as $1 = d^*$. But the injurer is negligent under the notion of negligence-up because if the injurer takes care equal to 2, expected loss will decrease by 1.1 and the cost of care will increase by 1; thus $d = 2$ is a cost-justified untaken care level.

Consider the situation when the victim is taking 0 care and the injurer is taking care $= 2$. Under the notion of negligence-dc, the injurer is nonnegligent as $2 > d^*$. The injurer is also nonnegligent under the notion of negligence-up because 2 being the highest feasible level of care, the question of there existing a cost-justified untaken precaution does not arise. $\Diamond$

Example 5.3. Let $C = D = \{0, 1\}$.
For $(c, d) \in C \times D$, let $L(c, d)$ be as given in the following array:

$L(c, d)$

	$d = 0$	$d = 1$
$c = 0$	10.0	8.5
$c = 1$	8.6	8.0

$\mathrm{TSC}(c, d)$, $(c, d) \in C \times D$, are as given in the following array:

$\mathrm{TSC}(c, d)$

	$d = 0$	$d = 1$
$c = 0$	10.0	9.5
$c = 1$	9.6	10.0

Thus $(0, 1)$ is the unique $\mathrm{TSC}(c, d)$-minimizing configuration of costs of care. If due care level for the injurer (d^*) has to be set appropriately from the perspective of minimization of social costs, then it has to be $d^* = 1$.

Now, consider the situation when the victim is taking care $c = 1$ and the injurer is taking care $d = 0$. Under the notion of negligence-dc, the injurer is negligent because $0 < d^*$. The injurer is however nonnegligent under the notion of negligence-up because if the injurer takes care equal to 1, expected loss will decrease by 0.6 only and the cost of care will increase by 1; thus there does not exist any cost-justified untaken precaution. ◊

Example 5.2 shows that it is possible for an injurer to be (i) negligent-dc as well as negligent-up, (ii) nonnegligent-dc and negligent-up, and (iii) nonnegligent-dc and nonnegligent-up. Example 5.3 shows that it is possible for an injurer to be (iv) negligent-dc and nonnegligent-up. The complete logical independence of the two notions therefore follows. ◊

5.2 Negligence as Existence of a Cost-Justified Untaken Precaution and Efficiency of Liability Rules: The General Impossibility Theorem

If negligence is defined as failure to take some cost-justified untaken precaution, then there is no liability rule which is efficient with respect to $\mathscr{A}^u$. The proof of this general impossibility theorem is given in the appendix to this chapter. The theorem is proved by showing the following: (i) A necessary condition for a liability rule to be efficient with respect to $\mathscr{A}^u$, when negligence is defined as failure to take some cost-justified precaution, is that the rule satisfies the condition of negligence liability. (ii) A necessary condition for a liability rule to be efficient with respect to $\mathscr{A}^u$, when negligence is defined as failure to take some cost-justified precaution, is that the rule violates the condition of negligence liability. The arguments establishing (i) and (ii) can informally be put as follows.

Suppose liability rule f violates the negligence liability condition, i.e. when one party is nonnegligent (say the victim) and the other party (the injurer) is negligent, the liability assignments are such that the negligent party (the injurer) is not required to bear the entire loss, i.e. for some $q_0 < 1, f(1, q_0) = (x_{q_0}, y_{q_0})$, where $y_{q_0} < 1$. Consider an application of f belonging to $\mathscr{A}^u$ such that (i) TSC-minimizing configuration (c_0, d_0) is unique and both c_0 and d_0 are positive; (ii) if both parties take optimal care, then the expected loss is zero; and (iii) $q_0 d_0$ is an element of D. Given that the victim is using c_0, if we consider a shift by the injurer from d_0 to $q_0 d_0$, then the increase in expected loss $L(c_0, q_0 d_0) - L(c_0, d_0) = L(c_0, q_0 d_0)$ must be greater than the reduction in the cost of care $(1 - q_0)d_0$ as TSC-minimizing configuration is unique. Consequently at $(c_0, q_0 d_0)$ for the injurer d_0 constitutes a cost-justified care level, and thus at $(c_0, q_0 d_0)$, q_0 and $(1 - q_0)$ would be the proportions of nonnegligence and negligence, respectively. The injurer bears only a part of the expected loss, but the entire decrease in the cost of care accrues to the injurer. Therefore, it follows that one can always find an application such that, in addition to (i)–(iii), $L(c_0, q_0 d_0) > (1 - q_0)d_0 > y_{q_0} L(c_0, q_0 d_0)$. This establishes the existence of an application for which the unique TSC-minimizing configuration is not a Nash equilibrium and consequently the existence of an application for which the rule is inefficient. Thus every liability rule violating the condition of negligence liability is inefficient with respect to $\mathscr{A}^u$.

Next, suppose liability rule f satisfies the negligence liability condition. Consider an application of f belonging to $\mathscr{A}^u$ such that (i) TSC-minimizing configuration (c_0, d_0) is unique and both care levels figuring in it are positive, (ii) it is feasible for the victim to take care at the level $c' > c_0$ and for the injurer to take care at the level $d' > d_0$, (iii) at (c', d_0) there does not exist any cost-justified untaken care level for the victim; and at (c_0, d') there does not exist any cost-justified untaken care level for the injurer, (iv) at (c', d') expected loss is positive and sufficiently large, and (v) if from the situation of one party taking more than the optimal care, c' or d' as the case may be, and the other party taking the optimal care, if the party taking the optimal care increases its care to d' or c' as the case may be, then the decrease in expected loss is greater than the increase in the cost of care. Therefore, at (c_0, d') the victim would be negligent as c' would constitute a cost-justified care level; and at (c', d_0) the injurer would be negligent as d' would constitute a cost-justified care level. As the liability share of at least one party when both parties are taking optimal care has to be positive, it follows that for a sufficiently large $L(c', d')$, one or the other party will find it advantageous to shift from the optimal level to c' or d' as the case may be so as to make the other party negligent and bear the entire loss. This establishes the existence of an application for which the unique TSC-minimizing configuration is not a Nash equilibrium and consequently the existence of an application for which the rule is inefficient. Thus, every liability rule satisfying the condition of negligence liability is inefficient with respect to $\mathscr{A}^u$.

From the general impossibility theorem, it follows that, like any other rule, the rule of strict liability with the defence of contributory negligence is not efficient for every application belonging to $\mathscr{A}^u$. The following example shows the inefficiency of strict liability with the defence of contributory negligence for a particular application.

Example 5.4. Consider the following application of the strict liability with the defence of contributory negligence:

$C = D = \{0, 1, 5\}$.

For $(c, d) \in C \times D$, let $L(c, d)$ be as given in the following array:

$L(c, d)$

	$d = 0$	$d = 1$	$d = 5$
$c = 0$	18	14	12
$c = 1$	14	10	8
$c = 5$	12	8	3

Here $(1, 1)$ is the unique TSC-minimizing configuration of costs of care.

For $(c, d) \in C \times D$, we have $C^u(c, d)$ as given in the following array:

$C^u(c, d)$

	$d = 0$	$d = 1$	$d = 5$
$c = 0$	$\{1, 5\}$	$\{1, 5\}$	$\{1, 5\}$
$c = 1$	$\emptyset$	$\emptyset$	$\{5\}$
$c = 5$	$\emptyset$	$\emptyset$	$\emptyset$

And the following array gives $D^u(c, d)$ for $(c, d) \in C \times D$:

$D^u(c, d)$

	$d = 0$	$d = 1$	$d = 5$
$c = 0$	$\{1, 5\}$	$\emptyset$	$\emptyset$
$c = 1$	$\{1, 5\}$	$\emptyset$	$\emptyset$
$c = 5$	$\{1, 5\}$	$\{5\}$	$\emptyset$

Therefore, the following array gives $\hat{c}(c, d)$ for $(c, d) \in C \times D$:

$\hat{c}(c, d)$

	$d = 0$	$d = 1$	$d = 5$
$c = 0$	5	5	5
$c = 1$	1	1	5
$c = 5$	5	5	5

And the following array gives $\hat{d}(c, d)$ for $(c, d) \in C \times D$:

$\hat{d}(c, d)$

	$d = 0$	$d = 1$	$d = 5$
$c = 0$	5	1	5
$c = 1$	5	1	5
$c = 5$	5	5	5

The following array gives, for $(c, d) \in C \times D$, $(EC_1(c, d), EC_2(c, d))$:

$(EC_1(c, d), EC_2(c, d))$

	$d = 0$	$d = 1$	$d = 5$
$c = 0$	$(18, \underline{0})$	$(14, 1)$	$(12, 5)$
$c = 1$	$(\underline{1}, 14)$	$(\underline{1}, 11)$	$(9, \underline{5})$
$c = 5$	$(5, 12)$	$(5, 9)$	$(\underline{5}, \underline{8})$

Therefore $(5, 5)$ is the only $(c, d) \in C \times D$ that is a Nash equilibrium. Thus the strict liability with the defence of contributory negligence rule is inefficient for this application. $\diamondsuit$

Analysis of Chap. 4 showed that internalization of harm by every party, while necessary for efficiency, is not sufficient. The closure with respect to the parties involved in the interaction giving rise to harm is also required. Analysis of this chapter points towards another potential source of inefficiency, namely strategic manipulation. When negligence is defined in the usual way, i.e. in terms of shortfall from the due care level, there is no possibility of strategic manipulation. However, if negligence is defined in terms of cost-justified untaken precautions, then depending on the liability rule, there can be strategic manipulation by one party or the other. In case cares by the two parties are substitutes for each other, in some instances one of the parties by taking less than the due care can render the other party negligent even when that party is taking the due care. In case of cares by the two parties being complements, at times it would be possible for one of the parties to render the other party negligent by taking more than the due care. Although there are liability rules which are not subject to strategic manipulation made possible by defining the notion of negligence in terms of cost-justified untaken precautions, it so happens that the liability rules which are efficient in the bilateral case when negligence is defined as shortfall from due care, i.e. the liability rules satisfying the condition of negligence liability, are all subject to strategic manipulation. Thus when negligence is defined as the existence of a cost-justified untaken precaution, then every liability rule satisfying the condition of negligence liability fails to be efficient because of strategic manipulation and every liability rule violating the condition of negligence liability fails to be efficient because every party is not made to internalize all of the harm, resulting in the impossibility theorem.

Concluding Remarks

Within the framework of the standard tort model, in view of the results of this chapter, a strong case can be made out for defining negligence as shortfall from the due care level. If the assumptions of the standard tort model do not hold, then it is not possible to claim superiority for defining negligence as shortfall from the due care level over defining negligence as the existence of a cost-justified untaken precaution. The costs of determining the socially optimal due care levels by courts can be quite high and may more than offset the efficiency gains. There is an additional point in favour of defining negligence in terms of cost-justified untaken precautions. This approach is more in harmony with the adversarial system than is the case with the due care level approach. In the due care level approach, the courts must necessarily play an active role to gather and process information for calculating the due care levels correctly. Under the untaken precaution approach, it is for the

(continued)

party which wants to establish the negligence of the other party to show the existence of a cost-justified precaution which the opposite party could have taken but did not.

If one wants to accommodate efficiency considerations within the framework of negligence as failure to take cost-justified precautions to the extent possible, one possible way to proceed would be to first define a quasi-ordering[4] over the set of all liability rules $\mathscr{L}$ and then consider the maximal elements of $\mathscr{L}$ with respect to the quasi-ordering as follows: Let f be a liability rule and let $\mathscr{A}_f^i \subseteq \mathscr{A}^u$ denote the set of applications for which f is inefficient. Define a binary relation R over the set of all liability rules by $[(\forall f, g \in \mathscr{L})[fRg \leftrightarrow \mathscr{A}_f^i \subseteq \mathscr{A}_g^i]]$. It is clear that R is reflexive and transitive, i.e. a quasi-ordering. Let $\mathscr{L}_m$ denote the set of maximal elements[5] of $\mathscr{L}$ with respect to R. If one wants to retain the framework of negligence as failure to take cost-justified precautions and has no requirement other than efficiency, then at first glance it would seem that the choice of a liability rule must be made from $\mathscr{L}_m$ only.

Appendix: The Impossibility Theorem

Proposition 5.1. *Let negligence be defined in terms of cost-justified untaken precautions. If a liability rule* $f : [0,1]^2 \mapsto [0,1]^2$ *is efficient for every application belonging to* $\mathscr{A}^u$*, then it satisfies the condition of negligence liability.*

Proof. Let f be any liability rule violating condition NL. As condition NL is violated, we must have:

$[\exists p \in [0,1)][f(p,1) \neq (1,0)] \ \vee \ [\exists q \in [0,1)][f(1,q) \neq (0,1)]$.

Suppose $[\exists q \in [0,1)][f(1,q) \neq (0,1)]$ holds.

Suppose for $q_0 \in [0,1)$ we have:

$f(1,q_0) = (x_{q_0}, y_{q_0}), y_{q_0} \neq 1$.

Let t be a positive number. As $y_{q_0} \in [0,1)$, we have $y_{q_0}t < t$. Choose a positive number r such that $y_{q_0}t < r < t$.

As $q_0 \neq 1, (1-q_0) \neq 0$. Let $d_0 = \frac{r}{1-q_0}$.

Let $0 < \epsilon$ and $0 < c_0$.

Now let C and D be specified as follows:

$C = \{0, c_0\}, D = \{0, q_0 d_0, d_0\}$.

---

[4] A binary relation is called a quasi-ordering iff it is reflexive and transitive.

[5] $l \in \mathscr{L}$ is maximal with respect to R iff $\sim (\exists l' \in \mathscr{L})(l' P l)$, where P is the asymmetric part of R.

For $(c, d) \in C \times D$, let $L(c, d)$ be as given in the following array:

$L(c, d)$

	$d = 0$	$d = q_0 d_0$	$d = d_0$
$c = 0$	$t + q_0 d_0 + c_0 + \epsilon$	$t + c_0 + \epsilon$	$c_0 + \epsilon$
$c = c_0$	$t + q_0 d_0$	t	0

It should be noted that the above specification of C, D, and L is consistent with (A1)–(A4). Furthermore, the specification of L is done in such a way that no inconsistency would arise even if $q_0 = 0$.

As $\epsilon > 0$ and $t > r = (1 - q_0)d_0$, it follows that (c_0, d_0) is the unique TSC-minimizing configuration.

The following array gives $C^u(c, d)$ for $(c, d) \in C \times D$:

$C^u(c, d)$

	$d = 0$	$d = q_0 d_0$	$d = d_0$
$c = 0$	$\{c_0\}$	$\{c_0\}$	$\{c_0\}$
$c = c_0$	$\emptyset$	$\emptyset$	$\emptyset$

And the following array gives $D^u(c, d)$ for $(c, d) \in C \times D$:

$D^u(c, d)$

	$d = 0$	$d = q_0 d_0$	$d = d_0$
$c = 0$	$\{d_0\}$	$\{d_0\}$	$\emptyset$
$c = c_0$	$\{d_0\}$	$\{d_0\}$	$\emptyset$

Therefore, we obtain:

$(\forall (c, d) \in C \times D)[\hat{c}(c, d) = c_0 \ \wedge \ \hat{d}(c, d) = d_0]$.

Now, expected costs of the injurer at $(c_0, q_0 d_0) = EC_2(c_0, q_0 d_0)$

$= q_0 d_0 + y_{q_0} L(c_0, q_0 d_0)$

$= q_0 d_0 + y_{q_0} t$

$EC_2(c_0, d_0) = d_0$

$EC_2(c_0, d_0) - EC_2(c_0, q_0 d_0)$

$= d_0 - q_0 d_0 - y_{q_0} t$

$= (r - y_{q_0} t)$

$> 0.$

This establishes that (c_0, d_0) is not a Nash equilibrium. Thus f is not efficient for the application under consideration. Consequently, it is not the case that f is efficient for every application belonging to $\mathscr{A}^u$. In case $[\exists p \in [0,1)][f(p,1) \neq (1,0)]$ holds, an analogous argument shows that it is not the case that f is an efficient liability rule for every application belonging to $\mathscr{A}^u$.

Thus it follows that if f is efficient for every application belonging to $\mathscr{A}^u$, then it must satisfy condition NL, establishing the proposition. □

Proposition 5.2. *Let negligence be defined in terms of cost-justified untaken precautions. If a liability rule* $f : [0,1]^2 \mapsto [0,1]^2$ *is efficient for every application belonging to* $\mathscr{A}^u$*, then it violates the condition of negligence liability.*

Proof. Let f be any liability rule satisfying condition NL.
Let $f(1,1) = (x^*, y^*)$.
First suppose that $x^* > 0$.
Let t be a positive integer such that $\frac{1}{t} < x^*$.[6]
Choose positive numbers $c_0, d_0, \theta_1, \theta_2, L_0, \epsilon_1, \epsilon_2, \epsilon_3, \epsilon_4, \epsilon_5$ such that:

(i) $\epsilon_3 < \epsilon_2$
(ii) $\epsilon_4 < \epsilon_1$
(iii) $\epsilon_1 < \epsilon_4 + \epsilon_5$
(iv) $\epsilon_2 < \epsilon_3 + \epsilon_5$
(v) $\epsilon_3 + \epsilon_4 + \epsilon_5 < \epsilon_1 + \epsilon_2$
(vi) $\epsilon_1 - \epsilon_4 < \theta_1$
(vii) $\epsilon_2 - \epsilon_3 < \theta_2$
(viii) $L_0 > t\epsilon$, where ϵ is any number greater than 2 [max $\{\epsilon_1, \epsilon_2, \epsilon_3, \epsilon_4, \epsilon_5\}$].[7]

Now consider the following application belonging to $\mathscr{A}^u$:
$C = \{0, c_0, c_0 + \epsilon_1\}, D = \{0, d_0, d_0 + \epsilon_2\}$.
For $(c,d) \in C \times D$, let $L(c,d)$ be as given in the following array:

$L(c,d)$

	$d = 0$	$d = d_0$	$d = d_0 + \epsilon_2$
$c = 0$	$c_0 + d_0 + \theta_1 + \theta_2 + L_0$	$c_0 + \theta_1 + L_0$	$c_0 + \theta_1 + L_0 - \epsilon_3$
$c = c_0$	$d_0 + \theta_2 + L_0$	L_0	$L_0 - \epsilon_3$
$c = c_0 + \epsilon_1$	$d_0 + \theta_2 + L_0 - \epsilon_4$	$L_0 - \epsilon_4$	$L_0 - \epsilon_3 - \epsilon_4 - \epsilon_5$

(c_0, d_0) is the unique TSC-minimizing configuration.

[6]It should be noted that as $x^* \in (0,1]$, it follows that $t > 1$.

[7]This can always be done. $\epsilon_1 = \epsilon_2 = 4, \epsilon_3 = \epsilon_4 = 2, \epsilon_5 = 3, \theta_1 = \theta_2 = 3, \epsilon = 10$, provides a simple example.

We obtain $C^u(c,d)$, $(c,d) \in C \times D$, as given in the following array:

$C^u(c,d)$

	$d=0$	$d=d_0$	$d=d_0+\epsilon_2$
$c=0$	$\{c_0, c_0+\epsilon_1\}$	$\{c_0, c_0+\epsilon_1\}$	$\{c_0, c_0+\epsilon_1\}$
$c=c_0$	$\emptyset$	$\emptyset$	$\{c_0+\epsilon_1\}$
$c=c_0+\epsilon_1$	$\emptyset$	$\emptyset$	$\emptyset$

and $D^u(c,d)$, $(c,d) \in C \times D$, as given in the following array:

$D^u(c,d)$

	$d=0$	$d=d_0$	$d=d_0+\epsilon_2$
$c=0$	$\{d_0, d_0+\epsilon_2\}$	$\emptyset$	$\emptyset$
$c=c_0$	$\{d_0, d_0+\epsilon_2\}$	$\emptyset$	$\emptyset$
$c=c_0+\epsilon_1$	$\{d_0, d_0+\epsilon_2\}$	$\{d_0+\epsilon_2\}$	$\emptyset$

Therefore, we obtain $\hat{c}(c,d)$, $(c,d) \in C \times D$, as given in the following array:

$\hat{c}(c,d)$

	$d=0$	$d=d_0$	$d=d_0+\epsilon_2$
$c=0$	$c_0+\epsilon_1$	$c_0+\epsilon_1$	$c_0+\epsilon_1$
$c=c_0$	c_0	c_0	$c_0+\epsilon_1$
$c=c_0+\epsilon_1$	$c_0+\epsilon_1$	$c_0+\epsilon_1$	$c_0+\epsilon_1$

And $\hat{d}(c,d)$, $(c,d) \in C \times D$, as given in the following array:

$\hat{d}(c,d)$

	$d=0$	$d=d_0$	$d=d_0+\epsilon_2$
$c=0$	$d_0+\epsilon_2$	d_0	$d_0+\epsilon_2$
$c=c_0$	$d_0+\epsilon_2$	d_0	$d_0+\epsilon_2$
$c=c_0+\epsilon_1$	$d_0+\epsilon_2$	$d_0+\epsilon_2$	$d_0+\epsilon_2$

Now, expected costs of the victim at $(c_0, d_0) = EC_1(c_0, d_0)$
$= c_0 + x^* L(c_0, d_0)$
$= c_0 + x^* L_0$

$EC_1(c_0+\epsilon_1, d_0)$

$$= c_0 + \epsilon_1 + x(1, \frac{d_0}{d_0+\epsilon_2})L(c_0+\epsilon_1, d_0)$$

$= c_0 + \epsilon_1$

as $x(1, \frac{d_0}{d_0+\epsilon_2}) = 0$ by condition NL.

$$EC_1(c_0, d_0) - EC_1(c_0 + \epsilon_1, d_0)$$
$$= c_0 + x^* L_0 - c_0 - \epsilon_1$$
$$= x^* L_0 - \epsilon_1$$

Now,
$$x^* > \frac{1}{t} \wedge L_0 > t\epsilon \wedge t > 0 \wedge \epsilon > 0 \rightarrow x^* L_0 > \epsilon$$
$$\rightarrow x^* L_0 > \epsilon_1$$
$$\rightarrow x^* L_0 - \epsilon_1 > 0.$$

Therefore it follows that (c_0, d_0) is not a Nash equilibrium. Thus f is not efficient for the application under consideration. Consequently, it is not the case that f is an efficient liability rule for every application belonging to $\mathscr{A}^u$. By an analogous argument it can be shown that if $y^* > 0$, then it is not the case that f is an efficient rule for every application belonging to $\mathscr{A}^u$. As we must have $(x^* > 0 \vee y^* > 0)$, it follows that it is not the case that f is efficient for every application belonging to $\mathscr{A}^u$.

This establishes that if f is efficient for every application belonging to $\mathscr{A}^u$, then it must violate condition NL. □

Theorem 5.1. *There is no liability rule which is efficient for every application belonging to* $\mathscr{A}^u$.

Proof. Follows immediately from Propositions 5.1 and 5.2. □

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Chapter 6
The Structure of Incremental Liability Rules

Under the negligence rule, as it is usually defined, the entire accident loss falls on the injurer if the injurer is negligent; and the entire loss is borne by the victim if the injurer is nonnegligent. There is, however, another version of negligence rule under which the injurer, if nonnegligent, bears no liability; and, if negligent, bears liability equal to the harm which can be ascribed to his negligence. Suppose the injurer has two options of either taking care which would cost him 1 or not taking care. Furthermore, suppose that injurer's taking care reduces harm from 100 to 90; and that he would be deemed to be negligent iff he does not take care. Then, under the former version of the negligence rule, which we can term as the standard version, if the injurer takes care then the loss of 90 would be borne by the victim; and if he does not take care then the loss of 100 would be borne by him. Under the latter version, which we will call the incremental version, if the injurer takes care then the loss of 90 would be borne by the victim; and if the injurer does not take care then the liability of the injurer would be 10 and the remaining loss of 90 would be borne by the victim. When the injurer does not take care and loss of 100 takes place, only the loss of 10 can be ascribed to the negligence of the injurer as the remaining loss of 90 would have taken place even if the injurer had been nonnegligent.

Incremental negligence rule is a member of a large class of rules which can be termed as the class of incremental liability rules. A standard liability rule determines the proportions, in which the two parties are required to bear the loss in case of occurrence of accident, as a function of proportions of nonnegligence of the two parties. On the other hand, an incremental liability rule is a rule which, as a function of proportions of nonnegligence of the two parties, specifies (i) which of the two parties, the victim or the injurer, is to be the non-residual liability holder and (ii) the proportion of the incremental loss, which can be ascribed to the negligence of the non-residual liability holder, to be borne by the non-residual party. Under the incremental negligence rule, the injurer is the non-residual liability holder for all configurations of proportions of nonnegligence of the two parties; and whenever the injurer is negligent, he is liable for the entire loss which is due to his negligence.

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The main concern of this chapter is with the question of efficiency of incremental liability rules. The necessary and sufficient conditions for an incremental liability rule to be efficient derived in this chapter can be stated as follows. When both parties are nonnegligent, designate the party that is the residual liability holder as r and the party that is the non-residual liability holder as nr. An incremental liability rule is efficient for every admissible application, i.e. every application belonging to $\mathscr{A}$, iff its structure is such that: (i) If party r is negligent and party nr is nonnegligent, then party r must remain the residual liability holder. (ii) If party nr is negligent and party r is nonnegligent, then party nr must either become the residual liability holder or liability of nr must be equal to the entire incremental loss which can be ascribed to the negligence of nr. The efficiency of incremental negligence rule follows as a simple corollary of this general characterization result.

The chapter is divided into three sections.[1] The first section is concerned with the definition and elaboration of the notion of an incremental liability rule. The characterization of efficient incremental liability rules is discussed in the second section. The third section is concerned with the necessary and sufficient conditions for efficiency of incremental liability rules when care is unilateral. These conditions are obtained as special cases of the conditions which characterize efficient incremental liability rules in the bilateral care context. The formal statements and proofs of propositions are relegated to the appendix to the chapter.

6.1 The Notion of an Incremental Liability Rule

Corresponding to each $(c, d) \in C \times D$, we define:

$$\begin{aligned} \hat{L}_1(c,d) &= L(c,d) - L(c^*,d) && \text{if } L(c,d) - L(c^*,d) > 0 \\ &= 0 && \text{if } L(c,d) - L(c^*,d) \leq 0 \end{aligned}$$

and

$$\begin{aligned} \hat{L}_2(c,d) &= L(c,d) - L(c,d^*) && \text{if } L(c,d) - L(c,d^*) > 0 \\ &= 0 && \text{if } L(c,d) - L(c,d^*) \leq 0 \end{aligned}$$

Thus, $\hat{L}_1(c,d)$ is the expected loss which can be ascribed to the negligence of the victim; and $\hat{L}_2(c,d)$ the expected loss which can be ascribed to the negligence of the injurer.

An incremental liability rule is a rule which, as a function of proportions of nonnegligence of the two parties, specifies (i) which of the two parties, the victim (1) or the injurer (2), is to be the non-residual liability holder; and (ii) the proportion of the incremental loss, which can be ascribed to the negligence of the non-residual liability holder, to be borne by the non-residual liability holder.

[1] This chapter draws on Jain (2009).

Formally, an incremental liability rule is a function f from $[0,1]^2$ to $\{1,2\} \times [0,1]$, $f : [0,1]^2 \mapsto \{1,2\} \times [0,1]$, such that $f(p,q) = (a, b_a)$.

We will write x for b_1 and y for b_2.

Remark 6.1. If the nonnegligence proportion of the non-residual party (a) is 1, then $\hat{L}_a = 0$; and consequently the entire expected loss would be borne by the residual party regardless of what b_a is. Therefore, an incremental liability rule can be written in an infinite number of different but equivalent ways. $\diamondsuit$

We denote by EL_1 the expected liability of the victim and by EL_2 the expected liability of the injurer.

Example 6.1. In the notation of this text, the incremental version of the negligence rule is defined by:[2]
$(\forall p \in [0,1])(\forall q \in [0,1))[f(p,q) = (2,1)] \wedge (\forall p \in [0,1])[f(p,1) = (2,0)]$.

In the tabular form, the incremental negligence rule can be written as follows.

Incremental negligence rule

	Injurer negligent $q < 1$ $(d < d^*)$	Injurer nonnegligent $q = 1$ $(d \geq d^*)$
Victim negligent $p < 1$ $(c < c^*)$	$a(p,q) = 2; y = 1$ $EL_1 = L(c,d^*)$; $EL_2 = L(c,d) - L(c,d^*)$	$a(p,1) = 2; y = 0$ $EL_1 = L(c,d); EL_2 = 0$
Victim nonnegligent $p = 1$ $(c \geq c^*)$	$a(1,q) = 2; y = 1$ $EL_1 = L(c,d^*)$; $EL_2 = L(c,d) - L(c,d^*)$	$a(1,1) = 2; y = 0$ $EL_1 = L(c,d); EL_2 = 0$

$\diamondsuit$

Let C, D, π, and H be given. If accident takes place and loss of $H(c,d)$ materializes, then:

If $a[p(c), q(d)] = 1$, then the loss borne by the victim will be:
$= x[p(c), q(d)] \left[H(c,d) - H(c^*,d) \dfrac{\pi(c^*,d)}{\pi(c,d)} \right]$ if $H(c,d) - H(c^*,d) \dfrac{\pi(c^*,d)}{\pi(c,d)} > 0$ and $\pi(c,d) \neq 0$

$= 0$ otherwise;

and the remainder of the loss will be borne by the injurer.

[2] An equivalent, but more concise, way to define incremental negligence rule is to define it by: $(\forall p \in [0,1])(\forall q \in [0,1])[f(p,q) = (2,1)]$.

If $a[p(c), q(d)] = 2$ then the loss borne by the injurer will be:

$= y[p(c), q(d)] \left[H(c,d) - H(c,d^*)\dfrac{\pi(c,d^*)}{\pi(c,d)} \right]$ if $H(c,d) - H(c,d^*)\dfrac{\pi(c,d^*)}{\pi(c,d)} > 0$

and $\pi(c,d) \neq 0$

$= 0$ otherwise;

and the remainder of the loss will be borne by the victim.

The expected costs of the victim and the injurer, to be denoted by EC_1 and EC_2, respectively, therefore, are:

$\left[c + x[p(c), q(d)]\hat{L}_1(c,d)\right]$ and $\left[d + L(c,d) - x[p(c), q(d)]\hat{L}_1(c,d)\right]$, respectively, if $a[p(c), q(d)] = 1$; and

$\left[c + L(c,d) - y[p(c), q(d)]\hat{L}_2(c,d)\right]$ and $\left[d + y[p(c), q(d)]\hat{L}_2(c,d)\right]$, respectively, if $a[p(c), q(d)] = 2$.

Example 6.2. Consider, for the incremental negligence rule, the application given below.

Let $C = D = \{0, 1, 2\}$.
For $(c,d) \in C \times D$, let $L(c,d)$ be as given in the following array:

$L(c,d)$

	$d=0$	$d=1$	$d=2$
$c=0$	20.0	18.0	16.5
$c=1$	18.0	16.5	15.4
$c=2$	16.5	15.5	15.0

Total social costs $(TSC)(c,d)$, $(c,d) \in C \times D$, are as given in the following array:

$TSC(c,d)$

	$d=0$	$d=1$	$d=2$
$c=0$	20.0	19.0	18.5
$c=1$	19.0	18.5	18.4
$c=2$	18.5	18.5	19.0

Thus $(1,2)$ is the unique TSC-minimizing configuration of costs of care.
Let $(c^*, d^*) = (1, 2)$.

The following array gives, for $(c, d) \in C \times D$, $(EL_1(c, d), EL_2(c, d))$:

$(EL_1(c, d), EL_2(c, d))$

	$d = 0$	$d = 1$	$d = 2$
$c = 0$	$(16.5, 3.5)$	$(16.5, 1.5)$	$(16.5, 0)$
$c = 1$	$(15.4, 2.6)$	$(15.4, 1.1)$	$(15.4, 0)$
$c = 2$	$(15.0, 1.5)$	$(15.0, 0.5)$	$(15.0, 0)$

The following array gives, for $(c, d) \in C \times D$, $(EC_1(c, d), EC_2(c, d))$:

$(EC_1(c, d), EC_2(c, d))$

	$d = 0$	$d = 1$	$d = 2$
$c = 0$	$(16.5, 3.5)$	$(16.5, 2.5)$	$(16.5, \underline{2.0})$
$c = 1$	$(\underline{16.4}, 2.6)$	$(\underline{16.4}, 2.1)$	$(\underline{16.4}, \underline{2.0})$
$c = 2$	$(17.0, \underline{1.5})$	$(17.0, \underline{1.5})$	$(17.0, 2.0)$

Therefore $(1, 2)$ is the only $(c, d) \in C \times D$ which is a Nash equilibrium. Thus the incremental negligence rule is efficient for the application considered here. $\Diamond$

6.2 Characterization of Efficient Incremental Liability Rules

Incremental liability rules which are efficient for all applications belonging to $\mathscr{A}$ are characterized by the following conditions:

If $a(1, 1) = 1$, then:

(i) $[a(1, q) = 1]$ for every $q < 1$ and
(ii) $[a(p, 1) = 1 \rightarrow x(p, 1) = 1]$ for every $p < 1$.

If $a(1, 1) = 2$, then:

(i) $[a(p, 1) = 2]$ for every $p < 1$ and
(ii) $[a(1, q) = 2 \rightarrow y(1, q) = 1]$ for every $q < 1$.

Schematically these conditions can be represented as follows (Tables 6.1 and 6.2):

Table 6.1 Non-residual liability holder when both parties nonnegligent = victim

	Injurer negligent $q < 1$	Injurer nonnegligent $q = 1$
Victim negligent $p < 1$		$a(p, 1) = 1 \rightarrow x(p, 1) = 1$
Victim nonnegligent $p = 1$	$a(1, q) = 1$	

Table 6.2 Non-residual liability holder when both parties nonnegligent = injurer

	Injurer negligent $q < 1$	Injurer nonnegligent $q = 1$
Victim negligent $p < 1$		$a(p, 1) = 2$
Victim nonnegligent $p = 1$	$a(1, q) = 2 \rightarrow y(1, q) = 1$	

The above necessary and sufficient conditions for efficiency can be restated as follows: Let the party which is the residual liability holder when both parties are nonnegligent be designated by r and the other party (non-residual liability holder) by nr. An incremental liability rule is efficient for every application belonging to $\mathscr{A}$ iff its structure is such that: (i) If party r is negligent and party nr is nonnegligent, then party r must remain the residual liability holder. (ii) If party nr is negligent and party r is nonnegligent, then party nr must either become the residual liability holder or liability of nr must be equal to the entire incremental loss which can be ascribed to its negligence.

In view of the fact that when the party which is the non-residual liability holder is nonnegligent, the entire loss falls on the residual liability holder, the necessary and sufficient conditions for efficiency imply that an incremental liability rule is efficient for all applications belonging to $\mathscr{A}$ iff: (i) If party r is negligent and party nr is nonnegligent, then the entire loss must be borne by party r. (ii) If party nr is negligent and party r is nonnegligent, then party nr must either bear the entire loss or must bear the entire incremental loss which can be ascribed to its negligence.

The formal statement of the characterization theorem and its proof are given in the appendix to this chapter. The characterization theorem is proved via six propositions; we discuss the logic of these propositions here informally, formal proofs being given in the appendix.

Proposition 6.1 establishes that a necessary condition for an incremental liability rule to be efficient with respect to $\mathscr{A}$ is: $[a(1,1) = 1 \rightarrow (\forall q \in [0,1))[a(1,q) = 1]]$. That is to say, if the non-residual liability holder, when both parties are nonnegligent, is the victim, then whenever the victim is nonnegligent and the injurer is negligent, the victim must be the non-residual liability holder. Suppose the liability rule f is such that when both parties are nonnegligent, the victim is the non-residual liability holder; but the injurer is the non-residual liability holder in some situation when the victim is nonnegligent and the injurer is negligent, i.e. $a(1,1) = 1$ but for some $q' < 1$, $a(1,q') = 2$. Consider an application of f belonging to $\mathscr{A}$ such that: (i) TSC-minimizing configuration is unique, and both care levels figuring in it are positive; (ii) when both parties take optimal care, the expected loss $L(c^*, d^*) = L^*$ is positive; and (iii) $q'd^*$ is an element of D. Given that the victim is using c^*, if we consider a shift by the injurer from d^* to $q'd^*$, then the increase in expected loss $L(c^*, q'd^*) - L(c^*, d^*)$, which is equal to the expected loss which can be ascribed to the negligence of the injurer, must be greater than the reduction in the cost of care $(1 - q')d^*$. The shift, however, will be beneficial to the injurer if the excess of increase in expected loss over reduction in the cost of care $L(c^*, q'd^*) - L(c^*, d^*) - (1 - q')d^*$ is less than L^*. As one can always find an application such that, in addition to (i)–(iii), $L(c^*, q'd^*) - L(c^*, d^*) - (1 - q')d^* < L^*$, it follows that there exists an application for which the unique TSC-minimizing configuration is not a Nash equilibrium. This establishes the existence of an application for which the rule is inefficient and therefore establishes the necessity of $[a(1,1) = 1 \to (\forall q \in [0,1))[a(1,q) = 1]]$ for efficiency.

Proposition 6.2 which states that a necessary condition for an incremental liability rule to be efficient with respect to $\mathscr{A}$ is: $[a(1,1) = 2 \to (\forall p \in [0,1))[a(p,1) = 2]]$, i.e. if the non-residual liability holder, when both parties are nonnegligent, is the injurer, then whenever the injurer is nonnegligent and the victim is negligent, the injurer must be the non-residual liability holder, is established analogously.

Proposition 6.3 establishes that a necessary condition for an incremental liability rule to be efficient with respect to $\mathscr{A}$ is: $[a(1,1) = 1 \to (\forall p \in [0,1))[a(p,1) = 1 \to x(p,1) = 1]]$. That is to say, if the non-residual liability holder, when both parties are nonnegligent, is the victim, then whenever the victim is negligent and the injurer is nonnegligent, in case the victim is the non-residual liability holder, the victim's liability must be equal to the entire incremental loss which can be ascribed to his negligence. Suppose the liability rule f is such that when both parties are nonnegligent the victim is the non-residual liability holder; but in some situation when the victim is negligent and the injurer is nonnegligent, the victim is the non-residual liability holder and his liability is less than the incremental loss which can be ascribed to his negligence, i.e. $a(1,1) = 1$ and for some $p' < 1$, $a(p',1) = 1 \wedge x(p',1) < 1$. Consider an application of f belonging to $\mathscr{A}$ such that: (i) TSC-minimizing configuration is unique, and both care levels figuring in it are positive, and (ii) $p'c^*$ is an element of C. Given that the injurer is using d^*, if we consider a shift by the victim from c^* to $p'c^*$, then the increase in expected loss $L(p'c^*, d^*) - L(c^*, d^*)$, which is equal to the expected loss which can be ascribed to the negligence of the victim, must be greater than the reduction in the cost of care $(1 - p')c^*$. The shift, however, can be beneficial to the victim as the victim bears only a part of the increase in expected loss. As one can always find an application such that, in addition to (i)–(ii), we have $L(p'c^*, d^*) - L(c^*, d^*) > (1 - p')c^* > x(p',1)[L(p'c^*, d^*) - L(c^*, d^*)]$, it follows that there exists an application for which the unique TSC-minimizing configuration is not a Nash equilibrium. This establishes the existence of an application for which the rule is inefficient and therefore establishes the necessity of $[a(1,1) = 1 \to (\forall p \in [0,1))[a(p,1) = 1 \to x(p,1) = 1]]$ for efficiency.

Proposition 6.4 which states that a necessary condition for an incremental liability rule to be efficient with respect to $\mathscr{A}$ is: $[a(1,1) = 2 \to (\forall q \in [0,1))[a(1,q) = 2 \to y(1,q) = 1]]$, i.e. if the non-residual liability holder, when both parties are nonnegligent, is the injurer, then whenever the injurer is negligent and the victim is nonnegligent, in case the injurer is the non-residual liability holder, he must be liable for the entire incremental loss which can be ascribed to his negligence, is established analogously.

Proposition 6.5 establishes that if an incremental liability rule satisfies the following four properties: (i) $a(1,1) = 1 \to (\forall q \in [0,1))[a(1,q) = 1]$, (ii) $a(1,1) = 2 \to (\forall p \in [0,1))[a(p,1) = 2]$, (iii) $a(1,1) = 1 \to (\forall p \in [0,1))[a(p,1) = 1 \to x(p,1) = 1]$, and (iv) $a(1,1) = 2 \to (\forall q \in [0,1))[a(1,q) = 2 \to y(1,q) = 1]$, then no matter which application $< C, D, \pi, H, (c^*, d^*) \in M >$

belonging to $\mathscr{A}$ is considered, (c^*, d^*) constitutes a Nash equilibrium. Suppose the injurer is using d^*. Consider a change by the victim from c^* to some $c > c^*$. Now, the reduction in expected loss $L(c^*, d^*) - L(c, d^*)$ cannot exceed the increase in cost of care $c - c^*$; otherwise (c^*, d^*) could not have been a TSC-minimizing configuration. While the increase in the cost of care falls on the victim, the benefit of decrease in expected loss accrues to the injurer in case $a(1, 1) = 1$ and to the victim in case $a(1, 1) = 2$. In either case, the victim derives no benefit by moving from c^* to some $c > c^*$. Next, consider a change from c^* to some $c < c^*$. The increase in expected loss $L(c, d^*) - L(c^*, d^*)$ must be at least as large as the decrease in cost of care $c^* - c$; otherwise (c^*, d^*) could not have been TSC-minimizing. If $a(1, 1) = 1$ as well as $a\left(\frac{c}{c^*}, 1\right) = 1$, then by (iii) the increase in victim's expected liability will be equal to the increase in expected loss. Thus, the move from c^* to some $c < c^*$ would not be advantageous to him. If $a(1, 1) = 1$ and $a\left(\frac{c}{c^*}, 1\right) = 2$, then the increase in victim's expected liability would be equal to the entire expected loss at (c, d^*), not merely the increase in expected loss $L(c, d^*) - L(c^*, d^*)$. If $a(1, 1) = 2$, then by (ii), the entire liability is on the victim both at c^* and at c. Consequently, whether $\left[a(1, 1) = 1 \wedge a\left(\frac{c}{c^*}, 1\right) = 2\right]$ or $a(1, 1) = 2$, a move from c^* to some $c < c^*$ could not be advantageous to him in view of the increase in expected loss being at least as large as the decrease in cost of care. Thus, given the choice of d^* by the injurer, c^* is best for the victim. Analogously, one shows that, given the choice of c^* by the victim, d^* is best for the injurer.

Proposition 6.6 establishes that if an incremental liability rule satisfies the following four properties: (i) $a(1, 1) = 1 \to (\forall q \in [0, 1))[a(1, q) = 1]$, (ii) $a(1, 1) = 2 \to (\forall p \in [0, 1))[a(p, 1) = 2]$, (iii) $a(1, 1) = 1 \to (\forall p \in [0, 1))[a(p, 1) = 1 \to x(p, 1) = 1]$, and (iv) $a(1, 1) = 2 \to (\forall q \in [0, 1))[a(1, q) = 2 \to y(1, q) = 1]$, then no matter which application $< C, D, \pi, H, (c^*, d^*) \in M >$ belonging to $\mathscr{A}$ is considered, all Nash equilibria would be TSC-minimizing. Suppose $(\overline{c}, \overline{d})$ is a Nash equilibrium. First, consider the case when $a(1, 1) = 1$. We calculate the expected costs of the victim EC_1 when his care is c^* and injurer's care is $\overline{d}$ for different values of $\overline{d}$. If $\overline{d} \geq d^*$, then the victim's expected liability would be 0; and if $\overline{d} < d^*$, then again the victim's expected liability would be 0 in view of (i). Therefore $EC_1(c^*, \overline{d}) = c^*$ regardless of the value of $\overline{d}$. Next, we calculate the expected costs of the injurer EC_2 when his care is d^* and victim's care is $\overline{c}$ for different values of $\overline{c}$. If $\overline{c} \geq c^*$, then the injurer's expected liability would be $L(\overline{c}, d^*) \leq L(c^*, d^*)$; if $\overline{c} < c^*$ and $a\left(\frac{\overline{c}}{c^*}, 1\right) = 1$, then the injurer's expected liability would be $L(c^*, d^*)$ in view of (iii), and if $\overline{c} < c^*$ and $a\left(\frac{\overline{c}}{c^*}, 1\right) = 2$, then the injurer's expected liability would be 0. Therefore, $EC_2(\overline{c}, d^*)$ would be at most equal to $d^* + L^*$. Thus we see that the sum of $EC_1(c^*, \overline{d})$ and $EC_2(\overline{c}, d^*)$ cannot exceed $c^* + d^* + L^* = TSC(c^*, d^*)$. In view of the fact that $(\overline{c}, \overline{d})$ is a

Nash equilibrium, we have $EC_1(\overline{c},\overline{d}) \leq EC_1(c^*,\overline{d})$ and $EC_2(\overline{c},\overline{d}) \leq EC_2(\overline{c},d^*)$. Therefore $TSC(\overline{c},\overline{d}) = TSC(c^*,d^*)$, as minimum of TSC is attained at (c^*,d^*).

Propositions 6.5 and 6.6 establish the sufficiency of (i)–(iv) for efficiency; and Propositions 6.1–6.4 their necessity. Therefore it follows that efficiency for incremental liability rules is characterized by conditions (i)–(iv).

It is interesting to compare the necessary and sufficient conditions for efficiency of incremental liability rules with the necessary and sufficient conditions for efficiency of standard liability rules. As shown in Chap. 3, a standard liability rule is efficient for all applications belonging to $\mathscr{A}$ iff it satisfies the condition of negligence liability. While the efficiency-characterizing conditions in the two cases are similar, one important difference should be noted. Negligence liability condition does not impose any restrictions on the liability assignments when both parties are nonnegligent or both parties are negligent. For incremental liability rules also the necessary and sufficient conditions for efficiency do not impose any restrictions on liability assignments when both parties are negligent. However, when both parties are nonnegligent, from the very definition of an incremental liability rule, it follows that the entire loss must be borne by one of the two parties alone.

It is interesting to analyse the structure of efficient incremental liability rules when the non-residual liability holder is the same for all $(p,q) \in [0,1]^2$. The necessary and sufficient conditions for efficiency reduce to:

$\big(\forall(p,q) \in [0,1]^2\big)\ [a(p,q) = 1] \rightarrow (\forall p \in [0,1))[x(p,1) = 1]$
and
$\big(\forall(p,q) \in [0,1]^2\big)\ [a(p,q) = 2] \rightarrow (\forall q \in [0,1))[y(1,q) = 1]$.

In other words, given that the victim is the non-residual liability holder regardless of nonnegligence proportions of the two parties, an incremental liability rule is efficient iff it is the incremental version of the strict liability with the defence of contributory negligence. And, given that the injurer is the non-residual liability holder regardless of nonnegligence proportions of the two parties, an incremental liability rule is efficient iff only it is the incremental version of the negligence rule.

6.3 Unilateral Care

The case of unilateral care obtains when only one party can take care or optimal care by one party is zero. If the victim cannot take care or optimal care by him is zero, then it is impossible for him to be negligent. Thus for analysing the unilateral case in which only injurer's care matters, it is convenient to restrict the domain of rules to situations where $p = 1$. Similarly, for analysing the unilateral case in which only the victim's care matters, it would be convenient to restrict the domain of rules to situations where $q = 1$.

If one considers the restricted domain $\{(1, q) \mid q \in [0, 1]\}$, then the necessary and sufficient conditions for efficiency of incremental liability rules reduce to:

If $a(1, 1) = 1$, then : $[a(1, q) = 1]$ for every $q < 1$
If $a(1, 1) = 2$, then : $[a(1, q) = 2 \rightarrow y(1, q) = 1]$ for every $q < 1$.

In other words, if only the injurer's care matters, then an incremental liability rule is efficient iff:

(i) Victim is the non-residual liability holder regardless of nonnegligence proportion of the injurer. In other words, the rule is that of strict liability. Given that $p = 1$, the rule can also be viewed as the incremental version of strict liability with the defence of contributory negligence.
Or
(ii) Injurer is the non-residual liability holder when he is nonnegligent. When he is negligent then: either the victim is the non-residual liability holder or alternatively if the injurer is the non-residual liability holder then his liability is equal to the incremental loss which can be ascribed to his negligence.

Given that $p = 1$, holding of non-residual liability by the victim is equivalent to saying that the entire liability falls on the injurer. Thus the statement that 'either the victim is the non-residual liability holder or alternatively if the injurer is the non-residual liability holder then his liability is equal to the incremental loss which can be ascribed to his negligence' is equivalent to saying that either injurer is liable for the entire loss or he is liable for the incremental loss which can be ascribed to his negligence. Therefore, (ii) can alternatively be stated as follows:

(ii) If injurer is nonnegligent, then the entire liability falls on the victim. If injurer is negligent, then his liability is equal to the entire loss or equal to the incremental loss which can be ascribed to his negligence.

One implication of (ii) is that for different values of $q \in [0, 1)$, one can arbitrarily assign full liability to the injurer or liability equal to the incremental loss which can be ascribed to his negligence without in any way affecting efficiency. For instance, the rule which is identical to the incremental version of the negligence rule defined for the restricted domain $\{1\} \times [0, 1]$ everywhere excepting for $q = \frac{1}{2}$, where the liability of the injurer is equal to the entire loss, is an efficient rule.

Analogous results hold for the unilateral care case when only the victim's care matters. Necessary and sufficient conditions for efficiency of an incremental liability rule when the domain of the rule is restricted to $\{(p, 1) \mid p \in [0, 1]\}$ are given by:

If $a(1, 1) = 1$, then : $[a(p, 1) = 1 \rightarrow x(p, 1) = 1]$ for every $p < 1$
If $a(1, 1) = 2$, then : $[a(p, 1) = 2]$ for every $p < 1$.

Thus, when only the victim's care matters, an incremental liability rule is efficient iff:

(i) If victim is nonnegligent, then the entire liability falls on the injurer. If victim is negligent, then his liability is equal to the entire loss or equal to the incremental loss which can be ascribed to his negligence.
Or
(ii) Injurer is the non-residual liability holder regardless of nonnegligence proportion of the victim. In other words, the rule is that of no liability. Given that $q = 1$, the rule can also be viewed as the incremental version of negligence rule.

Concluding Remarks

One implication of the results of this chapter on necessary and sufficient conditions for efficiency for the class of incremental liability rules is that the set of efficient rules is much larger than the set of efficient standard liability rules. If efficiency were the only normative criterion which had significance then, an efficient rule from efficiency perspective being as good as any other efficient rule, such an implication would be of no or little value. But that obviously is not the case. The importance of non-efficiency values would be recognized even by those who would give primacy to efficiency over all other values. If non-efficiency values do matter, then it is clear that the larger the class of efficient rules from which the society can choose, the better in general would be the possibilities of attaining the desired values. Instances of non-efficiency considerations which might be considered important are provided by distributive and fairness criteria. The evaluation of different efficient rules would differ depending on which non-efficiency normative criteria are considered important. As in different contexts different non-efficiency criteria might be considered relevant, the importance of having a large class of efficient rules cannot be overemphasized.

In the context of non-efficiency criteria, it is important to consider the cases of liability rules working perfectly and working imperfectly separately. While different perfectly working liability rules in general have different distributive implications, it is also possible that the two rules which have identical distributive implications when they work perfectly have very different distributive implications when they work only imperfectly. For instance, both standard and incremental versions of the negligence rule have the same distributive implications when they work perfectly. Under both, both parties take optimal care and the entire loss is borne by the victim. However, if the rules work only imperfectly, then the incremental negligence rule tends to favour injurers more compared to the standard negligence rule. If an injurer makes a mistake regarding due care and is consequently found negligent, his liability under the standard negligence rule in general would be greater than his liability under the incremental negligence rule. Similarly, the distributive

(continued)

implications of the two versions, standard and incremental, of strict liability with the defence of contributory negligence are identical when they work perfectly; but not when they work imperfectly. The incremental version tends to favour victims more compared to the standard version when victims are found negligent because of mistakes regarding due care. The point is a general one. Even when two efficient rules do not differ with respect to some non-efficiency criterion when they function perfectly, they may differ with respect to the criterion when they function imperfectly. As in most real situations, some imperfection in the working of rules can be expected, not only the implications of the rules from the perspective of non-efficiency values when they work perfectly but also those when they work imperfectly are relevant from the point of view of making social choices regarding liability rules. Once again, it is clear that the larger the set of efficient rules, the better it is likely to be from the perspective of non-efficiency values.

Even from the perspective of efficiency, rules which are efficient under ideal conditions perform differently when there are errors, biases, uncertainty, or some other imperfection. For instance, if due care is moderately greater than the optimal care, then under the standard negligence rule the injurer would take the due care, but under the incremental negligence rule the injurer would take the optimal care. Thus, under errors of moderate overestimation of due care, the incremental negligence rule is superior to the standard negligence rule from efficiency perspective.

From the above considerations the following general point emerges. Suppose that of the two rules f_1 and f_2, both efficient under ideal conditions, under one kind of errors f_1 performs better than f_2 from the perspective of efficiency, and under another kind of errors f_2 performs better than f_1; then, if efficiency is the sole objective or alternatively if the two rules are likely to perform equally well with respect to relevant non-efficiency criteria, then when errors of the former kind are likely to predominate f_1 should be used in preference to f_2, and when errors of the latter type are likely to predominate then f_2 should be used in preference of f_1. Indeed, whenever two rules, efficient under ideal conditions, perform differently under mistakes, errors or biases from the perspective of efficiency, one obtains a policy implication. Again it is evident that for any specific kind of errors, the larger the set of rules which one has at one's disposal, the better one is likely to do in terms of the social goal of efficiency.

Unlike the case of liability rules, what is required for efficiency of incremental liability rules, although not much dissimilar from what is required for efficiency of ordinary liability rules, is asymmetric with respect to the two parties. This asymmetry is potentially of considerable significance. If one wants to view the harmful interaction as essentially asymmetric in the sense of being caused by the injurer, then the requirement for efficiency in the case

(continued)

of standard liability rules can easily conflict with the intuitive requirement for justice. The notion of incremental liability rule is potentially of great use in such contexts as by making the victim as the non-residual liability holder, one can have efficiency without any significant sacrifice of requirement of justice.

The asymmetry of incremental liability rules is of significance not only for reducing conflicts between efficiency and intuitive notions of justice but also for enhancing efficiency in certain contexts. For instance, if the context is such that in the event of a large loss one of the parties is likely to be judgment-proof then, by the choice of an appropriate incremental liability rule in which the party likely to be judgment-proof is the non-residual liability holder, the possibilities of socially inappropriate behaviour on the part of the party likely to be judgment-proof could be attenuated or even eliminated altogether. Although both the incremental and the ordinary variants of the negligence rule are efficient under the standard assumptions and neither is efficient in the presence of errors in the determination of actual care levels, it can be argued that in the presence of uncertainty in the determination of negligence the inducement for taking socially wasteful excessive care is less in the case of the incremental negligence rule than in the case of the ordinary negligence rule.[3] Thus, potentially both the asymmetric and the incremental aspects of incremental liability rules can be made use of for enhancing efficiency in contexts where all of the assumptions of the standard tort model do not hold.

Appendix

Proposition 6.1. *Let incremental liability rule f be efficient with respect to $\mathscr{A}$. Then we must have:*
$[a(1,1) = 1 \rightarrow (\forall q \in [0,1))[a(1,q) = 1]]$.

Proof. Let f be an incremental liability rule. Suppose $[a(1,1) = 1 \wedge (\exists q \in [0,1))[a(1,q) = 2]]$. Assume $a(1,q_0) = 2, q_0 < 1$.
Let (i) $0 < \epsilon_1, \epsilon_2, c_0, d_0$ and (ii) $\epsilon_2 < L_0$.
Let C and D be specified as follows:
$C = \{0, c_0\}, D = \{0, q_0 d_0, d_0\}$.

[3] See Grady (1983, 1984) and Dari-Mattiacci (2005), among others.

For $(c,d) \in C \times D$, let $L(c,d)$ be as given in the following array:

$L(c,d)$

	$d=0$	$d=q_0d_0$	$d=d_0$
$c=0$	$c_0+\epsilon_1+d_0+\epsilon_2+L_0$	$c_0+\epsilon_1+(1-q_0)d_0+\epsilon_2+L_0$	$c_0+\epsilon_1+L_0$
$c=c_0$	$d_0+\epsilon_2+L_0$	$(1-q_0)d_0+\epsilon_2+L_0$	L_0

It should be noted that the above specification of C, D, and L is consistent with (A1)–(A4). Furthermore, the specification of L is done in such a way that no inconsistency would arise even if $q_0 = 0$.

As $\epsilon_1 > 0$ and $\epsilon_2 > 0$, it follows that (c_0, d_0) is the unique TSC-minimizing configuration. Let $(c^*, d^*) = (c_0, d_0)$.

Now, expected costs of the injurer at $(c_0, q_0d_0) = EC_2(c_0, q_0d_0)$
$= q_0d_0 + y(1,q_0)\hat{L}_2(c_0, q_0d_0)$, as $a(1,q_0) = 2$

$= q_0d_0 + y(1,q_0)[L(c_0,q_0d_0) - L(c_0,d_0)]$

$= q_0d_0 + y(1,q_0)[L_0 + (1-q_0)d_0 + \epsilon_2 - L_0]$

$\leq q_0d_0 + (1-q_0)d_0 + \epsilon_2$

$= d_0 + \epsilon_2$

$EC_2(c_0, d_0)$
$= d_0 + L(c_0,d_0) - x(1,1)\hat{L}_1(c_0,d_0)$, as $a(1,1) = 1$

$= d_0 + L_0$

Therefore,
$EC_2(c_0,d_0) - EC_2(c_0,q_0d_0)$

$\geq L_0 - \epsilon_2$

$> 0.$

This establishes that (c_0, d_0) is not a Nash equilibrium. Consequently f is not an efficient incremental liability rule for every application belonging to $\mathscr{A}$, establishing the proposition. $\square$

Analogously the following proposition can be established.

Proposition 6.2. *Let incremental liability rule f be efficient with respect to $\mathscr{A}$. Then we must have:*
$[a(1,1) = 2 \rightarrow (\forall p \in [0,1))[a(p,1) = 2]]$.

Proposition 6.3. *Let incremental liability rule f be efficient with respect to $\mathscr{A}$. Then we must have:*
$[a(1,1)=1 \rightarrow (\forall p \in [0,1))[a(p,1)=1 \rightarrow x(p,1)=1]]$.

Proof. Let f be an incremental liability rule. Suppose $[a(1,1) = 1 \wedge (\exists p \in [0,1))[a(p,1)=1 \wedge x(p,1)<1]]$. Assume $[a(p_0,1)=1 \wedge x(p_0,1)<1], p_0<1$.
Choose positive numbers $c_0, d_0, L_0, \epsilon_1, \epsilon_2$ such that:
if $x(p_0,1)>0$ then $\epsilon_1 < (1-p_0)c_0\frac{1-x(p_0,1)}{x(p_0,1)}$.

Now consider the following application belonging to $\mathscr{A}$:
$C=\{0, p_0c_0, c_0\}, D=\{0,d_0\}$.
For $(c,d) \in C \times D$, let $L(c,d)$ be as given in the following array:

$L(c,d)$

	$d=0$	$d=d_0$
$c=0$	$c_0+d_0+\epsilon_1+\epsilon_2+L_0$	$c_0+\epsilon_1+L_0$
$c=p_0c_0$	$(1-p_0)c_0+d_0+\epsilon_1+\epsilon_2+L_0$	$(1-p_0)c_0+\epsilon_1+L_0$
$c=c_0$	$d_0+\epsilon_2+L_0$	L_0

In view of the fact that $\epsilon_1>0$ and $\epsilon_2>0$, it follows that (c_0,d_0) is the unique TSC-minimizing configuration. Let $(c^*,d^*)=(c_0,d_0)$.
Expected costs of the victim at $(c_0,d_0) = EC_1(c_0,d_0)=c_0$, as $a(1,1)=1$

$$EC_1(p_0c_0,d_0)$$
$$= p_0c_0 + x(p_0,1)\hat{L}_1(p_0c_0,d_0), \text{as } a(p_0,1)=1$$
$$= p_0c_0 + x(p_0,1)[L(p_0c_0,d_0)-L(c_0,d_0)]$$
$$= p_0c_0 + x(p_0,1)[(1-p_0)c_0+\epsilon_1]$$

$$EC_1(c_0,d_0)-EC_1(p_0c_0,d_0)$$
$$= c_0 - p_0c_0 - x(p_0,1)[(1-p_0)c_0+\epsilon_1]$$
$$= (1-p_0)c_0 - x(p_0,1)[(1-p_0)c_0+\epsilon_1]$$
$$= (1-x(p_0,1))(1-p_0)c_0 - x(p_0,1)\epsilon_1$$

If $x(p_0,1)>0$, then $(1-x(p_0,1))(1-p_0)c_0 - x(p_0,1)\epsilon_1 = x(p_0,1)\left[\frac{(1-x(p_0,1))}{x(p_0,1)}(1-p_0)c_0-\epsilon_1\right]>0$.
If $x(p_0,1)=0$, then $(1-x(p_0,1))(1-p_0)c_0 - x(p_0,1)\epsilon_1 = (1-p_0)c_0>0$.

Thus $EC_1(c_0, d_0) - EC_1(p_0 c_0, d_0) > 0$; and consequently it follows that (c_0, d_0), unique TSC-minimizing configuration, is not a Nash equilibrium. Therefore f is not an efficient incremental liability rule for every application belonging to $\mathscr{A}$, establishing the proposition. □

Analogously the following propositions can be established.

Proposition 6.4. *Let incremental liability rule f be efficient with respect to $\mathscr{A}$. Then we must have:*
$[a(1,1) = 2 \rightarrow (\forall q \in [0,1))[a(1,q) = 2 \rightarrow y(1,q) = 1]]$.

Proposition 6.5. *Let f be an incremental liability rule satisfying the following four conditions:*

(i) $a(1,1) = 1 \rightarrow (\forall q \in [0,1))[a(1,q) = 1]$
(ii) $a(1,1) = 2 \rightarrow (\forall p \in [0,1))[a(p,1) = 2]$
(iii) $a(1,1) = 1 \rightarrow (\forall p \in [0,1))[a(p,1) = 1 \rightarrow x(p,1) = 1]$
(iv) $a(1,1) = 2 \rightarrow (\forall q \in [0,1))[a(1,q) = 2 \rightarrow y(1,q) = 1]$.

Let $< C, D, \pi, H, (c^*, d^*) \in M >$ be an application belonging to $\mathscr{A}$. Then (c^*, d^*) is a Nash equilibrium.

Proof. Let f be an incremental liability rule satisfying conditions (i)–(iv); and let $< C, D, \pi, H, (c^*, d^*) \in M >$ be an application belonging to $\mathscr{A}$.
First suppose $a(1,1) = 1$.

$EC_1(c^*, d^*) = c^*$

$c < c^* \wedge a\left(\frac{c}{c^*}, 1\right) = 1 \rightarrow EC_1(c, d^*) = c + [L(c, d^*) - L(c^*, d^*)]$, by (iii)

Therefore, $c < c^* \wedge a\left(\frac{c}{c^*}, 1\right) = 1 \rightarrow EC_1(c, d^*) - EC_1(c^*, d^*) = [c + L(c, d^*)]$
$- [c^* + L(c^*, d^*)] = [c + d^* + L(c, d^*)] - [c^* + d^* + L(c^*, d^*)] = TSC(c, d^*)$
$- TSC(c^*, d^*) \geq 0$ (6.1)

$c < c^* \wedge a\left(\frac{c}{c^*}, 1\right) = 2 \rightarrow EC_1(c, d^*) = c + L(c, d^*)$
Therefore, $c < c^* \wedge a\left(\frac{c}{c^*}, 1\right) = 2 \rightarrow EC_1(c, d^*) - EC_1(c^*, d^*) = [c + L(c, d^*)]$
$- [c^*] = [c + d^* + L(c, d^*)] - [c^* + d^*] \geq [c + d^* + L(c, d^*)]$
$- [c^* + d^* + L(c^*, d^*)] = TSC(c, d^*) - TSC(c^*, d^*) \geq 0$ (6.2)

$c > c^* \rightarrow EC_1(c, d^*) = c$

Therefore, $c > c^* \rightarrow EC_1(c, d^*) - EC_1(c^*, d^*) = c - c^* > 0$ (6.3)

$EC_2(c^*, d^*) = d^* + L(c^*, d^*)$

$d < d^* \to EC_2(c^*, d) = d + L(c^*, d)$, as $a\left(1, \dfrac{d}{d^*}\right) = 1$ by (i).

Therefore, $d < d^* \to EC_2(c^*, d) - EC_2(c^*, d^*) = [d + L(c^*, d)] - [d^* + L(c^*, d^*)]$

$$= [c^* + d + L(c^*, d)] - [c^* + d^* + L(c^*, d^*)] = TSC(c^*, d) - TSC(c^*, d^*) \geq 0 \tag{6.4}$$

$d > d^* \to EC_2(c^*, d) = d + L(c^*, d)$

Therefore, $d > d^* \to EC_2(c^*, d) - EC_2(c^*, d^*) = [d + L(c^*, d)] - [d^* + L(c^*, d^*)]$

$$= [c^* + d + L(c^*, d)] - [c^* + d^* + L(c^*, d^*)] = TSC(c^*, d) - TSC(c^*, d^*) \geq 0 \tag{6.5}$$

(6.1)–(6.5) establish that if $a(1, 1) = 1,$ then (c^*, d^*) is a Nash equilibrium. (6.6)

By an analogous argument it can be shown that if $a(1, 1) = 2,$ then (c^*, d^*) is a Nash equilibrium. (6.7)

(6.6) and (6.7) establish the proposition. □

Proposition 6.6. *Let f be an incremental liability rule satisfying the following four conditions:*

(i) $a(1, 1) = 1 \to (\forall q \in [0, 1))[a(1, q) = 1]$
(ii) $a(1, 1) = 2 \to (\forall p \in [0, 1))[a(p, 1) = 2]$
(iii) $a(1, 1) = 1 \to (\forall p \in [0, 1))[a(p, 1) = 1 \to x(p, 1) = 1]$
(iv) $a(1, 1) = 2 \to (\forall q \in [0, 1))[a(1, q) = 2 \to y(1, q) = 1]$.

Let $< C, D, \pi, H, (c^*, d^*) \in M >$ *be an application belonging to* $\mathscr{A}$. *Then:* $(\forall(\overline{c}, \overline{d}) \in C \times D)[(\overline{c}, \overline{d})$ *is a Nash equilibrium* $\to$ $(\overline{c}, \overline{d}) \in M]$.

Proof. Let f be an incremental liability rule satisfying (i)–(iv); and let $< C, D, \pi, H, (c^*, d^*) \in M >$ be an application belonging to $\mathscr{A}$. Suppose $(\overline{c}, \overline{d})$ is a Nash equilibrium.

$(\overline{c}, \overline{d})$ is a Nash equilibrium implies

$$EC_1(\overline{c}, \overline{d}) \leq EC_1(c^*, \overline{d}) \tag{6.8}$$

and

$$EC_2(\overline{c}, \overline{d}) \leq EC_2(\overline{c}, d^*) \tag{6.9}$$

$$(6.8) \wedge (6.9) \to \overline{c} + \overline{d} + L(\overline{c}, \overline{d}) \leq EC_1(c^*, \overline{d}) + EC_2(\overline{c}, d^*) \tag{6.10}$$

First suppose $a(1,1) = 1$.

Now,

$$EC_1(c^*, \overline{d}) = c^* \text{ if } \overline{d} < d^*; \text{as } a\left(1, \frac{\overline{d}}{d^*}\right) = 1 \text{ by } (i)$$
$$= c^* \text{ if } \overline{d} \geq d^*; \text{as } a(1,1) = 1$$

Thus $EC_1(c^*, \overline{d}) = c^*$ (6.11)

$$\begin{aligned} EC_2(\overline{c}, d^*) &= d^* + L(\overline{c}, d^*) \leq d^* + L(c^*, d^*) && \text{if } \overline{c} \geq c^*; \text{as } a(1,1) = 1 \\ &= d^* + L(\overline{c}, d^*) - \hat{L}_1(\overline{c}, d^*) = d^* \\ &\quad + L(c^*, d^*) && \text{if } \overline{c} < c^* \text{and } a\left(\tfrac{\overline{c}}{c^*}, 1\right) = 1 \\ &= d^* && \text{if } \overline{c} < c^* \text{and } a\left(\tfrac{\overline{c}}{c^*}, 1\right) = 2 \end{aligned}$$

Thus, $EC_2(\overline{c}, d^*) \leq d^* + L(c^*, d^*)$ (6.12)

(6.10)–(6.12) imply that:

$$TSC(\overline{c}, \overline{d}) = \overline{c} + \overline{d} + L(\overline{c}, \overline{d}) \leq EC_1(c^*, \overline{d}) + EC_2(\overline{c}, d^*) \leq c^* + d^* + L(c^*, d^*)$$
$$= TSC(c^*, d^*) \qquad (6.13)$$

As TSC are minimized at (c^*, d^*), it follows that $TSC(\overline{c}, \overline{d}) = TSC(c^*, d^*)$; and consequently we must have $(\overline{c}, \overline{d}) \in M$. An analogous argument shows that if $a(1,1) = 2$, then also we must have $(\overline{c}, \overline{d}) \in M$. The proposition therefore stands established. □

Theorem 6.1. *An incremental liability rule is efficient for every application belonging to $\mathscr{A}$ iff it satisfies the following four conditions:*

(i) $a(1,1) = 1 \rightarrow (\forall q \in [0,1))[a(1,q) = 1]$
(ii) $a(1,1) = 2 \rightarrow (\forall p \in [0,1))[a(p,1) = 2]$
(iii) $a(1,1) = 1 \rightarrow (\forall p \in [0,1))[a(p,1) = 1 \rightarrow x(p,1) = 1]$
(iv) $a(1,1) = 2 \rightarrow (\forall q \in [0,1))[a(1,q) = 2 \rightarrow y(1,q) = 1]$.

Proof. Let f be an incremental liability rule; and suppose that f satisfies (i)–(iv). Let $< C, D, \pi, H, (c^*, d^*) \in M >$ be an arbitrary application belonging to $\mathscr{A}$. Then, (c^*, d^*) is a Nash equilibrium by Proposition 6.5, and we have $(\forall(\overline{c}, \overline{d}) \in C \times D)[(\overline{c}, \overline{d})$ is a Nash equilibrium $\rightarrow (\overline{c}, \overline{d}) \in M]$ by Proposition 6.6. Thus, f is efficient for every application belonging to $\mathscr{A}$. This establishes sufficiency of (i)–(iv) for an incremental liability rule to be efficient with respect to $\mathscr{A}$. Propositions 6.1–6.4 establish the necessity of (i)–(iv), respectively, for an incremental liability rule to be efficient for every application belonging to $\mathscr{A}$. □

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Chapter 7
The Negligence Rule

There are three versions of the negligence rule, one of the most important liability-apportionment rules of tort law, which are used in practice. These three versions are as follows: (i) Standard negligence rule under which the injurer is liable for the entire loss iff he is negligent, and not at all liable iff he is nonnegligent, and negligence is defined as failure to take at least the due care. (ii) Incremental negligence rule under which the injurer is liable for the loss which can be attributed to his negligence iff he is negligent, and not at all liable iff he is nonnegligent, and negligence is defined as failure to take at least the due care. (iii) Incremental negligence rule under which the injurer is liable for the loss which can be attributed to his negligence iff he is negligent, and not at all liable iff he is nonnegligent, and negligence is defined as the existence of a cost-justified untaken precaution. These three rules will be designated as negligence rule $(s\text{-}dc)$, negligence rule $(i\text{-}dc)$, and negligence rule $(i\text{-}up)$, respectively. The rule under which the injurer is liable for the entire loss iff he is negligent, and not at all liable iff he is nonnegligent, and negligence is defined as the existence of a cost-justified untaken precaution, i.e. negligence rule $(s\text{-}up)$, is not used in practice.

This chapter looks at the efficiency or otherwise of different versions of the negligence rule. Although some of the results on the efficiency of different versions follow from the theorems of the previous chapters, in view of the great importance of negligence rule, in this chapter self-contained efficiency analysis is provided for each version. This chapter is divided into five sections. The first four sections are concerned with the efficiency analysis of negligence rule $(s\text{-}dc)$, negligence rule $(i\text{-}dc)$, negligence rule $(i\text{-}up)$, and negligence rule $(s\text{-}up)$, respectively. The last section discusses the problem of strategic manipulation inherent in the notion of negligence.[1]

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[1]The first three sections of this chapter draw on Jain (2010a) and the last section on Jain (2010b).

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7.1 Negligence Rule $(s\text{-}dc)$

Efficiency of the standard version of the negligence rule when negligence is defined as failure to take at least the due care was established by Brown (1973). In view of the fact that the negligence rule satisfies the negligence liability condition, the efficiency of the negligence rule also follows as a corollary of Theorem 3.4 which states that, given that the negligence is defined as failure to take at least the due care, a liability rule $[0,1]^2 \mapsto [0,1]^2$ is efficient iff it satisfies the negligence liability condition. For the sake of completeness, we provide the proof of efficiency of negligence rule $(s\text{-}dc)$ here.

Theorem 7.1. *If negligence is defined as shortfall from due care, then the standard version of the negligence rule is efficient for every application belonging to $\mathscr{A}$.*

Proof. Let $< C, D, \pi, H, (c^*, d^*) \in M >$ be an application belonging to $\mathscr{A}$.

First we show that (c^*, d^*) is a Nash equilibrium.
$EC_1(c^*, d^*) = c^* + L(c^*, d^*)$

$c \neq c^* \rightarrow EC_1(c, d^*) = c + L(c, d^*)$

Therefore, $c \neq c^* \rightarrow EC_1(c, d^*) - EC_1(c^*, d^*) = [c + L(c, d^*)]$
$- [c^* + L(c^*, d^*)] = [c + d^* + L(c, d^*)] - [c^* + d^* + L(c^*, d^*)] = \mathrm{TSC}(c, d^*)$
$- \mathrm{TSC}(c^*, d^*) \geq 0$ (7.1)

$EC_2(c^*, d^*) = d^*$

$d < d^* \rightarrow EC_2(c^*, d) = d + L(c^*, d)$

Therefore, $d < d^* \rightarrow EC_2(c^*, d) - EC_2(c^*, d^*) = d + L(c^*, d) - d^* \geq d$
$+ L(c^*, d) - d^* - L(c^*, d^*) = [c^* + d + L(c^*, d)] - [c^* + d^* + L(c^*, d^*)]$
$= \mathrm{TSC}(c^*, d) - \mathrm{TSC}(c^*, d^*) \geq 0$ (7.2)

$d > d^* \rightarrow EC_2(c^*, d) = d$

Therefore, $d > d^* \rightarrow EC_2(c^*, d) - EC_2(c^*, d^*) = d - d^* > 0$ (7.3)

(7.1)–(7.3) establish that (c^*, d^*) is a Nash equilibrium. (7.4)

Next we show that if $(\overline{c}, \overline{d}) \in C \times D$ is a Nash equilibrium, then it must be TSC minimizing.
Suppose $(\overline{c}, \overline{d})$ is a Nash equilibrium.

$(\overline{c}, \overline{d})$ is a Nash equilibrium implies

$$EC_1(\overline{c}, \overline{d}) \leq EC_1(c^*, \overline{d}) \tag{7.5}$$

and

$$EC_2(\overline{c},\overline{d}) \leq EC_2(\overline{c},d^*) \tag{7.6}$$

$$(7.5) \wedge (7.6) \rightarrow \overline{c}+\overline{d}+L(\overline{c},\overline{d}) \leq EC_1(c^*,\overline{d})+EC_2(\overline{c},d^*) \tag{7.7}$$

Now,

$$\begin{aligned} EC_1(c^*,\overline{d}) &= c^* && \text{if } \overline{d} < d^* \\ &= c^* + L(c^*,\overline{d}) \leq c^* + L(c^*,d^*) && \text{if } \overline{d} \geq d^* \end{aligned}$$

Thus, $EC_1(c^*,\overline{d}) \leq c^* + L(c^*,d^*)$ (7.8)

$$EC_2(\overline{c},d^*) = d^* \tag{7.9}$$

(7.7)–(7.9) imply that:
$\mathrm{TSC}(\overline{c},\overline{d}) = \overline{c}+\overline{d}+L(\overline{c},\overline{d}) \leq EC_1(c^*,\overline{d})+EC_2(\overline{c},d^*) \leq c^*+d^*+L(c^*,d^*)$
$= \mathrm{TSC}(c^*,d^*)$ (7.10)

As total social costs (TSC) are minimized at (c^*,d^*), it follows that $\mathrm{TSC}(\overline{c},\overline{d})$
$= \mathrm{TSC}(c^*,d^*)$; and consequently we must have $(\overline{c},\overline{d}) \in M$. (7.11)

(7.4) and (7.11) establish the theorem. □

7.2 Negligence Rule $(i\text{-}dc)$

In the previous chapter it was established that, given that negligence is defined as failure to take at least the due care, an incremental liability rule is efficient with respect to $\mathscr{A}$ iff it satisfies conditions (i)–(iv): (i) $a(1,1) = 1 \rightarrow (\forall q \in [0,1))[a(1,q) = 1]$; (ii) $a(1,1) = 2 \rightarrow (\forall p \in [0,1))[a(p,1) = 2]$; (iii) $a(1,1) = 1 \rightarrow (\forall p \in [0,1))[a(p,1) = 1 \rightarrow x(p,1) = 1]$; (iv) $a(1,1) = 2 \rightarrow (\forall q \in [0,1))[a(1,q) = 2 \rightarrow y(1,q) = 1]$. In the case of incremental negligence rule, we have $a(p,q) = 2$, for all $p,q \in [0,1]$. Therefore, (i) and (iii) are inapplicable and both (ii) and (iv) are satisfied. Consequently, the efficiency of negligence rule $(i\text{-}dc)$ follows as a corollary of Theorem 6.1. A self-contained proof of the efficiency of negligence rule $(i\text{-}dc)$ is given below.

Theorem 7.2. *If negligence is defined as shortfall from due care, then the incremental version of the negligence rule is efficient for every application belonging to $\mathscr{A}$.*

Proof. Let $< C, D, \pi, H, (c^*,d^*) \in M >$ be an application belonging to $\mathscr{A}$.

First we show that (c^*, d^*) is a Nash equilibrium.

$EC_1(c^*, d^*) = c^* + L(c^*, d^*)$

$c \neq c^* \rightarrow EC_1(c, d^*) = c + L(c, d^*)$

Therefore, $c \neq c^* \rightarrow EC_1(c, d^*) - EC_1(c^*, d^*) = [c + L(c, d^*)]$
$- [c^* + L(c^*, d^*)] = [c + d^* + L(c, d^*)] - [c^* + d^* + L(c^*, d^*)] = \text{TSC}(c, d^*)$
$- \text{TSC}(c^*, d^*) \geq 0$ (7.12)

$EC_2(c^*, d^*) = d^*$

$d < d^* \rightarrow EC_2(c^*, d) = d + L(c^*, d) - L(c^*, d^*)$

Therefore, $d < d^* \rightarrow EC_2(c^*, d) - EC_2(c^*, d^*) = d + L(c^*, d)$
$- L(c^*, d^*) - d^* = [c^* + d + L(c^*, d)] - [c^* + d^* + L(c^*, d^*)] = \text{TSC}(c^*, d)$
$- \text{TSC}(c^*, d^*) \geq 0$ (7.13)

$d > d^* \rightarrow EC_2(c^*, d) = d$

Therefore, $d > d^* \rightarrow EC_2(c^*, d) - EC_2(c^*, d^*) = d - d^* > 0$ (7.14)

(7.12)–(7.14) establish that (c^*, d^*) is a Nash equilibrium. (7.15)

Next we show that if $(\overline{c}, \overline{d}) \in C \times D$ is a Nash equilibrium, then it must be TSC minimizing.
Suppose $(\overline{c}, \overline{d})$ is a Nash equilibrium.
$(\overline{c}, \overline{d})$ is a Nash equilibrium implies

$$EC_1(\overline{c}, \overline{d}) \leq EC_1(c^*, \overline{d}) \tag{7.16}$$

and

$$EC_2(\overline{c}, \overline{d}) \leq EC_2(\overline{c}, d^*) \tag{7.17}$$

(7.16) $\wedge$ (7.17) $\rightarrow \overline{c} + \overline{d} + L(\overline{c}, \overline{d}) \leq EC_1(c^*, \overline{d}) + EC_2(\overline{c}, d^*)$ (7.18)

Now,

$$\begin{aligned} EC_1(c^*, \overline{d}) &= c^* + L(c^*, d^*) & \text{if } \overline{d} < d^* \\ &= c^* + L(c^*, \overline{d}) \leq c^* + L(c^*, d^*) & \text{if } \overline{d} \geq d^* \end{aligned}$$

Thus, $EC_1(c^*, \overline{d}) \leq c^* + L(c^*, d^*)$ (7.19)

$$EC_2(\overline{c}, d^*) = d^* \tag{7.20}$$

(7.18)–(7.20) imply that:

$$\mathrm{TSC}(\overline{c},\overline{d}) = \overline{c} + \overline{d} + L(\overline{c},\overline{d}) \leq EC_1(c^*,\overline{d}) + EC_2(\overline{c},d^*) \leq c^* + d^* + L(c^*,d^*)$$
$$= \mathrm{TSC}(c^*,d^*) \tag{7.21}$$

As TSC are minimized at (c^*,d^*), it follows that $\mathrm{TSC}(\overline{c},\overline{d}) = \mathrm{TSC}(c^*,d^*)$;

and consequently we must have $(\overline{c},\overline{d}) \in M$. (7.22)

(7.15) and (7.22) establish the theorem. □

7.3 Negligence Rule $(i\text{-}up)$

Corresponding to each $(c,d) \in C \times D$, we define

$$\mathscr{L}_1^u(c,d) = \{L(c,d) - L(c^u,d) \mid c^u \in C^u(c,d)\}$$
$$\mathscr{L}_2^u(c,d) = \{L(c,d) - L(c,d^u) \mid d^u \in D^u(c,d)\}$$

We define

$$\begin{aligned}\hat{L}_1^u(c,d) &= \sup \mathscr{L}_1^u(c,d) && \text{if } C^u(c,d) \neq \emptyset\\ &= 0 && \text{if } C^u(c,d) = \emptyset\end{aligned}$$

and

$$\begin{aligned}\hat{L}_2^u(c,d) &= \sup \mathscr{L}_2^u(c,d) && \text{if } D^u(c,d) \neq \emptyset\\ &= 0 && \text{if } D^u(c,d) = \emptyset\end{aligned}$$

Thus, when negligence is defined in terms of cost-justified untaken precautions, then at (c,d), $\hat{L}_1^u(c,d)$ is the expected loss which can be ascribed to the negligence of the victim; and $\hat{L}_2^u(c,d)$ the expected loss which can be ascribed to the negligence of the injurer.

If negligence is defined as the existence of a cost-justified untaken precaution, then the incremental version of the negligence rule is not efficient for every application belonging to $\mathscr{A}^u$ as the following theorem shows.[2]

[2] In Chap. 5 it was shown that there is no liability rule which is efficient with respect to $\mathscr{A}^u$. A similar proposition holds for incremental liability rules. In Jain (2005. Efficiency of incremental liability rules with negligence as existence of a cost-justified untaken precaution, unpublished manuscript) it has been shown that there is no incremental liability rule which is efficient with respect to $\mathscr{A}^u$.

Theorem 7.3. *If negligence is defined as the existence of a cost-justified untaken precaution, then the incremental negligence rule is not efficient for every application belonging to* $\mathscr{A}^u$.

Proof. Consider the following application belonging to $\mathscr{A}^u$.
Let $C = \{0, p_0 c_0, c_0\}$; $D = \left\{0, d_0, \frac{d_0}{q_0}\right\}$; and $L(c,d)$, $(c,d) \in C \times D$, be as given in the following array:

$L(c,d)$

	$d = 0$	$d = d_0$	$d = \frac{d_0}{q_0}$
$c = 0$	$c_0 + \epsilon_1 + \frac{d_0}{q_0} + \epsilon_2$	$c_0 + \epsilon_1 + \left(\frac{1}{q_0} - 1\right) d_0$	$c_0 + \epsilon_1 - \epsilon_3$
$c = p_0 c_0$	$(1 - p_0)c_0 + \epsilon_1 + \frac{d_0}{q_0} + \epsilon_2$	$(1 - p_0)c_0 + \epsilon_1 + \left(\frac{1}{q_0} - 1\right) d_0$	$(1 - p_0)c_0 + \epsilon_1 - \epsilon_3$
$c = c_0$	$\frac{d_0}{q_0} + \epsilon_2$	$\left(\frac{1}{q_0} - 1\right) d_0$	ϵ_4

where $p_0, q_0, c_0, d_0, \epsilon_1, \epsilon_2, \epsilon_3, \epsilon_4 > 0$ are such that[3]:

(i) $p_0 < 1$ and $q_0 < 1$
(ii) $\epsilon_3 < \epsilon_1$
(iii) $\epsilon_4 < \min\{\epsilon_2, (\epsilon_1 - \epsilon_3)\}$
(iv) $\epsilon_1 < \left(\frac{1}{q_0} - 1\right) d_0$.

(c_0, d_0) is the unique TSC-minimizing configuration.
We obtain $D^u(c,d)$, $(c,d) \in C \times D$, as given in the following array:

$D^u(c,d)$

	$d = 0$	$d = d_0$	$d = \frac{d_0}{q_0}$
$c = 0$	$\left\{d_0, \frac{d_0}{q_0}\right\}$	$\left\{\frac{d_0}{q_0}\right\}$	$\emptyset$
$c = p_0 c_0$	$\left\{d_0, \frac{d_0}{q_0}\right\}$	$\left\{\frac{d_0}{q_0}\right\}$	$\emptyset$
$c = c_0$	$\left\{d_0, \frac{d_0}{q_0}\right\}$	$\emptyset$	$\emptyset$

Therefore, the injurer is negligent or nonnegligent at $(c,d) \in C \times D$, as given in the following array:

Negligence/nonnegligence of the injurer

	$d = 0$	$d = d_0$	$d = \frac{d_0}{q_0}$
$c = 0$	Negligent	Negligent	Nonnegligent
$c = p_0 c_0$	Negligent	Negligent	Nonnegligent
$c = c_0$	Negligent	Nonnegligent	Nonnegligent

[3] $p_0 = q_0 = \frac{1}{2}, c_0 = d_0 = 10, \epsilon_1 = \epsilon_2 = 3, \epsilon_3 = \epsilon_4 = 1$ satisfy (i)–(iv).

Now, expected costs of the victim at $(c_0, d_0) = EC_1(c_0, d_0)$

$= c_0 + L(c_0, d_0)$

$= c_0 + \left(\frac{1}{q_0} - 1\right) d_0$

$EC_1(p_0c_0, d_0)$

$= p_0c_0 + L(p_0c_0, d_0) - \hat{L}_2^u(p_0c_0, d_0)$

$= p_0c_0 + \left[(1 - p_0)c_0 + \epsilon_1 + \left(\frac{1}{q_0} - 1\right) d_0\right] - \left[\left(\frac{1}{q_0} - 1\right) d_0 + \epsilon_3\right]$

$= c_0 + \epsilon_1 - \epsilon_3$

$EC_1(c_0, d_0) - EC_1(p_0c_0, d_0)$

$= \left[\left(\frac{1}{q_0} - 1\right) d_0 - \epsilon_1\right] + \epsilon_3$

$> 0.$

Therefore, it follows that (c_0, d_0) is not a Nash equilibrium. Consequently negligence rule $(i\text{-}up)$ is not efficient for every application belonging to $\mathscr{A}^u$. $\square$

7.3.1 Negligence Rule $(i\text{-}up)$: Unilateral Care

As inefficiency of a rule with respect to a class of applications merely entails that the rule is inefficient for at least one application belonging to the class, it may be the case that the rule might be efficient with respect to some interesting subclasses of the class of applications in question. There is one subclass of $\mathscr{A}^u$ which is worth exploring in the context of the negligence rule $(i\text{-}up)$: the subclass of applications where the optimal care by the victim is zero. In what follows we explore the efficiency of the negligence rule $(i\text{-}up)$ in this unilateral case. Let $\mathscr{A}_1^{u0} = \{< C, D, \pi, H >\in \mathscr{A}^u | (\forall(\overline{c}, \overline{d}) \in C \times D)[(\overline{c}, \overline{d}) \in M \rightarrow \overline{c} = 0]\}$; and $\mathscr{A}_2^{u0} = \{< C, D, \pi, H >\in \mathscr{A}^u | (\forall(\overline{c}, \overline{d}) \in C \times D)[(\overline{c}, \overline{d}) \in M \rightarrow \overline{d} = 0]\}$.

Theorem 7.4. *If negligence is defined as the existence of a cost-justified untaken precaution, then the incremental negligence rule is not efficient for every application belonging to* $\mathscr{A}_1^{u0}$.

Proof. Consider the following application belonging to $\mathscr{A}_1^{u0}$.

Let $C = \{0, \epsilon_1\}; D = \left\{0, d_0, \frac{d_0}{q_0}\right\}$; and $L(c, d)$, $(c, d) \in C \times D$, be as given in the following array:

$L(c, d)$

	$d = 0$	$d = d_0$	$d = \frac{d_0}{q_0}$
$c = 0$	$\frac{d_0}{q_0} + \theta + L_0$	$L_0 + \left(\frac{1}{q_0} - 1\right) d_0$	$L_0 + \epsilon_2$
$c = \epsilon_1$	$\frac{d_0}{q_0} + \theta + L_0 - \epsilon_3$	$L_0 + \left(\frac{1}{q_0} - 1\right) d_0 - \epsilon_3$	$L_0 + \epsilon_2 - \epsilon_3 - \epsilon_4$

where $q_0, d_0, L_0, \theta, \epsilon_1, \epsilon_2, \epsilon_3, \epsilon_4 > 0$ are such that[4]:

(i) $q_0 < 1$
(ii) $\epsilon_3 + \epsilon_4 < \epsilon_1$
(iii) $\epsilon_2 < \min\{\theta, \epsilon_4\}$
(iv) $\epsilon_1 < L_0$
(v) $\epsilon_1 + \epsilon_2 < \left(\frac{1}{q_0} - 1\right) d_0$.

$(0, d_0)$ is the unique TSC-minimizing configuration.
We obtain $D^u(c, d)$, $(c, d) \in C \times D$, as given in the following array:

$D^u(c, d)$

	$d = 0$	$d = d_0$	$d = \frac{d_0}{q_0}$
$c = 0$	$\left\{d_0, \frac{d_0}{q_0}\right\}$	$\emptyset$	$\emptyset$
$c = \epsilon_1$	$\left\{d_0, \frac{d_0}{q_0}\right\}$	$\left\{\frac{d_0}{q_0}\right\}$	$\emptyset$

Therefore, the injurer is negligent or nonnegligent at $(c, d) \in C \times D$, as given in the following array:

Negligence/nonnegligence of the injurer

	$d = 0$	$d = d_0$	$d = \frac{d_0}{q_0}$
$c = 0$	Negligent	Nonnegligent	Nonnegligent
$c = \epsilon_1$	Negligent	Negligent	Nonnegligent

Now, expected costs of the victim at $(0, d_0) = EC_1(0, d_0)$
$= 0 + L(0, d_0)$

$$= L_0 + \left(\frac{1}{q_0} - 1\right) d_0$$

[4] $q_0 = \frac{1}{2}, d_0 = 10, L_0 = 5, \theta = 4, \epsilon_1 = 4, \epsilon_2 = 1, \epsilon_3 = 1, \epsilon_4 = 2$ satisfy (i)–(v).

$$
\begin{aligned}
&EC_1(\epsilon_1, d_0)\\
&= \epsilon_1 + L(\epsilon_1, d_0) - \hat{L}_2^u(\epsilon_1, d_0)\\
&= \epsilon_1 + L(\epsilon_1, d_0) - \left[L(\epsilon_1, d_0) - L\left(\epsilon_1, \frac{d_0}{q_0}\right)\right]\\
&= \epsilon_1 + L_0 + \epsilon_2 - \epsilon_3 - \epsilon_4\\
&EC_1(0, d_0) - EC_1(\epsilon_1, d_0) = \left(\frac{1}{q_0} - 1\right) d_0 - \epsilon_1 - \epsilon_2 + \epsilon_3 + \epsilon_4 > 0.
\end{aligned}
$$

This establishes that $(0, d_0)$ is not a Nash equilibrium. Consequently incremental negligence rule with negligence defined as existence of a cost-justified precaution is not an efficient liability rule for every application belonging to $\mathscr{A}_1^{u0}$.[5] □

The main reason why incremental negligence rule is not efficient even in the unilateral case when negligence is defined in terms of existence of cost-justified untaken precautions is, as mentioned earlier in Chap. 5, that this way of defining negligence introduces strategic manipulability in the system. When negligence is defined in terms of existence of cost-justified untaken precautions, whether one is negligent or not depends not only on one's own care level but also on the care level of the other party. Consequently, the possibility of the victim being in a position, by taking care which is socially deficient or socially excessive, to render an injurer who is taking socially optimal level of care negligent cannot be ruled out. The application which was considered to establish the preceding proposition was such that when both the injurer and the victim are taking socially optimal amounts of care, d_0 and 0, respectively, the injurer is nonnegligent. However, by taking from a social perspective excessive care, the victim can make the injurer with care level d_0 negligent and thereby benefit himself by bringing about a reduction in his expected costs.

Depending on the application, the manipulation of a situation by the victim can be done by taking a socially deficient level of precaution or by taking a socially excessive level of precaution. However, if one is considering only applications belonging to $\mathscr{A}_1^{u0}$, then of course manipulation by the victim by taking a socially deficient level of care is not possible; the only way the victim can manipulate is by taking a socially excessive level of care. If a situation is such that the victim can manipulate it by taking a socially excessive level of care, then it must be the case that there is some complementarity in the victim's and injurer's precautions. In the application which was considered in the context of the preceding theorem, there is a unique TSC-minimizing configuration with the victim taking 0 care and the injurer d_0 care. Given that the victim is taking 0 care if the injurer increases his care from d_0 to $\frac{d_0}{q_0}$, reduction in expected loss is less than the increase of $\left(\frac{1}{q_0} - 1\right) d_0$ in care

[5] It can also be shown that negligence rule (i-up) is not efficient with respect to $\mathscr{A}_2^{u0}$.

cost. However, if the victim is taking $\epsilon_1 > 0$ care, then the injurer's increasing care from d_0 to $\frac{d_0}{q_0}$ results in a decrease in expected loss greater than the increase in care cost.

Thus, it seems that if one is considering an application belonging to $\mathscr{A}_1^{u0}$ in which this kind of complementarities are not there, then the victim would not be able to manipulate and the source of inefficiency would no longer be there. While it is true that if applications belonging to $\mathscr{A}_1^{u0}$ are suitably restricted to rule out complementarities, then manipulation by the victim would no longer be possible, efficiency is still not guaranteed as is shown in Theorem 7.5. The idea of absence of complementarities in the victim's and injurer's precautions can be formalized as follows:

Let an application be called S-restricted iff $(\forall c, c' \in C)(\forall d, d' \in D)[c > c' \wedge d > d' \rightarrow [L(c', d) - L(c, d) \leq L(c', d') - L(c, d')] \wedge [L(c, d') - L(c, d) \leq L(c', d') - L(c', d)]]$. Let the subset of all S-restricted applications belonging to $\mathscr{A}^u$ be denoted by $\mathscr{A}^{uS}$.

Theorem 7.5. *If negligence is defined as the existence of a cost-justified untaken precaution, then the incremental negligence rule is not efficient for every application belonging to* $\mathscr{A}_1^{u0} \cap \mathscr{A}^{uS}$.

Proof. Consider the following application belonging to $\mathscr{A}_1^{u0} \cap \mathscr{A}^{uS}$.
Let $C = \{0\}$; $D = [0, 2\epsilon], \epsilon > 0$; and

$$L(0, d) = 2\epsilon + \theta,\ 0 \leq d \leq \epsilon$$
$$= 2\epsilon - d,\ \epsilon < d \leq 2\epsilon;$$

where $\theta > 0$.
We obtain

$$(\forall d \in [0, \epsilon])[D^u(0, d) = (\epsilon, 2\epsilon]] \wedge (\forall d \in (\epsilon, 2\epsilon])[D^u(0, d) = \emptyset];$$

and consequently the injurer is negligent at every $d \in [0, \epsilon]$ and nonnegligent at every $d \in (\epsilon, 2\epsilon]$.
Now, for $0 \leq d \leq \epsilon$,

$$EC_2(0, d) = d + \hat{L}_2^u(0, d)$$
$$= d + 2\epsilon + \theta > 2\epsilon$$

$$\text{Thus, for } 0 \leq d \leq \epsilon \text{ we have: } EC_2(0, d) > 2\epsilon \tag{7.23}$$

For $\epsilon < d \leq 2\epsilon$,

$$EC_2(0, d) = d \leq 2\epsilon \tag{7.24}$$

From (7.23) and (7.24) it follows that no $(0, d), 0 \leq d \leq \epsilon$, can be a Nash equilibrium. From (7.24), it follows that no $(0, d), \epsilon < d \leq 2\epsilon$, can be a Nash equilibrium. Thus, there does not exist any $(0, d) \in \{0\} \times D$ which is a Nash equilibrium, establishing the proposition. □

A stronger assumption than ruling out complementarities is to assume that the only feasible care level for the victim is zero. Let the set of applications where the victim cannot take any positive level of care be denoted by $\mathscr{A}_1^{u0'} \subset \mathscr{A}_1^{u0} \cap \mathscr{A}^{uS}$. Thus, $\mathscr{A}_1^{u0'} = \{< C, D, \pi, H >\in \mathscr{A}^u \mid C = \{0\}\}$. Even restricting the set of applications to $\mathscr{A}_1^{u0'}$ does not ensure efficiency as is clear from the proof of Theorem 7.5 above. Therefore, the following theorem follows in view of the proof of Theorem 7.5.

Theorem 7.6. *If negligence is defined as the existence of a cost-justified untaken precaution, then the incremental negligence rule is not efficient for every application belonging to* $\mathscr{A}_1^{u0'}$.

When care is unilateral and applications are S-restricted, strategic considerations introduced by the idea of negligence as existence of a cost-justified precaution get eliminated so that the victim is not in a position to manipulate and bring about inefficiency. The reason why inefficiency can still be there is due to another problem which appears because of the way the notion of negligence is defined. As there is no specified level of care which injurer has to take in order to be nonnegligent, a rational injurer would like to choose the minimum amount of care consistent with being nonnegligent. This, however, may not be possible if set $D_M = \{d \in D \mid (\exists c \in C)[(c, d) \in M]\}$ has no minimum. This problem does not arise when negligence is defined as failure to take at least the due care regardless of whether $\min\ D_M$ exists or not because of fixity of due care level. If we consider only those applications belonging to $\mathscr{A}_1^{u0} \cap \mathscr{A}^{uS}$ such that $\min\ D_M$ exists, then it can be shown that incremental negligence rule is efficient for these applications as is done in the theorem which follows.

Let $\mathscr{A}_2^{um} \subset \mathscr{A}^u$ denote the subset of applications which are such that $\min\ D_M$ exists.

Theorem 7.7. *If negligence is defined as the existence of a cost-justified untaken precaution, then the incremental negligence rule is efficient for every application belonging to* $\mathscr{A}_1^{u0} \cap \mathscr{A}^{uS} \cap \mathscr{A}_2^{um}$.

Proof. Consider any application $< C, D, \pi, H >$ belonging to $\mathscr{A}_1^{u0} \cap \mathscr{A}^{uS} \cap \mathscr{A}_2^{um}$. Let $\min\ D_M = d_m$.

Consider any $(0, d) \in C \times D,\ d < d_m$.
As $(0, d) \notin M$ and $(0, d_m) \in M$, it follows that $d + L(0, d) > d_m + L(0, d_m)$ implying $L(0, d) - L(0, d_m) > d_m - d$. Which in turn implies that the injurer is negligent at $(0, d)$. Therefore,

$$\begin{aligned} &EC_2(0, d) = d + \hat{L}_2^u(0, d) \\ &\geq d + L(0, d) - L(0, d_m),\ \text{as } d_m \in D^u(0, d) \\ &> d_m. \end{aligned} \tag{7.25}$$

As $(0, d_m) \in M,$ it follows that the injurer is nonnegligent at$(0, d_m)$.Therefore his expected costs at $(0, d_m)$ are d_m. (7.26)

From (7.25) and (7.26), it follows that given that the victim takes care $= 0$,

for injurer d_m is better than any $d < d_m$. (7.27)

Next consider any $(0, d) \in C \times D, d > d_m$.

$$\begin{aligned} EC_2(0, d) &= d + \hat{L}_2^u(0, d) \text{ if injurer is negligent at}(0, d) \\ &= d \qquad\qquad\quad \text{ if injurer is nonnegligent at}(0, d) \end{aligned}$$

Thus injurer's expected costs at $(0, d)$ would be greater than d_m regardless of whether he is negligent or nonnegligent at $(0, d)$. As injurer's expected costs at $(0, d_m)$ are d_m, it follows that:

Given that the victim takes care $= 0$, for injurer d_m is better than any $d > d_m$.

(7.28)

Next consider $(c, d_m) \in C \times D, c > 0$.
As the application under consideration belongs to $\mathscr{A}^{uS}$, it follows that for any $d > d_m$:
$L(c, d_m) - L(c, d) \leq L(0, d_m) - L(0, d)$.
As $L(0, d_m) - L(0, d) \leq d - d_m$ in view of the fact that $(0, d) \in M$, it follows that we have
$L(c, d_m) - L(c, d) \leq d - d_m$
which implies that the injurer is nonnegligent at $(c, d_m), c > 0$.
Consequently, $EC_1(c, d_m) - EC_1(0, d_m) = [c + L(c, d_m)] - [0 + L(0, d_m)] = [c + d_m + L(c, d_m)] - [0 + d_m + L(0, d_m)] = \text{TSC}(c, d_m) - \text{TSC}(0, d_m) > 0$, as $(c, d_m) \notin M$ and $(0, d_m) \in M$.
Therefore, we obtain:

Given that the injurer takes care $= d_m$, for the victim 0 is better than any $c > 0$.
(7.29)

(7.27)–(7.29) establish that $(0, d_m)$ is a Nash equilibrium. (7.30)

Suppose $(\overline{c}, \overline{d}) \in C \times D$ is a Nash equilibrium.
$(\overline{c}, \overline{d})$ being a Nash equilibrium implies:

$EC_1(\overline{c}, \overline{d}) \leq EC_1(0, \overline{d})$; and (7.31)

$EC_2(\overline{c}, \overline{d}) \leq EC_2(\overline{c}, d_m)$ (7.32)

(7.31) and (7.32) imply that:

$EC_1(\overline{c}, \overline{d}) + EC_2(\overline{c}, \overline{d}) = \overline{c} + \overline{d} + L(\overline{c}, \overline{d}) \leq EC_1(0, \overline{d}) + EC_2(\overline{c}, d_m)$. (7.33)

First consider $\overline{d} < d_m$.
The injurer is negligent at $(0, \overline{d})$. Therefore, $EC_1(0, \overline{d}) = L(0, \overline{d}) - \hat{L}_2^u(0, \overline{d})$

$\hat{L}_2^u(0, \overline{d}) \geq L(0, \overline{d}) - L(0, d_m),$ as $d_m \in D^u(0, \overline{d})$ in view of the facts that $(0, \overline{d})$ $\notin M$ and $(0, d_m) \in M$

Therefore, $\overline{d} < d_m \rightarrow EC_1(0, \overline{d}) \leq L(0, d_m)$. (7.34)

Next consider $\overline{d} > d_m$:

If at $(0, \overline{d})$ injurer is nonnegligent, then $EC_1(0, \overline{d}) = L(0, \overline{d})$

If at $(0, \overline{d})$ injurer is negligent, then $EC_1(0, \overline{d}) = L(0, \overline{d}) - \hat{L}_2^u(0, \overline{d})$

Thus, $EC_1(0, \overline{d}) \leq L(0, \overline{d}) \leq L(0, d_m)$

Therefore, $\overline{d} > d_m \rightarrow EC_1(0, \overline{d}) \leq L(0, d_m)$. (7.35)

(7.34) and (7.35) establish that $EC_1(0, \overline{d}) \leq L(0, d_m)$ (7.36)

As has been seen above, injurer is nonnegligent at $(\overline{c}, d_m), c \geq 0$.

Therefore, $EC_2(\overline{c}, d_m) = d_m$. (7.37)

(7.33), (7.36), and (7.37) imply:

$EC_1(\overline{c}, \overline{d}) + EC_2(\overline{c}, \overline{d}) = \overline{c} + \overline{d} + L(\overline{c}, \overline{d}) \leq EC_1(0, \overline{d}) + EC_2(\overline{c}, d_m)$
$\leq L(0, d_m) + d_m$. (7.38)

(7.38) $\rightarrow (\overline{c}, \overline{d}) \in M$. (7.39)

(7.39) establishes that all $(\overline{c}, \overline{d}) \in C \times D$ which are Nash equilibria are TSC minimizing; and (7.30) establishes that there is at least one $(c, d) \in C \times D$ which is a Nash equilibrium.

This establishes the efficiency of incremental negligence rule with respect to $\mathscr{A}_1^{u0} \cap \mathscr{A}^{uS} \cap \mathscr{A}_2^{um}$. □

As $\mathscr{A}_1^{u0'} \subset \mathscr{A}_1^{u0} \cap \mathscr{A}^{uS}$, the following theorem follows as a corollary of Theorem 7.7.

Theorem 7.8. *If negligence is defined as the existence of a cost-justified untaken precaution, then the incremental negligence rule is efficient for every application belonging to* $\mathscr{A}_1^{u0\prime} \cap \mathscr{A}_2^{um}$.

7.4 Negligence Rule $(s\text{-}up)$

In Chap. 5 (Theorem 5.1), it was shown that if negligence is defined in terms of cost-justified untaken precautions, then there is no liability rule which is efficient with respect to $\mathscr{A}$. Inefficiency of negligence rule $(s\text{-}up)$ therefore follows as a corollary of Theorem 5.1. Negligence rule $(s\text{-}up)$ is efficient neither with respect to $\mathscr{A}_1^{u0}$ nor with respect to $\mathscr{A}_2^{u0}$. For negligence rule $(s\text{-}up)$ propositions corresponding to Theorems 7.5–7.8 also hold.

7.5 Negligence Notions: Strategic Considerations

We saw above that both versions of the negligence rule are efficient if negligence is defined as failure to take at least the due care, and inefficient if negligence is defined as failure to take some cost-justified precaution. The inefficiency arises because the latter way of defining negligence renders it open to strategic manipulation. Although negligence defined in terms of due care is not subject to strategic manipulation of the kind to which the negligence defined in terms of failure to take some cost-justified precaution is, there is another kind of strategic manipulation which both the notions are subject to. We now discuss this second kind of strategic manipulability of negligence notions.

From the perspective of minimization of social costs, it is clearly desirable that those who are in a position to undertake reduction of expected loss at a lower cost than others are the ones who should take greater care compared to those who can do so only at a higher cost. Therefore, if the victim's costs of taking care remain the same but the injurer's costs of taking care vary, one would expect the care burden on the injurer tending to increase as his costs of taking various levels of care decrease. The following example illustrates the point.

Example 7.1. Suppose both the victim and the injurer have three alternatives: no care, moderate care, and high care; and let these three alternative levels of care cost each of them 0, 1, and 2, respectively. Thus,
$C = D = \{0, 1, 2\}$.
For $(c, d) \in C \times D$, let $L(c, d)$ be as given in the following array:

$L(c, d)$

	No care $(d = 0)$	Moderate care $(d = 1)$	High care $(d = 2)$
No care $(c = 0)$	10.00	8.50	7.75
Moderate care $(c = 1)$	8.50	7.00	6.25
High care $(c = 2)$	7.75	6.25	5.50

Social costs $[c+d+L(c,d)]$, $(c,d) \in C \times D$, therefore, are as given in the following array:

$\text{TSC}(c,d)$

	No care $(d=0)$	Moderate care $(d=1)$	High care $(d=2)$
No care $(c=0)$	10.00	9.50	9.75
Moderate care $(c=1)$	9.50	9.00	9.25
High care $(c=2)$	9.75	9.25	9.50

Thus, $(1,1)$, corresponding to (moderate care, moderate care), is the unique social costs minimizing configuration of costs of care.

For efficiency the due care for the injurer must be set at the level of moderate care costing 1. With due care for the injurer set at moderate care, under the standard negligence rule and under the incremental negligence rule, expected costs of the victim and the injurer $(EC_1(c,d), EC_2(c,d))$ will be as given, respectively, in the following arrays:

Negligence rule $(s\text{-}dc) : (EC_1(c,d), EC_2(c,d))$

	No care $(d=0)$	Moderate care $(d=1)$	High care $(d=2)$
No care $(c=0)$	$(\underline{0}, 10)$	$(8.5, \underline{1})$	$(7.75, 2)$
Moderate care $(c=1)$	$(1, 8.5)$	$(\underline{8}, \underline{1})$	$(\underline{7.25}, 2)$
High care $(c=2)$	$(2, 7.75)$	$(8.25, \underline{1})$	$(7.5, 2)$

Negligence rule $(i\text{-}dc) : (EC_1(c,d), EC_2(c,d))$

	No care $(d=0)$	Moderate care $(d=1)$	High care $(d=2)$
No care $(c=0)$	$(8.5, 1.5)$	$(8.5, \underline{1})$	$(7.75, 2)$
Moderate care $(c=1)$	$(\underline{8}, 1.5)$	$(\underline{8}, \underline{1})$	$(\underline{7.25}, 2)$
High care $(c=2)$	$(8.25, 1.5)$	$(8.25, \underline{1})$	$(7.5, 2)$

If negligence is defined in terms of the existence of cost-justified untaken precautions, then regardless of the victim's care level, for the injurer at no care level, both moderate care and high care constitute cost-justified precautions; and at moderate care and high care levels, there does not exist any cost-justified precaution. Therefore, regardless of the victim's care level, the injurer will be negligent if he takes no care and nonnegligent if he takes moderate or high care. Consequently, for the negligence rule $(s\text{-}up)$ expected costs of the victim and the injurer $(EC_1(c,d), EC_2(c,d))$ will be the same as in the negligence rule $(s\text{-}dc)$ case. For negligence rule $(i\text{-}up)$ $(EC_1(c,d), EC_2(c,d))$ will be as given below.

Negligence rule $(i\text{-}up) : (EC_1(c,d), EC_2(c,d))$

	No care $(d=0)$	Moderate care $(d=1)$	High care $(d=2)$
No care $(c=0)$	$(7.75, 2.25)$	$(8.5, \underline{1})$	$(7.75, 2)$
Moderate care $(c=1)$	$(\underline{7.25}, 2.25)$	$(\underline{8}, \underline{1})$	$(\underline{7.25}, 2)$
High care $(c=2)$	$(7.5, 2.25)$	$(8.25, \underline{1})$	$(7.5, 2)$

Thus, in all four cases of negligence rule, $(s\text{-}dc)$, $(i\text{-}dc)$, $(s\text{-}up)$, and $(i\text{-}up)$, (moderate care, moderate care) will constitute the unique Nash equilibrium; and the injurer's expected costs in all cases will be 1.

Now, suppose there is no change in costs of taking alternative levels of care by the victim; but for the injurer the cost of taking any particular level of care now is only 0.7 of what it was earlier. Then we have:

$C = \{0, 1, 2\}, D = \{0, 0.7, 1.4\}$.

Social costs $[c + d + L(c,d)]$, $(c,d) \in C \times D$, are as given in the following array:

$\text{TSC}(c,d)$

	No care $(d=0)$	Moderate care $(d=0.7)$	High care $(d=1.4)$
No care $(c=0)$	10.00	9.20	9.15
Moderate care $(c=1)$	9.50	8.70	8.65
High care $(c=2)$	9.75	8.95	8.90

Thus, $(1, 1.4)$, corresponding to (moderate care, high care), is the unique social costs minimizing configuration of costs of care.

For efficiency the due care for the injurer must be set at the level of high care costing 1.4. With due care for the injurer set at high care, under the standard negligence rule and under the incremental negligence rule, expected costs of the victim and the injurer $(EC_1(c,d), EC_2(c,d))$ will be as given, respectively, in the following arrays:

Negligence rule $(s\text{-}dc) : (EC_1(c,d), EC_2(c,d))$

	No care $(d=0)$	Moderate care $(d=0.7)$	High care $(d=1.4)$
No care $(c=0)$	$(\underline{0}, 10)$	$(\underline{0}, 9.2)$	$(7.75, \underline{1.4})$
Moderate care $(c=1)$	$(1, 8.5)$	$(1, 7.7)$	$(\underline{7.25}, \underline{1.4})$
High care $(c=2)$	$(2, 7.75)$	$(2, 6.95)$	$(7.5, \underline{1.4})$

Negligence rule $(i\text{-}dc) : (EC_1(c,d), EC_2(c,d))$

	No care $(d=0)$	Moderate care $(d=0.7)$	High care $(d=1.4)$
No care $(c=0)$	$(7.75, 2.25)$	$(7.75, 1.45)$	$(7.75, \underline{1.4})$
Moderate care $(c=1)$	$(\underline{7.25}, 2.25)$	$(\underline{7.25}, 1.45)$	$(\underline{7.25}, \underline{1.4})$
High care $(c=2)$	$(7.5, 2.25)$	$(7.5, 1.45)$	$(7.5, \underline{1.4})$

If negligence is defined in terms of the existence of cost-justified untaken precautions, then regardless of the victim's care level, for the injurer at no care level, both moderate care and high care constitute cost-justified precautions; at moderate care level, high care constitutes cost-justified care level; and at high care level, there does not exist any cost-justified precaution. Therefore, regardless of the victim's care level, the injurer will be negligent if he takes no care or moderate care and nonnegligent if he takes high care. Consequently, both for the standard negligence rule and the incremental negligence rule, expected costs of the victim and the injurer $(EC_1(c,d), EC_2(c,d))$ will be the same as in the negligence as failure to take at least the due care case.

Thus, in all four cases of negligence rule, $(s\text{-}dc)$, $(i\text{-}dc)$, $(s\text{-}up)$, and $(i\text{-}up)$, (moderate care, high care) will constitute the unique Nash equilibrium; and the injurer's expected costs in all cases will be 1.4. $\diamondsuit$

Thus all four variants of negligence rule tend to put greater burden on the more efficient individuals. The obverse side of this feature of negligence rule is that it tends to punish dexterity and reward incompetence. This perverse feature of the negligence rule is in some ways quite similar to the perversity of utilitarianism which would allocate most of the good things of life to gluttons with a keen sense of enjoyment rather than to those who are industrious but without highly cultivated tastes. Utilitarianism tends to reward investment of time in cultivating taste and punish investment of time in productive activities. The negligence rule tends to reward the class of injurers who are inefficient and punish those who are efficient.

In view of this perverse feature of the negligence rule if an injurer is in a position to bring about a reduction in his cost of care by expending an amount which is cost-justified, there is no guarantee that such a reduction would be undertaken. In the context of the example discussed above, improved precaution technology is resulting in reduction of social costs from 9.00 to 8.65, a net gain of 0.35. If such an improvement can be brought about at an expense of less than 0.35, then from a social perspective it would be worthwhile. But, the injurer who might be in a position to bring about such an improvement would be a loser if he does so as he will have to not only bear the expenses which would bring about such an improvement in the precaution technology in the first place but also greater cost of care as it will go up from 1 to 1.4. Needless to say, no rational injurer would undertake such an improvement in precaution technology.

In fact, in general, there would be incentive for individuals with greater dexterity to hide their superior abilities and pretend that they also have similar abilities as the less dexterous ones. If they can successfully misrepresent their abilities of taking care, they could end up paying less than otherwise. Thus, it might pay to invest time and effort in misleading the courts regarding one's ability to take care. As an illustration suppose that the actual accident context is that of Example 7.1 with improved precaution technology. If there is no misrepresentation, then the injurer's expected costs would be 1.4. If the injurer is successful in pretending that his D is $\{0, 1, 2\}$ rather than $\{0, 0.7, 1.4\}$, then his gain will be 0.4. Therefore as long as he can successfully misrepresent by expending an amount less than 0.4, he would do so.

Thus we see that the rule of negligence can at times provide disincentives for increasing one's efficiency by undertaking cost-justified expenses for the purpose and incentives for expending resources for the purpose of misleading courts.

The property of both negligence rule $(s\text{-}dc)$ and negligence rule $(i\text{-}dc)$ of minimizing the sum of precaution costs and expected accident loss crucially depends on the ability of courts to fix due care level on a case-by-case basis. This by itself would require expending of considerable resources. Moreover, as discussed above, parties would normally have incentives to misrepresent their abilities to take precaution for accident prevention, which would tend to further increase the costs of eliciting correct information for the purpose of fixing due care levels on a case-by-case basis.

Under the negligence rule there is one further source of strategic manipulation. This arises from the fact that taking care in one context can have beneficial implications in another context. Suppose injurer's taking care not only brings about a reduction in expected accident loss in the context of victim-injurer interaction under consideration but also in another context. Then, in calculating the optimal amount of care, one should take into account not only the context under consideration but also the other context or contexts. In general, more contexts one includes, the greater would be the care which would minimize social costs. Thus, from the perspective of the injurer, the narrower the scope, the better it is for him. As greater foreseeability would result in general greater care being required, injurers would have incentive to misrepresent their ability to foresee various contingencies.

One possible way that might be considered for eliminating strategic aspects is to abandon individualized determination of negligence, whether in terms of due care or in terms of existence of cost-justified untaken precautions and go in for a uniform specification for all injurers notwithstanding differences in costs of taking care. It is immediately clear that if there is a uniform due care for all injurers, then there is no incentive for any injurer to expend resources for misrepresenting his abilities of taking care. Thus the wasteful use of resources discussed above in the context of individualized specifications would not arise if there is a uniform due care for all injurers. Also, if an injurer can reduce costs of care by expending resources in a cost-effective manner, he would do so as both the gains and costs would accrue to him. Another advantage of a uniform due care level is that the costs of using the legal system would tend to be smaller compared to the case when for each accident case the due care level has to be determined separately on the basis of effectiveness of the parties in taking various levels of care.

On the minus side, if there is a uniform due care level for all injurers, then the property of negligence rule $(s\text{-}dc)$ and negligence rule $(i\text{-}dc)$ of minimizing the sum of precaution costs and expected accident loss would no longer hold. Furthermore, it is not the case that with uniform due care there is no scope for strategic considerations. While, given the fixity of uniform due care, no individual has any incentive to misrepresent his abilities or disincentive to undertake cost-justified measures for increasing effectiveness of care, the set of injurers as a whole has all the incentives for strategic manipulations which have been discussed above. To see this, first we note that if negligence rule is to play some role in reducing costs of accidents, then in fixing a uniform due care, the average effectiveness of caretaking by injurers vis-a-vis the average effectiveness of caretaking by victims has to be taken into account. But then for the set of injurers as a whole, all the points, on which individual injurers in the case of individualized specifications found it beneficial to behave strategically, become relevant. Therefore, in most instances, one can expect some kind of collective or organization of injurers to emerge which would attempt to do what individuals in a setting of individualized specifications can be expected to do.

To sum up, the strategic aspects have been discussed under three different notions of negligence. If the notion of negligence is defined as shortfall from due care which is set in an individualized way and at a social costs minimizing level, then at least three sources of strategic manipulation arise:

(i) Individuals may not undertake cost-justified measures to increase effectiveness of their caretaking.
(ii) Individuals may expend resources for the purpose of misrepresenting their caretaking abilities, such expending of resources being socially wasteful.
(iii) Individuals may expend resources for socially harmful purpose of misrepresenting their abilities of foreseeing multiple risks.

When due care is set at a uniform level for all injurers, but still with a view to minimize social costs of accidents, then while individuals will have no incentive to manipulate strategically, the set of individuals as a whole will continue to have incentives to manipulate as in (i)–(iii). Whether a collective organization of injurers will emerge or not will of course depend on many factors including potential gains from manipulation and transaction costs involved. When the notion of negligence is defined in terms of the existence of cost-justified untaken precautions, then in addition to (i)–(iii), the possibilities of manipulation emerge also on account of the fact that negligence or otherwise of one party now may depend on what the other party does.

Although here strategic manipulation has been discussed only in the context of the negligence rule, what has been discussed is relevant for a wide variety of liability rules and incremental liability rules. Indeed, the strategic considerations discussed here are relevant for every liability rule and every incremental liability rule which under the standard assumptions is efficient. All the three notions of negligence which have been discussed are such that they give rise to strategic considerations. All the three notions incorporate in some way the objective of minimization of social costs of accidents. Minimization of social costs requires allocating caretaking in such a way that those who are better at it do more of it compared to the others. To induce more able persons to take more care, the negligence standards must be chosen in such a way that more able persons have to take greater care in order to escape from being adjudged negligent compared to the others. Thus notions of negligence having the perverse implication of punishing ability and dexterity are a direct consequence of attempting to use the idea for minimization of social costs. This perverse implication, however, provides inappropriate signals to rational individuals giving rise to possibilities of strategic behaviour which will work in the opposite direction, i.e. against the objective of minimization of social costs. Thus it appears to be the case that achieving the objective of minimization of social costs by the use of liability rules and incremental liability rules may be much more difficult than is generally thought to be the case.

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Chapter 8
Decomposition of Loss and a Class of Negligence Rules

Negligence rule is one of the most important liability rules for apportioning accident loss between victim and injurer. While negligence rule is efficient if negligence is defined as failure to take at least the due care and the due care is set appropriately from the perspective of social costs minimization, the apportionment of loss under it has an all-or-none character. If injurer is negligent then the entire loss is borne by him; and if he is nonnegligent then the entire loss is borne by the victim. In this chapter we investigate whether distributive considerations can be brought in without affecting the efficiency of negligence rule.[1] It so turns out that in providing correct incentives to the parties, part of accident loss, equal to the optimal loss when both parties are taking care at levels at which total social costs (TSC) are minimized, suitably adjusted to take into account differing probabilities of accident with different care levels, seems to play no role and can therefore be apportioned between the two parties independently of their care levels. It is the apportionment of the accident loss over and above the adjusted optimal loss which turns out to be crucial from the point of view of providing correct incentives to the parties. An example may help illustrate the point.

Consider a two-party interaction in which the accident occurs with certainty, but the magnitude of accident loss depends on the care levels of the parties. Let the loss be 100 if neither party takes care, 98.5 if one party takes care and the other does not, and 97 if both parties take care. Let taking care by either party cost 1. As the negligence rule is an efficient liability rule, it would induce in the context of the scenario of this example both parties to take care and thus lead to the socially optimal outcome. It can be easily checked that the negligence rule would lead to the socially optimal outcome of both parties taking care even if part of the loss, equal to the optimal loss, which is 97 here, is assigned to the injurer regardless of whether he is negligent or nonnegligent.

[1]This chapter relies on Jain and Kundu (2011).

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This example makes it clear that the efficiency requirement does not preclude altogether a role for distributive considerations. Part of the accident loss can be assigned between the parties on non-efficiency considerations without affecting the efficiency property. For a systematic treatment of this question, the notion of negligence rule needs to be generalized so that all possible decompositions of accident loss could be considered to find out the precise constraints imposed by the efficiency requirement.

Let the loss which takes place in case of occurrence of accident when both parties are taking TSC-minimizing care levels be called optimal loss. Let θ be a nonnegative number. Let excess loss be defined as excess of total accident loss over θ times the adjusted optimal loss (adjusted to take into account differing probabilities of accident with different care levels) if total accident loss is greater than θ times the adjusted optimal loss; otherwise as zero. And let the specified loss be the difference between total accident loss and excess loss. Corresponding to negligence rule, one can define a two-parameter $(\lambda, \theta), \lambda \in [0, 1], \theta \geq 0$, family of rules in the following way: (i) The excess loss is to be borne by the injurer iff he is negligent; and the excess loss is to be borne by the victim iff the injurer is nonnegligent. (ii) The specified loss is to be assigned between victim and injurer in the fixed proportions of λ and $1 - \lambda$, respectively. This more general notion of a negligence rule would be called a (λ, θ)-negligence rule. In other words, while the standard negligence rule apportions the entire accident loss on the basis of whether injurer is negligent or nonnegligent, a (λ, θ)-negligence rule does so only for one part of the accident loss (excess loss), the other part (specified loss) being divided up between the two parties in fixed proportions. It should be noted that if $\theta = 0$, then the notion of a (λ, θ)-negligence rule coincides with that of the standard negligence rule.

We show that if $0 \leq \theta \leq 1$, then a (λ, θ)-negligence rule is efficient regardless of the value of λ. In other words, the efficiency of negligence rule continues to hold provided the quantum of loss that is assigned independently of the negligence or otherwise of the injurer does not exceed adjusted optimal loss. If $\theta > 1$, then it can be shown that (λ, θ)-negligence rule is not efficient for any value of λ. In view of the above, it is clear that the requirements imposed by efficiency considerations can be quite mild depending on the context.

The results regarding the class of (λ, θ)-negligence rules discussed in this chapter are special cases of more general results on the totality of all liability rules. As one can define a two-parameter $(\lambda, \theta), \lambda \in [0, 1], \theta \geq 0$, family of rules corresponding to the negligence rule, likewise one can do so for every liability rule. If f is a liability rule, one defines a two-parameter $(\lambda, \theta), \lambda \in [0, 1], \theta \geq 0$, family of rules corresponding to f by the following: g is a (λ, θ)-liability rule corresponding to f iff (i) $(\forall p, q \in [0, 1]^2)[g(p, q) = f(p, q)]$, (ii) the expected loss to be borne by the victim = $x(p, q)$(expected excess loss) + λ (expected specified loss), and (iii) the expected loss to be borne by the injurer = $y(p, q)$(expected excess loss) + $(1 - \lambda)$ (expected specified loss). It can be shown that: (i) If f is an efficient liability rule, then every (λ, θ)-liability rule corresponding to f with $0 \leq \theta \leq 1$ is efficient;

and every (λ, θ)-liability rule corresponding to f with $\theta > 1$ is inefficient; (ii) If f is an inefficient liability rule, then every (λ, θ)-liability rule corresponding to f is inefficient.[2]

The chapter is divided into two sections. The first section discusses and formalizes the idea of decomposition of accident loss. The second section states and proves that if $0 \leq \theta \leq 1$, then (λ, θ)-negligence rule is efficient for every $0 \leq \lambda \leq 1$.

8.1 Decomposition of Accident Loss

Given that the configuration of costs of care is (c, d), if accident takes place then the loss of $H(c, d)$ would materialize. In what follows we decompose this loss in two parts such that while apportionment of one part between the two parties is relevant from the perspective of efficiency, the apportionment of the other part is not. Let $\lambda \in [0, 1]$ and $\theta \geq 0$. We will use the term 'multiple' for θ.

Define functions S and G as follows:

$$\begin{aligned} S(c,d) &= H(c,d) - \theta H^* \frac{\pi^*}{\pi(c,d)} \quad && \text{if } \pi(c,d) \neq 0 \ \wedge \ H(c,d) > \theta H^* \frac{\pi^*}{\pi(c,d)} \\ &= 0 && \text{otherwise.} \\ G(c,d) &= \theta H^* \frac{\pi^*}{\pi(c,d)} && \text{if } S(c,d) > 0 \\ &= H(c,d) && \text{if } S(c,d) = 0. \end{aligned}$$

H^* will be termed as optimal loss and $H^* \frac{\pi^*}{\pi(c,d)}$ as adjusted optimal loss. If total accident loss is greater than θ times the adjusted optimal loss, then the accident loss is decomposed into two parts: the excess of accident loss over θ times the adjusted optimal loss being the S part, to be called excess loss, and the θ times the adjusted optimal loss being the G part, to be called the specified loss; otherwise the entire accident loss is equated with G. It should be noted that S and G always sum to total accident loss: $(\forall (c, d) \in C \times D)[S(c, d) + G(c, d) = H(c, d)]$. Thus, (S, G) constitutes a decomposition of accident loss H. If adjusted optimal loss is positive, then by varying θ between 0 and a sufficiently large positive number all possible decompositions of accident loss in two parts can be obtained. If adjusted optimal loss is 0, then regardless of the value of θ, S would be equal to the total accident loss and G equal to 0.

Next, we decompose expected loss as follows:

$$\begin{aligned} R(c,d) &= \pi(c,d) S(c,d); \text{ and} \\ F(c,d) &= \pi(c,d) G(c,d). \end{aligned}$$

[2] See Jain and Kundu (2004).

Thus,

$$
\begin{aligned}
R(c,d) &= L(c,d) - \theta L^* \quad && \text{if } L(c,d) > \theta L^* \\
&= 0 && \text{otherwise.} \\
F(c,d) &= \theta L^* && \text{if } L(c,d) > \theta L^* \\
&= L(c,d) && \text{if } L(c,d) \leq \theta L^*.
\end{aligned}
$$

Therefore we have

$$
\begin{aligned}
R(c,d) &= \max\{L(c,d) - \theta L^*,\ 0\} \\
F(c,d) &= \min\{L(c,d),\ \theta L^*\}.
\end{aligned}
$$

From the definitions of functions R and F, it follows that we have $(\forall (c,d) \in C \times D)[R(c,d) + F(c,d) = L(c,d)]$. Thus, (R, F) constitutes a decomposition of expected accident loss L.

Consistent with the nomenclature of G and S, $F(c,d)$ will be referred to as the expected specified loss and $R(c,d)$ as the expected excess loss.

The following example illustrates some of the definitions which have been introduced above.

Example 8.1. Let $C = D = \{0, 1, 2\}$.
For $(c,d) \in C \times D$, let $L(c,d)$ be as given in the following array:

$L(c,d)$

	$d=0$	$d=1$	$d=2$
$c=0$	20.0	18.0	16.5
$c=1$	18.0	16.5	15.4
$c=2$	16.5	15.5	15.0

$\text{TSC}(c,d)$, $(c,d) \in C \times D$, are as given in the following array:

$\text{TSC}(c,d)$

	$d=0$	$d=1$	$d=2$
$c=0$	20.0	19.0	18.5
$c=1$	19.0	18.5	18.4
$c=2$	18.5	18.5	19.0

Thus $(1,2)$ is the unique TSC-minimizing configuration of costs of care. Let $(c^*, d^*) = (1,2)$.

Let $\theta = 1$. Then we obtain $R(c,d)$, $(c,d) \in C \times D$, as given in the following array:

$R(c,d)$

	$d=0$	$d=1$	$d=2$
$c=0$	4.6	2.6	1.1
$c=1$	2.6	1.1	0
$c=2$	1.1	0.1	0

And, $F(c,d)$, $(c,d) \in C \times D$, as given in the following array:

$F(c,d)$

	$d=0$	$d=1$	$d=2$
$c=0$	15.4	15.4	15.4
$c=1$	15.4	15.4	15.4
$c=2$	15.4	15.4	15.0

$\Diamond$

8.1.1 A Class of Negligence Rules

Let the due care for injurer be $d^* \in D$, where d^* is such that $(\exists c^* \in C)[(c^*, d^*) \in M]$. Under the standard negligence rule, in case of occurrence of accident, if injurer's care level d is less than d^* then the entire accident loss is borne by the injurer; otherwise the entire accident loss is borne by the victim. We now define a two-parameter family of negligence rules. Let $0 \leq \lambda \leq 1$ and $0 \leq \theta$. A (λ, θ)-negligence rule is defined by the following: In case of occurrence of accident (i) regardless of injurer's care level, $\lambda G(c,d)$ is borne by the victim and $(1-\lambda)G(c,d)$ is borne by the injurer; (ii) If injurer's care level d is less than d^* then the entire $S(c,d)$ is borne by the injurer; otherwise the entire $S(c,d)$ is borne by the victim.

Thus, given C, D, π, H and $(c^*, d^*) \in M$, if accident takes place and loss of $H(c,d)$ materializes, then:

If $d < d^*$ then $\lambda G(c,d)$ will be borne by the victim and $S(c,d) + (1-\lambda)G(c,d)$ by the injurer.
If $d \geq d^*$ then $S(c,d) + \lambda G(c,d)$ will be borne by the victim and $(1-\lambda)G(c,d)$ by the injurer.
Therefore, if $d < d^*$, then $EC_1(c,d) = c + \lambda F(c,d)$ and $EC_2(c,d) = d + R(c,d) + (1-\lambda)F(c,d)$.
And, if $d \geq d^*$, then $EC_1(c,d) = c + R(c,d) + \lambda F(c,d)$ and $EC_2(c,d) = d + (1-\lambda)F(c,d)$.

Remark 8.1. It should be noted that if $\theta = 0$, the definition of a (λ, θ)-negligence rule reduces to that of the standard negligence rule. $\Diamond$

The following example illustrates some of the definitions discussed above.

Example 8.2. Consider the (λ, θ)-negligence rule f with $\lambda = \frac{1}{2}, \theta = 1$.
Let C, D, L, and (c^*, d^*) be as in Example 8.1.
We obtain $(EC_1(c,d), EC_2(c,d))$, $(c,d) \in C \times D$, as given in the following array:

$(EC_1(c,d), EC_2(c,d))$

	$d=0$	$d=1$	$d=2$
$c=0$	$(\underline{7.7}, 12.3)$	$(\underline{7.7}, 11.3)$	$(8.8, \underline{9.7})$
$c=1$	$(8.7, 10.3)$	$(8.7, 9.8)$	$(\underline{8.7}, \underline{9.7})$
$c=2$	$(9.7, \underline{8.8})$	$(9.7, \underline{8.8})$	$(9.5, 9.5)$

We find that the only $(c,d) \in C \times D$ which is a Nash equilibrium is (1,2). As (1,2) is TSC minimizing, we conclude that the $\left(\frac{1}{2}, 1\right)$-negligence rule is efficient for the application considered here.[3] $\Diamond$

8.2 Efficiency of (λ, θ)-Negligence Rules

Theorem 8.1. *If* $0 \le \theta \le 1$, *then* (λ, θ)*-negligence rule is efficient with respect to* $\mathscr{A}$.

Proof. Let f be a (λ, θ)-negligence rule, where $0 \le \theta \le 1$. Let $< C, D, \pi, H, (c^*, d^*) \in M >$, where d^* is the due care level for the injurer, belong to $\mathscr{A}$.
Suppose (c^*, d^*) is not a Nash equilibrium. This implies:

$$(\exists c' \in C)[EC_1(c', d^*) < EC_1(c^*, d^*)] \vee (\exists d' \in D)[EC_2(c^*, d') < EC_2(c^*, d^*)]. \tag{8.1}$$

First suppose $(\exists c' \in C)[EC_1(c', d^*) < EC_1(c^*, d^*)]$ holds. (8.2)

We first consider the case $c' < c^*$.

As $(c^*, d^*) \in M$ it follows that $c' < c^* \;\rightarrow\; L(c', d^*) > L^*$

$\rightarrow L(c', d^*) - \theta L^* > 0,$ as $0 \le \theta \le 1$

$$\rightarrow R(c', d^*) = L(c', d^*) - \theta L^* \;\wedge\; F(c', d^*) = \theta L^*. \tag{8.3}$$

[3] By Theorem 8.1, $\left(\frac{1}{2}, 1\right)$-negligence rule is efficient for all applications belonging to $\mathscr{A}$.

In view of (8.3), (8.2) implies:

$$c' + L(c', d^*) - \theta L^* + \lambda\theta L^* < c^* + (1-\theta)L^* + \lambda\theta L^*. \tag{8.4}$$

$$(8.4) \;\rightarrow\; c' + d^* + L(c', d^*) < c^* + d^* + L^*.$$

This is a contradiction as $(c^*, d^*) \in M$, and therefore $TSC(c', d^*)$ cannot be less than $TSC(c^*, d^*)$.

This contradiction establishes that $c' < c^* \;\rightarrow\;$ (8.2) cannot hold. (8.5)

Next consider the case when $c' > c^* \;\wedge\; L(c', d^*) > \theta L^*$.
If $c' > c^* \;\wedge\; L(c', d^*) > \theta L^*$, then (8.2) implies:
$c' + L(c', d^*) - \theta L^* + \lambda\theta L^* < c^* + (1-\theta)L^* + \lambda\theta L^*$

$\rightarrow c' + d^* + L(c', d^*) < c^* + d^* + L^*$.
This is a contradiction. Therefore:
$c' > c^* \;\wedge\; L(c', d^*) > \theta L^* \;\rightarrow\;$ (8.2) cannot hold. (8.6)

Next consider the case when $c' > c^* \;\wedge\; L(c', d^*) \leq \theta L^*$.
If $c' > c^* \;\wedge\; L(c', d^*) \leq \theta L^*$, then (8.2) implies:

$$c' + \lambda L(c', d^*) < c^* + (1-\theta)L^* + \lambda\theta L^*. \tag{8.7}$$

$$(8.7) \;\rightarrow\; (1-\lambda)c' + \lambda[c' + d^* + L(c', d^*)] < (1-\lambda)c^* + \lambda[c^* + d^* + L^*]$$

$$-\,\lambda L^* + (1-\theta)L^* + \lambda\theta L^*$$

$$\rightarrow (1-\lambda)c' < (1-\lambda)c^* - \lambda L^* + (1-\theta)L^* + \lambda\theta L^*,$$

as $\mathrm{TSC}(c', d^*) \geq \mathrm{TSC}(c^*, d^*)$ and $\lambda \geq 0$

$$\rightarrow (1-\lambda)c' < (1-\lambda)c^* + (1-\theta)(1-\lambda)L^* \tag{8.8}$$

If $(1-\lambda) = 0$ then [(8.8) $\rightarrow$ $0 < 0$, a contradiction]. (8.9)

If $(1-\lambda) > 0$ then [(8.8) $\rightarrow$ $c' < c^* + (1-\theta)L^*$]. (8.10)

But we have $L^* - L(c', d^*) \leq c' - c^*$, as $(c^*, d^*) \in M$;
and
$L(c', d^*) \leq \theta L^*$, by hypothesis.
Consequently, $L^* \leq c' - c^* + \theta L^*$

$$\rightarrow c^* + (1-\theta)L^* \leq c', \tag{8.11}$$

contradicting $c' < c^* + (1-\theta)L^*$.

In view of (8.9)–(8.11), it follows that:

$$c' > c^* \ \wedge\ L(c', d^*) \leq \theta L^* \ \rightarrow\ (8.2) \text{ cannot hold.} \tag{8.12}$$

$$(8.5), (8.6), \text{ and } (8.12) \text{ establish that } (8.2) \text{ cannot hold.} \tag{8.13}$$

$$\text{Next suppose } (\exists d' \in D)[EC_2(c^*, d') < EC_2(c^*, d^*)] \text{ holds.} \tag{8.14}$$

We first consider the case $d' < d^*$.
As $(c^*, d^*) \in M$ it follows that $d' < d^* \ \rightarrow\ L(c^*, d') > L^*$

$$\rightarrow L(c^*, d') - \theta L^* > 0, \text{ as } 0 \leq \theta \leq 1$$
$$\rightarrow R(c^*, d') = L(c^*, d') - \theta L^* \ \wedge\ F(c^*, d') = \theta L^*. \tag{8.15}$$

In view of (8.15), (8.14) implies:

$$d' + L(c^*, d') - \theta L^* + (1 - \lambda)\theta L^* < d^* + (1 - \lambda)\theta L^*. \tag{8.16}$$
$$(8.16) \ \rightarrow\ c^* + d' + L(c^*, d') < c^* + d^* + \theta L^*$$
$$\rightarrow c^* + d' + L(c', d^*) < c^* + d^* + L^*, \text{ as } 0 \leq \theta \leq 1.$$

This is a contradiction as $(c^*, d^*) \in M$, and therefore $TSC(c^*, d')$ cannot be less than $TSC(c^*, d^*)$.

$$\text{This contradiction establishes that } d' < d^* \ \rightarrow\ (8.14) \text{ cannot hold.} \tag{8.17}$$

Next consider the case when $d' > d^* \ \wedge\ L(c^*, d') > \theta L^*$.
If $d' > d^* \ \wedge\ L(c^*, d') > \theta L^*$, then (8.14) implies:
$d' + (1 - \lambda)\theta L^* < d^* + (1 - \lambda)\theta L^*$
$\rightarrow d' < d^*$

This is a contradiction as $d' > d^*$. Therefore:

$$d' > d^* \ \wedge\ L(c^*, d') > \theta L^* \ \rightarrow\ (8.14) \text{ cannot hold.} \tag{8.18}$$

Finally consider the case when $d' > d^* \ \wedge\ L(c^*, d') \leq \theta L^*$.
If $d' > d^* \ \wedge\ L(c^*, d') \leq \theta L^*$, then (8.14) implies:

$$d' + (1 - \lambda)L(c^*, d') < d^* + (1 - \lambda)\theta L^*. \tag{8.19}$$
$$(8.19) \ \rightarrow\ \lambda d' + (1 - \lambda)[c^* + d' + L(c^*, d')] < \lambda d^* + (1 - \lambda)[c^* + d^* + L^*]$$
$$-\ (1 - \lambda)L^* + (1 - \lambda)\theta L^*$$
$$\rightarrow \lambda d' < \lambda d^* - (1 - \lambda)L^* + (1 - \lambda)\theta L^*,$$

as $\text{TSC}(c^*, d') \geq \text{TSC}(c^*, d^*)$ and $(1 - \lambda) \geq 0$

$$\rightarrow \lambda d' < \lambda d^* - (1-\theta)(1-\lambda)L^*$$

$$\rightarrow \lambda d' < \lambda d^*, \text{ as } (1-\theta)(1-\lambda)L^* \geq 0. \tag{8.20}$$

$$(8.20) \wedge \lambda > 0 \;\rightarrow\; d' < d^*, \text{ a contradiction as } d' > d^* \text{ by hypothesis} \tag{8.21}$$

$$(8.20) \wedge \lambda = 0 \;\rightarrow\; 0 < 0, \text{ a contradiction.} \tag{8.22}$$

(8.21) and (8.22) establish that:

$$d' > d^* \;\wedge\; L(c^*, d') \leq \theta L^* \rightarrow (8.14) \text{ cannot hold.} \tag{8.23}$$

(8.17), (8.18), and (8.23) establish that (8.14) cannot hold. (8.24)

(8.13) and (8.24) establish that (c^*, d^*) is a Nash equilibrium. (8.25)

Let $(\overline{c}, \overline{d})$ be a Nash equilibrium. $(\overline{c}, \overline{d})$ being a Nash equilibrium implies:

$$(\forall c \in C)[EC_1(\overline{c}, \overline{d}) \leq EC_1(c, \overline{d})] \tag{8.26}$$

and

$$(\forall d \in D)[EC_2(\overline{c}, \overline{d}) \leq EC_2(\overline{c}, d)] \tag{8.27}$$

$$(8.26) \;\rightarrow\; [EC_1(\overline{c}, \overline{d}) \leq EC_1(c^*, \overline{d})] \tag{8.28}$$

$$(8.27) \;\rightarrow\; [EC_2(\overline{c}, \overline{d}) \leq EC_2(\overline{c}, d^*)] \tag{8.29}$$

Adding inequalities (8.28) and (8.29), we obtain:

$$EC_1(\overline{c}, \overline{d}) + EC_2(\overline{c}, \overline{d}) = \overline{c} + \overline{d} + L(\overline{c}, \overline{d}) \leq EC_1(c^*, \overline{d}) + EC_2(\overline{c}, d^*). \tag{8.30}$$

$$\overline{d} < d^* \rightarrow EC_1(c^*, \overline{d}) = c^* + \lambda F(c^*, \overline{d})$$

Also, $\overline{d} < d^* \rightarrow F(c^*, \overline{d}) = \theta L^*$

Therefore, $\overline{d} < d^* \rightarrow EC_1(c^*, \overline{d}) = c^* + \lambda\theta L^*$

As $(1-\theta)L^* \geq 0,$ we conclude:

$$\overline{d} < d^* \rightarrow EC_1(c^*, \overline{d}) \leq c^* + (1-\theta)L^* + \lambda\theta L^* \tag{8.31}$$

$$\overline{d} \geq d^* \rightarrow EC_1(c^*, \overline{d}) = c^* + R(c^*, \overline{d}) + \lambda F(c^*, \overline{d})$$

$$L(c^*, \overline{d}) > \theta L^* \rightarrow R(c^*, \overline{d}) = L(c^*, \overline{d}) - \theta L^* \wedge F(c^*, \overline{d}) = \theta L^*$$

$$\rightarrow EC_1(c^*, \overline{d}) = c^* + L(c^*, \overline{d}) - \theta L^* + \lambda\theta L^*$$

$$\rightarrow EC_1(c^*, \overline{d}) \leq c^* + (1-\theta)L^* + \lambda\theta L^*, \text{ as } L(c^*, \overline{d}) \leq L^*$$

$L(c^*,\overline{d}) \leq \theta L^* \rightarrow R(c^*,\overline{d}) = 0 \wedge F(c^*,\overline{d}) = L(c^*,\overline{d})$

$\rightarrow EC_1(c^*,\overline{d}) = c^* + \lambda L(c^*,\overline{d}) \leq c^* + \lambda\theta L^* \leq c^* + (1-\theta)L^* + \lambda\theta L^*,$

as $(1-\theta)L^* \geq 0$

Therefore, $\overline{d} \geq d^* \rightarrow EC_1(c^*,\overline{d}) \leq c^* + (1-\theta)L^* + \lambda\theta L^*$ (8.32)

From (8.31) and (8.32) we conclude: $EC_1(c^*,\overline{d}) \leq c^*+(1-\theta)L^*+\lambda\theta L^*$ (8.33)

$EC_2(\overline{c},d^*) = d^*+(1-\lambda)F(\overline{c},d^*) \leq d^*+(1-\lambda)\theta L^*,$ as $F(\overline{c},d^*) \leq \theta L^*$ (8.34)

In view of (8.33) and (8.34), (8.30) reduces to:

$\overline{c} + \overline{d} + L(\overline{c},\overline{d}) \leq EC_1(c^*,\overline{d}) + EC_2(\overline{c},d^*) \leq c^* + (1-\theta)L^* + \lambda\theta L^* + d^*$

$+ (1-\lambda)\theta L^*$

$\rightarrow \overline{c} + \overline{d} + L(\overline{c},\overline{d}) \leq c^* + d^* + L^*.$

As TSC is minimized at (c^*,d^*), it follows that it must be the case that:

$\overline{c} + \overline{d} + L(\overline{c},\overline{d}) = c^* + d^* + L^*.$

This establishes that if $(\overline{c},\overline{d})$ is a Nash equilibrium, then $(\overline{c},\overline{d}) \in M.$ (8.35)

(8.25) and (8.35) establish the theorem. □

It can be shown that if $\theta > 1$ then (λ,θ)-negligence rule is not efficient with respect to $\mathscr{A}$ for any value of λ, $0 \leq \lambda \leq 1$. The following example showing the inefficiency of a particular (λ,θ)-negligence rule where $\theta > 1$ illustrates the logic because of which if $\theta > 1$ then (λ,θ)-negligence rule is not efficient with respect to $\mathscr{A}$ for any value of λ, $0 \leq \lambda \leq 1$.

Example 8.3. Consider the (λ,θ)-negligence rule with $\theta = 1.1$ and $\lambda = \frac{1}{2}$ in the context of the same application as that of Examples 8.2 and 8.3.

We obtain the array of $(EC_1(c,d), EC_2(c,d))$, $(c,d) \in C \times D$, as given below:

$(EC_1(c,d), EC_2(c,d))$

	$d=0$	$d=1$	$d=2$
$c=0$	$(\underline{8.47}, 11.53)$	$(\underline{8.47}, 10.53)$	$(\underline{8.25}, \underline{10.25})$
$c=1$	$(9.47, 9.53)$	$(9.25, \underline{9.25})$	$(8.7, 9.7)$
$c=2$	$(10.25, \underline{8.25})$	$(9.75, 8.75)$	$(9.5, 9.5)$

The only $(c, d) \in C \times D$ which is a Nash equilibrium is $(0, 2)$, and the only $(c, d) \in C \times D$ at which TSC is minimized is $(1, 2)$. Therefore, we conclude that the (λ, θ)-negligence rule in question is inefficient for the application considered here and consequently is not efficient with respect to $\mathscr{A}$. $\Diamond$

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Chapter 9
Multiple Injurers and Victims

So far we have been concerned with two-party interactions involving one victim and one injurer. In this chapter we look at multiparty interactions. In the first section we consider accidents involving one victim and n injurers.[1] When there are one victim and n injurers, we show that the condition of collective negligence liability is sufficient for efficiency. A liability rule defined for one victim and n injurers satisfies the condition of collective negligence liability iff its structure is such that whenever some individuals are negligent, no nonnegligent individual bears any loss in case of occurrence of accident. For the case of one victim and one injurer, the condition of collective negligence liability reduces to that of negligence liability. Thus the condition of collective negligence liability can be viewed as a generalization of the condition of negligence liability when there are one victim and n injurers; and the proposition that the condition of collective negligence liability is sufficient for efficiency of any liability rule defined for one victim and n injurers can be viewed as a generalization of the proposition that the condition of negligence liability is sufficient for efficiency of any liability rule defined for one victim and one injurer. The question whether this condition is necessary for efficiency is an open question.[2]

In Sect. 9.2 we consider the case of multiple victims and one injurer.[3] Unlike the case of one victim and multiple injurers, in the case of multiple victims and one injurer, it turns out that there is no liability rule which is efficient for all applications. The fact that there is no rule which is efficient for all applications does not of

[1]Parts of Sect. 9.1 rely on Jain and Kundu (2006).

[2]The first result with one victim and n injurers is due to Landes and Posner (1980). They showed the efficiency of the rule of negligence with one victim and multiple injurers. Multiple-tortfeasor context was also analyzed in Tietenberg (1989), Kornhauser and Revesz (1989), and Miceli and Segerson (1991). In Jain and Kundu (2006) it has been shown that the condition of negligence liability is necessary and sufficient for the subclass of simple liability rules defined for one victim and n injurers.

[3]Section 9.2 draws on Jain (2009).

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course in any way preclude the possibility of a rule being efficient with respect to some subclass of applications which may be of interest. We consider the important subclass of applications β' such that the expected loss of a particular victim depends only on the care level taken by that victim and the care level taken by the injurer. We show that a sufficient condition for a multiple-victim one-injurer liability rule to be efficient with respect to subclass β' of applications is that its structure be such that: (i) whenever the injurer is negligent and a particular victim is nonnegligent, the entire loss incurred by that victim must be borne by the injurer; and (ii) whenever a particular victim is negligent and the injurer is nonnegligent, the entire loss incurred by that victim must be borne by the victim himself. In fact, for an important subclass of multiple-victim one-injurer liability rules, characterized by the condition that the proportions in which the loss incurred by a particular victim are to be borne by the injurer and that victim must depend only on the nonnegligence proportions of the injurer and that victim, the above condition ($(n, 1)$-negligence liability) is both necessary and sufficient for efficiency with respect to subclass of applications β'.

Like the condition of collective negligence liability, the condition of $(n, 1)$-negligence liability also reduces to that of negligence liability for the case of one victim and one injurer. Thus the condition of $(n, 1)$-negligence liability can be thought of as a generalization of negligence liability condition for liability rules defined for multiple victims and one injurer.

In the context of one victim and multiple injurers, there exist liability rules which are efficient for all applications. On the other hand, there are no liability rules which are efficient for all applications when there are multiple victims and one injurer. The reason for this difference lies in the fact that for efficiency what is required is that all parties involved internalize the totality of harm resulting from the interaction. This is possible when there are multiple injurers and one victim, but not when there are multiple victims and one injurer. Regardless of how much care is taken by a victim, he can at most be made to bear his own loss in entirety, but not any part of loss incurred by another victim. There are contexts in which the loss that a particular victim suffers depends not only on the care taken by himself and the injurer but also on the care levels of other victims. In such situations, given that the victims can at most be made to bear their own losses in entirety, there is no way that a victim could be made to internalize the loss incurred by another victim. Consequently, unlike the case of one victim and multiple injurers, in the case of one injurer and multiple victims, one gets an impossibility theorem. If one considers only those applications where expected loss of a victim depends only on his own care level and the care level of the injurer, but not on the care levels of other victims, then it becomes possible to make all parties internalize all losses which they are in a position to affect; and one obtains possibility theorems.

In view of the fact that there is no liability rule defined for multiple victims and one injurer which is efficient for all applications, it follows that there does not exist any liability rule defined for multiple injurers and multiple victims which is efficient for all applications. However, from the results regarding the efficiency of liability rules defined for one victim and multiple injurers and the results regarding the efficiency of liability rules defined for multiple victims and one injurer, it appears that if efficiency is considered with respect to appropriately defined restricted domains, then possibility results should obtain for multiple-victim multi-injurer liability rules.

9.1 One Victim and Multiple Injurers

First we consider the case of one victim and n injurers. The victim will be designated as individual 1; and the n injurers as individuals $2, \ldots, (n+1)$, where $n \geq 1$. It will be assumed that the entire loss, to begin with, falls on the victim. Let $N = \{1, \ldots, n+1\}$. We denote by $c_i \geq 0$ the cost of care taken by individual $i, i \in N$. We assume that for each $i \in N$, c_i is a strictly increasing function of the level of care taken by individual i. This of course implies that for every individual $i, i \in N$, c_i itself can be taken as the index of the level of care of individual i. Let for each $i \in N$:

$C_i = \{c_i \mid c_i$ is the cost of some feasible level of care which can be taken by individual $i\}$.

We assume

$$(\forall i \in N)[0 \in C_i]. \tag{$\alpha1$}$$

$c_i = 0$ will be identified as no care by individual i. Assumption ($\alpha1$) merely says that taking no care is always a feasible option for each individual belonging to N.

Let π denote the probability of occurrence of accident and $H \geq 0$ the loss in case of occurrence of accident. Both π and H will be assumed to be functions of $c_1, \ldots, c_{n+1}$; $\pi = \pi(c_1, \ldots, c_{n+1})$, $H = H(c_1, \ldots, c_{n+1})$. Let $L = \pi H$. L is thus a function of $c_1, \ldots, c_{n+1}$, $L = L(c_1, \ldots, c_{n+1})$; and denotes the expected loss due to accident. We assume:

$$(\forall (c_1, \ldots, c_{n+1}), \left(c'_1, \ldots, c'_{n+1}\right) \in C_1 \times \ldots \times C_{n+1})(\forall j \in N)\Big[(\forall i \in N)$$
$$\left(i \neq j \;\; \rightarrow \;\; c_i = c'_i\right) \;\wedge\; c_j > c'_j \;\; \rightarrow \;\; \pi(c_1, \ldots, c_{n+1}) \leq \pi\left(c'_1, \ldots, c'_{n+1}\right)\Big]. \tag{$\alpha2$}$$

and

$$(\forall (c_1, \ldots, c_{n+1}), \left(c'_1, \ldots, c'_{n+1}\right) \in C_1 \times \ldots \times C_{n+1})(\forall j \in N)\Big[(\forall i \in N)$$
$$\left(i \neq j \;\; \rightarrow \;\; c_i = c'_i\right) \;\wedge\; c_j > c'_j \;\; \rightarrow \;\; H(c_1, \ldots, c_{n+1}) \leq H\left(c'_1, \ldots, c'_{n+1}\right)\Big]. \tag{$\alpha3$}$$

That is to say, it is assumed that greater care by an individual, given the levels of care of all other individuals, does not result in higher probability of accident or of greater quantum of loss. From ($\alpha 2$) and ($\alpha 3$), it follows that:

$$\left(\forall (c_1, \ldots, c_{n+1}), \left(c_1', \ldots, c_{n+1}'\right) \in C_1 \times \ldots \times C_{n+1}\right) (\forall j \in N) \Big[(\forall i \in N) \left(i \neq j \;\; \rightarrow \;\; c_i = c_i'\right) \wedge \; c_j > c_j' \;\; \rightarrow \;\; L(c_1, \ldots, c_{n+1}) \leq L\left(c_1', \ldots, c_{n+1}'\right) \Big].$$

In other words, a larger expenditure on care by an individual, given the expenditure on care by all other individuals, results in lesser or equal expected accident loss.

Total social costs (TSC) are defined to be the sum of costs of care of all the individuals and the expected loss due to accident; TSC $= [\Sigma_{i \in N} c_i] + L(c_1, \ldots, c_{n+1})$. TSC are thus a function of $c_1, \ldots, c_{n+1}$. Let $M = \Big\{ \left(c_1', \ldots, c_{n+1}'\right) \in C_1 \times \ldots \times C_{n+1} \mid [\Sigma_{i \in N} c_i'] + L\left(c_1', \ldots, c_{n+1}'\right)$ is minimum of $\{[\Sigma_{i \in N} c_i] + L(c_1, \ldots, c_{n+1}) \mid c_i \in C_i, i \in N\} \Big\}$. Thus M is the set of all costs of care configurations $\left(c_1', \ldots, c_{n+1}'\right)$ which are TSC minimizing. It will be assumed that:

$$C_1, \ldots, C_{n+1}, \text{ and } L \text{ are such that } M \text{ is nonempty.} \tag{$\alpha 4$}$$

Let $\left(c_1^*, \ldots, c_{n+1}^*\right) \in M$. Given $c_1^*, \ldots, c_{n+1}^*$, we define for each $i \in N$, function p_i, as follows:
$p_i : C_i \mapsto [0, 1]$
$p_i(c_i) = 1$ if $c_i \geq c_i^*$
$p_i(c_i) = \frac{c_i}{c_i^*}$ if $c_i < c_i^*$.

Depending on the liability rule, there could be legally specified due care levels for all individuals, or for some of them or for none of them. If the liability rule specifies the due care level for individual $i, i \in N$, then c_i^* used in the definition of p_i would be taken to be identical with the legally specified due care level. If the liability rule does not specify the due care level for individual i, then c_i^* used in the definition of p_i can be taken to be any $c_i^* \in C_i$ subject to the requirement that $\left(c_1^*, \ldots, c_{n+1}^*\right) \in M$. Thus in all cases, for each individual i, c_i^* would denote the legally binding due care level for individual i whenever the idea of legally binding due care level for individual i is applicable.

If $p_i(c_i) = 1$, individual i would be called nonnegligent; and if $p_i(c_i) < 1$, individual i would be called negligent.

A liability rule defined for one victim and n injurers is a rule which specifies the proportions in which $n + 1$ individuals are to bear the loss in case of occurrence of accident as a function of their nonnegligence proportions. Formally, a liability rule defined for one victim and n injurers is a function f from $[0, 1]^{n+1}$ to $[0, 1]^{n+1}$, $f : [0, 1]^{n+1} \mapsto [0, 1]^{n+1}$, such that $f(p_1, \ldots, p_{n+1}) = f[p_1(c_1), \ldots, p_{n+1}(c_{n+1})] = (x_1, \ldots, x_{n+1}) = [x_1(p_1(c_1), \ldots, p_{n+1}(c_{n+1})), \ldots, x_{n+1}(p_1(c_1), \ldots, p_{n+1}(c_{n+1}))]$, where

$\Sigma_{i \in N} x_i = 1$. If accident takes place and loss of $H(c_1, \ldots, c_{n+1})$ materializes, then $x_i[p_1(c_1), \ldots, p_{n+1}(c_{n+1})]\ H(c_1, \ldots, c_{n+1})$ will be borne by individual i. The expected costs EC_i of individual $i, i \in N$, therefore, are given by $EC_i = c_i + x_i[p_1(c_1), \ldots, p_{n+1}(c_{n+1})]\ L(c_1, \ldots, c_{n+1})$. Every individual is assumed to regard an outcome to be at least as good as another outcome iff expected costs for the individual under the former are less than or equal to expected costs under the latter.

Let the set of applications $< C_1, \ldots, C_{n+1}, \pi, H, \left(c_1^*, \ldots, c_{n+1}^*\right) \in M >$ satisfying ($\alpha 1$)–($\alpha 4$) be denoted by α. A liability rule f defined for one victim and n injurers is efficient for the application $< C_1, \ldots, C_{n+1}, \pi, H, \left(c_1^*, \ldots, c_{n+1}^*\right) \in M >$ belonging to α iff $[(\forall(\overline{c}_1, \ldots, \overline{c}_{n+1}) \in C_1 \times \ldots \times C_{n+1})[(\overline{c}_1, \ldots, \overline{c}_{n+1})$ is a Nash equilibrium $\rightarrow (\overline{c}_1, \ldots, \overline{c}_{n+1}) \in M] \wedge (\exists(\overline{c}_1, \ldots, \overline{c}_{n+1}) \in C_1 \times \ldots \times C_{n+1})[(\overline{c}_1, \ldots, \overline{c}_{n+1})$ is a Nash equilibrium$]]$. f is defined to be efficient for a set of applications iff it is efficient for every application belonging to the set.

The following examples illustrate the definitions given above.

Example 9.1. Let f defined for one victim and two injurers be given by $(\forall(p_1, p_2, p_3) \in [0,1]^3)[[p_1 < 1 \rightarrow f(p_1, p_2, p_3) = (1, 0, 0)] \wedge [p_1 = 1 \rightarrow f(p_1, p_2, p_3) = (0, \frac{1}{2}, \frac{1}{2})]]$.
Consider the following application:
$(\forall i \in N)[C_i = \{0, 1\}]$
$L(0,0,0) = 3.5, L(0,0,1) = L(0,1,0) = 2, L(1,0,0) = 3, L(0,1,1) = 0.5, L(1,0,1) = L(1,1,0) = 1.5, L(1,1,1) = 0.$
$(0,1,1)$ is the unique TSC-minimizing configuration of costs of care. Let $(c_1^*, c_2^*, c_3^*) = (0, 1, 1)$.
Here $(0,0,0)$, which is not TSC minimizing, is the only Nash equilibrium.
Thus f is inefficient for the given application. $\lozenge$

Example 9.2. Let f defined for one victim and two injurers be given by $(\forall(p_1, p_2, p_3) \in [0,1]^3)[[p_2 < 1 \rightarrow f(p_1, p_2, p_3) = (0, 1, 0)] \wedge [p_2 = 1 \wedge p_3 < 1 \rightarrow f(p_1, p_2, p_3) = (0, 0, 1)] \wedge [p_2 = 1 \wedge p_3 = 1 \rightarrow f(p_1, p_2, p_3) = (1, 0, 0)]]$.
Consider the same application as in Example 9.1.
Here $(0,1,1)$, which is TSC minimizing, is the only Nash equilibrium.
Thus f is efficient for the given application. $\lozenge$

9.1.1 A Sufficient Condition for Efficiency of Liability Rules Defined for One Victim and n Injurers

First we state the condition of collective negligence liability.

Condition of collective negligence liability (CNL): A liability rule f defined for one victim and n injurers satisfies the condition of collective negligence liability iff $\left(\forall(p_1, \ldots, p_{n+1}) \in [0,1]^{n+1}\right) [(p_1, \ldots, p_{n+1}) \neq (1, \ldots, 1) \rightarrow (\forall i \in N)(p_i = 1 \rightarrow x_i(p_1, \ldots, p_{n+1}) = 0)]$.

In other words, a liability rule defined for one victim and n injurers satisfies the condition of collective negligence liability iff its structure is such that whenever some individuals are negligent, no nonnegligent individual bears any loss in case of occurrence of accident.

It should be noted that for the case of $n = 1$, the condition of collective negligence liability reduces to that of the condition of negligence liability. Therefore the condition of collective negligence liability can be viewed as a generalization of the condition of negligence liability for the case of more than one injurer.

The theorem that follows shows that the condition of collective negligence liability is sufficient to ensure efficiency when there are one victim and multiple injurers.

Theorem 9.1. *If a liability rule f defined for one victim and n injurers satisfies the condition of collective negligence liability, then it is efficient with respect to the set of applications α.*

Proof. Let liability rule f satisfy condition CNL. Take any $C_1, \ldots, C_{n+1}, \pi, H$ and $\left(c_1^*, \ldots, c_{n+1}^*\right) \in M$ satisfying ($\alpha 1$)–($\alpha 4$). Denote $f(1, \ldots, 1)$ by $\left(x_1^*, \ldots, x_{n+1}^*\right)$. Suppose $\left(c_1^*, \ldots, c_{n+1}^*\right)$ is not a Nash equilibrium. Then, for some $k \in N$ there is some $c_k' \in C_k$ which is a better strategy for individual k than c_k^*, given that every other individual j uses $c_j^*, j \in N, j \neq k$. That is to say:

$$
\begin{aligned}
&(\exists k \in N)\left(\exists c_k' \in C_k\right)\left[c_k' + x_k\left[p_1\left(c_1^*\right), \ldots, p_k\left(c_k'\right), \ldots, p_{n+1}\left(c_{n+1}^*\right)\right]\right.\\
&L\left(c_1^*, \ldots, c_k', \ldots, c_{n+1}^*\right) < c_k^* + x_k\left[p_1\left(c_1^*\right), \ldots, p_k\left(c_k^*\right), \ldots, p_{n+1}\left(c_{n+1}^*\right)\right]\\
&\left.L\left(c_1^*, \ldots, c_{n+1}^*\right)\right].
\end{aligned} \tag{9.1}
$$

First consider the case $c_k' < c_k^*$.
If $c_k' < c_k^*$, then $x_k\left[p_1\left(c_1^*\right), \ldots, p_k\left(c_k'\right), \ldots, p_{n+1}\left(c_{n+1}^*\right)\right] = 1$, by condition CNL. Therefore:

$$(9.1) \;\rightarrow\; c_k' + L\left(c_1^*, \ldots, c_k', \ldots, c_{n+1}^*\right) < c_k^* + x_k^* L\left(c_1^*, \ldots, c_k^*, \ldots, c_{n+1}^*\right).$$

As $0 \leq x_k^* \leq 1$, we obtain:

$$c_k' + L\left(c_1^*, \ldots, c_k', \ldots, c_{n+1}^*\right) < c_k^* + L\left(c_1^*, \ldots, c_{n+1}^*\right).$$

Adding $\Sigma_{j \in N-\{k\}} c_j^*$ to both sides of the above inequality, one obtains:

$$\left[\Sigma_{j \in N-\{k\}} c_j^*\right] + c_k' + L\left(c_1^*, \ldots, c_k', \ldots, c_{n+1}^*\right) < \left[\Sigma_{j \in N} c_j^*\right] + L\left(c_1^*, \ldots, c_{n+1}^*\right). \tag{9.2}$$

Inequality (9.2) says that TSC at $\left(c_1^*, \ldots, c_k', \ldots, c_{n+1}^*\right)$ are less than the TSC at $\left(c_1^*, \ldots, c_{n+1}^*\right)$. But TSC attain their minimum at $\left(c_1^*, \ldots, c_{n+1}^*\right)$.

This contradiction establishes that if $c_k' < c_k^*$, then (9.1) cannot hold. (9.3)

Next consider the case $c'_k > c^*_k$.
If $c'_k > c^*_k$, then:

(9.1) $\rightarrow$ $c'_k + x^*_k L\left(c^*_1, \ldots, c'_k, \ldots, c^*_{n+1}\right) < c^*_k + x^*_k L\left(c^*_1, \ldots, c^*_{n+1}\right)$,

as $x_k\left[p_1\left(c^*_1\right), \ldots, p_k\left(c'_k\right), \ldots, p_{n+1}\left(c^*_{n+1}\right)\right] = x^*_k$.
Adding $x^*_k \Sigma_{j \in N-\{k\}} c^*_j$ to both sides of the above inequality, one obtains:

$$\left(1 - x^*_k\right) c'_k + x^*_k\left[\left[\Sigma_{j \in N-\{k\}} c^*_j\right] + c'_k + L\left(c^*_1, \ldots, c'_k, \ldots, c^*_{n+1}\right)\right] <$$
$$\left(1 - x^*_k\right) c^*_k + x^*_k\left[\left[\Sigma_{j \in N} c^*_j\right] + L\left(c^*_1, \ldots, c^*_{n+1}\right)\right].$$

As the minimum of TSC = $\left[\Sigma_{j \in N} c^*_j\right] + L\left(c^*_1, \ldots, c^*_{n+1}\right)$, it must be the case that:

$$\left[\Sigma_{j \in N-\{k\}} c^*_j\right] + c'_k + L\left(c^*_1, \ldots, c'_k, \ldots, c^*_{n+1}\right) \geq \left[\Sigma_{j \in N} c^*_j\right] + L\left(c^*_1, \ldots, c^*_{n+1}\right);$$

and consequently:

$$x^*_k\left[\left[\Sigma_{j \in N-\{k\}} c^*_j\right] + c'_k + L\left(c^*_1, \ldots, c'_k, \ldots, c^*_{n+1}\right)\right] \geq x^*_k\left[\left[\Sigma_{j \in N} c^*_j\right]\right.$$
$$\left.+ L\left(c^*_1, \ldots, c^*_{n+1}\right)\right].$$

Therefore we conclude:
$\left(1 - x^*_k\right) c'_k < \left(1 - x^*_k\right) c^*_k$.
If $\left(1 - x^*_k\right) > 0$, then $c'_k < c^*_k$, contradicting the hypothesis that $c'_k > c^*_k$. If $\left(1 - x^*_k\right) = 0$, then we obtain $0 < 0$, a contradiction.

This establishes that if $c'_k > c^*_k$, then (9.1) cannot hold. (9.4)

(9.3) and (9.4) establish that $\left(c^*_1, \ldots, c^*_{n+1}\right)$ is a Nash equilibrium. (9.5)

Let $(\bar{c}_1, \ldots, \bar{c}_{n+1})$ be a Nash equilibrium.
$(\bar{c}_1, \ldots, \bar{c}_{n+1})$ being a Nash equilibrium implies:

$$(\forall i \in N)(\forall c_i \in C_i)[\bar{c}_i + x_i[p_1(\bar{c}_1), \ldots, p_{n+1}(\bar{c}_{n+1})]L(\bar{c}_1, \ldots, \bar{c}_{n+1}) \leq c_i + x_i$$
$$[p_1(\bar{c}_1), \ldots, p_i(c_i), \ldots, p_{n+1}(\bar{c}_{n+1})]L(\bar{c}_1, \ldots, c_i, \ldots, \bar{c}_{n+1})]. \quad (9.6)$$

$$(9.6) \rightarrow (\forall i \in N)\Big[\bar{c}_i + x_i[p_1(\bar{c}_1), \ldots, p_{n+1}(\bar{c}_{n+1})]L(\bar{c}_1, \ldots, \bar{c}_{n+1}) \leq c^*_i + x_i$$
$$[p_1(\bar{c}_1), \ldots, p_i\left(c^*_i\right), \ldots, p_{n+1}(\bar{c}_{n+1})]L\left(\bar{c}_1, \ldots, c^*_i, \ldots, \bar{c}_{n+1}\right)\Big]. \quad (9.7)$$

$$(9.7) \rightarrow [\Sigma_{i \in N} \bar{c}_i] + L(\bar{c}_1, \ldots, \bar{c}_{n+1}) \leq [\Sigma_{i \in N} c^*_i] + \Sigma_{i \in N} x_i\Big[(\forall j \in N - \{i\})$$
$$(p_j = p_j(\bar{c}_j)) \wedge p_i = p_i\left(c^*_i\right)\Big] L\left[(\forall j \in N - \{i\})(c_j = \bar{c}_j) \wedge c_i = c^*_i\right], \quad (9.8)$$

as $\Sigma_{i \in N} x_i[p_1(\bar{c}_1), \ldots, p_{n+1}(\bar{c}_{n+1})] = 1$.

Let $\{i \in N \mid \bar{c}_i < c_i^*\}$ be designated by N_0.

First consider the case when $N_0 = \emptyset$.
$N_0 = \emptyset \ \rightarrow \ (\forall i \in N)$
$[\bar{c}_i \geq c_i^* \ \wedge \ x_i \left[(\forall j \in N - \{i\})(p_j = p_j(\bar{c}_j)) \wedge \ p_i = p_i\left(c_i^*\right)\right] = x_i^*]$.
(9.8), therefore, reduces to:

$$[\Sigma_{i \in N}\bar{c}_i] + L(\bar{c}_1, \ldots, \bar{c}_{n+1}) \leq [\Sigma_{i \in N} c_i^*] + \Sigma_{i \in N} x_i^* L\left[(\forall j \in N - \{i\})(c_j = \bar{c}_j) \wedge \ c_i = c_i^*\right]. \tag{9.9}$$

As $(\forall i \in N)\, [\bar{c}_i \geq c_i^*]$, we have:

$$(\forall i \in N)\left[L\left[(\forall j \in N - \{i\})(c_j = \bar{c}_j) \ \wedge \ c_i = c_i^*\right] \leq L\left(c_1^*, \ldots, c_{n+1}^*\right)\right]. \tag{9.10}$$

(9.9) and (9.10) $\ \rightarrow \ [\Sigma_{i \in N}\bar{c}_i] + L(\bar{c}_1, \ldots, \bar{c}_{n+1}) \leq [\Sigma_{i \in N} c_i^*] +$

$$\Sigma_{i \in N} x_i^* L\left(c_1^*, \ldots, c_{n+1}^*\right) = [\Sigma_{i \in N} c_i^*] + L\left(c_1^*, \ldots, c_{n+1}^*\right). \tag{9.11}$$

(9.11) says that TSC at $(\bar{c}_1, \ldots, \bar{c}_{n+1})$ are less than or equal to TSC at $\left(c_1^*, \ldots, c_{n+1}^*\right)$. As TSC at $\left(c_1^*, \ldots, c_{n+1}^*\right)$ are minimum, it must be the case that TSC at $(\bar{c}_1, \ldots, \bar{c}_{n+1})$ are equal to TSC at $\left(c_1^*, \ldots, c_{n+1}^*\right)$. Therefore we conclude:

$$(\bar{c}_1, \ldots, \bar{c}_{n+1}) \text{ is a Nash equilibrium } \wedge \ N_0 = \emptyset \ \rightarrow \ (\bar{c}_1, \ldots, \bar{c}_{n+1}) \in M. \tag{9.12}$$

Next consider the case $\# N_0 = 1$. Let $N_0 = \{k\}$.

$$N_0 = \{k\} \ \rightarrow \ (\forall i \in N - \{k\}) \left[x_i \left[(\forall j \in N - \{i\})(p_j = p_j(\bar{c}_j)) \wedge \ p_i = p_i\left(c_i^*\right)\right] = 0\right], \tag{9.13}$$

by condition CNL.
In view of (9.13), (9.8) reduces to:

$$[\Sigma_{i \in N}\bar{c}_i] + L(\bar{c}_1, \ldots, \bar{c}_{n+1}) \leq [\Sigma_{i \in N} c_i^*] + x_k^* L\left(\bar{c}_1, \ldots, c_k^*, \ldots, \bar{c}_{n+1}\right), \tag{9.14}$$

as $x_k\left[p_1(\bar{c}_1), \ldots, p_k\left(c_k^*\right), \ldots, p_{n+1}(\bar{c}_{n+1})\right] = x_k^*$.
Now, $(\forall i \in N - \{k\})\, [\bar{c}_i \geq c_i^*] \ \rightarrow \ L\left(\bar{c}_1, \ldots, c_k^*, \ldots, \bar{c}_{n+1}\right) \leq L(c_1^*, \ldots, c_{n+1}^*)$.
Consequently, (9.14) implies:

$$[\Sigma_{i \in N}\bar{c}_i] + L(\bar{c}_1, \ldots, \bar{c}_{n+1}) \leq [\Sigma_{i \in N} c_i^*] + x_k^* L\left(c_1^*, \ldots, c_{n+1}^*\right) \leq [\Sigma_{i \in N} c_i^*] + L\left(c_1^*, \ldots, c_{n+1}^*\right) \tag{9.15}$$

(9.15) establishes that:

$$(\overline{c}_1, \ldots, \overline{c}_{n+1}) \text{ is a Nash equilibrium and } \#N_0 = 1 \;\rightarrow\; (\overline{c}_1, \ldots, \overline{c}_{n+1}) \in M. \tag{9.16}$$

Finally consider the case when $\#N_0 > 1$.

$$\#N_0 > 1 \;\rightarrow\; (\forall i \in N)\left[x_i\left[(\forall j \in N - \{i\})(p_j = p_j(\overline{c}_j)) \;\wedge\; p_i = p_i\left(c_i^*\right)\right] = 0\right], \tag{9.17}$$

by condition CNL.
In view of (9.17), (9.8) reduces to:

$$[\Sigma_{i \in N}\overline{c}_i] + L(\overline{c}_1, \ldots, \overline{c}_{n+1}) \leq \Sigma_{i \in N} c_i^*. \tag{9.18}$$

(9.18) in turn implies $[\Sigma_{i \in N}\overline{c}_i] + L(\overline{c}_1, \ldots, \overline{c}_{n+1}) \leq [\Sigma_{i \in N} c_i^*] + L\left(c_1^*, \ldots, c_{n+1}^*\right).$ (9.19)

(9.19) establishes that:

$$(\overline{c}_1, \ldots, \overline{c}_{n+1}) \text{ is a Nash equilibrium and } \#N_0 > 1 \;\rightarrow\; (\overline{c}_1, \ldots, \overline{c}_{n+1}) \in M. \tag{9.20}$$

(9.12), (9.16), and (9.20) establish that all $(\overline{c}_1, \ldots, \overline{c}_{n+1}) \in C_1 \times \ldots \times C_{n+1}$

which are Nash equilibria are TSC minimizing. (9.21)

(9.5) and (9.21) establish the theorem. □

9.1.2 The Problem of Characterization of Efficient Liability Rules Defined for One Victim and Multiple Injurers

Theorem 9.1 establishes that every liability rule defined for one victim and multiple injurers which satisfies the condition of collective negligence liability is efficient. The question whether collective negligence liability is necessary for efficiency is an open one. In Jain and Kundu (2006), it has been shown that collective negligence liability is both necessary and sufficient for efficiency for the class of simple liability rules defined for one victim and multiple injurers. Simple liability rules do not differentiate between no care and care levels which are below the due care; that is to say, treat all nonnegligence proportions belonging to $[0, 1)$ the same.[4] In view of the

[4]More formally, $f : [0,1]^{n+1} \mapsto [0,1]^{n+1}$ is a simple liability rule iff $\left(\forall (p_1, \ldots, p_{n+1}) \in [0,1]^{n+1}\right)(\forall i \in N)[p_i < 1 \rightarrow f(p_1, \ldots, p_{i-1}, p_i, p_{i+1}, \ldots, p_{n+1}) = f(p_1, \ldots, p_{i-1}, 0, p_{i+1}, \ldots, p_{n+1})]$.

necessity and sufficiency of collective negligence liability for efficiency of simple liability rules, it follows that for the class of liability rules defined for one victim and multiple injurers there are only two possibilities. These possibilities are as follows: (i) the condition of collective negligence liability is both necessary and sufficient for efficiency with respect to α for any liability rule, and (ii) for the class of liability rules as a whole, there does not exist any condition which is both necessary and sufficient for efficiency with respect to α. It is an open question as to which of these two possibilities in fact holds.

9.1.3 Negligence Rule and Strict Liability with the Defense of Contributory Negligence

Consider the class of simple liability rules defined for one victim and multiple injurers by: (a) If an injurer is nonnegligent, then he is not at all liable. (b) If at least one injurer is negligent, then the set of all injurers taken together is fully liable. This class of rules can be thought of as embodying the essential features of the negligence rule in the multi-injurer context in a natural way. From the definition of the negligence rule it follows that in the two-party context it is characterized by the following four implications: (i) If the injurer is negligent, then he is fully liable. (ii) If the injurer is fully liable, then he is negligent. (iii) If the injurer is nonnegligent, then he is not at all liable. And (iv) if the injurer is not at all liable, then he is nonnegligent. It is clear that in a multi-injurer context, (i) cannot be satisfied. It is equally clear that one cannot abandon (i) altogether without giving up the very idea behind the negligence rule. (b), however, seems a natural way to retain the idea of (i) in the multi-injurer context. (ii) and (iii) also must be retained if the essential idea behind the negligence rule is to remain intact. (a) requires for every injurer what (iii) requires for the single injurer in the two-party context. It should be noted that (a) implies (ii) for every injurer. In the multi-injurer context, with respect to (iv), there is some leeway; nothing essential seems to hinge on whether it holds or not. Thus it seems appropriate to term the class of simple liability rules defined by (a) and (b) as the class of negligence rules.[5] It is obvious that (a) and (b) together imply satisfaction of collective negligence liability. Thus as a corollary of Theorem 9.1, it follows that every variant of negligence rule in a multi-injurer context is efficient.

Unlike the rule of negligence, in the case of strict liability with the defense of contributory negligence, all the four implications characterizing the rule in the

[5]The rule of Example 9.2 belongs to the class of negligence rules. This rule satisfies the condition of collective negligence liability.

two-party context can continue to hold in the multi-injurer context. Indeed, the implications: (1) If the victim is negligent then he is fully liable, (2) If the victim is fully liable then he is negligent, (3) If the victim is nonnegligent then he is not at all liable, and (4) If the victim is not at all liable then he is nonnegligent; must be satisfied if the essential features of strict liability with the defense of contributory negligence are to be retained. Now, every rule in the class of simple liability rules satisfying (1)–(4), which we can term as the class of strict liability with the defense of contributory negligence rules, does not satisfy the condition of collective negligence liability.[6] On the other hand, it is clear that there is a subclass of the class of simple liability rules satisfying (1)–(4) which does satisfy the condition of collective negligence liability. Thus, in the multi-injurer context, while every variant of the negligence rule is efficient, only some variants of strict liability with the defense of contributory negligence are efficient.[7]

9.2 Multiple Victims and One Injurer

In this section we consider the case of n victims (individuals $1, \ldots, n$), $n \geq 1$, and one injurer (individual $n+1$). Let $N = \{1, \ldots, n, n+1\}$ and $N_V = \{1, \ldots, n\}$. It would be assumed that the losses, to begin with, fall on the victims. We denote by $c_i \geq 0$ the cost of care taken by individual $i, i \in N$. We assume that for each $i \in N$, c_i is a strictly increasing function of the level of care taken by individual i. This implies that for every individual $i, i \in N$, c_i itself can be taken as the index of the level of care of individual i. Let for each $i \in N$:
$C_i = \{c_i \mid c_i$ is the cost of some feasible level of care which can be taken by individual $i\}$.
We assume

$$(\forall i \in N)[0 \in C_i]. \qquad (\beta 1)$$

$c_i = 0$ will be identified as no care by individual i. Assumption ($\beta 1$) merely says that taking no care is always a feasible option for each individual belonging to N.

[6]The rule of Example 9.1 belongs to the class of strict liability with the defense of contributory negligence rules. The rule does not satisfy the condition of collective negligence liability.

[7]The observation that not all variants of the rule of strict liability with the defense of contributory negligence in the one-victim multi-injurer context are inefficient was first made by Kornhauser and Revesz (1989). As all variants of the rule of strict liability with the defense of contributory negligence in the one-victim multi-injurer context are simple liability rules defined for one victim and multiple injurers, the general characterization theorem proved in Jain and Kundu (2006) enables one to demarcate the efficient variants of the rule of strict liability with the defense of contributory negligence from those variants which are inefficient.

Let π denote the probability of occurrence of accident and $H_i \geq 0$ the loss to individual $i \in N_V$ in case of occurrence of accident. π and $H_i, i \in N_V$, will be assumed to be functions of $c_1, \ldots, c_n, c_{n+1}$; $\pi = \pi(c_1, \ldots, c_n, c_{n+1})$; $H_i = H_i(c_1, \ldots, c_n, c_{n+1})$. Let $\mathbb{L}_i = \pi H_i, i \in N_V$. $\mathbb{L}_i, i \in N_V$, is thus a function of $c_1, \ldots, c_n, c_{n+1}$ and denotes the expected loss to individual i. We assume:

$$(\forall (c_1, \ldots, c_n, c_{n+1}), \left(c'_1, \ldots, c'_n, c'_{n+1}\right) \in C_1 \times \ldots \times C_n \times C_{n+1})(\forall j \in N)$$
$$\Big[(\forall i \in N)\big(i \neq j \;\; \rightarrow \;\; c_i = c'_i\big) \;\wedge\; c_j > c'_j \;\; \rightarrow \;\; \pi(c_1, \ldots, c_n, c_{n+1})$$
$$\leq \pi\left(c'_1, \ldots, c'_n, c'_{n+1}\right)\Big]. \qquad (\beta 2)$$

$$(\forall k \in N_V)(\forall (c_1, \ldots, c_n, c_{n+1}), \left(c'_1, \ldots, c'_n, c'_{n+1}\right) \in C_1 \times \ldots \times C_n \times C_{n+1})$$
$$(\forall j \in N)\Big[(\forall i \in N)\,(i \neq j \;\; \rightarrow \;\; c_i = c'_i) \;\wedge\; c_j > c'_j \;\; \rightarrow \;\; H_k(c_1, \ldots, c_n, c_{n+1})$$
$$\leq H_k\left(c'_1, \ldots, c'_n, c'_{n+1}\right)\Big]. \qquad (\beta 3)$$

That is to say, greater care by an individual, given the care levels of all other individuals, does not result in greater probability of accident or greater loss to some victim in case of accident.

Assumptions ($\beta 2$) and ($\beta 3$) imply:
$$(\forall k \in N_V)(\forall (c_1, \ldots, c_n, c_{n+1}), \left(c'_1, \ldots, c'_n, c'_{n+1}\right) \in C_1 \times \ldots \times C_n \times C_{n+1})$$
$$(\forall j \in N)\Big[(\forall i \in N)\,(i \neq j \;\; \rightarrow \;\; c_i = c'_i) \;\wedge\; c_j > c'_j \;\; \rightarrow \;\; \mathbb{L}_k(c_1, \ldots, c_n, c_{n+1})$$
$$\leq \mathbb{L}_k\left(c'_1, \ldots, c'_n, c'_{n+1}\right)\Big].$$
That is to say, greater care by an individual, given the levels of care of all other individuals, results, for every $k \in N_V$, in lesser or equal expected accident loss.

TSC are defined to be the sum of costs of care of all the individuals and expected losses of the victims; TSC $= \Sigma_{i \in N} c_i + \Sigma_{i \in N_V} \mathbb{L}_i(c_1, \ldots, c_n, c_{n+1})$. TSC are thus a function of $c_1, \ldots, c_n, c_{n+1}$. Let $M = \left\{\left(c'_1, \ldots, c'_n, c'_{n+1}\right) \mid \Sigma_{i \in N} c'_i + \Sigma_{i \in N_V} \mathbb{L}_i \left(c'_1, \ldots, c'_n, c'_{n+1}\right)\right.$ is minimum of $\{\Sigma_{i \in N} c_i + \Sigma_{i \in N_V} \mathbb{L}_i(c_1, \ldots, c_n, c_{n+1}) \mid (c_1, \ldots, c_n, c_{n+1}) \in C_1 \times \ldots \times C_n \times C_{n+1}\}\}$. Thus M is the set of all costs of care configurations $\left(c'_1, \ldots, c'_n, c'_{n+1}\right)$ which are TSC minimizing. It will be assumed that:

$$C_1, \ldots, C_n, C_{n+1}; \mathbb{L}_1, \mathbb{L}_2, \ldots, \mathbb{L}_n \text{ are such that } M \text{ is nonempty.} \qquad (\beta 4)$$

Let $\left(c^*_1, \ldots, c^*_n, c^*_{n+1}\right) \in M$. Given $c^*_1, \ldots, c^*_n, c^*_{n+1}$, we define for each $i \in N$ function p_i, $p_i : C_i \mapsto [0, 1]$, as follows:

$p_i(c_i) = 1$ if $c_i \geq c^*_i$
$p_i(c_i) = \frac{c_i}{c^*_i}$ if $c_i < c^*_i$.

Depending on the liability rule, there could be legally specified due care levels for all individuals, or for some of them or for none of them. If there is a legally specified due care level for individual $i, i \in N$, then c_i^* used in the definition of p_i would be taken to be identical with the legally specified due care level. If there is no legally specified due care level for individual i, then c_i^* used in the definition of p_i can be taken to be any $c_i^* \in C_i$ subject to the requirement that $\left(c_1^*, \ldots, c_n^*, c_{n+1}^*\right) \in M$. Thus in all cases, for each individual i, c_i^* would denote the legally binding due care level for individual i whenever the idea of legally binding due care level for individual i is applicable. If $p_i(c_i) = 1$, individual i would be called nonnegligent; and if $p_i(c_i) < 1$, individual i would be called negligent.

A multiple-victim one-injurer liability rule is a rule which specifies the proportions in which each victim's loss is to be divided between the victim in question and the injurer in case of occurrence of accident as a function of proportions of nonnegligence of individuals. Formally, a liability rule defined for n victims and one injurer is a function f from $[0,1]^{n+1}$ to $[0,1]^n$, $f : [0,1]^{n+1} \mapsto [0,1]^n$, such that $f(p_1, \ldots, p_n, p_{n+1}) = f[p_1(c_1), \ldots, p_n(c_n), p_{n+1}(c_{n+1})] = (x_1, \ldots, x_n) = [x_1(p_1(c_1), \ldots, p_n(c_n), p_{n+1}(c_{n+1})), \ldots, \quad x_n(p_1(c_1), \ldots, p_n(c_n), p_{n+1}(c_{n+1}))]$, where $x_i, i \in N_V$, is the proportion of loss to the victim i which is borne by victim i; and $(1 - x_i) = y_i$ the proportion to be borne by the injurer.

If accident takes place and losses of $H_1, \ldots, H_n$ are incurred by victims $1, \ldots, n$, respectively, then $x_1[p_1(c_1), \ldots, p_n(c_n), p_{n+1}(c_{n+1})]H_1(c_1, \ldots, c_n, c_{n+1}), \ldots,$ $x_n[p_1(c_1), \ldots, p_n(c_n), p_{n+1}(c_{n+1})]H_n(c_1, \ldots, c_n, c_{n+1})$ will be borne by individuals $1, \ldots, n$, respectively, and $\Sigma_{i \in N_V} y_i[p_1(c_1), \ldots, p_n(c_n), p_{n+1}(c_{n+1})]H_i$ $(c_1, \ldots, c_n, c_{n+1})$ will be borne by the injurer. Victim i's, $i \in N_V$, expected costs therefore are:

$c_i + x_i[p_1(c_1), \ldots, p_n(c_n), p_{n+1}(c_{n+1})]\mathbb{L}_i(c_1, \ldots, c_n, c_{n+1})$;

and injurer's expected costs are:

$c_{n+1} + \Sigma_{i \in N_V} y_i[p_1(c_1), \ldots, p_n(c_n), p_{n+1}(c_{n+1})]\mathbb{L}_i(c_1, \ldots, c_n, c_{n+1})$.

Every individual belonging to N is assumed to regard an outcome to be at least as good as another outcome iff expected costs of the individual under the former are less than or equal to expected costs under the latter.

The context in which a liability rule f defined for n victims and one injurer is applied is completely specified by $C_1, \ldots, C_n, C_{n+1}$; π; $H_1, \ldots, H_n$; and $\left(c_1^*, \ldots, c_n^*, c_{n+1}^*\right) \in M$. The set of all applications $< C_1, \ldots, C_n, C_{n+1}; \pi; H_1, \ldots, H_n; \left(c_1^*, \ldots, c_n^*, c_{n+1}^*\right) \in M >$ satisfying assumptions ($\beta 1$)–($\beta 4$) will be denoted by β.

A liability rule f defined for n victims and one injurer is defined to be efficient for a given application $< C_1, \ldots, C_n, C_{n+1}; \pi; H_1, \ldots, H_n; \left(c_1^*, \ldots, c_n^*, c_{n+1}^*\right) \in M >$ iff $(\forall(\overline{c}_1, \ldots, \overline{c}_n, \overline{c}_{n+1}) \in C_1 \times \ldots \times C_n \times C_{n+1})[(\overline{c}_1, \ldots, \overline{c}_n, \overline{c}_{n+1})$ is a Nash equilibrium $\rightarrow$ $(\overline{c}_1, \ldots, \overline{c}_n, \overline{c}_{n+1}) \in M]$ $\wedge$ $(\exists(\overline{c}_1, \ldots, \overline{c}_n, \overline{c}_{n+1}) \in C_1 \times \ldots \times C_n \times C_{n+1})[(\overline{c}_1, \ldots, \overline{c}_n, \overline{c}_{n+1})$ is a Nash equilibrium].

The following examples illustrate the definitions given above.

Example 9.3. Consider the liability rule f defined for two victims and one injurer given by $(\forall(p_1, p_2, p_3) \in [0,1]^3)[[p_3 < 1 \rightarrow x_1(p_1, p_2, p_3) = 0 \wedge x_2(p_1, p_2, p_3) = 0] \wedge [p_3 = 1 \rightarrow x_1(p_1, p_2, p_3) = 1 \wedge x_2(p_1, p_2, p_3) = 1]]$.[8]

Consider an application of the above rule such that:
$C_1 = C_2 = C_3 = \{0, 1\}$;
$\mathbb{L}_1(0,0,0) = \mathbb{L}_1(0,1,0) = 2; \mathbb{L}_1(1,0,0) = \mathbb{L}_1(1,1,0) = .75; \mathbb{L}_1(0,0,1) = \mathbb{L}_1(0,1,1) = 1.25; \mathbb{L}_1(1,0,1) = \mathbb{L}_1(1,1,1) = 0$;
$\mathbb{L}_2(0,0,0) = \mathbb{L}_2(1,0,0) = 2; \mathbb{L}_2(0,1,0) = \mathbb{L}_2(1,1,0) = .75; \mathbb{L}_2(0,0,1) = \mathbb{L}_2(1,0,1) = 1.25; \mathbb{L}_2(0,1,1) = \mathbb{L}_2(1,1,1) = 0$.
$(1,1,1)$ is the unique TSC-minimizing configuration of costs of care. Let $(c_1^*, c_2^*, c_3^*) = (1,1,1)$.
It can easily be checked that $(1,1,1)$ is the only $(c_1, c_2, c_3) \in C_1 \times C_2 \times C_3$, which is a Nash equilibrium. The rule is therefore efficient for the application under consideration. $\diamondsuit$

Example 9.4. Consider an application of the rule of Example 9.3 such that:
$C_1 = C_2 = C_3 = \{0, 1\}$;
$(\forall(c_1, c_2, c_3) \in C_1 \times C_2 \times C_3)[\mathbb{L}_1(c_1, c_2, c_3) = \mathbb{L}_2(c_1, c_2, c_3) = 2.25 - 0.75\Sigma_{i \in N} c_i]$.
$(1,1,1)$ is the unique TSC-minimizing configuration of costs of care. Let $(c_1^*, c_2^*, c_3^*) = (1,1,1)$.
Here $(0,0,1)$ is the only $(c_1, c_2, c_3) \in C_1 \times C_2 \times C_3$, which is a Nash equilibrium. The rule is therefore inefficient for the application under consideration. $\diamondsuit$

9.2.1 An Impossibility Theorem

First we establish that there is no liability rule defined for multiple victims and one injurer which invariably gives rise to efficient outcomes.

Theorem 9.2. *There is no liability rule defined for $n \geq 2$ victims and one injurer which is efficient for all applications belonging to β.*

Proof. Let f be any liability rule defined for $n \geq 2$ victims and one injurer.
Consider an application belonging to β specified by:
$(\forall i \in N)[C_i = \{0, 1\}]$.
Let:
$\frac{1}{2} < \theta < 1$;
$0 < \epsilon < \frac{1-\theta}{n}$.

[8]This rule can be thought of as the negligence rule defined for two victims and one injurer.

Let:
$(\forall i \in N_V - \{1,2\})(\forall (c_1,\ldots,c_n,c_{n+1}) \in C_1 \times \ldots \times C_n \times C_{n+1})$
$[\mathbb{L}_i(c_1,\ldots,c_n,c_{n+1}) = \theta + \epsilon - \epsilon c_{n+1} - \theta c_i]$;
$(\forall i \in \{1,2\})(\forall (c_1,\ldots,c_n,c_{n+1}) \in C_1 \times \ldots \times C_n \times C_{n+1})$
$[\mathbb{L}_i(c_1,\ldots,c_n,c_{n+1}) = 2\theta + \epsilon - \epsilon c_{n+1} - \theta c_1 - \theta c_2]$.

Therefore, we obtain:
$\mathrm{TSC}(c_1,\ldots,c_n,c_{n+1}) = \Sigma_{i\in N} c_i + \Sigma_{i\in\{1,2\}}[2\theta + \epsilon - \epsilon c_{n+1} - \theta c_1 - \theta c_2]$
$+ \Sigma_{i\in N_V-\{1,2\}}[\theta + \epsilon - \epsilon c_{n+1} - \theta c_i]$
$= (n+2)\theta + n\epsilon + (1-n\epsilon)c_{n+1} + (1-2\theta)c_1 + (1-2\theta)c_2 + \Sigma_{i\in N_V-\{1,2\}}(1-\theta)c_i$.
$(1-n\epsilon) > 0,\ (1-2\theta) < 0$, and $(1-\theta) > 0$ imply that TSC is uniquely minimized at $[c_1 = 1, c_2 = 1, (\forall i \in N_V - \{1,2\})(c_i = 0), c_{n+1} = 0]$.
Let $\left[c_1^* = 1 \wedge c_2^* = 1 \wedge (\forall i \in N_V - \{1,2\})\,(c_i^* = 0) \wedge c_{n+1}^* = 0\right]$.
Consider the configuration $\left(c_1^*,\ldots,c_n^*,c_{n+1}^*\right)$.
Given that every $i \in N, i \neq 1,$ is going to use $c_i = c_i^*$;

If victim 1 uses $c_1 = 0,$ his expected costs $= EC_1\left(0, c_2^*,\ldots,c_n^*,c_{n+1}^*\right) =$
$0 + x_1[p_1(0), p_2\left(c_2^*\right),\ldots,p_n\left(c_n^*\right),p_{n+1}\left(c_{n+1}^*\right)]\mathbb{L}_1\left[0, c_2^*,\ldots,c_n^*,c_{n+1}^*\right]$
$= x_1\left[p_1(0), p_2\left(c_2^*\right),\ldots,p_n\left(c_n^*\right),p_{n+1}\left(c_{n+1}^*\right)\right](\theta+\epsilon)$
$\leq \theta + \epsilon,$ as $0 \leq x_1\left[p_1(0), p_2\left(c_2^*\right),\ldots,p_n\left(c_n^*\right),p_{n+1}\left(c_{n+1}^*\right)\right] \leq 1$
$< 1,$ as $\epsilon < 1 - \theta$

If victim 1 uses $c_1 = 1 = c_1^*,$ his expected costs $= EC_1\left(c_1^*, c_2^*,\ldots,c_n^*,c_{n+1}^*\right) =$
$1 + x_1\left[p_1\left(c_1^*\right), p_2\left(c_2^*\right),\ldots,p_n\left(c_n^*\right),p_{n+1}\left(c_{n+1}^*\right)\right]\mathbb{L}_1\left[c_1^*, c_2^*,\ldots,c_n^*,c_{n+1}^*\right]$
$= 1 + x_1\left[p_1\left(c_1^*\right), p_2\left(c_2^*\right),\ldots,p_n\left(c_n^*\right),p_{n+1}\left(c_{n+1}^*\right)\right]\epsilon$
$\geq 1.$

Thus, given that every $i \in N, i \neq 1$, is going to use $c_i = c_i^*$, for victim 1 $c_1 = 0$ is better than $c_1 = 1 = c_1^*$. Therefore it follows that the unique TSC-minimizing configuration of care levels $\left(c_1^*,\ldots,c_n^*,c_{n+1}^*\right)$ is not a Nash equilibrium. f is therefore inefficient with respect to β. □

9.2.2 Restricted Domain and Efficiency

Now we state a condition on expected loss functions. Victims' expected loss functions satisfy the condition of mutual independence (CMI) iff $(\forall k \in N_V)$ $(\forall (c_1,\ldots,c_n,c_{n+1}), \left(c_1',\ldots,c_n',c_{n+1}'\right) \in C_1 \times \ldots \times C_n \times C_{n+1})$ $\left[c_k = c_k' \wedge c_{n+1} = c_{n+1}' \rightarrow \mathbb{L}_k\left(c_1,\ldots,c_n,c_{n+1}\right) = \mathbb{L}_k(c_1',\ldots,c_n',c_{n+1}')\right]$.

In other words, victims' expected loss functions satisfy the condition of mutual independence iff every victim's expected loss depends only on his own care level and the care level of the injurer.[9]

The set of all applications $< C_1, \ldots, C_n, C_{n+1};\ \pi;\ H_1, \ldots, H_n; (c_1^*, \ldots, c_n^*, c_{n+1}^*) \in M >$ satisfying assumptions ($\beta 1$)–($\beta 4$) and CMI will be denoted by β'.

If victims' expected loss functions satisfy CMI, then we will write $\mathbb{L}_i(c_1, \ldots, c_n, c_{n+1})$ as $L_i(c_i, c_{n+1}), i \in N_V$.

Next we state a condition on liability rules defined for multiple victims and one injurer similar to the condition of negligence liability.

Condition of $(n, 1)$-negligence liability [$(n, 1)$-NL]: A liability rule f defined for n victims and one injurer satisfies the condition of $(n, 1)$-negligence liability iff $(\forall k \in N_V)\left(\forall (p_1, \ldots, p_n, p_{n+1}) \in [0,1]^{n+1}\right)[[p_k = 1 \wedge p_{n+1} < 1 \rightarrow x_k(p_1, \ldots, p_n, p_{n+1}) = 0] \wedge [p_k < 1 \wedge p_{n+1} = 1 \rightarrow x_k(p_1, \ldots, p_n, p_{n+1}) = 1]]$.

In other words, a liability rule defined for n victims and one injurer satisfies the condition of $(n, 1)$-negligence liability iff its structure is such that for every $k \in N_V$: (i) whenever the injurer is negligent and victim k is nonnegligent, then the entire loss incurred by victim k must be borne by the injurer; and (ii) whenever the injurer is nonnegligent and victim k is negligent, then the entire loss incurred by victim k must be borne by victim k himself. It is immediate that the condition of $(n, 1)$-negligence liability reduces that of negligence liability for the case of $n = 1$. Thus the condition can be viewed as a generalization of negligence liability for the case of multiple victims and one injurer.

It can be shown that $(n, 1)$-negligence liability is a sufficient condition for efficiency with respect to β'. The proof of this theorem (Theorem 9.3) is given in the Appendix.

Let $\mathscr{F}$ designate the set of all liability rules defined for n victims and one injurer. Let the subclass $\mathscr{F}'$ of $\mathscr{F}$ be defined by the condition: $(\forall k \in N_V)\left(\forall (p_1, \ldots, p_n, p_{n+1}), (p'_1, \ldots, p'_n, p'_{n+1}) \in [0,1]^{n+1}\right)\left[p_{n+1} = p'_{n+1} \wedge p_k = p'_k \rightarrow x_k(p_1, \ldots, p_n, p_{n+1}) = x_k\left(p'_1, \ldots, p'_n, p'_{n+1}\right)\right]$. Thus, if a liability rule defined for n victims and one injurer belongs to $\mathscr{F}'$, then the proportions in which victim k's loss is to be divided between the injurer and victim k in case of accident are entirely determined by the nonnegligence proportions of the injurer and victim k.

By Theorem 9.3, the condition of $(n, 1)$-negligence liability is a sufficient condition for any liability rule defined for n victims and one injurer to be efficient with respect to β'. Applications belonging to β' have the characteristic that expected loss of a particular victim depends only on his own care level and the care level of the injurer. When expected loss of a victim depends only on his own care level and the care level of the injurer, the use of a liability rule defined for n victims and one injurer belonging to $\mathscr{F}'$ seems particularly appropriate, where the proportion of loss that a particular victim has to bear depends only on the nonnegligence proportions of the injurer and the victim in question. Theorem 9.4,

[9]If condition CMI holds, then it must be the case that the probability of accident π depends only on the care level of the injurer.

the proof of which is given the Appendix, shows that if we consider the subclass $\mathscr{F}'$ of liability rules defined for n victims and one injurer, then the condition of $(n,1)$-negligence liability is both necessary and sufficient for efficiency with respect to β'. In other words, the rules in $\mathscr{F}'$ which are efficient with respect to $\beta'$are characterized by the condition of $(n,1)$-negligence liability. Whether $(n,1)$-negligence liability is necessary for any liability rule defined for n victims and one injurer belonging to $\mathscr{F}$ to be efficient with respect to β' is an open question.

Appendix

Theorem 9.3. *Let liability rule f defined for n victims and one injurer satisfy the condition of $(n,1)$-negligence liability. Then f is efficient with respect to β'.*

Proof. Let liability rule f satisfy the condition of $(n,1)$-negligence liability. Consider any application $< C_1,\ldots,C_n,C_{n+1};\ \pi;\ H_1,\ldots,H_n;\left(c_1^*,\ldots,c_n^*,c_{n+1}^*\right) \in M >$ belonging to β'. Suppose $\left(c_1^*,\ldots,c_n^*,c_{n+1}^*\right)$ is not a Nash equilibrium. Then, for some $k \in N$, there is some $c_k' \in C_k$ which is a better strategy for individual k than c_k^*, given that every other individual i uses $c_i^*, i \in N, i \neq k$. That is to say, if $k = n+1$, we must have:

$$\left(\exists c_{n+1}' \in C_{n+1}\right)\left[c_{n+1}' + \Sigma_{i\in N_V} y_i\left[p_1\left(c_1^*\right),\ldots,p_n\left(c_n^*\right),p_{n+1}\left(c_{n+1}'\right)\right]\right.$$
$$\left. L_i\left(c_i^*,c_{n+1}'\right) < c_{n+1}^* + \Sigma_{i\in N_V} y_i\left[p_1\left(c_1^*\right),\ldots,p_n\left(c_n^*\right),p_{n+1}\left(c_{n+1}^*\right)\right] L_i\left(c_i^*,c_{n+1}^*\right)\right] \tag{9.22}$$

and if $k \in N_V$, we must have:

$$\left(\exists c_k' \in C_k\right)\left[c_k' + x_k\left[p_1\left(c_1^*\right),\ldots,p_k\left(c_k'\right),\ldots,p_n\left(c_n^*\right),p_{n+1}\left(c_{n+1}^*\right)\right]\right.$$
$$L_k\left(c_k',c_{n+1}^*\right) \ < \ c_k^* + x_k\left[p_1\left(c_1^*\right),\ldots,p_k\left(c_k^*\right),\ldots,p_n\left(c_n^*\right),p_{n+1}\left(c_{n+1}^*\right)\right]$$
$$\left. L_k\left(c_k^*,c_{n+1}^*\right)\right]. \tag{9.23}$$

Suppose (9.22) holds and $c_{n+1}' < c_{n+1}^*$.

$$c_{n+1}' < c_{n+1}^* \to \left(\forall i \in N_V\right)\ \left[y_i\left[p_1\left(c_1^*\right),\ldots,p_n\left(c_n^*\right),p_{n+1}\left(c_{n+1}'\right)\right] = 1\right],$$

by condition $(n,1)$-NL. Therefore:

$$(9.22) \wedge c_{n+1}' < c_{n+1}^* \ \to \ c_{n+1}' + \Sigma_{i\in N_V} L_i\left(c_i^*,c_{n+1}'\right) \ < \ c_{n+1}^* + \Sigma_{i\in N_V} y_i$$
$$\left[p_1\left(c_1^*\right),\ldots,p_n\left(c_n^*\right),p_{n+1}\left(c_{n+1}^*\right)\right] L_i\left(c_i^*,c_{n+1}^*\right)$$
$$\to \ c_{n+1}' + \Sigma_{i\in N_V} L_i\left(c_i^*,c_{n+1}'\right) \ < \ c_{n+1}^* + \Sigma_{i\in N_V} L_i\left(c_i^*,c_{n+1}^*\right),\ \text{as } 0 \le y_i$$
$$\left[p_1\left(c_1^*\right),\ldots,p_n\left(c_n^*\right),p_{n+1}\left(c_{n+1}^*\right)\right] \le 1, i \in N_V.$$

Adding $\Sigma_{i \in N_V} c_i^*$ to both sides, we obtain:

$$\text{TSC}\left(c_1^*, \ldots, c_n^*, c'_{n+1}\right) < \text{TSC}\left(c_1^*, \ldots, c_n^*, c_{n+1}^*\right),$$

$$\text{a contradiction as TSC is minimum at } \left(c_1^*, \ldots, c_n^*, c_{n+1}^*\right). \tag{9.24}$$

Next suppose (9.22) holds and $c'_{n+1} > c_{n+1}^*$.
First we note that $\left[p_1\left(c_1^*\right), \ldots, p_n\left(c_n^*\right), p_{n+1}\left(c'_{n+1}\right)\right] = \left[p_1\left(c_1^*\right), \ldots, p_n\left(c_n^*\right), p_{n+1}\left(c_{n+1}^*\right)\right] = (1, \ldots, 1)$. For $i \in N_V$, designate $x_i(1, \ldots, 1)$ by x_i^*, and $y_i(1, \ldots, 1)$ by y_i^*.
(9.22) and $c'_{n+1} > c_{n+1}^* \rightarrow c'_{n+1} + \Sigma_{i \in N_V} y_i^* L_i\left(c_i^*, c'_{n+1}\right) < c_{n+1}^* + \Sigma_{i \in N_V} y_i^* L_i\left(c_i^*, c_{n+1}^*\right)$
$\rightarrow c'_{n+1} < c_{n+1}^* + \Sigma_{i \in N_V} y_i^* \left[L_i\left(c_i^*, c_{n+1}^*\right) - L_i\left(c_i^*, c'_{n+1}\right)\right]$.
Now, $(\forall i \in N_V)\left[L_i\left(c_i^*, c_{n+1}^*\right) - L_i\left(c_i^*, c'_{n+1}\right) \geq 0\right]$, as $c'_{n+1} > c_i^*$.
As $(\forall i \in N_V)\left[0 \leq y_i^* \leq 1\right]$,
$c'_{n+1} < c_{n+1}^* + \Sigma_{i \in N_V} y_i^* \left[L_i\left(c_i^*, c_{n+1}^*\right) - L_i\left(c_i^*, c'_{n+1}\right)\right] \rightarrow c'_{n+1} < c_{n+1}^* + \Sigma_{i \in N_V} \left[L_i\left(c_i^*, c_{n+1}^*\right) - L_i\left(c_i^*, c'_{n+1}\right)\right]$
$\rightarrow c'_{n+1} + \Sigma_{i \in N_V} L_i\left(c_i^*, c'_{n+1}\right) < c_{n+1}^* + \Sigma_{i \in N_V} L_i\left(c_i^*, c_{n+1}^*\right)$.
Adding $\Sigma_{i \in N_V} c_i^*$ to both sides, we obtain:

$$\text{TSC}\left(c_1^*, \ldots, c_n^*, c'_{n+1}\right) < \text{TSC}\left(c_1^*, \ldots, c_n^*, c_{n+1}^*\right), \text{ a contradiction.} \tag{9.25}$$

Next suppose (9.23) holds and $c'_k < c_k^*, k \in N_V$.
$c'_k < c_k^* \rightarrow x_k\left[p_1\left(c_1^*\right), \ldots, p_k\left(c'_k\right), \ldots, p_n\left(c_n^*\right), p_{n+1}\left(c_{n+1}^*\right)\right] = 1$, by condition $(n, 1)$-NL. Therefore:
(9.23) $\wedge\ c'_k < c_k^* \rightarrow c'_k + L_k\left(c'_k, c_{n+1}^*\right) < c_k^* + x_k^* L_k\left(c_k^*, c_{n+1}^*\right)$,
as $x_k\left[p_1\left(c_1^*\right), \ldots, p_k\left(c_k^*\right), \ldots, p_n\left(c_n^*\right), p_{n+1}\left(c_{n+1}^*\right)\right] = x_k^*$
$\rightarrow c'_k + L_k\left(c'_k, c_{n+1}^*\right) < c_k^* + L_k\left(c_k^*, c_{n+1}^*\right)$, as $0 \leq x_k^* \leq 1$.
Adding $\Sigma_{i \in N - \{k\}} c_i^* + \Sigma_{i \in N_V - \{k\}} L_i\left(c_i^*, c_{n+1}^*\right)$ to both sides, we obtain:

$$\text{TSC}\left(c_1^*, \ldots, c'_k, \ldots, c_n^*, c_{n+1}^*\right) < \text{TSC}\left(c_1^*, \ldots, c_k^*, \ldots, c_n^*, c_{n+1}^*\right), \text{ a contradiction.} \tag{9.26}$$

Finally suppose (9.23) holds and $c'_k > c_k^*, k \in N_V$.
$c'_k > c_k^* \rightarrow x_k\left[p_1\left(c_1^*\right), \ldots, p_k\left(c'_k\right), \ldots, p_n\left(c_n^*\right), p_{n+1}\left(c_{n+1}^*\right)\right] = x_k \left[p_1\left(c_1^*\right), \ldots, p_k\left(c_k^*\right), \ldots, p_n\left(c_n^*\right), p_{n+1}\left(c_{n+1}^*\right)\right] = x_k^*$.
Therefore:
(9.23) $\wedge\ c'_k > c_k^* \rightarrow \left(1 - x_k^*\right) c'_k + x_k^* \left[c'_k + L_k\left(c'_k, c_{n+1}^*\right)\right] < \left(1 - x_k^*\right) c_k^* + x_k^* \left[c_k^* + L_k\left(c_k^*, c_{n+1}^*\right)\right]$.
Adding $\Sigma_{i \in N - \{k\}} x_k^* c_i^* + \Sigma_{i \in N_V - \{k\}} x_k^* L_i\left(c_i^*, c_{n+1}^*\right)$ to both sides, we obtain:
$\left(1 - x_k^*\right) c'_k + x_k^* \text{TSC}\left(c_1^*, \ldots, c'_k, \ldots, c_n^*, c_{n+1}^*\right) < \left(1 - x_k^*\right) c_k^* + x_k^* \text{TSC}\left(c_1^*, \ldots, c_k^*, \ldots, c_n^*, c_{n+1}^*\right)$.
$\rightarrow \left(1 - x_k^*\right) c'_k < \left(1 - x_k^*\right) c_k^*$, as $\text{TSC}\left(c_1^*, \ldots, c'_k, \ldots, c_n^*, c_{n+1}^*\right) \geq \text{TSC}\left(c_1^*, \ldots, c_k^*, \ldots, c_n^*, c_{n+1}^*\right)$ and $x_k^* \geq 0$.

$$\left(1 - x_k^*\right) c'_k < \left(1 - x_k^*\right) c_k^* \rightarrow 0 < 0, \text{ if } \left(1 - x_k^*\right) = 0; \text{ a contradiction.} \tag{9.27}$$

$(1 - x_k^*)\, c_k' < (1 - x_k^*)\, c_k^* \to c_k' < c_k^*$, if $(1 - x_k^*) > 0$, contradicting the hypothesis that $c_k' > c_k^*$. (9.28)

(9.27) and (9.28) establish that (9.23) cannot hold with $c_k' > c_k^*, k \in N_V$. (9.29)

(9.24)–(9.26), and (9.29) establish that $\left(c_1^*, \ldots, c_n^*, c_{n+1}^*\right)$ is a Nash equilibrium. (9.30)

Let $(\overline{c}_1, \ldots, \overline{c}_n, \overline{c}_{n+1})$ be a Nash equilibrium. $(\overline{c}_1, \ldots, \overline{c}_n, \overline{c}_{n+1})$ being a Nash equilibrium implies:

$$(\forall c_{n+1} \in C_{n+1})[\overline{c}_{n+1} + \Sigma_{i \in N_V} y_i[p_1(\overline{c}_1), \ldots, p_n(\overline{c}_n), p_{n+1}(\overline{c}_{n+1})] L_i(\overline{c}_i, \overline{c}_{n+1}) \leq$$
$$c_{n+1} + \Sigma_{i \in N_V} y_i[p_1(\overline{c}_1), \ldots, p_n(\overline{c}_n), p_{n+1}(c_{n+1})] L_i(\overline{c}_i, c_{n+1})];\ \text{and} \tag{9.31}$$

$$(\forall i \in N_V)(\forall c_i \in C_i)[\overline{c}_i + x_i[p_1(\overline{c}_1), \ldots, p_i(\overline{c}_i), \ldots, p_n(\overline{c}_n), p_{n+1}(\overline{c}_{n+1})]$$
$$L_i(\overline{c}_i, \overline{c}_{n+1}) \leq c_i + x_i[p_1(\overline{c}_1), \ldots, p_i(c_i), \ldots, p_n(\overline{c}_n), p_{n+1}(\overline{c}_{n+1})] L_i(c_i, \overline{c}_{n+1})]. \tag{9.32}$$

$$(9.31) \ \to \ \left[\overline{c}_{n+1} + \Sigma_{i \in N_V} y_i[p_1(\overline{c}_1), \ldots, p_n(\overline{c}_n), p_{n+1}(\overline{c}_{n+1})] L_i(\overline{c}_i, \overline{c}_{n+1}) \leq\right.$$
$$\left. c_{n+1}^* + \Sigma_{i \in N_V} y_i[p_1(\overline{c}_1), \ldots, p_n(\overline{c}_n), p_{n+1}\left(c_{n+1}^*\right)] L_i\left(\overline{c}_i, c_{n+1}^*\right)\right]. \tag{9.33}$$

$$(9.32) \ \to \ (\forall i \in N_V)\left[\overline{c}_i + x_i[p_1(\overline{c}_1), \ldots, p_i(\overline{c}_i), \ldots, p_n(\overline{c}_n), p_{n+1}(\overline{c}_{n+1})]\right.$$
$$\left. L_i(\overline{c}_i, \overline{c}_{n+1}) \leq c_i^* + x_i[p_1(\overline{c}_1), \ldots, p_i\left(c_i^*\right), \ldots, p_n(\overline{c}_n), p_{n+1}(\overline{c}_{n+1})] L_i\left(c_i^*, \overline{c}_{n+1}\right)\right]. \tag{9.34}$$

$$(9.33) \text{ and } (9.34) \ \to \ \Sigma_{i \in N} \overline{c}_i + \Sigma_{i \in N_V} L_i(\overline{c}_i, \overline{c}_{n+1}) \leq \Sigma_{i \in N} c_i^* + \Sigma_{i \in N_V} x_i$$
$$[(\forall j \in N - \{i\})(p_j = p_j(\overline{c}_j)) \ \wedge \ p_i = p_i\left(c_i^*\right)] L_i\left[c_i^*, \overline{c}_{n+1}\right] + \Sigma_{i \in N_V} y_i$$
$$[(\forall j \in N_V)(p_j = p_j(\overline{c}_j)) \ \wedge \ p_{n+1} = p_{n+1}\left(c_{n+1}^*\right)] L_i\left[\overline{c}_i, c_{n+1}^*\right]. \tag{9.35}$$

For $i \in N_V$ we have:
$\overline{c}_{n+1} < c_{n+1}^* \wedge \overline{c}_i < c_i^* \to x_i\left[(\forall j \in N - \{i\})(p_j = p_j(\overline{c}_j)) \ \wedge \ p_i = p_i\left(c_i^*\right)\right]$
$= 0 \ \wedge y_i\left[(\forall j \in N_V)(p_j = p_j(\overline{c}_j)) \ \wedge \ p_{n+1} = p_{n+1}\left(c_{n+1}^*\right)\right] = 0$, by condition $(n, 1)$-NL.
Therefore, $x_i\left[(\forall j \in N - \{i\})(p_j = p_j(\overline{c}_j)) \ \wedge \ p_i = p_i\left(c_i^*\right)\right] L_i\left[c_i^*, \overline{c}_{n+1}\right] + y_i$
$\left[(\forall j \in N_V)(p_j = p_j(\overline{c}_j)) \ \wedge \ p_{n+1} = p_{n+1}\left(c_{n+1}^*\right)\right] L_i\left[\overline{c}_i, c_{n+1}^*\right] = 0$ (9.36)

$\overline{c}_{n+1} < c^*_{n+1} \wedge \overline{c}_i \geq c^*_i \rightarrow x_i\left[(\forall j \in N - \{i\})(p_j = p_j(\overline{c}_j)) \;\wedge\; p_i = p_i\left(c^*_i\right)\right] = 0$
by condition $(n, 1)$-NL.
Therefore, $x_i\left[(\forall j \in N - \{i\})(p_j = p_j(\overline{c}_j)) \;\wedge\; p_i = p_i\left(c^*_i\right)\right] L_i\left[c^*_i, \overline{c}_{n+1}\right]$
$+y_i\left[(\forall j \in N_V)(p_j = p_j(\overline{c}_j)) \;\wedge\; p_{n+1} = p_{n+1}\left(c^*_{n+1}\right)\right] L_i\left[\overline{c}_i, c^*_{n+1}\right]$
$= y_i\left[(\forall j \in N_V)(p_j = p_j(\overline{c}_j)) \;\wedge\; p_{n+1} = p_{n+1}\left(c^*_{n+1}\right)\right] L_i\left[\overline{c}_i, c^*_{n+1}\right]$
$\leq L_i\left[\overline{c}_i, c^*_{n+1}\right]$
$\leq L_i\left[c^*_i, c^*_{n+1}\right]$, by assumptions ($\beta 2$) and ($\beta 3$). (9.37)

$\overline{c}_{n+1} \geq c^*_{n+1} \wedge \overline{c}_i < c^*_i \rightarrow y_i\left[(\forall j \in N_V)(p_j = p_j(\overline{c}_j)) \;\wedge\; p_{n+1} = p_{n+1}\left(c^*_{n+1}\right)\right]$
$= 0,$ by condition $(n, 1)$-NL.
Therefore, $x_i\left[(\forall j \in N - \{i\})(p_j = p_j(\overline{c}_j)) \;\wedge\; p_i = p_i\left(c^*_i\right)\right] L_i\left[c^*_i, \overline{c}_{n+1}\right]$
$+ y_i\left[(\forall j \in N_V)(p_j = p_j(\overline{c}_j)) \;\wedge\; p_{n+1} = p_{n+1}\left(c^*_{n+1}\right)\right] L_i\left[\overline{c}_i, c^*_{n+1}\right]$
$= x_i\left[(\forall j \in N - \{i\})(p_j = p_j(\overline{c}_j)) \;\wedge p_i = p_i\left(c^*_i\right)\right] L_i\left[c^*_i, \overline{c}_{n+1}\right]$
$\leq L_i\left[c^*_i, \overline{c}_{n+1}\right]$
$\leq L_i\left[c^*_i, c^*_{n+1}\right]$, by assumptions ($\beta 2$) and ($\beta 3$). (9.38)

$\overline{c}_{n+1} \geq c^*_{n+1} \wedge \overline{c}_i \geq c^*_i \rightarrow \left[(\forall j \in N - \{i\})(p_j = p_j(\overline{c}_j)) \;\wedge\; p_i = p_i\left(c^*_i\right)\right]$
$= [p_1(\overline{c}_1), \ldots, p_i = 1, \ldots, p_n(\overline{c}_n), p_{n+1} = 1]$
$= \left[(\forall j \in N_V)(p_j = p_j(\overline{c}_j)) \;\wedge\; p_{n+1} = p_{n+1}\left(c^*_{n+1}\right)\right]$
Therefore, $x_i\left[(\forall j \in N - \{i\})(p_j = p_j(\overline{c}_j)) \;\wedge\; p_i = p_i\left(c^*_i\right)\right] L_i\left[c^*_i, \overline{c}_{n+1}\right]$
$+ y_i\left[(\forall j \in N_V)(p_j = p_j(\overline{c}_j)) \;\wedge\; p_{n+1} = p_{n+1}\left(c^*_{n+1}\right)\right] L_i\left[\overline{c}_i, c^*_{n+1}\right]$
$\leq x_i\left[(\forall j \in N - \{i\})(p_j = p_j(\overline{c}_j)) \;\wedge p_i = p_i\left(c^*_i\right)\right] L_i\left[c^*_i, c^*_{n+1}\right]$
$+ y_i\left[(\forall j \in N_V)(p_j = p_j(\overline{c}_j)) \;\wedge\; p_{n+1} = p_{n+1}\left(c^*_{n+1}\right)\right] L_i\left[c^*_i, c^*_{n+1}\right]$
$= L_i\left[c^*_i, c^*_{n+1}\right].$ (9.39)

In view of (9.36)–(9.39), (9.35) implies:
$\Sigma_{i \in N}\overline{c}_i + \Sigma_{i \in N_V} L_i(\overline{c}_i, \overline{c}_{n+1}) \leq \Sigma_{i \in N} c^*_i + \Sigma_{i \in N_V} L_i\left[c^*_i, c^*_{n+1}\right]$
$\rightarrow \mathrm{TSC}(\overline{c}_1, \ldots, \overline{c}_n, \overline{c}_{n+1}) \leq \mathrm{TSC}\left(c^*_1, \ldots, c^*_n, c^*_{n+1}\right).$
As TSC is minimum at $\left(c^*_1, \ldots, c^*_n, c^*_{n+1}\right)$, we in fact must have:
$\mathrm{TSC}(\overline{c}_1, \ldots, \overline{c}_n, \overline{c}_{n+1}) = \mathrm{TSC}\left(c^*_1, \ldots, c^*_n, c^*_{n+1}\right),$
which implies that $(\overline{c}_1, \ldots, \overline{c}_n, \overline{c}_{n+1}) \in M$.

This establishes that $(\forall(\overline{c}_1, \ldots, \overline{c}_n, \overline{c}_{n+1}) \in C_1 \times \ldots \times C_n \times C_{n+1})$
$[(\overline{c}_1, \ldots, \overline{c}_n, \overline{c}_{n+1})$ is a Nash equilibrium $\rightarrow (\overline{c}_1, \ldots, \overline{c}_n, \overline{c}_{n+1}) \in M]$. (9.40)

(9.30) and (9.40) establish the theorem. □

Theorem 9.4. *Let liability rule f defined for $n \geq 2$ victims and one injurer belong to $\mathscr{F}'$. Then f is efficient for every application belonging to β' iff it satisfies the condition of $(n, 1)$-negligence liability.*

Proof. Let f belong to $\mathscr{F}'$. Suppose f violates $(n, 1)$-NL. Then we must have: $(\exists k \in N_V)\left(\exists(p_1, \ldots, p_n, p_{n+1}) \in [0,1]^{n+1}\right)[[p_{n+1} < 1 \wedge p_k = 1 \wedge x_k(p_1, \ldots, p_n, p_{n+1}) > 0] \vee [p_{n+1} = 1 \wedge p_k < 1 \wedge y_k(p_1, \ldots, p_n, p_{n+1}) > 0]]$.
First suppose $(\exists k \in N_V)\left(\exists(p_1, \ldots, p_n, p_{n+1}) \in [0,1]^{n+1}\right)[p_{n+1} < 1 \wedge p_k = 1 \wedge x_k(p_1, \ldots, p_n, p_{n+1}) > 0]$.
Let this $p_{n+1} < 1$ be designated by $\overline{p}_{n+1}$. As f belongs to $\mathscr{F}'$, it follows that we must have $x_k[(\forall i \in N_V)(p_i = 1), p_{n+1} = \overline{p}_{n+1}] = \overline{x}_k > 0$.

Now consider an application belonging to β'as specified below:
Let $t > 0$.
Choose r_1, r_2 such that $(1 - \overline{x}_k)t = \overline{y}_k t < r_1 < r_2 < t$.
Let $\overline{c}_{n+1} = \frac{r_2}{1-\overline{p}_{n+1}}$.
Let $(\forall j \in N_V)(\overline{c}_j > 0 \wedge \delta_j > 0)$.
Choose ϵ such that $0 < \epsilon < \frac{r_2 - r_1}{n-1}$.
Let $C_{n+1} = \{0, \overline{p}_{n+1}\overline{c}_{n+1}, \overline{c}_{n+1}\}$; $C_k = \{0, \overline{c}_k\}$; $(\forall i \in N_V - \{k\})[C_i = \{0, \overline{c}_i\}]$.
Let $L_k(c_k, c_{n+1})$ and $L_i(c_i, c_{n+1}), i \in N_V - \{k\}$, be as specified in the following arrays, respectively.[10]

$L_k(c_k, c_{n+1})$

	$c_{n+1} = 0$	$c_{n+1} = \overline{p}_{n+1}\overline{c}_{n+1}$	$c_{n+1} = \overline{c}_{n+1}$
$c_k = 0$	$\overline{c}_k + \delta_k + \dfrac{\overline{p}_{n+1}\overline{c}_{n+1}}{n} + t$	$\overline{c}_k + \delta_k + t$	$\overline{c}_k + \delta_k$
$c_k = \overline{c}_k$	$\dfrac{\overline{p}_{n+1}\overline{c}_{n+1}}{n} + t$	t	0

$L_i(c_i, c_{n+1})$

	$c_{n+1} = 0$	$c_{n+1} = \overline{p}_{n+1}\overline{c}_{n+1}$	$c_{n+1} = \overline{c}_{n+1}$
$c_i = 0$	$\overline{c}_i + \delta_i + \dfrac{\overline{p}_{n+1}\overline{c}_{n+1}}{n} + \epsilon$	$\overline{c}_i + \delta_i + \epsilon$	$\overline{c}_i + \delta_i$
$c_i = \overline{c}_i$	$\dfrac{\overline{p}_{n+1}\overline{c}_{n+1}}{n} + \epsilon$	ϵ	0

$(\forall j \in N_V)(\delta_j > 0)$ and $t > r_2$ imply that $(\forall i \in N)(c_i = \overline{c}_i)$ is the unique TSC-minimizing configuration of care levels.
Let $\left(c_1^*, \ldots, c_n^*, c_{n+1}^*\right) = (\forall i \in N)(c_i = \overline{c}_i)$.

Now, given that every $i \in N, i \neq n+1$, is using $c_i = c_i^*$;
If the injurer uses $c_{n+1} = \overline{c}_{n+1}$, then his expected costs $= EC_{n+1}(\overline{c}_1, \ldots, \overline{c}_n, \overline{c}_{n+1}) = \overline{c}_{n+1}$, as $(\forall i \in N_V)(L_i(\overline{c}_i, \overline{c}_{n+1}) = 0)$.

[10] Specifications of $L_k(c_k, c_{n+1})$ and $L_i(c_i, c_{n+1}), i \in N_V - \{k\}$, are done in such a way that no inconsistency would arise even if $\overline{p}_{n+1} = 0$.

If the injurer uses $c_{n+1} = \overline{p}_{n+1}\overline{c}_{n+1}$, then his expected costs $= EC_{n+1}(\overline{c}_1, \ldots, \overline{c}_n, \overline{p}_{n+1}\overline{c}_{n+1}) = \overline{p}_{n+1}\overline{c}_{n+1} + \overline{y}_k t + \Sigma_{i \in N_V - \{k\}} y_i(1, \ldots, 1, \overline{p}_{n+1})\epsilon$.
$EC_{n+1}(\overline{c}_1, \ldots, \overline{c}_n, \overline{c}_{n+1}) - EC_{n+1}(\overline{c}_1, \ldots, \overline{c}_n, \overline{p}_{n+1}\overline{c}_{n+1}) = \overline{c}_{n+1} - [\overline{p}_{n+1}\overline{c}_{n+1} + \overline{y}_k t + \Sigma_{i \in N_V - \{k\}}\, y_i(1, \ldots, 1, \overline{p}_{n+1})\epsilon]$
$\geq (1 - \overline{p}_{n+1})\overline{c}_{n+1} - \overline{y}_k t - (n-1)\epsilon$
$> (1 - \overline{p}_{n+1})\overline{c}_{n+1} - \overline{y}_k t - (r_2 - r_1)$
$= r_1 - \overline{y}_k t$
> 0.

Thus, given that every $i \in N, i \neq n+1$, is using $c_i = c_i^*$, for the injurer $c_{n+1} = \overline{p}_{n+1}\overline{c}_{n+1}$ is better than $c_{n+1} = \overline{c}_{n+1}$. Thus the only TSC-minimizing configuration is not a Nash equilibrium. This establishes that f is not efficient with respect to β'.

(9.41)

Next suppose that $(\exists k \in N_V)\left(\exists(p_1, \ldots, p_n, p_{n+1}) \in [0,1]^{n+1}\right)[p_{n+1} = 1 \ \wedge\ p_k < 1 \ \wedge\ y_k(p_1, \ldots, p_n, p_{n+1}) > 0]$.
Let this $p_k < 1$ be designated by $\overline{p}_k$. As f belongs to $\mathscr{F}'$, it follows that we must have $y_k[(\forall i \in N - \{k\})(p_i = 1) \wedge p_k = \overline{p}_k] = \overline{y}_k > 0$.

Now consider an application belonging to β'as specified below:
Let $t > 0$.
Choose r such that $(1 - \overline{y}_k)t = \overline{x}_k t < r < t$.
Let $\overline{c}_k = \frac{r}{1-\overline{p}_k}$.
Let $(\forall i \in N_V - \{k\})(\overline{c}_i > 0 \wedge \delta_i > 0)$.
Choose $\overline{c}_{n+1}$ and ϵ such that $0 < \frac{\overline{c}_{n+1}}{n} < \epsilon$.
Let $C_{n+1} = \{0, \overline{c}_{n+1}\}$; $(\forall i \in N_V - \{k\})(C_i = \{0, \overline{c}_i\})$; $C_k = \{0, \overline{p}_k\overline{c}_k, \overline{c}_k\}$.
Let $L_k(c_k, c_{n+1})$ and $L_i(c_i, c_{n+1}), i \in N_V - \{k\}$, be as specified in the following arrays respectively.[11]

$L_k(c_k, c_{n+1})$

	$c_{n+1} = 0$	$c_{n+1} = \overline{c}_{n+1}$
$c_k = 0$	$\overline{p}_k\overline{c}_k + t + \epsilon$	$\overline{p}_k\overline{c}_k + t$
$c_k = \overline{p}_k\overline{c}_k$	$t + \epsilon$	t
$c_k = \overline{c}_k$	ϵ	0

$L_i(c_i, c_{n+1})$

	$c_{n+1} = 0$	$c_{n+1} = \overline{c}_{n+1}$
$c_i = 0$	$\overline{c}_i + \delta_i + \epsilon$	$\overline{c}_i + \delta_i$
$c_i = \overline{c}_i$	ϵ	0

[11]Specification of $L_k(c_k, c_{n+1})$ is done in such a way that no inconsistency would arise even if $\overline{p}_k = 0$.

$\overline{c}_{n+1} < n\epsilon$, $(\forall i \in N_V - \{k\})(\delta_i > 0)$ and $t > r$ imply that $(\forall i \in N)(c_i = \overline{c}_i)$ is the unique TSC-minimizing configuration of care levels.
Let $(c_1^*, \ldots, c_n^*, c_{n+1}^*) = (\forall i \in N)(c_i = \overline{c}_i)$.
Now, given that every $i \in N, i \neq k$, is using $c_i = c_i^*$;
If victim k uses $c_k = \overline{c}_k$, then his expected costs = $EC_k(\overline{c}_1, \ldots, \overline{c}_n, \overline{c}_{n+1}) = \overline{c}_k$, as $L_k(\overline{c}_k, \overline{c}_{n+1}) = 0$
If victim k uses $c_k = \overline{p}_k \overline{c}_k$, then his expected costs = $EC_k(\overline{c}_1, \ldots, \overline{p}_k \overline{c}_k, \ldots, \overline{c}_n, \overline{c}_{n+1}) = \overline{p}_k \overline{c}_k + \overline{x}_k t$.
$EC_k(\overline{c}_1, \ldots, \overline{c}_k, \ldots, \overline{c}_n, \overline{c}_{n+1}) - EC_k(\overline{c}_1, \ldots, \overline{p}_k \overline{c}_k, \ldots, \overline{c}_n, \overline{c}_{n+1}) = \overline{c}_k - \overline{p}_k \overline{c}_k - \overline{x}_k t$
$= (1 - \overline{p}_k)\overline{c}_k - \overline{x}_k t$
$= r - \overline{x}_k t$
$> 0.$

Thus, given that every $i \in N, i \neq k,$ is using $c_i = c_i^*,$ for victim k $c_k = \overline{p}_k \overline{c}_k$ is better than $c_k = \overline{c}_k$. Thus the only TSC-minimizing configuration of care levels is not a Nash equilibrium. This establishes that f is not efficient with respect to β'.

(9.42)

(9.41) and (9.42) establish the necessity of $(n, 1)$-negligence liability for a rule belonging to $\mathscr{F}'$ to be efficient with respect to β'. The sufficiency of $(n, 1)$-negligence liability for a rule belonging to $\mathscr{F}'$ to be efficient with respect to β' follows from Theorem 9.3. □

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Chapter 10
Epilogue

This text has been almost exclusively concerned with the efficiency analysis of liability rules. In this concluding chapter, apart from summarizing the main results which have been derived and analyzed, we discuss the significance thereof. We also discuss some issues concerning positive and normative analysis of law from the efficiency perspective.

10.1 Summary

An analysis of the totality of liability rules, within a framework essentially that of the standard tort model, shows that a necessary and sufficient condition for a liability rule to be efficient is that it satisfy the condition of negligence liability. Negligence liability condition requires that if one party is negligent and the other nonnegligent, then the entire loss in case of accident must be borne by the negligent party. This result can be generalized by considering decomposed liability rules. A liability rule apportions the entire loss on the basis of the nonnegligence proportions of the parties; but a decomposed liability rule apportions one part of the loss on the basis of the nonnegligence proportions of the parties and the other part in fixed proportions. Corresponding to each liability rule, there exists a class of decomposed liability rules. If the quantum of loss that is assigned independently of the nonnegligence proportions does not exceed the adjusted optimal loss, then the necessity and sufficiency of negligence liability condition for efficiency continues to hold. For the class of incremental liability rules, efficiency obtains iff (i) if the party, which is the non-residual holder of liability when both parties are nonnegligent, is nonnegligent and the other party negligent, then it continues to remain the non-residual liability holder; and (ii) if the party, which is the non-residual holder of liability when both parties are nonnegligent, is negligent and the other party nonnegligent, then it either

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becomes the residual liability holder or its liability is equal to the entire incremental loss which can be attributed to its negligence.

The results pertaining to the efficiency of rules apportioning liability are derived under the assumption that the activity levels of the parties are fixed. If both the care levels and activity levels can be varied, then there is no liability rule which yields efficient outcomes for all applications. Within the framework of the standard tort model the notion of negligence is defined as failure to take at least the due care; and it is assumed that the due care levels are set appropriately from the perspective of minimization of social costs. If negligence is defined in terms of existence of cost-justified untaken precautions, then it turns out that there is no liability rule which is efficient for all applications. Neither is there an incremental liability rule with the property of yielding efficient outcomes for all applications when negligence is defined in terms of existence of cost-justified untaken precautions. One important feature of tort law is that the payment to the victim equals the liability of the injurer. It turns out that if such is not the case, i.e. if the liability is decoupled, then there is no rule which is efficient for all applications.

From the propositions establishing the efficiency or otherwise of the rules, one can isolate five different reasons for inefficiency: (i) Inefficiency can occur because of failure of one or both parties to internalize the entire harm resulting from the interaction. If a liability rule does not satisfy the condition of negligence liability, then the proportion of the loss which is borne by one of the two parties when it is negligent, the other party being nonnegligent, is less than 1, and consequently the party fails to internalize the entire harm. It is because of this failure on the part of at least one party to internalize the entire loss that a rule violating the negligence liability condition fails to yield efficient outcomes in all cases. (ii) When negligence is defined as failure to take at least the due care, whether one is negligent or not depends entirely on one's own actions. On the other hand, if negligence is construed as failure to take some cost-justified precaution, whether one is negligent or not depends partly on one's own actions and partly on the other party's actions. This introduces in the system possibilities of strategic manipulation. It is because of strategic manipulability that no liability rule or incremental liability rule can invariably give rise to efficient outcomes when negligence is defined as existence of a cost-justified untaken precaution. (iii) While the care levels taken by the parties are generally observable, their activity levels are not. Furthermore, while optimal care levels, being based on objective information, can in principle be determined by the courts, the same is not possible for the activity levels, as the information required for calculating the optimal activity levels is subjective and therefore private. Consequently, if both care and activity levels can vary then there is no liability rule which can ensure efficiency in all cases. If both optimal care levels and optimal activity levels of the parties were determinable by the courts, the negligence could have been defined as failure to take at least the due care or failure to take at most the optimal activity level. Then any liability rule satisfying the negligence liability condition would have yielded efficient outcomes in all cases. It is because of the impossibility of objective determination of optimal activity levels that there does not exist any rule which is efficient with both care and activity levels being variable. (iv) When liability is coupled, as is the case with liability rules as well as with incremental liability rules, there is closure with respect to the parties involved in the negative externality generating interaction. When liability is decoupled, then the closure with respect to the parties involved in the negative externality generating interaction does not take place. As a consequence, some harm is externalized away from the parties or alternatively liability borne by the parties together gets multiplied. The incentives to take care therefore get distorted. There can be either too little or too much care. It is because of parties together facing too little or too much liability that with decoupled liability no rule can be invariably efficient. (v) The essential idea behind negligence is that those who are more dexterous should take greater amount of care compared to the less dexterous ones. Thus being more dexterous imposes greater costs. This aspect of negligence creates perverse incentives with respect to cost-justified dexterity-enhancing investments and for wasteful expenditures for misrepresenting one's abilities. If the due care is fixed, a cost-justified dexterity-enhancing investment would be undertaken as both the gains and costs accrue to the individual concerned. But, if the due care as a consequence of enhanced dexterity is going to increase, then some or all or even more than all of the gains would get frittered away although all the costs would still be borne by the individual. Thus, determining the due care with a view to minimize the social costs of accidents also creates perverse incentives, and many cost-justified dexterity-enhancing investments would not be undertaken. As misrepresentation of one's capabilities can benefit individuals, socially wasteful, but privately beneficial, expenditures for the purpose of misrepresenting one's capabilities would also be undertaken.

10.2 The Efficiency Thesis

Economic analysis of common law has revealed a connection between it and efficiency. Within the framework of the standard tort model, most of the liability rules used in practice turn out to be efficient. Economic analysis of contract law has shown that the most commonly used damage measure in cases of breach of contract, namely, expectation damages, is an efficient damage measure. In view of these and similar results pertaining to efficiency of laws, procedures, and doctrines of common law, a view has emerged that efficiency provides a unified explanation for the whole of common law. According to this view, common law can best be understood by hypothesizing that judges decide cases so as to promote efficiency.[1] A more moderate view has been articulated by Calabresi (1980, pp. 561–562):

> I find it hard to explain judicial behaviour in America simply in terms of wealth maximization. But I find it equally hard to explain it only in terms of ultimates or principles. In other words, my guess would be that courts do decide policies as well as principles, and perhaps

[1] See, for instance, Posner (1987), and Landes and Posner (1987).

more often the first than the second. The policies are based, and again I am guessing, on that mixture of efficiency and distribution that in the particular context is thought by the court to be instrumental toward justice and, in particular, does not violate any fairly precisely defined rights or veto points. Whether that is an appropriate task for that institution is another matter and one that I wish to pass for now. My only thought on that at this point is that such a discussion works better in context than in the abstract. I find it difficult, in other words, to consider that issue apart from the capabilities of other institutions. For that reason, discussions of the role of courts that do not distinguish England from America (let alone both of these countries from Italy and France) seem to me prima facie suspect.

In connection with the question as to what extent efficiency provides a positive theory of common law, two important points need to be considered. One point relates to purely theoretical considerations regarding the kind of connection that is required for one to assert that efficiency constitutes an explanation for common law. The second point relates to whether the connection between efficiency and common law that has been established is of the kind required by the theoretical considerations.

Suppose one is able to establish that: (i) all rules of a legal system satisfy a particular property; and (ii) these rules are the only ones which satisfy the property in question. In such a case, it is immediate that the property in question constitutes a complete explanation of the rules of the legal system, as a rule is part of the legal system iff it satisfies the property in question. On the other hand, if all rules of the legal system satisfy the property, but there are other rules, not part of the legal system, which also satisfy the property, then it would not be accurate to say that the property constitutes a complete explanation of the rules of the legal system. In such a case, the property in question constitutes only a partial explanation of the rules of the legal system. For a full explanation, one would need to know the basis on which the selection of rules has been made from the set of rules satisfying the property. Consider, for instance, a hypothetical situation such that all rules of a particular kind which are used in practice are efficient, but the rules used in practice constitute only a proper subset of all efficient rules of the kind under consideration. Suppose also that all rules that are used in practice satisfy a particular condition other than efficiency, and every efficient rule which is not used in practice violates this condition, then it would be correct to say that efficiency and this other condition together constitute a complete explanation for the rules which are used in practice of the kind under consideration. Within the framework of the standard tort model, there are an infinite number of efficient liability rules, an infinite number of efficient decomposed liability rules, and an infinite number of efficient incremental liability rules. Thus assuming that the liability rules used in practice are by and large efficient, one can assert that efficiency provides a partial explanation for liability rules used in practice. The only way to establish efficiency as a complete explanation for the liability rules used in practice would be to formulate a model within the framework of which the liability rules used in practice are the only liability rules which are efficient; and also show that the assumptions of the model have a close correspondence with reality.

When both care and activity levels can be varied, there is no liability rule which is invariably efficient; so is the case when negligence is defined as existence of some cost-justified untaken precaution. These impossibility theorems do not necessarily pose difficulties for the efficiency thesis of common law. This is because the possibility that the rules which are usually employed may be among those least inefficient is not ruled out. Only an analysis of the degrees of inefficiency of rules can establish whether these impossibility theorems have any negative implications for the efficiency thesis.

The analysis of the no distinction between the due care and more than the due care feature carried out in the text showed that this feature cannot be explained solely in terms of efficiency. There are efficient as well as inefficient liability rules satisfying the no distinction between the due care and more than the due care feature; and also efficient as well as inefficient liability rules violating the no distinction between the due care and more than the due care feature. Thus the no distinction between the due care and more than the due care feature is logically completely independent of efficiency; and consequently it is not possible to provide an explanation of the feature solely in terms of efficiency. From this it follows that there are at least some features of tort law which cannot possibly have an explanation solely in terms of efficiency.

Thus it appears that a more moderate claim, possibly along the lines suggested by Calabresi, may be more appropriate. While efficiency may figure in a prominent way in a positive theory of law, it is unlikely that a fully satisfactory theory is possible without bringing other values or criteria.

10.3 Efficiency, Justice, and Other Values

Wealth maximization is clearly a desirable objective. Flowing from it, the desirability of having laws which are efficient is taken for granted. Coase, for instance, in the concluding passage of his classic paper ‘The Problem of Social Cost’ says:

> It would clearly be desirable if the only actions performed were those in which what was gained was worth more than what was lost. But in choosing between social arrangements within the context of which individual decisions are made, we have to bear in mind that a change in the existing system which will lead to an improvement in some decisions may well lead to a worsening of others. Furthermore we have to take into account the costs involved in operating the various social arrangements (whether it be the working of a market or of a government department), as well as the costs involved in moving to a new system. In devising and choosing between social arrangements we should have regard for the total effect. This, above all, is the change in approach which I am advocating. (p. 44)

If laws are efficient or designed to be efficient, then the question arises as to what the implications of it would be for other values in general and for justice in particular. One viewpoint that has emerged in the aftermath of economic analysis of law seems to equate justice with efficiency. Posner (1981) puts the point as follows:

> A second meaning of ‘justice’, and the most common I would argue, is simply ‘efficiency’. When we describe as ‘unjust’ convicting a person without a trial, taking property without just compensation, or failing to require a negligent automobile driver to answer in damages to the victim of his carelessness, we can be interpreted as meaning simply that the conduct or practice in question wastes resources. It is no surprise that in a world of scarce resources, waste is regarded as immoral.

If justice is given a meaning which makes it a value invariably implying efficiency, then the question of there being contexts in which the two values would be in conflict with each other does not arise. On the other hand, if one imputes to justice a meaning close to what is the commonly held view of it, then there can be contexts in which the values of efficiency and justice would be in conflict with each other. When there is a conflict of values, how the conflict would be resolved depends on the nature of institutions. If one invariably goes in for wealth maximization in the design of institutions, then the conflicts would be resolved in favour of wealth.

If one considers an activity with negative externalities but in the aggregate socially beneficial, then under the wealth maximization criterion undertaking of the activity would be socially better than not undertaking it. If individuals on whom the costs of negative externalities fall are not compensated for harm, then depending on the context, it might be a case of injustice as these individuals are made to bear the costs of harm without any fault of theirs. One may, however, condone it on the ground that the situation under discussion involves a trade-off between fairness considerations on the one hand and wealth considerations on the other and that in social contexts such trade-offs are inevitable. In the context of trade-offs between values, it is important to remember that by their very nature basic rights do not permit any trade-offs. In Ronald Dworkin's terminology[2] they are trumps. Willingness to make a trade-off between wealth and fairness in some situations does not imply willingness to make a trade-off between wealth and basic rights. One may countenance the losses of those suffering negative externalities for the sake of much bigger gains of those undertaking the activity if the losses are such that they do not have any serious consequences for those suffering harm. However, if the losses are such that they impinge on the basic rights of the victims then obviously the situation is radically different and not at all appropriate for making any trade-offs. One cannot make a recommendation for the activity to be undertaken in such a scenario without abandoning commitment to basic rights. Wealth maximization is an aggregative criterion and basic rights are quintessentially non-aggregative in character; consequently they are bound to conflict in some situations. Whether in a particular social context efficiency in the sense of wealth maximization would conflict with basic rights or not depends on the scope of the efficiency criterion.

In the context of societal institutions in general, and legal institutions in particular, there are bound to be a multiplicity of independent values. Although economic method has been used to analyze institutions only from the efficiency perspective, given its generality there is no reason why it cannot be applied to analyze institutions from the perspective of other important values. After all, the economic method consists of analyzing the implications of actions undertaken by purposive individuals within the constraint of given rules of the game defined by the institutional structure. Once one has determined what would happen as a consequence of totality of actions undertaken by the individuals, in principle it could be analyzed from the perspective of any desired value or criterion. This kind of

[2] Dworkin (1977).

analyses can bring about clarity regarding when certain values considered important are likely to be nonconflictive with each other and when likely to be in conflict.

One important limitation of economic analysis of law needs to be noted. Economic analysis of law is essentially consequentialist in nature. Value analysis of laws can be done from two perspectives. One can, as is done in economic analysis, analyze the characteristics of the outcomes which would result when rational individuals act within the framework of the law under study. One can associate the values embodied in the outcomes with the law in question. Alternatively, one can do a textual analysis of the law to determine which values are being proclaimed through it. It is possible that two different laws have identical consequences but textually embody different values. A rule may be unacceptable because of what it expresses and not because of what it results in. For instance, if a law specifies one kind of penalty for one set of people and another kind of penalty for another set of people for the same infringement, most people would have no hesitation in terming the law as unjust. The existence of a law which specifies unequal treatment for different sets of people does not, however, necessarily imply the existence of legal cases involving unequal treatment. This can happen for any number of reasons. Suppose, for instance, the penalty for one set of people is so harsh compared to the likely benefits of the activity proscribed by the legal rule in question that it proves to be an absolute deterrent, and consequently the legal rule is violated by no person belonging to the category for which the penalty is harsh. Thus all violators would be from the category of those for whom the penalty is light. Thus as far as the actual outcome is concerned, all violators of the rule would have been treated equally. In this example it is evident that the absence of cases with unequal treatment would not make the rule in question any less unjust. For the purpose of determining which values are expressed by legal rules, the textual analysis of a body of laws would have to look at not only individual rules, subsets of rules, and the entire set of rules but also their interrelationships. For instance, each of the two legal rules dealing with transgressions of similar gravity taken singly may be fine but together may be objectionable on the ground of disproportionate punishments for crimes of more or less similar magnitude. In view of these and similar considerations, it seems fairly clear that for a proper understanding of the relationship between laws and values, consequentialist as well as non-consequentialist (procedural, textual, etc.) analyses are required.

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