Incentive compatibility

Asked 3 years, 7 months ago Active 3 years, 3 months ago Viewed 119 times

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I have a set of objects $A = \{a_1, a_2, a_3, \cdots, a_m\}$ $A = \{a_1, a_2, a_3, \cdots, a_m\}$ and one player, who proposes a vector of valuations $t = (t_1\ t_2\ t_3\ \cdots\ t_m)$ $t = (t_1\ t_2\ t_3\ \cdots\ t_m)$ for the m m objects. T T is the set of all such valuation types. Given T T I design an allocation function

$$f : T \to [0, 1]^m$$

$$f\colon T \to [0,\ 1]^m$$

and a payment function

$$p : T \to \mathbb{R}$$

$$p\colon T \to \mathrm{R}$$

The mechanism (f, p) (f, p) is said to be *Incentive compatible* if

$$t(f(t)) - p(t) \geq t(f(t')) - p(t') \quad \forall t, t' \in T \qquad (1)$$

$$t(f(t)) - p(t) \geq t(f(t^{'})) - p(t^{'}) \quad \forall t, t^{'} \in T \qquad (1)$$

One can easily derive from the above eqaution that incentive compatibility implies the following:

$$t(f(t)) + t'(f(t')) \geq t(f(t')) + t'(f(t)) \qquad (2)$$

$$t(f(t)) + t^{'}(f(t^{'})) \geq t(f(t^{'})) + t^{'}(f(t)) \qquad (2)$$

that is, $(1) \Rightarrow (2) (1) \Rightarrow (2)$.

Is the converse $((2) \Rightarrow (1)) (2) \Rightarrow (1))$ true?

mechanism-design

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edited Feb 14 '17 at 2:19
Bayesian
1,708 ● 1 ● 8 ● 31

asked Oct 17 '16 at 14:37
Abishanka Saha
213 ● 1 ● 4

I have derived that $p(t)_{f(t)}$ $p(t)_{f(t)}$ is a function of $f(t)_{f(t)}$, that is if $f(t) = f(t')_{f(t) = f(t^{'})}$ then $p(t) = p(t')$ $p(t) = p(t^{'})$. I am trying to analytically write $p(t)_{p(t)}$ $p(t)_{p(t)}$ as an implicit function of $f(t)_{f(t)}$. Please help – Abishanka Saha Oct 18 '16 at 5:50

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First of all, the general form of the problem you got there is extremely demanding. In multidimensional screening problems, analytically often all hell breaks loose in a sense that it is just not tractable. One way out might be this recent approach by Gabriel Carrol: Robustness and Separation in Multidimensional Screening (also provides an introduction that hints at the intractibility of the problem).

Coming back to your question: $(2) (2)$ does not imply $(1) (1)$, because $(2) (2)$ only depends on $f f$ and you can add a transfer rule that is not IC. As a counterexample, consider a single good, $m = 1$ $m = 1$. Take some strictly IC mechanism, $(f, p) (f, p)$ and consider two types, $t t$ and $t' t^{'}$. So, $(1) (1)$ is

$$U(t) := f(t)t - p(t) > f(t')t - p(t')$$
$$U(t') := f(t')t' - p(t') > f(t)t' - p(t)$$

$$U(t) := f(t)t - p(t) > f(t^{'})t - p(t^{'}) U(t^{'}) := f(t^{'})t^{'} - p(t^{'}) > f(t)t^{'} - p(t)$$

and by construction $(2) (2)$ holds as well. Let $t' > t t^{'} > t$ and a lager allocation probability $f(t') > f(t) f(t^{'}) > f(t)$ is accompanied by a larger expected transfer, $p(t') > p(t) p(t^{'}) > p(t)$ - otherwise $t t$ would love to fake being $t' t^{'}$.

Now, construct $\widetilde{f}(s) = f(s) \widetilde{f}(s) = f(s)$ for all $s \in T s \in T$ and let $\widetilde{p}(t) = p(t') \widetilde{p}(t) = p(t^{'})$ and $\widetilde{p}(t') = p(t) \widetilde{p}(t^{'}) = p(t)$. Since $(2) (2)$ only depends on the allocation function $f f$ and these functions are the same in both mechanisms, $(2) (2)$ holds for $(\widetilde{f}, \widetilde{p}) (\widetilde{f}, \widetilde{p})$. Obviously, $(1) (1)$ is violated for the reason named above:

$$\widetilde{f}(t')t - \widetilde{p}(t') > \widetilde{f}(t)t - \widetilde{p}(t).$$

$$\widetilde{f}(t^{'})t - \widetilde{p}(t^{'}) > \widetilde{f}(t)t - \widetilde{p}(t).$$

To address your other problem (in your comment), use the integral formulation of expected utility. Rewriting $(1) (1)$ yields,

$$f(t)(t - t') \geq U(t) - U(t') \geq f(t')(t - t')$$

$$f(t)(t - t^{'}) \geq U(t) - U(t^{'}) \geq f(t^{'})(t - t^{'})$$

implying that $U U$ is Lipschitz continuous, implying $U U$ is differentiable a.e., and equals the integral over its derivative:

$$U(t) = U(\underline{t}) + \int_{\underline{t}}^{t} f(s)ds$$

$$U(t) = U(\underline{t}) + \int_{\underline{t}}^{t} f(s)ds$$

where $\underline{t} \underline{t}$ is the lowest possible type. Then, you rewrite,

$$f(t)t - p(t) = U(\underline{t}) + \int_{\underline{t}}^{t} f(s)ds$$

$$f(t)t - U(\underline{t}) - \int_{\underline{t}}^{t} f(s)ds = p(t).$$

$$f(t)t - p(t) = U(\underline{t}) + \int_{\underline{t}}^{t} f(s)ds f(t)t - U(\underline{t}) - \int_{\underline{t}}^{t} f(s)ds = p(t).$$

Suppose $\underline{t} = 0 \underline{t} = 0$, then

$$p(t) = p(0) + f(t)t - \int_{0}^{t} f(s)ds.$$

$$p(t) = p(0) + f(t)t - \int_{0}^{t} f(s)ds.$$

This implies the revenue equivalence theorem: If two auctions have the same allocation rule, payment functions (and thus revenue) can only differ by a constant. The same trick works for multi-unit auctions with multi-unit demand, see, e.g., Krishna, Ch 14.

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answered Feb 13 '17 at 12:52
Bayesian
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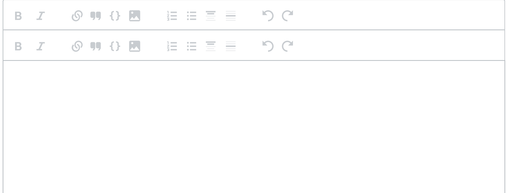

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