

No. 2165

LOCATION AS A SIGNAL OF QUALITY

Nikolaos Vettas

INDUSTRIAL ORGANIZATION



Centre for Economic Policy Research

LOCATION AS A SIGNAL OF QUALITY

Nikolaos Vettas

Discussion Paper No. 2165
June 1999

Centre for Economic Policy Research
90–98 Goswell Rd
London EC1V 7RR
Tel: (44 171) 878 2900
Fax: (44 171) 878 2999
Email: cepr@cepr.org

This Discussion Paper is issued under the auspices of the Centre's research programme in **Industrial Organization**. Any opinions expressed here are those of the author(s) and not those of the Centre for Economic Policy Research. Research disseminated by CEPR may include views on policy, but the Centre itself takes no institutional policy positions.

The Centre for Economic Policy Research was established in 1983 as a private educational charity, to promote independent analysis and public discussion of open economies and the relations among them. It is pluralist and non-partisan, bringing economic research to bear on the analysis of medium- and long-run policy questions. Institutional (core) finance for the Centre has been provided through major grants from the Economic and Social Research Council, under which an ESRC Resource Centre operates within CEPR; the Esmée Fairbairn Charitable Trust; and the Bank of England. These organizations do not give prior review to the Centre's publications, nor do they necessarily endorse the views expressed therein.

These Discussion Papers often represent preliminary or incomplete work, circulated to encourage discussion and comment. Citation and use of such a paper should take account of its provisional character.

Copyright: Nikolaos Vettas

June 1999

ABSTRACT

Location as a Signal of Quality*

We examine a horizontal product differentiation duopoly model where firms are also differentiated with respect to the quality of their products. Firms first choose their locations (or product characteristics) and then compete in prices. Under full information, it is shown that, whereas the low quality firm prefers to locate as far as possible from its competitor, the same is not true for the high quality firm, unless the quality difference is small enough. The Paper then suggests an explanation for spatial agglomeration based on incomplete information considerations. Because it is less costly for a high quality firm than for a low quality firm to locate close to a rival firm, choosing a location closer to a rival signals high quality.

JEL Classification: D4, D8, L1

Keywords: location, quality, horizontal and vertical differentiation, spatial agglomeration, signalling

Nikolaos Vettas
The Fuqua School of Business
Duke University
Durham
NC 27708-0120
USA
Tel: (1 919) 660 7756
Fax: (1 919) 684 2818
Email: nikos.vettas@duke.edu

*I thank Simon Anderson, Jim Anton, Kyle Bagwell, Jim Friedman, Wes Magat, George Mailath, Jim Peck, and workshop participants at the University of Virginia, the Ohio State University, the Duke-UNC microtheory seminar, the Midwest Economic Theory meetings at Northwestern University and the Southeast Economic Theory meetings at Virginia Tech for helpful comments and discussions. Of course, all errors are mine.

Submitted 1 April 1999

NON-TECHNICAL SUMMARY

In most markets there is both vertical (quality) and horizontal (variety) product differentiation. Quality refers to characteristics of a product that all consumers find desirable. Variety reflects the fact that consumers may have different preferences with respect to some other product characteristic. This Paper attempts to improve our understanding of markets where both types of differentiation are present, examining in particular the effect of quality differences on the horizontal differentiation decisions of firms.

Some examples may illustrate how both horizontal and vertical characteristics are taken into consideration when consumers select among different products. Consumers may choose a restaurant either because it offers high quality of food and service or because of its convenient location. But, they would rarely travel a long distance to dine at a low quality restaurant (at least unless the price was sufficiently low). Consumers may choose to purchase a car based on either 'objective quality' characteristics, such as fuel efficiency, reliability and safety, or because they particularly like its colour, design, or 'feel of the road'. In academia, researchers are likely to pay attention to work close to their field of expertise, as well as to work that is outside their particular area of interest but is produced by someone known as an outstanding researcher. In our analysis we take the quality aspect of firms' products as given and examine the implications for their horizontal product differentiation and pricing strategies. For example, in the situations mentioned before, how does the optimal location of a restaurant vary, depending on whether the quality of its food is high or low? Given that an automobile manufacturing company has higher overall quality than its competitors, how should it differentiate its product with respect to the design or other characteristics about which consumers' tastes may differ? Or, how should a gifted young researcher decide (early in graduate school) which, among many different fields, to pursue?

The model introduces quality differences in a standard linear-city duopoly model. Location in the linear city can be interpreted either as geographical location or as a choice of variety. Firms choose first their locations and then compete in prices. Consumers make their purchasing decisions after they observe the final 'delivered' prices, that is, prices adjusted for quality plus a transportation cost. The first part of the Paper considers the full information case. We find that it is more costly for a low quality firm than for a high quality firm to locate closer to a rival firm. Whereas the low quality firm prefers to locate as far as possible from its competitor, the same is not true for the high quality firm, unless the quality difference is small enough. Put differently, the low quality firm seeks to move away in product space, in an effort to reduce

price competition. A firm that enjoys a large quality advantage, however, seeks to press its vertical advantage by moving towards its rival. In this sense, the 'maximum differentiation' result does not hold when the quality differences are significant.

In the second part of the analysis, we recognize that firms may have private information about the quality of their products. Potential customers can observe the location of a restaurant, but they may not know the quality of the food until after they have consumed it. Similarly, consumers may be able to inspect the design characteristics of an automobile, but they may not have precise information about its reliability until after they have driven it for some time. Or, when evaluating a researcher's application for academic employment, it may be difficult to judge their overall 'quality', whereas their field of specialization is immediately observable. In these examples, the agents making the location choices may have initially better information about their quality than other agents. This information may become known to others in the market only after some time, whereas the location choices are immediately observable.

The main insight in the Paper is that a firm's location can be used as a *signal* of its quality. This is because it is more costly for a low quality firm than for a high quality firm to be located close to a competitor. Thus, when location has a literal geographical interpretation, the model provides an explanation for spatial agglomeration due to incomplete information considerations: a high quality firm may choose to signal its quality to the consumers by locating close to competition (whereas a low quality firm would choose to operate in more 'distanced' markets). When location is interpreted as variety, another interpretation of this behaviour is that firms who offer high quality products may avoid substantial product differentiation in the variety dimension. In the context of the examples introduced above, a new restaurant may choose to locate in a 'restaurant district', even though it would face stronger competition there, to signal high quality. The inference here is that, for a low quality restaurant, choosing a location that implies intense competition with other restaurants would lead to low profit (in the middle run, after it is revealed that its quality is not high). Choosing to serve a particular segment of the market, for example some local neighbourhood demand, would, therefore, be a better strategy for a low quality firm (some consumers would still want to eat there, even if it is a low quality restaurant, because of its convenient location). As a result, locating close to competition (and, in particular, closer to competition than under full information), signals that this is a high quality restaurant. Similarly, for an automobile manufacturer, avoiding differentiation of a model's design from 'the mainstream' may signal high quality, whereas a low quality manufacturer may find it more profitable to follow a 'niche' strategy. And graduate students may choose academic fields where there is very intense

‘competition’, simply because choosing other fields may be perceived as a sign of weakness (or ‘low quality’).

As a by-product of the analysis in this Paper, we also obtain new insights with respect to the role of quality in oligopolistic competition that is different from the role in models of pure vertical differentiation. In such models, increasing the quality of the high quality firm increases the profit of both the high quality and the low quality firm. This result is due to the fact that, in these models, firms selling different qualities coexist in the market because consumers differ with respect to how much they value quality. In this Paper, firms selling different qualities coexist in the market because there is also differentiation along an additional (horizontal) dimension: low quality firms survive by specializing in offering a variety that some consumers find attractive. Then, increasing the quality difference between firms has a very different implication: while it increases the high quality firm’s profit, it decreases the low quality firm’s demand and profit. This may be a more realistic representation of some markets.

1. Introduction

In most markets there is both vertical (quality) and horizontal (variety) product differentiation. Quality refers to characteristics of a product that all consumers find desirable. Variety reflects the fact that consumers may have different preferences with respect to some other product characteristic. This paper attempts to improve our understanding of markets where both types of differentiation are present, examining in particular the effect of quality differences on the horizontal differentiation decisions of firms.¹

Some examples may illustrate how both horizontal and vertical characteristics are taken into consideration when consumers select among different products. Consumers may choose a restaurant either because it offers high quality of food and service or because of its convenient location. But they would rarely travel a long distance to dine at a low-quality restaurant (at least unless the price was sufficiently low). Consumers may choose to purchase a car based on either “objective quality” characteristics, such as fuel efficiency, reliability, and safety, or because they particularly like its color, design, or “feel of the road.” In academia, researchers are likely to pay attention to work close to their field of expertise, as well as to work that is outside their particular area of interest but is produced by someone known as an outstanding researcher. In our analysis we take the quality aspect of firms’ products as given and examine the implications for their horizontal product differentiation and pricing strategies. For example, in the situations mentioned before, how does the optimal location of a restaurant vary, depending on whether the quality of its food is high or low? Given that an automobile manufacturing company has higher overall quality than its competitors, how should it differentiate its product with respect to the design or other characteristics about which consumers’ tastes may differ? Or, how should a

¹In most of the literature, the vertical and the horizontal dimensions have been examined separately. There are few exceptions where papers incorporate both types of differentiation (see the discussion below).

gifted young researcher decide (early in graduate school) which among many different fields to pursue?²

The model introduces quality differences in the d’Aspremont *et al.* [7] reformulation of Hotelling’s [17] linear-city duopoly model. Location in the linear city can be interpreted, as usual, either literally as geographical location or as a choice of variety. Firms choose first their locations and then compete in prices. Consumers make their purchasing decisions after they observe the final “delivered” prices, that is, prices adjusted for quality plus a transportation cost. The first part of the paper considers the full-information case. We consider sequential, as well as simultaneous location choices of firms. A first key insight is that it is more costly for a low-quality firm than for a high-quality firm to locate closer to a rival firm. Whereas the low-quality firm prefers to locate as far as possible from its competitor, the same is not true for the high-quality firm, unless the quality difference is small enough. Put differently, the low-quality firm seeks to move away in product space, in an effort to reduce price competition. A firm that enjoys a large quality advantage, however, seeks to press its vertical advantage by moving towards its rival. In this sense, the “maximum differentiation” result does not hold when the quality differences are significant.

In the second part of the analysis, we recognize that firms may have private information about the quality of their products. Potential customers can observe the location of a restaurant but they may not know the quality of the food until after they have consumed it. Similarly, consumers may be able to inspect the design characteristics of an automobile, but they may not have precise information about its reliability until after they have driven it for some time. Or, when evaluating a researcher’s application for academic employment, it may be difficult to judge her overall “quality”, whereas her field of specialization is immediately observable. In these examples, the agents making the location choices may have initially better

²In many situations quality may be also a choice variable. Here we focus on the case where quality is already determined (or, equivalently, is much more difficult to change than other characteristics).

information about their quality that other agents. This information may only become known to others in the market only after some time, whereas the location choices are immediately observable.

The main insight in the paper is that a firm's location can be used as a *signal* of its quality. This is because it is more costly for a low-quality firm than for a high-quality firm to be located close to a competitor. Thus, when location has a literal geographical interpretation, the model provides an explanation for spatial agglomeration due to incomplete information considerations: a high-quality firm may choose to signal its quality to the consumers by locating close to competition (whereas a low-quality firm would choose to operate in more "distanced" markets). When location is interpreted as variety, another interpretation of this behavior is that firms that offer high-quality products may avoid substantial product differentiation in the variety dimension.³ In the context of the examples introduced above, a new restaurant may choose to locate in a "restaurant district," even though it would face stronger competition there, to signal high quality. The inference here is that, for a low-quality restaurant, choosing a location that implies intense competition with other restaurants would lead to low profit (in the middle-run, after it is revealed that its quality is not high). Choosing to serve a particular segment of the market, say some local neighborhood demand, would, therefore, be a better strategy for a low-quality firm (some consumers would still want to eat there, even if it is a low-quality restaurant, because of its convenient location). As a result, locating close to competition (and, in particular, closer to competition than under full-information), signals that this is a high-quality restaurant. Similarly, for an automobile manufacturer, avoiding to differentiate a model's design from "the mainstream" may signal high quality, whereas a low-quality manufacturer may find it more profitable to follow a "niche" strategy. And graduate students may choose academic fields where there is very intense "competition," simply because choosing other fields may be perceived as a sign of weakness (or "low quality").

³The literature has established that there are strong agglomeration forces in many markets.

As a by-product of the analysis in this paper, we also obtain new insights with respect to the role of quality in firms' competition. This role is different from the role in existing vertical differentiation models. In such models (following e.g. the well-known Shaked and Sutton [27] formulation) increasing the quality of the high-quality firm increases the profit of *both* the high-quality and the low-quality firm. This result is due to the fact that, in these models, firms selling different qualities coexist in the market because consumers differ with respect to how much they value quality. Instead, in the present formulation, firms selling different qualities coexist because there is also differentiation along an additional (horizontal) dimension: low-quality firms survive by specializing in offering a variety that some consumers find attractive. Then, increasing the quality difference between firms has a very different implication: while it increases the high-quality firm's profit, it decreases the low-quality firm's demand and profit. This may be a more realistic representation of some markets.

The analysis is related to three different and well-known literatures. First, it is related to the horizontal differentiation or location literature. The pioneering work here is by Hotelling [17] who proposed minimum differentiation as the expected outcome. His work has been shown to ignore that the existence of a pure-strategy equilibrium may be a problem with linear transportation costs, and that an incentive to relax price competition may lead to maximum rather than minimum differentiation. The analysis of d'Aspremont *et al.* [7] showed that the introduction of a quadratic transportation cost guarantees existence of equilibrium and, at the same time, delivers maximum differentiation. A number of subsequent papers have further examined this issue. In general, it appears a relatively robust result that minimum differentiation does not hold in this class of models.⁴ The basic intuition is that, once firms locate close to one another, price competition becomes intense and decreases profits. Second, the paper is related to the literature modelling vertically differentiated markets, including

⁴See Gabszewicz and Thisse [16] for a survey.

Shaked and Sutton [27], [28].⁵ Finally, the paper is related to the signaling literature, and in particular on firms signaling product quality.

More specifically, since this paper suggests that incomplete information may contribute to agglomeration, it is related to work that offers alternative explanations for agglomeration. Whereas, as discussed above, the tendency to relax price competition contributes towards maximal differentiation, other forces may make firms prefer to locate near each other. In a number of papers, starting with de Palma *et al.* [8], large enough heterogeneity in consumers' tastes relaxes price competition and may lead to minimum differentiation. Having quantity rather than price as the strategic variable may also contribute to agglomeration (see Anderson and Neven [2]). Collusion is another mechanism that, by keeping prices high, may contribute to agglomeration (see e.g. Jehiel [19] and Friedman and Thisse [13]). Other explanations are based not on relaxing price competition, but on features of the demand.⁶

With respect to two-dimensional differentiation, there is some work on markets where firms can choose to differentiate with respect to either variety or quality. Economides [11] shows that, in a model with linear transportation costs and firms choosing first their locations and then their qualities and prices, the equilibrium exhibits maximum variety and minimum quality differentiation. Similarly, in Neven and Thisse [24] firms choose maximum (minimum) horizontal differentiation only when they choose minimum (maximum) vertical differentiation. Tabuchi [29] allows firms to locate on two-dimensional space and finds a similar result, that firms maximize their distance in one dimension and minimize it in the other. Irmen and Thisse [18] generalize this

⁵See Gabszewicz and Thisse [15] for a comparison of horizontally and vertically differentiated oligopolies.

⁶If consumers' tastes are not uniformly distributed, firms may have a tendency to locate closer to each other because they tend to locate near the demand (see the early work of Neven [25] on non-uniform distributions). When consumers are imperfectly informed about prices (Dudey [9]) or product characteristics (Wolinsky [31]), economies of scale in search may also contribute towards agglomeration.

intuition and show that, in a model with multiple characteristics, firms maximize differentiation in the “dominant” characteristic and minimize differentiation in all others.⁷ Parts of the analysis concerning the mechanics of the market with simultaneous moves and under perfect information are closely related to Ziss [32] who studies horizontal differentiation when the competitors have different costs. A recent paper by Bester [5] also examines location and signaling. Whereas he also concludes that signaling may lead to agglomeration, his analysis is very different from the present paper. Bester’s insight is that maximal differentiation may not occur when firms use price to signal quality, because signaling limits the extent of price competition.⁸ In particular, in his model quality is a choice variable and only high quality can be profitably sold in the market. Price competition is mitigated by moral-hazard considerations: prices become downward rigid, because low prices do not offer enough incentives in equilibrium for high quality to be produced. In contrast, the focus of the present paper is on the different strategic incentives for low and high quality firms (with both qualities coexisting in the market) and on the ability of location itself to signal quality.

The remainder of the paper is as follows. Section 2 sets up the notation and presents the analysis of the basic model under full information. We characterize the game where firms choose their locations and then their prices. We first consider sequential and then simultaneous location choices. We also examine how profits vary as the quality difference between the two firms increases. Section 3 represents the core of the paper: it introduces incomplete information about the quality and examines location as a signal of quality. Section 4 examines a number of extensions

⁷Ferreira and Thisse [12] consider a horizontal differentiation model where the firms’ transportation technologies may be different, thus introducing an element of vertical differentiation. They show that we may have minimum (maximum) vertical differentiation when horizontal differentiation is large (small).

⁸In this sense, by restricting price competition, price signaling is shown to have effects similar to collusion (as in the papers mentioned above).

and alternative formulations: allowing firms to choose locations outside the interval where consumers are located; adding consumer heterogeneity with respect to quality; considering the effect of an outside option so that consumers do not always purchase (in equilibrium) one unit of the good (from either firm); and discussing other signaling possibilities. Section 5 contains concluding remarks.

2. The basic model: analysis with known qualities

2.1. Set-up and Notation

There is a continuum of consumers uniformly distributed on a unit-length interval. There are two firms, with locations A and B in $[0,1]$. Consumers have unit demands. A consumer located at $x \in [0,1]$ who decides to purchase (one unit) from firm i , has surplus

$$R + q_i - p_i - t \cdot (i - x)^2, \quad (2.1)$$

$i = A, B$, where q_i and p_i are the quality and price respectively for firm i , $t > 0$ is an index of the transportation cost and $R \geq 0$ is the basic reservation utility obtained by any consumer who purchases one of the two products.⁹ The outside option, if neither of the two products is purchased, is normalized to zero. We will assume that R is large enough so that, in equilibrium, consumers always choose to purchase from A or B .¹⁰ Without loss of generality, assume that firm B has higher quality and that

$$q_A = 0, \quad q_B = q > 0.$$

⁹We adopt the quadratic transportation cost formulation, which allows the results to be directly comparable with d'Aspremont *et al.* [7]. A linear cost formulation would result to well-understood existence problems, at least as the quality difference becomes small.

¹⁰Equivalently, consumers always have enough income to purchase the product from one of the two firms. This assumption, standard in the literature, is maintained throughout most of the analysis. The implications of relaxing it are discussed in Section 4.3.

For simplicity, the unit cost of production is assumed zero for both firms.¹¹

The basic game we analyze under perfect information is as follows. The two firms first choose their locations. We entertain two alternative assumptions here, sequential and simultaneous location choices. Then the firms simultaneously choose their prices and consumers decide whether to purchase from firm A or B . We are looking for a subgame perfect equilibrium when firms maximize their profits and consumers their utility.

2.2. Equilibrium Prices

First, we analyze the equilibrium behavior for fixed locations of the two firms. Suppose, without loss of generality, that $0 \leq A \leq B \leq 1$.

Let $z \in [0, 1]$ be the demand for firm A (and $1 - z$ for B). For $z \in (0, 1)$, z is determined by the location of a consumer indifferent between purchasing from A or B . Then equating the quality-adjusted final delivered prices we have

$$p_A + t(z - A)^2 = p_B - q + t(B - z)^2$$

and, solving for z we obtain:¹²

$$z = \frac{A + B}{2} + \frac{(p_B - q) - p_A}{2t(B - A)} \quad (2.2)$$

if $p_B - q + t[B^2 - A^2 + 2(A - B)] \leq p_A \leq p_B - q + t(B^2 - A^2)$. We have a corner solution $z = 0$ if $p_A + tA^2 \geq p_B - q + tB^2$ and $z = 1$ if $p_A + t(1 - A)^2 \leq p_B - q + t(1 - B)^2$.

The profit functions are

$$\pi_A = p_A \cdot z \text{ and } \pi_B = p_B \cdot (1 - z) \quad (2.3)$$

and we can now state the first result.

¹¹Thus, the unit cost is assumed independent of quality. This corresponds, for example, to the case where higher quality is a result of more successful past R&D, rather than the use of better materials.

¹²This expression is not well defined when the two locations coincide. Then, there is no horizontal differentiation and the firm with the lower quality-adjusted price captures the entire market.

Proposition 1. For arbitrary locations of the two firms $0 \leq A \leq B \leq 1$, the equilibrium prices are:

$$p_A^*(A, B) = \begin{cases} 0 & \text{if } B \leq \hat{B} \\ \frac{t}{3}[B^2 - A^2 + 2(B - A) - (q/t)] & \text{if } B \geq \hat{B} \end{cases} \quad (2.4)$$

and

$$p_B^*(A, B) = \begin{cases} q - t(B^2 - A^2) & \text{if } B \leq \hat{B} \\ \frac{t}{3}[A^2 - B^2 + 4(B - A) + (q/t)] & \text{if } B \geq \hat{B} \end{cases} \quad (2.5)$$

where

$$\hat{B} \equiv \hat{B}(A) \equiv -1 + \sqrt{(A+1)^2 + (q/t)} > A. \quad (2.6)$$

Proof. To characterize the equilibrium, we first proceed assuming that $z \in (0, 1)$. Then from the first order conditions

$$\partial \pi_A / \partial p_A = 0 \text{ and } \partial \pi_B / \partial p_B = 0$$

we obtain the reaction functions:

$$p_A(p_B) = \frac{1}{2}[t(B^2 - A^2) + p_B - q],$$

$$p_B(p_A) = \frac{1}{2}[t(A^2 - B^2) + p_A + 2t(B - A) + q]$$

and the equilibrium prices

$$p_A^* = \frac{t}{3}[B^2 - A^2 + 2(B - A) - \frac{q}{t}], \quad (2.7)$$

$$p_B^* = \frac{t}{3}[A^2 - B^2 + 4(B - A) + \frac{q}{t}]. \quad (2.8)$$

However, so far we have ignored the possibility of a corner solution ($z = 0$ or 1). In particular, from (2.7) and substituting (2.7) and (2.8) into (2.2) we have

$$p_A^* \geq 0 \Leftrightarrow z \geq 0 \Leftrightarrow B^2 + 2B - [A^2 + 2A + (q/t)] \geq 0 \Leftrightarrow B \geq \hat{B} \quad (2.9)$$

where \hat{B} is the larger root of the quadratic and the last equivalency is true because the smaller root is negative.¹³

Now we look at the equilibrium when $B \leq \hat{B}$. First, it is easy to see that, if $p_B^* = q - t(B^2 - A^2)$, firm A has zero demand if it charges a non negative price (see Figure 1). Thus, the maximum profit it can possibly obtain is zero and so the price specified in the suggested strategy ($p_A^* = 0$) is optimal. Next we need to show that $p_B^* = q - t(B^2 - A^2)$ is a best response. Given $p_A^* = 0$, firm B is solving $\max_{p_B} \pi^B(p_B) \equiv p_B(1 - z)$, where

$$z = \begin{cases} 0 & \text{if } p_B \leq q - t(B^2 - A^2) \\ \frac{A+B}{2} + \frac{p_B - q}{2t(B-A)} & \text{if } p_B \geq q - t(B^2 - A^2) \end{cases}.$$

Clearly, as long as $p_B \leq q - t(B^2 - A^2)$ the optimal price is $p_B = q - t(B^2 - A^2)$. Decreasing the price any further would not increase demand, since the entire market already purchases from B . Furthermore, substitutions and simple calculations show that $\pi^B(p_B)$ is continuous at $q - t(B^2 - A^2)$ and decreasing when $p_B \geq q - t(B^2 - A^2)$.¹⁴ It follows that the best response of B to $p_A^* = 0$ is $p_B^* = q - t(B^2 - A^2)$, the maximum price that gives the entire market to B . It is also easy to see that this is the unique equilibrium as long as prices have to be positive.¹⁵ ■

The following remarks are in order. First, note that for $A = B$ we have $p_A^* = 0$ and $p_B^* = q$ which is the standard outcome in a duopoly where differentiation is only vertical. Second, note that \hat{B} is an increasing function of A , with $\hat{B} > A$. It is also increasing in q/t , and $\hat{B} \rightarrow A$ as $q \rightarrow 0$. Third, note that \hat{B} can be either above or below 1. Clearly, if $\hat{B} \geq 1$, which is equivalent to $(q/t) \geq 3 - (A^2 + 2A)$, the quality

¹³It is easy to check that the other corner condition, $z \leq 1$, is equivalent to $p_B^* \geq 0$ and is true for all $0 \leq A \leq B \leq 1$.

¹⁴The condition $B < \hat{B}$ needs to be used in the proof of this claim.

¹⁵If prices are allowed to be negative (more generally, below unit cost), it is known from the vertical differentiation literature that there is also a continuum of “implausible” equilibria where the low quality firm charges a negative price and all the consumers purchase from the high quality firm.

difference is so large (relative to the transportation cost) that the low-quality firm has zero demand even if its competitor is located as far away as possible (to its right).

Figure 2 presents the equilibrium prices as functions of B , for $A = 0$.¹⁶ Note that for $B > \hat{B}$ differentiation relaxes price competition and so both prices are increasing in B . However, for $B < \hat{B}$, p_B^* is decreasing in B . The intuition for this last property is as follows. When $B < \hat{B}$, B is close enough to A that B finds it profitable to capture the entire market (even when A chooses a zero price). Furthermore as B moves from \hat{B} towards A , B can increase its price without losing any of the in-between consumers. In other words, as B approaches A , A 's local monopoly power decreases and B can exploit more easily the power it derives from its higher quality.

2.3. Location incentives

We can now examine the firms' location choices. Before proceeding to the details, it is useful to summarize the main intuition. There are two effects relevant to the location decisions. First, there is a "direct" demand effect which contributes towards minimum differentiation. For fixed prices, each firm would like to minimize the distance from its competitor: As shown below, for $z \in (A, B)$ we have $\partial z / \partial A > 0$ and $\partial(1 - z) / \partial B < 0$. Second, there is the price competition or "strategic" effect. Higher differentiation relaxes price competition: $(\partial z / \partial p_B^*)(\partial p_B^* / \partial A) < 0$ and $(\partial(1 - z) / \partial p_A^*)(\partial p_A^* / \partial B) > 0$. For the low-quality firm, the threat of competition is more severe and the second effect always dominates. The high-quality firm is less concerned about the competition and, depending on the parameters, may want to minimize the distance from the other firm.

We now turn to the formal analysis. Substituting the optimal prices into (2.3) we obtain the profits π_A^* and π_B^* as functions of the locations.

¹⁶Note that, if $A = 0$, $p_A^* < p_B^*$. In other words, the low quality firm charges a lower price than its competitor. This is because, if $B \geq \hat{B}$, from (2.7) and (2.8) we have $p_A^* < p_B^* \Leftrightarrow (A^2 - A) - (B^2 - B) + (q/t) > 0$ which is true, whereas if $B \leq \hat{B}$ it is easy to see that $p_B^* = q - t(B^2 - A^2) > 0 = p_A^*$.

Lemma 1. *The low-quality firm maximizes its (equilibrium) profit by maximizing its distance from the high-quality firm, that is, for $0 \leq A \leq B \leq 1$, $\partial\pi_A^*/\partial A \leq 0$. More precisely, for $B \geq \hat{B}$ we have*

$$\partial\pi_A^*/\partial A < 0. \quad (2.10)$$

For $B < \hat{B}$ we have $\partial\pi_A^*/\partial A = 0$, while A increases its profit if it can increase its distance from B enough so that $|B - A| > |\hat{B} - A|$.

Proof. The statement for the $B < \hat{B}$ case follows directly from Proposition 1. For $B \geq \hat{B}$, and using the “envelope” theorem, we have:

$$\frac{\partial\pi_A^*}{\partial A} = p_A^* \left(\frac{\partial z}{\partial A} + \frac{\partial z}{\partial p_B^*} \frac{\partial p_B^*}{\partial A} \right). \quad (2.11)$$

Now

$$\frac{\partial z}{\partial A} = \frac{1}{2} + \frac{p_B - q - p_A}{2t(A - B)^2}$$

and

$$\frac{\partial z}{\partial p_B^*} \frac{\partial p_B^*}{\partial A} = \left(-\frac{1}{2t(A - B)} \right) \left(-\frac{4}{3}t + \frac{2}{3}tA \right) = \frac{1}{3} \left(\frac{2 - A}{A - B} \right) < 0.$$

Direct substitution and simple calculations show that for all $A < B$ and $B \geq \hat{B}$, (2.10) is true. ■

We now turn to the high-quality firm, B . Fix A and consider first $A \leq B \leq \hat{B}$. Then $\pi_B^* = p_B^* \cdot 1 = q - t(B^2 - A^2)$ which is decreasing in B . Thus, as long as it does not exceed \hat{B} , the optimal location for B is at A .

It remains to consider $B > \hat{B}$. We have:

$$\frac{\partial\pi_B^*}{\partial B} = p_B^* \left(-\frac{\partial z}{\partial B} - \frac{\partial z}{\partial p_A^*} \frac{\partial p_A^*}{\partial B} \right) = -p_B^* \left(\frac{\partial z}{\partial B} + \frac{\partial z}{\partial p_A^*} \frac{\partial p_A^*}{\partial B} \right), \quad (2.12)$$

with

$$\frac{\partial z}{\partial B} = \frac{1}{2} + \frac{p_A - p_B + q}{2t(A - B)^2}$$

and

$$\frac{\partial z}{\partial p_A^*} \frac{\partial p_A^*}{\partial B} = \frac{1}{2t(A-B)} \cdot \frac{2t(B+1)}{3} = \frac{B+1}{3(A-B)} < 0.$$

Substitution and some manipulation show that

$$\text{sign} \left(\frac{\partial \pi_B^*}{\partial B} \right) = \text{sign} \left(A - 3B + 4 + \frac{q}{t(A-B)} \right).$$

First note that if $q = 0$ then $\partial \pi_B^* / \partial B > 0$ (because $A - 3B + 4 > 0$). Next, since $A = 0$ is a requirement for an equilibrium (because $\partial \pi_A^* / \partial A < 0$ when $B > \hat{B}$), we focus on this case and have

$$\text{sign} \left(\frac{\partial \pi_B^*}{\partial B} \right) \Big|_{A=0} = \text{sign} \left(4 - 3B - \frac{q}{tB} \right).$$

Therefore

$$\frac{\partial \pi_B^*}{\partial B} \Big|_{A=0} \geq 0 \Leftrightarrow 4 - 3B - \frac{q}{tB} \geq 0 \Leftrightarrow 3B^2 - 4B + \frac{q}{t} \leq 0.$$

The last quadratic expression has positive discriminant if and only if $q/t < 4/3$ and roots

$$\underline{B} = \frac{2 - \sqrt{4 - 3(q/t)}}{3}, \quad \bar{B} = \frac{2 + \sqrt{4 - 3(q/t)}}{3}. \quad (2.13)$$

It follows that the only case when π_B^* is increasing in B is when $B > \hat{B}$ and B is between \underline{B} and \bar{B} (when these roots are real). Note that \bar{B} can be either below or above 1 and that $\bar{B} \geq 1 \Leftrightarrow q/t \leq 1$.

Collecting the above information we have the following.

Lemma 2. *If $q = 0$ then $\partial \pi_B^* / \partial B > 0$ for all $B \in [0, 1]$. If $q/t > 4/3$ and $A = 0$ then $\partial \pi_B^* / \partial B < 0$ for all $B \in [0, 1]$. If $0 < q/t \leq 4/3$ and $A = 0$ then*

$$\partial \pi_B^* / \partial B \geq 0 \Leftrightarrow \max\{\underline{B}, \hat{B}\} \leq B \leq \min\{\bar{B}, 1\}. \quad (2.14)$$

Lemma 3. *If $A = 0$ the optimal location for B is either $B^* = 0$ or $B^* = \min\{\bar{B}, 1\}$.*

Proof. From Lemma 2 note that π_B^* is strictly decreasing in B between 0 and $\max\{\underline{B}, \hat{B}\}$. Since $\hat{B} > 0$, $\max\{\underline{B}, \hat{B}\} > 0$ and 0 is a local maximum (of course, if $\max\{\underline{B}, \hat{B}\} \geq 1$, then 0 is automatically the global maximum, as well). π_B^* is increasing for $\max\{\underline{B}, \hat{B}\} \leq B \leq \min\{\bar{B}, 1\}$ (and decreasing for $B > \bar{B}$ when $\bar{B} < 1$). Thus $\min\{\bar{B}, 1\}$ is also a local maximum. Clearly, the globally optimal location will be either at 0 or at $\min\{\bar{B}, 1\}$. ■

See Figure 3 for B 's profit as a function of its location, and Figure 4 for a summary of some critical values for B 's location.

Lemma 4. *If $A = 0$ the optimal location for B is $B^* = 1$ if $q/t < \bar{Q}$ and $B^* = 0$ if $q/t > \bar{Q}$, where $\bar{Q} \equiv 6 - 3\sqrt{3}$ (≈ 0.804).*

Proof. When B is located at $A = 0$ we know that it charges price q and captures the entire demand. Therefore, its profit is q . By Lemma 3 we need to compare this profit level to B 's profit when it is located at $\min\{\bar{B}, 1\}$. We calculate first B 's profit at location 1. Assuming for the moment that $\hat{B} < 1$ (so that less than the entire market is captured by B) and substituting $A = 0$ into (2.2), (2.7), and (2.8), we have $\pi_B = p_B^* \cdot (1 - z)$, where

$$p_B^* = \frac{t}{3}(4B - B^2 + \frac{q}{t})$$

and

$$z = (B^2 + 2B - \frac{q}{t})/6B.$$

The relevant inequality is then $\pi_B \geq q$, or

$$(4B - B^2 + \frac{q}{t})^2 \geq 18B\frac{q}{t}. \quad (2.15)$$

Substituting now $B = 1$, (2.15) gives

$$(\frac{q}{t})^2 - 12\frac{q}{t} + 9 \geq 0$$

which implies $\frac{q}{t} \leq 6 - 3\sqrt{3} \approx 0.804$ or $\frac{q}{t} \geq 6 + 3\sqrt{3} \approx 11.2$. Recall, however, that we have assumed that $\hat{B} < 1$, which can be easily shown to be equivalent to $q/t < 3$.

Thus the only parameters for which B has higher profit at 1 than at 0 satisfy

$$\frac{q}{t} \leq 6 - 3\sqrt{3} \equiv \bar{Q}.$$

The next step is to examine if there are parameters that make $\bar{B} < 1$ the optimal location. We need to substitute $B = \bar{B} = (2 + \sqrt{4 - 3(q/t)})/3$ into (2.15). The relevant range for q/t is now $[1, 4/3]$ because $q/t < 1 \Rightarrow \bar{B} > 1$ and \bar{B} is not real for $q/t > 4/3$. Straightforward calculations show that, for $q/t \in [1, 4/3]$, profit at \bar{B} is always less than q . ■

Having analyzed how profits vary with firms' location choices, we next turn to an analysis of the equilibrium of the game where firms choose locations and then prices. We first consider sequential location choices and then turn to simultaneous location choices.

2.4. Sequential location choices

A specification where the location choices of firms are sequential rather than simultaneous, is a more reasonable description of some real-world cases.¹⁷ In addition, given the strategic incentives discussed in the previous section, the case of sequential choices exhibits richer behavior. It will also be the basis for our analysis of incomplete information later in the paper, and so we discuss it first. Specifically, we now examine the sequential location problem, where prices are again chosen simultaneously, after both locations have been chosen. We look, of course, for a subgame perfect equilibrium. Clearly, when location choices are sequential and the incentives are different for low and high-quality firms, the order of moves matters. To simplify the exposition, we assume for the following Proposition that if A is indifferent among several locations it chooses the one that maximizes its distance from B .¹⁸ The main results can be summarized as follows (see Figure 5).

¹⁷See also the early work of Prescott and Visscher [26] on sequential location choices.

¹⁸If this assumption does not hold, some additional location configurations can be supported in equilibrium.

Proposition 2. *Consider the game where the firms choose their locations sequentially (and then compete in prices).*

(i) *If $q/t < \bar{Q} \approx 0.804$, the equilibrium exhibits maximum differentiation independently of the order of moves: $[A^* = 0, B^* = 1]$ or $[A^* = 1, B^* = 0]$.*

(ii) *If $q/t > \bar{Q}$ and the low-quality firm moves first, the equilibrium locations are $A^* = B^* \in [0, 1]$.*

(iii) *If $q/t > \bar{Q}$ and the high-quality firm moves first, in equilibrium the high-quality firm chooses (depending on the parameters) a location between an endpoint and the midpoint ($1/2, \bar{B} = (2 + \sqrt{4 - 3(q/t)})/3$, or 1), while the low quality firm chooses to locate at the other endpoint. More precisely, if $\bar{Q} < q/t \leq 1$ we have in equilibrium $[B^* = 1, A^* = 0]$, or $[B^* = 0, A^* = 1]$. If $1 < q/t \leq (29\sqrt{145} - 187)/128 \approx 1.267$ we have $[B^* = \bar{B} \text{ and } A^* = 0]$ or, symmetrically, $[B^* = 1 - \bar{B} \text{ and } A^* = 1]$. If $q/t > (29\sqrt{145} - 187)/128 \approx 1.267$ we have $B^* = 1/2$ and $A^* \in \{0, 1\}$.*

Proof. (i) When $q/t < \bar{Q}$ we know, from Lemmas 1 and 4, that both firms want to maximize the distance between them. Thus the only equilibrium is for the first mover to choose one extreme location and for the follower to choose the other extreme.

(ii) When $q/t > \bar{Q}$ the high-quality firm wants to minimize, while the low-quality firm to maximize their distance. Thus, when B moves second, it chooses to locate at the same point as A . In turn, A 's location choice when it moves first is irrelevant for A (that is, any choice is weakly optimal): whatever this choice is, B will also move there and leave A with zero profit.

(iii) When B moves first, B anticipates that A will locate as far away as possible. Since A can choose to locate on either extreme, this implies that none of the values that involve locations closer than $1/2$ from A are attainable for B . If $q/t > 4/3$ then we know from Lemma 2 that B 's profit is monotonically decreasing in its distance from A . Given that A will locate as far away as possible, the choice that yields the minimum distance for B , is $B^* = 1/2$ (and A responds with A^* equal either to 0 or 1).

When $\bar{Q} < q/t < 4/3$, it is no longer true that B 's profit is monotonically decreasing in its distance from A . Again, it is not possible for B to locate itself closer than $1/2$ from A , but now $\min\{\bar{B}, 1\}$ is a local maximum that needs to be considered (as well as, symmetrically, $\max\{0, 1 - \bar{B}\}$). In addition, $1/2$ is also a local maximum if and only if B 's profit is decreasing at $1/2$. Thus, in this case, B will choose as its location either $\min\{\bar{B}, 1\}$ or $1/2$, the one that gives the higher profit, given that A is expected to locate at 0. In particular, it is easy to show that $\hat{B} \leq 1/2 \Leftrightarrow \underline{B} \leq \hat{B} \Leftrightarrow q/t \leq 5/4$. Thus, from Lemma 2 it follows that, when $\bar{Q} < q/t \leq 5/4$, B 's profit function is increasing at $1/2$, which implies that $1/2$ is not an optimal location for B when it moves first, but instead it chooses $\min\{\bar{B}, 1\}$ (or, symmetrically, $\max\{0, 1 - \bar{B}\}$). Finally, for the case $5/4 < q/t < 4/3$ again a comparison of B 's profit at $1/2$ and at $\min\{\bar{B}, 1\}$ is in order. Since in this case $q/t > 1$, it follows from (2.13) that $\bar{B} < 1$, and so to find the optimal we only need to compare B 's profit at $1/2$ and at \bar{B} (given that firm A will locate at 0). From the analysis earlier in the paper it is easy to see that B 's profit at $1/2$ is $q - t/4$ (since $\hat{B} > 1/2$ and B captures the entire market with price equal to $q - t/4$) while profit at \bar{B} can be calculated using (2.2), (2.7), (2.8), and (2.2). Direct calculations then show that profit at $1/2$ is lower (and thus \bar{B} is the preferred location between these two) if and only if $5/4 < q/t \leq (29\sqrt{145} - 187)/128 \approx 1.267$. ■

The most important intuition from the above Proposition is as follows. When the high-quality firm moves first and the quality difference is high enough, we expect that the high-quality firm will choose the location at the center of the market, whereas the low-quality firm will “differentiate” itself as much as possible and locate at an extreme point. *As the quality difference decreases, the optimal location of a (high quality) firm which enters first moves away from the market center and towards an extreme point.* See Figure 6 for a graphical representation of the optimal location of a high quality firm which enters first (and anticipates that a low quality firm will locate at 0) as a function of the quality difference.

2.5. Simultaneous location choices

We now turn to a discussion of the equilibrium when firms choose their locations simultaneously. There are three possibilities here: maximal differentiation when the quality difference is small, non-existence of a pure strategy equilibrium for a larger quality difference (because the low-quality firm prefers maximal and the high-quality firm prefers minimal differentiation), and a “degenerate” case where the low-quality firm is indifferent among all locations, when the quality difference is even larger.

Proposition 3. *Suppose that firms choose their locations simultaneously (and then they compete in prices). If $q/t \leq \bar{Q}$ the equilibrium locations of the firms are $A^* = 0$ and $B^* = 1$ (or, symmetrically, $A^* = 1$ and $B^* = 0$). If $\bar{Q} < q/t < 5/4$ there is no equilibrium in pure strategies. If $5/4 \leq q/t \leq 3$ then any $A^* = B^* \in [2 - \sqrt{1 + (q/t)}, -1 + \sqrt{1 + (q/t)}]$ is an equilibrium in locations. If $q/t \geq 3$ any $A^* = B^* \in [0, 1]$ is an equilibrium in locations.*

Proof. It follows from the analysis in section 2.3. From Lemma 1, we know that A always wants to maximize its distance from B . (Note that for $q/t \leq \bar{Q} < 3$, with $A = 0$, we have that $\hat{B} < 1$ which means that $B = 1 > \hat{B}$ is possible and the maximum differentiation incentive for A is strict.) From Lemma 4, if $q/t \leq \bar{Q}$ maximum differentiation is optimal for B , as well. Thus the equilibrium locations are the two extremes.

If $q/t > \bar{Q}$ minimum differentiation is optimal for B . (Lemma 4 has shown that this is true when A is at an extreme location. It is easy to show that the threshold \bar{Q} is decreasing as A increases from 0 to $1/2$ and *a fortiori* minimum differentiation is optimal for B when A has an interior location.) Thus if A strictly prefers to be at one of the extremes then, clearly, no equilibrium exists in pure strategies.

The only case where a pure strategy equilibrium exists when minimum differentiation is optimal for B is when A is indifferent among all locations (making zero profits). Consider, first a candidate equilibrium with $A^* = B^* \in [1/2, 1]$, where A makes zero

profit. The most profitable deviation for A is to maximize its distance from B^* by locating at 0. This deviation would increase A 's profit to some positive level if and only if the critical value \hat{B} for $A = 0$ is less than B^* . Otherwise, we know from Proposition 1 that B would still capture the entire market. Thus for the proposed deviation not to be profitable, it is needed that $B^* < \hat{B}$, which from (2.6) is equivalent to $q/t > (B^* + 1)^2 - 1$. Symmetrically, for $A^* = B^* \in [0, 1/2]$ the most profitable deviation for A is to locate at 1 and the condition is $B^* > 1 - \hat{B}$ (with \hat{B} evaluated at $A = 0$) which is equivalent to $q/t > (2 - B^*)^2 - 1$. Inverting these conditions we find that, for a fixed value of q/t , $A^* = B^* \in [1 - \hat{B}, \hat{B}] = [2 - \sqrt{1 + (q/t)}, -1 + \sqrt{1 + (q/t)}]$ constitute an equilibrium. From the above conditions it follows that the minimum value of q/t that allows existence of such an equilibrium is $5/4$, with equilibrium locations at $1/2$. For values of q/t below $5/4$ we have $\hat{B} < 1/2$ which means that firm A can always make positive sales by locating at an endpoint. It also follows that for $q/t \geq 3$ any $A^* = B^* \in [0, 1]$ is an equilibrium. ■

Note that in the “degenerate” equilibria described in Proposition 3 where firms choose the same location, the low-quality firm's profit is always zero and the key is that the quality difference is large enough that even if it were to deviate to the most remote location it would still not increase its profit. We do not seek to characterize mixed strategy equilibria in the case when a pure strategy equilibrium fails to exist. Instead, we turn to other aspects of the problem.

2.6. The effect of quality on profits

It is important to examine how the firms' profits vary as the quality difference, q , increases. As shown below, the profit of the high-quality firms increases whereas the profit of the low-quality firm decreases, as q increases. This may seem an intuitive result, but it should be emphasized that it is exactly the opposite of what happens in the vertical differentiation models following the Shaked and Sutton ([27], [28])

formulation.¹⁹ In those models increasing the quality differential increases the profit of *both* firms. It is thus important to study the issue and understand why these models give different answers.²⁰

Proposition 4. *An increase in q decreases the low-quality (A) firm's and increases the high-quality (B) firm's profit, provided that both firms have positive market shares ($A < \hat{B} < B$). If only B has positive market share ($A < B < \hat{B}$), A 's profit remains zero and B 's profit increases.*

Proof. We proceed in two steps. First consider what happens when q increases but the firms' locations do not change. Let $A < \hat{B} < B$. Consider the effect on A 's profit. For $A < \hat{B} \leq B$ and using the “envelope” theorem, we have:

$$\frac{\partial \pi_A^*}{\partial q} = p_A^* \left(\frac{\partial z}{\partial q} + \frac{\partial z}{\partial p_B^*} \frac{\partial p_B^*}{\partial q} \right)$$

with

$$\frac{\partial z}{\partial q} = \frac{1}{2t(A - B)} < 0, \quad \frac{\partial z}{\partial p_B^*} = -\frac{1}{2t(A - B)} > 0, \quad \text{and} \quad \frac{\partial p_B^*}{\partial q} = \frac{1}{3} > 0$$

and therefore

$$\frac{\partial \pi_A^*}{\partial q} = p_A^* \frac{1}{3t(A - B)} < 0. \tag{2.16}$$

Similarly, it is easy to show that $\partial \pi_B^* / \partial q > 0$. If, now we start from $A < B < \hat{B}$, so that only B has positive market share, then clearly an increase in q leaves A 's profit unchanged at a zero level while B 's profit, $q - t(B^2 - A^2)$, increases.

Next allow the firms' locations to change when q changes. It follows from the above analysis of equilibrium locations that the only possible change is that the high-quality firm may want to locate closer to its competitor if a large enough increase in q takes

¹⁹Their demand formulation is based on ideas that can be found in Mussa and Rosen [23] and Gabszewicz and Thisse [14].

²⁰See, e.g., Shaked and Sutton [27], p.8, Lemma 4.

place. Such a location change will contribute towards increasing the high-quality firm's profit and decreasing the low-quality firm's profit. ■

As mentioned above, there is a key difference between the present model and some other oligopoly models that involve vertical differentiation. In particular, in the Shaked and Sutton class of models (see [27], [28]) an increase in the quality of a high-quality competitor is “good news” for a low-quality competitor. In contrast, here an increase in the quality of a high-quality competitor is “bad news” for a low-quality competitor. The intuition is as follows.

In Shaked and Sutton, firms selling different qualities can coexist in the market because consumers differ with respect to how much they value quality (or, equivalently, with respect to their income). As a result, some consumers find the high-quality and high-price combination more attractive while others prefer the low-quality and low price combination. A low-quality firm makes positive sales by focusing on consumers with low valuation for quality. Increasing the quality difference has thus the effect that the high-quality firm finds it more desirable to focus on the high end of the market and allows the low-quality firm to increase its profit. This is the sense in which firms can “relax price competition through product differentiation”.

In the present paper, the reason why firms selling different qualities coexist is because there is also differentiation along an additional (horizontal) dimension. Consumers do not differ with respect to their valuation of quality.²¹ Thus when firms are not differentiated with respect to the horizontal characteristic, all consumers purchase from the high-quality firm. Increasing the quality difference between firms in our framework has a very different implication: it makes the high-quality firm's product relatively more desirable for all consumers, so that now the quality difference dominates the horizontal preference for some consumers, and decreases the low-quality

²¹See the discussion in section 4.2.

firm's demand and profit.²²

Both formulations, pure vertical differentiation and the one presented in this paper, seem reasonable for different market situations. The key difference, that increasing the quality difference benefits the low-quality firm in one framework and hurts it in the other, may offer some guidance as to which is the most appropriate model for a specific market.

3. Incomplete information: location as a signal of quality

The main idea here is that, since it is more costly for a low-quality firm than for a high-quality firm to be located close to a competitor, location can be used as a signal of product quality.

3.1. Preliminary analysis: an example

To see the basic argument, it is helpful to start the analysis by returning to our study of sequential entry under full information. Suppose that an entrant, say firm B , with quality $q \geq 0$, chooses its location and is then followed by another firm, say firm A , with quality 0. As Figure 6 demonstrates, the first entrant's location will be closer to the center of the market the higher the quality of its product, q , is (while the second entrant chooses an extreme point). Since the first firm's location depends on its quality, one may ask whether, when the firm has private information about its quality, its location choice can reveal information about its quality.

To fix ideas, suppose that the quality of the first entrant's product, $q \in \{0, \bar{q}\}$, can be either low (the same as the product quality of the follower, that is, 0) with

²²Formally, in Shaked and Sutton we can write the (indirect) utility of consumer i who purchases from firm j as $u_{ij} = R + \theta_i q_j - p_j$ with θ_i distributed across the population of consumers, while here we have $u_{ij} = R + q_j - p_j - t(i - j)^2$ where what differs across consumers is their location i relative to the firm's location.

probability $1 - \rho$, or high, $\bar{q} > 0$, with probability ρ .²³ The first entrant has private information about the quality of its product: B knows, before choosing its location, if $q = 0$ or $q = \bar{q}$. The consumers and firm A , however, do not know B 's quality. They use ρ as their prior beliefs and can update their beliefs after they observe B 's location choice.

To simplify the notation, assume here and for the remainder of the section that the transportation cost index is $t = 1$. For illustration purposes, suppose that \bar{q} is in the interval where location is strictly monotonic in quality (see Figure 6). For example, let $\bar{q} = 1.2$. Then, it is easy to see that *under full information*, when $q = 0$ firm B chooses to locate at 1 (and then firm A locates at 0; by symmetry we may have, of course, the reverse configuration). The two firms then share the market, each with profit equal to 0.5. If $q = \bar{q}$, on the other hand, it follows from Proposition 2 that, *under full information*, B locates at $\bar{B} = (2 + \sqrt{4 - 3(\bar{q}/t)})/3 = (2 + \sqrt{4 - 3(1.2)})/3 \approx 0.877$ and simple calculations show that it enjoys profit approximately equal to 0.983 (observe that, as expected, the entrant has higher profit when $q = \bar{q}$ than when $q = 0$).²⁴

Now consider the case where firm B has private information about its quality. Even if its true quality is low ($q = 0$) why would it not pretend that it is high ($q = \bar{q}$) by locating at $\bar{B} \approx 0.877$ instead of 1? Unless there is some “disciplining” mechanism, it is clear that, regardless of quality, the entrant will always choose the location that the high-quality type would have chosen under perfect information.

What could this “disciplining” mechanism be? A natural mechanism is that quality becomes known after some consumers purchase the product (essentially, the assumption here is that this happens more quickly than the firm may adjust its loca-

²³We can think of firm B 's product quality as either “average” (the same as the quality of the follower) or “exceptional”.

²⁴The low-quality type follows a “niche” strategy, trying to differentiate itself as much as possible from the competition. In contrast, the high-quality type chooses to bring to the market a product that corresponds more to the “mainstream” or to the average consumer.

tion). In particular, we consider the following game. Nature chooses the quality of firm B 's product, $q \in \{0, \bar{q}\}$. Firm B observes the quality of its product and chooses a location. Then firm A chooses its location. The two firms choose their first-period prices (simultaneously) and consumers make their purchasing decisions. At the end of period one, B 's quality is revealed to everyone in the market. In each of the subsequent periods, the two firms choose their prices and consumers decide from which firm to purchase. We assume that the firms live forever and have one-period discount factor $\delta \in [0, 1)$. Note that locations are chosen at the beginning of the first period and cannot be changed subsequently in the game. Prices, on the other hand, are allowed to differ from period to period. This formulation captures the idea that it is more difficult to change a firm's geographical location or product characteristics than its prices. Also note that, in this game, a low-quality entrant has both a benefit and a cost if it chooses the location that under full information corresponds to the high-quality entrant.

We consider Perfect Bayesian Equilibria of the game. At a *separating* equilibrium the two types of firm B choose different locations. Firm A and the consumers can “invert” the location choice and learn what B 's true quality is. At a pooling equilibrium both types choose the same location and consumers posterior beliefs are the same as their priors. We focus now on separating equilibria.²⁵

Specifically, let us first consider the following question. Can we construct a separating equilibrium of the incomplete information game constructed above where the two types choose their full-information locations, that is, the entrant locates at 1 if $q = 0$ and at $\bar{B} \approx 0.877$ if $q = \bar{q}$? Suppose we have such an equilibrium. Then, based on the arguments above, the high-quality type has profit $0.983/(1 - \delta)$ and the low quality type has profit $0.5/(1 - \delta)$. Consider a deviation where the low-quality type mimics the high-quality type and locates at 0.877. Then, its first-period profit

²⁵As argued below, it is not possible to have pooling with respect to locations and separation with respect to prices.

increases to 0.983. However, since its location cannot be changed and its quality is revealed to be low after the first period, its profit will be lower than 0.5 in all subsequent periods. More precisely, its profit will be approximately equal to 0.475.²⁶ Note that the reason that profit decreases in subsequent periods from 0.5 to 0.475 is because, with equal qualities, profits are highest under maximal differentiation and the entrant is now located too close to its competitor. It should be clear that this deviation with respect to locations is profitable for the low-quality firm if and only if

$$\frac{0.5}{1-\delta} < 0.983 + \frac{\delta}{1-\delta} 0.475 \Leftrightarrow \delta < 0.951.$$

The conclusion here is that, unless the discount factor is higher than 0.951, a separating equilibrium where the two types choose their perfect information locations does not exist.

To continue with the same scenario, (even) for discount factors lower than the threshold derived above, perhaps it would be possible to construct a separating equilibrium where the high-quality type's location is *closer* to its competitor (and, consequently, closer to the market center) than the full information location. By locating closer to competition (that is to firm *A*), the high-quality type raises the cost of mimicking for the low-quality type. As an example, consider a configuration where the high-quality type locates at 1/2 while the low-quality type locates at 1 (the choice of 1/2 here is arbitrary). It can be shown that these locations can be supported as a separating equilibrium (even for values of δ that are below the threshold given above). To show this result, we calculate first the high-quality type's profit if located at 1/2. This is equal to 0.967 per period (notice that this profit is, by construction, lower than 0.983, the profit if located at the full-information optimal point $\bar{B} \approx 0.877$). Similarly, if the low type is located at 1/2, its profit can be calculated to be 0.34.

²⁶This is calculated as the full information equilibrium profit of a firm located at $\bar{B} \approx 0.877$ when it is competing with a firm located at 0 and both have quality equal to 0.

Then, proceeding as before, the low type has a profitable deviation if and only if

$$\frac{0.5}{1-\delta} < 0.967 + \frac{\delta}{1-\delta} 0.34 \Leftrightarrow \delta < 0.745.$$

Thus, we see that, by locating closer to competition, the high-quality type discourages the low-quality type from mimicking its strategy. We also need to guarantee that the high-quality type does not wish to deviate to some other location. For example, it should be clear that when $\delta = 1$ the high-quality type would have no incentive to distort its location choice since (under our assumptions) quality is revealed after the first period. Thus, it is needed that conditions are placed on δ so that it is not too close to 1. Suppose that, according to posterior beliefs, any location other than $1/2$ is interpreted as low quality (note that this assumption is of consequence only for the first period as subsequently the quality becomes known). What is the most profitable deviation? At the one extreme, as already discussed above, when $\delta = 1$ the high-quality type would choose to deviate to \bar{B} . This choice would allow the firm to have the highest possible profit in each of the periods after the quality is revealed. At the other extreme, when $\delta = 0$, the most profitable deviation is to 1, given that only the first period matters and that both firms are viewed as offering low quality. For $\delta \in (0, 1)$ the most profitable deviation is to a point in $[\bar{B}, 1]$ (and the higher δ is, the more likely that the most profitable deviation is at or near point \bar{B}). While we cannot have a simple expression for the most profitable deviation, we can find an upper bound to the profit in any deviation and, since here we are simply analyzing a suggestive example, this would be sufficient.²⁷ Clearly, an upper bound for profit if the high-quality firm deviates from the equilibrium is $0.5 + \frac{\delta}{1-\delta} 0.983$. The calculation here is that 0.5 is attainable in the first period, if located at 1, while 0.983 is attainable, after quality becomes known, if located at $\bar{B} \approx 0.877$.²⁸ It is easy

²⁷See the analysis below for details.

²⁸We are calculating an upper bound here, this is why we consider the most favorable location in the first period and the most favorable location in the subsequent periods. The firm can only choose a single location.

to calculate that this upper bound is less than the equilibrium profit, $0.967/(1 - \delta)$, if and only if $\delta < 0.968$.

We can summarize the results from this example as follows. Suppose that $0.745 < \delta < 0.968$, $t = 1$, and the first entrant's quality is $q \in \{0, 1.2\}$. Then there is a separating equilibrium where the low-quality type locates at 1 and the high-quality type locates at $1/2$.

3.2. Generalizing the analysis

In the above analysis, the choice of $1/2$ as the location for the high-quality type was arbitrary (and extreme). While it may be necessary for a high-quality type to locate closer to competition (than under full information) to separate itself from the low-quality type, there may be other locations (between $1/2$ and the full information location \bar{B}) that allow separation. In general, we can derive conditions so that the high-quality type chooses a location $x \in [1/2, 1]$ in a separating equilibrium. Further, we look for the value of x that satisfies these conditions while minimizing the signaling distortion or, equivalently, minimizing the distance from the full information location. As shown below, given our construction, the value of x that allows us to obtain separation while minimizing the distortion (equivalently, minimizing the distance from the full-information location) is the unique separating equilibrium that satisfies the “intuitive criterion” of Cho and Kreps [6].

We can now generalize the above analysis. Let us denote by $\pi(b, \ell; \bar{q})$ the per-period profit of the first entrant (firm B) that is believed to have high quality (\bar{q}) with probability b and is located at point ℓ (and under the assumption that the second mover will locate at 0). Note that this profit is indexed by the high-quality type's quality, \bar{q} . Also note that $\pi(0, x; \bar{q})$ is the same for all \bar{q} and, so, when $b = 0$ we simply write $\pi(b, x)$ instead of $\pi(b, x; \bar{q})$. Suppose that we have a separating equilibrium. First, it is easy to see that, in any separating equilibrium, the low-quality type chooses to locate at the extreme point 1. So, we can write $B(L) = 1$ (to indicate that the

low-quality type of the first entrant, firm B , is at 1). Similarly, suppose that, in equilibrium, the high-quality type selects location x : $B(H) = x$. Then, the high-quality type has equilibrium profit $\pi(1, x; \bar{q})/(1 - \delta)$ and the low type has profit $\pi(0, 1)/(1 - \delta)$. The condition for the low-quality type not to mimic the high-quality type is

$$\frac{\pi(0, 1)}{1 - \delta} \geq \pi(1, x; \bar{q}) + \frac{\delta}{1 - \delta} \pi(0, x) \Leftrightarrow \frac{\delta}{1 - \delta} \geq \frac{\pi(1, x; \bar{q}) - \pi(0, 1)}{\pi(0, 1) - \pi(0, x)} \quad (3.17)$$

Furthermore, based on our analysis under full-information, we can calculate that (assuming $t = 1$) we have $\pi(0, 1) = 0.5$, $\pi(0, x) = \frac{(4x - x^2)^2}{18x}$, and $\pi(1, x; \bar{q}) = \frac{(4x - x^2 + \bar{q})^2}{18x}$.²⁹ Substitution of the relevant expressions into (3.17) gives

$$\phi(x, \bar{q}) \equiv \frac{(4x - x^2 + \bar{q})^2 - 9x}{9x - (4x - x^2)^2} \leq \frac{\delta}{1 - \delta}. \quad (3.18)$$

The meaning of this expression is that, for a given value of δ (and \bar{q}), a value of x that satisfies (3.18) represents a location for the high-quality type that the low-quality type does not find profitable to mimic.

To minimize the distortion for the high-quality type, we look for the largest value of x that satisfies (3.18) with equality. Thus, we need to study (3.18) more closely, to establish that such a value is well defined. The analysis is in the Appendix and the results are as follows.³⁰

Remark 1. (At least, in the range $\bar{q} \in [1, 1.267]$ that we are focusing on.) For any \bar{q} , there is a unique value $\delta_o(\bar{q})$ (defined by $\min_x \phi(x, \bar{q}) = \delta_o(\bar{q})/(1 - \delta_o(\bar{q}))$) such that: for $\delta < \delta_o(\bar{q})$ there is no value of $x \in [0, 1]$ that satisfies (3.18); for $\delta > \delta_o(\bar{q})$ the

²⁹To obtain these expressions, substitute $B = x$, $A = 0$, and $t = 1$ into (2.4), (2.5) and use (2.2) and (2.5) to calculate profits. The last two of the expressions here are under the assumption that x is greater than the threshold \hat{B} , otherwise the first mover captures the entire demand and we have $\pi(1, x; \bar{q}) = \bar{q} - x^2$.

³⁰See also figure 7.

maximum value of x that satisfies (3.18) is uniquely defined. This critical value of x is between $1/2$ and \bar{B} . As δ increases, the critical value of x increases as well.

Intuitively, when the discount factor is low, the low-quality firm always finds it in its best interest to mimic the high-quality firm's location and thus separation is not possible. As the discount factor increases, the low-quality type starts putting more weight on the future periods (when it will be competing in the market after it has been revealed that its quality is low) and the temptation to mimic the high-quality firm's location becomes weaker, allowing, in turn, the high-quality firm to achieve separation with a smaller distortion relative to the full-information location \bar{B} .

While any x that satisfies (3.18) means that the low-quality firm would not find it in its interest to mimic, we are primarily interested in equilibria where the high-quality type chooses the location, $B(H)$, that satisfies (3.18) with equality. As mentioned above, this is the unique separating equilibrium that satisfies the “intuitive criterion” of Cho and Kreps [6]. The idea is as follows. Consider an equilibrium where the high-quality type locates at x , with $x < B(H) < \bar{B}$. Consider now a deviation to location x' , with $x < x' < B(H)$. What should be the beliefs after observing such a deviation? According to the “intuitive criterion,” the beliefs should assign zero probability to the low-quality type. The reason is that the low-quality type would not choose that location even under the most favorable beliefs, that is, even if it is interpreted to be high-quality with probability one (in the first period). We know that this is true because, by construction, any location at the left of $B(H)$ means that the low quality firm would not choose to locate there even if it were interpreted as high quality. Given now that the low-quality firm would never choose x' , beliefs should assign probability one to the high type after observing a location x' . But then, if it is to be interpreted as a high-quality type, the high-quality type would prefer to be located at x' rather than at x , that is, the location at x' would give higher profit to the high-quality type than the profit under the proposed equilibrium. We conclude that an equilibrium where the high-quality type locates at x , with $x < B(H)$ fails

the “intuitive criterion,” and that the only equilibrium that survives the “intuitive criterion” assigns location $B(H)$ to the high quality-type.³¹

Note that (3.18) guarantees that the low-quality type does not wish to mimic the high-quality type. Similarly, we can write down conditions for the high-quality type not to have a profitable deviation. As in the example discussed above, if $\delta = 1$ and $B(H) < \bar{B}$, it is immediate that the full-information optimal location \bar{B} represents a profitable deviation for the high-quality type: then there is no reason to distort the first period location given that quality is revealed after the first period. On the other hand, if beliefs at any location other than $B(H)$ are that quality is low, if $\delta = 0$ the deviation that gives the highest profit is to point 1. More generally, for the high-quality firm not to have an incentive to deviate from the equilibrium location $B(H)$, we require that

$$\begin{aligned} \max_{x \in [\bar{B}, 1]} \left\{ \pi(0, x) + \frac{\delta}{1 - \delta} \pi(1, x; \bar{q}) \right\} &\leq \frac{\pi(1, B(H), \bar{q})}{1 - \delta}, \quad \text{or} \quad (3.19) \\ \max_{x \in [\bar{B}, 1]} \left\{ \frac{(4x - x^2)^2}{18x} + \frac{\delta}{1 - \delta} \frac{(4x - x^2 + \bar{q})^2}{18x} \right\} &\leq \frac{1}{1 - \delta} \frac{(4B(H) - (B(H))^2 + \bar{q})^2}{18B(H)}. \end{aligned}$$

From the above discussion it should be clear how a separating equilibrium can be constructing. We summarize the discussion as follows.

Proposition 5. *Location may be a signal of quality. Consider the incomplete information game defined above. When the discount factor, δ , is close to zero, a separating equilibrium does not exist (since the type that has lower equilibrium profit*

³¹This is also the equilibrium that satisfies the “undefeated” refinement of Mailath, Okuno-Fujiwara, and Postlewaite [21] that places restrictions on beliefs across equilibria. The equilibrium where the high-quality type locates at $B(H)$ “defeats” the equilibria where the high-quality type locates at x , $x < B(H)$, that is, more distanced from the full information location, \bar{B} . Roughly speaking, given that there is also an equilibrium where the high-quality type locates at x' with $x < x' < B(H)$, when considering the original equilibrium, a deviation to x' should be interpreted as “a proposal” for the second equilibrium. In particular, off-the equilibrium beliefs should be consistent with beliefs at the other equilibrium. But then, clearly a profitable deviation exists (to x') that upsets the equilibrium with location at x .

would mimic the other type's location). When δ is close to 1, there is separation with each type selecting their full-information locations (and, so, separation involves no distortion of location choices). When δ is not too close to 1 or too close to zero, there is a unique separating equilibrium that satisfies the “intuitive criterion”. In this equilibrium, the high-quality type may choose to locate closer to competition than under full-information.

It is helpful to present here some numerical examples. Using (3.18) we can calculate that the critical location (the one satisfying (3.18) with equality) is $x = 0.875$ if $\delta = 0.95$ and $\bar{q} = 1.2$, $x = 0.771$ if $\delta = 0.9$ and $\bar{q} = 1.2$, $x = 0.592$ if $\delta = 0.8$ and $\bar{q} = 1.2$, and $x = 0.885$ if $\delta = 0.95$ and $\bar{q} = 1.1$. As discussed above, we can then construct separating equilibria (that satisfy the “intuitive criterion”).³² For example, when $\delta = 0.9$ and $\bar{q} = 1.2$ we have $B(H) = 0.771$ and $B(L) = 1$. Note that, under full information, a first mover with quality $\bar{q} = 1.2$ would choose to locate at 0.877 and thus separation here implies choosing to locate *closer* to competition (than under full information) in order to signal high quality. Note that, for a given \bar{q} , a higher value of δ leads to a higher x (and closer to the full information point \bar{B}). A higher δ makes mimicking more costly for the low-quality type and, so, the high-quality type does not need to distort its location by much for the low-quality type not to be able to mimic.

³²It can be also checked that, for these values, the high-quality type has no profitable deviation. For example, consider $\delta = 0.9$, $\bar{q} = 1.2$ and an equilibrium with $B(H) = 0.771$ and $B(L) = 1$. It can be calculated that, in equilibrium, the high-quality type has per-period profit 0.98089, with a total profit of $0.98089/(1 - 0.90) = 9.8089$. A deviation to 1 would give profit $0.5 + \frac{0.90}{1-0.90}(.98) = 9.32 < 9.8089$, while a deviation to $\bar{B} = 0.877$ (and assuming that it is interpreted as a low quality in the first period) gives overall profit $0.47519 + \frac{0.90}{1-0.90}(.98281) = 9.3205 < 9.8089$. Similarly, it can be shown that any other deviation gives lower profit than 0.98089, and thus this configuration constitutes an equilibrium.

3.3. Further observations

Our analysis above has considered only some of the aspects of location in an environment with incomplete information about quality. Our goal was to demonstrate how location may serve as a signal of quality, and we chose to do this within a framework that we have tried to keep as simple as possible. Of course, many additional aspects of the problem remain open, including alternative timing assumptions in the game, with some of these discussed later in the paper.

A number of observations are in order regarding the above analysis. First, we have examined how location may signal quality. It is important to note that, in principle, beliefs can be influenced, of course, not only by a firm's location but also by its price (or other choice variables). So, in more general settings, we could have signaling through location, signaling through price, or both. Our formulation requires beliefs to be formed after the entrant's location is observed, and then prices to be determined in the subgame given these beliefs.³³ In particular, in the model considered it is not possible for firms to separate with respect to prices when they are pooling with respect to locations. This is because there is no mechanism in the paper that prevents a low quality firm from mimicking the high quality firm's strategy. Mechanisms that allow prices to function as signals of quality, for example the presence of some informed consumers, cost differences, and repeat purchases, have been explored in the literature in a variety of models.³⁴ To focus on the main point, we have kept the model here as simple as possible. Our formulation appears a reasonable starting point which allows us to focus on the information role of locations. However, extensions of this work, employing more general formulations, might allow us to also consider issues related to multi-dimensional signaling.³⁵ In our case, the situation is complicated by the

³³This is consistent with the notion that location (or product characteristics) are more difficult to change than prices.

³⁴See, for example, Milgrom and Roberts [22] and Bagwell and Riordan [4].

³⁵See, for example, Milgrom and Roberts [22] on signaling through a combination of advertising

fact that, after the entrant chooses its location, prices are determined in equilibrium by both the informed and the uninformed firm, instead of all uninformed agents (consumers and a firm) moving only after they observe the location and price of the firm with private information. Note also that a key difference between the two strategic variables in the model is that location cannot be changed from period to period while the price can be changed.

An additional point concerning pooling is the following. Consider again the game where the first entrant has quality $q \in \{0, \bar{q}\}$ but now suppose that \bar{q} is slightly below 1 so that both types of the first mover would have chosen to locate at point 1 under full information. But this does not necessarily mean that both firms with location at point 1 represents a “plausible” pooling equilibrium of the incomplete information game. In particular, the high-quality type may have an incentive (if the discount factor is not too close to 1) to signal its quality by locating closer to competition. We briefly discuss this point here. Suppose that we have a pooling equilibrium where both types locate at 1. This may indeed be supported as a pooling equilibrium, with posterior beliefs following the prior distribution at each location. But these “off-the-equilibrium” beliefs may be problematic and the equilibrium may fail the “intuitive criterion.” The idea is again that a deviation from 1 to some other location may have to be interpreted as coming from a high-quality type (because the low quality type would never choose to deviate to such a location), and then a high-quality firm may indeed have an incentive to deviate to that location.³⁶

and prices, and Wilson [30] for multi-dimensional signaling results.

³⁶More formally, equilibrium payoffs would be $\pi(\rho, 1; \bar{q}) + \frac{\delta}{1-\delta}\pi(1, 1; \bar{q})$ for the high-quality type, and $\pi(\rho, 1; \bar{q}) + \frac{\delta}{1-\delta}\pi(0, 1)$ for the low-quality type. Consider a deviation to a location $x' < 1$. For some parameter values, we have that the low-quality type does not want to move to x' , even if interpreted as high-quality (in the first period): $\pi(1, x'; \bar{q}) + \frac{\delta}{1-\delta}\pi(0, x') \leq \pi(\rho, 1; \bar{q}) + \frac{\delta}{1-\delta}\pi(0, 1)$. On the other hand, the high quality type may want to move to x' if beliefs put probability one to high quality: $\frac{\pi(1, x'; \bar{q})}{1-\delta} \geq \pi(\rho, 1; \bar{q}) + \frac{\delta}{1-\delta}\pi(1, 1; \bar{q})$. In such a case, the “intuitive criterion” fails.

4. Extensions and alternative formulations

Thus far the paper has presented the main intuition and results, in a framework that is as simple as possible and makes the analysis easy to compare to existing research in the area. There are a number of alternative formulations, however, and possible extensions of the analysis that is useful to consider in order to obtain a more complete understanding of the problem.

4.1. Locations outside the unit interval

Our analysis has followed past work on location choices that, with very few exceptions, has assumed that firms cannot locate outside the (unit) interval where consumers are located. However, in some situations it is reasonable to assume that firms can locate outside the interval of consumers' locations. For completeness, we now relax this assumption. The goal is to understand how location incentives vary as a function of the quality difference between the firms in the game with enlarged strategy spaces. As shown below, whereas it still remains true that the low-quality firm has a stronger incentive than the high-quality firm to move away from the competition, allowing the firms to locate outside the unit interval generates new insights. With respect to the formal analysis, we maintain all the assumptions made in the basic model, except that firms can now choose to locate anywhere on the real line.³⁷

Proposition 6. *Suppose that the two firms, A and B , can choose any location on the real line and that they choose their locations simultaneously. In equilibrium, the low quality firm, A , has positive price (and market share) if and only if $q/t \leq 9/4$. For $q/t \leq 9/4$ the equilibrium locations are*

³⁷See Anderson, Goeree, and Ramer [1] for a characterization of the equilibrium (when firms can be located outside the unit interval) for more general consumer densities (this work, however, does not examine how firms' incentives change when firms differ with respect to quality, which is our focus).

$$A^* = -\frac{1}{4} - \frac{q}{3t}, \quad B^* = \frac{5}{4} - \frac{q}{3t} \quad (4.20)$$

and the equilibrium prices are

$$p_A^* = \frac{3}{2}t - \frac{2}{3}q, \quad p_B^* = \frac{3}{2}t + \frac{2}{3}q. \quad (4.21)$$

(Of course, we also have a symmetric equilibrium by exchanging the locations of A and B .)

Proof. Simple arguments show that the equilibrium price expressions in Proposition 1 are still valid when firms choose locations outside the $[0,1]$ interval. Then we can proceed as before to obtain the expressions for profits as functions of the firms' locations and using the envelope theorem we have

$$\frac{\partial \pi_A^*}{\partial A} = p_A^* \frac{(A - B)(3A - B + 2) - q/t}{6(A - B)^2}$$

and

$$\frac{\partial \pi_B^*}{\partial B} = -p_B^* \frac{(A - B)(4 + A - 3B) + q/t}{6(A - B)^2}.$$

Substituting the relevant price expressions and solving the system of the two first-order conditions,³⁸ we obtain the equilibrium locations (4.20) and, substituting into the price expressions in Proposition 1, we obtain the equilibrium prices. We only need to consider the possibility that the high-quality firm captures the entire market. Straightforward calculations show that this does not happen if and only if $q/t \leq 9/4$ (this condition is equivalent to $p_A^* \geq 0$). ■

It follows from the above Proposition, that when there is no quality difference ($q = 0$), the firms locate at $-1/4$ and $5/4$ (with prices equal to $3t/2$). Also, the distance of the two firms, $B - A = 3/2$ is constant for all q . As q increases, the high-quality firm, B , moves closer to $1/2$, while firm A is pushed further away to the

³⁸The system of the first-order conditions becomes $2(A - B) + 3A^2 - 4AB + B^2 = q/t$ and $4(B - A + AB) - 3B^2 - A^2 = q/t$.

left. In addition, as q increases, p_B^* increases while p_A^* decreases. When $q/t = 9/4$, the high-quality firm is located at $1/2$ (with price $3t$), while the low quality firm is located at -1 (with zero price). Figure 8 represents the equilibrium locations.³⁹

Observe that this analysis has implications that are different from when firms are constrained to locate within the unit interval. In particular, the distance between the two firms does not change (at least as long as the quality difference is not too large). More importantly, from the point of view of this paper, the high quality firm's equilibrium location is a monotonic function of quality and the high quality firm chooses locations that are closer to the midpoint of the unit interval than the low quality firm's locations. This monotonicity makes it easy to consider location as a signal to quality. One can have an analysis similar to the one presented earlier in the paper and see that, when firms have private information about their quality, firms can use their location as a signal, with high-quality firms choosing to locate closer to the midpoint of the unit interval that they would under full information.⁴⁰

³⁹As in the case when locations are restricted to be in $[0, 1]$ we could also consider parameter values beyond the point where the high-quality firm captures the entire market. In particular, it can be shown that, as q/t increases beyond $9/4$, A has an incentive to move further to the left (past the point $1/2$). The analysis is not essential for our results (and is complicated since it involves a number of corner solutions), and is thus omitted.

We can also derive the location choices under sequential moves. Again consider $q/t \leq 9/4$ and suppose that firm B moves first. It is straightforward to show that the optimal location for the follower, A , is $\frac{1}{3}[(2B - 1) - \sqrt{(B + 1)^2 + (3q/t)}]$ and that, substituting this choice into firm B 's profit, we obtain $\partial\pi_B/\partial B = p_B[(1/\sqrt{(1 + B)^2 + (3q/t)}) - (2/3)]$ which is negative in the relevant range ($B \geq 1/2$). We conclude that, for $q/t \leq 9/4$, when firm B moves first it chooses to locate at $1/2$ while A locates at $-\sqrt{(1/4) + (q/3t)}$. The follower's location, therefore, ranges from $-1/2$ (for $q/t = 0$) to -1 (for $q/t = 9/4$).

⁴⁰Notice that, depending on whether choices are sequential or simultaneous, another effect may be important here, that a firm may want to induce its rival to locate as far as possible from "its market". The simplest situation in which one can examine the signaling role of prices is when one of the firms, say A , has already an established location in the market and firm B , which has private information about its quality, chooses its location (again with quality becoming known after the first

4.2. Consumer heterogeneity with respect to quality

In the model, we have assumed that consumers are only differentiated with respect to one of the product dimensions and we have assumed that quality enters all consumers' utility function in the same way (see expression (1)). In general, while all consumers agree on the ranking of different qualities, they may differ with respect to their valuation for quality. More precisely and following Mussa and Rosen [23], in (1) we could pre-multiply q_i by a parameter θ , with θ distributed across consumers.⁴¹ Such a formulation would change the analysis in the following way. With enough differentiation in consumers' tastes for quality, a low-quality firm could make positive profit even if it had the same location as its high-quality rival. The idea, which is central in much of the vertical differentiation literature, is that some consumers buy high quality (at high price) and others low quality (at low price). Thus, for any given firm, the need to differentiate in the horizontal dimension is now not as imperative and the strategic incentives explored in the paper become relatively weaker.⁴² However, even in this "enriched" model, it would be true that a low-quality firm has a *stronger* incentive than a high-quality firm to differentiate itself from a rival. Thus, at least when consumer heterogeneity with respect to the quality characteristic is not too great, the basic intuition and results of the paper would remain valid.

period). Then, by locating closer to firm A than under full information, firm B may signal high quality.

⁴¹Consumers are then distributed on a two-dimensional space, while in the model they are distributed on a line.

⁴²In Neven and Thisse [24], for some parameter values, the "direct" effect dominates the "price-competition" effect and firms with different qualities choose to locate at the midpoint of the unit interval. See also footnote 22 and the discussion in Section 2.6 above.

4.3. Low values relative to the outside option

In the analysis we have assumed that all consumers prefer to purchase one unit of the good to not purchasing at all. In other words, we have assumed that R in expression (2.1) is large enough that, in equilibrium, consumers always choose to purchase from A or B . While such an assumption is standard in the literature and appears reasonable for our analysis, it is interesting to discuss the implications of relaxing it.⁴³ The point here is that when firms cannot take as given that all consumers will purchase from one firm or the other, there is an additional cost associated with moving to an extreme location, because such a strategy implies that some consumers have to pay a high transportation cost. In particular, the above analysis has shown that, when the quality difference is large, the high-quality firm prefers to minimize the distance from its low-quality rival even if the rival is located at an endpoint. Contributing to this result is that consumers only compare high-quality to low-quality. When R is low, however, a high-quality firm competes not only with its low-quality rival but also with the outside option and, therefore, may choose not to move to an endpoint.⁴⁴

Let us briefly revisit the location and pricing problem of the firms, allowing now R to be small. For example, in Proposition 1 we saw that for R high enough, if $A = B = 0$, the equilibrium prices are $p_A^* = 0$ and $p_B^* = q$. Since consumers also pay the transportation cost, if R were low relative to the outside option, only some of the consumers would receive a positive surplus. For all consumers to purchase we need $R > t \cdot 1^2 = t$, in other words, the value of the good has to be higher than the transportation cost even for the most remotely located consumer. If $R < t$ then, for $p_A^* = 0$ and $p_B^* = q$, the high-quality firm demand is less than 1, and it is not guaranteed that these locations and prices constitute an equilibrium. In fact, we can show that, for some parameter values, this is not an equilibrium. To see this, first

⁴³Economides [10] relaxes this assumption and shows that the non-existence problem in the Hotelling model with linear transportation cost can be solved.

⁴⁴Note that the optimal location for a monopolist is the midpoint of the market interval.

observe that when we explicitly recognize that consumers have an outside option, a consumer located at x may choose to purchase either from A or from B or not purchase at all, according to $\max\{R - p_A - t(x - A)^2, R + q - p_B - t(x - B)^2, 0\}$. Let now $R < \min\{t, q/2\}$. Then we can show that when $A = B = 0$ and $p_A = 0$, the optimal price for B is $p_B^* < q$, with $p_B^* = \frac{2}{3}(R + q)$ if $R + q \leq 3t$ and $p_B^* = R + q - t$ if $R + q > 3t$, while B 's profit is $\pi_B^* = \frac{2}{3}(R + q)$ if $R + q \leq 3t$ and $\pi_B^* = R + q - t\sqrt{\frac{R+q}{3t}}$ if $R + q > 3t$. Then, it is easy to see that when $A = 0$, 0 is not an optimal location for B .⁴⁵

We thus see that when consumers have an outside option, for some parameter values there is an incentive for the firms to move closer to the market center than our previous analysis suggests. A complete analysis of the equilibria when R is arbitrarily low requires considering a number of simultaneous corner constraints and is outside the scope of this paper. Examples indicate that there are equilibria where, for R low, the high quality-firm chooses an interior location (with a low-quality firm at an endpoint). The point of the discussion here is that bringing an outside option into the picture may affect the dynamics of competition between the two firms.⁴⁶ Regarding the incomplete information analysis, by making a corner solution less desirable and bringing the high-quality firm to an interior location, an outside option can be helpful in considering how location can be used as a signal.

⁴⁵The proof is as follows. When B charges $p < q$, no consumer prefers to purchase from A rather than B . A consumer indifferent between purchasing one unit of the good or not is located at $d = \sqrt{(R + q - p)/t}$. Total demand for B is $\min\{d, 1\}$. B 's profit is $p\sqrt{(R + q - p)/t}$ if $p > R + q - t$ and p if $p \leq R + q - t$ and optimization gives the result. $R < \min\{t, q/2\}$ guarantees that the consumers prefer to purchase from A rather than from B . If this condition fails, the optimal price for B is q . Under these conditions and proposed prices, all consumers strictly prefer to purchase from B than from A . Even if B decreased its price a little, 0 would still be an optimal price for A . On the other hand, by moving from $B = 0$ slightly to its right, B relaxes the outside option constraint and can strictly increase its demand without decreasing its price, thus increasing its profit.

⁴⁶It appears likely that similar considerations arise in a model where three, rather than two firms, compete in the market.

4.4. Other signaling possibilities

There are several additional dimensions and extensions of the problem under incomplete information. We briefly discuss some of them here. An issue that has been left open in the previous analysis is two-way signaling. In the model, only one of the firms has private information about its quality, whereas there is symmetric information concerning the quality of the second firm. It seems interesting to study firms' competition and the information structure when two firms, each with private information about their own quality, can choose their locations.⁴⁷ Again, there is a number of possible formulations (location decisions, for example, could be simultaneous or sequential).

A related interesting possibility is that, when a firm chooses to locate near another firm, consumers take that location to signal not only the entrant's impression of its own quality but also the entrant's impression of the incumbent's quality.⁴⁸

More generally, the idea that location (or variety) can be a signal of quality can be studied in frameworks that differ from the one presented here. For example, suppose that demand is not uniformly distributed with respect to different varieties. Then the decision of a firm to choose a location or a variety that is not very popular for "horizontal" reasons may also be viewed as a signal of high quality.

5. Conclusion

This paper has examined aspects of oligopolistic competition in markets where products are differentiated both horizontally and vertically. Consumers may be attracted to a product because its overall quality is high, or because it is of a special variety that satisfies their individual tastes. This formulation allows us to highlight aspects

⁴⁷See Mailath [20] on simultaneous signaling by firms with private information about their own costs.

⁴⁸This possibility raises a multi-sender signaling issue as, for example, in Bagwell and Ramey [3].

of the role that quality plays in markets that are different from that in models of pure vertical differentiation. From the point of view of a low-quality firm, in this paper it is “bad news” if the quality of the high-quality firm increases. In contrast, in a pure vertical differentiation model this would typically be “good news”.

The main idea in the paper is that, if firms have private information about product quality, their choice of variety (or location) can be used as a signal of quality. In the model, high-quality firms find it less costly than low-quality firms to locate closer to rival firms. Thus, locating closer to other firms (moving towards spatial agglomeration) or, equivalently, not differentiating the product much in the horizontal dimension, may be a signal of high quality. For example, the location of a restaurant in a city’s “restaurant district” may be a signal of high quality, in the sense that its quality is high enough to survive intense competition. A researcher who chooses to work on a topic that many other researchers are studying may be sending a signal about her overall research ability. More generally, this logic may help explain why, in a large number of markets, many products look very much alike in certain dimensions.

Generally speaking, this paper suggests that a product characteristic may be viewed as a signal of some other characteristic of the product that consumers care about. Moreover, firms may choose to “distort” a given characteristic of a product (choosing a different product design than under full information), in order to signal some other characteristic of the product (such as “high quality”). It is not being argued here, of course, that the logic in this paper is the only possible explanation, in all markets, for why firms may locate close to other firms in product or geographical space. Other reasons, such as search, have been presented in the literature and a different combination of reasons may be more relevant for each real-life case. However, the logic presented here is an intriguing one, as it involves potential distortions in product characteristics and appears plausible to the extent that agglomeration or other observed regularities in horizontal space appear correlated with vertical differentiation characteristics.

In addition to the extensions discussed in the previous section, there is a number of other potentially interesting ideas for future work. For example, in some markets, firms may be able to price discriminate among consumers. While past work has considered location models with price discrimination, it appears interesting to revisit the issue when there is also quality differentiation. It also seems interesting to examine the relation between the behavior explored in this paper and the idea, presented in the literature, that agglomeration may be due to search externalities. We can revisit the idea that imperfectly informed consumers prefer to visit locations with multiple firms, when there is also quality differentiation. The point here is that high-quality firms may have a higher incentive to move to a “cluster”, whereas low-quality firms prefer to move to a different location because, within a cluster, the comparison with other firms is not favorable. This comparison, in turn, implies that average quality may be higher in a cluster than elsewhere, which may attract more consumers there, thus further strengthening firms’ incentives to move to a cluster (as well as price competition within a cluster). This mechanism is related to but different from the idea in the present paper, where the analysis is driven by signaling, not search. In both cases, however, a central consideration is that high-quality firms find it relatively less costly than low-quality firms to locate closer to competition.

Appendix

Study of inequality (3.18)

The left-hand side of (3.18) can be rewritten as

$$\phi(x, \bar{q}) = \bar{q}f(x, \bar{q}) - 1,$$

where

$$f(x, \bar{q}) \equiv \frac{\bar{q} + 2(4x - x^2)}{9x - (4x - x^2)^2}.$$

It is straightforward to check that the following statements are true for $\phi(x, \bar{q})$. It is continuous in $x \in (0, 1)$ and has asymptotes at $x = 0$ and $x = 1$. In particular, for any $\bar{q} > 0$ we have

$$\lim_{x \rightarrow 0^+} \phi(x, \bar{q}) = \lim_{x \rightarrow 1^-} \phi(x, \bar{q}) = +\infty.$$

Further, ϕ is U-shaped in $(0, 1)$. Its unique minimum in this interval occurs at point x that solves $\partial\phi(x, \bar{q})/\partial x = 0$ or equivalently (for $\bar{q} \neq 0$) $\partial f(x, \bar{q})/\partial x = 0$. This last condition can be written as

$$\frac{\partial f(x, \bar{q})}{\partial x} = \frac{(4x^3 - 24x^2 + 32x - 9)\bar{q} - 2x^2(2x^3 - 20x^2 + 64x - 55)}{x^2(1 - x)^2(x^2 - 7x + 9)^2} = 0.$$

It can be checked that, for \bar{q} positive, this equation has a unique solution. This solution satisfies

$$\bar{q} = \frac{2x^2(2x^3 - 20x^2 + 64x - 55)}{4x^3 - 24x^2 + 32x - 9}$$

and $x < 0.3856$ (0.3856 is a root of the expression that pre-multiplies \bar{q} in the numerator of $\partial f(x, \bar{q})/\partial x$).

While it is messy to express x as a function of \bar{q} , it is easy to show that in the relevant range $x \in (0, 1)$ and $\bar{q} > 0$ it represents an one-to-one relation between the two variables. In other words, for a given \bar{q} we can find the value of x that minimizes

$\phi(x, \bar{q})$. Let us call this value $\hat{x}(\bar{q})$. Substituting this value, we obtain the minimum value of the left-hand side of (3.18), $\phi(\hat{x}(\bar{q}), \bar{q})$.

With respect to the main arguments in our analysis, there are two important implications here. First, ϕ is continuous and strictly increasing in the interval $(\hat{x}(\bar{q}), 1)$ and tends to $+\infty$ as x approaches 1. Thus as long as δ is not too low, we can determine uniquely the maximum value of x that satisfies (3.18) with equality. In particular, in the relevant range $\bar{q} \in (1, 1.267)$ given in Proposition 2, this value of x will be always in between $1/2$ and \bar{B} . Second, once we obtain the minimum value $\phi(\hat{x}(\bar{q}), \bar{q})$ we can solve $\phi(\hat{x}(\bar{q}), \bar{q}) = \delta/(1 - \delta)$ to obtain value $\delta_o(\bar{q})$, the minimum value of δ required for a separating equilibrium. For $\delta < \delta_o(\bar{q})$ the low-quality firm always wants to mimic the high-quality firm strategy. For $\delta > \delta_o(\bar{q})$ if the high-quality firm chooses a location x , with $x > \frac{1}{2} > \hat{x}(\bar{q})$, the low-quality firm would prefer to locate at point 1 rather than mimicking the high-quality strategy.

References

- [1] Simon P. Anderson, Jacob K. Goeree, and Roald Ramer. Location, location, location. *Journal of Economic Theory*, 77:102–127, 1997.
- [2] Simon P. Anderson and Damien J. Neven. Cournot competition yields spatial agglomeration. *International Economic Review*, 32:793–808, 1991.
- [3] Kyle Bagwell and Garey Ramey. Oligopoly limit pricing. *Rand Journal of Economics*, 22:155–172, 1992.
- [4] Kyle Bagwell and Michael H. Riordan. High and declining prices signal product quality. *American Economic Review*, 81:224–239, 1991.
- [5] Helmut Bester. Quality uncertainty mitigates product differentiation. *Rand Journal of Economics*, 29:828–844, 1998.
- [6] In-Koo Cho and David M. Kreps. Signaling games and stable equilibria. *Quarterly Journal of Economics*, 102:179–221, 1987.
- [7] Claude d’Aspremont, Jean Jaskold Gabszewicz, and Jacques-François Thisse. On Hotelling’s ‘stability in competition’. *Econometrica*, 47:1145–1150, 1979.
- [8] André de Palma, Victor Ginsburgh, Yorgos Y. Papageorgiou, and Jacques-François Thisse. The principle of minimum differentiation holds under sufficient heterogeneity. *Econometrica*, 53:767–781, 1985.
- [9] Marc Dudev. Competition by choice: The effect of consumer search on firm location decisions. *American Economic Review*, 80:1092–1104, 1990.
- [10] Nicholas Economides. The principle of minimum differentiation revisited. *European Economic Review*, 24:345–368, 1984.
- [11] Nicholas Economides. Quality variations and maximal variety differentiation. *Regional Science and Urban Economics*, 19:21–29, 1989.

- [12] Rodolphe Dos Santos Ferreira and Jacques-François Thisse. Horizontal and vertical differentiation: The Launhardt model. *International Journal of Industrial Organization*, 14:485–506, 1996.
- [13] James W. Friedman and Jacques-François Thisse. Partial collusion fosters minimum differentiation. *Rand Journal of Economics*, 24:631–645, 1993.
- [14] Jean J. Gabszewicz and Jacques-François Thisse. Price competition, quality and income disparities. *Journal of Economic Theory*, 20:340–359, 1979.
- [15] Jean J. Gabszewicz and Jacques-François Thisse. On the nature of competition with differentiated goods. *Economic Journal*, 96:160–172, 1986.
- [16] Jean J. Gabszewicz and Jacques-François Thisse. Location. In Robert J. Aumann and Sergiu Hart, editors, *Handbook of Game Theory*, chapter 9. North-Holland, 1992.
- [17] Harold Hotelling. Stability in competition. *Economic Journal*, 39:41–57, 1929.
- [18] Andreas Irmen and Jacques-François Thisse. Competition in multi-characteristics spaces: Hotelling was almost right. *Journal of Economic Theory*, 78:76–102, 1998.
- [19] Philippe Jehiel. Product differentiation and price collusion. *International Journal of Industrial Organization*, 10:633–643, 1992.
- [20] George J. Mailath. Simultaneous signaling in an oligopoly model. *Quarterly Journal of Economics*, 103:417–427, 1989.
- [21] George J. Mailath, Masahiro Okuno-Fujiwara, and Andrew Postlewaite. Belief-based refinements in signalling games. *Journal of Economic Theory*, 60:241–276, 1993.

- [22] Paul Milgrom and John Roberts. Price and advertising signals of product quality. *Journal of Political Economy*, 94:796–821, 1986.
- [23] Michael Mussa and Sherwin Rosen. Monopoly and product quality. *Journal of Economic Theory*, 18:301–317, 1978.
- [24] Damien Neven and Jacques-François Thisse. On quality and variety competition. In J.J. Gabszewicz, J.-F. Richard, and L.A. Wolsey, editors, *Economic Decision-Making: Games, Econometrics and Optimization. Contributions in Honour of Jacques Drèze*, chapter 9, pages 175–199. North-Holland, 1990.
- [25] Damien J. Neven. On Hotelling’s competition with non-uniform customer distributions. *Economics Letters*, 21:121–126, 1986.
- [26] Edward C. Prescott and Michael Visscher. Sequential location among firms with foresight. *Bell Journal of Economics*, 8:378–393, 1977.
- [27] Avner Shaked and John Sutton. Relaxing price competition through product differentiation. *Review of Economic Studies*, 49:3–13, 1982.
- [28] Avner Shaked and John Sutton. Natural oligopolies. *Econometrica*, 51:1469–1483, 1983.
- [29] Takatoshi Tabuchi. Two-stage two-dimensional spatial competition between two firms. *Regional Science and Urban Economics*, 24:207–227, 1994.
- [30] Robert Wilson. Multi-dimensional signalling. *Economics Letters*, 19:17–21, 1985.
- [31] Asher Wolinsky. Retail trade concentration due to consumers’ imperfect information. *Bell Journal of Economics*, 14:275–282, 1983.
- [32] Steffen Ziss. Entry deterrence, cost advantage and horizontal product differentiation. *Regional Science and Urban Economics*, 23:523–543, 1993.

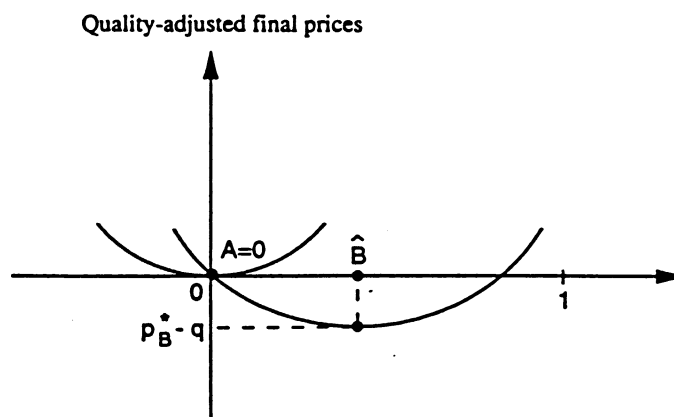


FIGURE 1. At $B = \hat{B}$: $p_B^* - q + tB^2 = p_A^* = 0$, $z = 0$.

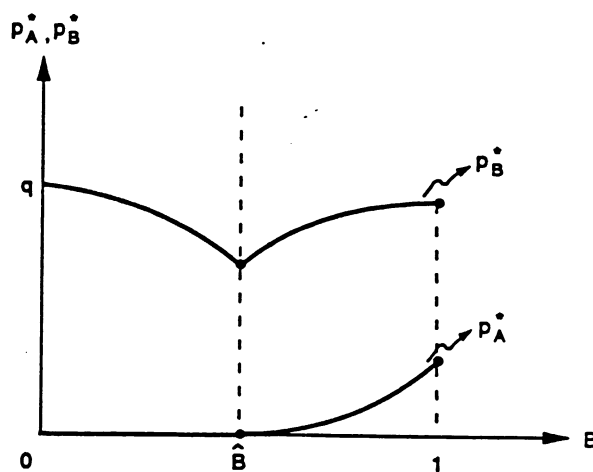


FIGURE 2. p_A^* and p_B^* as functions of B , for $A = 0$.

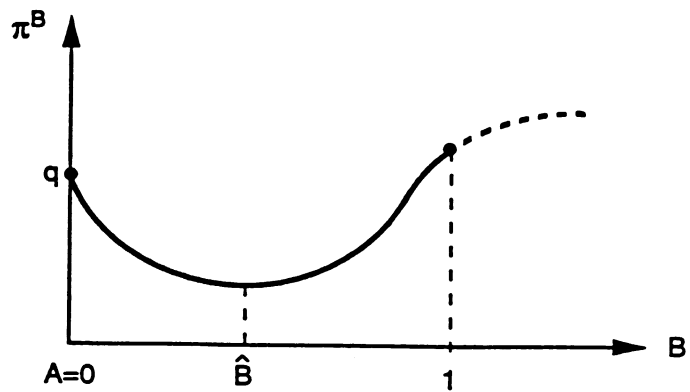
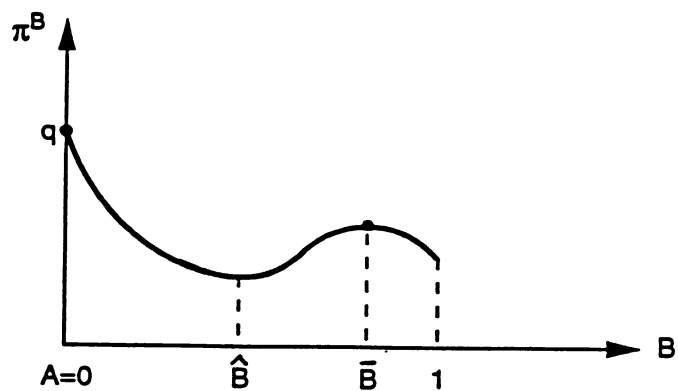


FIGURE 3. For $A = 0$ and R high enough, the optimal location for B is either 0 or $\min\{\bar{B}, 1\}$.

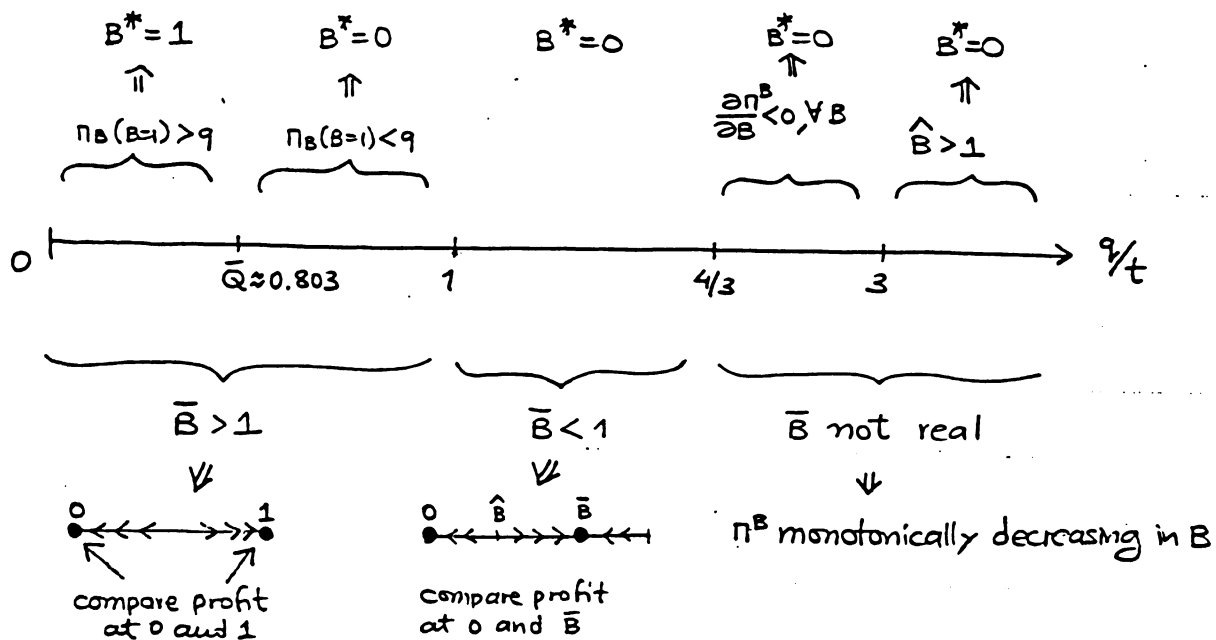
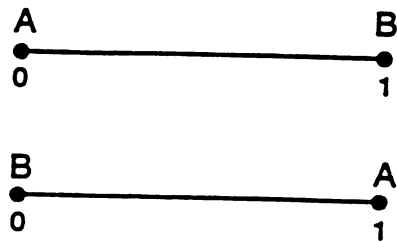
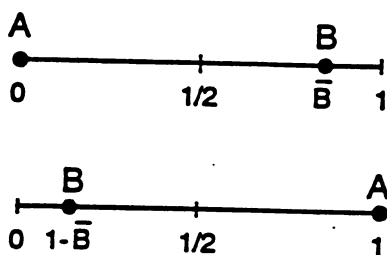


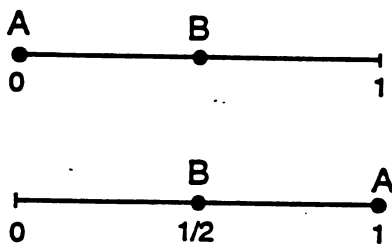
FIGURE 4. B 's location as a best response to $A = 0$.



Equilibrium locations when $q/t < \bar{Q}$



Equilibrium locations when $1 < q/t < 5/4$ and B moves first



Equilibrium locations when $q/t > 4/3$ and B moves first

FIGURE 5. Equilibrium locations with sequential entry.

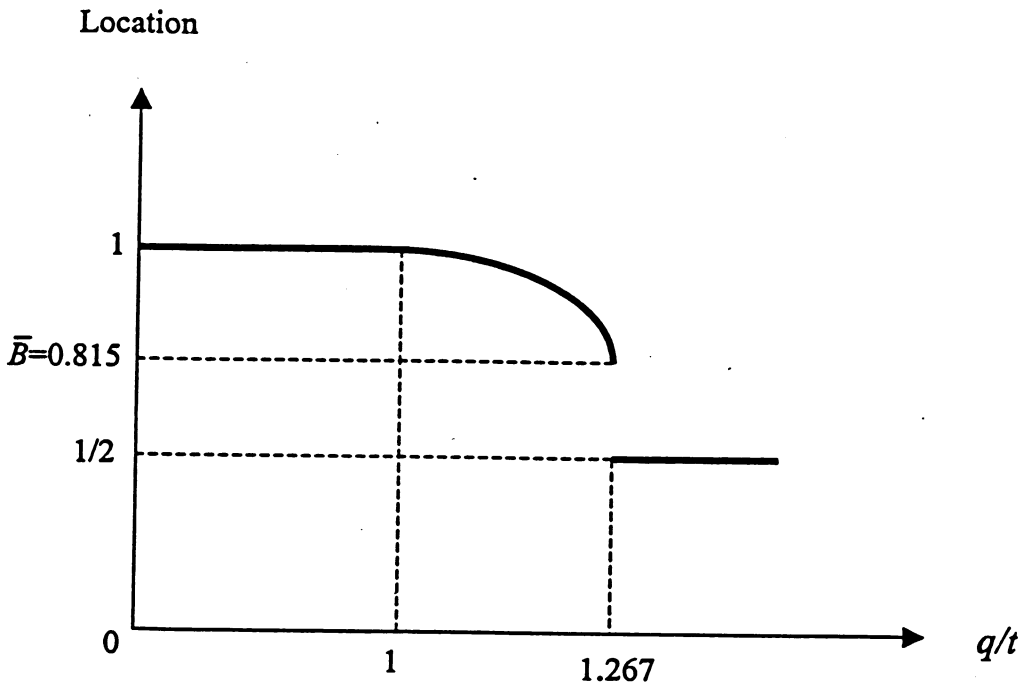


FIGURE 6. Location of first mover under sequential entry.

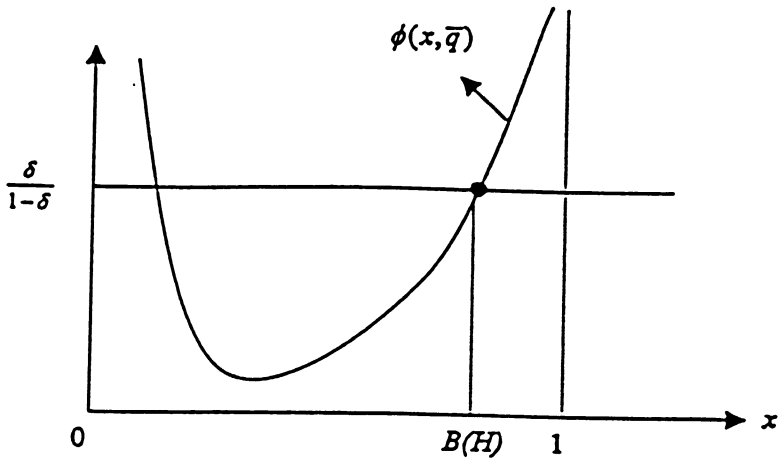


FIGURE 7. A separating equilibrium.

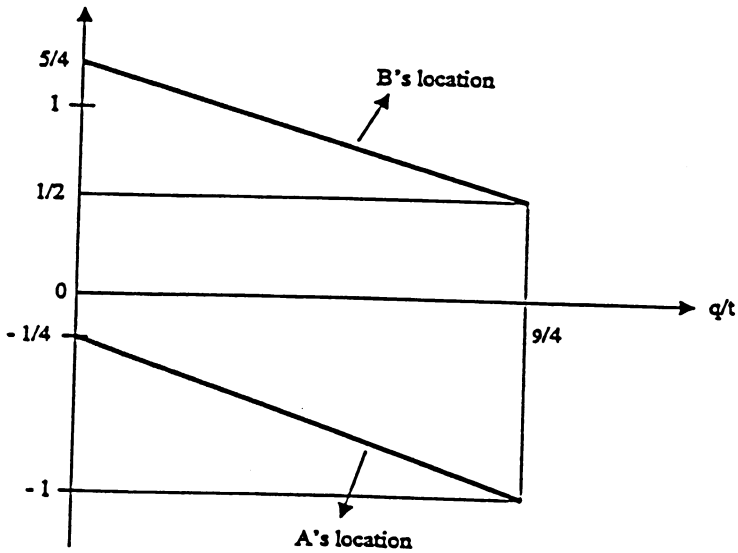


FIGURE 8. Equilibrium locations with simultaneous moves when locations outside the $[0,1]$ interval are possible.