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Second Opinions and Price Competition: Inefficiency in the Market for Expert Advice

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We consider a market in which an expert must exert costly but unobservable effort to identify the service that meets the consumer's need. In our model, experts offer competing contracts and the consumer may gather multiple opinions. We explore the incentives that a competitive sampling of prices and opinions provides for experts to exert effort and find that there is a tension between price competition and the equilibrium effort. In particular, the equilibrium fails to realize the second best welfare optimum. An intervention, that limits price competition via price control, increases welfare.

1. INTRODUCTION

This paper analyses the provision of a "credence" service. A credence service has the property that the consumer (the *principal*) must rely on *experts* to identify the correct type of service. Medical services, repair services and various types of consulting and advisory services belong to this broad category.

The provision of credence services is beset by a number of information problems. Here, we are concerned with a situation where the expert's diagnostic effort is unobservable and the final success of the service is not contractible (say, because it is not easily or objectively measurable). We focus on the role of a specific mechanism—the gathering of multiple opinions—in mitigating the information problem and disciplining the expert's behaviour.

Consider a home owner who finds his air conditioning does not work properly. He decides to call a contractor to diagnose the problem. The contractor may send a skilled worker at a high cost or an unskilled worker at a low cost. The skilled worker recommends the appropriate repair whereas the unskilled worker makes a recommendation that does not solve the problem. The home owner cannot distinguish the skilled from the unskilled worker. He may, however, ask another contractor for a second opinion. If the second opinion matches the recommendation of the first contractor then it is likely that the recommendations are indeed correct. If the two recommendations do not match then the home owner is less confident that either of the two recommendations are correct. In that case, he may consult a third contractor. By consulting multiple experts, the home owner learns which contractor has provided a correct recommendation.

There are of course other potential information problems, such as the unobservability of the expert's actions in the provision of the service. Moreover, there are other forces, such as reputation, that work to mitigate these problems. We disregard these issues and corrective forces in the interest of isolating the particular information problem and particular corrective force outlined above. We do not underestimate the importance of these missing elements. However, some of them have been discussed in the literature[1] and the manner in which reputation might work is relatively well understood from different contexts.

A process of gathering recommendations and bids as described in the home repair scenario above is typical for markets of many credence goods such as repair services, consulting or medical services. We analyse a stylized model of this process. A principal is in need of a service but is uncertain as to which of a continuum of possible services matches his need. There is one correct service which gives the principal a payoff of $V > 0$; any other service yields a payoff of zero. The set of possible services is modelled as a continuum to assure that an unguided guess will not yield the right choice with positive probability. Experts can identify the correct service if they choose a high effort and incur a cost $c > 0$. The principal can consult experts, but does not observe whether the experts incurred the cost.

Experts are sampled sequentially from a large population. A sampled expert offers the principal a contract. Upon observing this contract, the principal decides whether to consult the expert or to continue sampling. Consulting an expert is costly for the principal. This cost represents the time it takes to visit a doctor, take the car to a mechanic, or wait for a contractor. Once the principal agrees to be diagnosed, the expert decides whether to incur the cost of effort and then provides a recommendation. After learning the recommendation the principal either buys the service or continues his search.

We assume that provision of the correct service requires the correct diagnosis. Thus, the principal cannot simply learn the information and then instruct some other expert to conduct the service. This assumption is more compelling for services that require specialized knowledge, such as medical services. Section 7 briefly discusses a contracting environment in which this assumption is relaxed.

The contract between the principal and the expert stipulates two prices, a diagnosis fee and a price for the service. The diagnosis fee is paid by the principal up front. In exchange, the expert makes a recommendation. The principal then has the option to buy the recommended service at the price stipulated in the contract. We do not allow contracts to depend on the success of the treatment, that is, the principal's payoff. This assumption is plausible if the success of the treatment is difficult to verify; for example, if only the principal can observe his payoff. Of course, if contracts could depend on the principal's payoff, then the incentive problem could easily be solved.

We analyse equilibrium behaviour and welfare in our model. In particular, we give conditions under which, in equilibrium, experts choose high effort with positive probability. We then ask whether the intervention of a planner could improve welfare.

The first best outcome in our model is for the service to be performed without wasteful search and duplication of the diagnoses. Obviously, the first best outcome cannot be sustained in equilibrium. Second best outcomes are Pareto optimal among those outcomes that can be sustained when the planner controls prices but effort and search decisions are made by the experts and the principal. This is a natural benchmark since it involves feasible and observed interventions in these markets.

We show that equilibrium behaviour does not lead to second best outcomes. This is perhaps the main qualitative insight of this paper. The source of this inefficiency is an externality of experts' effort on other experts' incentives. To reward effort, the principal must verify, at least with some positive probability, whether an expert's recommendation is correct. The cost of

1. See, *e.g.* Pitchik and Schotter (1987, 1989), Wolinsky (1993) and Emons (1997) for analyses of the case where the expert's actions are unobservable.

verification depends on the effort level of other experts. If, for example, other experts rarely provide high-quality recommendations, then verification is very costly. In that case, it is not optimal for the principal to verify the recommendation and hence experts have no incentive to incur the cost of effort.

In a second best outcome each expert provides a correct diagnosis with high probability and it is relatively easy for the principal to verify the correctness of a diagnosis. As a consequence, when an expert deviates to a lower price, the principal has an incentive to accept the deviating offer even when it leads to a slightly lower probability of a correct diagnosis. After all, the principal can verify the correctness of the recommendation of the deviating expert at a low cost. But then a slightly lower price leads to higher profits because the expert is more likely to make a sale when he offers the service at a lower price than the other experts. Hence, the high probability of a correct diagnosis cannot be sustained in the presence of price competition. Experts will free ride on the information provided by other experts and ensure themselves a sale of the service by undercutting the price.

We conclude that competition may be in conflict with good expert incentives. In particular, a floor on the price of the service may improve welfare.

Related literature

The related literature on credence goods focuses on situations where the expert can provide multiple services that vary in their cost of provision. For example, Pitchik and Schotter (1987, 1989) and Wolinsky (1993) analyse a situation where the principal cannot observe the type of service provided, so the expert may defraud the principal by misrepresenting a low-cost service as a costly one. Darby and Karni (1973) and Emons (1997) analyse the situation in which the principal observes the service provided but cannot observe whether the more costly service was actually needed.[2] Here too the expert may bias the recommendation towards the more costly service if that service is more profitable. Dulleck and Kerschbaumer (2001) provide a synthesis of some of the findings of this literature.

In these models, experts obtain the correct diagnosis without special efforts (it is either not costly or a by-product of the provision of the service), and the success of the service is verifiable. The focus is on the experts' incentives to induce principals to get more costly services than they actually need. In contrast, our model abstracts away from this issue by making all types of the service equally costly so that experts have no incentive to misrepresent the correct diagnosis in case they know it. The incentive problem in our model stems from the unobservability of the expert's diagnostic effort.

Our analysis is also related to the literature on procurement (Laffont and Tirole, 1993). In a typical procurement scenario the government invites potential suppliers to participate in competitive bidding over the design and subsequent supply of custom made equipment. The objective is to create price competition and at the same time to encourage firms to invest in R&D. This is similar to our model, where the principal is interested in obtaining a low price but also wishes to induce the experts to invest in diagnostic effort. However, there are two key differences. In the situation analysed here, the principal must rely on the diagnosis of other experts to determine whether a recommendation is correct. Hence, there is an *informational externality* in the expert's choice of diagnostic effort. In the procurement literature, the government knows its needs and can evaluate the quality of each proposal directly without relying on other proposals. Furthermore, the procurement literature allows the principal to commit to a mechanism. In our model, the principal has no such commitment power. In particular, the principal cannot choose

2. See also Taylor (1995), Alger and Salanie (2001) and Emons (2001).

a limited number of experts *ex ante* and commit to contracting only with those experts. This assumption seems appropriate for situations in which the principal is a small buyer of medical, repair or other professional services.

Our conclusion that a minimum price may enhance efficiency is reminiscent of Telser's (1960) argument in favour of a minimum retail price. Telser points out that a retailer has little incentive to provide the consumer with information if other retailers provide this service. Hence retailers may free ride on one another's services. In contrast, experts only have an incentive to provide a high-quality recommendation if other experts confirm the recommendation, that is, also provide a high-quality recommendation. Hence, the source of the inefficiency in our setting is quite different from the retail setting.

2. THE MODEL

The principal is in need of a service but is uncertain as to which service meets his need. The range of possible services is [0, 1]. The principal benefits from the service a only if it matches his type $\alpha \in [0, 1]$, hence his utility is

$$\begin{cases} V & \text{if } \alpha = a \\ 0 & \text{if } \alpha \neq a \end{cases}$$

where $V > 0$. The principal does not know his own type α and has a uniform prior on [0, 1]. The set of possible types is modelled as a continuum to assure that an unguided guess will not yield the right choice with positive probability.

There is an infinite population of identical experts, indexed by $k \in [0, 1]$. Experts serve a dual role: they recommend a service to the principal and, if chosen by the principal, perform the recommended service. The expert must choose high or low diagnostic effort. High diagnostic effort always leads to the correct recommendation α. Low diagnostic effort leads to a random recommendation drawn from a uniform distribution on [0, 1]. The cost of high diagnostic effort is $c > 0$ and the cost of low diagnostic effort is zero. We assume that the cost of performing any of the potential services is zero. Notice that the model does not leave the experts discretion over their reporting: if the expert learns α, the recommendation must be correct, otherwise it must be a random draw from a uniform distribution. This is done to avoid dealing with uninteresting multiplicities in the communication between expert and principal.

The basic incentive problem studied in this paper stems from the fact that the principal does not observe the experts' effort and the experts do not know the history of the principal.

There is an infinite number of discrete periods. Within each period events unfold in the following order:

(1) An expert is chosen at random and offers a contract (d, p). If accepted, a contract requires the principal to pay d to the expert. In return, the expert recommends a service and the principal has the option to buy the recommended service at the price p at any future date.
(2) The principal decides on one of the following actions: (i) accept the contract; (ii) sample a new expert; (iii) buy the service from an expert whose contract the principal previously accepted; (iv) quit the process without purchase. The decision to buy the service and the decision to quit the game.
(3) If the principal accepts the contract, he pays the fee d and incurs a cost $s > 0$.
(4) Next, the expert chooses the diagnostic effort $e \in \{0, 1\}$ where $e = 1$ denotes high effort. We allow the expert to randomize between the two effort levels and denote with $x \in [0, 1]$ the probability of high diagnostic effort.
(5) Finally, the principal learns the recommendation $r \in [0, 1]$.

The model incorporates two features that strengthen price competition. First, the principal observes the expert's contract (d, p) at no cost. The search cost s and the fee d are only paid if the principal accepts the contract offer. Second, prior to the decision to purchase the service, the principal observes the contract offered by a new expert. The first feature eliminates the familiar paradox that even small search costs give the sellers with monopoly power (Diamond, 1971). The second feature ensures that the sequential manner in which the prices are observed does not dampen price competition compared to a situation in which the principal observes a few price offers simultaneously.

To relate these assumptions to a particular environment consider the example of a home owner in need of the services of a contractor. The cost s reflects the time it takes to wait for and supervise the contractor when he diagnoses the problem. On the other hand, we assume that the principal learns the contractor's prices at no cost, for example through an advertisement or over the phone. In addition, any of the contractors who previously made a recommendation can be hired to do the job.

We assume that the service can only be purchased from an expert who recommends it. This assumption ensures that the principal cannot first learn the appropriate service and then instruct an arbitrary expert (who did not provide a high-quality recommendation) to perform the service. This is justified in a setting where the recommendation does not uniquely identify the service to be performed. In the mechanic example, the recommendation may not contain all the necessary instructions for an unskilled worker to implement the repair. This can be formalized by modelling the principal's need as a point in a two-dimensional space. An informed expert identifies both dimensions but communicates only the first dimension to the principal. In Section 7, we briefly discuss an environment where this assumption is relaxed.

Suppose the principal is of type α, received recommendations from n experts whose fees were $d_1, \ldots, d_n$, and purchases from an expert who recommends a. Then, the principal's utility is

$$\begin{cases} V - p - \sum_{i=1}^{n} d_i - ns & \text{if } \alpha = a \\ -p - \sum_{i=1}^{n} d_i - ns & \text{if } \alpha \neq a. \end{cases}$$

If the principal quits after n recommendations without purchase, then his utility is $-\sum_{i=1}^{n} d_i - ns$. The principal seeks to maximize his expected utility.

An expert who operates under the contract (d, p) and exerts effort $e \in \{0, 1\}$, receives the following payoff:

$$\begin{cases} d - e \cdot c + p & \text{if the principal purchases the service from this expert } \textit{in some period} \\ d - e \cdot c & \text{if the principal does not purchase the service from this expert } \textit{in any period}. \end{cases}$$

Experts seek to maximize the expected profit.

The relevant past *history* of the principal records the sequence of experts whose contract he accepted,[3] their initial offers and recommendations.

Every period, after sampling a new expert the principal chooses one of the available options. After n recommendations and after observing a new contract offer, the principal must choose from $n + 3$ options: quit, buy from any one of the n experts who previously made a recommendation, accept the new contract offer, or continue sampling.

Formally, the principal's strategy σ is a sequence of functions $\sigma = (\sigma_n)_{n=0}^{\infty}$, where the function σ_n takes as input a history of length n (that records the encounters with the n previously

3. If the principal samples an expert but decides not to get a recommendation from this expert, the contract offered by this expert is not recorded as part of the relevant history. This is done for notational simplicity and has no consequences for the subsequent analysis because we focus on symmetric equilibria in what follows.

sampled experts) and a newly sampled offer of the form (d, p), and prescribes a probability distribution (an element of the $n + 3$ dimensional simplex) over the $n + 3$ available options.

Experts do not observe the history of the principal. A *strategy* for expert k consists of a contract offer $(d_k, p_k) \in R^2_+$ and an effort choice should the principal accept the contract offer. This effort choice depends on the contract offered and is denoted by $\xi_k : R^2_+ \to [0, 1]$, where $\xi_k(d_k, p_k)$ is the probability of a high diagnostic effort. In this paper, we analyse *symmetric perfect Bayesian equilibria*. In a symmetric perfect Bayesian equilibrium, experts must form beliefs over the set of possible histories of the principal *conditional* on being sampled by the principal. The principal's and the experts' strategies determine a stopping rule defined over histories. If the associated stopping time has a finite expectation, then the strategies determine a well defined probability distribution over histories conditional on a particular expert being sampled.[4] In equilibrium, the experts' beliefs must coincide with this probability distribution along the equilibrium path.

3. OPTIMAL SEARCH AND DIAGNOSTIC EFFORT

We begin by analysing the optimal search strategy of the principal. Consider the fixed symmetric strategy profile (d, p, ξ) of experts and let $x = \xi(d, p)$ be the probability of high diagnostic effort when this strategy is followed. The symmetry of the profile implies that only the experts' recommendations vary along the principal's search history. Optimal behaviour for the principal is therefore characterized by a stopping rule applied to sequences of recommendations.

Lemma 1 shows that the principal's best response is of a simple form. If participation is worthwhile, then the principal either buys after the first recommendation or he searches for a matching pair of recommendations and buys from one of the two experts making that recommendation. All proofs are given in the Appendix.

Lemma 1. *Every pure best response to (d, p, x) is one of the following three strategies:* (i) *quit;* (ii) *accept one contract and purchase the recommended service;* (iii) *accept contracts until two recommendations match. Then, purchase from one of the two experts who provided the matching recommendations.*

An implication of Lemma 1 is that the principal purchases the service after receiving two matching recommendations. The intuition is straightforward. Since search is costly $(d + s > 0)$, the payoff from buying the correct service must be strictly positive. Furthermore, two matching recommendations reveal the correct diagnosis. Hence, the principal has nothing to gain from continued search and he purchases the service from one of the experts making the matching recommendations.

A further implication of Lemma 1 is that the principal continues search after two or more conflicting recommendations. The intuition is that, in the presence of conflicting recommendations, the probability that any given recommendation is correct is revised downward and hence is lower than x. Therefore, the expected benefit from stopping and making a purchase after $n > 1$ recommendations is lower than the benefit of stopping after only one recommendation. On the other hand, the probability that one of the $n > 1$ recommendations is correct is greater than x. Therefore, the probability that the next recommendation leads to a matching pair of recommendations is higher after $n > 1$ recommendations than it is after

4. Note that if the expectation of the stopping time induced by σ and (d, p, ξ) is not finite, then this probability distribution may not be well defined. However, since search is costly, this situation will not occur when σ is a best response.

the first recommendation. Thus, if it is weakly optimal to continue searching after the first recommendation, it must be strictly optimal to continue after $n > 1$ conflicting observations, since the benefit of stopping is lower while the benefit of continued search is higher.

Suppose that the principal decides to participate, that is, he does not quit at the beginning of the game. Lemma 1 implies that the best response to (d, p, x) can be described by a single parameter: the probability that the principal stops after the first recommendation. We denote this probability by f. With probability $1 - f$ the principal searches for two matching recommendations.

If the principal stops after the first recommendation his payoff is

$$xV - p - (s + d). \tag{1}$$

Since a randomly sampled expert makes the correct recommendation with probability x, the expected duration of the search for one correct recommendation is $1/x$. Therefore, the expected duration of the search for two correct recommendations is $2/x$ and the corresponding expected search and diagnosis cost is $2(s + d)/x$. Therefore, the principal's payoff when he searches until two matching recommendations are found is

$$V - p - 2\frac{s + d}{x}. \tag{2}$$

The principal participates if the value of (1) or (2) is non-negative. The principal stops after the first recommendation ($f = 1$) if (1) is greater than (2) and he searches for a matching recommendation ($f = 0$) if (2) is greater than (1). If (2) is equal to (1) then any $f \in [0, 1]$ is optimal.

We now analyse the experts' effort decision for a given fee–price pair (d, p). To determine the optimal behaviour of the expert we must specify her belief about the principal's history. If the principal has not previously been diagnosed then he may stop after the first recommendation. By contrast, if the principal has been diagnosed by other experts, he will search for a matching recommendation. The probability B denotes the expert's belief that the principal has not been diagnosed by other experts. Note that B describes the expert's belief *conditional on the principal accepting the contract*.

If an expert incurs the cost c, she provides the principal with the correct diagnosis. The expected profit in this case is

$$d + pfB + (1 - fB)\frac{p}{2} - c \tag{3}$$

where fB is the probability that the principal has never sampled before and stops after the first recommendation and $(1 - fB)$ is the probability that the principal searches for a matching recommendation. In the latter case, the expert makes a sale with probability $1/2$. This follows because the principal samples experts in random order and the principal's strategy does not depend on the identity of the expert.[5]

On the other hand, if the expert does not incur the cost c, she will make an incorrect recommendation. The expected profit in this case is

$$d + pfB \tag{4}$$

since she only sells the service to a principal who buys the service after the first recommendation. Thus, the expert chooses high diagnostic effort ($x = 1$) when (4) is greater than (3) and low

5. Notice that we are not assuming here that the principal chooses with equal probability between the two matching experts. Conditional on being one of the two experts who provided correct recommendations, an expert is the first of those with probability $1/2$, regardless of how the principal randomizes between these two experts.

diagnostic effort ($x = 0$) when (4) is less than (3). When (4) is equal to (3) then the expert is indifferent and any $x \in [0, 1]$ is optimal.

4. FIXED PRICE EQUILIBRIUM

We next characterize "equilibrium" outcomes when prices are fixed. This is an intermediate step to our characterization of full equilibria which can be found in the following section. For the remainder of this section, we assume that prices are fixed at (p, d) and experts can only choose diagnostic effort.

The profile (d, p, x, f) is a *fixed price equilibrium* if the principal's search strategy, f, is optimal given (d, p, x) and the experts' effort decision $x \in [0, 1]$ is optimal given (d, p, x, f) and the belief B. Moreover, each expert's belief must be consistent with the strategy profile (d, p, x, f). We say that (d, p, x, f) is *non-degenerate* if experts choose high diagnostic effort with positive probability, *i.e.* if $x > 0$. In a degenerate fixed price equilibrium diagnostic effort is low with probability 1, *i.e.* $x = 0$.

In a fixed price equilibrium, B must be consistent with the search behaviour of the principal (described by f) and the effort choice of experts (described by x). Recall that B represents the experts' belief that conditional on accepting the contract the principal has not previously been diagnosed. Lemma 2 shows that for the expert's belief to be consistent B must be equal to the inverse of the expected duration of search. Note that with probability f the duration of search is one period and with probability $1 - f$ the duration of search is $2/x$, the number of periods it takes to find two matching recommendations. Hence, the expected duration of search is

$$f + (1 - f)\frac{2}{x}.$$

Lemma 2 follows from the fact that experts are sampled in random order. The proof is given in the Appendix.

Lemma 2. *The expert's beliefs are consistent with (d, p, x, f) if and only if*

$$B = \frac{x}{fx + 2(1 - f)}.$$

In a non-degenerate fixed price equilibrium, high diagnostic effort must yield a profit not smaller than low diagnostic effort and hence

$$fBp + (1 - fB)\frac{p}{2} - c \geq fBp. \tag{5}$$

Note that (5) requires that $p \geq 2c$, that is, the price must be at least twice the cost of high diagnostic effort. If the principal always buys after the first recommendation ($f = 1$), then $B = 1$ and $1 - fB = 0$ and (5) cannot hold. Therefore, $f < 1$ is a necessary condition for a non-degenerate fixed price equilibrium. Hence, it must be the case that the principal weakly prefers to search for two matching recommendations, *i.e.*

$$V - p - 2\frac{s + d}{x} \geq xV - p - (s + d). \tag{6}$$

Inequality (6) can be solved for $x \in [0, 1]$ when $d + s \leq \bar{s} \equiv V/(2\sqrt{2} + 3)$, that is, when the search cost and the diagnostic fee are not too large. In that case, (6) is satisfied for all $x \in [\underline{x}(d), \overline{x}(d)]$ where $\underline{x}(d) < 1$ and $\overline{x}(d) < 1$ are the two roots of the quadratic equation implied by (6) when it holds with equality.

Since $x < 1$ it follows that experts must be indifferent between their two effort levels and therefore (5) must hold with equality in any non-degenerate fixed price equilibrium.

$$fBp + (1 - fB)\frac{p}{2} - c = fBp. \tag{7}$$

Finally, the principal's participation requires

$$V - p - 2\frac{s + d}{x} \geq 0. \tag{8}$$

We conclude that on the path of a non-degenerate fixed price equilibrium the system (6)–(8) must hold.

Lemma 3 characterizes fixed price equilibria. For all parameters there are degenerate fixed price equilibria. Non-degenerate equilibria exist only if the search cost s and the cost of effort c are not too large. Inequality (6) implies that the principal can only be induced to search for a matching recommendation if $s \leq \bar{s} \equiv V/(2\sqrt{2} + 3)$. Hence, non-degenerate equilibria can exist only when $s \leq \bar{s}$. Inequality (5) implies that experts will only choose high effort if $p \geq 2c$. Therefore, existence of a non-degenerate equilibrium requires that $2c$ be smaller than V net of the expected search cost $2s/x$.

Lemma 3. (i) *There always exist degenerate fixed price equilibria;* (ii) *For $s \leq \bar{s}$ and $c \leq V/2 - s/\bar{x}(0)$ there exist non-degenerate fixed price equilibria;* (iii) *The profile (d, p, x, f) is a non-degenerate fixed price equilibrium iff $d + s \leq \bar{s}$, $p \leq V - 2(s + d)/x$, $f = (p - 2c)/(p - 2c + xc)$ and either $p > 2c$, and $x \in \{\underline{x}(d), \bar{x}(d)\}$ or $p = 2c$ and $x \in [\underline{x}(d), \bar{x}(d)]$.*

It is straightforward to see that there always are degenerate fixed price equilibria in which experts do not invest in the diagnosis (*i.e.* $x = 0$) and the principal quits immediately. In a degenerate fixed price equilibrium, on and off the path, an expert expects other experts not to invest in the diagnosis and hence she has no incentive to do so. Degenerate equilibria come about because of the strategic complementarity between experts' diagnostic effort. If other experts never provide the correct diagnosis, then the principal has no way of observing an expert's deviation to a high diagnostic effort.

Lemma 3 distinguishes two types of non-degenerate fixed price equilibria. In the first type, price is greater than twice the cost of diagnostic effort, $p > 2c$. In that case experts' profits are strictly positive since high diagnostic effort ensures that the principal buys with probability at least $1/2$. This implies that (6) must hold with equality. Otherwise, the principal strictly prefers to search for a matching recommendation (resulting in $f = 0$). But then the expert strictly prefers high effort and (7) is violated. Hence, the principal searches for a matching recommendation with a probability less than 1 ($f \in (0, 1)$), and therefore must be indifferent between searching and stopping after the first recommendation. Since (6) holds with equality, x can take only one of two values: x must be either $\underline{x}(d) \in (0, 1)$ or $\bar{x}(d) \in (0, 1)$. Using Lemma 2 to substitute for B we can then solve (7) to yield $f = (p - 2c)/(p - 2c + xc)$.

In the second type of equilibrium the experts' profit is zero, $p = 2c$ and $f = 0$. In this case, the principal searches until he gets matching recommendations, and x may take on any value in the interval $[\underline{x}(d), \bar{x}(d)]$. When $p = 2c$ and $f = 0$, experts are indifferent between high and low effort. In this case, every effort probability x that satisfies (6) and (8) is compatible with a fixed price equilibrium.

5. EQUILIBRIUM WITH PRICE COMPETITION

We next turn to the characterization of symmetric equilibria when there is unconstrained price competition among experts.

Recall that an equilibrium consists of a strategy for experts (d, p, ξ), a strategy for the principal σ, and conditional beliefs for experts. Lemma 1 implies that we can describe the strategy of the principal on the equilibrium path by the probability of stopping search after one recommendation, f. We say that (d, p, x, f) is an *equilibrium outcome* if there are equilibrium strategies (d, p, ξ) and σ and conditional beliefs such that on the equilibrium path the experts choose $(d, p, x) = (d, p, \xi(d, p))$ and the principal's strategy is characterized by f.

There always exist degenerate equilibria. In fact, every degenerate fixed price equilibrium is also a full equilibrium. When search and diagnosis costs are sufficiently low, there also exist non-degenerate equilibria. Of course, a non-degenerate equilibrium must satisfy the conditions of non-degenerate fixed price equilibria in Lemma 3. In addition, price deviations by experts must be unprofitable. Consequently, non-degenerate equilibria exist for a smaller region of the parameter space than do non-degenerate fixed price equilibria.

Proposition 1 shows that in a non-degenerate equilibrium, the diagnosis fee must be zero; the probability of high diagnostic effort is $\underline{x}(0)$, which is the lowest value of x in any non-degenerate fixed price equilibrium. The equilibrium price is at least two times the cost of high diagnostic effort and at most equal to the total surplus when the principal searches for two matching recommendations.

Proposition 1. (i) *There always exists a degenerate equilibrium;* (ii) *There is* $\widetilde{s} \in (0, \overline{s})$ *such that non-degenerate equilibrium exists iff* $s \le \widetilde{s}, c \le V/2 - s/\underline{x}(0)$*;* (iii) *If* (d, p, x, f) *is a non-degenerate equilibrium outcome then*

$$\begin{aligned} &d = 0 \\ &x = \underline{x}(0) \\ &p \in \left[2c, V - \frac{2s}{x}\right] \\ &f = \frac{p - 2c}{p - 2c + xc}. \end{aligned}$$

Next, we briefly explain the strategies that support non-degenerate equilibrium outcomes. The expert's effort decision and the principal's search decision are determined in the equilibrium of the continuation game that follows the principal's acceptance of an expert's contract offer. The diagnosis fee d is sunk by the time the expert decides on the diagnostic effort and hence d does not affect this decision. Therefore, a lower d is always attractive for the principal and, by a standard Bertrand style argument, d must be 0 in equilibrium.

Unlike the diagnosis fee, the price p affects the expert's effort incentives. Therefore, the standard Bertrand style argument is not applicable for p. An expert who deviates from the equilibrium contract to a higher price is ignored by the principal who expects her to choose low effort. Such an expert indeed has no incentive to choose high effort since—in case the principal elects to verify her recommendation through search—the principal purchases from another expert who charges a lower price. A deviation from the equilibrium contract to a lower price is ignored by the principal because the price discount is more than offset by a reduced probability of high effort. In more specific terms, consider a deviation from a non-degenerate equilibrium to a contract $(0, p')$, with $c < p' < p$. In the equilibrium of the continuation game following this deviation, the expert's optimal effort $y = \xi(0, p')$ satisfies

$$yV - p' = V - \left((1-y)\frac{2}{x} + y\frac{1}{x}\right)s - yp' - (1-y)p. \tag{9}$$

This is the counterpart of (6) for the case where the first expert sampled has deviated to $(0, p')$. Thus, y is chosen so that a principal who has not previously received a recommendation is indifferent between stopping after the recommendation of the deviating expert and continuing search.

We must ensure that for this choice of y the principal has no incentive to accept the contract offer of the deviating expert. Solving (9) yields

$$y = \frac{V - 2s/x - (p - p')}{V - s/x - (p - p')} < \frac{V - 2s/x}{V - s/x} = x \tag{10}$$

where the second equality follows from the equilibrium condition (6) for $x = \underline{x}(0)$. In the Proof of Proposition 1 we show that after any history the principal's loss from the reduced probability of effort (y rather than $\underline{x}(0)$) is larger than the gain from the price discount (p' rather than p). Thus, the principal has no incentive to get diagnosed by the deviating expert and the expert has no incentive to deviate.

A related argument explains why other fixed price equilibria do not survive price competition. Consider for example a fixed price equilibrium with $d = 0$, $p = 2c$ and $x \in (\underline{x}(0), \overline{x}(0))$ and consider a deviation to $(0, p')$, where p' is slightly lower than p. The expert's weakest incentive for high diagnostic effort is generated by the belief that the principal has previously received no recommendation. In this case $y = \xi(0, p')$ must again satisfy (9). However, since (6) holds with strict inequality for $x \in (\underline{x}(0), \overline{x}(0))$, it follows that $x < (V - 2\frac{s}{x})/(V - \frac{s}{x})$ so that for p' just below p, we have $y > x$. This means that such a deviation will be attractive for the principal. It would also be profitable for the expert, since it yields a payoff of $p' - c > 0$ instead of the 0 profit associated with this fixed price equilibrium. Now, since this deviation is attractive even with the most detrimental belief, there is no equilibrium that supports this outcome.

Notice that the non-degenerate equilibria differ from one another only in the price p and the probability f that the principal stops after the first recommendation: a higher p is associated with a higher f. Since price undercutting is precluded by the decrease in effort that it would induce, it is not surprising that different price levels are compatible with equilibrium. The positive relationship between p and f keeps the expert's incentives balanced, as equilibrium requires. A higher price, which is more conducive to effort, must be associated in equilibrium with a larger value of f, which in turn weakens the incentives to exert effort.

The interaction between the principal and an expert, following the acceptance of this expert's contract, has the flavour of an inspection game. The expert chooses between the high and low effort and the principal decides whether or not to verify this expert's recommendation through search. However, since this simple game is embedded here in a dynamic process of search and price competition, the present model is significantly richer. First, the parameters of the simple inspection game are determined endogenously by the contract offered by this expert as well as the offers and actions of other experts. Second, the inspection component itself is in fact a game of incomplete information, since the expert is uncertain about the principal's history. These additional ingredients create interdependencies between the effort levels of experts and—as the next section demonstrates—lead to an inefficiently low equilibrium effort level.

6. WELFARE

In this section, we analyse the welfare properties of equilibria. Pareto optimality requires that the correct service is provided at a minimal cost. Hence, in a Pareto optimal outcome one expert chooses high diagnostic effort and provides the service to the principal.

Clearly, Pareto optimal outcomes are not incentive compatible and hence cannot be sustained in equilibrium. An expert cannot be induced to choose high effort unless the principal verifies the recommendation with positive probability, which in turn requires high effort by other experts. A more interesting question is whether the equilibrium outcome achieves an appropriately constrained welfare optimum—the second best.

Notice that there are several potential notions of a second best. For example, if we only impose the experts' incentive compatibility and participation constraints, then we can attain welfare levels that are nearly Pareto optimal. To see this, suppose that the principal and two experts commit to the following game. The first expert is asked to provide a recommendation. The second expert does not observe this recommendation and, with a small probability $\alpha > 0$, is asked to provide a recommendation as well. If only one expert is asked or if both experts agree, then the expert(s) are paid a price greater or equal to c. If the two experts provide conflicting recommendations, then both are fined heavily. Clearly, if the fines are sufficiently high relative to α, this game has an equilibrium in which the expert(s) choose high effort. Since α is small there is little duplication of effort and hence the welfare levels are close to first best. However, the implementation of this outcome requires substantial commitment and enforcement capabilities and it may therefore not be a useful benchmark for evaluating the welfare performance of the equilibrium.

Our notion of constrained optimum assumes a social planner with more modest capabilities. We say that an outcome is *second best* if it is Pareto optimal among all those outcomes that can be sustained by a fixed price equilibrium. In other words, these are outcomes achievable by regulating the prices but leaving the participants' behaviour unconstrained otherwise. Since in a fixed price equilibrium each expert chooses an optimal diagnostic effort and the principal searches optimally for recommendations, the second best respects the principal's and the experts' incentive constraints.

In a non-degenerate fixed price equilibrium (d, p, x, f) the principal must (weakly) prefer to search for a matching recommendation. Therefore, when evaluating the principal's payoff, denoted $u(d, p, x, f)$, we can assume he searches for a matching recommendation. Hence,

$$u(d, p, x, f) = V - p - 2\frac{s+d}{x}.$$

The total surplus, that is, the sum of the payoffs of experts and the principal depends on x, the probability of high diagnostic effort, and on f, the probability that search stops after the first recommendation. We denote the total surplus with $U(x, f)$ where

$$U(x, f) = f(xV - s - xc) + (1 - f)\left(V - 2\frac{s}{x} - 2c\right).$$

The first term represents the surplus when the principal buys the service after the first recommendation. In that case, the high diagnostic effort and the correct service are provided with probability x and the search cost is s. The second term represents the surplus when the principal searches for a matching recommendation. In this case, the correct service is provided with certainty, two experts provide high diagnostic effort, and the expected search cost is $2\frac{s}{x}$.

The sum of the payoffs received by experts is denoted $\pi(d, p, x, f)$ and is simply the difference between the total surplus $U(x, f)$ and the principal's payoff $u(d, p, x, f)$. Hence

$$\pi(d, p, x, f) = U(x, f) - u(d, p, x, f).$$

Definition (Second best). The fixed price equilibrium (d, p, x, f) is second best for the welfare weight $\lambda \in [0, 1]$ if $\lambda u(d, p, x, f) + (1 - \lambda)\pi(d, p, x, f) \geq \lambda u(d', p', x', f') + (1 - \lambda)\pi(d', p', x', f')$ for all fixed price equilibria (d', p', x', f'). We say that (d, p, x, f) is second best if it is second best for some $\lambda \in [0, 1]$.

Note that for $\lambda = 1/2$ the second best outcome maximizes total surplus whereas for $\lambda = 1$ the second best outcome maximizes the principal's payoff.

The characterization of fixed price equilibria in Lemma 3 implies that $x \in [\underline{x}(d), \overline{x}(d)]$ where $\underline{x}(d)$ and $\overline{x}(d)$ are the two roots of the quadratic equation

$$x^2 V - x(s + d + V) - 2(s + d) = 0.$$

Hence, $\overline{x}(d)$ is the maximal probability of high diagnostic effort in a fixed price equilibrium with diagnosis fee d. Moreover, since $\overline{x}(d)$ is decreasing in d, it follows that $\overline{x}(0)$ is the maximal probability of high diagnostic effort in any fixed price equilibrium. Proposition 2 below shows that in any second best outcome the probability of high diagnostic effort is $\overline{x}(0)$.

Lemma 3 establishes that in a non-degenerate fixed price equilibrium the probability f that the principal stops after the first recommendation satisfies

$$f = \frac{p - 2c}{p - 2c + xc}$$

and hence is increasing in the price. For $f > 0$, the principal's participation constraint implies that the maximal price is

$$\overline{p} = xV - (d + s).$$

Setting $d = 0$, $x = \overline{x}(0)$ and plugging the maximal price in the above formula for f we get

$$\overline{f} = \frac{\overline{x}(0)V - s - 2c}{\overline{x}(0)V - s - 2c + \overline{x}(0)c}. \tag{11}$$

Hence, $\overline{f}$ is the maximal probability that the principal stops after the first recommendation consistent with a fixed price equilibrium and diagnostic effort $\overline{x}(0)$.

Proposition 2 shows that every second best fixed price equilibrium has experts choose high effort with probability $\overline{x}(0)$ and charge a diagnosis fee d of zero. The price of the service in second best outcomes depends on the welfare weights. If the goal is to maximize total surplus ($\lambda = 1/2$) or if the experts' payoff is given a larger weight than the principal's payoff then the price is the maximal price among all fixed price equilibria ($p = \overline{p}$). Correspondingly, the probability that the principal searches for a matching recommendation is minimal ($f = \overline{f}$). When all the weight is on the principal's payoff then the price is equal to $2c$, the smallest price among all fixed price equilibria. In that case, the principal searches for a matching recommendation with probability 1 ($f = 0$).

Proposition 2. *If* (d, p, x, f) *is second best then* $x = \overline{x}(0)$ *and* $d = 0$*;* $(0, \overline{p}, \overline{x}(0), \overline{f})$ *is second best for* $\lambda \leq 1/2$ *and* $(0, 2c, \overline{x}(0), 0)$ *is second best for* $\lambda = 1$.

As a corollary to Proposition 2 we conclude that equilibria in our model are not second best. The equilibrium probability of high diagnostic effort is $\underline{x}(0)$ which is less than the diagnostic effort required for a second best outcome, $\overline{x}(0)$.

Corollary 1. *Suppose* $s \leq \overline{s}$, $c \leq V/2 - s/\overline{x}(0)$ *and* (d, p, x, f) *is an equilibrium outcome. Then* (d, p, x, f) *is not second best.*

Equilibria are not second best because the experts' effort level is too low. At any fixed price equilibrium (d, p, x, f) with the second best effort level $\overline{x}(0)$ experts have an incentive to undercut the equilibrium price. Next, we provide an intuition for this fact. Suppose an expert deviates to a slightly lower price p'. Whether or not the principal finds the deviating contract offer attractive depends on the probability with which the deviating expert chooses high effort. This probability is determined by the equilibrium of the subgame following acceptance of the offer by the principal. That equilibrium, in turn, depends on the effort choices of other experts as these effort choices affect the cost at which the principal can verify the expert's recommendation by searching for a match. When the other experts choose high effort with probability $\overline{x}(0)$ then verification has a relatively low cost. As a result, in the subgame following the deviation, the effort level of the deviating expert remains high enough to be attractive for the principal. Hence, the principal finds the deviating contract offer attractive and price undercutting cannot be deterred.

To see more specifically why the second best effort level $\overline{x}(0)$ is not sustained in equilibrium while the lower level $\underline{x}(0)$ can be sustained, consider a fixed price equilibrium (d, p, x, f) with $d = 0$ and $x \in \{\underline{x}(0), \overline{x}(0)\}$ and consider a deviating offer $(0, p')$ with $p' < p$. As explained in Section 5, the deviating expert will choose the high diagnostic effort with the probability $y < x$ given by (10).[6] To underline the fact that y depends on x and on the magnitude of the price discount $\Delta = p - p'$, let us write $y = y(x, \Delta)$. The principal's expected gain from accepting the deviating contract is $\Delta - [x - y(x, \Delta)]V$, *i.e.* the price reduction minus the expected loss due to the reduced effort. It follows from (10) that, for a given Δ, the difference $x - y(x, \Delta)$ is an increasing function of s/x (which is the expected cost of verifying a correct recommendation when all other experts choose high effort with probability x). This implies that $\Delta - [\overline{x}(0) - y(\overline{x}(0), \Delta)]V > \Delta - [\underline{x}(0) - y(\underline{x}(0), \Delta)]V$. In other words, a given price discount Δ is more attractive for the principal when $x = \overline{x}(0)$ than it is when $x = \underline{x}(0)$. For this reason the fixed price equilibrium configuration with $\overline{x}(0)$ is more vulnerable to price deviations than the configuration with $\underline{x}(0)$. For sufficiently small Δ we have that $\Delta - [\overline{x}(0) - y(\overline{x}(0), \Delta)]V > 0$ and hence the principal gains from accepting the deviating contract when $x = \overline{x}(0)$. This deviation is also profitable for the deviating expert, since by choosing high effort she can ensure a profit of at least $p' - c$ which exceeds the equilibrium profit $pfB + (1 - fB)p/2 - c$ when p' is close to p. Hence, the second best contracts cannot be sustained in equilibrium. In contrast when $x = \underline{x}(0)$, we have $\Delta - [\underline{x}(0) - y(\underline{x}(0), \Delta)]V < 0$, for any Δ, and hence the principal does not benefit from a price discount.

The inefficiency of the equilibrium in this second best sense stands in contrast to a common intuition derived from other scenarios with competition in incentive contracts. Consider, for example, the following variation of our model. The principal samples sellers as in our model. Each seller may offer a high- or low-quality product and—at a fixed cost $k > 0$—the principal can verify the quality of the offered product. The sequential structure of the model is unchanged. The difference to our model is that the quality of product is verified directly (by incurring a fixed cost) rather than by searching for matching recommendations. In this modified model competition among the experts leads to a second best outcome that maximizes the principal's welfare. The reason is that in this modified model, other experts' effort choices have no effect on the cost of verification.[7]

6. Notice that, while this was shown there for $x = \underline{x}(0)$, it holds for $x = \overline{x}(0)$ as well since $\overline{x}(0)$ also satisfies (6) with equality.

7. To see the difference more clearly, we construct a simple equilibrium in this modified game. First, note that as in our model, sellers offer a contract (d, p). The principal sequentially samples contracts and decides whether or not to verify the product quality. The search strategy of the principal is now a simple stationary strategy. Independent of the history, the principal verifies the product quality with probability $1 - f$. The principal buys the good if the product is verified to have high quality or if he choose not to verify (with probability $1 - f$). A straightforward calculation reveals

The inefficiency result is rather robust. As we mention in Section 7, this result survives alternative specifications of the mechanism by which price is determined and alternative specifications of the diagnosis technology. The inefficiency of equilibria might be somewhat alleviated under weakened competition. For example, a softer price competition due to an additional dimension of differentiation among experts might allow equilibria with a higher effort level. However, there is no reason to expect that such a modification will necessarily eliminate the inefficiency.

Of course, our model is too stylized to provide firm grounds for regulatory intervention. However, the analysis suggests that restrictions on price competition might be beneficial when the forces highlighted in the model seem important. Note that a price regulation can be beneficial even if the price is fixed at a level at which experts make zero profits ($p = 2c$), *i.e.* the lowest price in any non-degenerate fixed price equilibrium. The key advantage of a regulated price is that effort levels can remain high without triggering a deviation by experts. Suppose, for example, the price is regulated at $p = 2c$ which is compatible with the second best outcome that maximizes the principal's welfare. If this market is deregulated and experts are free to offer any contract then prices will stay the same (or rise) but the equilibrium effort level x must fall from its second best level to deter price undercutting. The resulting equilibrium outcome with price competition will be worse for the principal.

7. DISCUSSION

In this section we revisit some of the modelling assumptions and discuss alternatives.

The sale of recommendations

We have assumed that the correct service can only be provided by an expert who made the correct recommendation. However, there are situations where it is straightforward to perform the service once the correct diagnosis is known. For example, if an automobile part has been found to be defective, any mechanic and perhaps the car owner himself can replace it. In Pesendorfer and Wolinsky (1998), we consider such an environment. In particular, we assume the recommended service can be performed by the principal. This is then an environment in which the recommendation itself (rather than the service) is the good being sold.

When the principal does not need the expert to implement the recommendation, the contracts studied in this paper cannot provide experts with incentives to choose high effort. Thus, the provision of incentives requires additional contractible information. In Pesendorfer and Wolinsky (1998), we assume that the service that the principal ultimately chooses is verifiable to all experts, and hence the contract may stipulate payments conditional on that choice. In this environment, a similar analysis to the one conducted in this paper reveals that the competitive outcome is inefficient. Analogous to the results given in this paper, the welfare maximizing fixed price equilibrium cannot be supported as a full equilibrium with price competition.

The 0–1 *nature of the diagnosis*

The model assumes that by choosing high effort the expert learns the correct diagnosis with certainty, whereas by choosing low effort the expert learns nothing. We adopt this assumption

that $d = 0$, $p = c$, $x = 1 - \sqrt{\frac{v}{V}}$, $f = 0$ is an equilibrium of this modified game. Moreover, this equilibrium maximizes the principal's welfare among all fixed price equilibria. Hence, it is second best.

because it leads to a very simple form of the optimal search. Notice, however, that the zero value of the information obtained without effort is merely a normalization. One may think instead that the zero level is already valuable for the principal and that the effort discussed here is just the extra effort required for further refinement of the diagnosis.

In a more elaborate model the value of the service would depend monotonically on its distance from the true problem and the diagnosis would be imperfect. Clearly, in such a model the stopping rule would be more complicated. However, the general form will remain similar: the search stops once a sufficient number of close recommendations is obtained. More importantly, the informational externality at the core of the described inefficiency in this market would still be present.

The diagnosis fee

Recall that the diagnosis fee d is restricted to be non-negative. A previous version (Pesendorfer and Wolinsky, 1998) allowed d to be negative (*i.e.* experts pay the principal to be examined by them). The results obtained there were similar as long as d is restricted to be greater or equal to $-s$ (*i.e.* so long as the principal does not profit just from being diagnosed). The rationale for placing a lower bound on d (be it 0 as in this paper or $-s$) is that contracts that make a sufficiently large up-front payment are vulnerable to abuse. Consider a model with a fringe of principals who are not interested in the service but would take advantage of an expert that offers up-front payment, then such an offer cannot be profitable. Without a lower bound on d at or above $-s$, non-trivial equilibria with $x > 0$ do not exist.

Alternative interpretation of the model

The model and the analysis were developed in the context of a single principal who samples sequentially from a population of experts. It is possible to embed this basic scenario in a market setting with a population of principals. In such a model, we envision the market as operating over time without beginning or end. In any period, the principals who obtained the service depart from the market and there is a flow of new principals into the market. Thus, at any time the population consists of principals who have experienced different search histories. In a steady state, the distribution of histories over the principal population remains constant over time (although the principals themselves change). A steady state equilibrium of this model would correspond to the equilibrium of the model analysed above, where the beliefs coincide with the equilibrium steady state distribution of the principal population.

Finally, observe that we could allow the principal to observe a number of contracts simultaneously, similar to an auction setting. This would not alter our analysis or results, nor would it affect the model with a population of principals described in the previous paragraph.

APPENDIX

Proof of Lemma 1. The existence of an optimal stopping rule is standard in this problem. We first show that stopping after two or more non-matching recommendations (with or without purchase) cannot be optimal. Consider a sample of n experts giving different recommendations. Let $\varphi(n)$ be the probability that a randomly drawn expert out of these n has the correct diagnosis.

$$\varphi(n) = \frac{(1-x)^{n-1}x}{(1-x)^n + n(1-x)^{n-1}x} = \frac{x}{1+(n-1)x}.$$

Observe that $\varphi(n)$ is decreasing. Let $\pi(n)$ be the probability that the $(n+1)$-st recommendation will match one of the first n recommendations,

$$\pi(n) = \frac{x^2 n}{1 + (n-1)x}.$$

Let W^n be the expected continuation value of the optimal search after receiving n different recommendations. Observe that

$$W^n \geq -(s+d) + Vx - p$$

since the principal can always buy from the last expert. Optimality requires that

$$W^n = \max\{0, \varphi(n)V - p, -(s+d) + (1-\pi(n))W^{n+1} + \pi(n)(V-p)\}.$$

Assume, that it is optimal to purchase after $n+1$ distinct recommendations and hence

$$W^{n+1} = \varphi(n+1)V - p.$$

Then,

$$\begin{aligned} W^n &= \max\{0, \varphi(n)V - p, -(s+d) + (1-\pi(n))(\varphi(n+1)V - p) + \pi(n)(V-p)\} \\ &= \max\{0, \varphi(n)V - p, -(s+d) + Vx - p\}. \end{aligned}$$

Since $\varphi(n+1)V - p = W^{n+1} \geq \max\{0, -(s+d) + Vx - p\}$ and since $\varphi(n)$ is decreasing in n, it follows that

$$\varphi(n)V - p > \max\{0, -(s+d) + Vx - p\}.$$

Therefore, it is optimal to purchase after n distinct recommendations. Proceeding inductively it follows that it is never optimal to purchase after two or more distinct recommendations.

Assume, that it is optimal to quit without purchase after $n+1$ distinct recommendations. In that case,

$$W^{n+1} = 0.$$

Then,

$$W^n = \max\{0, \varphi(n)V - p, -(s+d) + \pi(n)(V-p)\}.$$

Since $0 \geq W^{n+1} \geq -(s+d) + \pi(n+1)(V-p)$ and since $\pi(n)$ is increasing in n, it follows that

$$W^n = \max\{0, \varphi(n)V - p\} > -(s+d) + \pi(n)(V-p).$$

Therefore, it is optimal to stop search after n recommendations. Proceeding inductively, it follows that it is never optimal to stop and quit after two or more distinct recommendations.

Since $s+d > 0$, searching beyond two matching recommendations is never optimal. Therefore, the optimal search policy may prescribe only stopping (with purchase) after two matching observations or after the first observation, or stopping (with quit) before any observations were obtained. ||

Proof of Lemma 2. Fix (x, f). Let n denote the number of recommendations received prior to the sampling of expert k. Let H_k be the set of histories such that k is sampled and let $P(n \mid H_k)$ be the probability that principal receives n recommendations prior to sampling expert k (conditional on k being sampled).

Let T denote the random stopping time of the search over the set of experts excluding k. We compute $P(n \mid H_k)$ by decomposing the sampling process over experts other than k into the disjoint events, $T = 1, 2, \ldots$. This yields,

$$P(n \mid H_k) = \sum_{m \geq n+1} P(n \mid T = m; H_k) \Pr(T = m \mid H_k). \tag{A.1}$$

Notice that

$$P(n \mid T = m; H_k) = \begin{cases} 1/m & \text{if } 0 \leq n \leq m-1 \\ 0 & \text{otherwise.} \end{cases} \tag{A.2}$$

Furthermore,

$$\begin{aligned} \Pr(T = m \mid H_k) &= \frac{\Pr(H_k \mid T = m)\Pr(T = m)}{\sum_{n \geq 1} \Pr(H_k \mid T = n)\Pr(T = n)} \\ &= \frac{m \Pr(T = m)}{\sum_{n \geq 1} n \Pr(T = n)} \end{aligned} \tag{A.3}$$

where the last equality uses the fact that

$$\frac{\Pr(H_k \mid T = m)}{\Pr(H_k \mid T = n)} = \frac{m}{n}. \tag{A.4}$$

(To compute the likelihood ratio of two zero probability events we take the limit of shrinking neighbourhoods of positive probability. Hence,

$$\frac{\Pr(H_k \mid T = m)}{\Pr(H_k \mid T = n)} = \lim_{\varepsilon\to 0} \frac{\Pr(\{H_{\tilde{k}} \mid \tilde{k} \in [k, k+\varepsilon]\} \mid T = m)}{\Pr(\{H_{\tilde{k}} \mid \tilde{k} \in [k, k+\varepsilon]\} \mid T = n)} = \lim_{\varepsilon\to 0} \frac{1-(1-\varepsilon)^m}{1-(1-\varepsilon)^n} = \frac{m}{n}$$

and therefore equation (A.4) follows.)

Substituting (A.3) and (A.2) into (A.1) yields

$$B = P(0 \mid H_k) = \frac{\sum_{m\geq 1} \Pr(T = m)}{\sum_{n\geq 1} n \Pr(T = n)} = \frac{1}{f + (1-f)\frac{2}{x}}$$

where the last equality follows since $\sum_{m\geq 1} \Pr(T = m) = 1$ and $\sum_{n\geq 1} n \Pr(T = n) = f + (1-f)\frac{2}{x}$. ‖

Proof of Lemma 3. (i) Suppose on and off the path experts never choose high effort. In that case, after any history it is optimal for the principal to quit without purchase. Hence, it is optimal for each expert to choose low effort. (ii) Let $(d, p, x, f) = (0, 2c, \overline{x}(0), 0)$. It is straightforward to show that (d, p, x, f) satisfies the equilibrium conditions (6), (7) and (8) for the indicated range of parameters. (iii) We first prove necessity. If a profile (d, p, x, f) with $x > 0$ is a fixed price equilibrium, then it satisfies (6), (7) and (8). (6) implies $d \leq \overline{s} - s$. Lemma 2 and equation (7) yield

$$f = (p - 2c)/(p - 2c + xc)$$

and hence $f < 1$ and $p \geq 2c$. From (8), $p \leq V - (s + d)/x$. If $p > 2c$ then $f > 0$, which in turn implies that (6) holds with equality and hence $x \in \{\underline{x}(d), \overline{x}(d)\}$. If $p = 2c$ then $f = 0$ and (6) may hold with inequality, implying that x has to be in $[\underline{x}(d), \overline{x}(d)]$.

To show sufficiency, suppose that a profile (d, p, x, f) with $x > 0$ is one of the profiles described in (iii). Then, $x \in [\underline{x}(d), \overline{x}(d)]$, $f = (p - 2c)/[p - 2c + xc]$ and $p \leq V - (s + d)/x$. Now, $x \in [\underline{x}(d), \overline{x}(d)]$ implies that (6) holds, $f = (p - 2c)/[p - 2c + xc]$ implies that (5) holds, and $p \leq p(d, x)$ implies that (8) holds. So (d, p, x, f) is a fixed price equilibrium. ‖

Proof of Proposition 1. Let (d, p, x, f) be a fixed price equilibrium of the form described in the proposition. To show that it is an equilibrium, we must complete the description of the strategies and beliefs in the continuation game after the principal encounters a deviating offer (d', p'):

(1) The principal does not accept (d', p'). If the principal is searching for a matching recommendation, he continues sampling the next expert. Otherwise, he buys from a previously sampled expert.
(2) In the continuation game following the acceptance of (d', p') the following strategies are used. The deviating expert believes with probability 1 that this is the principal's first sample. If (d', p') is the first offer accepted by the principal, he stops searching and purchases from this expert with probability f'. With probability $1 - f'$ the principal searches for a matching recommendation. The principal breaks ties in favour of experts who did not deviate. If the principal accepts (d', p'), the expert chooses $e = 1$ with probability $\xi\left(d', p'\right) = y'$, where y is the smallest probability of high diagnostic effort consistent with the expert's incentives.

First, note that $y' = 0$ if $p' \leq c$ since the expert weakly prefers low diagnostic effort. When $p' \geq p$, then the principal buys from an expert who has not deviated whenever he searches for a matching recommendation. Hence, the deviating expert has a strict incentive to choose low diagnostic effort ($y' = 0$) when $p' \geq p$. Since for $y' = 0$ the principal has no incentive to accept (d', p') we may assume that $p' \in (c, p)$ in the following.

For $y > 0$ and $p \in (c, p)$ the counterpart of (5) must hold, that is,

$$f'p' \leq p' - c. \tag{A.5}$$

Observe that (A.5) implies $f' < 1$ and therefore the counterpart of (6) must hold

$$yV - p' \leq V - \left((1-y)\frac{2}{x} + y\frac{1}{x}\right)(s + d) - yp' - (1-y)p. \tag{A.6}$$

Observe that (A.6) requires $y < 1$. A strict inequality in (A.6) implies $f' = 0$. But $p' > c$ and $f' = 0$ imply that (A.5) holds strictly and hence $y = 1$. Therefore, $f' \in (0, 1)$ and (A.6) holds with equality.

Claim 1. *Let $W^n(d', p', y)$ be the principal's expected continuation value if the principal has n distinct recommendations, samples the deviating expert and continues optimally thereafter.*

$$W^n(d', p', y) = V - (s+d)\left(\frac{2-y}{x} - \frac{n}{1+(n-1)x}\right) - (s+d') - (yp' + (1-y)p). \tag{A.7}$$

Proof of Claim 1. Recall that y and p' are such that,

$$\begin{aligned} W^0(d', p', y) &= Vy - (s+d') - p' \\ &= V - (s+d') - \left(\frac{2-y}{x}\right)(s+d) - \big(yp' + (1-y)p\big) \end{aligned} \tag{A.8}$$

where $Vy - (s+d') - p'$ is the expected utility of being diagnosed by the deviating expert and purchasing from her immediately. Let $\psi(n)$ denote the probability that one of the n past recommendations is correct.

$$\psi(n) = \frac{n(1-x)^{n-1}x}{(1-x)^n + n(1-x)^{n-1}x} = \frac{nx}{1+(n-1)x}.$$

If a principal with n distinct recommendations plans to search for two matching recommendations then the expected search duration is

$$\begin{aligned} &\psi(n)(y + (1-y)(1+1/x)) + (1-\psi(n))(y(1+1/x) + (1-y)(1+2/x)) \\ &\quad = 1 + \frac{2-y-\psi(n)}{x}. \end{aligned}$$

The expected price that the principal pays under this plan is $yp' + (1-y)p$, since with probability y the deviating expert provides the correct recommendation and $p' < p$. The principal's expected utility if she samples the deviating expert and continues searching until a matching recommendation is obtained, is therefore

$$V - (s+d') - \left(\frac{2-y-\psi(n)}{x}\right)(s+d) - (yp' + (1-y)p) > Vy - (s+d') - p' \tag{A.9}$$

where the inequality follows from $\psi(n) > 0$ for $n \geq 1$ and (A.8). Therefore,

$$W^n(d', p', y) = V - (s+d') - \left(\frac{2-y-\psi(n)}{x}\right)(s+d) - (yp' + (1-y)p). \tag{A.10}$$

Substitution for $\psi(n)$ gives the required expression. ‖

After a history of n distinct observations, the principal strictly prefers to sample the deviating expert, if $W^n(d', p', y) > W^n(d, p, x)$. From (A.7)

$$W^n(d', p', y) - W^n(d, p, x) = \frac{y-x}{x}(s+d) + y(p-p') + d - d'.$$

Recall that (A.6) holds with equality. Solve it for y, and substitute the result above to get

$$\begin{aligned} \Delta(d', p'; d, p, x) &\equiv W^n(d', p', y) - W^n(d, p, x) \\ &= \left(\frac{s+d}{x} + p - p'\right)\frac{V - 2(s+d)/x + p' - p}{V - (s+d)/x + p' - p} - s - d'. \end{aligned}$$

Claim 2. (i) *If $d = 0$, $x = \underline{x}(s)$ and $x \leq \sqrt{s/V}$, then $\Delta(d', p'; d, p, x) \leq 0$, for all d' and $p' \leq p$.*

(ii) *If any of these conditions fails, there are p', d' arbitrarily close to p, d with $p' \in (c, p)$ such that $\Delta(d', p'; d, p, x) > 0$.*

Proof of Claim 2. (ii): If $x \in (\underline{x}(s+d), \overline{x}(s+d))$, then (6) holds with strict inequality. Hence,

$$x < \frac{V - 2(s+d)/x}{V - (s+d)/x}$$

and, for p' close to p,

$$x < \frac{V - 2(s+d)/x + p' - p}{V - (s+d)/x + p' - p} = y.$$

It follows that, for p' close to p and $d' = d$, $\Delta(d', p'; d, p, x) > 0$.

If $x \in \{\underline{x}(d), \overline{x}(d)\}$ then $\Delta(d, p; d, p, x) = 0$. Therefore, if $d > 0$, then for $d' < d$ and p' sufficiently close to p, $\Delta(d', p'; d, p, x) > 0$. Assume, therefore, $d = 0$. Since

$$\frac{\partial}{\partial p'}\Delta(d', p'; 0, p, x)\Big|_{p'=p} = -1 + V\frac{s}{x} \Big/ \left(V - \frac{s}{x}\right)^2,$$

it follows that $\frac{\partial}{\partial p'}\Delta(d', p'; 0, p, x)\Big|_{p'=p} < 0$ iff $x > \sqrt{s/V}$. Hence, if $x > \sqrt{s/V}$ there are $p' < p$ arbitrarily close to p such that $\Delta(d, p'; 0, p, x) > 0$. Since x satisfies (6) with equality, we have

$$x \in \{\underline{x}(0), \overline{x}(0)\} = \left\{\frac{V + s \pm \sqrt{(V+s)^2 - 8sV}}{2V}\right\}. \tag{A.11}$$

It is easy to verify that only $\underline{x}(0)$ satisfies $x \leq \sqrt{s/V}$. This completes the proof of part (ii).

To prove (i) observe that $\Delta(d', p'; d, p, x)$ is a concave function of p'. Since $d = 0$ and $x \leq \sqrt{s/V}$, $\frac{\partial}{\partial p'}\Delta(d', p'; d, p, x)\Big|_{p'=p} \geq 0$. Hence, $\Delta(d', p'; d, p, x)$ is maximized at $p' = p$ and $d' = 0$, over all $d' \geq 0$ and $p' \leq p$. Since $x = \underline{x}(s)$, $\Delta(d, p; d, p, x) = 0$ and hence $\Delta(d', p'; d, p, x) \leq 0$, for all d' and $p' < p$. ‖

Part (i) of Claim 2 implies that any fixed price equilibrium with $d = 0$, $x = \underline{x}(s)$ and $x \leq \sqrt{s/V}$ is a full equilibrium since it would be optimal for the principal to ignore any deviation.

We now argue that there are no other equilibria. Part (ii) of the claim implies that when the conditions of the proposition are violated then a principal would always (for every history) accept the deviating contract offer under the assumed beliefs of the expert and the assumed continuation strategies. However, note that we have chosen continuation strategies and beliefs that make the deviation least attractive. To see this, note that the expert believes that if the principal accepts the contract, he has previously not been diagnosed. These beliefs give the expert the lowest incentive to choose high diagnostic effort because a principal who has previously been diagnosed has a greater incentive to search for a matching recommendation than a principal who has not previously been diagnosed. Claim 1 demonstrates this fact. Furthermore, we have broken ties so that x' is minimized.

We may therefore conclude that if any of the conditions of the Proposition are violated it follows that for *any* belief and *any* sequentially rational continuation strategy there is a (p', d') that the principal would strictly prefer. Moreover, $p' < p$, and p', d' can be chosen arbitrarily close to p, d.

It remains to show that such a deviation is profitable for the expert. Note that if the deviating expert chooses high diagnostic effort he makes a sale with probability one after the deviating offer with $p' < p, d' < d$ is accepted. This is so because the principal must purchase from the expert with the lower price. But this, in turn implies that the expert increases the probability of a sale from 1/2 to 1 by deviating. Since $p \geq 2c$ this implies greater profits for p' sufficiently close to p. Hence it follows that the deviation is profitable for any belief and any sequentially rational continuation strategy. Therefore, there cannot be a non-degenerate equilibrium.

Finally, the range of s for which such equilibria exist is such that $\underline{x}(s) \leq \sqrt{s/V}$. Now, using (A.11) to express $\underline{x}(s)$ and rearranging, $\underline{x}(s) \leq \sqrt{s/V}$ is equivalent to $V - 3\sqrt{sV} + s \geq 0$. Let $\tilde{s}$ and $\tilde{\tilde{s}}$ denote the two solutions of this inequality when it holds with equality. The inequality is satisfied for $s \leq \tilde{s}$ and $s \geq \tilde{\tilde{s}}$. Observe that $\tilde{s} > 0$ and $\tilde{\tilde{s}} > V$. Thus, the relevant range of s for which there exists an equilibrium with $x > 0$ is $s \leq \tilde{s}$. ‖

Proof of Proposition 2. *Case* 1*:* $\lambda \geq 1/2$. Note that

$$\lambda u + (1-\lambda)\pi = (1-\lambda)U + (2\lambda - 1)u.$$

Observe that U and u are strictly increasing in x and u is strictly decreasing in d. We show that if (d, p, x, f) with $x < \overline{x}(0)$ is a fixed price equilibrium, then there is p' such that $(0, p', \overline{x}(0), f)$ is also a fixed price equilibrium. Moreover, both U and u are strictly higher at $(0, p', \overline{x}(0), f)$. To see this, recall from Lemma 3 that $x \leq \overline{x}(0)$ and that $f = (p-2c)/[p-2c+xc]$. Define $p' = p + (p-2c)(\overline{x}(0) - x)/x$ and observe that $(p'-2c)/[p'-2c+\overline{x}(0)c] = (p-2c)/[p-2c+xc] = f$. Since U depends only on (x, f) it follows that $U(\overline{x}(0), f) > U(x, f)$. Now, if $f = 0$, then $p' = p = 2c$ and

$$u(0, p', \overline{x}(0), f) = V - 2s/\overline{x}(0) - p' > V - 2s/x - p = u(d, p, x, f).$$

If $f > 0$, then, by Lemma 3, $x \in \{\underline{x}(0), \overline{x}(0)\}$ and hence $u(d, p, x, f) = V - 2(s+d)/x - p = xV - s - d - p$. Therefore,

$$\begin{aligned} u(0, p', \overline{x}(0), f) &= \overline{x}(0)V - s - p' \\ &= [\overline{x}(0) - x][xV - p + 2c]/x + xV - p - s \end{aligned}$$

$$> xV - s - d - p$$
$$= u(d, p, x, f).$$

The choice of p' and $\overline{x}(0)$ guarantee that $(0, p', \overline{x}(0), f)$ satisfies (7) and (6). Hence $(0, p', \overline{x}(0), f)$ is a fixed price equilibrium with both u and U higher than at (d, p, x, f).

To prove the proposition for Case 1 it remains to show that $p = 2c$ when $\lambda = 1$. Note that $u = \overline{x}(0)V - p - s$ and hence the result follows since $p = 2c$ is the smallest price in any fixed price equilibrium.

Case 2*:* $\lambda \leq 1/2$. We first show that U is maximized at $(0, \overline{p}, \overline{x}(0), \overline{f})$. The argument in Case 1 shows that $x = \overline{x}(0)$ and $d = 0$ in any price equilibrium that maximizes U. Since from the definition of $\overline{x}(0)$ we have $\overline{x}(0)V - s = V - 2s/\overline{x}(0)$, it follows that $\overline{x}(0)V - s - \overline{x}(0)c > V - 2\frac{s}{\overline{x}(0)} - 2c$ and hence that $U(\overline{x}(0), f)$ is strictly increasing in f. Since f is an increasing function of p, the maximal f given $\overline{x}(0)$ is achieved at the maximal price consistent with (8), $p = V - 2s/\overline{x}(0) = \overline{x}(0)V - s$. Therefore, $U(\overline{x}(0), f)$ is maximized at

$$\overline{f} = \frac{\overline{x}(0)V - s - 2c}{\overline{x}(0)V - s - 2c + \overline{x}(0)c}.$$

Now observe that $u \geq 0$ in any fixed price equilibrium. Hence, $(1 - \lambda)U(x, f) + (2\lambda - 1)u(d, p, x, f) \leq (1 - \lambda)U(\overline{x}(0), \overline{f})$ for $\lambda \leq 1/2$. But $u(0, \overline{p}, \overline{x}(0), \overline{f}) = 0$ and hence this upper bound is attained at $(0, \overline{p}, \overline{x}(0), \overline{f})$.

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