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Eliciting information from multiple experts

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Abstract

A decision maker DM has to elicit information from informed experts regarding the desirability of a certain action. The experts share similar preferences, which differ significantly from those of DM, and they possess different pieces of information. The question is how much information DM can elicit, despite the difference in interests. The focus is on how DM can take advantage of the multiplicity of experts. For the case in which DM cannot commit to a mechanism, the main observation is that allowing partial communication among the experts might result in revelation of more information than either full communication or no communication. In the case in which DM can commit to a mechanism the maximum inducement for revelation is attained by mechanisms that are non-monotonic in the information. The analysis makes repeated use of the idea that experts understand that their reports matter only when they are pivotal.

JEL classification: C72; D73; D82

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1. Introduction

This paper considers a situation in which a decision maker has to elicit information from informed experts regarding the desirability of a certain action. The problem is that the preferences of these experts are different than those of the decision maker so that the experts might want to misrepresent their information. The question is how much information the decision maker can elicit, despite the differ-

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ence in interests. The focus here is on ways in which the decision maker can take advantage of the multiplicity of experts to overcome their bias to some extent. The special features of the environment are that the experts share similar preferences which differ significantly from those of the decision maker, and that the nature of the relations precludes the use of side payments to generate incentives.

Consider, for example, the problem of a king who has to decide whether to wage a war. He may consult his generals who have expert knowledge about different aspects of the military capabilities of theirs and the enemy's armies. It is not implausible to assume that the generals share similar preferences with regard to this issue and that they are significantly more eager to go to war than the king is. If all generals had the same information, it might be possible to get them to disclose it, but the problem is that each general knows only a certain piece of the relevant information. The question here concerns the extent of information that the king can collect under different regimes of organizing the transmission of information.

More specifically, suppose that a decision maker DM has to decide whether to undertake a certain project. The information regarding the desirability of this project is given by a signal $S \in \{0, \ldots, N\}$, with higher values of the signal conveying more favorable information about the project. DM does not observe S, but an expert observes S and can communicate the information to DM. However, DM and the expert do not share the same preferences: DM would like to undertake the project if $S \geqslant n_D$, but the expert is less eager and would like the project to be undertaken only when $S \geqslant n_E > n_D$ (i.e., the difference in preferences here is in the opposite direction than in the above story about the king and his generals). Clearly, in the absence of side payments, it is quite obvious that the expert cannot be induced to reveal information that will lead to the implementation of the project when $S < n_E$. Suppose, however, that DM faces several experts rather than just one. If all these experts observe S, it is possible to devise a mechanism such that in an equilibrium the experts would reveal S to DM. Suppose therefore that each of these experts observes an independent piece of the information that together combine to the signal S. The focus of the discussion here is on whether and how the multiplicity of the experts affects DM's ability to elicit information in comparison to the single expert case, even though the experts continue to share the same preferences.

We assume that the preferences of DM are sufficiently different from those of the experts and obtain the following observations. If DM cannot commit to a mechanism and there is no communication among the experts, then DM does not get any information in equilibrium. If DM can commit to a mechanism, it becomes possible to elicit from the experts substantially more information. In some cases, the information elicited is sufficient to implement DM's best outcome in all but one state. But even without commitment ability, DM can elicit some relevant information by partitioning the experts into groups and allowing the members of each group to communicate with each other before they report their information to DM. Obviously, if all experts are allowed to communicate, they can be induced to reveal the relevant information, at least, when their aggregate information makes it desirable for them to undertake the project. The more interesting observation is that, if communication among the experts can be restricted to certain subsets (e.g., DM can designate "working groups" and communication is restricted to within these groups), then even more information can be elicited (i.e., the experts end up revealing more information than their collective interest would justify). All of these observations are quite straightforward. They make repeated use of the idea that experts choose their report with the understanding that it matters only when they are pivotal. For example, in the case in which commitment to a mechanism can be made, the best mechanism from DM's perspective is usually not monotonic in the reported information. It manipulates experts' incentives to disclose their information by making them pivotal both at low values of the aggregate signal and at high values.

There is an established literature dealing with information transmission between an informed expert and an uninformed decision maker. Crawford and Sobel (1982) is perhaps the basic reference. A strand of that literature also considers the case of a decision maker who faces multiple experts (e.g., Gilligan and Krehbiel, 1989; Krishna and Morgan, 2001; Battaglini, 1999; and Austen-Smith, 1993). The first three of these papers consider a situation in which the experts observe the same information, but they differ from one another in their preferences. The present paper considers a somewhat different situation: The experts have different pieces of information so their reports cannot be confronted, but they share a similar bias relative to the decision maker. This direction complements the literature represented by these papers. In some sense the problem considered here is more significant, since the elicitation of common information is usually not very difficult, except when there are only two experts. Austen-Smith's paper considers the case of two experts with conditionally independent signals and allows for different configurations of their preferences relative to one another and the decision maker. The scenario in which the preferences of the two experts are similar to one another but sufficiently different than the decision maker's is reminiscent of the scenario considered in the present paper. However, Austen-Smith focuses on the comparison between simultaneous and sequential reporting for the different configurations of the preferences that may arise in his model. Thus, the questions discussed, by Austen-Smith are not directly related to questions discussed here. In fact, the fact that his model restricts the attention to two experts precludes some of the questions that the present paper addresses. The present paper is also related to the literature on decision making by juries composed of strategic jurors (see Austen-Smith and Banks, 1996 and Feddersen and Pesendorfer, 1998). Like the experts in the present model, the strategic jurors whose behavior is analyzed by those articles recognize that their votes matter only when they are pivotal. Whereas that literature focuses on the performance of different majority rules, the present paper looks at the somewhat broader question of organizing a decision making process that relies on such a panel of experts.

2. Model

A decision maker DM has to choose an action $a \in \{L, H\}$. It is convenient to think of H as undertaking some project (H stands for "high investment") and of L as the status quo of not undertaking it (L stands for "low investment"). The relative desirability of these actions depends on information that DM does not have. However, he consults with N experts. Each expert, $i \in N$, has a piece of information captured by a signal $s_i \in \{0, 1\}$, where $s_i = 1$ is more favorable information for H than $s_i = 0$. Let $s = (s_1, s_2, \ldots, s_N)$, $S = \sum_{i=1}^{N} s_i$, and $S_{-i} = S - s_i$. There are no transfers between DM and the experts. Thus, their respective payoffs depend only on the action a and the information s. The source of the tension is that the experts' views on the desirability of H might differ from those of DM and consequently they may attempt to distort their information. Following are the main assumptions:

1. The signals s_i are i.i.d. with $q = \Pr(s_i = 1)$.
2. $s_i = 1$ is verifiable. Thus, expert i can report that $s_i = 0$ when in fact $s_i = 1$, but cannot over-report.
3. DM's utility depends on a and s as follows:
$$\widetilde{V}(a \mid s) = \begin{cases} V(S), & \text{if } a = H, \\ 0, & \text{if } a = L, \end{cases}$$
where V is increasing and $E[V(S)] < 0$. Thus, in the absence of any information, DM's optimal choice is L.
4. All experts have the same preferences described by the utility function
$$\widetilde{U}(a \mid s) = \begin{cases} U(S), & \text{if } a = H, \\ 0, & \text{if } a = L, \end{cases}$$
where U is increasing.
5. $V(n-1) > U(n)$, for all $n = 1, \ldots, N$, i.e., DM is more eager than the experts to choose H.
6. $U(N) > 0$, i.e., for sufficiently high values of S both DM and the experts prefer H to L (notice that assumption 3 and 5 already imply that, for sufficiently low values, both prefer L to H).

The central ingredients are that the experts are less eager than DM and that they have identical preferences. The exact identity of preferences is not crucial in the sense that the main observations reported below will continue to hold if the experts' preferences are just sufficiently similar rather than identical. The important element is that all experts clearly differ from DM in a certain direction. Here they are assumed less eager, but of course the case in which they are more eager is conceptually similar.[1] This assumption intends to capture an important ingredient in a class of decision making situations in which the experts are individuals with similar background and culture and hence have similar biases. The introduction mentions the advice of military officers regarding the desirability of war. As another example, one may think of environmental experts working for the government. It is quite conceivable that individuals who chose this career share similar and predictable preferences. Furthermore, what is important here is not that the same bias is shared by *all* experts but rather that there is an identifiable subgroup of experts whose preferences are biased in such a manner. Since the signals are independent, the subproblem of eliciting the information from such subgroup is essentially the same as the problem discussed here.

A number of the assumptions imposed above are intended to simplify matters as much as possible in order to focus on the main points we have to make. In this category we have the symmetry of the signals, the verifiability of information, as well as the parameter restrictions $E[V(S)] < 0$ and $U(N) > 0$. The symmetry of the signals is reflected in identical distributions of the s_i's and the dependence of the payoffs on s only through S. The parameter restrictions eliminate the need of distinguishing different cases that do not pose different conceptual issues. For example, if $E[V(S)] > 0$, then in the absence of information, DM's default action is H rather than L, but this does not affect the essence of the analysis. The verifiability of $s_i = 1$ is also not crucial. Since the experts are less eager than DM, they naturally would not have an interest in exaggerating their reports. This assumption simply exempts us from considering situations when reporting $s_i = 1$ means in fact that $s_i = 0$.

3. The benchmark case of no commitment and no communication

In this benchmark case DM communicates with each expert separately and cannot commit to a mechanism. Thus, DM's decision is a best response to whatever information he gets from the reports of the experts. Expert i submits to DM the report $r_i \in \{0,1\}$, $r_i \leqslant s_i$. Then, based on $r = (r_1, \dots, r_N)$, DM chooses L or H. DM's strategy is a function, $x : \{0,1\}^N \to [0,1]$, where $x(r)$ is the probability with which H is chosen given the reports r. Expert i's strategy is a probability $y_i \in [0,1]$, of reporting $r_i = 1$ when $s_i = 1$, i.e., $y_i = \Pr(r_i = 1 \mid s_i = 1)$. Let $R = \sum_{i=1}^{N} r_i$ and $R_{-i} = R - r_i$.

An equilibrium consists of DM's strategy $x^* : \{0,1\}^N \to [0,1]$ and experts' strategies $y_i^* \in [0,1]$, such that, for all $r \in \{0,1\}^N$, $x^*(r)$ maximizes DM's

[1] This case will be the exact mirror image if the assumption on verifiability was also reversed. But since that assumption is not crucial anyway, even without changing it this case is conceptually similar.

expected utility, given $(y_i^*)_{i=1}^n$, and y_i^* maximizes expert i's utility, given $x^*(r)$ and $(y_i^*)_{j\neq i}$. In other words,

$$x^*(r) = \begin{cases} 1, & \text{if } \sum_{k\geqslant R} V(k)\Pr(S=k|r) > 0, \\ 0, & \text{if } \sum_{k\geqslant R} V(k)\Pr(S=k|r) < 0, \end{cases}$$

$y_i^* = 1$ only if

$$\sum_{r_{-i}} \sum_{k\geqslant R_{-i}} U(k+1)x^*(r_{-i}, r_i = 1)\Pr(S_{-i} = k \mid r_{-i}, y_{-i}^*)\Pr(r_{-i}|y_{-i}^*)$$
$$\geqslant \sum_{r_{-i}} \sum_{k\geqslant R_{-i}} U(k+1)x^*(r_{-i}, r_i = 0)\Pr(S_{-i} = k \mid r_{-i}, y_{-i}^*)\Pr(r_{-i}|y_{-i}^*)$$

and $y_i^* = 0$ only if this (weak) inequality is reversed.

The absence of commitment is captured of course by the condition that DM's equilibrium strategy x^* is a best response to the experts' reports. The following proposition establishes that in the benchmark case considered in this section, no meaningful information is elicited from the experts. This owes to the difference in preferences between DM and the experts and to the experts' understanding that their reports matter only when they are *pivotal*. That is, only when the individual expert's report might change the actual decision with positive probability.

Proposition 1. *In any equilibrium L is chosen with probability* 1.

Proof. Consider an equilibrium x^*, y^*. Expert i is said to be *pivotal* at r_{-i} if $x^*(r_{-i}, r_i = 0) = 0$ but $x^*(r_{-i}, r_i = 1) > 0$. In such a case $\sum_{k\geqslant R_{-i}} V(k)\Pr(S = k \mid r_{-i}, r_i = 0, y^*) \leqslant 0$ and $\sum_{k\geqslant R_{-i}+1} V(k)\Pr(S = k \mid r_{-i}, r_i = 1, y^*) > 0$ (or the first holds with $<$ while the second holds with $\geqslant$). Therefore, if i is pivotal at r_{-i}, then

$$\sum_{k\geqslant R_{-i}} U(k+1)\Pr(S_{-i} = k \mid r_{-i}, y_{-i}^*)$$
$$= \sum_{k\geqslant R_{-i}} U(k+1)\Pr(S = k+1 \mid r_{-i}, r_i = 1, y^*)$$
$$< \sum_{k\geqslant R_{-i}} V(k)\Pr(S = k+1 \mid r_{-i}, r_i = 1, y^*)$$
$$\leqslant \sum_{k\geqslant R_{-i}} V(k)\Pr(S = k \mid r_{-i}, r_i = 0, y^*) \leqslant 0,$$

where the equality and the weak inequality before last follow from $r_i = 1$, and the strict inequality owes to Assumption 4 above. It follows that, if $\Pr(r_{-i}|y_{-i}^*) > 0$ for some r_{-i} at which agent i is pivotal, then since $x^*(r_{-i}, r_i = 0) < x^*(r_{-i},$

$r_i = 1$),

$$\sum_{r_{-i}} x^*(r_{-i}, r_i = 1) \Pr(r_{-i} | y^*_{-i}) \sum_{k \geqslant R_{-i}} U(k+1) \Pr(S_{-i} = k \mid r_{-i}, y^*_{-i})$$
$$< \sum_{r_{-i}} x^*(r_{-i}, r_i = 0) \Pr(r_{-i} | y^*_{-i}) \sum_{k \geqslant R_{-i}} U(k+1) \Pr(S_{-i} = k \mid r_{-i}, y^*_{-i}).$$

This implies $y_i^* = 0$. Now suppose that there exists some r such that $\Pr(r|y^*) > 0$ and $x^*(r) > 0$. Then, since $x^*(r)$ is monotonically non-decreasing in r and since $x^*(0) = 0$, there exists i such that $r_i = 1$ and i is pivotal at some $r'_{-i} \leqslant r_{-i}$. By the above, $y_i^* = 0$ in contradiction to $\Pr(r|y^*) > 0$. Therefore, there does not exist r such that $\Pr(r|y^*) > 0$ and $x^*(r) > 0$. Hence, L is chosen with probability 1. □

Given $x(r)$, expert i is *pivotal* at r_{-i} if $x(r_{-i}, r_i = 0) = 0$ but $x(r_{-i}, r_i = 1) > 0$. Experts choose their report with the understanding that it would have an effect only when they are pivotal. Thus, given DM's and the other experts' behavior, expert i's best response is such that the incremental expected utility over the instances in which she is pivotal is non-negative:

$$\sum_{s_{-i}} \Pr(s_{-i}) U(S_{-i} + 1)$$
$$\times \sum_{r_{-i}} \Pr(r_{-i} | s_{-i}, y_{-i}) \big[x(r_{-i}, r_i = 1) - x(r_{-i}, r_i = 0)\big] \geqslant 0.$$

Owing to the difference in preferences between the experts and DM, in the instances in which experts might be pivotal they clearly prefer decision L. Therefore, experts are induced to suppress positive signals and this leads to the result of the proposition.

The observation that in any equilibrium DM always chooses L owes of course to the assumption that, in the absence of any information, DM prefers L to H. If this assumption were reversed, it would still be the case that no relevant information is revealed in equilibrium. This can be verified by simply noting that the main argument of the above proof establishing the suppression of the information by the experts continues to be valid. Consequently, in this case, DM would always choose H in equilibrium, since H is his preferred choice in the absence of information.[2]

In order to extract more information from the experts, DM has to be able to manipulate the instances in which they are pivotal in a way that will generate incentives to report positive signals. The following two sections discuss two

[2] To establish this point formally, we only have to make the following changes in the last paragraph of the proof. Replace "$x^*(r) > 0$" by "$x^*(r) < 1$," replace "$x^*(0) = 0$" by "$x^*(N) = 1$," and replace "$r'_{-i} \leqslant r_{-i}$" by "$r''_{-i} \geqslant r_{-i}$." Finally, of course, we have to replace "L" by "H" in the conclusion of the proposition.

ways in which the information transmission process might be modified to create stronger incentives for information disclosure by experts. One way is made possible if DM has the power to commit in advance to the response to the experts' reports, while the other is facilitated by allowing some communication among the experts prior to the reporting stage.

4. The full commitment case

Suppose that DM can commit to a mechanism $x:\{0,1\}^N \to [0,1]$, where $x(r)$ describes the probability with which H is chosen given the reports $r \in \{0,1\}^N$. The main observation of this section is that the maximum disclosure of information will be often achieved by a mechanism in which the probability of H is not monotonic in the number of positive signals. The non-monotonicity owes to the fact that effective inducement of information revelation is generated by spreading the points at which experts become pivotal. With such mechanisms the extent of information that experts reveal might be substantially beyond what they would like to reveal given their interests.

Formally, by the revelation principle, DM's problem is maximization of his expected utility subject to the experts' incentive compatibility constraints. That is,

$$\mathrm{Max}_x \sum_{s\in\{0,1\}^N} \Pr(s)x(s)V(S), \tag{1}$$

so that

$$\sum_{s_{-i}} \Pr(s_{-i})x(s_{-i}, s_i = 1)U(S_{-i}+1) \geqslant \sum_{s_{-i}} \Pr(s_{-i})x(s_{-i}, s_i = 0)U(S_{-i}+1).$$

The following claim establishes that, without loss of generality, we may restrict attention to a symmetric problem in which $x(s) = z(n)$ for all s such that $S = n$. Thus, DM's problem can be restated as

$$\mathrm{Max}_z \sum_{n=0}^{N} \Pr(S=n)z(n)V(n), \tag{2}$$

so that

$$\sum_{n=0}^{N} \Pr(S_{-i}=n)z(n+1)U(n+1) \geqslant \sum_{n=0}^{N} \Pr(S_{-i}=n)z(n)U(n+1).$$

Claim 2. *If z is a solution to problem* (2)*, then $x(s) = z(n)$ for all s such that $S = n$ is a solution to problem* (1).

The proof of this claim is relegated to Appendix A. Problem (2) is a linear program. The nature of the solution is illustrated below using an example with simple step utility functions. Before turning to the example, it is useful to outline the intuition that underlies the solution. The optimal mechanism from DM's perspective induces substantial disclosure of information by appropriate choice of the points in which experts are pivotal. To understand how this works, let n_D and n_E, $n_D < n_E$, denote the threshold levels for DM and the experts respectively. That is, DM prefers H to L iff $S \geqslant n_D$ and the experts prefer H iff $S \geqslant n_E$. Suppose that DM commits to choosing H if the number of positive reports is between n_D and $n_E - 1$, or is greater than n_E, and to choosing L if this number is exactly $n_E - 1$. In this manner, an expert is made pivotal at the three points in which the total number of positive reports by the others is $n_D - 1$, $n_E - 2$, or $n_E - 1$. Provided that all other experts report truthfully, an expert would like to conceal a positive signal if the others report a total of $n_D - 1$ positive signals, since this would result in H when she prefers L. But the expert would like to disclose a positive signal if the others report $n_E - 2$ or $n_E - 1$ positive signals, since then the decision will switch from H to L and from L to H, respectively, in accordance with the expert's preferences at these points. Thus, if the incremental gains to an expert from reporting a positive signal at $n_E - 2$ and at $n_E - 1$, weighted by their probabilities, exceed the incremental loss at $n_D - 1$, weighted by its probability, experts are induced to disclose their information. In this case DM's best decision ends up being implemented in all instances except when there are exactly $n_E - 1$ positive signals. If the incremental gains at $n_E - 2$ and $n_E - 1$ do not exceed the incremental loss at $n_D - 1$, DM's optimal scheme would shift the switching points to achieve the desired effect, but the general idea of the scheme remains the same.

Besides confirming the intuition outlined above concerning the effectiveness of commitment to a non-monotonic $z(n)$ function, the following example also shows that through commitment DM may be able to induce the experts to reveal significantly more information than their joint interest would entail.

Example.

$$V(n) = \begin{cases} -1, & \text{if } n < n_D, \\ 1, & \text{if } n \geqslant n_D, \end{cases} \qquad U(n) = \begin{cases} -1, & \text{if } n < n_E, \\ 1, & \text{if } n \geqslant n_E, \end{cases} \qquad n_E > n_D. \tag{3}$$

Thus, U and V are simple step functions differing in the threshold levels n_E and n_D at which H becomes the preferred action.

Claim 3. *The optimal solution in this example is characterized by a level* $\tilde{n} \in [n_D, n_E)$ *as follows*:

$$z(n) = \begin{cases} 0, & \text{for } n < n_D, \\ 1, & \text{for } n \in [n_D, \tilde{n}), \\ z \in [0,1], & \text{for } n = \tilde{n}, \\ 0, & \text{for } n \in (\tilde{n}, n_E), \\ 1, & \text{for } n \geqslant n_E, \end{cases} \tag{4}$$

where

$$\tilde{n} = \max\bigl\{n \in [n_D, n_E)\colon\ \Pr(S_{-i} = n_D - 1) \leqslant \Pr(S_{-i} = n - 1) + \Pr(S_{-i} = n_E - 1)\bigr\}. \tag{5}$$

The proof is relegated to Appendix A. Notice that z is not monotonic in n: The decision switches from L to H at n_D, then it switches back to L at $\tilde{n}$ and then back again to H at n_E. If $\Pr(S_{-i} = n_D - 1) \leqslant \Pr(S_{-i} = n_E - 2) + \Pr(S_{-i} = n_E - 1)$, then $\tilde{n} = n_E - 1$. In this case $z(n)$ coincides with the simple scheme described in the intuitive discussion above, and DM's best outcome is implemented at all n's except for $n_E - 1$. When this inequality does not hold, the probabilities of the events $S_{-i} = n_E - 2$ and $S_{-i} = n_E - 1$, in which the expert is pivotal and wants to disclose a positive signal, are not sufficiently high to offset the effect of the event $S_{-i} = n_D - 1$ in which the expert prefers to conceal a positive signal. To obtain this balance, the switching point from decision H to L has to be moved to from $n_E - 1$ to a lower level $\tilde{n}$ whose probability is higher. In such a case, DM is prevented from implementing H in an interval $[\tilde{n}, n_E - 1]$ rather than just at $n_E - 1$. With other parameters given, the latter situation would arise when the probability of a positive signal, q, is sufficiently low. In the extreme, when q is particularly low, $\tilde{n} = n_D$ in which case DM can do no better than follow the experts' preferences.

The non-monotonicity of the above solution is not merely an artifact of this example. The following proposition argues that, more generally, if a monotonic mechanism is optimal, then it essentially implements the experts' best outcome. In other words, the solution to DM's problem might be monotonic only in the extreme situations in which DM can do no better than adhere almost completely to the experts' preferences.

Proposition 4. *For a generic choice of* U *and* V, *if* DM*'s optimal* z *is non-decreasing in* n, *then it satisfies* $z(n) = 1$ *if* $n \geqslant n_E$ *and* $z(n) = 0$ *if* $n < n_E - 1$.

Notice that the z functions of the proposition yield the experts' best outcome everywhere except perhaps at $n_E - 1$. The family of monotonic mechanisms includes the very natural voting procedures that prescribe the choice of H if and only if the number of reported positive signals exceeds some quota. As it turns out, when DM and the experts differ systematically in their preferences, as assumed here, these procedures usually do not elicit the maximal amount of information.

5. Communication

If DM does not have commitment ability, the experts might still be induced to disclose positive signals by allowing them to communicate and pool information among themselves before they report it. Obviously, if all N experts are allowed to pool their information before reporting to DM, then they would want to disclose their information when they have n_E or more positive signals. Actually, the only relevant information that DM can elicit in this case is whether $n < n_E$ or $n \geqslant n_E$. Thus, with full communication among the experts, the best DM can achieve is the experts' best outcome which is implementing H if and only if $n \geqslant n_E$. This is of course better than the complete suppression of the information in the benchmark case that obtains in the absence of commitment and communication. But the question is whether DM can do even better by allowing only more limited communication.[3] The main observation of this section is that DM can often elicit more information by allowing communication only within certain subsets of experts. It is important to emphasize that we are not looking here for the optimal mechanism subject to DM's inability to commit (see more on this at the end of this section), but rather examine the performance of a specific natural form of organization whereby DM partitions the set of experts into groups.

Thus, the description now involves three stages. In the first stage DM partitions the set of experts into m groups. Let $G = \{G_1, \dots, G_m\}$ denote the group structure, let n_i denote the size of group i, let $s_{i_j} \in \{0, 1\}$ denote the signal obtained by the j's member of group G_i and let $s_i = s_{i_1} + \cdots + s_{i_{n_i}}$. Then, in the second stage, each group G_i reports to DM a number $r_i \in \{0, \dots, n_i\}$, interpreted as a number of positive signals its members got. As before,we impose the verifiability constraint $r_i \leqslant s_i$. Finally, in the third stage, given the vector of reports $r = (r_1, \dots, r_m)$, DM chooses L or H. Thus, a strategy for DM consists of a choice of a partition G and the probability $x(r, G)$ with which DM chooses H as a function of the reports r and the structure G. A strategy for expert group i is a probability distribution $y_i(s_i, G)$ over allowable reports r_i, i.e., $y_i(., G) : \{0, \dots, n_i\} \to \Delta(\{0, \dots, n_i\})$ such that $r_i \leqslant s_i$. The equilibrium notion here is perfect Bayesian equilibrium.

[3] This observation bears a distant relationship to an observation made by Dessein (1999). Dessein showed that in the context of the Crawford–Sobel model, it would sometimes be optimal for the principal to delegate the decision to the agent in order to avoid the loss of information caused by the strategic transmission of the information. The facilitation of full communication among the experts in our model plays a similar role to the delegation in Dessein's model.

The idea is that members of group i can freely communicate. Since they have the same preferences, there is no need to model the details of the information exchange within the group and we may simply assume that they will pool their information and will agree on a common report. Notice that in writing y_i as a function of G, we assume that the partition is publicly observable.

The following simple example of a scenario with four experts illustrates how the possibility of partitioning the experts into communicating sub groups serves the interests of DM.

Example 1.

$$N = 4, \qquad q = 1/4, \qquad V(n) = n - 1.1, \qquad U(n) = n - 2.4.$$

In the benchmark case with no commitment or communication, the only equilibrium outcome is the decision L, i.e., the project is never implemented. In the full communication case, the decision is H when $S \geqslant 3$ and it is L otherwise. This is of course the experts' best outcome. When it is possible to allow only partial communication, the best equilibria for DM, have the following form. The experts are divided into two groups of two. Each group makes a positive report only if both of its signals are 1. That is, $r_i = 2$ if $s_{i_1} + s_{i_2} = 2$ and $r_i = 0$ otherwise. DM chooses H if and only if at least one of the groups sends a positive report, i.e., DM chooses H if $r = (2, 0)$, $(0, 2)$ or $(2, 2)$ and chooses L if $r = (0, 0)$. In this equilibrium, the decision is H when $S \geqslant 3$ but also in those cases in which $S = 2$ and one of the groups happens to get the two positive signals, i.e., $s_{i_1} + s_{i_2} = 2$ for at least one i. Thus, H is implemented here in some instances in which the experts would rather have L.

It is natural to inquire how the optimal arrangement (from DM's perspective) is determined by the data of the situation. We do not have a general characterization of the optimal arrangement in terms of the primitive data, but we can expose two considerations that interact in shaping it. One consideration is that, to induce revelation, the expert groups have to be sufficiently large, relative to the discrepancy in preferences between DM and the experts. We have already seen that, in the extreme case of singleton groups, no revelation is possible under our assumption on the preferences. The following result uses a similar idea to bound the minimal group size necessary for some information revelation to occur. Given G, x, and y, group G_i is *pivotal* at r if $x(r, G) > 0$ but $x(r_{-i}, r_i = 0, G) = 0$.

Proposition 5. *Suppose that $U(k) < V(k - \ell)$, for all $k \geqslant \ell$, and let (G, x, y) be an equilibrium. If G_i is pivotal at some r such that $\Pr(r|y) > 0$, then $n_i > \ell$.*

Proof. Suppose that G_i is pivotal at $\bar{r}$ and $\Pr(\bar{r}|y) > 0$. This implies that $\sum_{k \geqslant R_{-i}} V(k) \Pr(S = k \mid \bar{r}_{-i}, r_i = 0, y) \leqslant 0$. Suppose now that $n_i \leqslant \ell$. Therefore, for any s_i,

$$\begin{aligned}
&\sum_{k \geqslant R_{-i}} U(k+s_i)\Pr(S_{-i}=k \mid \bar{r}_{-i}, y) \\
&\quad < \sum_{k \geqslant R_{-i}} V(k)\Pr(S_{-i}=k \mid \bar{r}_{-i}, y) \\
&\quad \leqslant \sum_{m=0}^{n_i} \Pr(s_i=m \mid r_i=0) \sum_{k \geqslant R_{-i}} V(k+m)\Pr(S_{-i}=k \mid \bar{r}_{-i}, y) \\
&\quad = \sum_{k \geqslant R_{-i}} V(k)\Pr(S=k \mid \bar{r}_{-i}, r_i=0, y) \leqslant 0.
\end{aligned}$$

Since $x(r,G)$ is monotonic in r (this is true in any equilibrium), it follows that for any s_i and any $j \leqslant s_i$ such that $x(\bar{r}_{-i}, r_i=j, G) > 0$,

$$\begin{aligned}
&\sum_{r_{-i}} x(r_{-i}, r_i=j, G)\Pr(r_{-i} \mid y_{-i}) \\
&\qquad \times \sum_{k \geqslant R_{-i}} U(k+s_i)\Pr(S_{-i}=k \mid r_{-i}, y_{-i}) \\
&\quad < \sum_{r_{-i}} x(r_{-i}, r_i=0, G)\Pr(r_{-i} \mid y_{-i}) \\
&\qquad \times \sum_{k \geqslant R_{-i}} U(k+s_i)\Pr(S_{-i}=k \mid r_{-i}, y_{-i}).
\end{aligned}$$

Notice that the inequality is strict since $\Pr(\bar{r}_{-i} \mid y_{-i}) > 0$. But the strict inequality and y being part of an equilibrium imply that y_i assigns zero probability to all $j \leqslant s_i$ such that $x(\bar{r}_{-i}, r_i=j, G) > 0$, in contradiction to $\Pr(\bar{r} \mid y) > 0$. Therefore, if G_i is pivotal at some r such that $\Pr(r \mid y) > 0$, then $n_i > \ell$. $\square$

Observe that the information of G_i affects the decision with positive probability only if it is pivotal at some r that occurs with positive probability. If G_i is not pivotal with positive probability, the reports of G_i do not matter, which means that there is an outcome equivalent equilibrium in which y_i is identically zero. Thus, if DM's preference for H is more intense than the experts' to the extent that, with ℓ fewer positive signals, DM's benefit is still greater than the experts' benefit, then for a group of experts to reveal meaningful information at equilibrium, it must be of size greater than ℓ.

Now, while a minimal group size is required to induce some revelation, the extent of information revealed does not necessarily increase with the group size. As we know from Example 1, more information is elicited when there are two separate groups than when all experts communicate with each other. The point is that a larger group has a greater opportunity to "manage" its information than a smaller group does. This consideration works to reduce the size of the groups that

DM would like to form. The following two examples demonstrate this point. In particular they show that, when the difference in preferences is more substantial in an otherwise similar situation, the optimal equilibrium arrangement may entail smaller expert groups.

Example 2.

$$N = 6, \qquad q = 1/2, \qquad V(n) = n - 3.4, \qquad U(n) = n - 4.2.$$

Again, in the benchmark case with no-communication, the only equilibrium outcome is the decision L. In the full communication case, the decision is H when $S \geqslant 5$ and it is L otherwise. When partial communication is possible, the best equilibrium for DM is such that the experts are grouped in two groups of three. In this equilibrium each group makes a positive report only if it has either 2 or 3 positive signals. That is, $r_i = 2$ if $s_{i_1} + s_{i_2} + s_{i3} \geqslant 2$ and $r_i = 0$ otherwise. DM chooses H if and only if at least two of the groups send a positive report (i.e., if $r = (2, 2)$). Thus, in this equilibrium, the decision is H when $S \geqslant 5$ but also in some instances in which $S = 4$, i.e., when $s_{i_1} + s_{i_2} + s_{i_3} \geqslant 2$ for both $i = 1$ and $i = 2$.

Example 3.

$$N = 6, \qquad q = 1/2, \qquad V(n) = n - 3.4, \qquad U(n) = n - 4.6.$$

This example differs from the previous one only in the specification of $U(n)$. Here it takes more favorable information to convince the experts of the desirability of the H decision. For this reason the scheme of Example 2 does not work here: Each of the expert groups would benefit from suppressing its information and reporting 0 when it has only 2 positive signals. Here the best equilibrium for DM is such that the experts are divided into three groups of two. In this equilibrium each group makes a positive report only if both of its signals are 1. That is, $r_i = 2$, if $s_{i_1} + s_{i_2} = 2$ and $r_i = 0$, otherwise. DM chooses H if and only if at least two of the groups send a positive report (i.e., if $r = (2, 2, 0)$, $(2, 0, 2)$, $(0, 2, 2)$, or $(2, 2, 2)$). Thus, in this equilibrium, the decision is H when $S \geqslant 5$ but also in those instances in which $S = 4$ and the four positive signals are concentrated in two of the groups, i.e., when $s_{i_1} + s_{i_2} + s_{j_1} + s_{j_2} = 4$ for some i and j.

Notice that the only difference between these examples is in the experts' preferences: The experts of Example 3 are less eager to have decision H than the experts of Example 2. In both examples DM learns about $S = 5$ or 6 whenever this is the case, and he learns about $S = 4$ in some fraction of the cases. DM prefers the arrangement of Example 2 in which he learns that $S = 4$ in $3/5$ of the cases that it is so. In contrast, in Example 3, DM learns about $S = 4$ in $1/5$ of the cases. However, the experts' weaker preferences for H in Example 3 cannot sustain the equilibrium of Example 2. If the experts in Example 3 were organized in two groups, each of the groups would choose to suppress the information when it observes only two signals. Therefore, to prevent the groups from "managing" the information in this way, the experts are partitioned into three groups of two.

As was mentioned before, experts choose their report with the understanding that it would have an effect only when they are pivotal. Owing to the difference in preferences between the experts and DM, in the absence of commitment and communication, experts might be pivotal only when they clearly prefer decision L, so they do not have an incentive to report a positive signal. The pooling of information in groups allows to change this situation. Since a group has a more significant piece of information, a sufficiently large group might be made pivotal in situations in which they prefer decision H, despite the differences in preferences between DM and the experts. But the size of the group should not be too large or else it would have incentives to suppress some of its information.

Let us conclude this section with a remark on its relations to the broader literature on mechanism design. Notice that this section considers a specific class of mechanisms for information gathering that might mitigate the problem created by DM's inability to commit. The subject of mechanism design without perfect commitment has been discussed by the literature on the 'Ratchet Effect,' for some special cases, and in more general terms by Bester and Strausz (2001). That literature provides characterizations of what can be achieved in certain contexts subject to the lack of commitment. Taking a more general approach, Barany (1992), Forges (1990), Ben-Porath (1998), and Gerardi (2001) provide systematic characterization of the set of outcomes that are implementable in the absence of commitment. The implication of this for the discussion of this section is that it may be possible to design a mechanism that will elicit more information than is elicited by the partition into groups. However, such mechanism might require the enforcement of elaborate communication structures. The analysis of this section did not enter these questions and chose instead to focus on the specific natural form of organizing the experts into separate groups.

6. Conclusion

This paper has discussed some of the issues arising in a situation in which a group of like minded informed experts are called to advise a decision maker who has different preferences. The focus was on the ways in which the decision maker can elicit the information in the most effective way. The benchmark observation was that, if the decision maker cannot commit to a mechanism and there is no communication among the experts, then no meaningful information is elicited from the experts in equilibrium. The subsequent observations concerned situations in which commitment or communication are possible. First, if DM can commit to a mechanism, it is sometimes possible to elicit substantially more information than the experts would like to reveal. The most effective mechanisms are typically not monotonic in the experts' reports. If commitment is impossible but the experts can be partitioned into groups such that the members of each group can communicate with each other before they report their information, then it is also possible to elicit more information than the experts would like to reveal. The most effective grouping for the purpose of eliciting information depends on the difference in preferences between the decision maker and the experts. All of these observations make repeated use of the idea that the experts choose their report with the understanding that it matters only when they are pivotal. Perhaps the more interesting observation out of those mentioned above concerns the enhanced inducement for revelation that partial communication creates in the no commitment case, both in comparison to the full communication case and in comparison to the no communication case.

Finally, it might be useful to mention some potentially interesting extensions. The above discussion was conducted under the somewhat extreme simplification that experts are identical in preferences the precision of information. It is natural to inquire how asymmetries in these aspects affect the design of the communication. It is intuitively clear that these ideas continue to hold when experts' preferences are sufficiently similar. It also appears intuitively clear that what is important here is not that the same bias is shared by *all* experts but rather that there is an identifiable subgroup of experts whose preferences are biased in such a manner. Since the signals are independent, the subproblem of eliciting the information from such a subgroup is essentially the same as the problem discussed above. It will be useful to explore how such subgroups might be selected.

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Appendix A

Proof of Claim 2. Define

$$z(n) = \sum_{\{s:\ S=n\}} x(s) \Big/ \binom{N}{n}. \tag{A.1}$$

This implies

$$\begin{aligned}\sum_s \Pr(s)x(s)V(S) &= \sum_{n=0}^{N} \sum_{\{s:\ S=n\}} \Pr(s)x(s)V(S) \\ &= \sum_{n=0}^{N} V(n)\left[\Pr(S=n) \Big/ \binom{N}{n}\right] \sum_{\{s:\ S=n\}} x(s) \\ &= \sum_{n=0}^{N} \Pr(S=n)z(n)V(n).\end{aligned}$$

Now,

$$\begin{aligned}
&\sum_{s_{-i}} \Pr(s_{-i}) x(s_{-i}, s_i = 1) U(S_{-i} + 1) \\
&\quad = \sum_{n=0}^{N-1} \sum_{\{s_{-i}:\, S_{-i}=n\}} \left[\Pr(S_{-i} = n) \Big/ \binom{N-1}{n} \right] x(s_{-i}, s_i = 1) U(S_{-i} + 1) \\
&\quad = \sum_{n=0}^{N-1} \Pr(S_{-i} = n) U(n+1) \sum_{\{s_{-i}:\, S_{-i}=n\}} \left[x(s_{-i}, s_i = 1) \Big/ \binom{N-1}{n} \right].
\end{aligned}$$

Summing the last expression over i, we get

$$\begin{aligned}
&\sum_{n=0}^{N-1} \Pr(S_{-i} = n) U(n+1) \sum_{i=1}^{N} \sum_{\{s_{-i}:\, S_{-i}=n\}} \left[x(s_{-i}, s_i = 1) \Big/ \binom{N-1}{n} \right] \\
&\quad = \sum_{n=0}^{N-1} \Pr(S_{-i} = n) U(n+1) \sum_{\{s:\, S=n+1\}} \left[(n+1) x(s) \Big/ \binom{N-1}{n} \right] \\
&\quad = N \sum_{n=0}^{N-1} \Pr(S_{-i} = n) U(n+1) \sum_{\{s:\, S=n+1\}} \left[x(s) \Big/ \binom{N}{n+1} \right] \\
&\quad = N \sum_{n=0}^{N-1} \Pr(S_{-i} = n) z(n+1) U(n+1).
\end{aligned}$$

Similarly,

$$\begin{aligned}
&\sum_{i=1}^{N} \sum_{s_{-i}} \Pr(s_{-i}) x(s_{-i}, s_i = 0) U(S_{-i} + 1) \\
&\quad = N \sum_{n=0}^{N-1} \Pr(S_{-i} = n) z(n) U(n+1).
\end{aligned}$$

Thus, if $x(s)$ is a solution for problem (1), then $z(n)$ as defined by (A.1) gives the same expected payoff to DM and it is a solution to problem (2). Conversely, any solution z to problem (2) induces a solution x to problem (1) by choosing $x(s) = z(S)$. This establishes the claim. □

Proof of Claim 3. The Lagrangian of the linear program (2) is

$$L\left[\{z(i), \mu(i), \nu(i)\}_{i=0}^{N}, \lambda\right] = \sum_{n=0}^{N} \Pr(S = n) z(n) V(n)$$

$$+\lambda\sum_{n=0}^{N}\Pr(S_{-i}=n)\bigl[z(n+1)-z(n)\bigr]U(n+1)$$
$$+\sum_{n=0}^{N}\mu(n)z(n)+\sum_{n=0}^{N}\nu(n)[1-z(n)],$$

where $\lambda>0$, $\mu(i)\geqslant 0$, $\nu(i)\geqslant 0$, $i=1,\ldots,N$, are Lagrange multipliers. The Kuhn–Tucker conditions are

$$\Pr(S=n)V(n)+\lambda\bigl[\Pr(S_{-i}=n-1)U(n)-\Pr(S_{-i}=n)U(n+1)\bigr]$$
$$+\mu(n)-\nu(n)=0, \tag{A.2}$$
$$\sum_{n=0}^{N}\Pr(S_{-i}=n)\bigl[z(n+1)-z(n)\bigr]U(n+1)=0, \tag{A.3}$$
$$\mu(n)z(n)=0; \qquad \nu(n)[1-z(n)]=0. \tag{A.4}$$

Using $\Pr(S=n)=\binom{N}{n}q^n(1-q)^{N-n}=(Nq/n)\Pr(S_{-i}=n-1)=[N(1-q)/(N-n)]\Pr(S_{-i}=n)$, condition (A.2) can be rewritten as

$$\frac{Nq}{n}V(n)+\lambda\left[U(n)-\frac{(N-n)q}{n(1-q)}U(n+1)\right]+\frac{\mu(n)-\nu(n)}{\Pr(S_{-i}=n-1)}=0. \tag{A.5}$$

Thus $z(n)=1$ if

$$\frac{Nq}{n}V(n)>\lambda\left[\frac{(N-n)q}{n(1-q)}U(n+1)-U(n)\right];$$

$z(n)=0$ if the reversed inequality holds, and $z(n)\in[0,1]$ if it holds with equality.

For the data of the example,

$$\frac{(N-n)q}{n(1-q)}U(n+1)-U(n)$$

is increasing over $[n_D,n_E-1]$ and is positive at n_E-1; it is negative for all $n\geqslant n_E$, if $Nq<n_E$, and it is non-negative for all $n\geqslant n_E$, if $Nq\geqslant n_E$. In addition $(Nq/n)V(n)$ is decreasing for $n\geqslant n_D$. Thus, in the case $Nq<n_E$, for any λ in the relevant range, there is a unique level $\tilde{n}$ such that

$$\frac{Nq}{n}V(n)=\lambda\left[\frac{(N-n)q}{n(1-q)}U(n+1)-U(n)\right].$$

This level determines $z(n)$ as described in (4). The level of λ and hence $\tilde{n}$ are pinned down by constraint (A.3). If $\Pr(S_{-i}=n_D-1)\leqslant\Pr(S_{-i}=n_E-2)+\Pr(S_{-i}=n_E-1)$, this constraint takes the form

$$-\Pr(S_{-i}=n_D-1)$$
$$+\bigl[\Pr(S_{-i}=n_E-2)+\Pr(S_{-i}=n_E-1)\bigr](1-z)=0. \tag{A.6}$$

In this case $\tilde{n} = n_E - 1$ and $z(\tilde{n}) = z$, where $z \in [0, 1]$ solves (A.6). Otherwise, this constraint is of the form

$$-\Pr(S_{-i} = n_D - 1) + \Pr(S_{-i} = \tilde{n} - 1)(1 - z) + \Pr(S_{-i} = \tilde{n})z \\ + \Pr(S_{-i} = n_E - 1) = 0,$$

where $\tilde{n}$ is the maximal $n \in [n_D, n_E - 1)$ that satisfies it for some $z \in [0, 1]$. In this case $z(\tilde{n}) = z$. Similarly, in the case $Nq \geqslant n_E$, we get $\tilde{n} = n_E - 1$ and $z(\tilde{n}) = z$, where z is determined by (A.6). □

Proof of Proposition 4. A monotonic $z(n)$ is characterized by n_1 and $n_2 > n_1$ such that $z(n) = 0$ for $n \leqslant n_1$, $z(n) \in (0, 1)$ for $n_1 < n < n_2$, and $z(n) = 1$ for $n \geqslant n_2$. If z is a solution to problem (2), it follows from the Kuhn–Tucker conditions derived in the proof of Claim 3 above (in particular see (A.5)) that, for $n_1 < n < n_2$,

$$\frac{Nq}{n} V(n) + \lambda \left[U(n) - \frac{(N-n)q}{n(1-q)} U(n+1) \right] = 0.$$

Now, for generic choice of U and V this equality is satisfied for at most one value of n. Thus, either $n_2 = n_1 + 1$, in which case $z(n)$ takes on only the values 0 and 1, or $n_2 = n_1 + 2$. Condition (A.3) from the proof of Claim 3 then implies that in both cases $n_2 = n_E$. Thus, in the former case $z(n)$ exactly implements the experts' best outcome, while in the latter case H is implemented with positive probability when there are $n_E - 1$ positive signals as well. □

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