# Proof of Propositions 3, 4 and 5

# Proof of Proposition 3

We first list possible equilibrium configurations and then determine the parameter values under which they hold.

(a) *Noncontingent real authority*

In this trivial equilibrium, the sender recommends action $A$, which is rubberstamped by the receiver without cue or issue-relevant communication. This equilibrium exists if and only if $\alpha \geq \alpha^*$.

(b) *No cue, issue-relevant communication*

In this equilibrium, absent issue-relevant communication, decision $A$ would not be taken by $R$. Equilibrium values for effort ($\overline{x}^*, \underline{x}^*$ and $y^*$) are then given by:

$$S'\left(\overline{x}^*\right) = y^* \overline{\alpha} s$$

$$S'\left(\underline{x}^*\right) = y^* \underline{\alpha} s$$

and:

$$R'\left(y^*\right) = (\gamma \overline{\alpha}\ \overline{x}^* + (1-\gamma)\underline{\alpha}\ \underline{x}^*) r_H.$$

For future reference, let $(\overline{U}_S^*, \underline{U}_S^*, U_R^*)$ denote the corresponding equilibrium payoffs.

(c) *Cue and issue-relevant communication*

Under cue communication, the receiver's effort depends on the revealed congruence.

*For low conguence* ($\widetilde{\alpha} = \underline{\alpha}$), issue-relevant efforts ($\underline{x}^c, \underline{y}^c$) are given by:

$$S'\left(\underline{x}^c\right) = \underline{y}^c \underline{\alpha} s$$

and:

$$R'\left(\underline{y}^c\right) = \underline{x}^c \underline{\alpha} r_H.$$

Let $(\underline{U}_S^c, \underline{U}_R^c)$ denote the associated payoffs (gross of the cue-communication costs). It can be shown that both efforts are lower than in the absence of cues.

*For high congruence* ($\widetilde{\alpha} = \overline{\alpha}$), two cases have to be distinguished. A first case is $\overline{\alpha} \geq \alpha^*$. Then, the receiver rubberstamps the sender's recommendation in the absence of issue-relevant communication: The sender enjoys "contingent real authority". Issue-relevant communication vanishes. *The two modes of communication are then substitutes.*

The second case is $\overline{\alpha} < \alpha^*$. Then the receiver does not pick action $A$ unless he is convinced through issue-relevant communication. The equilibrium is then given by:

$$S'(\overline{x}^c) = \overline{y}^c \overline{\alpha} s$$

and:

$$R'(\overline{y}^c) = \overline{x}^c \overline{\alpha} r_H.$$

It can be shown that both efforts are higher than in the absence of cues, and so *the two modes of communication are complements.*

Whether the high-congruence type enjoys real authority ($\overline{\alpha} \geq \alpha^*$) or not ($\overline{\alpha} < \alpha^*$), let $(\overline{U}^c_S, \overline{U}^c_R)$ denote the equilibrium utilities (gross of the cue-communication costs). One has:

$$\overline{U}^c_S > \overline{U}^*_S$$

$$\underline{U}^c_S < \underline{U}^*_S$$

$$U^c_R \equiv \gamma \overline{U}^c_R + (1-\gamma)\underline{U}^c_R \gtrless U^*_R.$$

The first two inequalities reflect the fact that cue communication removes "cross-subsidies" between the two types. In particular, the receiver does not try as hard when learning that congruence is low. By contrast, the receiver may gain or lose from the disclosure of cues, even though he can better fine-tune his effort. First, the cue may allow the high-congruence type to economize on issue-relevant communication ($\overline{\alpha} \geq \alpha^*$). Second, even if the high-congruence type does not enjoy real authority ($\overline{\alpha} < \alpha^*$), the reduction in the low-congruence type's effort may penalize the receiver more than the latter benefits from an increased effort by the high-congruence type.[1]

We are now in a position to determine the domain of existence of the various equilibria (we have already noted that the non-contingent real authority equilibrium exists if and only if $\alpha \geq \alpha^*$). We have until now applied the Pareto-dominance criterion to eliminate Pareto-dominated low-communication equilibria. Here, matters are more complex, as the low-congruence sender tends to lose from cue-communication. Still, we can rule out "cue-communication failures" between the high-congruence sender and the receiver.

(b) *No cue, issue-relevant communication*

For this to be an equilibrium, we of course first need equilibrium payoffs $(\overline{U}^*_S, \underline{U}^*_S, U^*_R)$ to be nonnegative. Second, as in subsection 2.2, the receiver must prefer not to take action $A$ in

[1] For example, think of the case in which $x$ is bounded above, and the high-congruence type's effort already hits the bound in the absence of the cue.

the absence of communication, and must find it worthwhile to exert effort (and, here, neither condition implies necessarily the other one).[2] Third, ruling out cue-communication failure, the high-congruence sender and the receiver should not have a joint interest in engaging in cue-communication.[3]

(c) *Cue and issue-relevant communication*

For this to be an equilibrium, we need equilibrium payoffs $(\underline{U}_S^c, \overline{U}_S^c - S_c, U_R^c - R_c)$ to be nonnegative. Moreover, the receiver should not prefer not to listen to the cue,[4] and the high-congruence sender should not prefer not to communicate the cue (knowing that it would prompt the receiver to believe she is the low-congruence type).[5] ■

## Proof of Proposition 4

(a) *No cues* (parts (i) and (ii) of the Proposition). Note first that, because of set-up costs $S(0)$ and $R(0)$, multiple equilibria can still occur, since when one party exerts no effort, the other one may not find it worthwhile exerting positive effort either.[6] A second observation is that, under supervisory decision-making, the possibility that communication vanishes when congruence becomes so high that $S$ enjoys real authority remains, when set-up costs are such that $x = 0$ implies $y = 0$. When communication does occur, a rise in any one party's stake in the project or in congruence increases communication intensity. Indeed, when $S$ does not know

[2] The first condition is equivalent to:

$$r_H(\gamma\overline{\alpha}(1 - \overline{x}^* y^*) + (1-\gamma)\underline{\alpha}(1 - \underline{x}^* y^*) + r_L(\gamma(1-\overline{\alpha})(1 - \overline{x}^* y^*) + (1-\gamma)(1-\underline{\alpha})(1 - \underline{x}^* y^*) < 0,$$

while the second condition is equivalent to:

$$U_R^* = r_H(\gamma\overline{\alpha}\overline{x}^* y^* + (1-\gamma)\underline{\alpha}\underline{x}^* y^*) - R(y^*) \geq \max\{0; \alpha r_H + (1-\alpha) r_L\},$$

and it can be checked that either constraint can be the more stringent one.

[3] In such a joint deviation, the low-congruence sender's effort would remain at $\underline{x}^*$, leading the receiver, upon not observing a favorable cue, to exert an effort $\widetilde{y}$ defined by:

$$R'(\widetilde{y}) = \underline{x}^* \underline{\alpha} r_H$$

and to have associated payoff $\widetilde{U}_R$. If on the other hand the sender does observe the favorable cue, efforts are $\overline{x}^c$ and $\overline{y}^c$. Therefore, ruling out a joint deviation requires either $\overline{U}_S^* \geq \overline{U}_S^c + S_c$ or $U_R^* \geq (\gamma\overline{U}_R^c + (1-\gamma)\widetilde{U}_R) + R_c$.

[4] This is equivalent to requiring:

$$\max_y \quad \{[(\gamma\overline{\alpha}\ \overline{x}^c + (1-\gamma)\underline{\alpha}\ \underline{x}^c)]\, y r_H - R(y)\} \leq U_R^c - R_c.$$

[5] This is equivalent to requiring:

$$\max_x \quad \{\underline{y}^c x\overline{\alpha}s - S(x)\} \leq \overline{U}_S^c - S_c.$$

[6] Because of set-up costs, there remains an element of strategic complementarity, which is responsible for the no-communication equilibrium. But the interior equilibrium is one where strategic substitutability is at work.

$R$'s payoff (as in section 2.1), the first-order conditions become:

$$p'(x^* + y^*)\alpha s = S'(x^*)$$

and:

$$p'(x^* + y^*)\alpha r_H = R'(y^*),$$

so that a rise in $s$, $r_H$ or $\alpha$ increases $x^* + y^*$. When $S$ does know $R$'s payoff (as in section 2.2), the first-order conditions become:

$$p'(x^* + y^*)s = S'(x^*)$$

and:

$$p'(x^* + y^*)\alpha r_H = R'(y^*).$$

Once again, a rise in $s$, $r_H$ or $\alpha$ increases $x^* + y^*$.

(b) *Cues* (part (iii) of the Proposition). Let us now turn to section 3 and the role of cues. In the benchmark where cues are not communicated, the first-order conditions for issue-relevant effort ($\overline{x}^*, \underline{x}^*$ and $y^*$) become:

$$S'\left(\overline{x}^*\right) = p'(\overline{x}^* + y^*)\overline{\alpha}s$$

$$S'\left(\underline{x}^*\right) = p'(\underline{x}^* + y^*)\underline{\alpha}s$$

and:

$$R'\left(y^*\right) = (\gamma\overline{\alpha}\ p'(\overline{x}^* + y^*) + (1-\gamma)\underline{\alpha}\ p'(\underline{x}^* + y^*))r_H.$$

Note that, from the two first-order conditions for $S$, we have $\overline{x}^* > \underline{x}^*$ and $p'(\overline{x}^* + y^*)\overline{\alpha} > p'(\underline{x}^* + y^*)\underline{\alpha}$. Consider now cue communication. For low congruence ($\widetilde{\alpha} = \underline{\alpha}$), issue-relevant efforts ($\underline{x}^c, \underline{y}^c$) are now given by:

$$S'\left(\underline{x}^c\right) = p'(\underline{x}^c + \underline{y}^c)\underline{\alpha}s$$

and:

$$R'\left(\underline{y}^c\right) = p'(\underline{x}^c + \underline{y}^c)\underline{\alpha}r_H.$$

Comparing the relevant first-order conditions, we obtain again that bad news about cues reduces effort, in that $\underline{x}^c + \underline{y}^c < \underline{x}^* + y^*$.[7]

For high congruence ($\widetilde{\alpha} = \overline{\alpha}$), cues can, as before, crowd out issue-relevant communication when they allow $S$ to enjoy real authority and set-up costs are such that $x = 0$ implies $y =$

[7]If not, we would have $\underline{x}^c < \underline{x}^*$ and therefore $\underline{y}^c > y^*$, a contradiction with $p'(\underline{x}^c + \underline{y}^c)\underline{\alpha} < p'(\underline{x}^* + y^*)\underline{\alpha} < p'(\overline{x}^* + y^*)\overline{\alpha}$ and the first-order conditions for $R$.

0. When congruence is not high enough to lead to real authority for $S$, cues again become complementary to issue-relevant communication. The equilibrium is given by:

$$S'\left(\overline{x}^c\right) = p'(\overline{x}^c + \overline{y}^c)\overline{\alpha}s$$

and:

$$R'\left(\overline{y}^c\right) = p'(\overline{x}^c + \overline{y}^c)\overline{\alpha}r_H.$$

Communication is higher than without cues in that $\overline{x}^c + \overline{y}^c > \overline{x}^* + y^*$ (by a similar reasoning than in the case of low congruence). ■

# Proof of Proposition 5

The receiver's payoff can be written as:

$$\rho r\left[x_H^1 y^1(M + x_H^2(S)y^2(S)) + (1 - x_H^1 y^1)(m + x_H^2(F)y^2(F))\right]$$
$$+(1-\rho)r\left[x_L^1 y^1(M + x_L^2(S)y^2(S)) + (1 - x_L^1 y^1)(m + x_L^2(F)y^2(F))\right]$$
$$-R_1(y^1) - (\rho x_H^1 y^1 + (1-\rho)x_L^1 y^1)R_2(y^2(S)) - (\rho(1 - x_H^1 y^1) + (1-\rho)(1 - x_L^1 y^1))R_2(y^2(F)).$$

As for the sender's payoff, it can be written as:

$$s_i\left[x_i^1 y^1(M + x_i^2(S)y^2(S)) + (1 - x_i^1 y^1)(m + x_i^2(F)y^2(F))\right]$$
$$-S_1(x_i^1) - x_i^1 y^1 S_2(x_i^2(S)) - (1 - x_i^1 y^1)S_2(x_i^2(F))$$

when her payoff from action $A$ is $s_i$ (with $i = H, L$). Taking derivatives with respect to the sender's period-2 choices yields:

$$s_i y^2(z) = S_2'(x_i^2(z))$$

for $z = S, F$. This implies:

$$x_H^2(z) > x_L^2(z).$$

Consider now the receiver's second-period choice. Given correct expectations of the sender's period-1 effort choices, her second-period beliefs are:

$$\rho'(S) = \frac{\rho x_H^1 y^1}{\rho x_H^1 y^1 + (1-\rho)x_L^1 y^1}$$

and:

$$\rho'(F) = \frac{\rho(1 - x_H^1 y^1)}{\rho(1 - x_H^1 y^1) + (1-\rho)(1 - x_L^1 y^1)}.$$

This means:

$$\rho'(S) > \rho'(F) \quad \text{iff} \quad x_H^1 > x_L^1.$$

Let us make for now the assumption:

$$x_H^1 > x_L^1.$$

Taking derivatives with respect to the receiver's period-2 choices yields:

$$r\left[\rho'(z)x_H^2(z) + (1-\rho'(z))x_L^2(z)\right] = R_2'(y^2(z))$$

for $z = S, F$. Given that $x_H^2(z) > x_L^2(z)$ and $\rho'(S) > \rho'(F)$, we have:

$$y^2(S) > y^2(F).$$

Coming back to the sender's period-2 choice:

$$s_i y^2(z) = S_2'(x_i^2(z)),$$

given that $y^2(S) > y^2(F)$, we have:

$$x_i^2(S) > x_i^2(F).$$

Taking now derivatives with respect to the sender's period-1 choice yields:

$$y^1\left[s_i(M - m + x_i^2(S)y^2(S) - x_i^2(F)y^2(F)) - S_2(x_i^2(S)) + S_2(x_i^2(F))\right] = S_1'(x_i^1).$$

The LHS of this condition is increasing in $s_i$ since $x_i^2(S)y^2(S) > x_i^2(F)y^2(F)$, so that we have:

$$x_H^1 > x_L^1,$$

which validates our starting assumption. We have thus proved parts (i) and (ii) of the Proposition. As for part (iii), note first that the sender's marginal gain from exerting effort in period 1, for a given period-1 effort level by the receiver, rises by:

$$y^1 s_i\left[x_i^2(S)y^2(S) - x_i^2(F)y^2(F)\right] > 0$$

because of period-2 communication. And the receiver's marginal gain from exerting effort in period 1, for given period-1 effort levels by the sender, rises by:

$$r\rho x_H^1\left[x_H^2(S)y^2(S) - x_H^2(F)y^2(F)\right] + r(1-\rho)x_L^1\left[x_L^2(S)y^2(S) - x_L^2(F)y^2(F)\right] > 0.$$

Taken together, these two increases in marginal gains imply part (iii) of the Proposition. ■