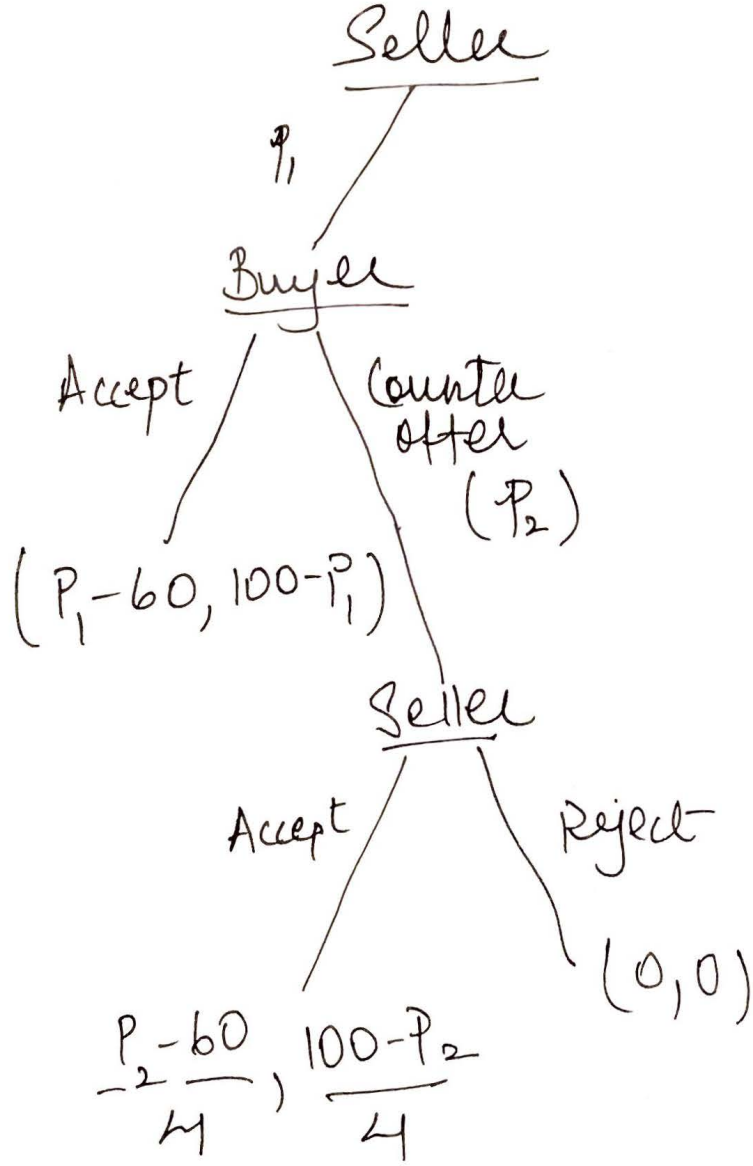
ASSIGNMENT - 3 - ~~Quest~~

Q2)

a) Game Tree



seller offer price = P_1
then if accepted
seller gets $P_1 - 60$
and buyers
utility is $100 - P_1$

Counter offer = P_2
given
$\delta_S = \delta_B = \frac{1}{4}$

∴ Seller gets
$\frac{P_2 - 60}{4}$
buyer
$\frac{100 - P_2}{4}$

b) To find subgame perfect NE
using backward induction.
for seller to accept in second round

ASSUMPTIONS

$P_2 \geq 60 \quad \left(\because \frac{P_2 - 60}{4} \geq 0\right)$

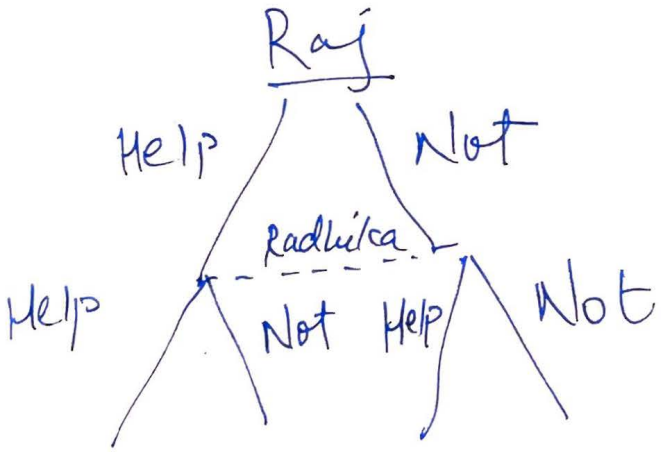
∴ Best response of consumer is to offer 60. ⇒ $\frac{100 - P_2}{4} \Rightarrow \frac{100 - 60}{4} = 10$.

for Buyer to accept P_1

$100 - P_1 \geq 10$

or $P_1 \leq 90$ ∴ Best response (optimal) for seller to offer $P_1^* = 90$. gives the SPNE.

(Q1) we can draw extensive tree for this simultaneous game.



Calculating payoffs. (Let Raj = (1)
Radhika = (2))

CASE - RAJ HELPS

Then $C_1(q_1) = 2q_1$ - Raj cost.

$\Pi_2(q_1) = 3q_1$ - Radhika profit

Similarly if Radhika helps

$C_2(q_2) = 2q_2$

$\Pi_1(q_2) = 3q_2$

Raj \ Radhika	H	N
H	$3q_2 - 2q_1$, $3q_1 - 2q_2$	$-2q_1, 3q_1$
N	$3q_2, -2q_2$	$0, 0$

In this 1 period game.

$BR_1(2) =$ N for H and N for N

$BR_2(1) =$ N for H and N for N

$\therefore$ unique NE = (0, 0) (Strictly dominates)

(11) In finite repetition SPNE is the NE also (show for T=2 using BI)

(c) Infinite version of game
for what δ is social optimal the SPNE.

$$U = u_1(q_1, q_2) + u_2(q_1, q_2)$$

max wrt q_1 and q_2 where $q_1, q_2 \in [0,1]$

$$U = -2q_1 + 3q_2 - 2q_2 + 3q_1$$

$$U = q_1 + q_2$$

$\therefore$ U is maximized at $q_1 = q_2 = 1$

This coordination can be achieved through grim trigger strategy

$q_i = 1$ if at all previous ~~times~~ nodes $q_i = 1, 1, 1 \ldots$ and $q_i = 0$ if otherwise

$\therefore u_1 = u_2 = 1 \Rightarrow 1 + \delta + \delta^2 + \ldots = \frac{1}{1-\delta}$
(when $q = 1 \forall t$)

If deviation ie $v_i = 0$

then at deviation pay off = 3 and 0 after every Time period.

ie $U_i = 3 + 0 + 0 \ldots = 3$

$\therefore$ for SPNE.

$$\frac{1}{1-\delta} \geq 3 \Rightarrow \boxed{\delta \geq \frac{2}{3}}$$