Information transmission in regulated markets

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Abstract. The seminal papers of Crawford and Sobel (1982) and Green and Stokey (1980) study models in which a signal about the state of the world is transmitted from a perfectly informed sender to an uninformed receiver. We study a model in which *multiple* signallers *compete* for consumers. The questions asked are: How much information is revealed? How does consumers' shopping around affect this information? How is each affected by incentives? An implication of the model is that in a price-regulated market for health care, patients should not be prohibited from gathering more than one opinion.

Transmission de l'information dans les marchés réglementés. Les mémoires fondateurs de Crawford et Sobel (1982) et de Green et Stokey (1980) ont étudié des modèles dans lesquels un signal quant à l'état du monde est transmis par un émetteur parfaitement informé à un récepteur mal informé. Les auteurs étudient un modèle dans lequel il y a de *multiples* émetteurs qui *luttent* pour l'attention des consommateurs. Les questions qu'on se pose sont: combien d'information est révélée? comment est-ce que le magasinage des consommateurs affecte cette information? comment est-ce-que chacun est affecté par les incitations? Un des résultats de ces analyses est que dans un marché réglementé comme celui des soins on ne devrait pas défendre aux patients d'obtenir plus qu'une opinion.

I. INTRODUCTION

The seminal papers of Crawford and Sobel (1982) and Green and Stokey (1980) study models of information transmission in which a signal about the state of the

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world is transmitted from a perfectly informed sender to an uninformed receiver. Upon receipt of the signal the receiver takes an action. The welfares of both the sender and the receiver are affected by the state of the world and by the action taken; their interests may be close or far apart. One situation that the model captures is the provision of medical services: a doctor provides a diagnosis (signal) and a patient decides which treatment to accept. The model also captures many situations of expert – client interaction: for example, automobile repair, the market for legal advice and action, and the markets for complex consumer goods (e.g., the expert is a computer salesperson and the client is a consumer).

The model omits one prominent feature of such situations, however: the fact that consumers can consult many experts before making a decision – in many situations *multiple* signallers *compete* for consumers. We study a model that incorporates this feature. There are many sellers (senders) and many consumers. Each seller knows more about the right match between any given consumer and the product than does the consumer. Each consumer searches sequentially among sellers for opinions before taking an action, patronizing the last seller who gave a signal consonant with the decision he[1] made. We focus on the following questions. How much information is revealed in such a market? How does consumers' shopping around affect the information that is revealed? How is the outcome affected by changing incentives?

In some markets price may be a signal.[2] In order to isolate the use of opinions as signals we consider a market in which the price is fixed. Our model thus applies to markets in which the price is regulated, like the markets for medical services in Canada, Great Britain, and Finland, and the markets for legal-aid clients in Canada and the United States. It also applies to markets in which prices are not determined directly by the signallers, or in which the most significant issue is the type of product to be purchased, not the price at which it sold. Computers with the same characteristics, for example, may sell at roughly the same price; the major issue in the market may be which features a consumer requires.

In our model each expert makes a higher profit if she sells an expensive product than if she sells an inexpensive one. However, the presence of competing experts means that a consumer who is advised to buy an expensive product may consult other experts before making a decision. The competition among the experts leads them to reveal some of their private information about the product that is appropriate for the consumer.

We find that in an equilibrium some (but not all) of the available information is released: experts do not uniformly advise consumers to purchase the most profitable product. More information is revealed in equilibrium if search is less costly for consumers. More information is also revealed in equilibrium if the interests of consumers and experts are closer, in that experts have less incentive to hide the facts from the consumer. It is well known that search involves an externality: the

1 For ease of exposition, we refer to an expert as 'she' and to a consumer as 'he.'

2 See Wolinsky (1983) and Wolinsky (1991) for examples of unregulated markets in which consumers use price as a signal of quality.

fact that consumers search may affect the equilibrium to the benefit of all who search. In the absence of search in our model all experts would simply recommend the most profitable product; it is the fact that consumers obtain 'second opinions' that polices the market. An implication is that in a regulated market for health care (as in Canada), for example, patients should not be prohibited from gathering more than one opinion. In our model the seller does not incur a cost when giving an opinion. In this case the presence of the externality implies that if consumers can obtain as many opinions as they wish, then they still search less than is optimal. It follows that if there exists a cost to the seller that is not too large, it remains optimal to impose no constraint at all on the number of opinions that consumers can obtain.

A regulator in our market can manipulate the prices of the services and the cost of search. We show that under some conditions a reduction in the price differential between the two types of service or a reduction in the cost of search leads experts to be more precise in equilibrium and reduces the aggregate expected loss of consumers. An implication is that the prices of services should be as close as possible given feasibility constraints, and that search should be made as easy as possible for consumers.

We consider extensions of the model in section v. In a model in which the number of experts is endogenous we find that as the interests of consumers and experts become closer, the number of experts increases. In a model in which the experts' skill is endogenous we find that as the interests of consumers and experts become closer, the experts become, on average, more competent.

II. THE MODEL

There are two populations of players, experts, and consumers, each identified with an interval in the real line. Consumers are of two types: those who need an expensive service (E-consumers) and those who need an inexpensive service (I-consumers). The fraction of E-consumers is $0 < \zeta < 1$. Each expert knows the type of every consumer, while each consumer knows only that the probability that he is an E-consumer is ζ.

Each consumer searches sequentially for opinions from randomly chosen experts; once he makes a decision he purchases a service from the last expert consulted whose opinion matches his decision. Consumers differ in their search cost; the unit cost of search is distributed according to the differentiable distribution function F. We assume that $F(0) = \phi > 0$: the search cost is zero for the fraction $\phi > 0$ of consumers.

An expensive service (E) fixes the problems of both an E- and an I-consumer, so that a consumer who purchases an E-service never learns his type (i.e., whether he needed an E- or an I-service).[3] An inexpensive service does not solve the problem of an E-consumer, however, and we assume that there is a liability rule

3 That is, an E-service is a 'credence good' (Darby and Karni 1973).

that penalizes an expert who signals I to an E-consumer. (In the medical context, malpractice laws are part of the liability rule.) We assume that the penalty is high enough that the only rational action for an expert who confronts an E-consumer is to signal E; thus we restrict experts to signal only E to E-consumers. The only choice that remains for an expert is whether to signal E ('cautious') or I ('precise') to an I-consumer.

As to the service *provided* by an expert to an I-consumer who requests an E-service, our model encompasses two cases. In one case, the service that is provided can be detected and consumers who request and pay for an E-service can sue experts who provide only I-services, making it optimal for experts to provide an E-service to any consumer who requests one. In the other case the service provided cannot be determined, and the expert provides the service *needed* independent of the service requested. (That is, sellers routinely cheat on the service they provide to I-consumers who request E-services.)

In either case, the experts' profits and consumers' losses depend only on the service needed and the service requested. We denote the loss of a consumer of type $s \in \{E, I\}$ who requests the service $d \in \{E, I\}$ by $C(s, d)$, and assume that[4]

$$C(I, I) = C(E, E) = 0 < C(I, E), C(E, I).$$

Each consumer takes the decision that minimizes his expected loss.

When confronted with a consumer an expert gives an opinion (either E or I). If patronized by a consumer of type s, an expert's unit pay-off is $\pi(s, d)$ if she is asked for the service $d \in \{E, I\}$. We assume that

$$\pi(I, E) > \pi(I, I) > 0 \quad \text{and} \quad \pi(I, E) > \pi(E, E) > 0.$$

Note that an expert obtains a profit from a consumer only if the consumer chooses to obtain the services from her. We assume that experts are risk-neutral profit-maximizers.

Knowing that an expert signals I only to an I-consumer, a consumer who obtains an I-signal knows that his type is I and thus immediately obtains an I-service. Thus in choosing a signal for an I-consumer an expert confronts a trade-off: a cautious signal (E) reaps more profit if the consumer is persuaded to choose an expensive service *and* returns to the expert to purchase it, while a precise signal (I) reaps a smaller profit with certainty. Note that if most experts give cautious opinions, then fixing the consumers' search behaviour, the probability that any particular expert will be patronized is low.

The expected loss of a consumer who chooses an E-service without search is $\zeta C(E, E) + (1 - \zeta)C(I, E) = (1 - \zeta)C(I, E)$, while that of a consumer who chooses an I-service without search is $\zeta C(E, I) + (1 - \zeta)C(I, I) = \zeta C(E, I)$. We assume that

$$\zeta > C(I, E)/[C(I, E) + C(E, I)],$$

4 The solution of the model remains a solution if $C(I, E) = 0$ or if $C(I, E) = C(E, I)$; however, other solutions may also exist.

so that it is optimal for a consumer who does not search to obtain an E-service. Thus the expected loss of a consumer who does not search is

$$\rho(\zeta) = (1 - \zeta)C(I, E). \tag{1}$$

Finally, we assume for simplicity that no consumer has a search cost so high that he would not search if all experts were precise: the support of the distribution F of search costs is $[0, (1 - \zeta)C(I, E)]$.

The behaviour of the experts results in a probability p that a consumer of type I receives the signal I. This probability determines each consumer's optimal sequential search and a decision rule, which feeds back into the experts' pay-off functions to determine their signalling rules.

The solution concept we use is Harsanyi's (1968) Bayesian Nash equilibrium: each type of each player acts optimally given her (correct) belief about the (probabilistic) behaviour of the other players. An equilibrium consists of a fraction p of experts who signal I to an I-consumer and a search and decision rule for each consumer.

1. Relation to the literature

Our model is related to those of Crawford and Sobel (1982), Green and Stokey (1980), Pitchik and Schotter (1987), and Wolinsky (1991). Crawford and Sobel analyse a model in which a signaller sends a signal to a receiver. They assume that there is a continuum of feasible signals and study how the signals vary as the preferences of the agents become more 'similar'; they find that the signals become finer as the interests of the agents become closer. In our model we can interpret a decrease in the spread between the prices[5] as bringing the interests of the agents closer; we provide a sufficient condition (theorem 4) under which the probability of precision increases as the interests of the agents become closer. We also provide an example in which the probability of precision decreases as the interests of the agents become closer. Green and Stokey analyse the effect of improved information on agents' welfares in a model similar to that of Crawford and Sobel.

Pitchik and Schotter study a model of information transmission in which there is one expert, one consumer of two possible types, and one signal. Though the consumer does not have the opportunity to search, he can opt to get out of the service-market altogether and simply obtain a do-it-yourself kit. (This option plays a role somewhat like that of search in the current model.) They also study how the signals differ as the preferences of the agents become more 'similar' (in a sense different from that of Crawford and Sobel) and obtain results different from those of Crawford and Sobel.

Wolinsky studies a model of information transmission in which consumers search and experts compete in both price and opinions. He finds that experts separate into two groups: those who provide only I-services and those who provide

5 A decrease in the spread between the prices will affect expert profit and consumer losses. Specifically, $\pi(I, E) - \pi(I, I)$ decreases and $C(I, E)$ decreases.

only E-services. The experts announce their intentions by price. Consumers first go to the low-priced expert to find out if they need an I-service. If consumers find that an I-service is not needed, they then patronize the group that provides the E-service.

III. EQUILIBRIA

1. Consumers

It is clear that the optimal behaviour of a consumer is either to obtain an E-service without search (which we refer to as the rule $S(0)$) or to adopt the following rule $S(n)$ for some value of n:

- If an I-opinion is received, then stop and obtain an I-service.
- If no I-opinion has been received and fewer than n E-opinions have been received, then continue searching.
- If no I-opinion has been received and n E-opinions have been received, then stop and obtain an E-service.

The optimal value of n is given in the following result.[6]

PROPOSITION 1. *An optimal rule for a consumer with search cost c is $S(N(\zeta, c, p))$, where*

$$N(\zeta, c, p) = \begin{cases} 0 & \text{if } c > 0 \text{ and } c \geqq pC(I, E) \\ N^*(\zeta, c, p) & \text{if } 0 < c < pC(I, E) \\ \infty & \text{if } c = 0, \end{cases}$$

and $N^*(\zeta, c, p)$ is the smallest non-negative integer n for which

$$(1-p)^n \leqq \frac{c\zeta}{(1-\zeta)[pC(I, E) - c]}, \tag{2}$$

with the convention that the left-hand side is 1 if $p = 1$ and $n = 0$.

Further, the rule $S(N\zeta, c, p))$ is the only optimal stopping rule unless $c = 0$ and $p \in \{0, 1\}$, in which case it is the unique dominant stopping rule among optimal rules. The expected loss of a consumer who uses this rule is

$$\rho_1(\zeta, c, p) = (1-\zeta)(1-p)^{N(\zeta,c,p)}C(I, E) + c\{\zeta N(\zeta, c, p) + (1-\zeta)[1-(1-p)^{N(\zeta,c,p)}]/p\}. \tag{3}$$

6 If $n = \infty$, then the rule requires that the consumer search indefinitely so long as no I-opinion is received. In this case the interpretation that should be given is that the population of experts is large but finite, and the consumer consults all experts before making a decision if no I-opinion is received.

Proof. Since the posterior probability that the consumer needs an I-service is 1 whenever an I-opinion is received, the consumer's optimal action in this case is to stop and obtain an I-service. It remains to determine the optimal number of searches in the case that only E-opinions are received.

$c = 0$: If $p \notin \{0, 1\}$ (i.e., if some experts are precise and some are cautious), then it is uniquely optimal for a consumer with a zero search cost to continue searching so long as no I-opinion has been received. If $p \in \{0, 1\}$, then either only E-opinions or only I-opinions are received, so that the expected loss is constant over all rules $S(n)$ with $n \geqq 0$ if $p = 0$ and over all rules $S(n)$ with $n \geqq 1$ if $p = 1$; the fact that $n = \infty$ is uniquely optimal for $p \notin \{0, 1\}$ means that the rule $S(\infty)$ is the unique dominant rule in these cases.

$c > 0$: We first find the expected loss of a consumer who uses the rule $S(n)$. This loss consists of the expected loss of the final decision plus the expected cost of search. If an I-opinion is received, then the expected loss associated with the final decision is zero. If only E-opinions are received, an event with probability $(1-p)^n$, then the consumer obtains an E-service; the expected loss of this final decision is $(1-\zeta)C(I, E)$.

The expected cost of search is simply the unit cost c multiplied by the expected number of searches. The expected number of searches is derived as follows. With probability ζ the consumer is of type E and therefore receives only E-opinions. In this case the consumer obtains n opinions. With probability $1-\zeta$ the consumer needs an I-service and therefore receives an I-opinion with probability p. In this case the number of searches is m with probability $p(1-p)^{m-1}$ if $m < n$ and is n with probability $(1-p)^n$, so that the expected number of searches is $p[1 + 2(1-p) + \ldots + (n-1)(1-p)^{n-2}] + n(1-p)^{n-1} = [1-(1-p)^n]/p$.

Hence the expected loss of a consumer who uses the rule $S(n)$ is

$$L(n) = (1-\zeta)(1-p)^n C(I, E) + c\{\zeta n + (1-\zeta)[1-(1-p)^n]/p\}.$$

The optimal value of n satisfies $L(m) \geqq L(n)$ for $m \neq n$, which is equivalent to $L(n-1) \geqq L(n)$ if $n \geqq 1$ and $L(n+1) \geqq L(n)$. These two inequalities imply that

$$(1-\zeta)(1-p)^n(pC(I, E)-c) \leqq c\zeta \leqq (1-\zeta)(1-p)^{n-1}(pC(I, E)-c).$$

Hence if $pC(I, E) - c \leqq 0$, the optimal value of n is 0, while if $pC(I, E) - c \geqq 0$, the optimal value of n satisfies

$$(1-p)^n \leqq c\zeta/\{(1-\zeta)[pC(I, E)-c]\} \leqq (1-p)^{n-1}.$$

Since $(1-p)^n$ is decreasing in n, this yields (2); (3) follows by substitution. ∎

In the sequel we assume that every consumer uses the stopping rule $S(N(\zeta, c, p))$ (though for the case $c = 0$ and $p = 0$ we could equally well assume that any rule is used). If $N(\zeta, c, p) = 0$, then this rule specifies that the consumer not search (but simply obtain an E-service). By definition we have $N(\zeta, c, p) = 0$ if $c \geqq pC(I, E)$; however, even if $c < pC(I, E)$, so long as c is not too small, we have $N(\zeta, c, p) = 0$, since the right-hand side of (2) in this case is larger than 1. Precisely,

$$N(\zeta,\ c,\ p) = 0 \text{ if and only if } c \geqq (1-\zeta)pC(I,\ E). \tag{4}$$

Note that N is decreasing in ζ and c. This is so because as the benefits of search decrease or the costs of search increase, consumers search less. Note also that N is first increasing and then decreasing in p. This change can be explained as follows. One effect of a higher value of p is to increase the trustworthiness of any opinions already received, which induces consumers to search less. However, a higher value of p also increases the trustworthiness of any future opinions to be obtained, which induces consumers to search more. When p is high and increases, the first effect dominates; when p is low and increases, the second effect dominates.

2. Experts
The following result gives the optimal behaviour of an expert.

PROPOSITION 2. *It is optimal for an expert to be precise if and only if* $\pi(I, I) \geqq H(p)\pi(I, E)$ *and it is optimal for her to be cautious if and only if* $\pi(I, I) \leqq H(p)\pi(I, E)$*, where*

$$H(p) = \int_{[0,\, p(1-\zeta)C(I,E)]} (1-p)^{N(\zeta,c,p)-1} dF(c), \tag{5}$$

with the convention that $0^0 = 1$.

Proof. An expert who signals I to an I-consumer obtains a unit profit of $\pi(I, I)$ for each consumer who solicits her opinion. An expert who signals E to an I-consumer makes a profit only on consumers with positive search costs who have so far obtained a sequence of $N(\zeta, c\,p) - 1$ E-opinions. The fraction of such consumers is $(1-p)^{N(\zeta,c,p)-1}$, so that the unit profit of an expert who gives a cautious signal to a consumer with a positive search cost who searches is $\pi(I,\ E)(1-p)^{N(\zeta,c,p)-1}$. Since a consumer searches if and only if $c \leqq (1-\zeta)pC(I, E)$, the result follows. (Note that an expert, independently of whether she signals I or E, obtains a profit also on consumers who do not search.) ■

3. Equilibria
We first consider an equilibrium in which all experts are precise ($p = 1$). We have $N(\zeta, c, 1) = 1$ if $0 < c \leqq (1-\zeta)C(I, E)$; given our simplifying assumption that the support of F is $[0,\ (1-\zeta)C(I, E)]$ we have $H(1) = 1 - \phi$ (the only consumers who do not patronize an expert who proposes an E-service are those with zero search cost[7]). Thus from proposition 2 it is optimal for an expert to be precise if and

7 Recall that we assume that a consumer with a zero search cost chooses the dominant search rule $S(\infty)$.

only if $(1-\phi)\pi(I,E) \leqq \pi(I,I)$: that is, if there are enough consumers with a zero search cost, who consequently do not accept the first E-opinion they encounter.

Thus there is an equilibrium in which $p = 1$ if and only if $\pi(I,I) \geqq (1-\phi)\pi(I,E)$. When such an equilibrium exists it is unique. (If $\pi(I,I) > (1-\phi)\pi(I,E)$ this is immediate, while if $\pi(I,I) = (1-\phi)\pi(I,E)$ uniqueness follows from the fact that $(1-\phi)\pi(I,E) > H(p)\pi(I,E)$ for all $p \neq 1$.)

We now consider the existence of an equilibrium in which all experts are cautious ($p = 0$). In this case the only consumers who search are those with zero search costs, and all of them search indefinitely: $H(0) = 0$. Hence an expert maximizes profit by being precise to consumers who search.[8] Thus, there is no equilibrium in which all experts are cautious to those who search.

Finally, when $\pi(I,I) < (1-\phi)\pi(I,E)$, we consider equilibria in which some experts are precise and some are cautious.

Theorem 1. *If $\pi(I,I) > (1-\phi)\pi(I,E)$, then $p = 1$ is the unique equilibrium fraction of experts who are precise. If $\pi(I,I) < (1-\phi)\pi(I,E)$, then for almost all exogenous parameters there is an odd finite number of equilibrium values of the fraction p of experts who are precise.*

Proof. The first case is shown above. To show the second case, first note that, in an equilibrium in which some experts are precise and some are cautious, the pay-off to each expert of being precise must be equal to the pay-off of being cautious: $\pi(I,I) = H(p)\pi(I,E)$ (see proposition 2). Since $H(0) = 0$ and $H(1) = 1-\phi$, we have $H(0) < \pi(I,I)/\pi(I,E) < H(1)$ when $\pi(I,I) < (1-\phi)\pi(I,E)$. Since H is continuous, it follows that there exists a value of p such that $\pi(I,I) = H(p)\pi(I,E)$. It remains to show that there is an odd finite number of such values of p.

Let $\#S$ denote the number of elements in the set S, let $H'(p)$ denote the derivative of H with respect to p, and let $H^{-1}(x) = \{p \in [0,1] : H(p) = x\}$. A regular value x of H is one such that $H'(p) \neq 0$ for any $p \in H^{-1}(x)$. It is easy to see that the values 0 and $1-\phi$ are regular values of H and that $\#H^{-1}(0) = \#H^{-1}(1-\phi) = 1$ for almost any set of exogenous parameters. In addition, since H and its derivative are continuous, the set of regular values of H is dense (Milnor 1976, 11) in $[0, 1-\phi]$. Thus, for almost all exogenous parameters $\pi(I,I)/\pi(I,E)$ is a regular value. Since H is continuous, $H^{-1}(\pi(I,I)/\pi(I,E))$ is closed (Royden 1988, 175). Since $[0,1]$ is compact, the closed set $H^{-1}(\pi(I,I)/\pi(I,E))$ is compact (ibid., 191). Since $H'(p) \neq 0$, H is also one-to-one in a neighbourhood of each $p \in H^{-1}(\pi(I,I)/\pi(I,E))$. Therefore the set $H^{-1}(\pi(I,I)/\pi(I,E))$ is discrete; a discrete compact set in a Hausdorf space is finite.

Since $H(0) < \pi(I,I)/\pi(I,E) < H(1)$, it follows from the Intermediate Value Theorem (ibid., 48) that H must increase at the minimum of the solution set and at the maximum of the solution set. Repeated use of the Intermediate Value Theorem

8 Experts can differentiate between a consumer who searches and one who does not as follows. Consumers who do not search immediately purchase an E-service. Consumers who search spend time asking questions regarding the type of service deemed appropriate for their needs.

shows that generically the finite number of solutions to $H(p) = \pi(I,I)/\pi(I,E)$ is odd. ■

Let $0 < p^* < 1$ be an equilibrium fraction of experts who are precise, so that $\pi(I,I) = H(p^*)\pi(I,E)$. Suppose that the fraction of experts who are precise increases slightly. Then, if H is increasing at p^*, it becomes optimal for experts to be cautious. Symmetrically, if the fraction of experts who are precise decreases slightly, then if H is increasing at p^*, it becomes optimal for experts to be precise. Thus if H is increasing at p^* and the system is perturbed, there is an incentive for it to return to the equilibrium. If H is decreasing at p^*, there is, on the contrary, a tendency for a movement away from equilibrium to be amplified. For this reason we refer to an equilibrium in which H is increasing at p^* as *stable*.

THEOREM 2. *If $\pi(I,I) < (1-\phi)\pi(I,E)$, then there exists a stable equilibrium, and, among the stable equilibria, that in which the fraction of precise experts is highest Pareto dominates all others.*

Proof. In this case an equilibrium value of p satisfies $H(p) = \pi(I, I)/\pi(I,E)$; as stated in the proof of theorem 2, it follows from the Intermediate Value Theorem that H is increasing at the smallest and largest solution. Thus the largest solution is a stable equilibrium. Now, in any equilibrium in which some experts are precise and some are cautious, the profit of every expert is the same, equal to $\pi(I,I)$. We complete the argument by showing that every consumer's expected loss decreases in p. To see this, note that from (3) every consumer's expected loss is non-increasing in p for a fixed value of N; if N adjusts to the optimum, then the loss cannot increase (and, for consumers who search, the loss in general decreases). ■

IV. COMPARATIVE STATICS

Parameters over which a regulator may have influence are the prices of the two types of service and the cost of search. We now study the effects of changes in these variables on market performance, which we measure by the aggregate expected loss of the consumers. Let $p^*(\zeta)$ be a stable equilibrium fraction of precise experts. A consumer whose search cost is zero has an expected loss of zero in any equilibrium in which $p^*(\zeta) > 0$. A consumer whose search cost is $0 < c < (1-\zeta)p^*(\zeta)C(I,E)$ has an expected loss of $\rho_1(\zeta, c, p^*(\zeta))$, while a consumer with a search cost $c > (1-\zeta)p^*(\zeta)C(I,E)$ does not search and has an expected loss of $(1-\zeta)C(I,E)$. Thus the aggregate expected loss in equilibrium is

$$\int_{(0,(1-\zeta)p^*(\zeta)C(I,E)]} \rho_1(\zeta, c, p^*(\zeta))F'(c)dc$$
$$+ (1-\zeta)C(I, E)\{1 - F[(1-\zeta)p^*(\zeta)C(I, E)]\}. \qquad (6)$$

In the following results we study the local comparative statics of stable equilibria. That is, for parameter values in the neighbourhood of some point we let $p^*(\zeta)$ be the equilibrium on some 'branch' of the correspondence that gives stable equilibria.

We want to consider the effect of changes in the prices of the two types of service. Such changes affect the profits $\pi(I,I)$, $\pi(I,E)$, and $\pi(E,E)$ and the consumer losses $C(I,E)$ and $C(E,I)$. However, changes in the profit $\pi(E,E)$ have no effect on the equilibrium; so as far as changes in the profits are concerned, we need to consider only changes in $\pi(I,I)$ and $\pi(I,E)$; we begin by studying the effects of these. In what follows we always assume that the changes are in keeping with the assumptions on the relationships among the costs and among the profits.

THEOREM 3. *Suppose that $\pi(I,I) < (1-\phi)\pi(I,E)$, so that the aggregate* stable *equilibrium fraction of precise experts is in* $(0,1)$. *If $\pi(I,I)/\pi(I,E)$ increases, then the (stable) equilibrium fraction of precise experts increases and the associated aggregate expected loss (as defined in (6)) decreases.*

Proof. The fact that $p^*(\zeta)$ is a stable equilibrium means that $H(p^*(\zeta)) = \pi(I,I)/\pi(I,E)$ and H is increasing at $p^*(\zeta)$; it is immediately apparent that $p^*(\zeta)$ increases as $\pi(I,I)/\pi(I,E)$ increases. Finally, as we argued in the proof of theorem 2, the expected loss of every consumer who searches is non-increasing in $p^*(\zeta)$. Some consumers who formerly did not search do search after the increase in $p^*(\zeta)$. The expected loss of these consumers clearly decreases. The expected loss is constant in $p^*(\zeta)$ for those consumers who do not search in either case. ■

Intuitively, as the ratio $\pi(I,I)/\pi(I,E)$ increases, there is less incentive for an expert to be cautious; experts are more precise in equilibrium and consumers are better off. The next result gives a sufficient condition for a decrease in the spread between the prices to achieve the same result. The sufficient condition is that $cF'(c)$ is increasing in c. The assumption of a decrease in the spread between the prices is equivalent to that of a decrease in $\pi(I,E)-\pi(I,I)$ and a decrease in $C(I,E)$.

THEOREM 4. *Suppose that $cF'(c)$ is increasing in c and that the aggregate* stable *equilibrium fraction of precise experts is between* 0 *and* 1. *If either the price of the I-service is increased or the price of the E-service is decreased, then the equilibrium fraction of precise experts increases and the aggregate expected loss decreases.*

Proof. The immediate effect of either price change is to increase $\pi(I,I)/\pi(I,E)$ and to decrease $C(I,E)$. We need to analyse the effect on H in order to then analyse the effect on the solution to $H(p) = \pi(I,I)/\pi(I,E)$. Recall that H is defined by equation (5) and that $N(\zeta,c,p)$ is the smallest non-negative integer satisfying inequality (2). As c increases from 0 to $(1-\zeta)C(I,E)$, $N(\zeta,c,p)$ decreases in discrete steps from ∞ to 0. Thus, equation (5) can be rewritten as

$$\sum_{N=1}^{\infty}\int_{G(N)C(I,E)}^{G(N-1)C(I,E)}(1-p)^{N-1}F'(c)dc = H(p), \tag{7}$$

where $G(x) = (1-\zeta)p(1-p)^x/[\zeta+(1-\zeta)(1-p)^x]$ and $G(x)C(I,E)$ is the cost of the marginal consumer who is indifferent between using the search rule x and the search rule $x+1$. Thus $G(x)C(I,E)$ is the value of c that solves $(1-p)^x = c\zeta/[(1-\zeta)(pC(I,E)-c)]$. Note that $G'(x) < 0$. The derivative with respect to $C(I,E)$ of the integral with exponent N on the left-hand side of equation (7) is $(1-p)^{N-1}[F'(G(N-1)C(I,E))G(N-1)-F'(G(N)C(I,E))G(N)] > 0$, since $cF'(c)$ is increasing in c and $G' < 0$. Thus, $H(p)$ shifts down if $C(I,E)$ decreases.

Thus the effect of either of the price changes given in the theorem is to increase $\pi(I,I)/\pi(I,E)$ and to shift H down. Since H is increasing in p at $p^*(\zeta)$, this implies that $p^*(\zeta)$ increases. It remains to argue that the aggregate expected loss decreases.

To see this, note that from (3) no consumer's expected loss can increase if p increases and $C(I,E)$ decreases for a fixed value of N; if N adjusts to the optimum, then the loss cannot increase. For consumers who do not search before or after the changes, the loss decreases, owing to the change in $C(I,E)$. For consumers who search in either case, the loss decreases, owing to the higher precision as well as the lower cost of making a mistake. ■

This result states that if $cF'(c)$ is increasing, then a lower price of the expensive repair results in higher equilibrium precision levels and lower aggregate expected losses. A lower price clearly reduces the incentives of the expert to cheat. In addition, it affects the group of consumers who face a cost of c and use any particular $N(\zeta,c,p)=n$, say. (Recall that $N(\zeta,c,p)$ is a step function.) The marginal consumer has less incentive to search if the price is lower. Thus, some low-cost consumers are dropped from the group. However, these low-cost consumers are replaced by high-cost consumers who have dropped out of the group that uses $N(\zeta,c,p)=n+1$. The assumption that $cF'(c)$ is increasing is equivalent to saying that as $C(I,E)$ increases, the weight of the consumers who are dropped is less than the weight of the consumers who are added. Thus, as $C(I,E)$ decreases, the probability of convincing a member of the group that uses $N(\zeta,c,p)=n$ to accept an E-opinion decreases if $cF'(c)$ increases.[9]

We now study the effect of a change in the cost of search. Suppose that the distribution function F of the search cost takes the form $F(c)=\phi+(1-\phi)G(c)$, where G is differentiable and $G(0)=0$. An increase in ϕ entails an increase in the fraction of the population with a search cost of zero and a decline in the population who face a positive search cost.

THEOREM 5. *Suppose that $\pi(I,I) < (1-\phi)\pi(I,E)$, so that the equilibrium fraction of precise experts lies in $(0,1)$. Then if ϕ increases, the fraction of precise experts increases and the aggregate expected loss decreases.*

Proof. It is immediate from (5) that H shifts down if ϕ increases. Since $H(p^*(\zeta)) = \pi(I,I)/\pi(I,E)$ in equilibrium and H is increasing at $p^*(\zeta)$, $p^*(\zeta)$ increases if ϕ increases. If $p^*(\zeta)$ increases, then the expected loss of consumers with positive

9 The sufficient condition essentially requires that $cF''(c)$ not be too negative relative to $F'(c)$. An example in which this is not so is $F(c) = \ln(\ln(c))$; in this case the equilibrium fraction of precise experts *decreases* if the price of the more expensive repair decreases.

search costs decreases, as shown in the proof of theorems 2 and 3. Thus the aggregate expected loss as measured by (6) decreases. ■

Intuitively, as more consumers face zero search costs, consumers become more vigilant, and so experts become more precise in equilibrium.

In summary, by reducing the price difference between an expensive and inexpensive service (in the case that $cF'(c)$ is increasing) or by reducing search costs for some consumers to zero, a regulator can induce the equilibrium fraction of precise experts to increase and the aggregate expected loss to decrease. An implication is that prices should be as close as possible given feasibility constraints and that search costs should be as small as possible.

V. EXTENSIONS

It is straightforward to extend the model by allowing experts to be uncertain about the exact fit between a consumer and a service. Assume that experts can identify an E-consumer with probability one but can identify an I-consumer with only probability $\mu < 1$. As long as experts are sufficiently knowledgeable, so that consumers prefer to search in the case that all experts are precise, the results are analogous to those for the basic model.

Another way to extend the model is to endogenize the skills of experts. Assume that prospective experts have skill levels parametrized by $\sigma \in [0, 1]$. At zero cost such an individual may become an expert who recognizes an E-consumer with probability one and an I-consumer with probability $\mu < 1$ (such experts are called *uncertain* experts or U-experts). To become an expert whose private knowledge is correct (i.e., a *certain* expert or C-expert) an individual of type σ must exert some effort, which costs $Z(1-\sigma)$, where Z is an increasing function. As profits are equalized in equilibrium and as we are interested in the equilibrium level of σ, we need derive only the equation that equates the equilibrium profits of a precise certain expert to those of a precise uncertain expert. A precise expert is one that gives an opinion consonant with the service that it estimates is needed. Let $\pi(s, d)$ be the profit of an expert when the estimated service needed is s and the service requested is d. We assume that the repair actually performed is the repair that is estimated to be needed. We also assume that the per unit profits do not depend on the expert's type. The profit of a precise C-expert is equal to that of a precise U-expert whenever the consumer is an E-consumer. Thus, we need to equate the profit of each type of expert over I-consumers. The per unit profit of a precise C-expert of type σ is $\pi(I, I)$ whenever the consumer is an I-consumer. The cost to an expert of type σ of becoming a C-expert is $Z(1-\sigma)$. With a suitable normalization over the size of the population of consumers, the profit of a precise C-expert of type σ over I-consumers is $\pi(I, I) - Z(1-\sigma)$. Denote this profit by $\Pi_C(\pi(I, I), \sigma)$.

To calculate the profit of a precise U-expert let p^* represent the equilibrium aggregate probability of precision among all experts. The profit of a precise U-expert of type σ over I-consumers is derived as follows. The revenue consists of those per unit profits realized from I-consumers who are estimated to need and request an E-service and from those who are estimated to need and request an I-service. The conditional probability that an I-consumer is estimated (by the U-expert) to need an E-service is the probability that the E-service is misdiagnosed by the U-expert: $(1-\mu)$. The conditional probability that an I-consumer (who receives an E-opinion) actually requests an E-service is $H(p^*)$. The probability that an I-consumer is estimated (by the U-expert) to need an I-service is the probability that an I-service is diagnosed correctly by the U-expert: μ. The conditional probability that an I-consumer (who receives an I-opinion) requests an I-service is 1. Thus, with a suitable normalization of the size of the population of consumers, the revenue of a U-expert over I-consumers is $\mu\pi(I,I)+(1-\mu)\pi(E,E)H(p^*)$. As the cost of becoming a U-expert is zero, the profit of a U-expert over I-consumers equals the revenue. Denote this profit by $\Pi_U(\pi(I,I))$. For σ^* to be the equilibrium proportion of U-experts we require that $\Pi_C(\pi(I,I),\sigma^*) = \Pi_U(\pi(I,I))$. In equilibrium, however, $\pi(I,I) = \pi(I,E)H(p^*)$. Thus, the above equation becomes $(1-\mu)H(p^*)[\pi(I,E)-\pi(E,E)] = Z(1-\sigma^*)$. Note that as $\pi(I,I)/\pi(I,E)$ increases, it is immediately apparent that $H(p^*)$ increases. It is then immediately apparent that if $\pi(I,I)/\pi(I,E)$ increases, σ^* decreases. As the interests of consumers and experts become closer, experts become, on average, more competent. Intuitively, as $\pi(I,I)/\pi(I,E)$ increases, the gap between the profit of a C-expert and that of a U-expert widens, so that it becomes worthwhile for the marginally skilled individual to exert the effort required to become a C-expert.

We can also endogenize the size of the expert population. Parametrize the expert-specific skills of prospective agents by $\sigma \in [0,1]$, as above. Assume that there is only one type of expert, say, a C-expert. An individual of type σ must exert effort to become an expert, which costs $Z(1-\sigma)$, where Z is an increasing function. Assume that the individuals are identical in all other respects so that the opportunity cost O of an individual is independent of σ. As above, if the size of the population of consumers is normalized to one, the equilibrium profit of a precise C-expert of type σ is $\zeta\pi(E,E)/(1-\sigma)+(1-\zeta)\pi(I,I)/(1-\sigma)-Z(1-\sigma)$. Denote this profit by $\Pi_E(\pi(I,I),\sigma)$. The equilibrium size of the population of experts is then $1-\sigma^*$, where $\Pi_E(\pi(I,I),\sigma^*) = O$. Since $D_1\Pi_E + D_1\Pi_E d\sigma^*/d\pi(I,I) = 0$, $D_1\Pi_E > 0$, and $D_2\Pi_E > 0$, it is immediately apparent that σ^* decreases as $\pi(I,I)$ increases. Thus, the size of the population of experts increases as $\pi(I,I)$ increases: as the interests of consumers and experts become closer, the number of experts increases. Intuitively, as the profit of a C-expert increases, it becomes worthwhile for the marginal individual to exert the effort to become a C-expert.

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