Rev. Econ. Design
DOI 10.1007/s10058-017-0203-y

ORIGINAL PAPER

Simple versus rich language in disclosure games

Jeanne Hagenbach[1] · **Frédéric Koessler**[2]

Received: 15 December 2016 / Accepted: 26 June 2017

Abstract This paper studies strategic information disclosure when the sender may not observe the payoff-relevant state, and the receiver may interpret messages naively. We characterize equilibria as a function of the language available to the sender. The language is simple if an informed sender can either fully disclose the state or nothing. The language is rich if he can disclose any closed interval containing the true state. We show that an informed sender and a strategic receiver get a higher ex-ante equilibrium payoff when the language is rich. The reverse holds for a naive receiver and an uninformed sender. Overall, our work suggests that the design of language is key in situations where disclosure is voluntary.

Keywords Information disclosure · Unravelling · Naive audience · Uninformed sender · Rich language · Persuasion

JEL Classification C72 · D82

We thank Alessandro Ispano, Navin Kartik, Peter Schwardmann and Vasiliki Skreta for helpful comments. We also thank an associate editor and two anonymous referees for constructive suggestions. The authors thank the *Agence National de la Recherche* (ANR-10-LABX-93-01; ANR DynaMITE) for financial support.

✉ Jeanne Hagenbach
jeanne.hagenbach@polytechnique.edu

Frédéric Koessler
frederic.koessler@psemail.eu

1 CNRS, École Polytechnique, Palaiseau, France

2 CNRS, Paris School of Economics, Paris, France

Published online: 06 July 2017

1 Introduction

Consider an informed party who would like to convince a decision-maker that a payoff-relevant state is as high as possible. If the informed party is able to certify information at no cost, the corresponding disclosure game leads to a fully revealing equilibrium. The logic behind this result follows the standard unravelling argument (Viscusi 1978; Grossman 1981; Milgrom 1981). Every disclosure strategy supporting such an equilibrium has the property that the message sent by the informed party in each state cannot be sent in lower states. In particular, a separating disclosure strategy can be made up of very precise messages, that is, messages available only in a given state, or made up of vaguer messages which are available in a given state but also in some higher states. As an example, consider a professor writing a recommendation letter to transmit information about the ranking of a student. Whether he states that the student is "in the top 10" or that he is "ranked number 10", a rational receiver should infer that the student is ranked exactly 10th. Hence, different sets of messages yield the same—fully-revealing—equilibrium outcome. In this paper, we study a disclosure game where the set of messages available affects how much information is transmitted in equilibrium.

In the former example, the vague statement "in the top 10" and the precise statement "ranked number 10" convey the same information in equilibrium because "in the top 10" is interpreted as ranked 10th. If the decision-maker (the receiver) interprets vague messages in a way which is not the least advantageous for the informed party (the sender), the unraveling argument ceases to work: an informed sender with sufficiently bad news does not disclose and relies on the high guess of the receiver. Several reasons can explain why the receiver does not interpret vague messages in a skeptical way. First, there may be a lack of sophistication on the receiver side. To account for this behavioral trait, we will simply consider that there is a positive probability that the receiver is naive and takes messages at face value. Second, the sender can be uninformed about the state with a positive probability (or, equivalently, unable to certify it for practical or behavioral reasons). In that case, a rational receiver cannot distinguish an informed sender who strategically decides not to disclose from an uninformed sender who has nothing to disclose. The strategic receiver then guesses a higher state after no disclosure than in the benchmark model. We will incorporate these two extensions in a standard disclosure game and examine precisely by how much full revelation fails as a function of the set of messages available. This set is called a language.

We say that the language is simple when an informed sender can either fully disclose the state or disclose nothing. The language is rich if he can disclose any closed interval containing the true state (which includes the possibility of reporting nothing). We characterize equilibria with both languages and show that they all have a threshold structure. When the language is simple, the informed sender never discloses states which are below a threshold state and fully disclose every state above. When the language is rich, the informed sender never discloses states which are below a threshold and reveals every state above using selective disclosure. We talk about selective disclosure when an informed sender uses a message certifying that the state is at least the true state. In fact, with a rich language, an informed sender who would like to reveal the state has several options to do so. If the receiver may take messages at face value, it is advantageous to use selective disclosure as it induces a higher guess from a naive receiver than full disclosure.

Selective disclosure can be more attractive than no disclosure in intermediate states, while no disclosure is more attractive than full disclosure in such states. It follows that the availability of selective disclosure pushes informed senders to use no disclosure less frequently when the language is rich. Formally, we show that the threshold state is lower with the rich language than with the simple one. Overall, the language available affects both the equilibrium threshold and the messages used to reveal information above it. These two aspects play differently on the equilibrium payoff of the different kinds of players. For example, a lower threshold implies that a strategic receiver learns the state more often, so he gets a higher equilibrium payoff under the rich language. In contrast, a naive receiver always prefers the simple language: even if a lower threshold implies that fewer states are not disclosed, selective disclosure is used above the threshold when the language is rich. For the uninformed sender, what matters is the interpretation of no disclosure by the receiver. When the threshold is lower, this interpretation of no disclosure is lower, so the uninformed sender prefers the simple language. The same is true for a sender informed of a low state, who uses no disclosure with both languages. If, instead, the sender is informed of a high state, he prefers the rich language as he can use selective disclosure to reveal his type.

While previous works have usually questioned the necessity of mandatory disclosure policies, our results suggest that the design of language might be key when disclosure is voluntary. In practice, a language can be called rich in markets where selective information disclosure can be freely chosen by the seller. For example, while disclosing false information is forbidden by the law, firms often enjoy a large flexibility when advertising their products. In contrast, a language can be called simple in environments where a seller who decides to disclose information is then forced to include a complete and standardized description. Take, for instance, hygiene grades of restaurants: owners choose either to disclose nothing or to post their exact grade on the window. Similarly, giving free samples of a food product may be a way to fully disclose its characteristics but nothing forces sellers to distribute samples.

Under this interpretation of languages, the analysis of our stylized model has the following simple empirical implications. First, as expected, a government intervention forcing a seller to use a simple language would successfully reduce seller's biases towards selective disclosure and therefore protect unwary consumers. Second, and more interestingly, our result that a strategic receiver learns the state more often when the language is rich suggests that policies aimed at protecting consumers by preventing the selective release of information may have a negative impact on sophisticated buyers. Hence, information asymmetries and adverse selection might be more severe with such a policy intervention.[1] This result also suggests that the possibility for asset issuers to observe multiple ratings and disclose only the most favorable (as reported in

[1] This prediction is consistent with some recent empirical studies. For example, taking as a case study eBay Motors, Lewis (2011) shows that sellers' selective disclosure through various online media such as photos, text, and graphics lead to information unravelling and therefore significantly reduce adverse selection in online used car markets. It would be interesting to study in a controlled environment (for example, in a laboratory experiment) whether restricting sellers to use a simple language would indeed reduce the forces of unravelling in such an environment. This is left for future research.

Skreta and Veldkamp 2009) is not necessarily detrimental to sophisticated investors. Finally, our result that an informed sender prefers the rich language when the state is high and the simple one when the state is low suggests that markets providing a richer and more flexible environment for information disclosure would attract higher quality sellers.

Related literature This is not the first article considering a variant of the classical disclosure game where the unravelling argument fails. In particular, Dye (1985) and Jung and Kwon (1988) consider a proportion of uninformed senders and the simple language scenario, and identify similar threshold equilibria to the ones we describe. Shin (1994) extends their analysis to two competing senders who have a privately known number of partially informative signals, and can decide which of these signals to disclose to the receiver. The unraveling argument also fails in the models of Glazer and Rubinstein (2001, 2004) and Sher (2011) where the sender is perfectly informed but is only able to disclose a limited number of partially informative signals to the receiver. More recently, Hummel et al. (2017) allow for uncertainty about whether the sender is informed or not in more general disclosure games than the ones studied in Dye (1985). Allowing for a rich language, they characterize which states are disclosed and which are not as a function of the state-dependent conflict of interest between the sender and the receiver. All these models do not consider naive receivers.

On the other hand, Milgrom and Roberts (1986) assume that the sender is always perfectly informed and that all receivers are naive. They focus on the role of competition between informed parties to get full information disclosure. Considering the rich language scenario, they already point out that naive receivers push senders to use selective disclosure. The effects of a proportion of naive receivers has been more extensively analyzed in cheap-talk games, for example in Ottaviani and Squintani (2006), Kartik et al. (2007) and Chen (2011). One explanation for some receivers taking messages at face value is that they have a wrong perception of the correlation between the sender's private information and messages sent. This interpretation is close in spirit to the bounded rationality ideas developed in Eyster and Rabin (2005), Jehiel (2005) or Esponda (2008). Alternatively, naive receivers can be viewed as agents who had access to few earlier recommendations as opposed to strategic receivers who can use past experience to read vague messages. We do not consider long-term communication relationships in the paper, but Sethi and Yildiz (2016) develop the idea that agents observed over time become better understood. In more applied contexts, the industrial organization literature has also considered naive consumers and studied their exploitation by firms. In Gabaix and Laibson (2006) for instance, information about the price of add-ons to a product is shrouded despite competition. In Ispano and Schwardmann (2016) where competitive firms simultaneous disclosure product quality and set their price, the presence of sophisticated consumers can enhance exploitation of naive ones.

Our paper is also related to several laboratory experiments reporting partial information revelation. In a labor market environment, Benndorf et al. (2015) show that low productivity workers under-reveal compared to what theory predicts. Their experiment involves computerized receivers who interpret no disclosure as the average productivity of workers who have hidden their private information. Implementing a classical disclosure game, Jin et al. (2016) show that low types do not disclose and receivers are not skeptical enough about no disclosure. King and Wallin (1991b) study a market environment where the potential buyers of an asset do not know whether the monopolist seller is informed about the value of the dividend or not. The seller can disclose the dividend when informed, but, as in our game, is forced to no disclosure when he has no private information. The authors show that the lowest dividend reported increases with the exogenous probability that the seller is uninformed. In all these experiments however, the language considered is simple. In contrast, King and Wallin (1991a) allow subjects to report a subset of possible dividends, with the true dividend being included. They report convergence to full disclosure provided that the set of possible disclosure is not too large and known to the receiver. In parallel, several empirical studies have documented that full disclosure is not observed in the field when voluntary. Several such cases can be found in Milgrom (2008) and Dranove and Jin (2010).[2]

Finally, recent studies have highlighted that partial information disclosure tends to be selective. For example, Luca and Smith (2015) show that most MBAs engage in some sort of shrouding when displaying information about their rankings on their websites. Conditional on displaying a ranking, the majority of schools coarsen information to make it seem more favorable. Isaac et al. (2016) document the same tendency for marketers who disclose brand information.

2 Model

The benchmark is the classical disclosure game of Milgrom (1981) and Grossman (1981). It involves a sender (S) and a receiver (R). In the beginning of the game, the sender privately observes a payoff-relevant state in $T = [0, 1]$ and sends a costless message to the receiver. The payoff-relevant state is distributed according to the full support distribution function F, with continuous, strictly positive density function f. We assume that the cumulative distribution function F is log-concave.[3] A generic state is denoted by t, and when there is a risk of confusion the associated random variable is denoted by $\boldsymbol{t}$. Upon receiving the sender's message, the receiver chooses an action $a \in \mathbb{R}$ which affects both players. The sender's payoff is equal to a and the receiver's payoff is such that his optimal action is equal to the expected value of the payoff-relevant state.

First, we depart from this benchmark by introducing the possibility that S does not observe the state: he is uninformed with probability $\lambda_S \in (0, 1)$. With probability $1 - \lambda_S$, he observes the state t as in the benchmark game. The event "the sender is informed" is denoted by I_S and the event "the sender is uninformed" is denoted by $\bar{I}_S$. Let $\mathcal{D}$ be a set of closed intervals contained in $[0, 1]$, with $T \in \mathcal{D}$. When the sender is informed about t, he can disclose any set in $\mathcal{D}(t) \equiv \{D \in \mathcal{D} : t \in D\}$. When uninformed, the sender can only disclose the set T. Messages certify information in that disclosing the interval D is hard evidence that $t \in D$. When there is a possible

[2] For example, Mathios (2000) reports that information about higher fat salad dressing was not disclosed until it became mandatory to do so. Brown et al. (2012) use data about cold openings of movies and document that audience fails to correctly infer the average quality of movies which have no critics.

[3] This assumption is used for equilibrium uniqueness and for our comparative statics results. It is a common assumption and it is satisfied by many distributions (see Bagnoli and Bergstrom 2005 for more details).

confusion between the set of types D and the decision to disclose the set D we denote the latter decision "D". A strategy for an informed sender is given by $\sigma_S : T \to \mathcal{D}$ such that $\sigma_S(t) \in \mathcal{D}(t)$ for every t. Without loss of generality we assume, as a tie-breaking rule, that a sender who is indifferent between disclosing D and T chooses T. The receiver observes the set D disclosed by the sender but does not know whether S is informed or not. Hence, the set of possible histories for the receiver is $\mathcal{D}$.

Second, we modify the benchmark game by assuming that the receiver is naive with probability $\lambda_R \in (0, 1)$. In that case, the receiver takes any disclosed set D at face value and chooses action $E(\boldsymbol{t} \mid \boldsymbol{t} \in D)$. With probability $1 - \lambda_R$, the receiver is strategic and his strategy is given by $\sigma_R : \mathcal{D} \to \mathbb{R}$. The sender does not know which type of receiver he faces.

We consider perfect Bayesian equilibria of this game. In equilibrium, the strategic receiver chooses

$$\sigma_R(D) = \begin{cases} E(\boldsymbol{t} \mid \text{“}D\text{”}) & \text{for a message “}D\text{” on the equilibrium path} \\ \text{any } t \in D & \text{for a message “}D\text{” off the equilibrium path,} \end{cases}$$

where $E(\boldsymbol{t} \mid \text{“}D\text{”})$ is computed with respect to the sender's strategy σ_S, the prior f, and the probability that the sender is uninformed, λ_S. When the informed sender discloses $\{t\}$, the receiver has a unique optimal action, $a = t$, whatever his belief about the sender's strategy. Hence, in equilibrium, $\sigma_R(\{t\}) = t$ for every t.

We say that the language is *simple* when $\mathcal{D} = \{T, \{t\}_{t \in T}\}$, that is, when an informed sender of type t can either disclose "T" or the singleton $\{t\}$. We say that the language is *rich* when $\mathcal{D}$ is the set of all closed intervals contained in $[0, 1]$, that is, when an informed sender of type t can disclose any closed interval containing t.[4]

3 Equilibria with rich language

We first characterize equilibria of the disclosure game when the language is simple.

Proposition 1 *Let the language be simple. Then, every equilibrium has the following properties. There exists a unique $t_s^* \in (0, E(\boldsymbol{t}))$ such that $\sigma_S(t) = T$ for $t \le t_s^*$ and $\sigma_S(t) = \{t\}$ for $t > t_s^*$. The threshold t_s^* is strictly increasing in λ_S and λ_R and satisfies*

$$\lambda_R(1-\lambda_S)\big[E(\boldsymbol{t})-t_s^*\big]+\lambda_S\frac{E(\boldsymbol{t})-t_s^*}{F(t_s^*)} = (1-\lambda_R)(1-\lambda_S)\big[t_s^*-E(\boldsymbol{t} \mid \boldsymbol{t} \le t_s^*)\big]. \quad (1)$$

Proof First note that, whatever the action of the receiver after no disclosure, the best response of the sender is a threshold disclosure strategy of the form $\sigma_S(t) = T$ if $t \le t_s^*$ and $\sigma_S(t) = \{t\}$ if $t > t_s^*$. Therefore, to characterize an equilibrium, it suffices

[4] Note that we exclude the possibility that $\mathcal{D}$ contains disjoint sets. In particular, this prevents the use of statements of the form "my type is either t_1 or above t_2", which we believe are less common than reports such as "my type is at least t" or "‘my type is in this range". If one has in mind labels or quality categories disclosed by firms to consumers, one can even question the availability of disjoint disclosure options.

to determine the action of the receiver after no disclosure and the threshold t_s^* used by the sender. When the strategic receiver faces "T", he takes the action

$$E(\boldsymbol{t} \mid \text{“}T\text{”}) = \Pr(I_S \mid \text{“}T\text{”}) E(\boldsymbol{t} \mid \boldsymbol{t} \leq t_s^*) + \Pr(\bar{I}_S \mid \text{“}T\text{”}) E(\boldsymbol{t}), \tag{2}$$

where

$$\Pr(I_S \mid \text{“}T\text{”}) = \frac{(1-\lambda_S)\Pr(\text{“}T\text{”} \mid I_S)}{(1-\lambda_S)\Pr(\text{“}T\text{”} \mid I_S) + \lambda_S \Pr(\text{“}T\text{”} \mid \bar{I}_S)} = \frac{(1-\lambda_S)F(t_s^*)}{(1-\lambda_S)F(t_s^*) + \lambda_S},$$

and

$$\Pr(\bar{I}_S \mid \text{“}T\text{”}) = \frac{\lambda_S}{(1-\lambda_S)F(t_s^*) + \lambda_S}.$$

Rewriting, we get

$$E(\boldsymbol{t} \mid \text{“}T\text{”}) = \frac{(1-\lambda_S)F(t_s^*)E(\boldsymbol{t} \mid \boldsymbol{t} \leq t_s^*) + \lambda_S E(\boldsymbol{t})}{(1-\lambda_S)F(t_s^*) + \lambda_S}. \tag{3}$$

When the receiver is naive, he takes action $E(\boldsymbol{t})$ when facing "T". It follows that, by withholding information, the sender gets the following expected action from the receiver:

$$\lambda_R E(\boldsymbol{t}) + (1-\lambda_R)\frac{(1-\lambda_S)F(t_s^*)E(\boldsymbol{t} \mid \boldsymbol{t} \leq t_s^*) + \lambda_S E(\boldsymbol{t})}{(1-\lambda_S)F(t_s^*) + \lambda_S}. \tag{4}$$

The sender of type t_s^* has to be indifferent between disclosing and withholding information, so t_s^* solves:

$$t_s^* = \lambda_R E(\boldsymbol{t}) + (1-\lambda_R)\frac{(1-\lambda_S)F(t_s^*)E(\boldsymbol{t} \mid \boldsymbol{t} \leq t_s^*) + \lambda_S E(\boldsymbol{t})}{(1-\lambda_S)F(t_s^*) + \lambda_S}.$$

Rearranging we get (1).

When t_s^* goes to zero the LHS of Eq. (1) tends to infinity and the RHS to zero. When $t_s^* \geq E(\boldsymbol{t})$ the LHS is negative and the RHS is strictly positive. Hence, there exists a solution $t_s^* \in (0, E(\boldsymbol{t}))$. Next, notice that the LHS is decreasing in t_s^*. By the log-concavity of F we also know from Bagnoli and Bergstrom (2005, Theorem 5) that the RHS is increasing in t_s^*. Hence, the solution t_s^* is unique. Finally, from the fact that $(1-\lambda_R)(1-\lambda_S)\big[t_s^* - E(\boldsymbol{t} \mid \boldsymbol{t} \leq t_s^*)\big] - \lambda_R(1-\lambda_S)\big[E(\boldsymbol{t}) - t_s^*\big] - \lambda_S \frac{E(\boldsymbol{t}) - t_s^*}{F(t_s^*)}$ is decreasing in λ_R and λ_S and increasing in t_s^*, we conclude that t_s^* is increasing in λ_S and λ_R. □

When λ_S and λ_R are both equal to 0, we are back to the benchmark disclosure game. In that case, the unraveling argument applies to the point where the (strategic) receiver perfectly learns the state in every equilibrium, and $t_s^* = 0$. At the other extreme, when λ_S or λ_R tends to 1, the equilibrium threshold tends to $t_s^* = E(\boldsymbol{t})$. The sender is nearly always uninformed or the receiver is nearly always naive, so the receiver's reaction to "T" tends to $E(t)$. In all intermediary cases, Proposition 1 establishes that some informed sender types below $E(t)$ withhold information. The reason is that there is a chance that the receiver interprets "T" as coming from every possible type of sender. To explain why, we now focus on the particular cases where $\lambda_R = 0$ or $\lambda_S = 0$.

When the sender is always informed ($\lambda_S = 0$) but the receiver may be naive, there is a chance λ_R that "T" is simply taken at face value. Therefore, the weight that an informed sender puts on reaction $E(t)$ when sending "T" is λ_R, and the threshold t_s^* satisfies

$$t_s^* = \lambda_R E(\boldsymbol{t}) + (1 - \lambda_R) E(\boldsymbol{t} \mid \boldsymbol{t} \leq t_s^*). \tag{5}$$

When the receiver is always strategic ($\lambda_R = 0$) but the sender may be uninformed, there is chance that "T" comes from an uninformed sender of any possible type. The weight that an informed sender puts on reaction $E(t)$ when sending "T" then corresponds to the probability, computed by the strategic receiver, that the sender is unformed when "T" is disclosed. This probability is given by $P(\bar{I}_S | \text{“}T\text{”}) = \frac{\lambda_S}{(1-\lambda_S)F(t_s^*)+\lambda_S}$. In this case, the threshold t_s^* satisfies

$$t_s^* = \frac{\lambda_S E(\boldsymbol{t}) + (1 - \lambda_S) F(t_s^*) E(\boldsymbol{t} | \boldsymbol{t} \leq t_s^*)}{(1 - \lambda_S) F(t_s^*) + \lambda_S}. \tag{6}$$

When $\lambda_R = 0$, the game studied is the one of Dye (1985) and Jung and Kwon (1988).

4 Equilibria when the message space is rich

We now characterize equilibria when the language is rich.

Proposition 2 *Let the language be rich. Then, every equilibrium has the following properties. There exists a unique $t_r^* \in (0, E[\boldsymbol{t}])$ such that $\sigma_S(t) = T$ for $t \leq t_r^*$ and $\sigma_S(t) = [t, 1]$ for $t > t_r^*$. The threshold t_r^* satisfies*

$$\begin{aligned} &\lambda_R (1 - \lambda_S)[E(\boldsymbol{t}) - E(\boldsymbol{t} | \boldsymbol{t} \geq t_r^*)] + \lambda_S \left[\frac{E(\boldsymbol{t}) - [\lambda_R E(\boldsymbol{t} | \boldsymbol{t} \geq t_r^*) + (1 - \lambda_R) t_r^*]}{F(t_r^*)} \right] \\ &\quad = (1 - \lambda_R)(1 - \lambda_S)[t_r^* - E(\boldsymbol{t} | \boldsymbol{t} \leq t_r^*)]. \end{aligned} \tag{7}$$

Proof We first argue that, in every equilibrium, informed sender types who do not disclose T use a separating strategy. Suppose by contradiction that, in equilibrium, there is some pooling on a disclosure set $D \neq T$. Let $\tilde{D}$ be the set of informed sender types who disclose D, and let t_{max} be the highest type among those. If type t_{max} deviates from D and discloses $[t_{max}, 1]$, then a strategic receiver will take an action which is higher than t_{max}, and therefore strictly higher than $\sigma_R(D) = E(\boldsymbol{t} \mid \boldsymbol{t} \in \tilde{D})$. In addition, following "D", a naive receiver takes action $E(\boldsymbol{t} | \boldsymbol{t} \in D)$. Given that D includes some types which are strictly lower than t_{max}, we have that $E(\boldsymbol{t} | \boldsymbol{t} \in D) < E(\boldsymbol{t} | \boldsymbol{t} \in [t_{max}, 1])$. So, the sender of type t_{max} has an interest in deviating to disclosing $[t_{max}, 1]$ instead of D.

Second, we argue that, in every equilibrium, an informed sender type t who fully reveals his type to a strategic receiver necessarily does so by disclosing $[t, 1]$. Indeed, consider by way of contradiction that there exists an equilibrium in which some informed sender type t reveals his type by disclosing $D \neq [t, 1]$. Following "D" the strategic receiver learns t and takes action t. Following "D" the naive receiver takes action $E(\boldsymbol{t}|\boldsymbol{t} \in D)$. If the informed sender type t deviates and discloses $[t, 1]$, then a strategic receiver chooses an action $a \geq t$ and a naive receiver chooses action $E(\boldsymbol{t}|\boldsymbol{t} \in [t, 1]) > E(\boldsymbol{t}|\boldsymbol{t} \in D)$. The deviation is therefore strictly profitable for type t.

At that point, we have established that, in every equilibrium, an informed sender type t either discloses T or $[t, 1]$, and the disclosure of $[t, 1]$ fully reveals the state to a strategic receiver. The informed sender's strategy must therefore be a threshold strategy: for some $t_r^* \in (0, 1)$, the sender discloses T for every $t \leq t_r^*$ and he discloses $[t, 1]$ for every $t > t_r^*$. We now show that such an equilibrium indeed exists. To do so, we check that there is no profitable deviation for an informed sender type t to a message off the equilibrium path "$[t', t'']$" with $t \in [t', t'']$ and $t'' < 1$. For such a message, let a strategic receiver assign probability 1 to t'. Hence, $\sigma_R([t', t'']) = t'$ for every t', t''. By revealing $[t, 1]$, an informed sender type t induces action $E(\boldsymbol{t} \mid \boldsymbol{t} \in [t, 1])$ from a naive receiver. Any deviation of the informed sender type t to a message "$[t', t'']$" with $t \in [t', t'']$ and $t'' < 1$ induces a smaller action from a naive receiver as $E(\boldsymbol{t}|\boldsymbol{t} \in [t, 1]) \geq E(\boldsymbol{t}|\boldsymbol{t} \in [t', t''])$. Any deviation to such a message also induces a smaller action from a strategic receiver who then take decision $t' \leq t$.

To complete the proof, we finally characterize the action taken by a strategic receiver when facing "T" in equilibrium, and the threshold t_r^*. When the strategic receiver faces "T", he takes the action

$$E(\boldsymbol{t} \mid \text{"}T\text{"}) = \frac{(1-\lambda_S)F(t_r^*)E(\boldsymbol{t} \mid \boldsymbol{t} \leq t_r^*) + \lambda_S E(\boldsymbol{t})}{(1-\lambda_S)F(t_r^*) + \lambda_S}, \tag{8}$$

as was established by Eq. (3) in the proof of Proposition 1. Now, the sender of type t_r^* must be indifferent between disclosing T or disclosing $[t_r^*, 1]$. So, t_r^* solves

$$\lambda_R E(\boldsymbol{t} \mid \boldsymbol{t} \geq t_r^*) + (1-\lambda_R)t_r^* = \lambda_R E(\boldsymbol{t}) + (1-\lambda_R)\frac{(1-\lambda_S)F(t_r^*)E(\boldsymbol{t} \mid \boldsymbol{t} \leq t_r^*) + \lambda_S E(\boldsymbol{t})}{(1-\lambda_S)F(t_r^*) + \lambda_S}.$$

Rearranging we get (7).

When t_r^* goes to zero the LHS of Eq. (7) tends to infinity and the RHS to zero. When $t_r^* \geq E(\boldsymbol{t})$, the LHS is negative and the RHS is positive. Hence, there exists a solution $t_r^* \in (0, E(\boldsymbol{t}))$. Next, since the LHS is decreasing in t_r^*, we can proceed exactly as in the proof of Proposition 1 to show that the RHS is increasing in t_r^* and thus the threshold t_r^* is unique. □

Proposition 2 establishes that the informed sender withholds states which are below a threshold t_r^* and uses the message "$[t, 1]$" to reveal every state $t > t_r^*$. We say that the sender uses *selective disclosure* above the threshold as he selects the messages which are the most advantageous in case the receiver takes messages at face value. The fact that the presence of naive receivers pushes the sender to select a particular separating strategy had already been pointed out in Milgrom and Roberts (1986). They studied the particular case where the sender is always informed ($\lambda_S = 0$) and the receiver naive, and consider a rich language. Then, there is a unique equilibrium such that every informed sender type t discloses $\sigma_S(t) = [t, 1]$.

Table 1 Informed sender's strategy

	Simple language	Rich language
$\lambda_S = 0, \lambda_R = 0$	$\sigma_S(t) = \{t\}$ for every t	Separating strategy for every t
$\lambda_S \in [0, 1), \lambda_R = 0$	$\sigma_S(t) = \begin{cases} T & \text{if } t \le t_s^* = t_r^* \\ \{t\} & \text{if } t > t_s^* = t_r^* \end{cases}$	$\sigma_S(t) = \begin{cases} T & \text{if } t \le t_s^* = t_r^* \\ \text{separating strategy} & \text{if } t > t_s^* = t_r^* \end{cases}$
$\lambda_S = 0, \lambda_R \in [0, 1)$	$\sigma_S(t) = \begin{cases} T & \text{if } t \le t_s^* \\ \{t\} & \text{if } t > t_s^* \end{cases}$	$\sigma_S(t) = [t, 1]$ for every t
$\lambda_S \in (0, 1), \lambda_R \in (0, 1)$	$\sigma_S(t) = \begin{cases} T & \text{if } t \le t_s^* \\ \{t\} & \text{if } t > t_s^* \end{cases}$	$\sigma_S(t) = \begin{cases} T & \text{if } t \le t_r^* \\ [t, 1] & \text{if } t > t_r^* \end{cases}$

In the particular case where the receiver is always strategic ($\lambda_R = 0$) but the sender may be uninformed, an informed sender of type t who reveals information does not necessarily do it by using message "$[t, 1]$". In that case, we have $\sigma_S(t) = T$ for $t \le t_r^*$, and $\sigma_S(t)$ for $t > t_r^*$ is such that $t' \notin \sigma_S(t)$ for every $t' < t$. The threshold t_r^* satisfies Eq. (6) and the only difference with the case of a simple language is that the sender has several options to reveal states which are above t_r^*.

5 Comparing simple and rich languages

Table 1 summarizes the informed sender's equilibrium strategies. As it appears in this table, the available language affects both the threshold below which an informed sender withholds information, and the messages used by this sender to fully reveal the information above the threshold. It follows that stating which language leads to more disclosure is not straightforward. This section will show how preferences for the simple or the rich language vary for the different types of players.

To compare the equilibrium welfare of the different types of sender and receiver as a function of the language, we start by ordering the thresholds t_s^* and t_r^*.

Proposition 3 *The threshold under which the informed sender withholds information is lower when the language is rich than when it is simple: $t_r^* \le t_s^*$ (with a strict inequality when $\lambda_R > 0$).*

Proof The RHS of Eqs. (1) and (7) are the same increasing functions of the thresholds. The LHS of Equations (1) and (7) both are decreasing functions of the thresholds. Noticing that, for any threshold, the LHS of (7) is lower (strictly lower when $\lambda_R > 0$) than the one of (1) completes the proof. □

Whether the language is rich or simple, for a fixed threshold t^*, every type of receiver interprets "T" in the same way. The result that $t_r^* \le t_s^*$ comes from the fact that, for an informed sender of type t, disclosing $[t, 1]$ is more attractive than disclosing $\{t\}$. The inequality is strict only when the receiver can potentially be naive.

Next, we say that a language is preferred if it leads to a higher interim expected equilibrium payoff.

Proposition 4 *For every t, a strategic receiver prefers the rich to the simple language while a naive receiver prefers the simple to the rich language. For every t, an uninformed sender prefers the simple to the rich language. There exists $\tilde{t} \in (t_r^*, t_s^*)$ such that an informed sender prefers the rich to the simple language when $t > \tilde{t}$ and prefers the simple to the rich language when $t < \tilde{t}$. The ex-ante expected payoff of an informed sender is higher with the rich than with the simple language.*

Proof Since $t_r^* < t_s^*$, a strategic receiver gets a finer partition of the informed sender's information when the language is rich than when it is simple.

In case the informed sender type is $t \leq t_r^*$, a naive receiver faces "T" with both languages; in case the informed sender type is $t \geq t_s^*$, a naive receiver prefers the simple to the rich language as he perfectly learns t if the language is simple; in case the informed sender type is $t_r^* < t < t_s^*$, the naive receiver faces "T" when the language is simple and takes action $E(\boldsymbol{t})$; he faces "$[t, 1]$" when the language is rich and takes action $E(\boldsymbol{t} \mid \boldsymbol{t} \in [t, 1])$, which is higher than $E(\boldsymbol{t})$. Since $t_s^* < E(\boldsymbol{t})$, a naive receiver prefers the simple language in this last case.

A strategic receiver takes a higher action facing "T" when the language is simple than when the language is rich. So an uninformed sender, who has no other option than disclosing T, prefers the simple to the rich language.

An informed sender who is of type $t < t_r^*$ discloses T with both languages, so he also prefers the simple to the rich language. An informed sender of type $t > t_s^*$ fully reveals his type to a strategic receiver with both languages. However, he does so using message "$[t, 1]$" when the language is rich and "$\{t\}$" when the language is simple. It follows that a naive receiver takes a higher action when the language is rich, which is preferred by an informed sender of type $t > t_s^*$. Now consider an informed sender who is of type $t \in (t_r^*, t_s^*)$. When the language is simple, his expected payoff is given by (4). When the language is rich, his expected payoff is given by

$$\lambda_R E(\boldsymbol{t} \mid \boldsymbol{t} \geq t) + (1 - \lambda_R)t. \tag{9}$$

The former is constant in t while the latter is increasing in t. Together with the previous comparisons, this implies that there exists $\tilde{t} \in (t_r^*, t_s^*)$ such that an informed sender prefers the rich to the simple language iff $t > \tilde{t}$.

To see that the ex-ante expected utility of an informed sender is higher when the language is rich than when the language is simple we first compare the expected utility of the sender before he knows whether he is informed or not. From his point of view, with probability $(1 - \lambda_R)$ the expected action of the receiver is $E(t)$ since the strategic receiver's action is unbiased on average. Hence, only the average action of a naive receiver matters for the comparison. When the sender is uninformed, the naive receiver's action is always $E(\mathbf{t})$ whatever the language. When the sender is informed and the language is simple, the naive receiver's action is $E(\mathbf{t})$ when $t < t_s^*$ and t when $t > t_s^*$. When the sender is informed and the language is rich the naive receiver's action is $E(\mathbf{t})$ when $t < t_r^*$ and $E(\boldsymbol{t} \mid \boldsymbol{t} \in [t, 1])$ when $t > t_r^*$. Since $E(\boldsymbol{t} \mid \boldsymbol{t} \in [t, 1]) \geq t$ and $t_r^* < t_s^*$, the naive receiver's action is always higher when the language is rich. We conclude that the expected utility of the sender before he knows whether he is informed or not is higher when the language is rich than when the language is simple.

Combined with the previous result that an uninformed sender prefers the simple to the rich language, this implies that an informed sender must prefer the rich language to the simple language. □

This proposition establishes that naive and strategic receivers disagree on the language they prefer. On the one hand, the rich language leads to fewer types withholding information, which is why strategic receivers prefer it. On the other hand, the simple language induces precise information disclosure when information is not withheld, which is why naive receivers prefer it. Clearly, the language affects several dimensions of information disclosure, and they do not matter the same way for all kinds of players.

6 Conclusion

By considering two simple variations of the standard disclosure game, we show that the choice of evidence that can be used to reveal information is not innocent. Precisely, the language available to the sender affects how much information is revealed in equilibrium, and different kinds of players do not agree on the language they prefer. The design of language is a novel issue and raises a different question than the one asking when mandatory disclosure is the appropriate way to reduce information asymmetries.

From an applied perspective, the choice between rich and simple languages could in fact be interpreted as a choice to implement or not a clause of conditional full disclosure. The idea is that, if the speaker makes a report, the report must be a complete report. To justify the restriction to hard evidence, one usually thinks of a dissuasive punishment in case of lying. Similarly, a high enough legal penalty could discourage the speaker of partially disclosing if he reports anything. If the clause is mandated, then the language is simple. It is practically interesting because, in presence of potentially uninformed senders, it becomes impossible to mandate full disclosure. This approach also naturally raises the question of mandating various standardized ways to disclose true information if anything is disclosed at all.

Finally, let us mention that the word *language* has several connotations that we do not incorporate in our analysis where it actually stands for the set of evidence available. Blume and Board (2013) examine the role of language from a different perspective. They focus on common interest games where players have private information about the cheap-talk messages they can use and understand. The authors find severe efficiency losses on coordination of actions when the language is private information, even if the language is in fact sufficient to attain efficiency. So, like in our setting, the amount of information revealed in equilibrium depends not only on the conflicts of interest but also on the language available.

References

Bagnoli M, Bergstrom T (2005) Log-concave probability and its applications. Econ Theory 26:445–469

Benndorf V, Kübler D, Normann H-T (2015) Privacy concerns, voluntary disclosure of information, and unraveling: an experiment. Eur Econ Rev 75:43–59

Blume A, Board O (2013) Language barriers. Econometrica 81:781–812

Brown AL, Camerer CF, Lovallo D (2012) To review or not to review? Strategic thinking at the box office. Am Econ J Microecon 4:1–26
Chen Y (2011) Perturbed communication games with honest senders and naive receivers. J Econ Theory 146:401–424
Dranove D, Jin GZ (2010) Quality disclosure and certification: theory and practice. J Econ Lit 48:935–963
Dye RA (1985) Disclosures of nonproprietary information. J Account Res 23:123–145
Esponda I (2008) Behavioral equilibrium in economies with adverse selection. Am Econ Rev 98:1269–1291
Eyster E, Rabin M (2005) Cursed equilibrium. Econometrica 73:1623–1672
Gabaix X, Laibson D (2006) Shrouded attributes, consumer myopia, and information suppression in competitive markets. Q J Econ 121:505–540
Glazer J, Rubinstein A (2001) Debates and decisions: on a rationale of argumentation rules. Games Econ Behav 36:158–173
Glazer J, Rubinstein A (2004) On optimal rules of persuasion. Econometrica 72:1715–1736
Grossman SJ (1981) The informational role of warranties and private disclosure about product quality. J Law Econ 24:461–483
Hummel P, Morgan J, Stocken PC (2017) Persuasion under second-order uncertainty. Working paper
Isaac MS, Brough AR, Grayson K (2016) Is top 10 better than top 9? The role of expectations in consumer response to imprecise rank claims. J Mark Res 53:338–353
Ispano A, Schwardmann P (2016) Competitive pricing and quality disclosure to cursed consumers. Working paper
Jehiel P (2005) Analogy-based expectation equilibrium. J Econ Theory 123:81–104
Jin GZ, Luca M, Martin D (2016) Is no news (perceived as) bad news? An experimental investigation of information disclosure. Technical report, National Bureau of Economic Research
Jung W-O, Kwon YK (1988) Disclosure when the market is unsure of information endowment of managers. J Account Res 26:146–153
Kartik N, Ottaviani M, Squintani F (2007) Credulity, lies, and costly talk. J Econ Theory 134:93–116
King R, Wallin D (1991a) Market-induced information disclosures: an experimental markets investigation. Contemp Account Res 8:170–197
King RR, Wallin DE (1991b) Voluntary disclosure when seller's level of information is unknown. J Account Res 29:96–108
Lewis G (2011) Asymmetric information, adverse selection and online disclosure: the case of eBay motors. Am Econ Rev 101:1535–1546
Luca M, Smith J (2015) Strategic disclosure: the case of business school rankings. J Econ Behav Organ 112:17–25
Mathios AD (2000) The impact of mandatory disclosure on product choices: an analysis of the salad dressing market. J Law Econ 43:651–677
Milgrom P (1981) Good news and bad news: representation theorems and applications. Bell J Econ 12:380–391
Milgrom P (2008) What the seller won't tell you: persuasion and disclosure in markets. J Econ Perspect 22:115–131
Milgrom P, Roberts J (1986) Relying on the information of interested parties. Rand J Econ 17:18–32
Ottaviani M, Squintani F (2006) Naive audience and communication bias. Int J Game Theory 2006:129–150
Sethi R, Yildiz M (2016) Communication with unknown perspectives. Econometrica 84:2029–2069
Sher I (2011) Credibility and determinism in a game of persuasion. Games Econ Behav 71:409–419
Shin HS (1994) The burden of proof in a game of persuasion. J Econ Theory 64:253–264
Skreta V, Veldkamp L (2009) Ratings shopping and asset complexity: a theory of ratings inflation. J Monet Econ 56:678–695
Viscusi WK (1978) A note on "lemons" markets with quality certification. Bell J Econ 9:277–279