Liability and reputation in credence goods markets

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HIGHLIGHTS

- We study the impact of liability on a credence-good seller's incentives to maintain a good reputation.
- Credence-good markets are characterized by information asymmetry about the value of sellers' services to consumers.
- Liability refers to the legal environment in which the seller is liable for fixing consumers' problems after charging them the price for his treatment.
- When the seller is short-lived, liability mitigates information asymmetry and facilitates trade.
- Nevertheless, liability may undermine a long-lived seller's incentive to maintain a good reputation and reduces market efficiency.

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ABSTRACT

This paper studies the impact of liability on a credence-good seller's incentives to maintain a good reputation. Credence-good markets are characterized by information asymmetry about the value of sellers' services to consumers who must rely on sellers for diagnosis and treatment provision. Liability refers to the legal environment in which the seller is liable for fixing consumers' problems after charging them the price for his treatment. When the seller is short-lived, liability mitigates information asymmetry and facilitates trade. Nevertheless, liability may undermine a long-lived seller's incentive to maintain a good reputation and reduces market efficiency.

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1. Introduction

We investigate the impact of liability on credence-good sellers' incentives to maintain a good reputation. In markets for credence goods, sellers diagnose consumers' problems and also provide treatment. Examples include health care, consulting and various repair services. The information asymmetry about the value of sellers' services exposes consumers to the risk of seller fraud which is widely documented in various markets.[1] The existing literature (see Dulleck and Kerschbamer, 2006 for a comprehensive review)

has shown that when sellers are liable for solving consumers' problems (Liability), trade takes place in the static setting.[2] Although Liability mitigates information asymmetry and facilitates trade in the one-shot game, it is unclear how it affects sellers' incentives to maintain a good reputation in a setting of infinitely repeated game.[3]

We study the role of Liability in a credence-good market in a repeated game. Contrary to the conventional wisdom, we find a seller may have the strongest incentive to provide the appropriate treatment and implement the first best outcome when there is no Liability. This suggests that legal protections for consumers

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[1] "Patients give horror stories as cancer doctor gets in prison", CNN, July 11, 2015. Auto repair scam is consistently listed as the number one of the top consumer complaints in the U.S. according to "Nation's Top Ten Consumer Complaints", Consumer Federation of America.
Balafoutas et al. (2013) and Liu et al. (2017) find evidence of seller fraud in taxi ride markets.

[2] Pitchik and Schotter (1987), Wolinsky (1993), Fong (2005), Liu (2011), Fong et al. (2014) and Taylor (1995).

[3] Dulleck et al. (2011) investigated the impact of reputation on expert fraud with and without Liability in a lab experiment. In their experiment, the game is repeated finitely many times and consumers learn from their own experience. We study infinitely repeated game and allow a consumer to learn from others' experiences.

in credence-good markets may undermine experts' incentives to maintain a good reputation, hence reducing the efficiency of the market.

The most closely related paper is Chen et al. (2017). Chen, et al. study optimal liability policy in a one-shot game. In their model, the expert needs to perform a costly and unobservable diagnosis to become informed. So, the optimal liability policy must solve both the adverse selection and the moral hazard problems. Our paper complements Chen, et al. in that we focus on the impact of Liability on the expert's incentives to maintain a good reputation in a repeated game and we do not consider the moral hazard problem. Also in a one-shot game, but with a different focus, Fong et al. (2014) compare the market outcomes under the liability and verifiability assumptions.

In a dynamic setting, Fong et al. (2017) studies credence-good sellers' trust building mechanism in monopoly and competitive markets. They assume throughout that the expert is liable for the treatment outcome and consider the case in which it is efficient to repair the serious problem but inefficient to repair the minor problem. By contrast, our study compares the legal environments with and without liability, and we consider the case when it is efficient to repair both the minor and serious problems.

2. Model

A risk neutral, long-lived expert interacts with an infinite sequence of risk neutral, short-lived consumers. Each period one consumer arrives with a problem which is either serious (s) or minor (m). Denote by l_i, $i = m, s$, the loss from problem i and assume $0 < l_m < l_s$. It is common knowledge that the problem is serious with probability $\alpha \in (0, 1)$. The consumer does not know the nature of the problem and consults the expert who can perfectly diagnose the consumer's problem at zero cost. The expert can fully prevent the loss l_i after incurring a treatment cost r_i, $i = m, s$, with $r_m < r_s$. We assume $r_i \le l_i$, $\forall i$. So, it is efficient to fix both types of problems. Furthermore, we assume $E(l) \equiv \alpha l_s + (1 - \alpha) l_m < r_s$,[4] which imposes an upper bound $\overline{\alpha} \equiv \frac{r_s - l_m}{l_s - l_m}$ on α.

A consumer maximizes her expected payoff. She receives utility $u = -l_i$, $i = m, s$, if problem i is left untreated and $u = -p$ if her problem is fixed at price p. Denote by $\delta \in (0, 1]$ the expert's discount factor. The expert maximizes the discounted expected life-time profit.

We summarize our model by describing the timeline of events. At the beginning of each period $t = 1, 2, \ldots$, the expert posts a price list (p_{mt}, p_{st}), with $p_{mt} \le p_{st}$, where p_{it}, $i = m, s$, is the quoted price for fixing problem i. Then, nature draws the loss of the consumer's problem. The consumer observes the price list and decides whether to consult the expert. If the consumer consults the expert, the expert perfectly diagnoses her problem. Then, the expert either proposes to fix the problem at one of the quoted prices or refuses to treat the consumer. Upon a treatment recommendation, the consumer decides whether or not to accept it.

Denote by $R_t \in \{p_{mt}, p_{st}, N\}$ the recommendation made by the expert, where N denotes refusal to treat the consumer. The expert's recommendation policy is $(\beta_{it}, \rho_{it}) \in [0, 1]^2$, $i = m, s$, where β_{it} is the probability that the expert recommends the expensive treatment (p_{st}) to problem i and ρ_{it} is the probability that the expert refuses to treat problem i. Denote by $\tau_{it} \in \{r_m, r_s\}$ the expert's treatment decision if a consumer accepts his recommendation. Let $a_t \in \{0, 1\}$ denote the consumer's acceptance decision, where 0

indicates rejection and 1 indicates acceptance. Finally, let $\gamma_{it} \in [0, 1]$, $i = m, s$, denote the probability that the consumer accepts price p_{it}. At the end of each period, the prices charged by the expert, his recommendation, the consumer's acceptance decision as well as her utility become public information.[5] Formally, we denote $h_t = \{p_{mt}, p_{st}, R_t, a_t, u_t\}$ as the public events that happen in period t and $h^t = \{h_n\}_{n=1}^{t-1}$ as a public history path at the beginning of the period, with $h^1 = \emptyset$. Let $H^t = \{h^t\}$ be the set of public history paths till time t. A public strategy of the expert is a sequence of functions $\{P_t, \beta_{mt}, \beta_{st}, \rho_{mt}, \rho_{st}, \tau_{it}\}_{t=1}^{\infty}$, where the pricing strategy P_t is a mapping from H^t to $\mathbb{R}_+^2$, and $(\beta_{mt}, \beta_{st}, \rho_{mt}, \rho_{st}, \tau_{it}) : H^t \cup \mathbb{R}_+^2 \cup \{m, s\} \rightarrow [0, 1]^5$. A public strategy of the consumer is $(\gamma_{mt}, \gamma_{st}) : H^t \cup R_t \rightarrow [0, 1]^2$.

We focus on *Stationary Perfect Public Equilibria* in which strategies are stationary and players use public strategies which constitute a Nash equilibrium following every public history.

3. Equilibrium

To investigate how liability affects the expert's incentives to maintain a good reputation and the implementation of the first-best outcome, we explicitly distinguish the following two legal environments:

Liability: *The expert must fix the consumer's problem if the consumer accepts his treatment recommendation and pays for it.*

No-Liability: *The expert must provide services to the consumer after she pays for the treatment but is not liable for fixing the consumer's problem.*[6]

When the expert is liable for the treatment outcome, he must provide the appropriate treatment for each type of problem. We can think of the expert's payoff being $-\infty$ when he fails to fix a problem he is paid to treat. In contrast, in the No-Liability environment, the expert is not liable to provide the appropriate treatment but needs to at least provide the minor treatment. His flow payoff is simply $p_i - r_j$ when he charges p_i and provides treatment j, irrespective of the actual problem.

3.1. Liability

Stage-Game Equilibrium and the Punishment Path Fong (2005) shows that in the one-shot game, there is a unique subgame perfect Nash equilibrium in which the expert sets $(p_m, p_s) = (l_m, l_s)$. In the recommendation subgame, the expert always truthfully reveals the nature of the problem ($\beta_m = 0, \beta_s = 1$) and never refuses to provide treatment ($\rho_m = \rho_s = 0$). The customer accepts a treatment at price p_m with probability $\gamma_m = 1$, and at price p_s with probability $\gamma_s = (p_m - r_m)/(p_s - r_m) = (l_m - r_m)/(l_s - r_m)$. In equilibrium, the expert earns a profit of

$$\pi_S^L \equiv \alpha (l_s - r_s) \frac{l_m - r_m}{l_s - r_m} + (1 - \alpha) (l_m - r_m). \tag{1}$$

Please refer to Proposition 1 in Fong (2005) for detailed discussion of the stage game Nash equilibrium.

We assume that if the punishment phase is triggered in the repeated game, players revert to the stage game Nash equilibrium perpetually.

Repeated game Define $\pi^{FB} := \alpha(l_s - r_s) + (1 - \alpha)(l_m - r_m)$ and $\underline{\delta}^L := \frac{r_s - E(l)}{r_s - E(l) + \pi^{FB} - \pi_S^L}$.

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[4] The case of $E(l) \ge r_s$ is trivial since there exists an equilibrium in which the expert charges $E(r)$ for both repairs and always fix the problem even in a static model.

[5] This assumption is motivated by the flourishing of websites like Angie's list, Yelp, and RateMDs on which consumers actively post and share reviews on experts' services.

[6] We thank the anonymous referee for the suggestions on the No-Liability legal environment.

Proposition 1. *When $\delta \geq \underline{\delta}^L$, the first best can be implemented by a single price $E(l)$. In each period, the expert always recommends to treat consumers at this price. Consumers accept the recommendation with probability one as long as the expert has not refused to treat anyone in the past. Otherwise, the game reverts to the stage game Nash equilibrium perpetually. When $\delta \leq \underline{\delta}^L$, the first best cannot be achieved in any stationary perfect public equilibrium.*

Note that the price $E(l)$ is greater than r_m but smaller than r_s. So, the expert makes a profit from fixing the minor problem but bears a loss from fixing the serious problem. This single price equilibrium fails to hold in the one-shot game because the expert will recommend this price only when consumers have the minor problem, yielding consumers a negative surplus since $l_m < E(l)$. So, consumers will not accept the price when the expert is short-lived. In the repeated-game setting, consumers expect the expert to repair both types of problems at $E(l)$, and refusal to treat a consumer will trigger the punishment phase, reducing the expert's future profit. When the expert is sufficiently patient, his future loss from refusal to treat the serious problem outweighs the current gain and hence it is optimal for the expert to fix both types of problems at $E(l)$. Although consumers are overcharged for fixing the minor problem ($l_m < E(l)$), they have a positive surplus $l_s - E(l)$ from fixing the serious problem, yielding them zero expected surplus. So, it is optimal for them to accept the treatment at $E(l)$ with probability one on the equilibrium path.

According to Proposition 1, the first best can be implemented when the expert is sufficiently patient. Because the single price allows the expert to fully extract the first-best surplus, it is also the most profitable equilibrium for the expert. The pricing and recommendation strategies of a long-lived expert (who is concerned about reputation) are qualitatively different from that of a short-lived expert (who is not concerned with future business). When the expert is long-lived, the most profitable equilibrium is fully pooling because he makes the same recommendation to both types of problems. In contrast, in the static counter part, the expert posts different prices and the equilibrium is fully separating because the expert honestly reports the consumer's problem.

3.2. No-liability

Stage-Game Equilibrium and the Punishment Path When the expert is not liable for the treatment outcome, he will always provide the less costly minor treatment if any treatment recommendation is accepted. Anticipating that the expert's treatment will only fix the minor problem, the consumer will accept a treatment if and only if she is charged $p_i \leq (1-\alpha)l_m$. This implies that there is no benefit of posting a price $p_i \notin [r_m, (1-\alpha)l_m]$, and if $r_m \leq p_m < p_s \leq (1-\alpha)l_m$, the expert will always recommend p_s. Therefore, it is WLOG to focus on single-price equilibria with $p_m = p_s \in [r_m, (1-\alpha)l_m]$. It is also clear that trade will occur if and only if $(1-\alpha)l_m \geq r_m$, or $\alpha \leq \alpha^* \equiv \frac{l_m - r_m}{l_m}$. When $\alpha \leq \alpha^*$, the expert maximizes his profit by charging $p_m = p_s = (1-\alpha)l_m$ and the associated profit is $\pi_S^{NL} \equiv (1-\alpha)l_m - r_m$. The following proposition immediately follows.

Proposition 2. *When $\alpha \leq \alpha^*$, the expert's profit is $\pi_S^{NL} = (1-\alpha)l_m - r_m$ in any subgame perfect Nash equilibrium. When $\alpha^* < \alpha$, there is no trade and the expert makes zero profit.*

Repeated game Define $\underline{\delta}_0^{NL}(\alpha) := \frac{r_s - r_m}{r_s - r_m + \pi^{FB}}$ and $\underline{\delta}_1^{NL}(\alpha) := \frac{r_s - r_m}{r_s - r_m + (\pi^{FB} - \pi_S^{NL})}$.

Proposition 3. *The first best can be implemented by the single price $E(l)$ if and only if*

$$\delta \geq \underline{\delta}^{NL}(\alpha) \equiv \begin{cases} \underline{\delta}_1^{NL}(\alpha) \ \forall \alpha \in (0, \alpha^*], \\ \underline{\delta}_0^{NL}(\alpha) \ \forall \alpha \in (\alpha^*, \overline{\alpha}). \end{cases}$$

Similar as the legal environment with liability, the first best can be implemented by the trigger strategy when the discount factor is sufficiently high. Because the expert is not liable for fixing the consumer's problem, he has an incentive to provide the minor treatment to save costs when the serious treatment is necessary to solve the consumer's problem. However, such a deviation is perfectly observable because the consumer's serious problem is not repaired. The game then reverts to the stage-game equilibrium perpetually from next period onward. The loss in future profits dominates the gain from current cost saving as long as the discount factor is sufficiently high. The minimum discount factor necessary to sustain the first best takes different functional forms when $\alpha \leq \alpha^*$ and when $\alpha^* < \alpha$ because trade occurs in the punishment phase in the former but does not occur in the latter case.

3.3. Liability and efficiency

Finally, we look at how the expert's liability to fix treated problem impacts the expert's incentives to maintain a good reputation and the implementation of the first best. Recall that the analysis in the legal environment with liability is conducted for $\alpha \in (0, \overline{\alpha})$, so in this section we compare the two legal regimes in this parameter range. When $l_m < \frac{l_s r_m}{l_s - r_s + r_m}$, $\alpha^* < \overline{\alpha}$. As a result, in the punishment phase in the No-Liability regime, there is no trade for $\alpha \in (\alpha^*, \overline{\alpha})$ and there is trade for $\alpha \in (0, \alpha^*]$. If $\frac{l_s r_m}{l_s - r_s + r_m} < l_m$, $\overline{\alpha} < \alpha^*$. Consequently, there is trade in the punishment phase of the No-Liability regime for all $\alpha \in (0, \overline{\alpha})$. Define $\widehat{l}_m \equiv r_m \times \left(\frac{(l_s - r_s)^2 + (l_s - r_m)r_s}{(l_s - r_s)^2 + (l_s - r_m)r_m} \right)$.

Proposition 4. *For $l_m < \widehat{l}_m$, there exists a cutoff $\widehat{\alpha}(l_m) \in (0, \overline{\alpha})$ such that $\underline{\delta}^{NL}(\alpha) < \underline{\delta}^L(\alpha)$ if and only if $\alpha < \widehat{\alpha}(l_m)$.*

Although no liability results in a huge efficiency loss when the expert has no reputation concern, interestingly, it may provide the expert the strongest incentives to implement the first best treatment. This is because while the expert gains more from shirking in providing the appropriate treatment in the No-Liability regime, he also bears a more severe punishment once he shirks. Which legal environment can better sustain the first best depends on the likelihood of the serious problem. When the expert is liable for treatment outcomes, he weighs between an immediate cost saving of $r_s - E(l)$ (refusing to provide the serious treatment at $E(l)$) and the loss of profit of $\pi^{FB} - \pi_S^L$ in each future period. In the No-Liability regime, the expert gains $r_s - r_m$ from providing the minor treatment instead of the serious treatment to the serious problem, but in each future period, he loses π^{FB} for $\alpha \in (\alpha^*, \overline{\alpha})$ and $\pi^{FB} - \pi_S^{NL}$ for $\alpha \in (0, \alpha^*]$. Because $r_m < E(l)$ and $\pi_S^{NL} < \pi_S^L$, the expert has more to gain but also more to lose from shirking in the No-Liability regime.

Suppose that α is arbitrarily close to zero. The expert's current gain per unit of future loss is $\frac{r_s - E(l)}{\pi^{FB} - \pi_S^L}$ when he is liable and is $\frac{r_s - r_m}{\pi^{FB} - \pi_S^{NL}}$ when he is not liable. It can be verified that $\frac{r_s - E(l)}{\pi^{FB} - \pi_S^L} > \frac{r_s - r_m}{\pi^{FB} - \pi_S^{NL}}$ when $l_m < \widehat{l}_m$. So, it is easier to sustain the first best in the No-Liability regime for very low α. Suppose that α is arbitrarily close to $\overline{\alpha}$. When the expert is liable, he has almost nothing to gain from the deviation because $r_s - E(l)$ converges to zero, but the expert's future loss from the deviation is bounded away from zero. So, the first best can be supported by a very small discount factor. In contrast, in the No-Liability regime, the expert's gain from the deviation is bounded away from zero and his loss from the deviation is also bounded. So, it requires a larger discount factor to sustain the first best.

One implication of Proposition 4 is that when the loss from the minor problem and the likelihood of the serious problem are relatively low in a market of expert services, it may be welfare enhancing to provide less legal protection for the consumers in that market.

4. Conclusion

Liability mitigates information asymmetry in market for credence goods when sellers are short-lived. Nevertheless, it may undermine a long-lived seller's incentives to maintain a good reputation and hence reduce efficiency. Our study finds that when consumers' loss from the minor problem is not too large, Liability strengthens the seller's incentives to maintain a good reputation if consumers are sufficiently likely to suffer the serious problem but undermines it, otherwise. This suggests that when introducing Liability in markets for credence goods, policy maker should take into account the role of reputation as well as the likelihood that consumers' problems will cause serious damages.

Acknowledgments

Appendix

Proof of Proposition 1. First, consider a single-price equilibrium $p_m = p_s = E(l)$. The expert's expected flow profit is $\pi^{FB} = E(l) - (\alpha r_s + (1 - \alpha) r_m)$. If the expert refuses to treat the serious problem, he saves the loss $E(l) - r_s$ in the current period but the punishment phase will be triggered, yielding the expert π_S^L in each future period. So, the no deviation condition in the case of a serious problem is

$$E(l) - r_s + \frac{\delta \pi^{FB}}{1 - \delta} \geq \frac{\delta \pi_S^L}{1 - \delta} \tag{2}$$
$$\delta \geq \underline{\delta}^L.$$

Because $r_m < r_s$, condition (2) ensures the expert will not refuse to treat the minor problem.

Given that the expert is liable for fixing the problem, the consumer's expected benefit from accepting the treatment is $E(l)$, which makes her just indifferent between accepting and rejecting the treatment. So, it is optimal for the consumer to accept the treatment with probability one.

Now show that for $\delta < \underline{\delta}^L$, the first best is not sustainable at a single price $p < E(l)$. To see this, the no deviation condition is

$$p - r_s + \frac{\delta}{1 - \delta}[p - (\alpha r_s + (1 - \alpha) r_m)] \geq \frac{\delta \pi_S^L}{1 - \delta}. \tag{3}$$

Since the left-hand-side of (3) is lower than the left-hand-side of (2) for $p < E(l)$ while the right-hand-sides are the same, (3) cannot be satisfied when $\delta < \underline{\delta}^L$.

Next, we show the first best cannot be implemented by a price list (p_m, p_s), with $p_m < p_s$, for $\delta < \underline{\delta}^L$, where both prices are recommended with positive probabilities, which can be stated as $\beta_m + \beta_s \in (0, 2)$. For the first best to be implemented, both prices must be accepted with probability one because otherwise at least one problem will be left unresolved with a positive probability. But if both prices are accepted with probability one, the expert will deviate to recommending p_s regardless of the problem ($\beta_m = \beta_s = 1$) which constitutes a contradiction. To see that, charging p_s instead of p_m gives the expert a higher payoff in the current period. Also, because the consumer's problem is treated with probability one, the consumer cannot detect the deviation, the deviation will not change the expert's continuation payoff. $\square$

Proof for Proposition 3. Define $\widehat{\pi}_S^{NL} := \max\{\pi_S^{NL}, 0\}$. So, $\widehat{\pi}_S^{NL} = \pi_S^{NL}$ for $\alpha \in (0, \alpha^*]$ and $\widehat{\pi}_S^{NL} = 0$ for $\alpha \in [\alpha^*, \overline{\alpha}]$.

Consider the single price $E(l)$. The expert's no deviation constraints when the consumer has the serious problem are:

$$E(l) - r_s + \frac{\delta \pi^{FB}}{1 - \delta} \geq \frac{\delta \widehat{\pi}_S^{NL}}{1 - \delta}, \tag{4}$$

$$E(l) - r_s + \frac{\delta \pi^{FB}}{1 - \delta} \geq E(l) - r_m + \frac{\delta \widehat{\pi}_S^{NL}}{1 - \delta}. \tag{5}$$

Condition (4) ensures that the expert will not refuse to treat the serious problem, which is similar to (2). Condition (5) ensures that the expert will not provide the minor treatment for the serious problem. Condition (5) implies (4) because $E(l) > r_m$. It can be directly verified that when $0 < \alpha \leq \alpha^*$, condition (5) is satisfied if and only if $\delta \geq \underline{\delta}_1^{NL}(\alpha)$, and when $\alpha^* < \alpha < \overline{\alpha}$, condition (5) is satisfied if and only if $\delta \geq \underline{\delta}_0^{NL}(\alpha)$. The same argument in the proof for Proposition 1 can be applied to show that the first best cannot be implemented if $\alpha \in (0, \alpha^*]$ and $\delta < \underline{\delta}_1^{NL}(\alpha)$ or if $\alpha \in [\alpha^*, \overline{\alpha}]$ and $\delta < \underline{\delta}_0^{NL}(\alpha)$.

Proof for Proposition 4. First consider $l_m < \frac{l_s r_m}{l_s - r_s + r_m}$. In this case, $\alpha^* \in (0, \overline{\alpha})$. So, in the No-Liability environment, the minimum discount factor necessary to sustain the first best is $\underline{\delta}_1^{NL}(\alpha)$ for $\alpha \in (0, \alpha^*]$ and $\underline{\delta}_0^{NL}(\alpha)$ for $\alpha \in [\alpha^*, \overline{\alpha})$.

Take the difference $\underline{\delta}^L - \underline{\delta}_0^{NL} = \frac{(r_s - r_m)\pi^{FB}}{[\pi^{FB} - \pi_S^L + r_s - E(l)][\pi^{FB} + r_s - r_m]} \times \left(\frac{\pi_S^L}{\pi^{FB}} - \frac{E(l) - r_m}{r_s - r_m} \right)$. The sign of $\underline{\delta}^L - \underline{\delta}_0^{NL}$ is determined by the sign of

$g(\alpha) \equiv \frac{\pi_S^L}{\pi^{FB}} - \frac{E(l) - r_m}{r_s - r_m}$. The derivative $\frac{\partial \left(\frac{\pi_S^L}{\pi^{FB}} \right)}{\partial \alpha} = \frac{-(l_s - l_m)(l_s - r_s)(l_m - r_m)}{(l_s - r_m)[\pi^{FB}]^2} < 0$, and $\frac{\partial \left(\frac{E(l) - r_m}{r_s - r_m} \right)}{\partial \alpha} = \frac{l_s - l_m}{r_s - r_m} > 0$. Hence, $g'(\alpha) < 0$. At $\alpha = 0$, $\frac{\pi_S^L}{\pi^{FB}} = 1 > \frac{l_m - r_m}{r_s - r_m} = \frac{E(l) - r_m}{r_s - r_m}$. At $\alpha = \overline{\alpha}$, $E(l) = r_s$ so $\frac{\pi_S^L}{\pi^{FB}} < 1 = \frac{E(l) - r_m}{r_s - r_m}$. Because $g(\alpha)$ is continuous in $(0, \overline{\alpha})$, there exists a cutoff $\alpha_0 \in (0, \overline{\alpha})$ such that $\underline{\delta}^L = \underline{\delta}_0^{NL}$ at α_0, $\underline{\delta}^L > \underline{\delta}_0^{NL}$ for $\alpha < \alpha_0$, and $\underline{\delta}^L < \underline{\delta}_0^{NL}$ for $\alpha > \alpha_0$.

Next, take the difference $\underline{\delta}^L - \underline{\delta}_1^{NL} = \frac{\alpha f(\alpha)}{[\pi^{FB} - \pi_S^L + r_s - E(l)][\pi^{FB} - \pi_S^{NL} + r_s - r_m]}$, where

$$f(\alpha) \equiv -(l_s - r_s + r_m) E(l) + \frac{(r_s - r_m)(l_s - r_s)(l_m - r_m)}{l_s - r_m} + r_m l_s.$$

The sign of $\underline{\delta}^L - \underline{\delta}_1^{NL}$ is determined by the sign of $f(\alpha)$. Note that $f'(\alpha) = -(l_s - r_s + r_m)(l_s - l_m) < 0$. Moreover, $f(0) = \frac{\left[(l_s - r_s)^2 + (l_s - r_m)r_s\right]r_m - l_m\left[(l_s - r_s)^2 + (l_s - r_m)r_m\right]}{l_s - r_m}$, which is positive if and only if $l_m < \widehat{l}_m$. Because $l_m < \frac{l_s r_m}{l_s - r_s + r_m} < \widehat{l}_m, f(0) > 0$. Lastly, $f(\overline{\alpha}) = -\frac{(l_s - r_s)(r_s - r_m)(l_s - l_m)}{l_s - r_m} < 0$. Since $\underline{\delta}^L - \underline{\delta}_1^{NL}$ is continuous in $(0, \overline{\alpha})$, there exists $\alpha_1 \in (0, \overline{\alpha})$ such that $\underline{\delta}^L = \underline{\delta}_1^{NL}$ at $\alpha = \alpha_1, \underline{\delta}^L > \underline{\delta}_1^{NL}$ for $\alpha < \alpha_1$ and $\underline{\delta}^L < \underline{\delta}_1^{NL}$ for $\alpha > \alpha_1$.

Next, we show for each $l_m < \frac{l_s r_m}{l_s - r_s + r_m}$, there exists a cutoff $\widehat{\alpha}(l_m)$ such that $\underline{\delta}^{NL} < \underline{\delta}^L$ if and only if $\alpha < \widehat{\alpha}(l_m)$. We can rewrite the difference $\underline{\delta}^L - \underline{\delta}_1^{NL} = \frac{\pi^{FB}(r_s - r_m)}{[\pi^{FB} - \pi_S^L + r_s - E(l)][\pi^{FB} - \pi_S^{NL} + r_s - r_m]} \times \left(\frac{\pi_S^L}{\pi^{FB}} - \frac{E(l) - r_m}{r_s - r_m} - \frac{\pi_S^{NL}(r_s - E(l))}{\pi^{FB}(r_s - r_m)} \right)$. Let $\widehat{g}(\alpha) \equiv \frac{\pi_S^L}{\pi^{FB}} - \frac{E(l) - r_m}{r_s - r_m} - \frac{\pi_S^{NL}(r_s - E(l))}{\pi^{FB}(r_s - r_m)}$. Note that $\pi_S^{NL} = 0$ at $\alpha = \alpha^*, \pi_S^{NL} > 0$ for $\alpha < \alpha^*$, and $\pi_S^{NL} < 0$ for $\alpha > \alpha^*$. Consequently, $\widehat{g}(\alpha) = g(\alpha)$ at $\alpha = \alpha^*, \widehat{g}(\alpha) < g(\alpha)$ for $\alpha < \alpha^*$, and $\widehat{g}(\alpha) > g(\alpha)$ for $\alpha > \alpha^*$. If $\alpha_0 = \alpha^*$, then $\alpha_1 = \alpha^*$ and hence $\widehat{\alpha}(l_m) = \alpha^*$. If $\alpha_0 > \alpha^*$, then $\widehat{g}(\alpha_0) > g(\alpha_0) = 0$. Since $\widehat{g}(\alpha)$ is decreasing, $\widehat{g}(\alpha) > 0, \forall \alpha \in (0, \alpha^*]$. So, $\widehat{\alpha}(l_m) = \alpha_0$. If $\alpha_0 < \alpha^*$, then $\widehat{g}(\alpha_0) < g(\alpha_0) = 0$ and $\alpha_1 < \alpha_0 < \alpha^*$. Since $g(\alpha)$ is decreasing in $\alpha, g(\alpha) < 0 \forall \alpha \in [\alpha^*, \overline{\alpha})$. In this case, $\widehat{\alpha}(l_m) = \alpha_1$.

Now, consider $\frac{l_s r_m}{l_s - r_s + r_m} < l_m < \widehat{l}_m$. In this case $\alpha^* > \overline{\alpha}$, so in the No-Liability environment, the minimum discount factor necessary to sustain the first best is $\underline{\delta}_1^{NL}(\alpha)$ for $\alpha \in (0, \overline{\alpha})$. The second paragraph of this proof has shown that in this case $\widehat{\alpha}(l_m) = \alpha_1$. $\square$

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