def 1 : A preference is called continuous if :

Say ~~le~~ we have 2 bundles x and y as functions of n (say $(1/n, 0)$ & $(1/n^2, 0)$)

(the sequence must be converging) (then we have 2 sequence $\{x_n\}$ & $\{y_n\}$)

then if $x_n \succeq y_n \;\forall n$

if continuous then $\lim_{n \to \infty} x_n \succeq \lim_{n \to \infty} y_n$

def 2 : A preference is called continuous if (n) the upper contour set for any bundle is closed.

[A set is closed if you take a sequence of points, then the limit of seq. is also included in the set]

def 1 => def 2 :

proof

Say we have an element x^* ; to show that the if there is a sequence of points $\{x^n\}$ such that $x^n \succeq x^* \;\forall n$ ~~then~~ $\lim_{n \to \infty} x^n \succeq x^*$.

In defn ① take $\{x^n\}$ as $\{x^n\}$ and $\{y^n\}$ as $\{x^*, x^*, x^* \ldots\}$

we see that if ~~x^n~~ $x_n \succeq x^* \;\forall n$

$\Rightarrow \lim x_n \succeq x^* \Rightarrow$ Hence proved.

* Connected set : the set which cannot be partitioned

To show ~~~~ lexicographic sequences ~~may be~~ are discontinuous

eg. $x^n = (2, 1)$ | $y^n = (2 - \frac{1}{n}, 2)$

$x^n \succ y^n \quad \forall n$; as $\frac{1}{n} \to 0 \; \forall n$

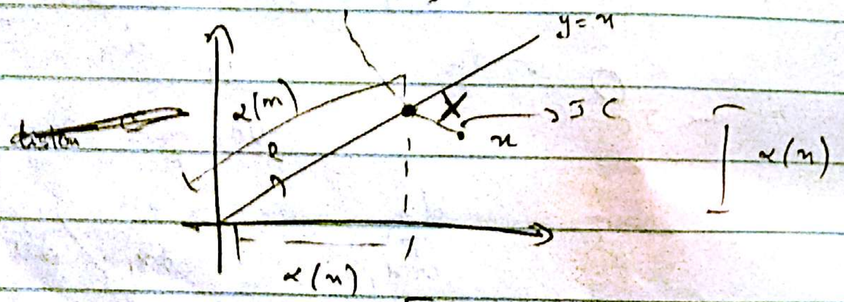
but $\lim_{n \to \infty} x^n \prec \lim_{n \to \infty} y^n$

Representation Theorem

Suppose that $\succeq$ is well behaved i.e. it is MON & convex. Additionally $\succeq$ is continuous (monotonic means ...)

(i) then $\exists$ u, s.t. u represent $\succeq$

(ii) u is continuous



The point x is $\alpha(x) e$

Clearly we have $\alpha(x) e \sim x$

~~Claim~~ $\alpha(x)$ can be the utility function.

First we have to show that such an α exists

Define a set & upper contour set of x $\cap$ Z ~~the line~~ the ray

$\{ \alpha \mid \alpha e \succeq x \}$

Lower contour set $\{ \alpha \mid \alpha e \preceq x \}$

The union of these two sets is entire line segment $y = x$

Due to the connected property there must be an element in their intersection. Hence $\{ \alpha \mid \alpha e \sim x \}$ is non empty and that $\alpha \sim x$ (since it lies in both sets)

Proof of Represent Th in Mascolell Whinston Green Microeconomic theory

Homothetic preference

$u(tx) = t\,u(x)$

$x \sim y \Rightarrow tx \sim ty$

Quasilinear

$x \sim y \Rightarrow x + t\binom{1}{0} \sim y + t\binom{1}{0}$

$u\left(x + t\binom{1}{0}\right) = u(x) + t\binom{1}{0}$

$u(x) = u(y)$

$u(x + t\binom{1}{0}) = u(\ldots)$

---

for same indifference curve

$u(x_1, x_2) = \text{constant}$

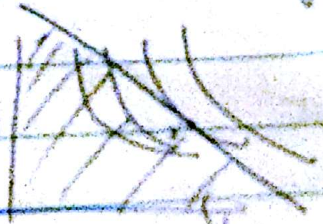
$du = \frac{\partial u}{\partial x_1} dx_1 + \frac{\partial u}{\partial x_2} dx_2$

for same I.C. $0 = du$

$\Rightarrow \frac{dx_2}{dx_1} = -\frac{\partial u}{\partial x_1} \Big/ \frac{\partial u}{\partial x_2}$

$\frac{\partial u}{\partial x_i}$ = so this is rate of change in utility to the change in x_i keeping the other constant.

This is called marginal utility of x_i



we have already said: The most preferable solution is point x where the budget line is tangent to IC.

budget set

How to mathematically formulate it?

THE SOCIAL NOTEBOOK

Utility max Problem.

$$\text{Max } u(x_1, x_2) \text{ s.t. } \vec{p}\cdot\vec{x} \leq M \quad \Big\} \; P > 0, \; M > 0$$

Budget set is compact

{We will use L.P.T.}

MTL100: Lagrangean Multiplier Technique

$$\mathcal{L}(x_1, x_2, \lambda) \qquad [\lambda \geq 0]$$

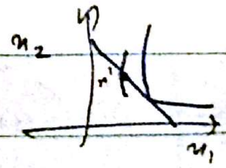
$$= u(x_1, x_2) + \lambda[M - P.x]$$

$\mathcal{L}$ is the function which you need to maximize

If you had another constraint, then add that also. (with another parameter)

The technique used to solve is Kuhn-Tucker

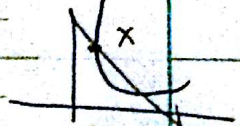
say at x' $\quad MRS \neq \frac{P_1}{P_2}$



$P_1x_1 + P_2x_2 = m$ ← budget line

→ from this statement can we say that this is not the point we're looking for?

Case 1: Slope of budget line is flatter than the slope of IC



Say we are at x, lets ask can I get some ~~x~~ more x_1 (by giving away some x_2).

Since the slope of ~~IC~~ budget line is less, according to our budget we need to loose lesser units of x_2 that what the MRS gives. Hence our utility increases since we would be left with more units of x_2 (if we follow budget line)

W. Rudin Mathematical analysis

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$MU_1 = \frac{\partial U}{\partial x_1}$ $MU_2 = \frac{\partial U}{\partial x_2}$ Max $U(x_1, x_2)$

s.t. $p \cdot x \leq M$

$\mathcal{L}(x, \lambda) = u(x) + \lambda [M - p \cdot x]$

inequality due to boundary

Conditions: $\frac{\partial \mathcal{L}}{\partial x_i} \leq 0$ $\frac{\partial \mathcal{L}}{\partial \lambda} \geq 0$

Kuhn Tucker Kuhl.

$\hookrightarrow \frac{\partial U}{\partial x_i} \leq \lambda P_i$ $M \geq P_x$

Corner sol^n

(Inequality might occur as corner sol^n, when consumption of atleast one good is 0)

if we get an interior solution

$\frac{\partial U}{\partial x_i} = \lambda P_i$

utility

If ~~the~~ ~~corner~~ lies ~~…~~

in ~~out~~ boundary $\frac{\partial u}{\partial x_i} \leq \lambda P_i$

$\Rightarrow$ Convexity of a set

~~for~~ for any $x \in X$ the upper

contour is convex $u()$ is quasiconcave.

$x, x' \in X$

$x'' = \alpha x + (1-\alpha) x'$

Now back to problem

Let x^* be the solution to the problem

At x^*, $\frac{\partial u}{\partial x_1} \leq \lambda P_1$ $\frac{\partial u}{\partial x_2} \leq \lambda P_2$

good sdⁿ

$M \geq P_1 x_1 + P_2 x_2$

if interior solution: $\frac{\partial u}{\partial x_1} = \lambda P_1$ $\frac{\partial u}{\partial x_2} = \lambda P_2$ $M = P x$

$\frac{\partial u}{\partial x_1} \Big/ \frac{\partial u}{\partial x_2} = P_1 / P_2$ $M = P x$

* Take a monotonic transformation of u to help solve

Sufficient conditions for maximization problem

(both for interior and exterior)

$u(\cdot)$ is negative semidefinite

Cobb - Douglas utility function

Actual steps in general

A) compute utility at corners

B) Solve by Lagrangian

C) If MRS strictly ↓, compare solutions in B and A

$u(x_1, x_2) = x_1^{\alpha} x_2^{\beta}$

$p_1 x_1 + p_2 x_2 = m$ Derive x^*

$\mathcal{L}(x_1, x_2, \lambda) = x_1^{\alpha} x_2^{\beta} - \lambda(p_1 x_1 + p_2 x_2 - m) = 0$

$\alpha x_1^{\alpha-1} x_2^{\beta} - \lambda p_1 = 0$

$\beta x_1^{\alpha} x_2^{\beta-1} - \lambda p_2 = 0$

$p_1 x_1 + p_2 x_2 = m$

~~$p_1 x_1 + p_2 x_2 = m$~~

$\alpha\beta \cancel{x_1^{\alpha-1} x_2^{\beta-1}} (\ldots) + m$

$x_1^{\alpha} x_2^{\beta}(\alpha+\beta) = m$

$x_1^{\alpha} x_2^{\beta} = \frac{m}{\alpha+\beta}$

$x_1^{\alpha/\beta}\left(\frac{m - p_1 x_1}{p_2}\right) = \left(\frac{m}{\alpha+\beta}\right)^{1/\beta}$

$U = \ln u = \alpha \ln x_1 + \beta \ln x_2$

$\frac{\alpha}{x_1} = \lambda p_1$

$\frac{\beta}{x_2} = \lambda p_2$

$x_1 = \frac{\alpha}{\lambda p_1} \quad | \quad x_2 = \frac{\beta}{\lambda p_2}$

$p_1 \frac{\alpha}{\lambda p_1} + p_2 \frac{\beta}{\lambda p_2} = m$

$\Rightarrow \lambda = \frac{\alpha+\beta}{m}$

$$x_1 = \frac{\alpha}{p_1}\left(\frac{m}{\alpha+\beta}\right) \quad \Big| \quad x_2 = \frac{\beta}{p_2}\left(\frac{m}{\alpha+\beta}\right)$$

$\gamma \to$ homogenous

This (x_1^*, x_2^*) is such a bundle which I can buy and want to buy. This x^* is an ordinary demand function (utility & a drop) it is a function of p_1, p_2 and m)

maximized level of utility when the prices / income are given by P_1, P_2, m;

$$= u\left(x_1^*(P, m),\ x_2^*(P, m)\right)$$

$$= v(P_1, P_2, m)$$

↳ for any P_1, P_2 and m, v gives the maximum level of utility

v is an indirect utility function

$\frac{\partial V}{\partial m}$ → is known as marginal utility of money

$$= \frac{\partial}{\partial m} u\left(x_1^*(P, m),\ x_2^*(P, m)\right)$$

$$= \left.\frac{\partial u}{\partial x_1}\right|_{x_1 = x_1^*} \frac{\partial x_1}{\partial M} + \left.\frac{\partial u}{\partial x_2}\right|_{x_2 = x_2^*} \frac{\partial x_2}{\partial M}$$

$$= \lambda P_1 \frac{\partial x_1}{\partial M} + \lambda P_2 \frac{\partial x_2}{\partial M} = \lambda\left(P_1 \frac{\partial x_1}{\partial M} + P_2 \frac{\partial x_2}{\partial M}\right)$$

but since $P_1 x_1 + P_2 x_2 = m \rightarrow \partial / \partial M$

$$= \lambda = \frac{\partial V}{\partial M}$$

Properties of ordinary demand function

(i) $x_i(P, m)$ is homogenous of degree 0 in prices and income.

(ii) $px^* = M$

↳ Walra's Law → market clearance; i.e. in equilibrium, we spend total money to get (buy) the best ~~project~~ product

(iii) $x^*(P, m)$ is a convex set. (set as, there can be multiple answers)

let $x, x' \in x^*$

let $x'' = \alpha x + (1-\alpha) x'$ using convexity, we can say utility is equal and x follows market clearance

↳ verify the proper

Task: · find demand functions for standard utility functions

· if ~~it is stri~~ preference is strictly convex, prove that demand function is a singleton set.

Another type of question:

your answer must be such that its utility $\geq$ some given u^*. You have to minimize the expenditure (x) [p_1 and p_2 are fixed]

This is the $\boxed{\text{Expenditure Minimisation Problem}}$

The answer : h is a function of (p_1, p_2, u^*)

it is called compensated demand function

How to solve

$$\text{Minimize } (p \cdot x) \quad \text{s.t. } u(x) \geq u^*$$

$$\mathcal{L}(x_1, x_2, \mu)$$

$$= p \cdot x + \mu\,(u(x) - u^*)$$

~~$p_1 + \mu$~~ First order condition:

let h be the final answer

$$u(h) = u^*$$

tangency $\Rightarrow$ $\frac{\partial u}{\partial h_1} = \frac{-1}{\mu} p_1$ $\qquad$ $\frac{\partial u}{\partial h_2} = \frac{-1}{\mu} p_2$ $\quad$ → dir wrt MP_1

$$\frac{\partial u / \partial h_1}{\partial u / \partial h_2} = MRS\big|_{h = h^*} = \frac{p_1}{p_2}$$

Can you link UMP and EMP?

Something to do with v and u

$$p_1 h_1^* + p_2 h_2^* = e(p_1, p_2, u^*)$$

↳ Expenditure function.

Given, the indirect utility function, (v), can we derive the ordinary demand function? Similarly with (e)

Just 1) Perfect substitutes

$$u(x_1, x_2) = \alpha x_1 + \beta x_2$$

$$p_1 x_1 + p_2 x_2 = m$$

find the ordinary demand function: (logically)

Consider $\frac{\alpha}{p_1}$, what is this

if $\frac{\alpha}{p_1}$ is more, that means x_1 is better as it gives higher utility at ~~for~~ lower price

$$\therefore x_1^* = \begin{cases} \frac{m}{p_1} & \text{if } \frac{\alpha}{p_1} > \frac{\beta}{p_2} \\ 0 & \text{if } \frac{\alpha}{p_1} < \frac{\beta}{p_2} \\ \underbrace{\left[0, \frac{m}{p_1}\right]}_{\text{any value}} & \text{if } \frac{\alpha}{p_1} = \frac{\beta}{p_2} \end{cases}$$

2) the ~~value~~ demand function (h^*) we get from the solution of EMP is called compensated demand function. It is homogenous ~~of~~ of degree 1 in prices.

$h^* \to$ Hicksian ~~//~~ $x^* \to$ Marshallian / Walrasian

Do something later $p \to h$ $p' \to h'$ $p'' \to h''$

$p'' = \alpha p + (1-\alpha) p'$

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to show $p''h'' \geq \alpha ph + (1-\alpha) p'h'$

Recoverability

1) UMP $\Rightarrow v(P, m)$ Indirect utility function

$$\frac{\partial V(P_1, P_2, m)}{\partial P_1} = \frac{\partial U(x_1^*(P_1, P_2, m), x_2^*(P_1, P_2, m))}{\partial P_1}$$

$$\boxed{\frac{\partial v}{\partial m} = \lambda}$$

$$= \frac{\partial U}{\partial x_1}\bigg| \frac{\partial x_1}{\partial P_1} + \frac{\partial U}{\partial x_2} \frac{\partial x_2}{\partial P_1}$$

at x^*

$$\frac{\partial U}{\partial x_i} = \lambda P_i$$

$$= \lambda \left[P_1 \frac{\partial x_1}{\partial P_1} + P_2 \frac{\partial x_2}{\partial P_1} \right]$$

$[P_1 x_1 + P_2 x_2 = m]$

~~$x_1 + P_2 \frac{\partial x_2}{\partial P_1} = 0$~~ $x_1 + P_1 \frac{\partial x_1}{\partial P_1} + P_2 \frac{\partial x_2}{\partial P_1} = 0$

$\therefore P_1 \frac{\partial x_1}{\partial P_1} + P_2 \frac{\partial x_2}{\partial P_2} = -x_1$

$= -\lambda x_1$

$$\therefore \frac{\partial V}{\partial P_1} \Big/ \frac{\partial V}{\partial M} = -x_1$$

ROY'S IDENTITY

$e(P_1, P_2, u^*) \to h_1(P_1, P_2, u^*)$

$e(P, m) = P_1 h_1(P, u^*) + P_2 h_2(P, u^*)$

$$\boxed{\frac{\partial e}{\partial P_1} = h_1}$$ Shepherd's lemma

{Slasky equation} ← on your own

CES utility function $U = (x_1^{\rho} + x_2^{\rho})^{1/\rho}$

How to express e in terms of u?

$u \to f(P, m) \Rightarrow$ express m in terms of u and p

quasiconcavity etc.

2 → Mathematical economical by Ch...

How to convert from v to u?

Given an indirect utility function $u^*(P_1, P_2, m)$, we can find $U(x_1, x_2)$ by:

$$U(x_1, x_2) = \min_{P_1, P_2} u^*(P_1, P_2, M) \text{ such that } P_1x_1 + P_2x_2 = M$$

How? Let $x = (x_1, x_2)$ be the demanded bundle when prices are (P_1, P_2) and income is M. Then, by definition, $u^*(P, M) = U(x)$. Let p' be another such vector with $p'x = M$. Since x is still affordable $u^*(P', M) \geq U(x)$ & $p'x = M$.