Teacher's Sign / Remarks

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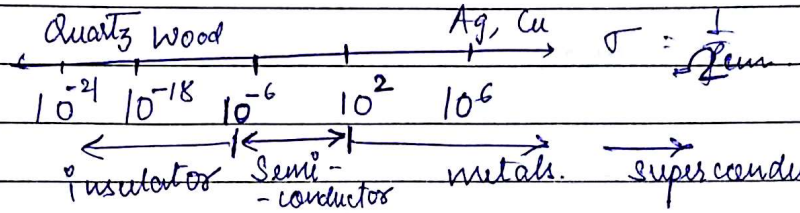
CONDUCTIVITY OF METALS

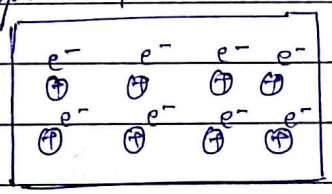
$\frac{V}{I} = R$ $\quad \rho = \frac{RA}{l}$ $\quad \sigma = \frac{1}{\rho}$

$J = \sigma E$ $\quad E = \frac{V}{l}$ $\quad J = \frac{I}{A}$.



FREE ELECTRON MODEL : (Conductivity of metals)

Copper : 4s

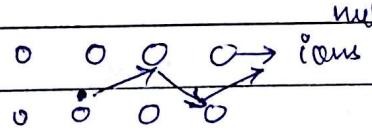


$N_a = \frac{N_o d}{M}$

N_o - Avagadro no.
d - density
M - atomic mass.

$N_a = n \equiv$ no. of e^- per unit volume
$10^{22}/cm^3$.

Neglected columbic force, hence free e^- model



Scattering of e^- cause resistance.

$$\boxed{m\frac{dv}{dt} + \gamma v = eE}$$

v - velocity.

takes care of scattering of e^-.

due to field, e^- accelerate and their

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drift velocity

velocity increase upto v_f. (terminal velocity)
So when $v \to v_f$, $dv/dt = 0$.

hence $\gamma v_f = eE$.

$$\gamma = \frac{eE}{v_f}$$

Hence,

$$m\frac{dv}{dt} + \frac{eE}{v_f} v = eE.$$

$\frac{dy}{dt} + ay = c$

$$\frac{dv}{dt} + \frac{eE}{mv_f} v = \frac{eE}{m},$$

$$v = v_f \left(1 - \exp^{-}\left(\frac{eE}{mv_f} t\right)\right).$$

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<u>Drift velocity</u> - avg. velocity with which e^- move

$$v_d = \frac{1}{N}(v_{x_1} + v_{x_2} + \dots v_{x_n})$$

$$J = eNv_d$$

<u>mean velocity</u> - velocity with which e^- move b/w two collision.

$v_m \gg v_d$. $10^6 m/s$.

** Independent of temperature.

<u>mobility</u> - $\mu = \frac{v_d}{E}$

$$\boxed{\frac{mv_f}{eE} = \tau}$$ τ: relaxation time.

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$V_f \equiv V_d$.

$$J = eNv_f = \frac{eN\tau eE}{m} = \left(\frac{Ne^2\tau}{m}\right)E.$$

$$J = \sigma E.$$

$$\boxed{\sigma = \frac{Ne^2\tau}{m}} \rightarrow (1)$$

N: no. of e^- per unit vol.

$$\mu = \frac{V_d}{E} = \frac{J}{eNE}. \Rightarrow \frac{J}{E} = \mu eN.$$

$$\Rightarrow \boxed{\sigma = \mu eN.} \rightarrow (2)$$

from (1) & (2).

$$\mu = \frac{e\tau}{m}.$$

$$\boxed{\tau = \frac{m\mu}{e}.}$$

Example:

Consider copper metal. $\mu = 43.4\ cm^2 V^{-1} s^{-1}$

$v = 10^6\ m/s$

for $V_d = 0.1\%$ of $10^6\ m/s$. What field to apply.

$V_d = 10^3\ m/s$.

$$\mu = \frac{V_d}{E} \Rightarrow E = \frac{10^3}{43.4 \times 10^{-4}}$$

$$= \frac{1}{50} \cdot 10^6 = 2 \times 10^5.$$

$$E \simeq 2 \times 10^5\ V/m.$$

200 kV/m.

Is it feasible? given $\sigma = 6 \times 10^7\ \Omega^{-1} m^{-1}$.

$$J = \sigma E = 12 \times 10^{12}$$

$$J \sim 10^{13}\ A/m^2.$$

$$= 10^{13-6}\ A/mm^2$$ Not feasible.

Cu burns.

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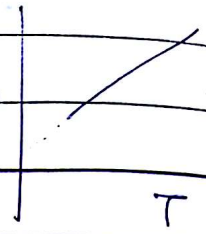
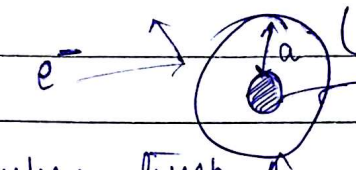
Dependence on temp:

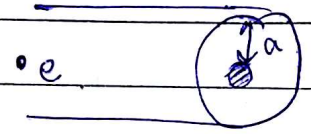
$$R_T = R_0(1+\alpha T)$$
$$\rho_T = \rho_0(1+\alpha T)$$



ion core vibrates.



when temp ↑, a ↑, resistance ↑.



$l = v\tau$.

$v \rightarrow$ mean velocity.

$l \equiv$ mean free path.

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$$\text{vol.} = v\tau\pi a^2.$$

N_s = No. of scattering centers per unit vol.

$$= \frac{1}{\pi a^2 v \tau}$$

$$\Rightarrow \tau \propto \frac{1}{\pi a^2}.$$

If temp. is more, more scattering.

$$a^2 \propto T$$ (equate energy of oscillator to thermal energy)

$$\therefore \tau \propto \frac{1}{T}$$

$$\mu = \frac{e\tau}{m}.$$

$$\rho = \frac{1}{\sigma} = \frac{1}{e\mu N} = \frac{m}{Ne^2\tau}.$$

$$\rho \propto T.$$

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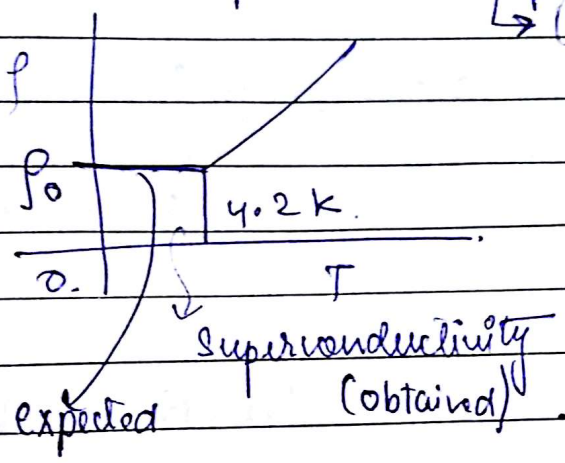
At lower temp., scattering (vibration) of ion stop so ideally it (ρ) should extrapolate to 0 at 0 temp. But defects are also present.
↳ (contribution to scattering)



ρ goes to 0 before 0 K.

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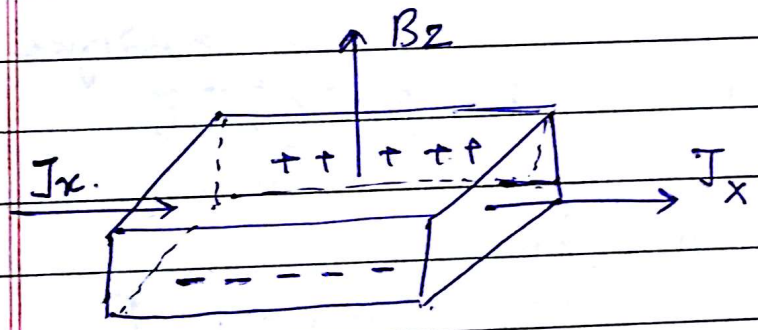
HALL EFFECT:

$\vec{F} = q\vec{V} \times \vec{B}$



$f_y = -e V_x B_z$.

In steady state, $eE_y = -e V_x B_z$

$J_x = n e V_x$.

$\Rightarrow V_x = \frac{J_x}{ne}$

$\Rightarrow \frac{E_y}{J_x B_z} = -\frac{1}{ne}$.

↳ $R_H = -\frac{1}{ne}$

R_H: Hall coefficient.

Thus we can find no. of e^- (n) per unit vol.

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Eg. Copper $\sigma = 5.9 \times 10^{7}\ \Omega m^{-1}$ $\mu_d = 3.2 \times 10^{-3}\ m^2 V^{-1} s^{-1}$

n = no. of e^- per unit vol.

$= \dfrac{\sigma}{e\mu_d} = 1.15 \times 10^{29}\ m^{-3}$

N_a = no. of Cu atom per unit vol.

$= 8.5 \times 10^{28}\ m^{-3}$ (found using density & molecular mass)

$1m^3 = 8m$

$\dfrac{n}{N_a} \approx 1.3$

A subst^n with more thermal conductivity has more elec. conductivity.

$$\boxed{\frac{K}{\sigma T} = \text{const.} = L}$$ (Lorentz number)

T – Temp. K = Thermal conductivity.

$\sigma = \dfrac{ne^2\tau}{m}$ $K = \dfrac{n\pi^2 k_B^2 T \tau}{3m}$ → Boltzman const.

$$\frac{K}{\sigma T} = \frac{\pi^2}{3}\left(\frac{k_B^2}{e^2}\right)$$ Wiedemann – Franz Law

Specific heat:

$C_v = \dfrac{du}{dt}$ u = average energy.

free e^- model couldn't explain similar sp. heats of metals & insulators.

Lattice part + Electronic part.

$3R$ + $\left(\dfrac{3}{2} k_B N_a\right)$ → X.

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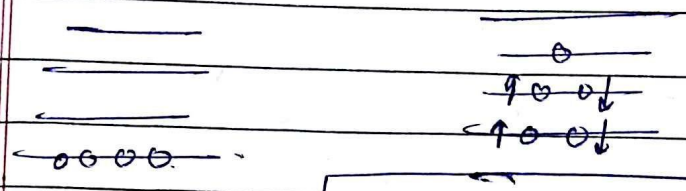
QUANTUM MECHANICAL CONSIDERATION:

1. e^- as a quantum particle.



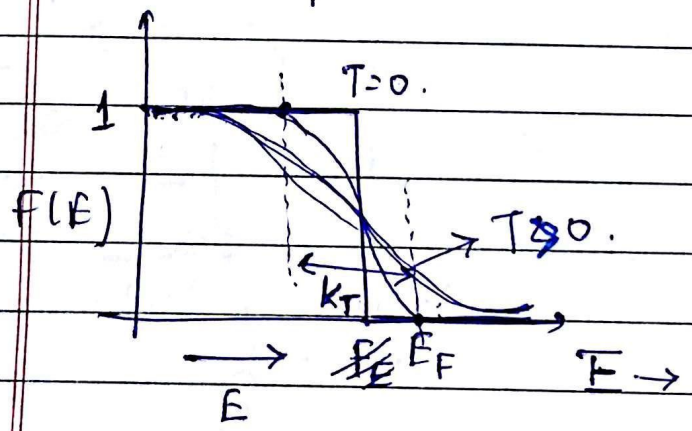
(Fermi-Dirac)

$T = 0$

$$\boxed{f(E) = \frac{1}{\left[\exp\left(\frac{E - E_F}{kT}\right) + 1\right]}}$$

$E_F \equiv$ fermi energy.

fermi level is state below which all levels are occupied at 0K.



F(E): propability of finding e^- at particular energy level.

for $E < E_F$, Prob. = $F(E) = 1 \Rightarrow$ all energy states filled till E_F.

At any particular temp., not all e^- participate. (Shown by k_T)

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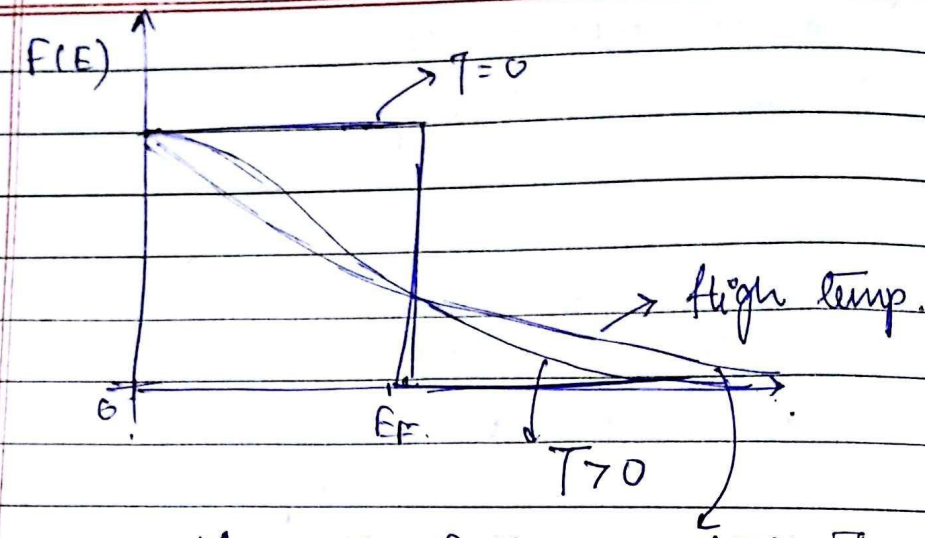


Maxwell - Boltzman distribution.

E_F : fermi Energy.

T_F : Fermi Temp. $\quad E_F = kT_F$.

v_f : Fermi velocity $\quad \frac{1}{2}mv_f^2 = E_f \Rightarrow v_F = \sqrt{\frac{2E_F}{m}}$

P_f : fermi momentum. $\quad E_F = \frac{P_F^2}{2m}$

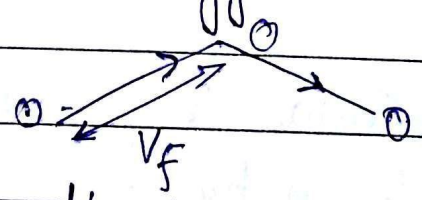
$\Rightarrow P_F = \sqrt{2mE_f}$

$\left(k = \frac{2\pi}{\lambda}\right)$ k_F : Wave vector. $\quad p = \hbar k$.

$E_F = \frac{\hbar^2 k_F^2}{2m}$.

Above fermi temp, fermi Dirac statistics fail.

v_f is velocity with which e^- move at fermi energy.



b/w these 2 pts.

e^- (scattering)

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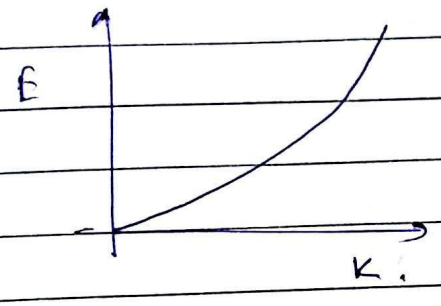
1. Free-e^- model : $V=0$.

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}E\psi = 0.$$

$$\psi = Ae^{i\alpha x}$$

$$\alpha = \sqrt{\frac{2mE}{\hbar^2}} \qquad E = \frac{\hbar^2\alpha^2}{2m}$$

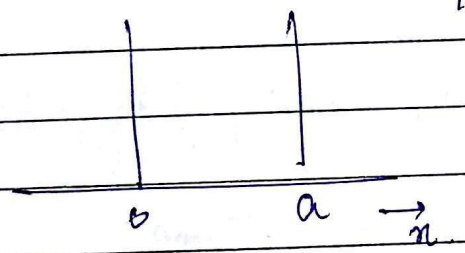
$\alpha \rightarrow$ wave vector K $\qquad (E = \hbar^2k^2/2m)$



All values of k allowed
$\Rightarrow$ All energy states allowed.

2. Bound e^-

$$V(x) = \begin{cases} 0 & 0<x<a \\ \infty & x\leq 0,\ x\geq a. \end{cases}$$



$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}E\psi = 0 \qquad 0<x<a$$

$$\psi = Ae^{i\alpha x} + Be^{-i\alpha x}.$$

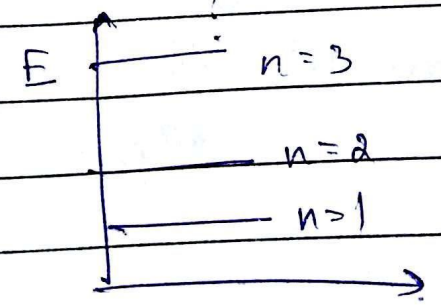
$\psi(0) = 0 \Rightarrow A = -B.$

$\psi(a) = 0 \Rightarrow A(e^{i\alpha a} - e^{-i\alpha a}) = 0.$

$\sin \alpha a = 0 \Rightarrow \alpha a = \pi n.$

$$\alpha K = n\pi$$

$$E_n = \frac{\alpha^2\hbar^2}{2m} = \frac{n^2\hbar^2\pi^2}{2ma^2}.$$



1D: $k_x a = n_x \pi$.

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3D: $\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)\psi+\frac{2mE}{\hbar^2}\psi=0.$

$\psi(x,y,z)$.

$k_x a = n_x \pi, \quad k_y a = n_y \pi \quad k_z a = n_z \pi.$

$$E = \frac{\hbar^2 \pi^2}{2ma^2}\left(n_x^2+n_y^2+n_z^2\right).$$

Degenerate States – Quantum no. are diff^n but E is same.

Eg: $(n_x, n_y, n_z) = (1,1,2), (1,2,1), (2,1,1).$

Periodic Boundary cond^n: $\psi(x)=\psi(x+a).$

$$\psi(r) = Ae^{i\vec{k}\cdot\vec{r}}$$

$$= A e^{ik_x x} \cdot e^{ik_y y} \cdot e^{ik_z z}.$$

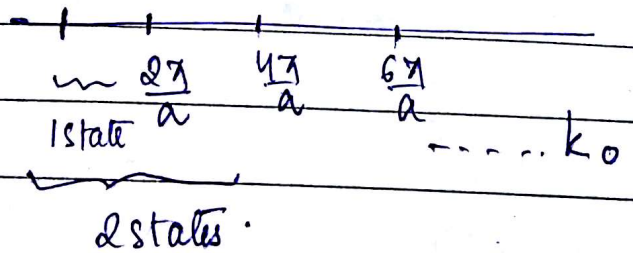
$$k_x = \frac{2\pi n_x}{a} \quad k_y = \frac{2\pi n_y}{a} \quad k_z = \frac{2\pi n_z}{a}$$

$\psi(x)=\psi(x+a)$, $\psi(y)=\psi(y+a)$.

$\psi(z)=\psi(z+a)$.

1D:

(momentum – Space).



k_0 corresponds to $\frac{k_0}{2\pi/a}$ states.

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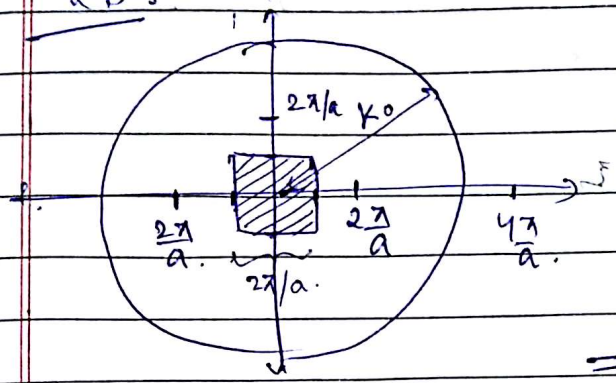
2D:



for circle corresponding to rad. $= k_0$,

no. of states $= \dfrac{\text{area of circle}}{\text{area of square}}$

$$= \frac{\pi k_0^2}{(2\pi/a)^2}.$$

3D:

Sphere of rad. $= k_0$

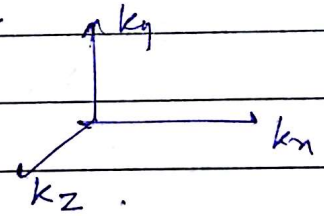
no. of states $= \dfrac{\frac{4\pi}{3} k_0^3}{(2\pi/a)^3} = \dfrac{a^3}{6\pi^2} k^3.$

4/8. E_F corresponding to k_F

$T = 0.$ N e^-.

no. of states b/w 0 to k_F. (~~non-degenerate~~)

$$= \frac{4/3\,\pi\, k_F^3}{(2\pi/a)^3}$$



(e⁻) $N = 2\left(\dfrac{\frac{4\pi}{3} k_F^3}{(2\pi/a)^3}\right) = \dfrac{a^3}{3}\dfrac{k_F^3}{\pi^2}.$

(each state hold 2 e^- opp. spin)

$$k_F = \left(3\pi^2 \frac{N}{a^3}\right)^{1/3}.$$

~~= No. of states~~

$\dfrac{N}{a^3} = n$ e^- density.

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$$E_F = \frac{\hbar^2 k_F^2}{2m}$$

$$= \frac{\hbar^2}{2m}(3\pi^2 n)^{2/3}$$

$$v_F = \frac{\hbar k_F}{m} = \frac{\hbar}{m}(3\pi^2 n)^{1/3}$$

$$T_F = \frac{E_F}{k_B} = \frac{\hbar^2}{2m k_B}(3\pi^2 n)^{1/3}$$

$k_B \rightarrow$ Boltzmann const.

$n \approx 1-5 \times 10^{22}/cm^3$

$E_F \sim 1-4\ eV$.

$v_F \sim 10^8\ cm/s$ $\quad T_F \sim 1-4 \times 10^4\ K$

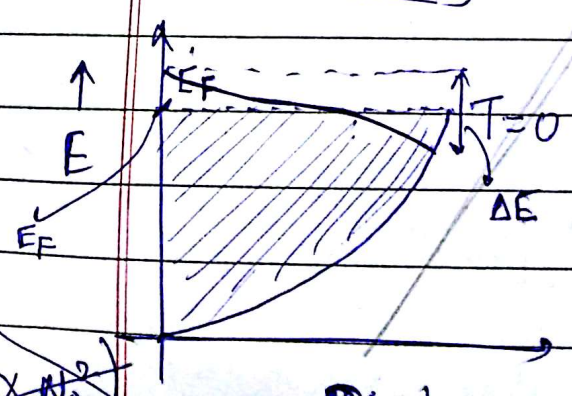
⇒ At room temp., apply Fermi-Dirac statistics.

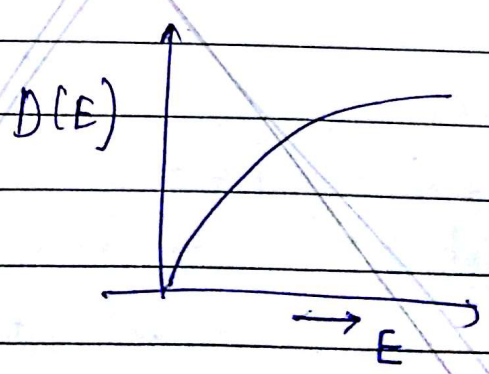
<u>Density of states</u> - No. of states per unit energy.

No. of state (N) $= \frac{a^3}{6}\frac{k^3}{\pi^2}$ $\quad E = \frac{\hbar^2 k^2}{2m}$

$$N_0 = \frac{a^3}{6}\frac{1}{\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2} E^{3/2}$$

$$\boxed{D(E) = \frac{dN_0}{dE}} = C E^{1/2}$$





$(E \propto n^2)$

$F(\varepsilon)$ - Fermi dirac dist fn.

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$$E = \int_0^\infty E\, D(\varepsilon) F(\varepsilon)\, d\varepsilon.$$

Total energy

$$n = 2 \int_0^\infty D(\varepsilon) f(\varepsilon)\, d\varepsilon.$$

$E \equiv \varepsilon$.

no. of e^-

$$f(\varepsilon) = \frac{1}{\exp\left(\frac{E - E_F}{k_T}\right) + 1}.$$

$\Delta E \rightarrow$ energy width $\rightarrow$ e^- participating in conduction state.

avg. E, $\langle E \rangle = \frac{E_{Total}}{n}$.

at $T=0$, $E_{Total} = \int_0^\infty E.D(E) F(E)\, dE$.

$\varepsilon \equiv E$.

$$= \int_0^\infty C E^{3/2} F(E)\, dE.$$

for 0 to E_F, $F(E) = 1$ rest $= 0$.

$$= \int_0^{E_F} C E^{3/2}\, dE.$$

$$= C E^{5/2} \times \frac{2}{5}.$$

$$n = \int_0^\infty D(E) F(E)\, dE = \cancel{C \frac{3}{2} E^{3/2}}$$

$$C \frac{2}{3} E^{3/2}.$$

$$\therefore \langle E \rangle = \frac{E_{Total}}{n} = \boxed{\frac{3}{5} E_F}.$$

for $T \neq 0$, $f(E) \neq 1$. Consider fermi-dirac fn.

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In quantum considerati,

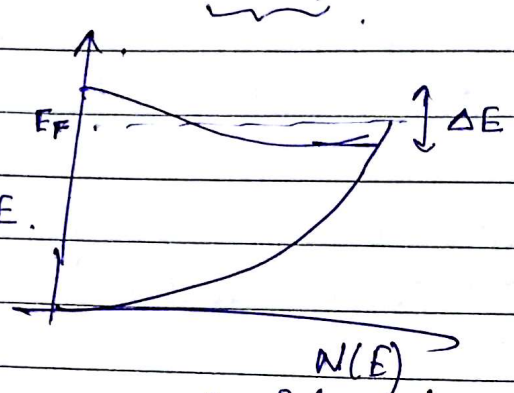
$$J = e N' v_F$$

no. of e^- near Fermi energy.

$J = e n v$

e^- density, drift velocity

$$N' = D(E_F) \cdot \Delta E$$



$$E = \frac{\hbar^2 k^2}{2m}$$

$$\left(\frac{dE}{dk}\right)_{k=k_F} = \hbar v_F$$

$$\Delta E \simeq \hbar v_F \cdot \Delta k \quad \text{--- (1)}$$

$$F = m\frac{dv}{dt} = eE$$

$$\hbar \frac{dk}{dt} = eE.$$

$$\Delta k = \frac{eE\tau}{\hbar}$$

$$\hbar \Delta k = eE\tau$$

$\therefore$ (1) $\Rightarrow$ $\Delta E = v_F e E \tau$.

$$\therefore J = e^2 v_F^2 D(E) \tau E$$

E → Electric field

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$0 \rightarrow k$.

No. of states $= \dfrac{\frac{4}{3}\pi k^3}{\left(\frac{2\pi}{a}\right)^3} = \dfrac{a^3}{6\pi^2} k^3$

No. of states for e^- (spin, anti-spin)

$$S(E) = \frac{a^3}{3\pi^2} k^3.$$

$$E = \frac{\hbar^2 k^2}{2m} \qquad S(E) = \frac{a^3}{3\pi^2}\left(\frac{2mE}{\hbar^2}\right)^{3/2}$$

$$= \frac{a^3}{3\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2} \cdot E^{3/2}.$$

Density of states $(D(E))$.
(states per unit energy)

$$D(E) = \frac{dS(E)}{dE} = \frac{a^3}{3\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2} \cdot \frac{3}{2}\sqrt{E}.$$

$$= \frac{a^3}{2\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2}\sqrt{E} = C E^{1/2}$$

Population density $N(E)$.

$$N(E) = D(E) \cdot F(E)$$

↳ fermi-dirac distrib.

(Occupancy of states depend on ↗).

for $T=0$, all states will be filled upto E_F but not for $T>0$.

$E \rightarrow E + dE$.

$$dN(E) = N(E) \cdot dE.$$

?

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$$N_T = \int_0^\infty dN(E) = \int_0^\infty D(E) \cdot F(E)\, dE.$$

$$= \int_0^\infty C E^{1/2} \frac{1}{\left(\exp\left(\frac{E-E_F}{k_T}\right)+1\right)} dE.$$

at $T = 0$,

$$N_T = \int_0^{E_F} C E^{1/2} dE = C \times \frac{2}{3} E^{3/2}$$

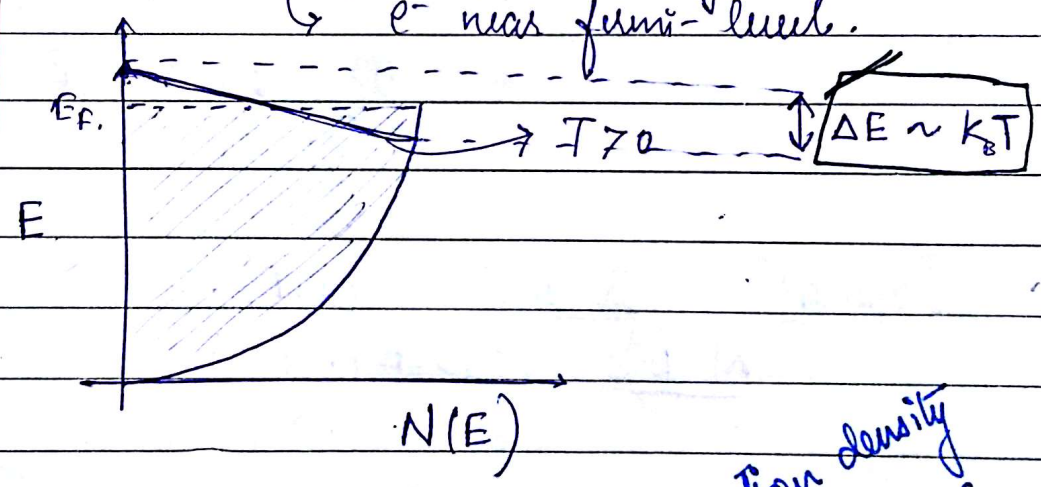
$$E_T \text{ (Total energy)} = \int_0^\infty E \cdot D(E) \cdot F(E)\, dE.$$

$$\langle E \rangle = \frac{E_T}{N_T} = \boxed{\left(\frac{3}{5}\right) E_F \text{ at } T = 0k.}$$

Quantum mech. picture:

$J = eN'v_F \rightarrow$ fermi velocity.

$N' \rightarrow$ e^- near fermi-level.



$N' = N(E_F) \cdot \Delta E$. = Population density $\times \Delta E$

$$J = e^2 v_F^2 N(E_F) \tau \mathcal{E}$$

$\mathcal{E} \rightarrow$ Electric field

* only e^- near fermi-level participate in conduction.

Specific heat → Electronic
→ lattice

Electronic Specific Heat

$$U = \frac{3}{2} k_B T N \Rightarrow (C_v)_e = \frac{\partial U}{\partial T}$$

$$= \frac{3}{2} k_B N.$$

(Independent of temp.)

Q. mech. picture :

$$U = \frac{3}{2} k_B T N'$$

$$N' = N(E_F) \cdot \Delta E = N(E_F) \cdot k_B T.$$

$$U = \frac{3}{2} k_B^2 T^2 N(E_F).$$

$$(C_v)_e = 3 k_B^2 T N(E_F). \quad \text{(temp. dependent)}$$

Success of free e⁻ model (Quan.)

σ, $R_H = -\frac{1}{ne}$, $(C_v)_e$.

Thermoelectric effect.

failure : - R_H → Hall coefficient (+ve)

- Thermoelectric effect sign.

- Couldn't explain insulator, semiconductor

→ optical properties: why metal shows diff^n lustre.

This is due to the assumption that $V = 0$. (e⁻ do experience potential).

There is periodic variation of potential in the lattice.

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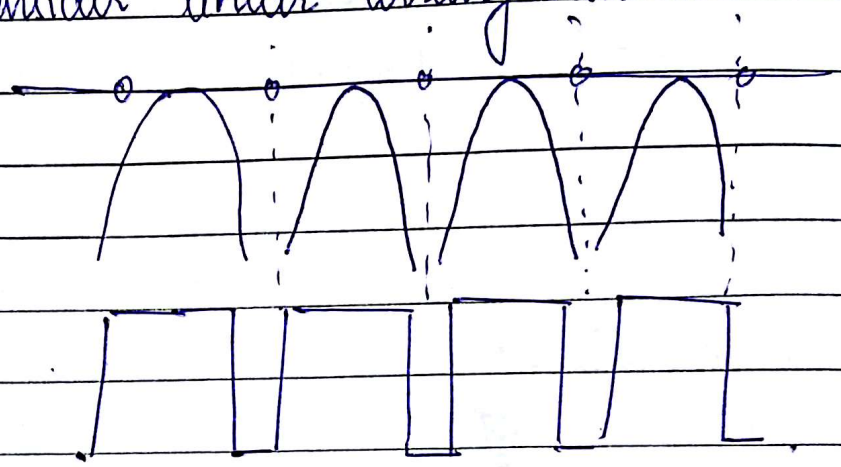
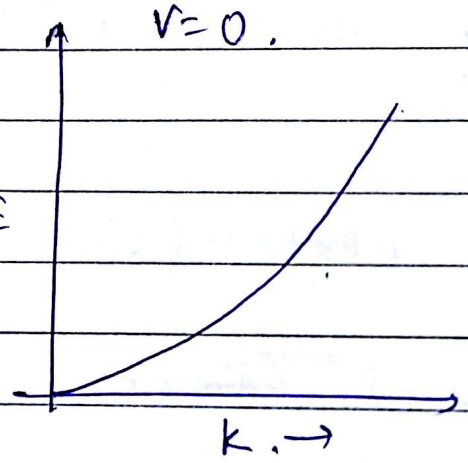
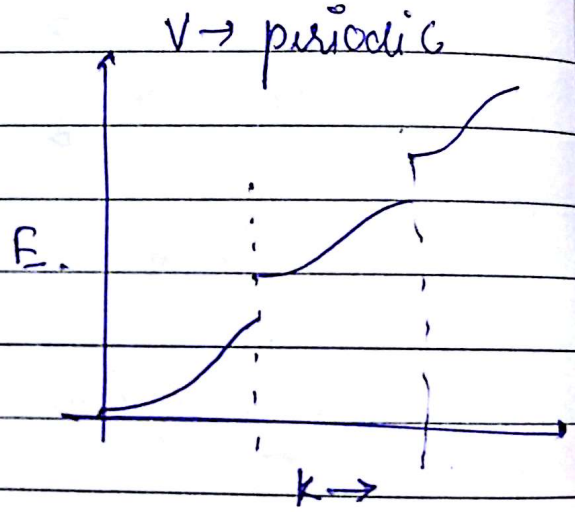
Consider linear arrangement:







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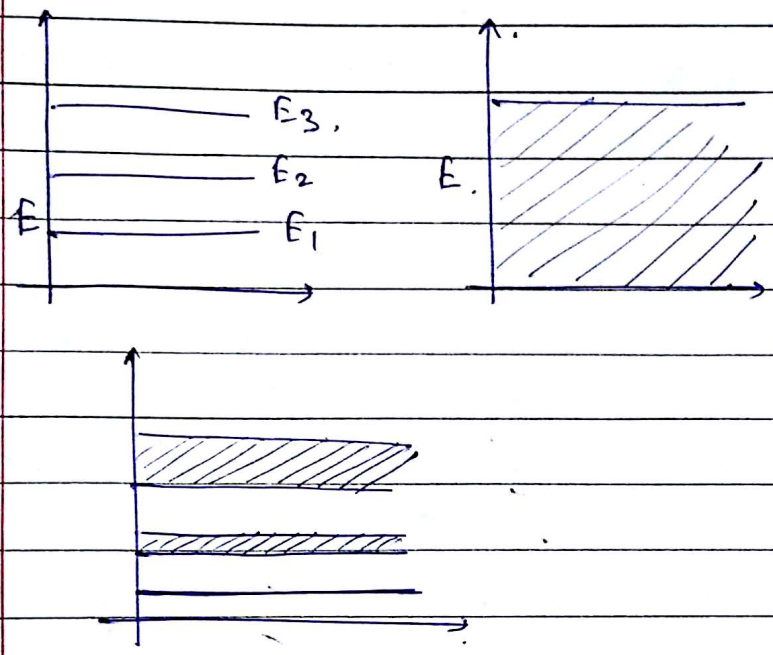
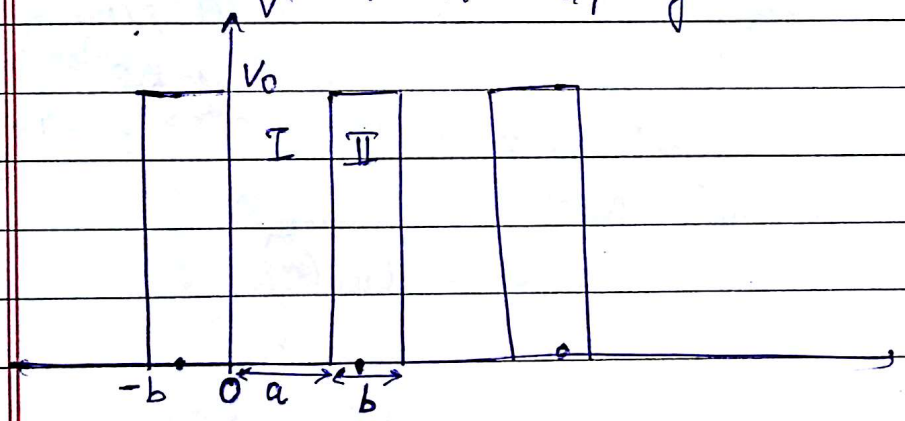


$V \rightarrow$ Potential exp. by e^-.



I : $V = 0$ II : $V = V_0$ $E < V_0$.

I : $V = 0$

$$\frac{d^2\psi}{dx^2} + \frac{2mE}{\hbar^2}\psi = 0. \qquad \alpha^2 = \frac{2mE}{\hbar^2}$$

II : $V = V_0$.

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}\overset{(E-V_0)}{\cancel{(V_0-E)}}\psi = 0.$$

$$\gamma^2 = \frac{2m(V_0-E)}{\hbar^2}$$

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Block fn cond^n

$$\psi(x) = u(x)\, e^{ikx}$$

$u(x)$ satisfies:

1) $u(x) = u(x+a+b)$ (periodicity)

$a+b$: lattice constant.

Substitute $\psi(x) = u(x)e^{ikx}$ in eq^n I & II.

$$\frac{d^2\psi}{dx^2} = \left(\frac{d^2u}{dx^2} + 2ik\frac{du}{dx} - k^2u\right)e^{ikx}$$

$$\text{I} \rightarrow \frac{d^2u}{dx^2} + 2ik\frac{du}{dx} - (k^2-a^2)u = 0$$

$$\text{II} \rightarrow \frac{d^2u}{dx^2} + 2ik\frac{du}{dx} - (k^2+\gamma^2)u = 0$$

(Damped-oscillation type eq^n).

$$u_I(x) = e^{-ikx}\left(Ae^{iax} + Be^{-iax}\right)$$

$$u_{II}(x) = e^{-ikx}\left(Ce^{-\gamma x} + De^{\gamma x}\right)$$

Continuity ~~Boundary~~ cond^n:

$$u_I(0) = u_{II}(0)$$

$$\rightarrow A + B = C + D \quad -(1)$$

?

$$\left(\frac{du_I}{dx}\right)_{x=0} = \left(\frac{du_{II}}{dx}\right)_{x=0}$$

$$A(ia-ik) + B(-ia-ik) = C(-\gamma-ik) + D(\gamma-ik) \quad -(2)$$

Cyclic cond^n (periodicity).

$$u_I(x=a) = u_{II}(x=-b)$$

$$Ae^{(ia-ik)a} + Be^{(ia-ik)a} = Ce^{-(-ik-\gamma)b} + De^{-(-ik+\gamma)b}$$

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$$\left(\frac{dU_I}{dx}\right)_{x=a} = \left(\frac{dU_{II}}{dx}\right)_{x=-b}$$

$$\frac{\gamma^2-\alpha^2}{2\alpha\gamma}\sinh(\gamma b)\sin(\alpha a) + \cosh(\gamma b)\cos(\alpha a) = \cos k(a+b) \quad (*)$$

Assumptions:

V_0 is very large, b is very small s.t. $V_0 b$ is finite.

$$\gamma^2 \approx \frac{2m}{\hbar^2}V_0 \qquad \gamma^2-\alpha^2 \approx \gamma^2$$

$$\gamma = \sqrt{\frac{2m}{\hbar^2}V_0}$$

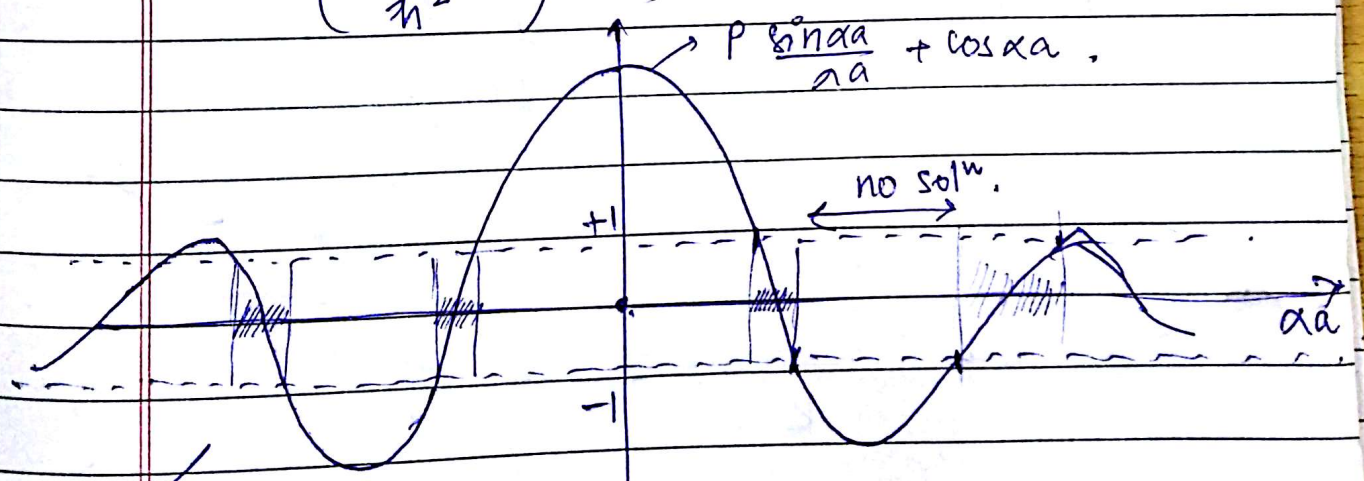
$$\gamma b = \sqrt{\frac{2mV_0}{\hbar^2}} \times b \quad : \text{very small no.}$$

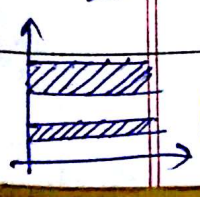
$$\sinh\gamma b \approx \gamma b \qquad \cosh\gamma b \approx 1.$$

$$(*) \Rightarrow \frac{mV_0 b}{\hbar^2\alpha}\sin\alpha a + \cos\alpha a = \cos k(a+b).$$

$$P\frac{\sin\alpha a}{\alpha a} + \cos\alpha a = \cos k(a+b)$$

$$P = \left(\frac{mV_0 ab}{\hbar^2}\right)$$





Sol^n exists for shaded value of αa.
(Energy bands). (some energy bands only allowed)

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$a + b \equiv A$.

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$$P\frac{\sin(\alpha a)}{\alpha a} + \cos \alpha a = \cos k(a+b)$$

b very small.

Suppose $P \to 0$.

$$\cos \alpha a = \cos k(a).$$

$$\alpha \cancel{a} = k \cancel{a}.$$

$$\alpha^2 = k^2$$

$$\alpha^2 = \frac{2mE}{\hbar^2} = k^2 \longrightarrow E = \frac{\hbar^2 k^2}{2m}$$

(Case of free e^-) $\rightarrow$ V_0 small

$P \to \infty$

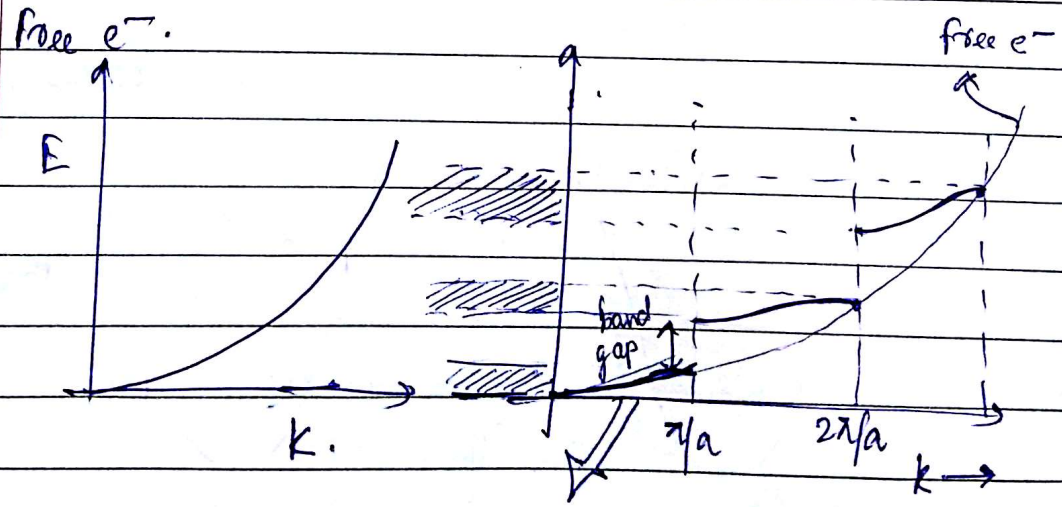
$$\frac{\sin(\alpha a)}{\alpha a} = 0.$$ (for $P\frac{\sin(\alpha a)}{\alpha a}$ to be finite)

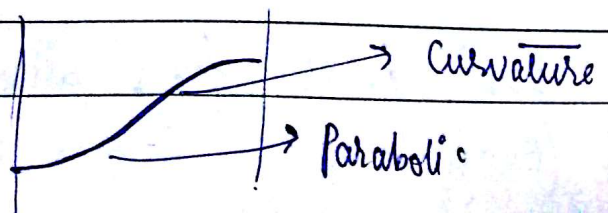
$$\alpha a = n\pi.$$

$$\frac{2mE}{\hbar^2} a^2 = n^2\pi^2$$

$$\Rightarrow E = \frac{n^2\pi^2\hbar^2}{2ma^2} \qquad n = 1, 2, 3 \ldots$$

(Case of bound e^-) $\rightarrow$ V_0 large.





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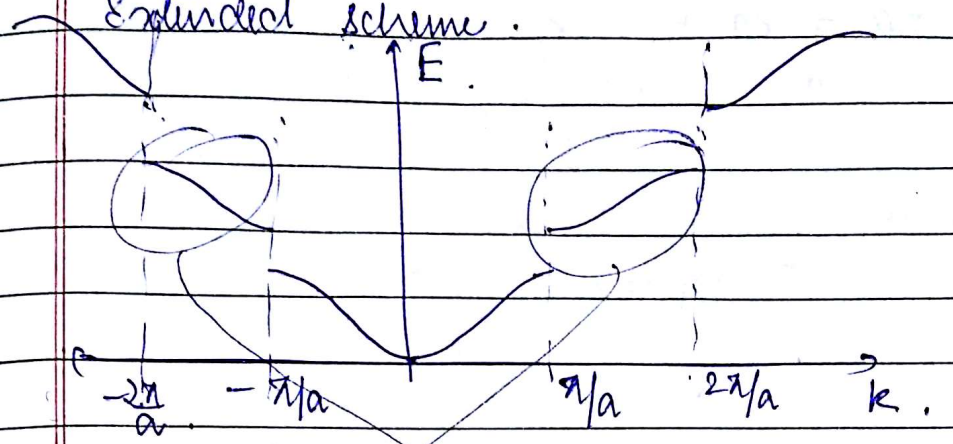
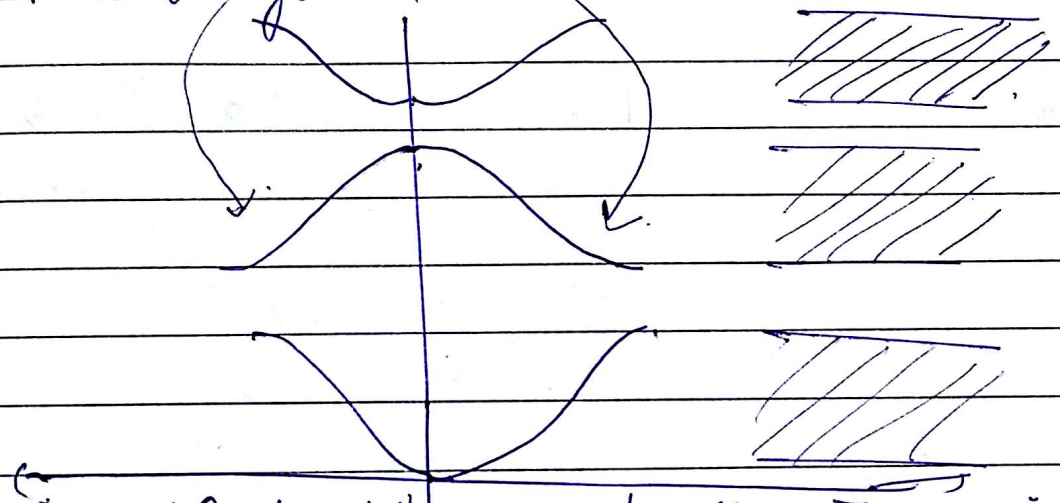
Extended scheme.



Reduced zone scheme:



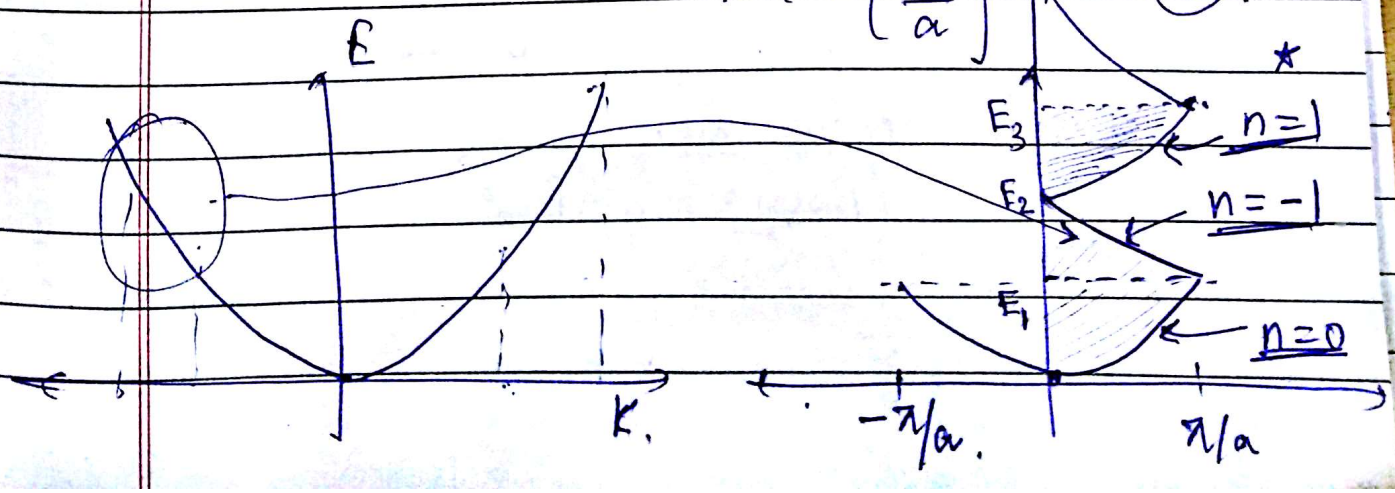
(Just shifting and alternative ~~mirror~~ ~~image~~ sigth to left and left wala to right.)

$P \to 0$. (free e⁻ situation)

$\cos \alpha a = \cos ka$.

$\alpha a = ka + 2n\pi$.

$\alpha = k + n\left(\frac{2\pi}{a}\right)$ (2).



$$\alpha a = ka + 2\pi n.$$

$$a = k + n\left(\frac{2\pi}{a}\right).$$

first band $\underline{n=0}$ E_{min}, E_{max}.

$\cancel{=0}$. $\alpha = k$ $\quad \alpha^2 = k^2$

$$E = \frac{\hbar^2 k^2}{2m}$$

$$\underset{(k=0)}{E_{min} = 0} \; ; \quad \underset{(k=\pi/a)}{E_{max} =} \frac{\hbar^2 \pi^2}{2ma^2}$$

Second band $\underline{n=-1}$. $E_{min} \rightarrow k = \pi/a$. (look from graph 2)

$E_{max} \rightarrow k = 0$.

$$E_{min} = \frac{\hbar^2}{2m}\left(\frac{\pi}{a} - \frac{2\pi}{a}\right)^2.$$

$$\underset{(k=0)}{E_{max}} = \frac{\hbar^2}{2m}\left(0 - \frac{2\pi}{a}\right)^2.$$

$$= \frac{4\hbar^2\pi^2}{2ma^2}.$$

2nd band width $= E_{max} - E_{min}$

$$= \left(\frac{4}{2} - \frac{1}{2}\right)\frac{\hbar^2\pi^2}{ma^2}$$

$$= \frac{3\hbar^2\pi^2}{2\,ma^2},$$

(n is choosen s.t.

$$(E_{max})_i = (E_{min})_{i+1}.$$

$i \in n$.

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E → energy $\mathcal{E}$ → Electric field.

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Effective mass.

free e^-. a = acceleration.

$am = -e\mathcal{E}_0$ → Electric field.

$a' m^* = -e\mathcal{E}_0$

$$\boxed{m^* = \frac{\hbar^2}{\frac{\partial^2 E}{\partial k^2}}}$$

(effec. mass)

$$a = \frac{dv_g}{dt} \qquad v_g = \frac{d\omega}{dk} = \frac{1}{\hbar}\frac{dE}{dk}$$

$$= \frac{1}{\hbar}\frac{d}{dt}\left(\frac{dE}{dk}\right)$$

$$= \frac{1}{\hbar}\frac{d^2E}{dk^2}\frac{dk}{dt}$$

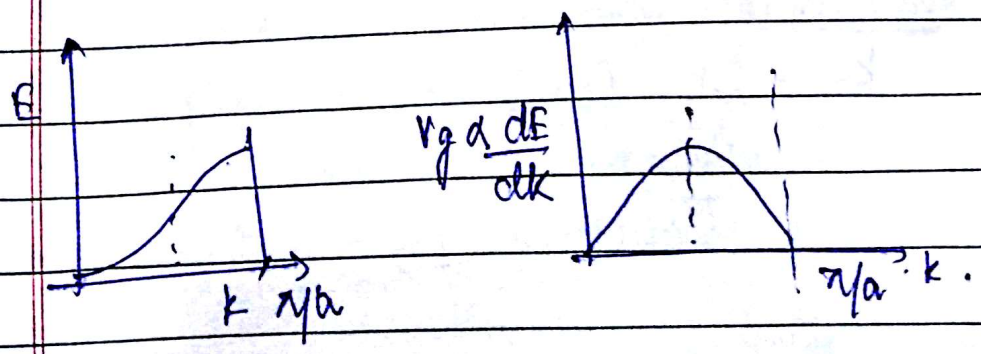
$$\delta E = -(e\mathcal{E}_0)\, v_g\, \delta t$$

$$= -(e\mathcal{E}_0)\frac{1}{\hbar}\left(\frac{dE}{dk}\right)\delta t$$

$$a' = \frac{1}{\hbar}\frac{d^2E}{dk^2}\left(\frac{-e\mathcal{E}_0}{\hbar}\right)$$

$$\frac{-e\mathcal{E}_0}{m^*} = \frac{-e\mathcal{E}_0}{\hbar^2}\left(\frac{d^2E}{dk^2}\right)$$

$$\Rightarrow m^* = \frac{\hbar^2}{\left(\frac{d^2E}{dk^2}\right)}$$



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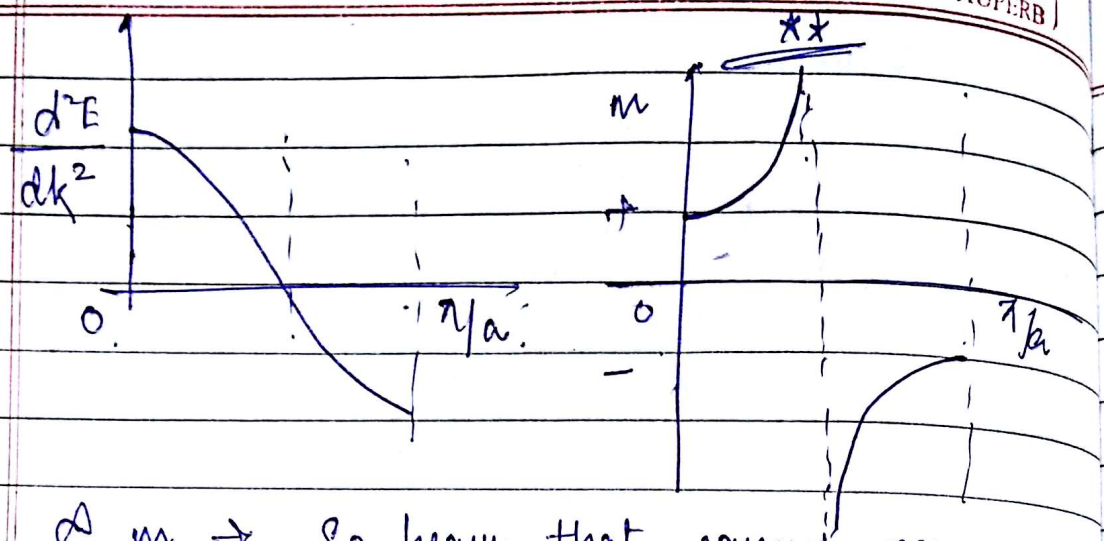


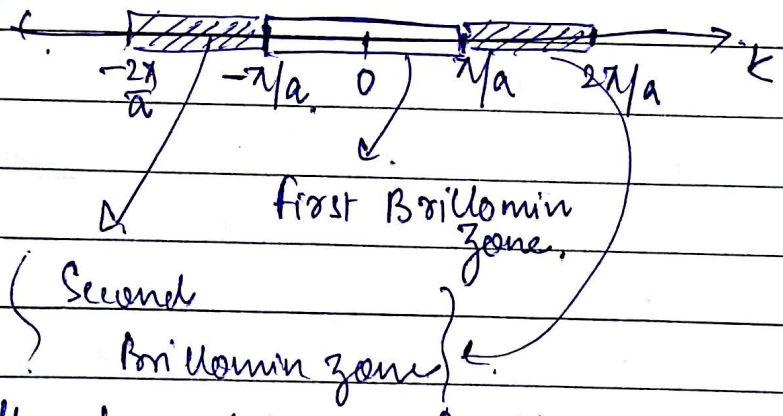
∞ m ⇒ so heavy that cannot move

(−ve) m ⇒ even on acc^n, it starts decelerating.

(This is just effective mass not real)

BRILLOMIN ZONE.

(doipoms.ac.uk/idlib/brillouin)



On the boundary of Brillomin zone, we get discontinuity in energy.

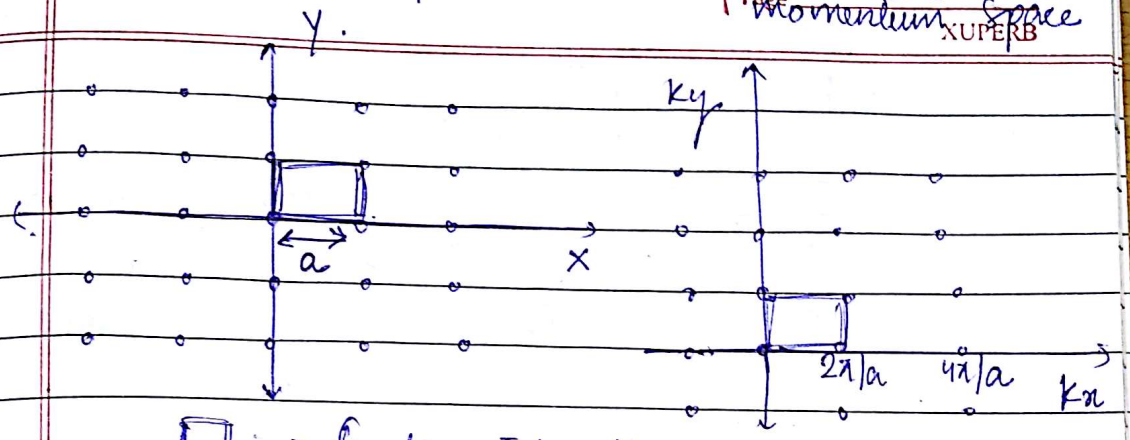
17/8 <u>momentum space</u>. (k_x, k_y, k_z)

$k_x = \frac{2\pi}{\lambda}$ (cm^{-1}) hence <u>reciprocal space</u>

fourier transform. (~~coordinate~~ real space → momentum space)

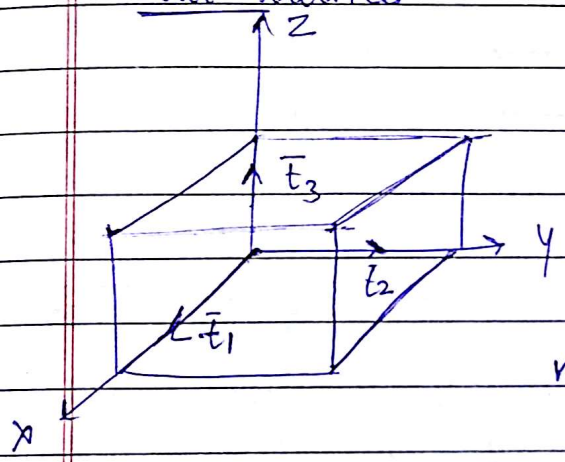
Lattice

real space | Momentum space



□ → fundamental unit

Real lattice



fundamental vector
$\bar{t}_1, \bar{t}_2, \bar{t}_3$

Translation vector
$\bar{R} = n_1\bar{t}_1 + n_2\bar{t}_2 + n_3\bar{t}_3$

$n_1, n_2, n_3 \rightarrow$ Integers.

Reciprocal space

fundamental vector: $\bar{b}_1, \bar{b}_2, \bar{b}_3$

$$\vec{G} = 2\pi(h_1\bar{b}_1 + h_2\bar{b}_2 + h_3\bar{b}_3)$$

$\bar{b}_1\cdot\bar{t}_1 = 1$	$\bar{b}_2\cdot\bar{t}_1 = 0$	$\bar{b}_3\cdot\bar{t}_1 = 0$
$\bar{b}_1\cdot\bar{t}_2 = 0$	$\bar{b}_2\cdot\bar{t}_2 = 0$	$\bar{b}_3\cdot\bar{t}_2 = 0$
$\bar{b}_1\cdot\bar{t}_3 = 0$	$\bar{b}_2\cdot\bar{t}_3 = 0$	$\bar{b}_3\cdot\bar{t}_3 = 1$

$$\bar{b}_m\cdot\bar{t}_n = \delta_{mn} \qquad \delta_{mn} = \begin{cases} 1 & m=n \\ 0 & m\neq n \end{cases}$$

Given $\bar{t}_1, \bar{t}_2, \bar{t}_3$, to find $\bar{b}_1, \bar{b}_2, \bar{b}_3$:

$$[t_1\ t_2\ t_3] = \bar{t}_1 \cdot (\bar{t}_2 \times \bar{t}_3)$$

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$$\bar{b}_1 = \frac{\bar{t}_2 \times \bar{t}_3}{[\bar{t}_1\ \bar{t}_2\ \bar{t}_3]}$$

$$\bar{b}_2 = \frac{\bar{t}_3 \times \bar{t}_1}{[t_1\ t_2\ t_3]}$$

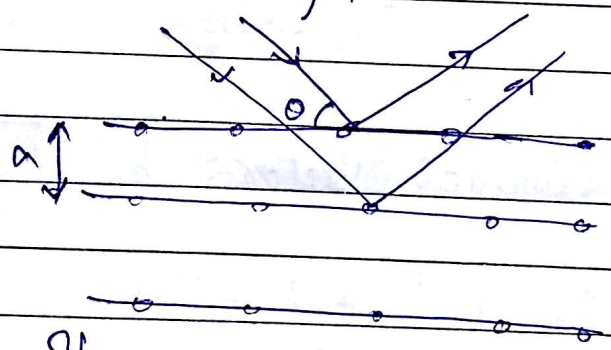
$$\bar{b}_3 = \frac{\bar{t}_1 \times \bar{t}_2}{[t_1\ t_2\ t_3]}$$

Real space → Reciprocal space

Real		Reciprocal
Simple cubic	→	Simple cubic
BCC	→	FCC
FCC	→	BCC.

BRAGG REFLECTION:

(In a crystal lattice, e^- are treated as waves).



The reflected waves interfere → either constructively or destructively.

Constructive:

$$2a\sin\theta = n\lambda \qquad \lambda = 2\pi/k$$

$$\cancel{2}a\sin\theta = \frac{\cancel{2}\pi n}{k}$$

$$k_{critical} = \frac{\pi n}{a\sin\theta}$$

$\theta = \pi/2$

1st zone

2nd zone

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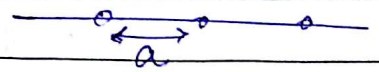
$$k_{crit.} = \frac{\pi n}{a}$$

Brillouin zone:

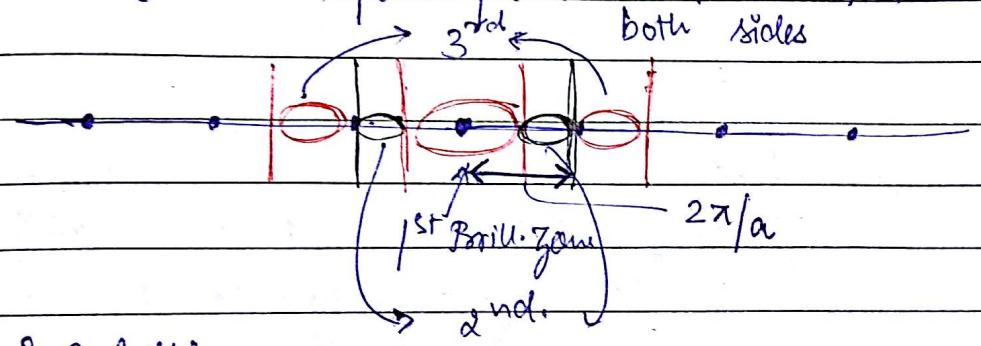
1D

Real space:

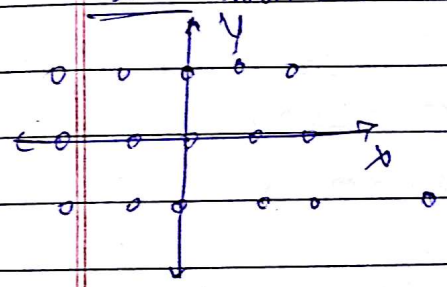


Reciprocal:

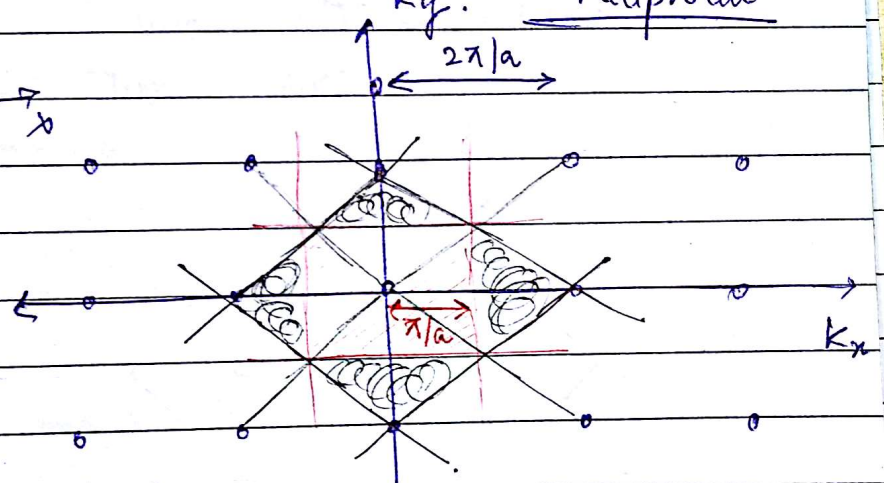
(Take midpoint from 0th to n^{th} both sides).



2-D lattice.



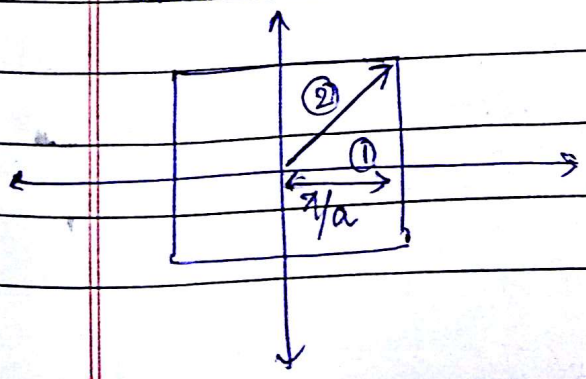
Reciprocal



1st zone: Go to first closest → draw ⊥r bisector

2nd zone: Go to 2nd closest (diagonal) → draw ⊥r bisector

Kcritical → max value of k possible in a given directn.



(1) $k_{crit.} = \pi/a$

(2) $k_{crit.} = \sqrt{2}\pi/a$

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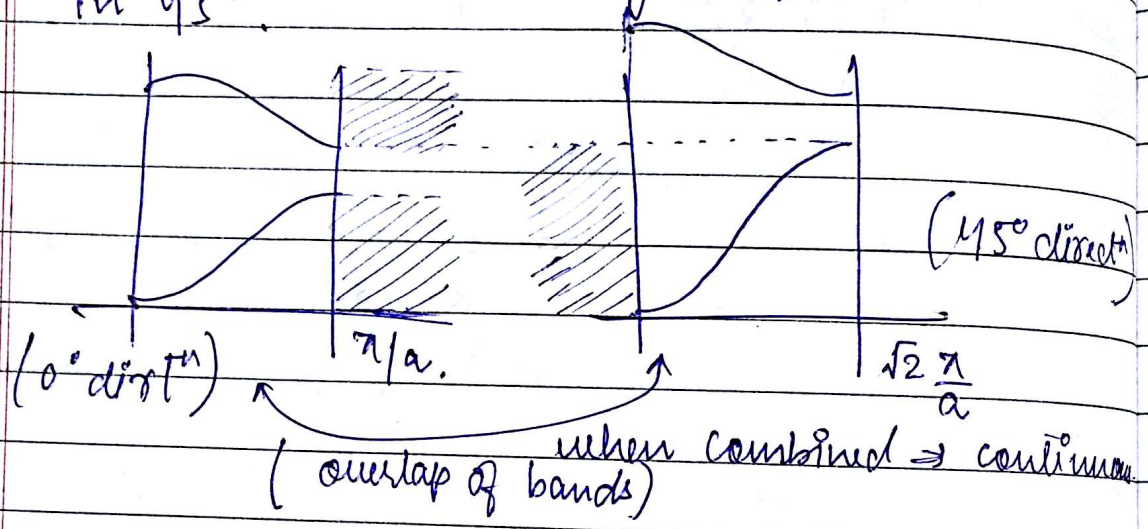
(T) E_{max} =.

e^- experiences gap after π/a if it moves in x dir^n. and a gap after $\sqrt{2}\pi/a$ in 45°.



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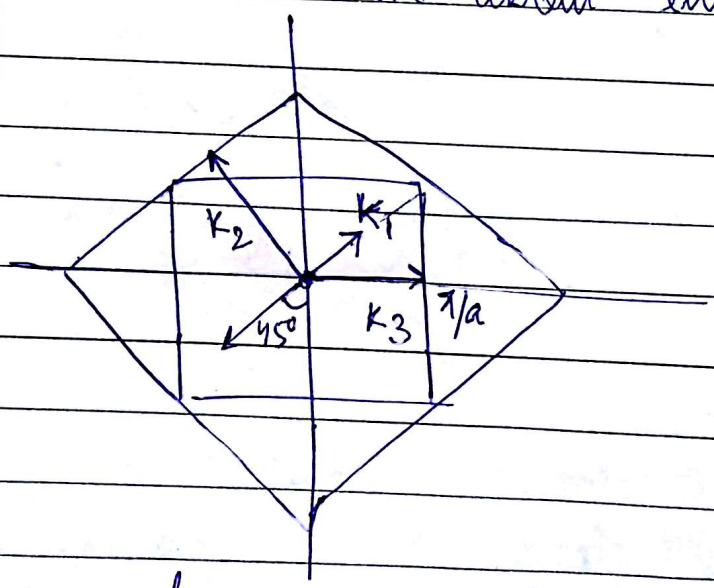
By drawing these zones in reciprocal space, we can talk about energy etc.



K_1 : free e^-. K_3 ($=\pi/a$) : reflect back)

* Anisotropy: Diff^n conductivity in diff^n directions.

∵ In diff^n dir^n, diff^n k_{max} values.

$\psi(x) = \psi(x+L)$. → Periodic condⁿ.

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for one state, vol. $= \left(\frac{2\pi}{L}\right)^3$.

Vol. of FBZ. $= \left(\frac{2\pi}{a}\right)^3$ $a \equiv$ lattice const.
first Brillouin zone

No. of states in FBZ $= \frac{(2\pi/a)^3}{(2\pi/L)^3} = \boxed{\frac{L^3}{a^3}}$.

No. of states for e^- $= \frac{2L^3}{a^3}$.

$= 2 \times$ (No. of unit cells)

Cubic crystal.

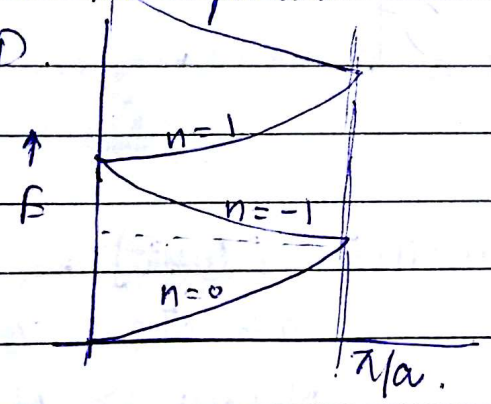
No. of atoms per unit cell $= 1$.

BCC. ——— " ——— $= 2$

FCC ——— " ——— $= 4$.

Free-e^- band picture.

1D.



$P \to 0$

$\cos\alpha a = \cos ka$.

$E = \frac{\hbar^2}{2m}\left(k_x + n\left(\frac{2\pi}{a}\right)\right)^2$.

$k' = \bar{k} + \bar{G}$.

$\bar{G} = 2\pi(h_1\bar{b}_1 + h_2\bar{b}_2 + h_3\bar{b}_3)$

Symmetry axis : → $(h_1 = 1, h_2 = 0, h_3 = 0)$

(3D). direction. [100] – Δ (Γ – X direction)

[110] – Σ (Γ – K direction)

[1,1,1] – Λ (Γ – L direction)

Γ → Gamma

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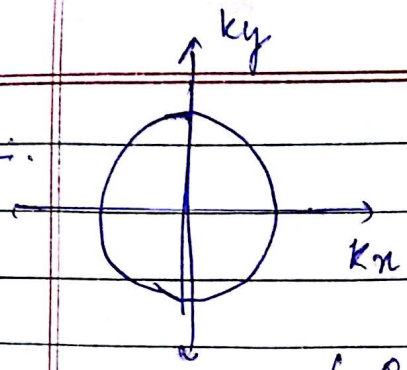
XUPERB

2D.

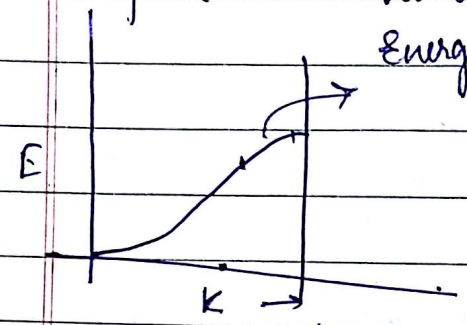


$$E = \frac{\hbar^2}{2m}\left(k_x^2 + k_y^2\right).$$

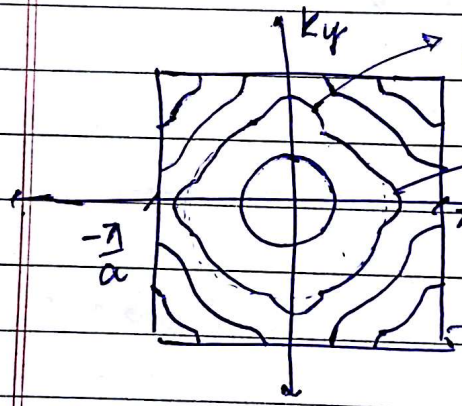
Points on circle → Same energy.

(for free e^-).

for e^- in variational potential:



Energy starts increasing slowly.



Bulging out.

Start getting effect of Brillouin zone.

Brillouin zone.

metal

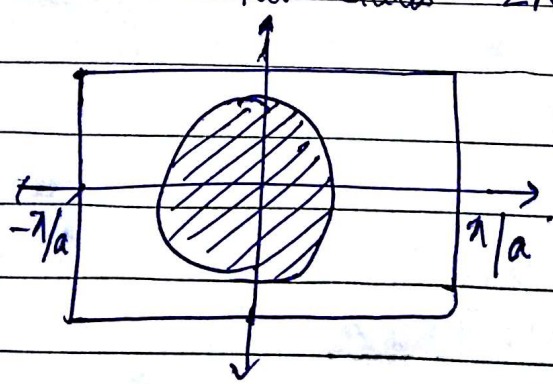
Atom: monovalent (cubic).
Na, Li

each atom contributes $1e^-$.

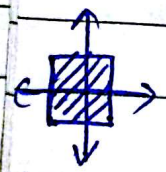
Half filled.

N cells ⇒ N states.
(Ne^-).

Total states = 2N.



Some states are unfilled.
⇒ conductivity.

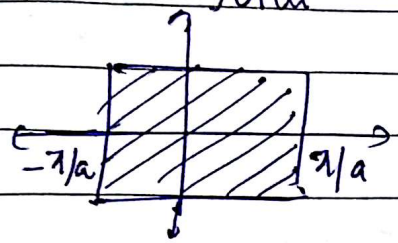


← for monovalent BCC, N cells ⇒ $2Ne^-$. (all filled states)
∴ overlap ⇒ conductor, no overlap ⇒ insulator

divalent:

N cells ⇒ 2N e^-

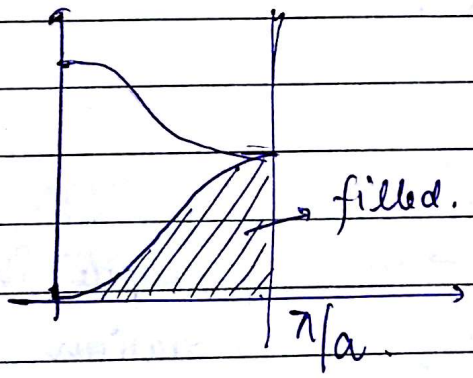
Total → 2N states. (all filled).

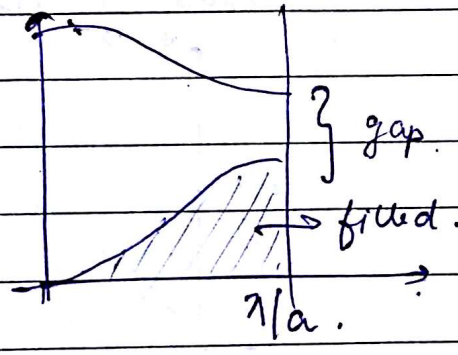


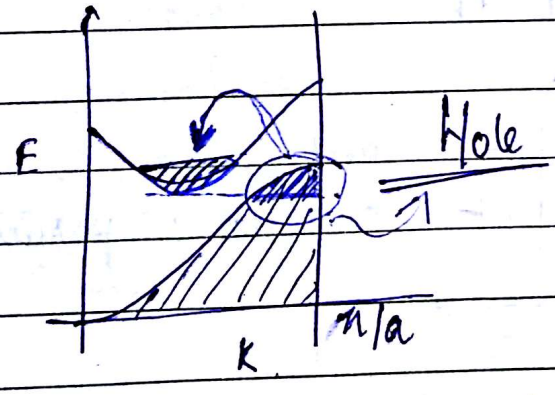
{classically, 2e^- per atom should lead to more conductivity.}

Due to overlapping of band, when electric field is applied, due to excitation, free states are available hence conduction.

If bands do not overlap ⇒ no conduction in this case.







contribution to conduction depends on effective mass (m^*)

if $m^* > m_{hole}$
⇒ hole contributes.

★★ This is the reason to get positive R_H (Hall's coefficient). $R_H = \frac{-1}{ne}$

$m^* = \frac{\hbar^2}{\frac{\partial^2 E}{\partial k^2}}$ → curvature of curve plays a role.

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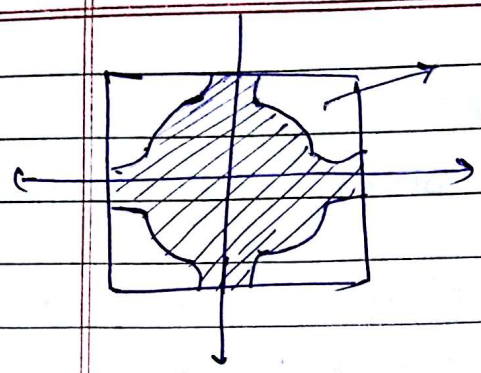
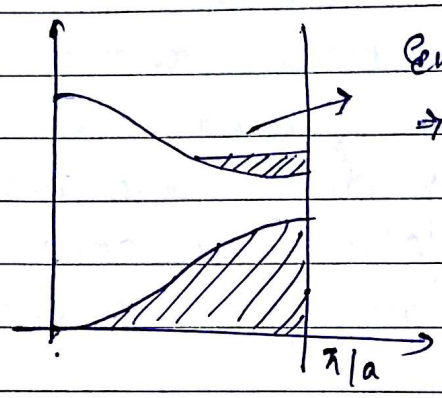
31/8.

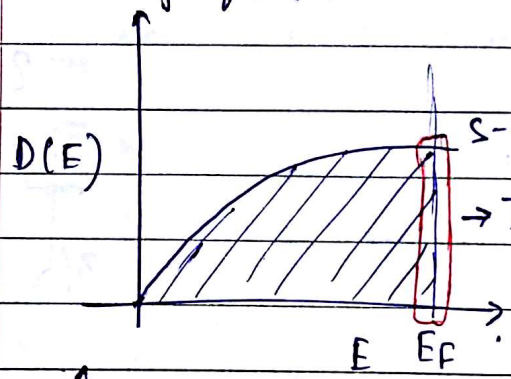


Empty states present.
⇒ metal
(conductor).

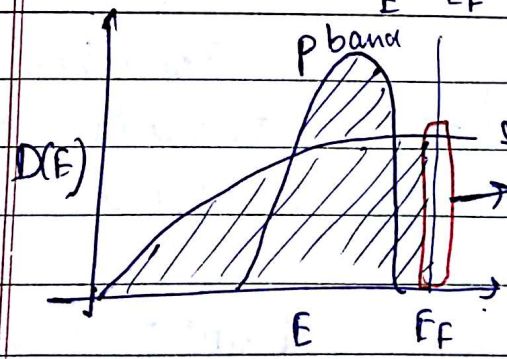


Empty states.
⇒ conductor.

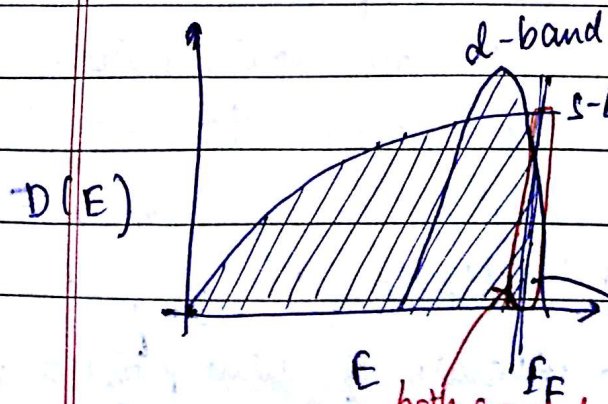
Density of state vs E.



s-band
→ These e^- participate in conduction.



s band.
→ s-band e^- participate.



D(E) and effective mass affects whether s e^- participate or d.

both s and d.
unfilled d.

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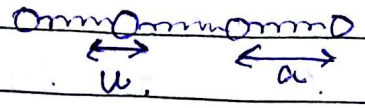
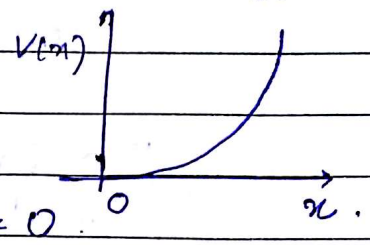
31/8. PHONONS. / Quantized form of lattice vibrations



$u \rightarrow$ amplitude of oscillator. $u << a$.

$V(x) = \frac{1}{2}\beta x^2$.

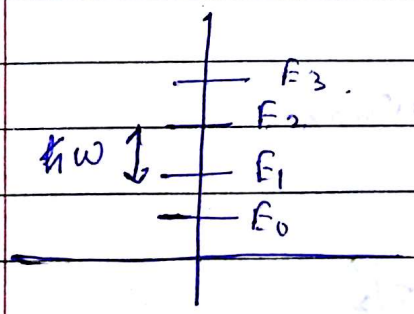


$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}(E - V(x))\psi = 0$$

$$E_n = \left(n + \frac{1}{2}\right)\hbar\omega. \qquad \omega = \sqrt{\frac{\beta}{m}}.$$

$\beta \rightarrow$ spring const.

$n = 0, 1, 2, \ldots$

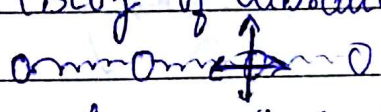


$n = 0$, ground state.

$E_0 = \frac{1}{2}(\hbar\omega)$

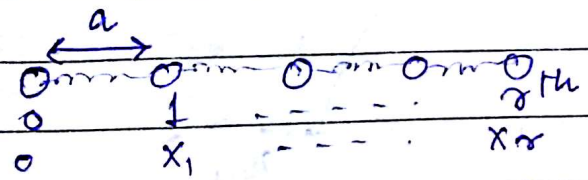
Lattice waves:

(Bcoz of vibration, wave forms).



Longitudinal (L-wave)

Transverse (T-wave)



$$\psi_r = A \exp(i(kx_r - \omega t)).$$

k = wave vector $= 2\pi/\lambda$. ω.

<u>Phonons:</u> Quantized lattice wave.

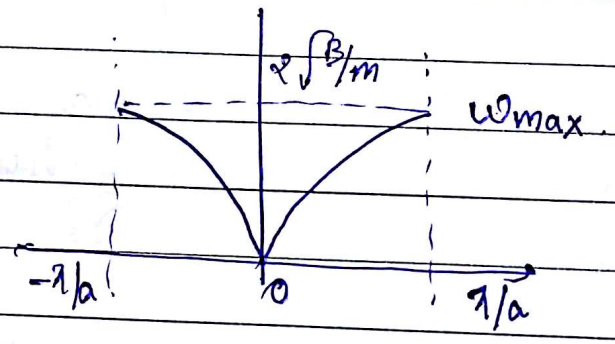
$E = \hbar\omega$ $p = \hbar k$.

Variation of ω with k.
(Dispersion relation)

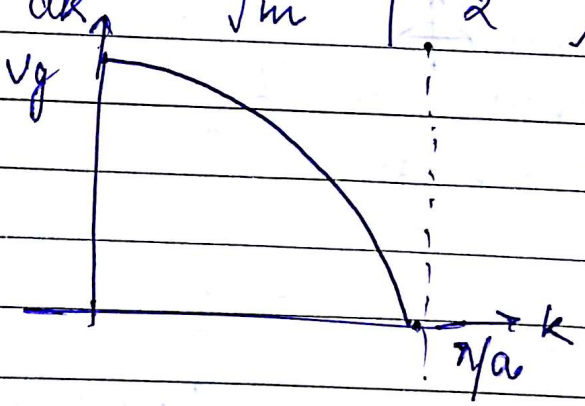
$$\omega = 2\sqrt{\frac{\beta}{m}}\left|\sin\frac{1}{2}ka\right|.$$

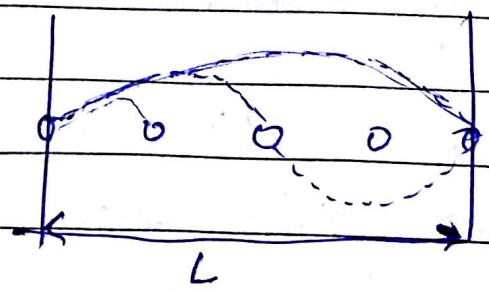
$\beta \rightarrow$ Spring constant
$m \rightarrow$ mass of atom.
$a \rightarrow$ lattice constant.

$k = \pi/a$ $\quad\quad$ $\omega \rightarrow \omega_{max} = 2\sqrt{\beta/m}$.
$k = 0$ $\quad\quad$ $\omega \rightarrow \omega_{min} = 0$.



$$V_g = \frac{d\omega}{dk} = \sqrt{\frac{\beta}{m}}\, a\left(\cos\frac{1}{2}ka\right)$$





$Na = L$.
N = no. of atoms
Max. no. of nodes $= N$

max: $\lambda/2 = L$
min: $\lambda/2 = a$.

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$$q\frac{\lambda}{2} = Na.$$

$$k_q = \frac{2\pi}{\lambda} = \frac{\pi q}{Na} \qquad q = 1, 2, \ldots$$

3D: L_x, L_y, L_z.
$\downarrow N_x \quad \downarrow N_y \quad \downarrow N_z$.

$$k_x = \frac{\pi q_x}{N_x a} \qquad k_y = \frac{\pi q_y}{N_y a} \qquad k_z = \frac{\pi q_z}{N_z a}$$

1D: $\left(\frac{2\pi}{L}\right) \rightarrow$ one mode

3D: $\left(\frac{2\pi}{L}\right)^3$ - one mode.

$N_p(s)$ = No. of modes = $\dfrac{\frac{4\pi}{3}k^3}{\left(\frac{2\pi}{L}\right)^3} = \dfrac{k^3L^3}{6\pi^2}$

N_p → phonon. s → states.

$D(\omega)$ = Density of states for phonons.
= No. of states per unit freq.

$$D(\omega) = \frac{dN}{d\omega} = \frac{Vk^2}{2\pi^2}\frac{dk}{d\omega} \qquad V = L^3$$

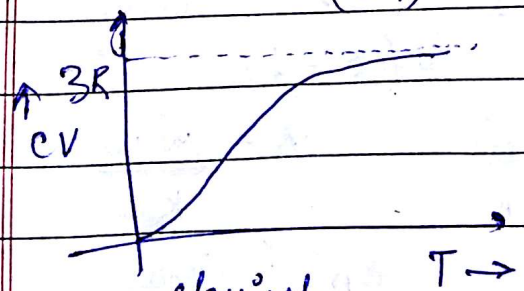
$\frac{dk}{d\omega}$ → depends on Dispersion relation.

Applications:

1. Specific heat.

$$C_v = \left(\frac{dU}{dT}\right)_V$$

$C_v = 3R$ (Dulong - Petit)
for solids at higher temp.



classical.

$(U)_{mol.} = 3k_BT \cdot N_a = 3RT$

$(C_v) = 3R$.

recall:

$$E_T = \int D(\omega) F(E) E \, dE.$$

for phonons,

$$E = \hbar\omega.$$

$$\langle n \rangle = \frac{1}{(e^{\hbar\omega/k_BT} - 1)}$$: distribution fn of phonons

At particular temp., how they are distributed with diff^n energy.

with inc. in temp. $\langle n \rangle$ increases.

$$D(\omega) = \frac{Vk^2}{2\pi^2} \frac{dk}{d\omega}.$$

Debye model:

$$\omega = vk$$ velocity of sound.

$$\frac{dk}{d\omega} = \frac{1}{v}.$$

$$D(\omega) = \frac{Vk^2}{2\pi^2 v}.$$

$$\therefore U = \int_0^{\omega_{max}} \hbar\omega \langle n \rangle D(\omega) \, d\omega.$$

$$= \int_0^{\omega_{max}} \hbar\omega \frac{1}{(e^{\hbar\omega/k_BT} - 1)} \left(\frac{Vk^2}{2\pi^2 v}\right) d\omega.$$

$$= \cancel{9NB}$$

$$= 9Nk_BT\left(\frac{T}{\theta}\right)^3 \int_0^{x_D} \frac{x^3}{(e^x - 1)} dx.$$

$$\theta = \frac{\hbar\omega_D}{k_B} \qquad x = \frac{\hbar\omega}{k_BT}$$

$$x_D = \frac{\hbar\omega_{max}}{kT}$$

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$$U = cT\left(\frac{T}{\theta}\right)^3.$$

$$C_v = \frac{dU}{dT} = 4c\left(\frac{T}{\theta}\right)^3.$$

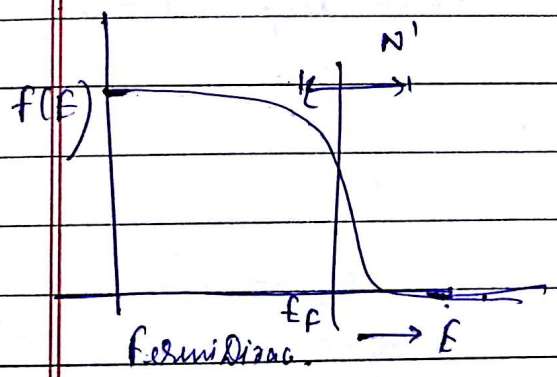
5/9. $\theta_D = \frac{\hbar\omega_D}{k_B}$: Debye Temp.

for $T > \theta_D$ $\quad C_v = 3R.$

$T < \theta_D$ $\quad C_v \propto T^3$

$$C_v = \underset{\text{Lattice}}{(C_v)_L} + \underset{e^-}{(C_v)_e} \rightarrow \frac{3}{2}kTN. \text{ (Classical)}$$

for insulator, as there are no free e^-
$\Rightarrow$ major contribution due to $(C_v)_L$.



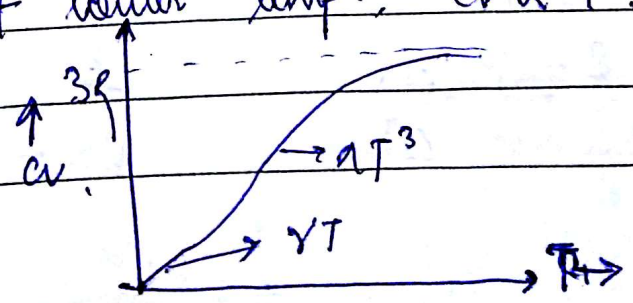
$$N' = \frac{N \cdot k_B T}{k_B T_F}$$ (e^- near to fermi ~~temp.~~ level.)

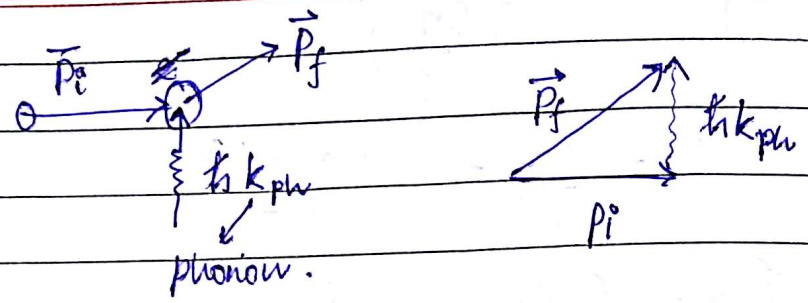
$$(C_v)_e = \frac{3}{2}kT \cdot \frac{NkT}{k_B T_F}.$$

$$C_v = \underset{\text{Phonon}}{\alpha T^3} + \underset{\text{conduct}^n\ e^-}{\gamma T}.$$

$\gamma \rightarrow$ very small. $\therefore$ major contribution from lattice. At very low temp., lattice vibration become small and phonon role become insignificant.

At lower temp., $C_v \propto T$. $\quad (C_v \propto \gamma T)$.





No. of collisions to change the ~~momen~~ direct^n:

initial $\rightarrow m_e v_F$ $\quad$ m_e: mass of e^-.

final $\leftarrow -m_e v_F$ $\quad$ v_F: fermi velocity.

$$N = \frac{2 m_e v_F}{\hbar k_{ph}}$$

Conductivity,

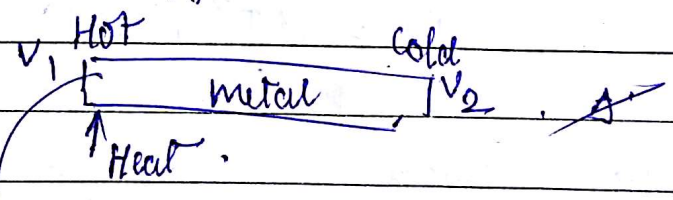
$$\sigma \propto \frac{1}{T}.$$

$$\sigma \propto \frac{1}{T^3} \rightarrow \text{at low temp. } (T < \theta_D)$$

$$n_{ph} \propto T, \ \sigma \propto \tau.$$

Thermoelectricity.

Seebeck effect:



e^- gain energy $\rightarrow$ diffusion.

Voltage develop across ends.

$$\Delta V = V_1 - V_2 \qquad \Delta T.$$

$$\Delta V \propto \Delta T.$$

$$S = -\frac{\Delta V}{\Delta T} \qquad V = -\int_{T_1}^{T_2} S\, dT.$$

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* for some metals, ΔS is (+ve).

$l \rightarrow$ mean free path.
$\hookrightarrow$ energy - dependent.

$n \rightarrow$ density of e^-.

Hot | cold

$\boxed{n, l \;\vdots\; n, l'}$

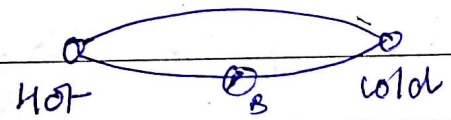
$\frac{1}{2} nl$ pass from hot to cold
& $\frac{1}{2} nl'$ from cold to hot.

Net diffusion $= \frac{1}{2} nl - \frac{1}{2} nl'$

$= \frac{1}{2} n (l - l')$.

$l > l' \;:\; S \rightarrow (-ve)$
$l < l' \;:\; S \rightarrow (+ve)$

Temp. sensor



$V \propto T^2$.

$$S = \frac{-\pi^2 k_B^2 T}{3 e E_F} x$$

Mott Jones Eqn.
material dependence.

$$V_{AB} = \int_{T_1}^{T_2} (S_A - S_B)\, dT.$$

$V_{AB} = V_A - V_B \qquad V_A = \int_{T_1}^{T_2} S_A\, dT.$

$$V_{AB} = \cancel{C}\int_{T_1}^{T_2} -\frac{\pi^2 k_B^2}{3e}\left(\frac{x_A}{E_{FA}} - \frac{x_B}{E_{FB}}\right) dT.$$

$$= \frac{\pi^2 k_B^2}{3e}\left(\frac{x_A}{E_{FA}} - \frac{x_B}{E_{FB}}\right)\left(T_1^2 - T_2^2\right)$$

$$V_{AB} = C\left(T_1^2 - T_2^2\right).$$

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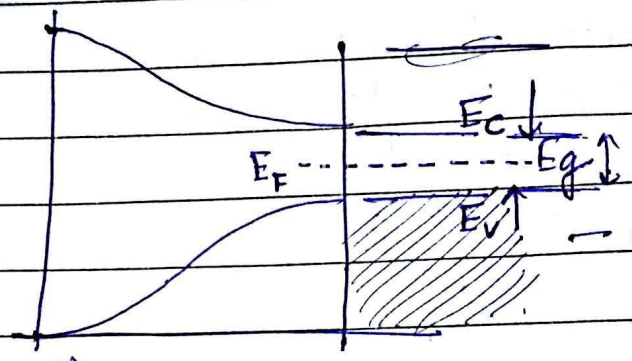
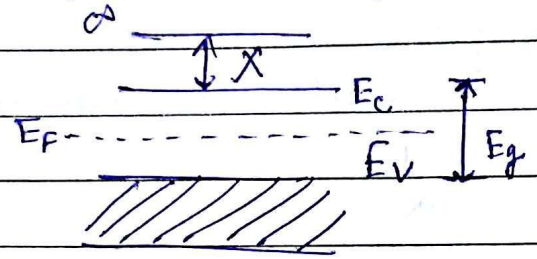
7/9. SEMICONDUCTOR.



— all available states occupied.



$\chi \rightarrow e^-$ affinity

GaAs - III - V Si - IV IV
CdS II - VI InP III - V

Intrinsic - pure, no doping.
Extrinsic - doping
Degnorate - doping. N_d is very large (doping density)

Intrinsic:

e^- from valence band goes to conduct^n band, thus creating hole.

$n_e = n_p$

Density of state

$$D(E) = \frac{a^3}{2\pi^2}\left(\frac{2m_e}{\hbar^2}\right)^{3/2} E^{1/2}$$

(States per unit energy) $a^3 = V \equiv$ volume

No. of states per unit energy, per unit vol.

$$g(E) = \frac{1}{2\pi^2}\left(\frac{2m_e}{\hbar^2}\right)^{3/2} E^{1/2}$$

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conduction band effective mass.

$$g_{CB}(E) = \frac{1}{2\pi^2}\left(\frac{2m_e^*}{\hbar^2}\right)^{3/2}(E-E_c)^{1/2}$$

$\because$ no e^- below E_c as no state is present.

Put $\hbar = h/2\pi$

$$g_{CB}(E) = \frac{8\sqrt{2}}{h^3}(m_e^*)^{3/2}(E-E_c)^{1/2}.$$

e^- in conduction band

$$n = \int_{E_c}^{\infty} g_{CB}(E)\cdot F(E)\,dE$$

where $f(E) = \dfrac{1}{e^{\left(\frac{E-E_F}{kT}\right)}+1}$

$\star\star$ $E - E_F \gg kT$

$$\therefore F(E) = e^{-(E-E_F)/kT}.$$

$$n = \int_{E_c}^{\infty} g_{CB}(E)\, e^{-(E-E_F)/kT}\cdot dE$$

$$= \frac{8\sqrt{2}}{h^3}(m_e^*)^{3/2}\int_{E_c}^{\infty}(E-E_c)^{1/2}\cdot e^{-(E-E_F)/kT}\,dE$$

$E - E_c = x$

$$= \frac{8\sqrt{2}}{h^3}(m_e^*)^{3/2}\, e^{-(E_c-E_F)/kT}\underbrace{\int_0^{\infty} x^{1/2}\cdot e^{-x/kT}\,dx}_{\frac{kT}{2}(\pi kT)^{1/2}}$$

$$n = 2\left(\frac{2\pi m_e^* k}{h^2}\right)^{3/2} T^{3/2}\, e^{-(E_c-E_F)/kT}$$

for no. of holes, prob. of occupancy $= \star$ $1-F(E)$

$$p = 2\left(\frac{2\pi m_h^* k}{h^2}\right)^{3/2} T^{3/2}\, e^{-(E_F-E_V)/kT}$$

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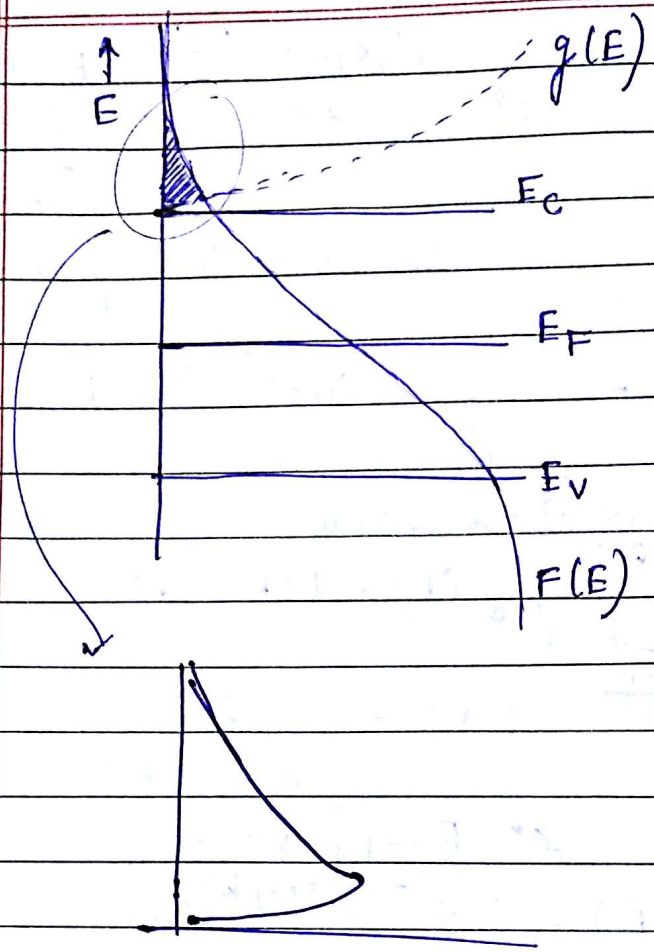
8/9.



For intrinsic semiconductor,

$$n = p$$

$$\Rightarrow m_e^{*3/2} \, e^{\frac{E_F - E_c}{kT}} = m_h^{*3/2} \cdot e^{\frac{E_v - E_F}{kT}}$$

$$\Rightarrow m_e^{*3/2} \frac{e^{E_F/kT}}{e^{E_c/kT}} = m_h^{*3/2} \frac{e^{E_v/kT}}{e^{E_F/kT}}$$

$$\Rightarrow \left(e^{E_F/kT}\right)^2 = \frac{\cancel{m_e^{*3/2}} \cdot m_h^{*3/2} \cdot e^{E_c/kT} \cdot e^{E_v/kT}}{m_e^{*3/2}}$$

$$e^{\frac{2E_F - (E_c + E_v)}{kT}} = \left(m_h^*/m_e^*\right)^{3/2}$$

$$\frac{2E_F - (E_c + E_v)}{kT} = \frac{3}{2} \ln\left(\frac{m_h^*}{m_e^*}\right)$$

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$$E_F = \frac{3}{4} kT \ln\left(\frac{m_h^*}{m_e^*}\right) + \frac{(E_c + E_v)}{2}$$

for $n > p$:

$$E_F > \frac{3}{4} kT \ln\left(\frac{m_h^*}{m_e^*}\right) + \frac{(E_c + E_v)}{2}$$

E_c
E_F * for $n > p$: E_F shifts towards conduction band.
E_v

E_c * for $p > n$: E_F shifts towards valence-band
E_F
E_v

→ Average energy of e^- in conduction band.

$$\underset{(\text{Total})}{E_T} = \int_{E_c}^{\infty} g_{CB}(E) \cdot F(E) \cdot E \, dE$$

$$= \int_{E_c}^{\infty} g_{CB}(E) \cdot E \, e^{-\left(\frac{E - E_F}{kT}\right)} dE$$

$$= \frac{8\sqrt{2}}{h^3} \left(m_e^{*3/2}\right) \int_{E_c}^{\infty} E \, (E - E_c)^{1/2} e^{-\left(\frac{E - E_F}{kT}\right)} dE.$$

$$= \frac{8\sqrt{2}}{h^3} \left(m_e^{3/2}\right) e^{E_F/kT} \int_{E_c}^{\infty} E \, (E - E_c) \, e^{-E/kT} dE.$$

$$\langle E \rangle = E_T / n.$$

$$\bar{E}_{cb} = E_c + \frac{3}{2} kT.$$

(for metal: $\bar{E} = \frac{3}{5} E_F$) → temp. independent.

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Why the diffn ?
Due to density difference.
Most states unoccupied in conduction band.

insulator, $n_i \sim 10^{11}/cm^3$
metal $n \sim 10^{24}/cm^3$.

for semi-conductor, (n, p) — e^-, holes. $\sigma_m = ne\mu$ (metal)

$$\sigma_s = ne\mu_e + pe\mu_p$$

$$\mu_e = \frac{e\tau_e}{m_e^*} \qquad \mu_h = \frac{e\tau_h}{m_h^*}$$

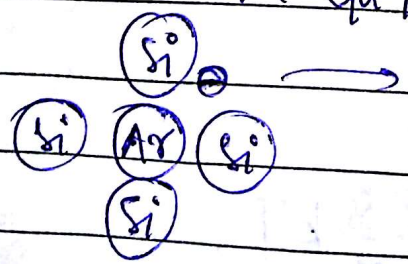
$$J_s = ne(V_d)_e + pe(V_d)_h$$

$(V_d)_e$ — drift vel. for e^-; $(V_d)_h$ — holes.

$J_m = neV_d$ (metal)

Recombination :
e^- come back from conduction band to valence band. The time for which it stays there is called lifetime.

19/9. Extrinsic :
Si + As (n-type)
Si + Ga / In (p-type).

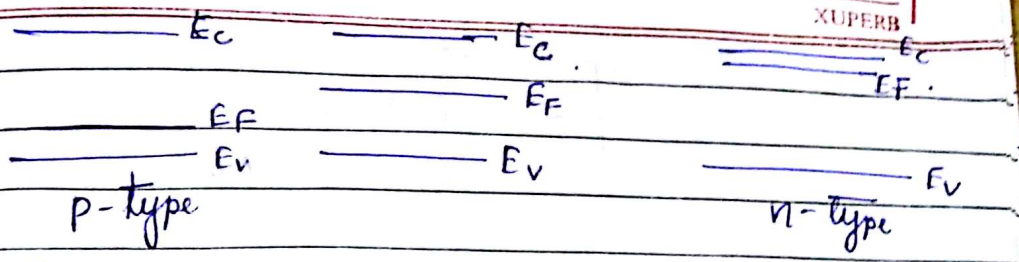


the e^- needs some energy to come out.
ionisation energy plays role here

n-typ

p-typ

atom



Hydrogen atom.

Binding energy, $E_b = \frac{m_e e^4}{8\varepsilon_0^2 h^2} = 13.6\ eV$

Binding energy of As in Si.

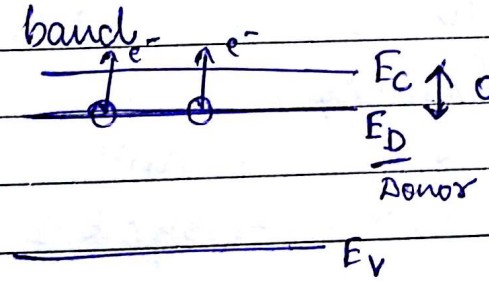
$$E_b = \frac{m_e^* e^4}{8\epsilon_r^2 \varepsilon_0^2 h^2} = 13.6\left(\frac{m_e^*}{m_e}\right)\left(\frac{1}{\varepsilon_r^2}\right)$$

$\varepsilon_r = 11.9 \qquad m_e^* = \frac{1}{3} m_e$.

$E_b = 0.032\ eV$

at room temp., $kT \sim$.

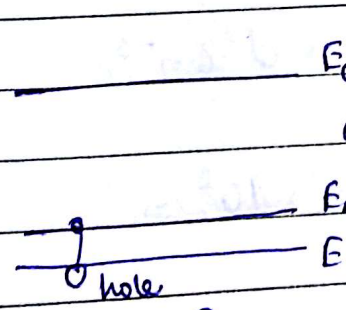
Hence, it ionises, goes to conduction band.



n-type

: At lower temp., ionisat^n do not take place

$\Rightarrow$ no conduction.



p-type

when temp. increases much, e^- from valence bands move to conduct^n band creating same no. of holes

$\rightarrow$ recombination

Donor density:

$N_D \rightarrow$ 1 part per million / billion.

$N_{Si} \rightarrow 5 \times 10^{22}\ cm^{-3}$

atom $\leftarrow N_D \sim \frac{5 \times 10^{22}}{10^9} = 5 \times 10^{13}\ cm^{-3}$.

no. of e^- $\quad n \simeq N_D$

$$n \cdot p = n_i^2$$

$$p = \frac{n_i^2}{n} = \frac{n_i^2}{N_D} \ll n_i$$

$$\sigma = ne\mu_n + pe\mu_p.$$

$n = p = n_i$ (Intrinsic semicond.)

$$\sigma = n_i e(\mu_n + \mu_p)$$

Extrinsic :

(n-type) $$\sigma = ne\mu_n + \left(\frac{n_i^2}{N_D}\right) e\mu_p$$

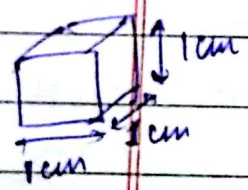
$$\simeq ne\mu_n.$$

Eg. 1 part per 10^9. doping.

$N = 5 \times 10^{22}\ cm^{-3}$

$\mu_e = 1350\ cm^2 V^{-1} s^{-1}$ $\quad$ hole $\mu_h (\mu_p) = 450\ cm^2 V^{-1} s^{-1}$

$n_i = 10^{10}\ cm^{-3}$



$$\sigma_i = n_i e(\mu_n + \mu_p)$$

$$= 2.88 \times 10^{-6}\ \Omega^{-1} cm^{-1}$$

$$R = \frac{L}{\sigma A} = 3.4 \times 10^5 = 340\ k\Omega.$$

$$N_D = n = \frac{N_{Si}}{10^9} = 5 \times 10^{13}\ cm^{-3}$$

$$p = \frac{n_i^2}{N_D} = 2 \times 10^6\ cm^{-3}$$

$$\sigma_n \simeq ne\mu_n. = 10^{-2}\ \Omega^{-1} cm^{-1}$$

$$R = 92\ \Omega$$

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Compensating doping (in p-n junction diodes)

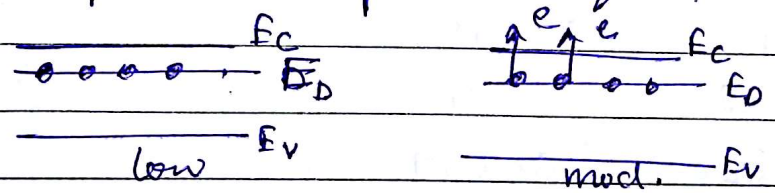
p-type → intrinsic → n-type.

N_D, N_A : donor, acceptor density

n-type: $n = N_D - N_A$ $\quad p = \frac{n_i^2}{(N_D - N_A)}$

p-type: $p = N_A - N_D$ $\quad n = \frac{n_i^2}{(N_A - N_D)}$

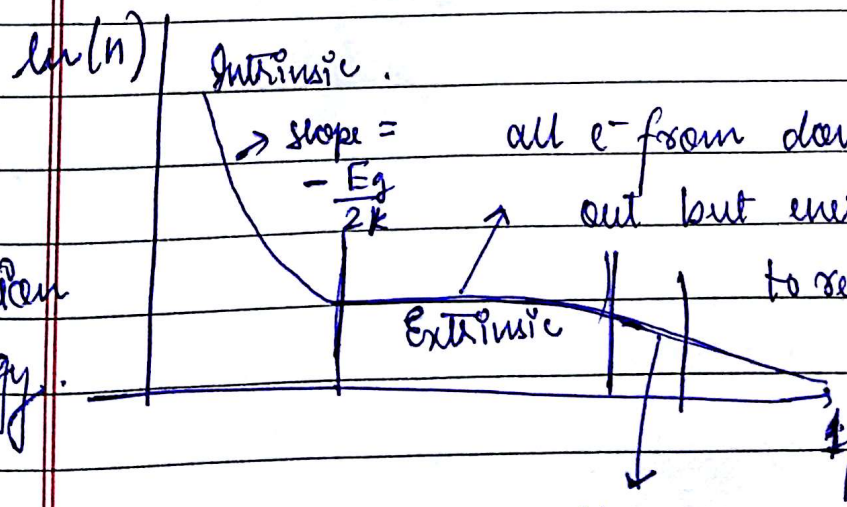
Temperature Dependence of n:



Very low temp. → no ionisation ⇒ no e^-.
⇒ behave as insulator.

moderate temp. → e^- come out ⇒ conduction
with ↑ in temp., more e^- come out
⇒ more conduction.

high temp. → with further increase,
e^- from valence band will come out,
there is formation of holes.



all e^- from donors have come out but energy isn't sufficient to remove from valence band.

Eg: ionisation energy of Si

slope $\alpha = -\frac{\Delta E}{2k}$ $\quad \Delta E = E_C - E_D$

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Intrinsic region

$$n = \sqrt{N_c N_v}\; e^{-\frac{E_g}{2kT}}$$

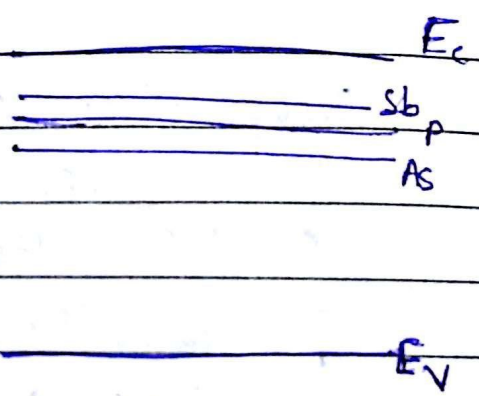
→ temp. depence

$$N_c = 2\left(\frac{2\pi m_e^* kT}{h^2}\right)^{3/2}$$

$$N_v = 2\left(\frac{2\pi m_h^* kT}{h^2}\right)^{3/2}$$

} derived earlier.

n- type. Ionization energy ~~E_D~~.



P	0.045
As	0.054
Sb	0.039

$E_c - E_D$

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$$\mu = \frac{e\tau}{m_e^*} \qquad \tau = \frac{1}{S\, v_{th}\, N_s}$$

temp. independent

$$S = \pi a^2 \propto T.$$

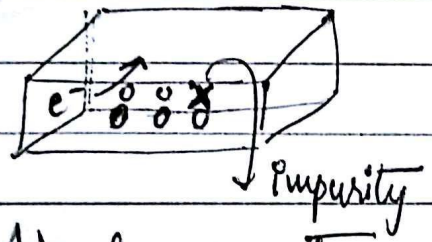
$$\frac{3}{2} kT = \frac{1}{2} m_e v_{th}^2.$$

$$v_{th} \propto \sqrt{T}$$

$$\mu_L \propto \frac{1}{T\sqrt{T}} \simeq T^{-3/2}$$

Lattice vibration limited mobility.

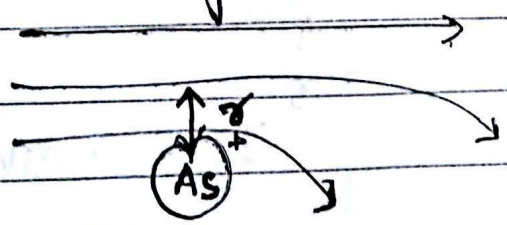
Extrinsic



* Another scattering is due to impurity.

At lower temp., scattering due to lattice becomes small and that due impurities become significant.

Scattering due to impurities:



$$|PE| = \frac{e^2}{4\pi\varepsilon_0\varepsilon_r r}.$$

at $r = r_c$ (critical distance)

$$\frac{3}{2} kT = \frac{e^2}{4\pi\varepsilon_0\varepsilon_r r_c}$$

$$r_c = \frac{2e^2}{3 \times 4\pi\varepsilon_0\varepsilon_r kT}.$$

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S_I = Scattering cross-section due to impurities.

$= \pi r_c^2 \propto T^{-2}$

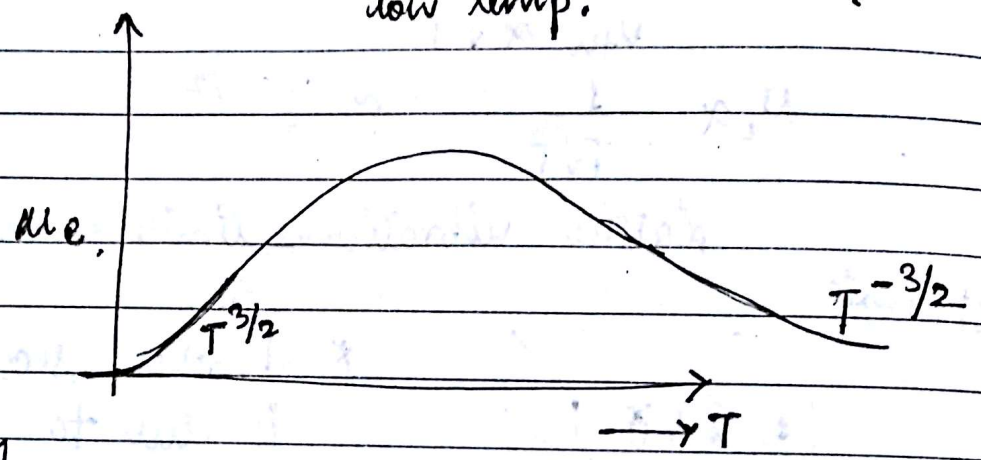
$V_{th} \propto \sqrt{T}$.

$\tau \propto T^{3/2}$

$\Rightarrow \mu_I \propto T^{3/2}$

low temp.

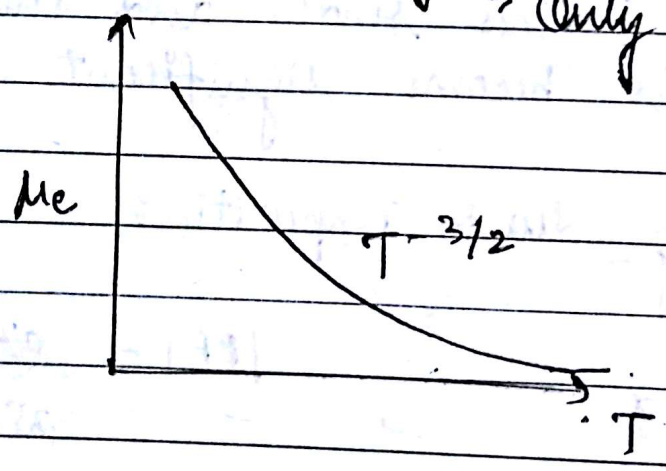
$\mu_L \propto T^{-3/2}$



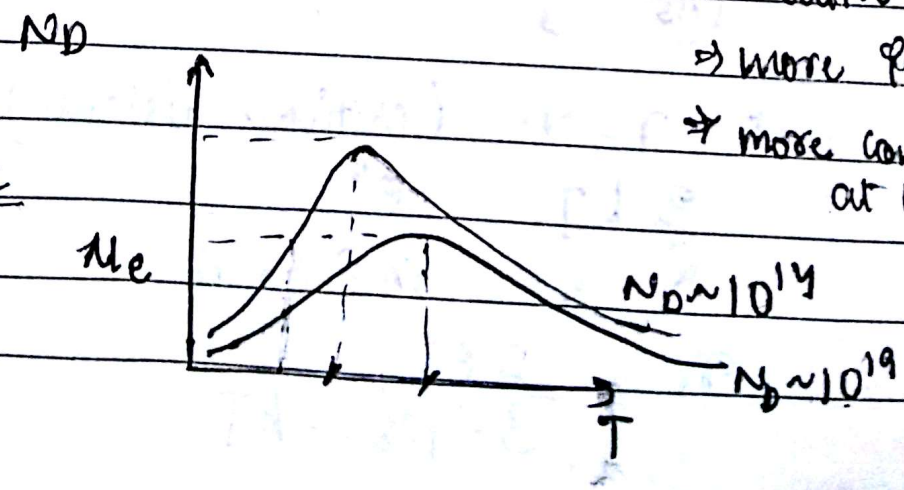
Intrinsic :

No impurity.

→ Only lattice contribution



Extrinsic



with more doping,

⇒ more impurity.

⇒ more cont. to scattering at low temp.

→ less μ.

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Instrinsic :

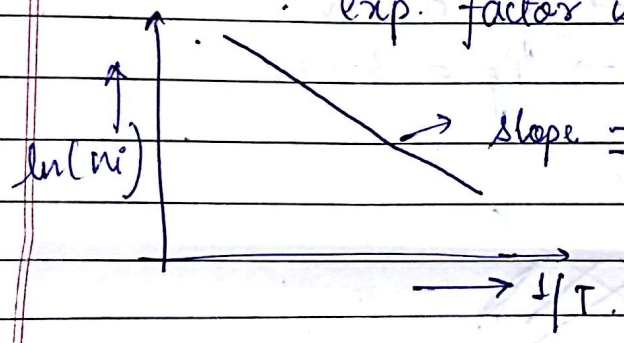
$$n = N_c \exp^{-}\left(\frac{E_c - E_F}{kT}\right) \qquad p = N_v \exp^{-}\left(\frac{E_F - E_v}{kT}\right).$$

$$n_i(T) = \sqrt{np}$$

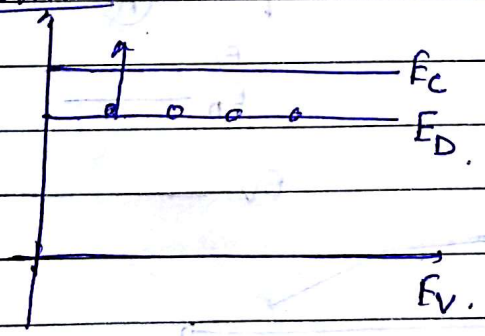
$$= \sqrt{N_c N_v} \exp^{-}\left(\frac{E_g}{2kT}\right) \qquad E_g = E_c - E_v$$

$$N_c = 2\left(\frac{2\pi m_e^* kT}{h^2}\right)^{3/2} \qquad N_v : m_e \to m_h.$$

Graph $\ln(n_i)$ as fn of $1/T$.
(neglected N_c, N_v temp. depen)
$\because$ exp. factor is dominant.



Extrinsic :



when temp. ↑, e^- comes from dopants, not the valence band
→ no hole formation.

~~Temp ↑~~

$$n = \sqrt{\frac{N_c N_D}{2}} \exp^{-}\left(\frac{E_c - E_D}{2kT}\right) \qquad \Delta E = E_c - E_D.$$

dopant : each state - $1e^-$
valence : —"— $2e^-$

N_D instead of N_V as contribution is from D rather than V.

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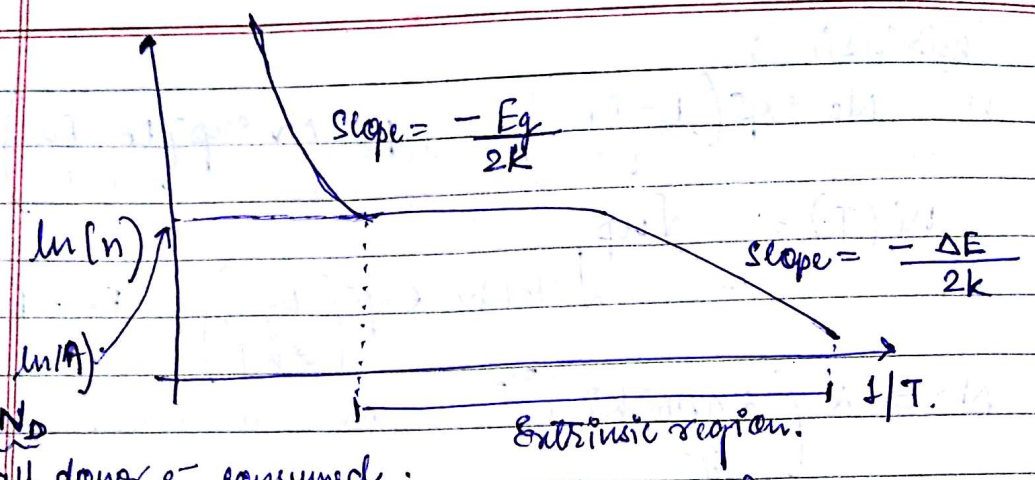
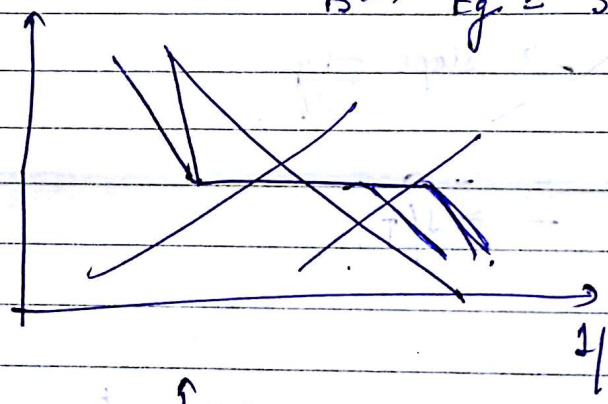
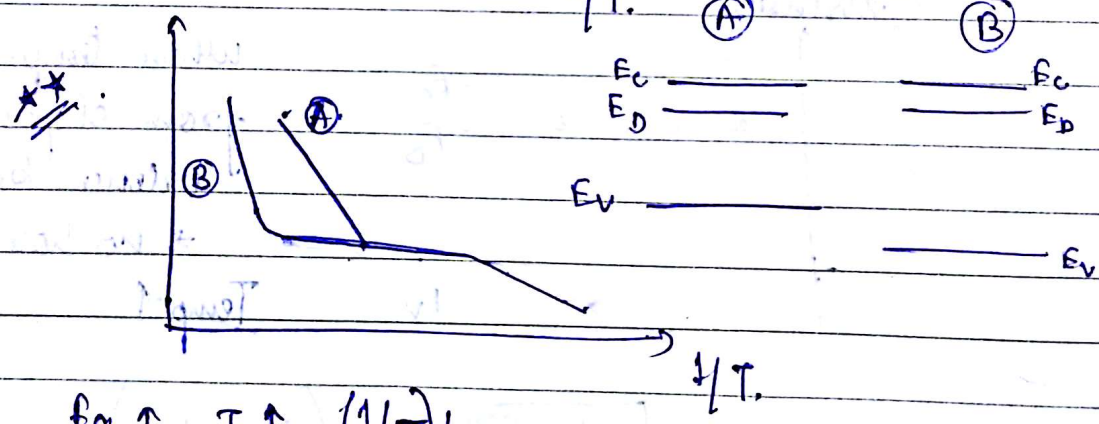


$A = N_D$
all donor e^- consumed.
(assuming each donor contributes one e^-).

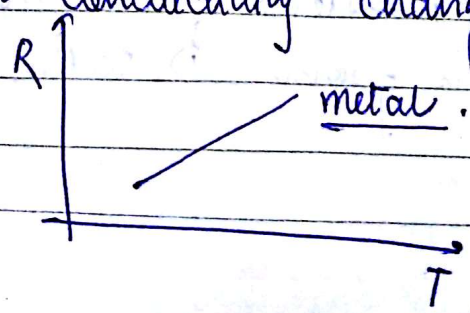
Eg. Semiconductor $A \rightarrow E_g = 0.3\,eV$
$B \rightarrow E_g = 3.5\,eV$





$E_g \uparrow \quad T \uparrow \quad (1/T) \downarrow$
(more temp. to excite).

How conductivity change with temp.?



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Semiconductor:

intrinsic $\sigma = ne\mu_n + pe\mu_p$.

$= n_i e(\mu_n + \mu_p)$.

Extrinsic $\sigma = ne\mu_n$ (n-type). $\mu = \frac{e\tau}{m}$.

we have seen dependence of n ~~of~~ on T.

$\mu_n = f(T) \rightarrow ?$

$\mu = \frac{e\tau}{m}$.

$\pi a^2 \sim l \times v$

$\tau = \frac{1}{S V_{th} N_s}$

S: scattering cross section

V_{th}: thermal velocity

N_s: scattering center per unit volume.

$\frac{3}{2}kT = \frac{1}{2} m_e V_{th}^2$.

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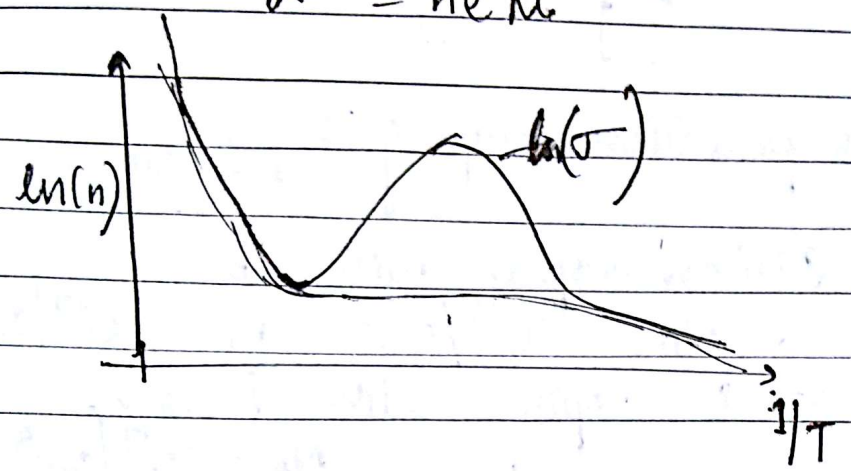
Dependence of σ with T.

$\sigma = ne\mu$



When T ↑, vel. ↑, also scattering ↑.

Degenarate Semiconductor ($N_D \sim 10^{20}/cm^3$)

$n < N_c$.

$$f(E) = \frac{1}{1 + \exp\left(\frac{E - E_F}{kT}\right)} \sim \exp^{-}\left(\frac{E_c - E_F}{kT}\right)$$

(Boltzaman).

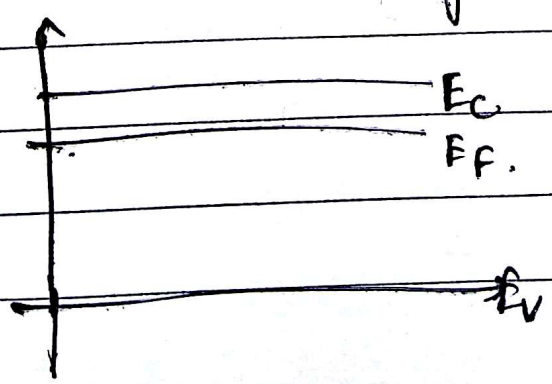
for $n_i \sim 10^{10}/cm^3$

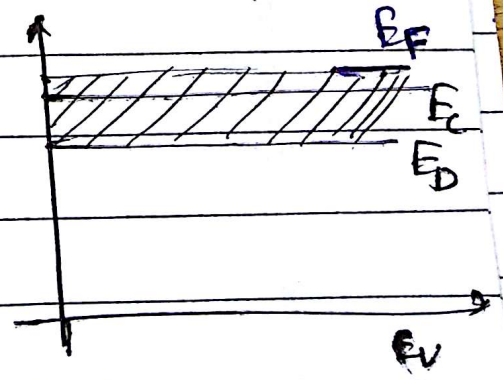
$N_D \sim 10^{16}/cm^3$ – moderate doping

$N_D \sim 10^{20}/cm^3$

E_D has discrete ~~energ~~ states. But when there is high doping (high density), there is band formation.

(in degenarate semiconductor)





$N_0 = n \approx N_c$

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Degenarate semicon : behave like metals
$\boxed{\rho \propto T}$

Compensation Doping : $n \rightarrow$ intrinsic $\rightarrow p$ (×)

Eg. Silicon doped with Sb.
$N_D = 10^{16}/cm^3$ $n_i = 10^{10}/cm^3$.
Start doping with Boron.
$N_A = 10^{17}/cm^3$.
Resultant will be n-type / p-type and what is carrier conc^n ?
$N_A > N_D$
$\therefore$ p type.
$p = N_A - N_D = 9 \times 10^{16}/cm^3$
$n = \frac{n_i^2}{p} = \frac{1}{9} \times 10^4 /cm^3$
$\sim 10^3/cm^3$.
initial $n = 10^{16}$ $(= N_D)$

$$n_i = N_c \exp\left(-\frac{E_c - E_{fi}}{kT}\right)$$

$$N_D = n = N_c \exp\left(-\frac{E_c - E_{fn}}{kT}\right)$$

$$\frac{N_D}{n_i} = \exp\left(\frac{E_{fn} - E_{fi}}{kT}\right)$$

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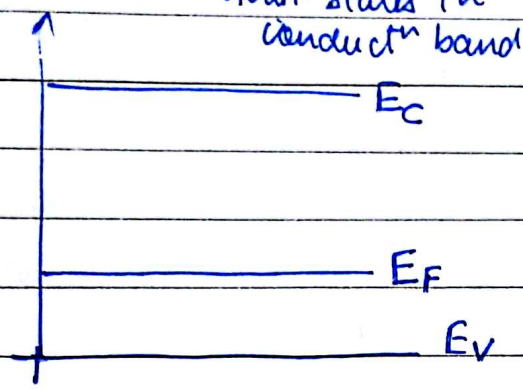
Non-Degenerate Semiconductor

i. Draw energy level diagram of a p-type non-degenerate semiconductor.

ii. What is relation b/w

n, N_c , p, N_v , n, p .

N_c → Avail. states in conduct^n band



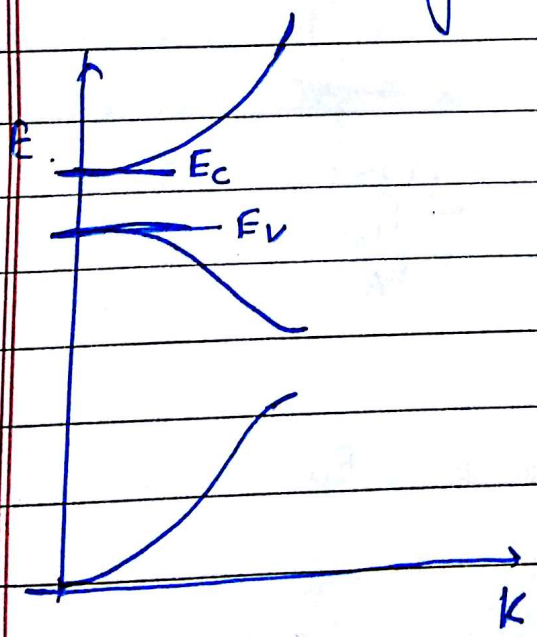
n-type.

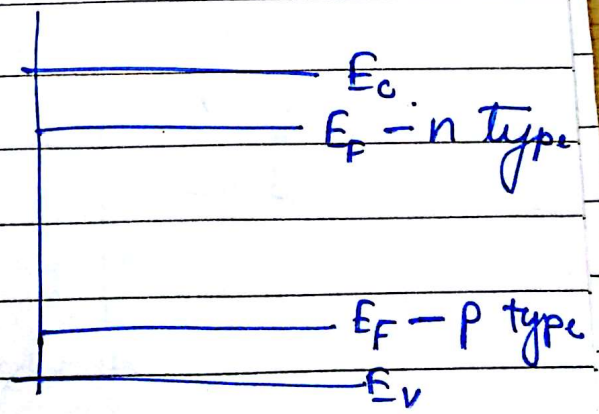
$n >> p$ $N_c >> n$

Semiconductor
↓ Doping
Extrinsic.

Low doping (1 ppm, 1 ppb) → Non-degenerate

High doping → Degenerate





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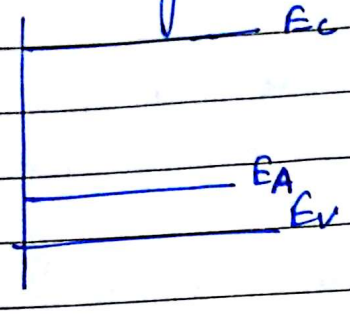
XUPERB

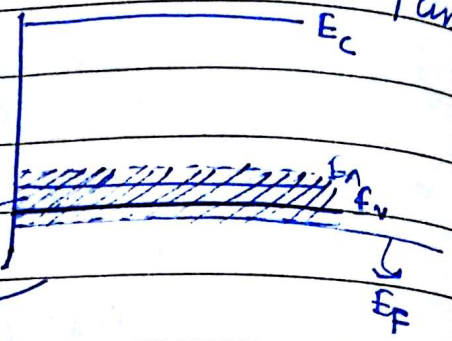
p-type

Non-degenerate

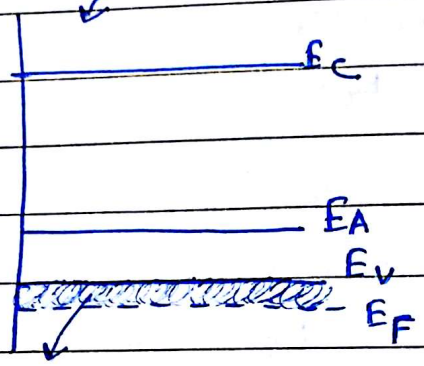


Degenerate

$N_A \approx 10^{20}/cm^3$



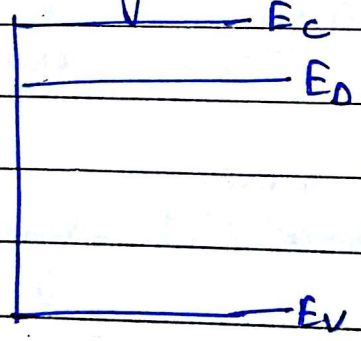
E_A band



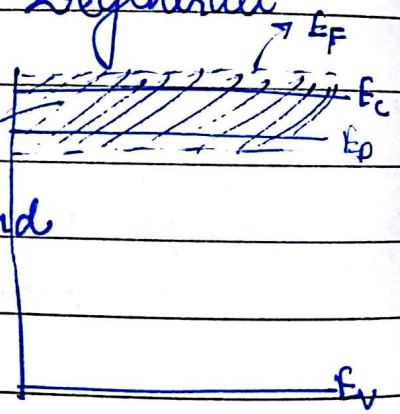
empty levels.

n-type

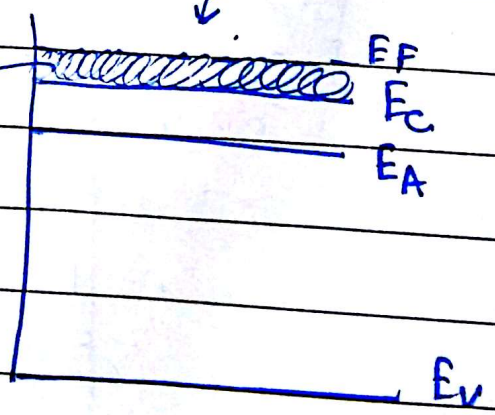
Non-degenerate



Degenerate



E_D band



filled states.

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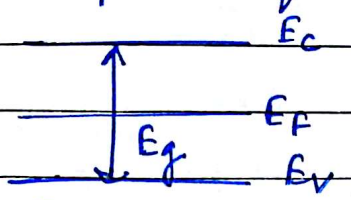
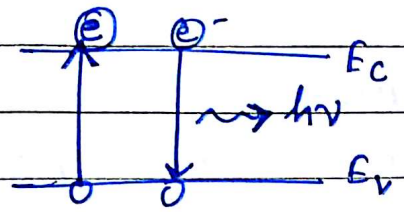
Absorption of light:



$h\nu < E_g$: no absorption
$h\nu > E_g$: absorption.



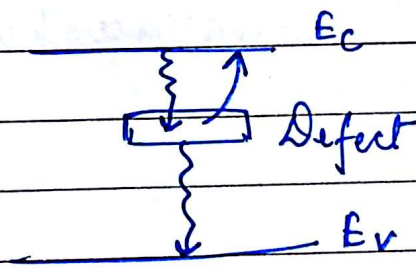
$h\nu =$

~~hole~~ e^- from conduction band to come back to valence band (lifetime) and combine with holes present:

Recombination (direct)

$h\nu = E_g$.

Indirect recombination: $h\nu' \neq E_g$



radiative

non-radiative

$h\nu' = E_D - E_v$
↳ Defect.

e^- comes to the defect.

Then either it goes back to conduct^n band or emits light by going to valence band.

If diff^n is so small (b/w E_c and defect) then the energy radiated ($E_c \rightarrow$ defect) will be consumed as thermal energy

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dark - $np = n_i^2$

when light illuminated,

$n \rightarrow n' = n + \Delta n$ } Due to photons

$p \rightarrow p' = p + \Delta p$

Consider n-type semiconductor.

$n_i = 10^{10}/cm^3$ $\quad n = 10^{17}/cm^3$ $\quad p = 10^3/cm^3$.

Suppose $\Delta n = 0.01$ of $n = 10^{15}$

$\Delta n << n$.

$\Delta p = \Delta n = 10^{15} >>> p$.

$p' = p + \Delta p$.

Photo-generated holes are much larger than already present.

Photons excite e^- from valence band and leaves hole there.

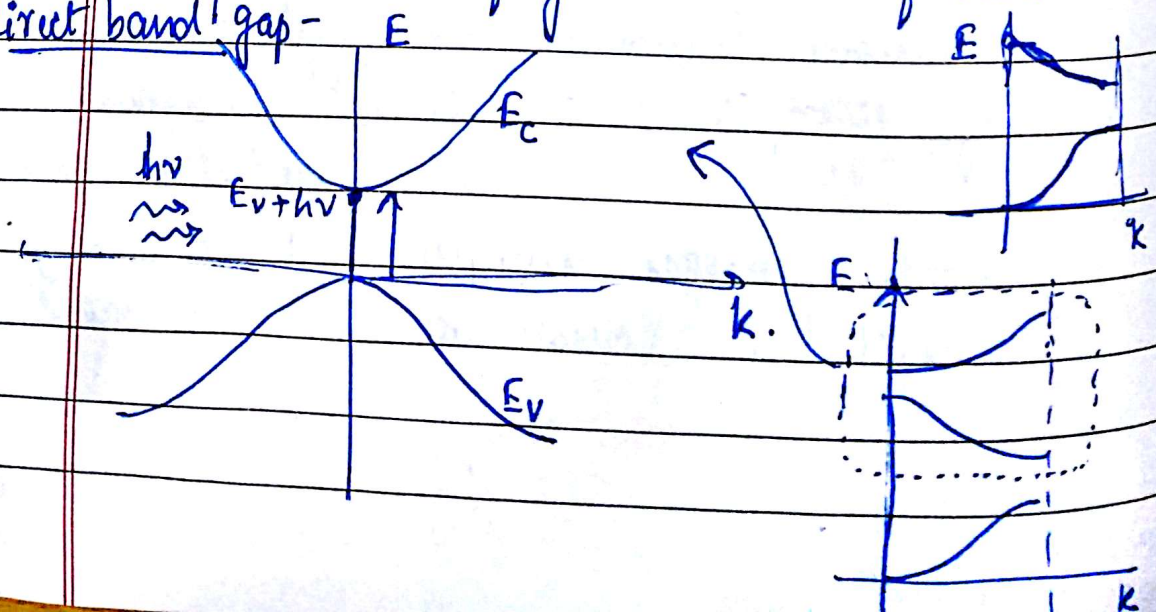
(Si not used

Generation $\rightarrow G_{ph}$

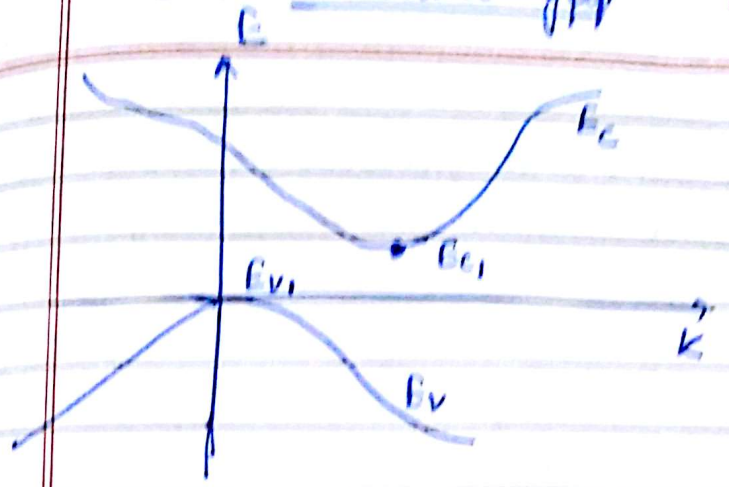
at what rate photons are falling and e^- are excited.

Equil. $\Rightarrow$ rate of generat^n = rate of recombinat^n

Direct band gap -



Indirect band-gap



Phonon's momentum come into picture to conserve momentum (slow process)

$$h\nu = E_{c_1} - E_{v_1}$$

$$(\hbar k)_{c_b} = (\hbar k)_{v_b} + \text{momentum of phonon.}$$

(si not used) → non-radiating type semiconductor hence cannot be used in LEDs.

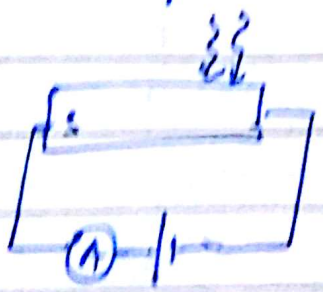
Larger thickness for indirect gap Luminous.

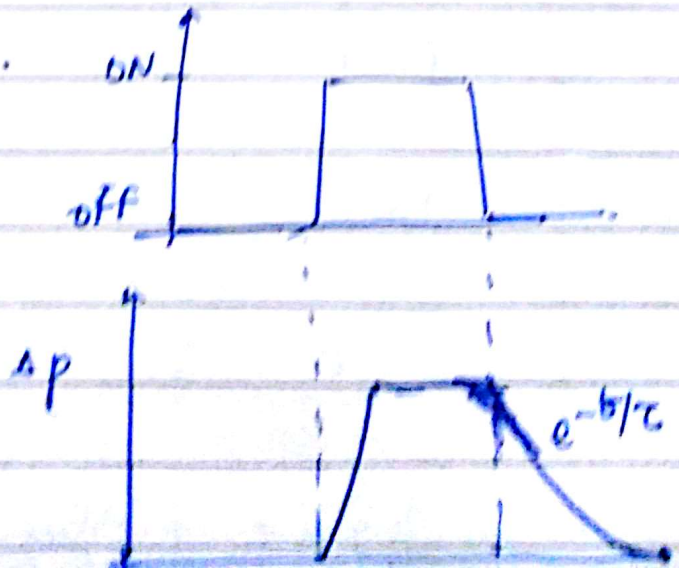
$$I = I_0 e^{-\alpha x}$$

α → absorption coeff.

Lifetime (τ)

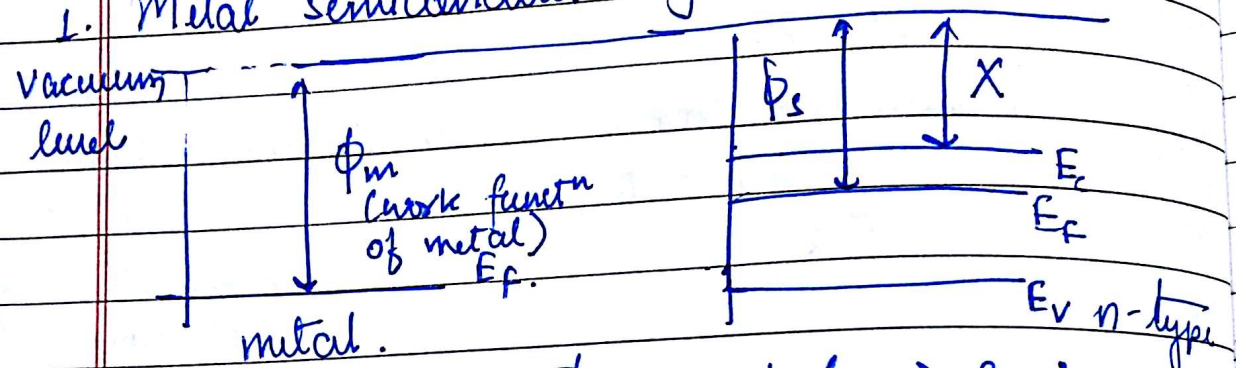
τ_p → holes.





Diffn diodes:
M-S : Metal semiconductor junctn ↳ (Schottky)
p-n juctn diode.
Zener diode.

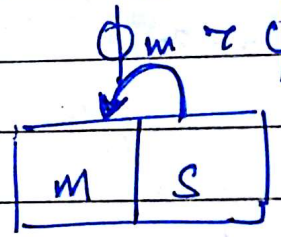
1. Metal Semiconductor Junctn.

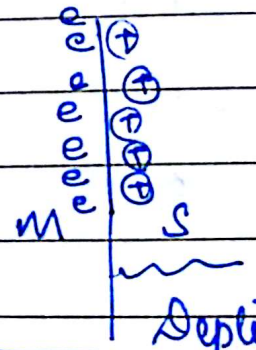


ϕ_s – work fn of Semicon.
X – e^- affinity.

Consider situatn where

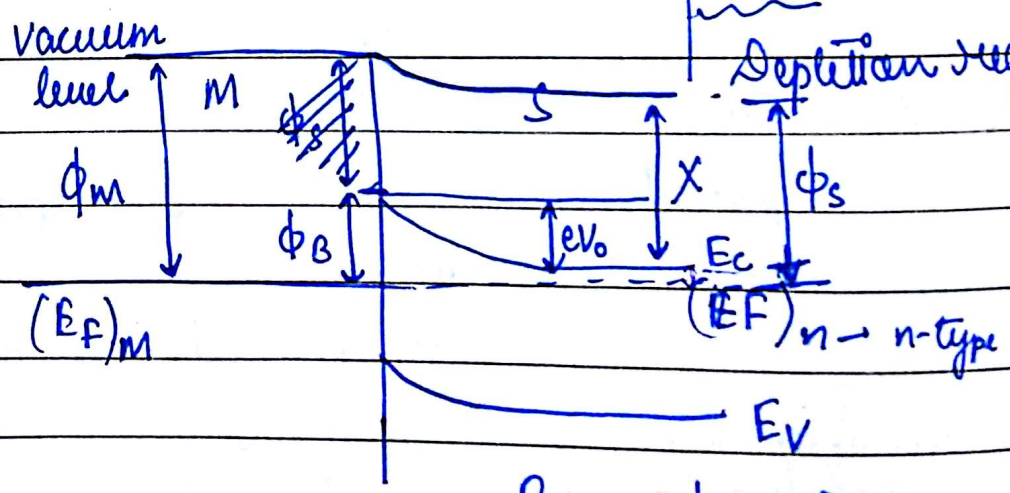
$\phi_m > \phi_s$





e^- come from semiconductor and go to metal side.

Case 1:
$\phi_m > X$



Band-bending

ϕ_B – Barrier height.
V_0 – built in voltage.

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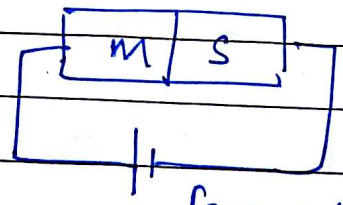
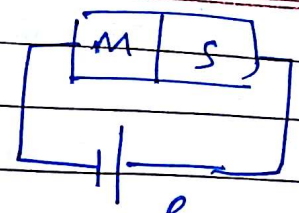


forward bias

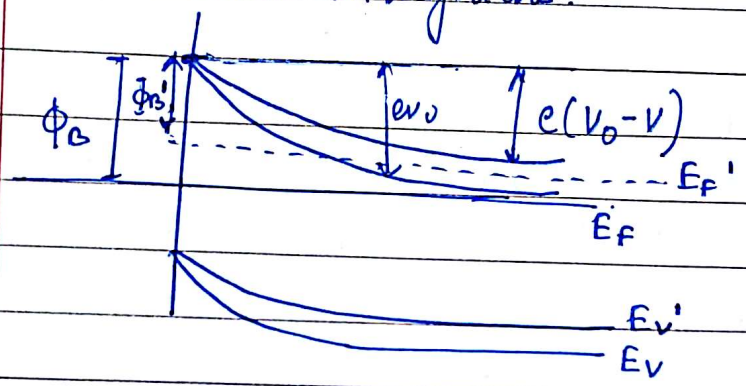


Reverse bias.

forward:

fermi-level shifts ↑.

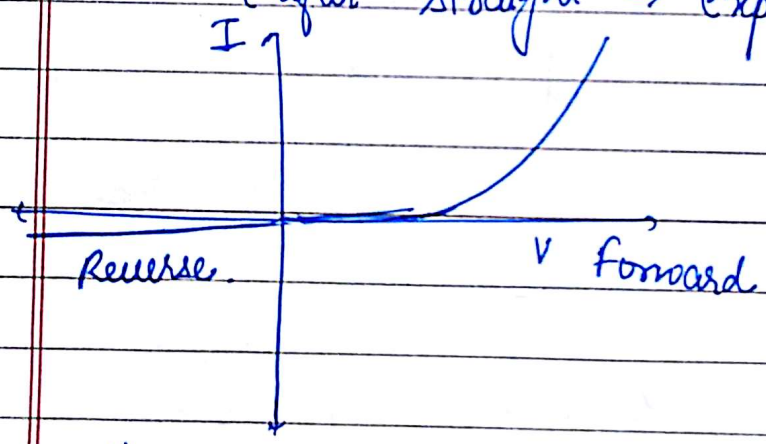
E_c band straightens.



ultimately band becomes straight.

(until bent → little or no current)

(after straight → exp. growth)



when reverse-biased, E_F shifts ↓

ϕ_B inc.

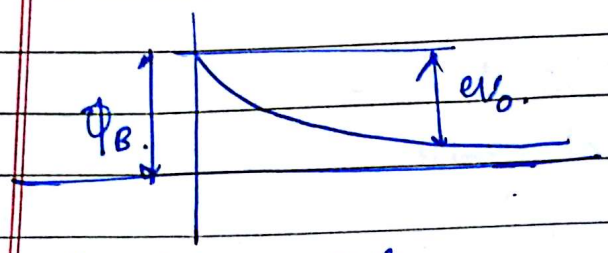
forward bias → $e(V_0-V)$

$$I = I_0 \exp\left[\frac{-e(V_0-V)}{kT}\right]$$

$V=0$ $\quad I_1 = I_0 \exp\left(\frac{-eV_0}{kT}\right)$ Current due to e^- flowing from S to M

$$I\,(\text{forward bias}) = I - I_1$$

$$= I_0 \exp\left(\frac{-eV_0}{kT}\right)\left[\exp\left(\frac{eV}{kT}\right) - 1\right]$$



Que. Consider metal,

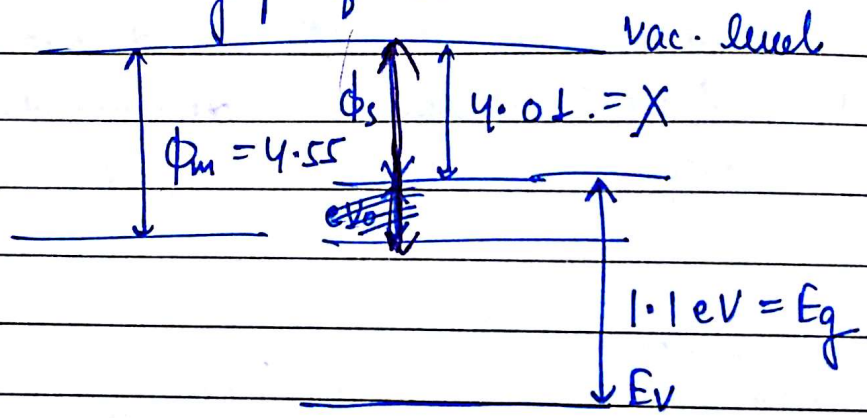
$\phi_m = 4.55\,eV$ $\quad \chi_s = 4.01\,eV$

$n = 10^{16}/cm^3$ $\quad N_c = 3\times10^{19}/cm^3$

for junct^n,

ϕ_B - ? $\quad eV_0$ - ?

band gap of Si = 1.1 eV



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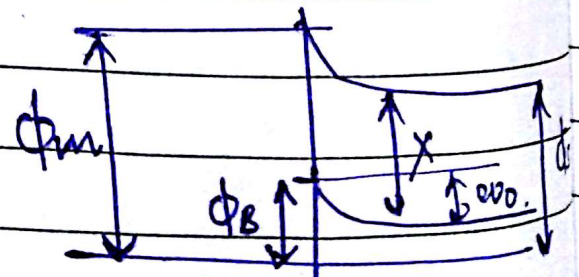
$$\phi_B = eV_0 + (E_c - E_{fn})$$

$$n = N_c \exp\left(-\left(\frac{E_c - E_{fn}}{kT}\right)\right)$$

? $\phi_m - \phi_s =$

$\phi_B = \cancel{\phi_m - \phi}$

$\cancel{\phi_s - \chi + eV_0}$.

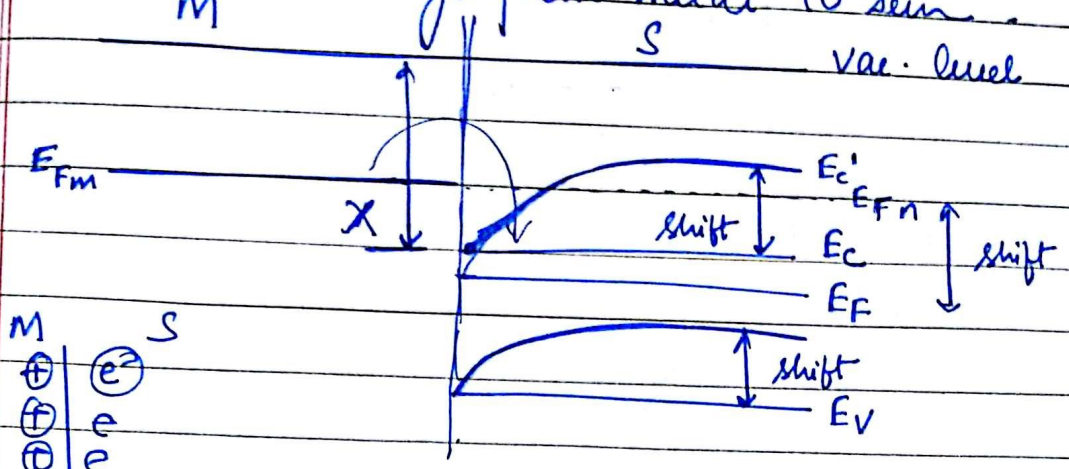


$\phi_B = \phi_m - \chi$

Case 2: $\phi_m < \chi$,

E_c lies below $(E_F)_m$.

e^- will go from metal to sem.



Accumulation region

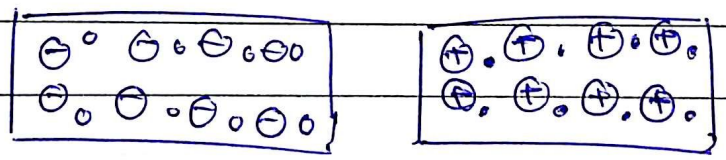
No barrier.

e^- easily flow.

$\phi_m > \chi$ → Schottky diode.

$\phi_m < \chi$ → Ohmic contact.

1/10 p-n junct^n diode.

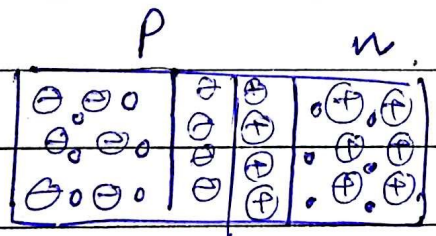


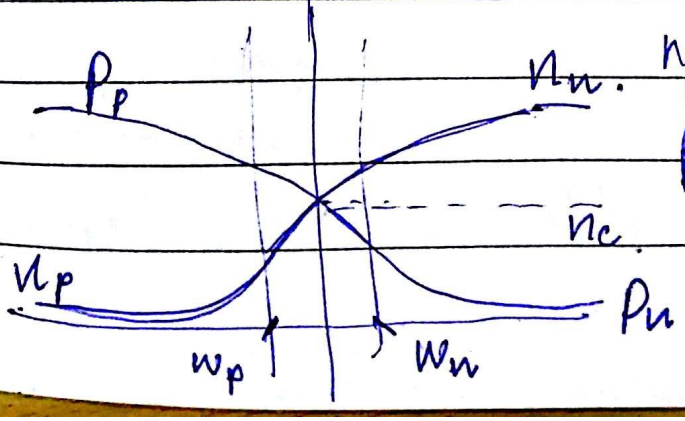
• - e^-

o - hole

⊕ Donor

⊖ Acceptor





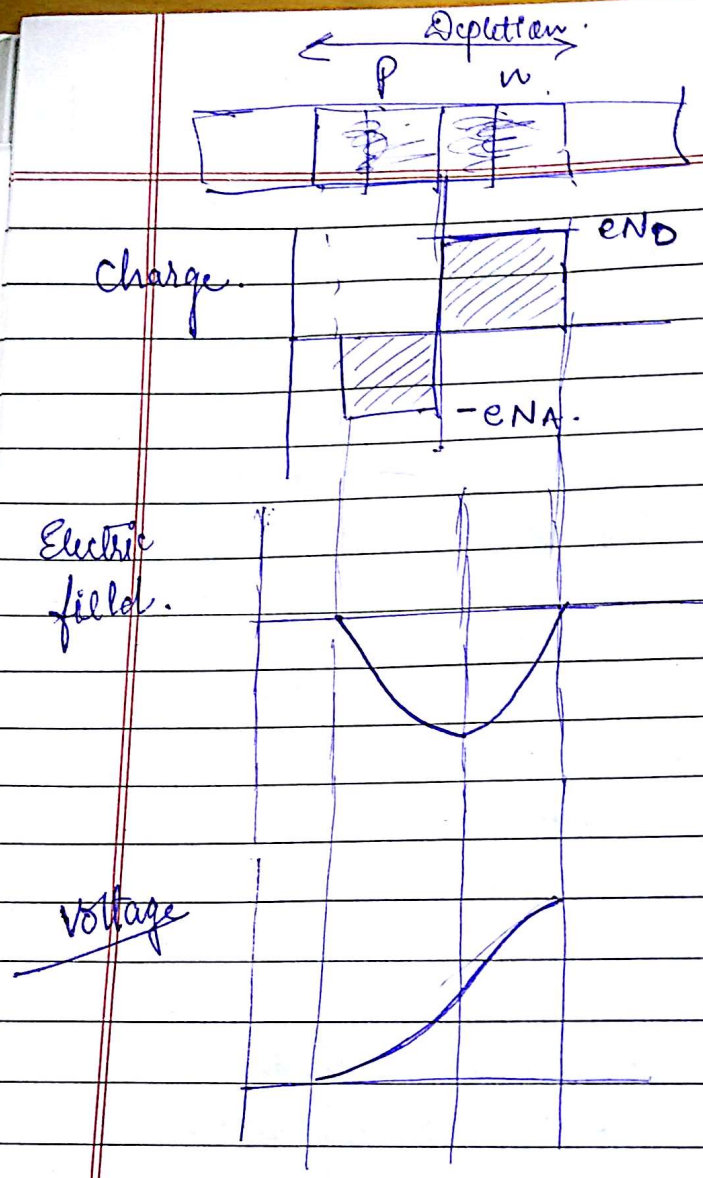
n_{n_0} → e^- conc^n at n-side

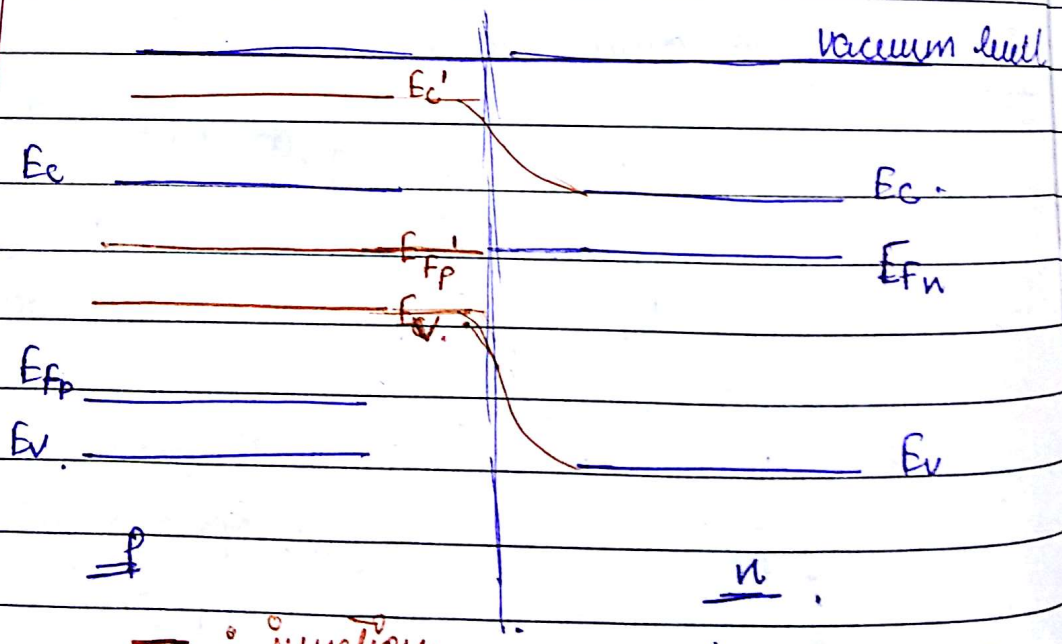
n_{p_0} → e^- at p side.

p_{p_0} → hole at p

p_{n_0} → hole at n.

$W_p + W_n$: Depletion region





keeping n fixed & shifting p-wala side.

when $V_0 = V_0$ band flatten out → conduction.

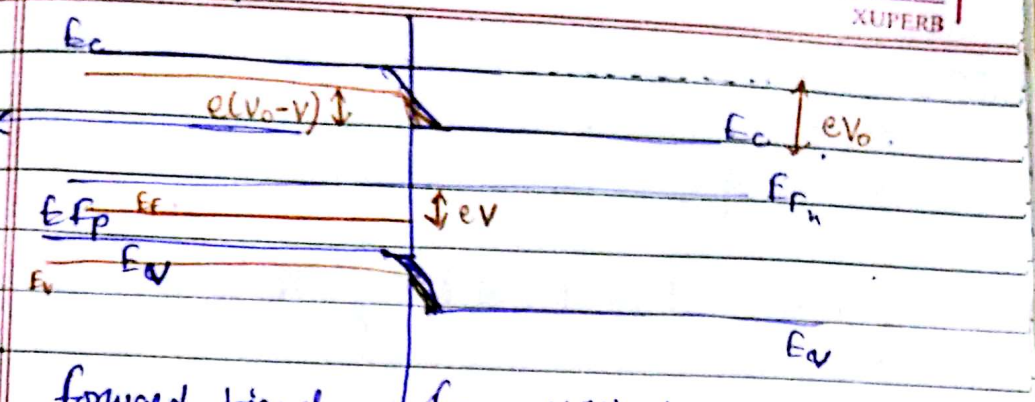
R.B:

E_{Fp}

★★ (n-type) minority charge carrier contributes

★ F-B: shift non-fixed side so that band-gap decreases

F.B:

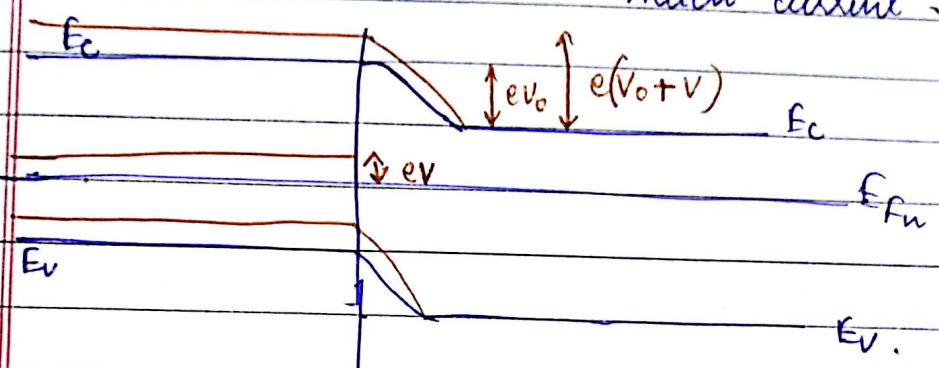


When $V_0 = V$, band flatten out → conduction.

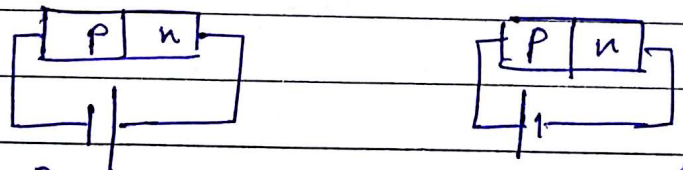
forward biased. E_{F_p} shifts ↓..

when more voltage → band flattens → much current.

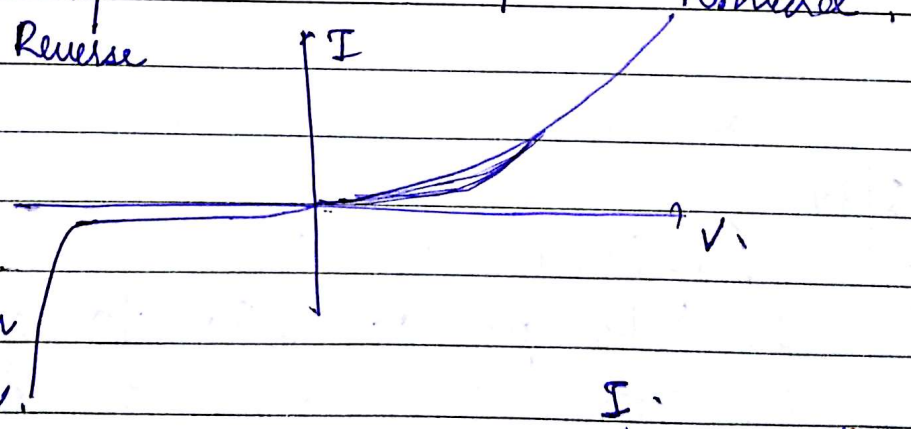
R.B:



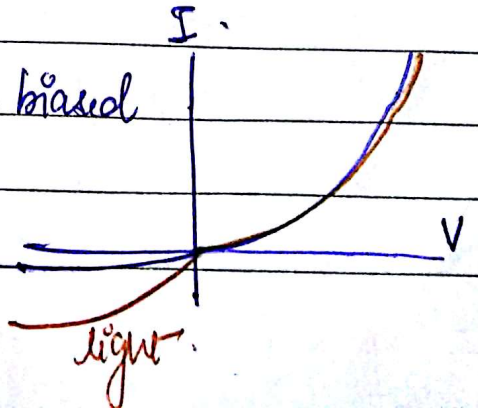
Reverse biased. E_{F_p} shifts ↑.



Reverse



Photodiode → reverse biased



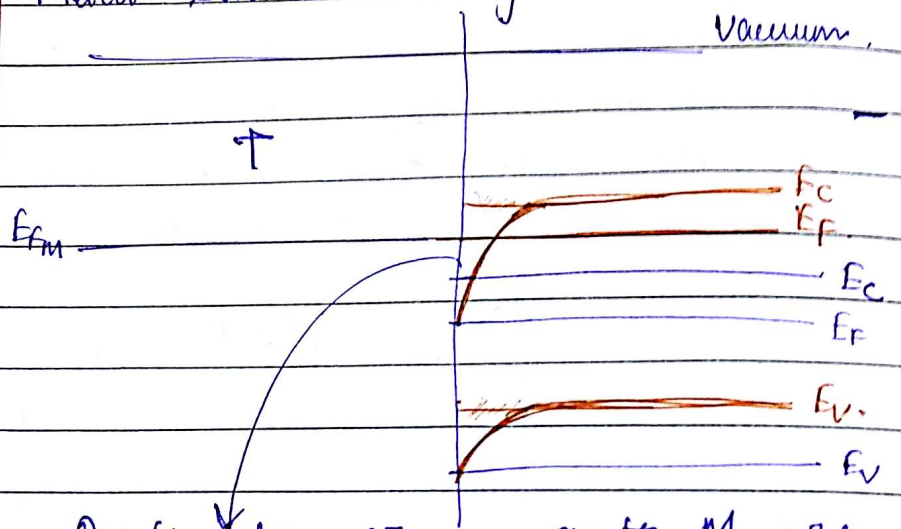
** (n-type) minority charge carrier contributes ← $n' = n_0 + \Delta n$

$p' = p_0 + \Delta p$

$\Delta n \ll n'$, $\Delta p \gg p_0$.

Eg: $n_0 = 10^{18}/cm^3$ $n_i = 10^{10}$.
$p_0 = 10^2/cm^3$.
$\Delta n << n$ $\Delta p >> p_0$.

Metal- semiconductor Jn:



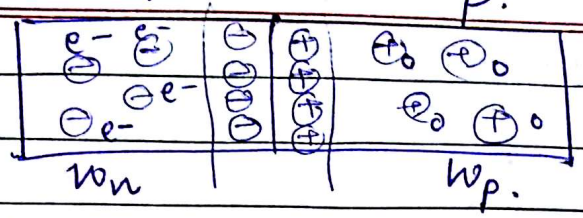
F.B: $V_0 \rightarrow V_0$
R.B: V_0

Due to slope, e^- can go to M-side.
if e^- has to go from metal to semiconductor, it overcomes hill.
So it absorbs energy / heat to go there.
∴ Gives cooling effect.
(Peltier cooler).

Diffusion length = lifetime × velocity.

5/10.

Depletion region:
When p-n connected, to ~~equalize~~ align fermi levels, there is transfer of e^-, hole.
This leads to depletion region
↓
only donor, acceptor present.
no holes / e^-.



⊖ : donor
⊕ : acceptor

Width :

$$W = w_n + w_p.$$

$$= \left(\frac{2\epsilon(N_A + N_D)V_0}{e N_A N_D}\right)^{1/2}$$

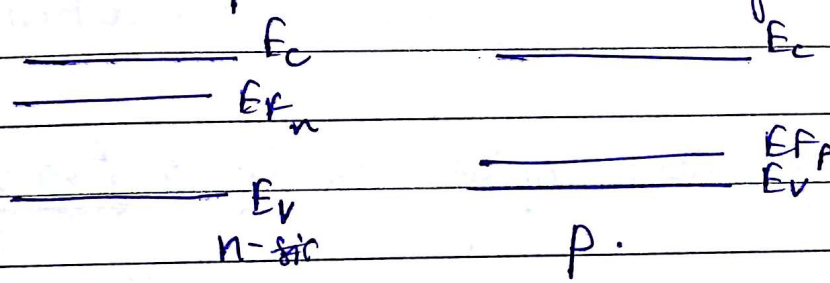
$$N_A = N_D = N \qquad = \left(\frac{4\epsilon V_0}{eN}\right)^{1/2}$$

$$W \propto \frac{1}{N^{1/2}}$$

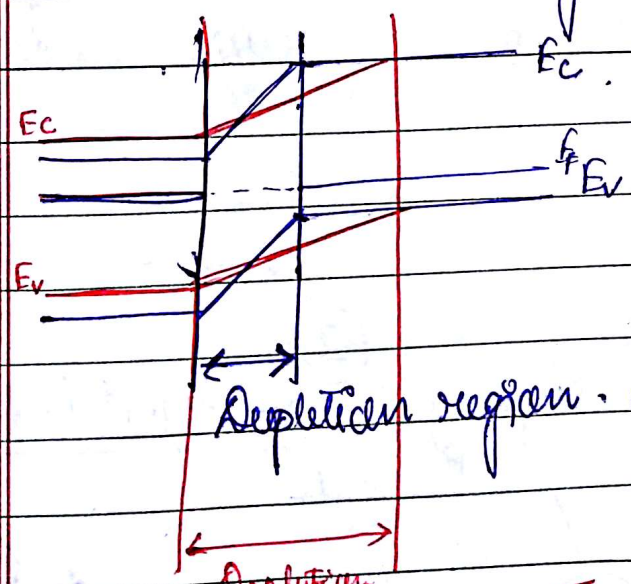
Depends on donor density.

F.B: $V_0 \rightarrow V_0 - V$.

R.B: $V_0 \rightarrow V_0 + V$

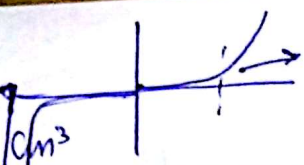


Lets keep p-side fixed.
Fermi level aligns.

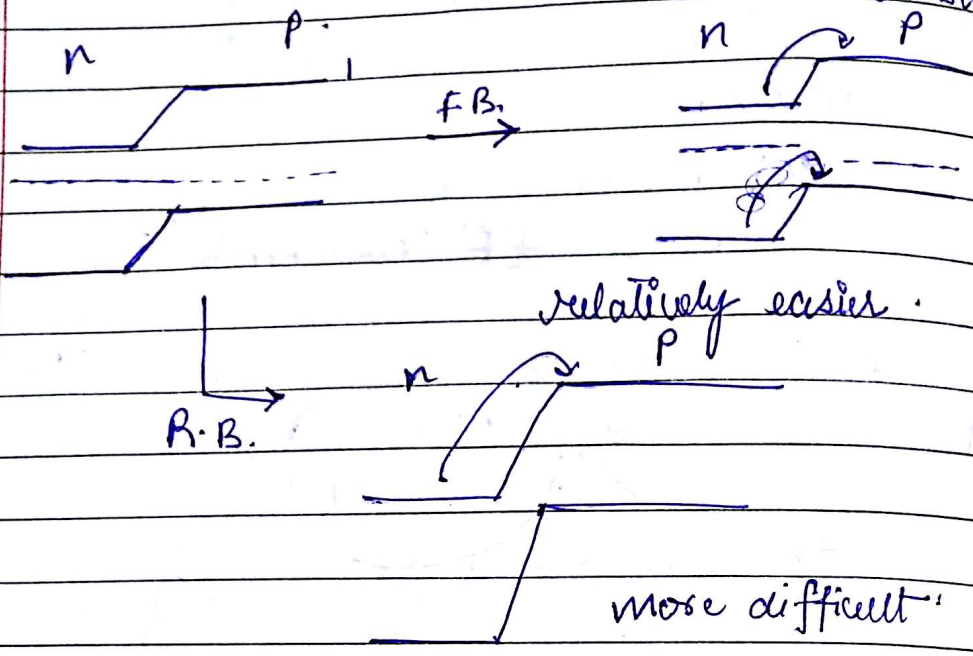


more doping
⇒ less depletion region

① $N_D \sim 10^{15} - 10^{17}/cm^3$

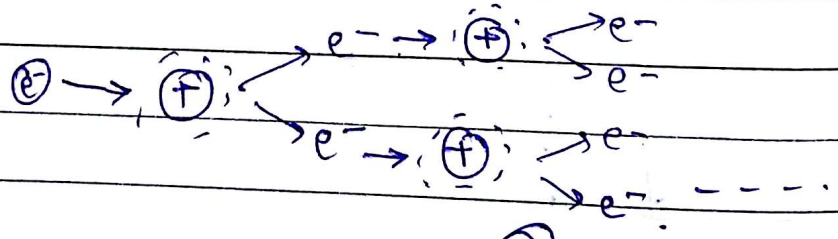


forward bias:



* When low doping density → Avalanch effect.

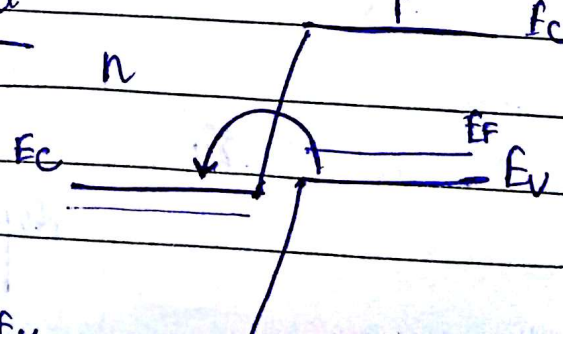
e^- in reverse bias experience large field and gets accelerated and knock down other atom generating more e^- & holes ⇒ current suddenly ↑.



②

When high doping, $N_D \sim 10^{18}/cm^3$.

Zener effect

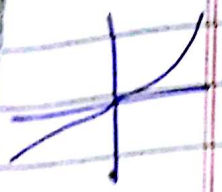


R.B.:

* e^- from E_v of p goes to n

(zener diode).

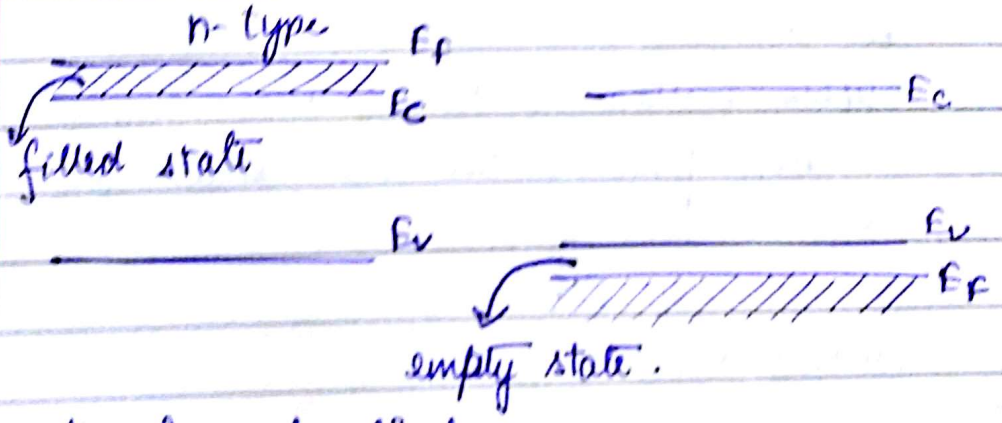
Depletion width becomes small.
By tunnelling, e^- from E_v (p) → E_c (n).



→ controlled current
This current is lower than that of Avlanch effect.

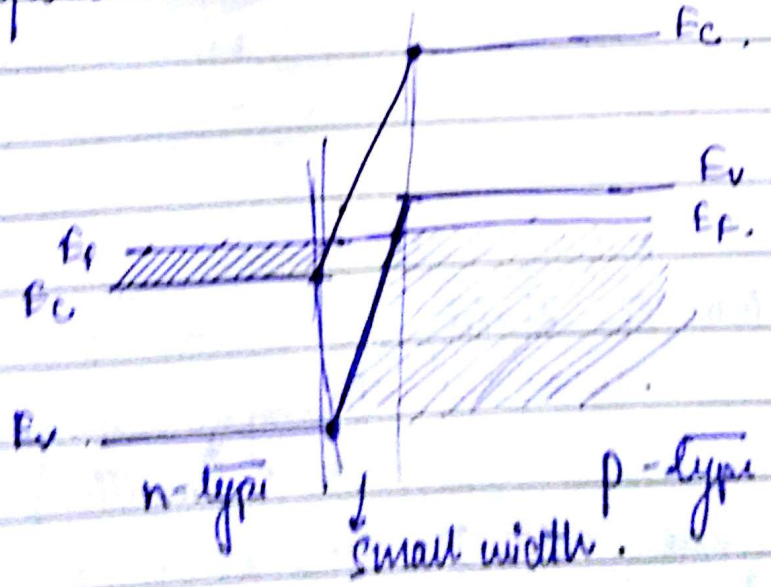
(3). $N_D \sim 10^{20}/cm^3$

Instead of discrete, there are continuous bands.



* Tunnel diode.

Equilibrium:



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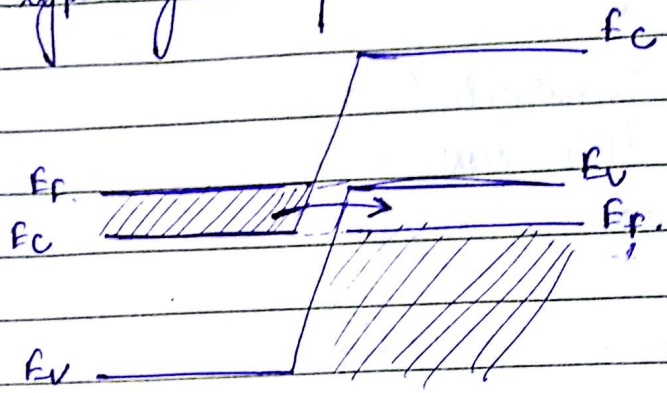
(a) When f.B. is applied, Fermi level of n type goes up.



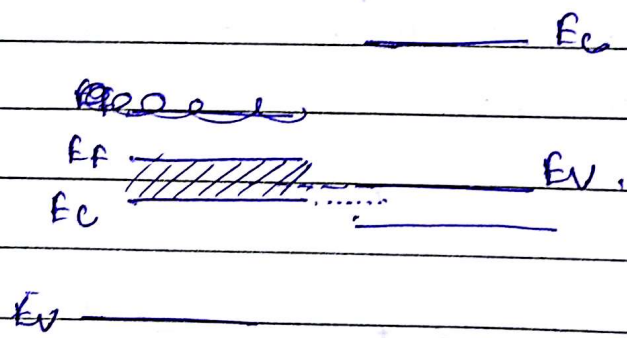
Through tunneling e⁻ from n-type band goes to p-side.

When barrier is small and empty states are present on one side and equal energy states.

(c).

Further more f.B.

(b)



No. of states decreased and hence ~~curr~~ e⁻ transfer decrease (due to tunelling).

Thus current decrease.

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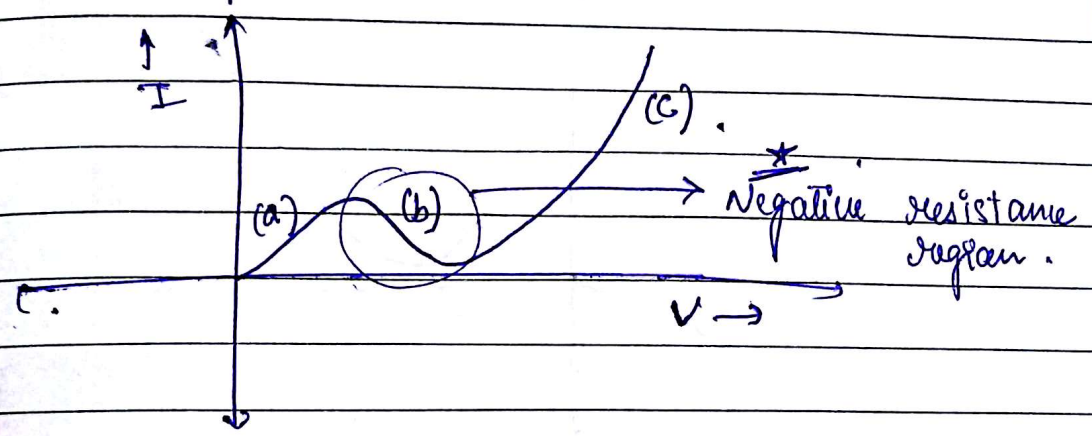
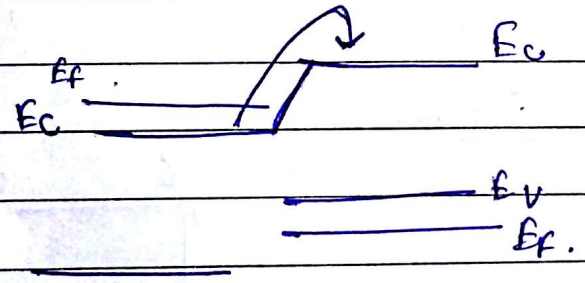
when further increase in F.B., e^- from ~~condu~~ E_c of n goes to E_c of p.

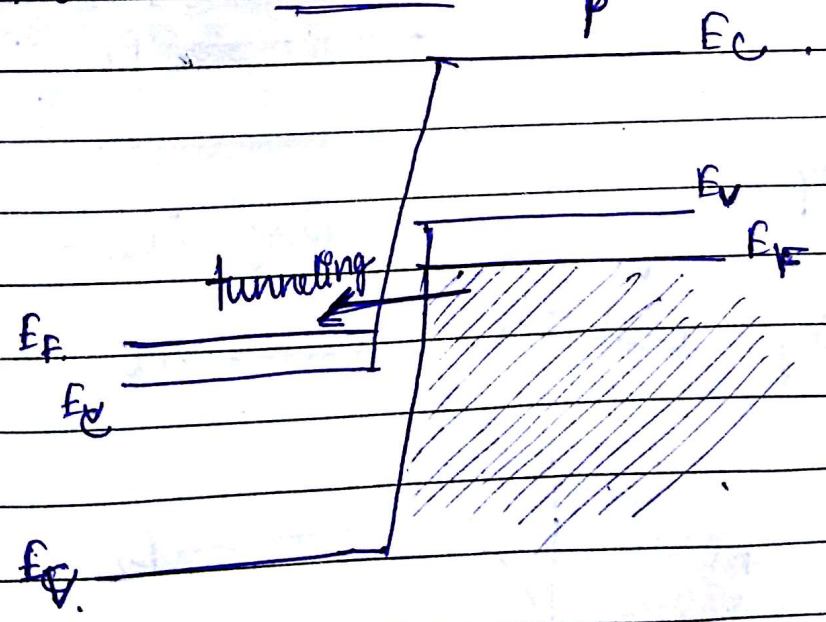
Thus slowly current increases as seen in previous case.



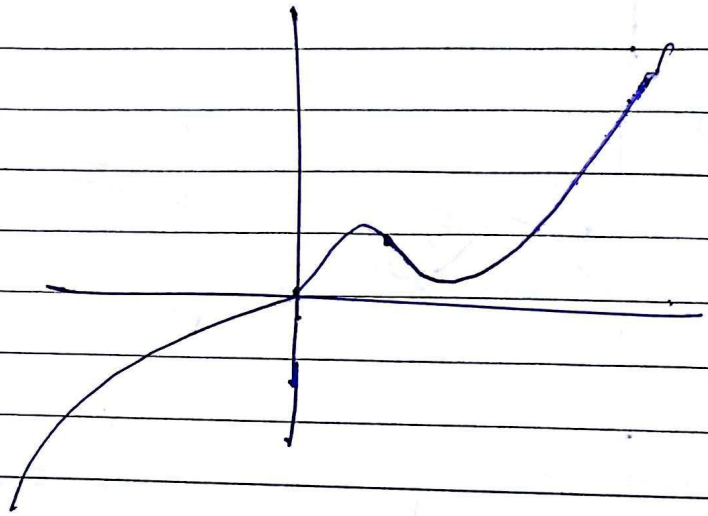
(c).



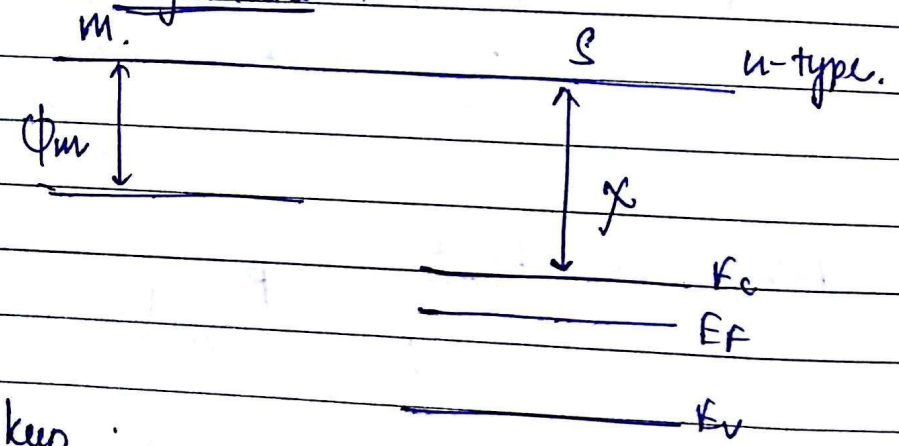
Now in R.B. :



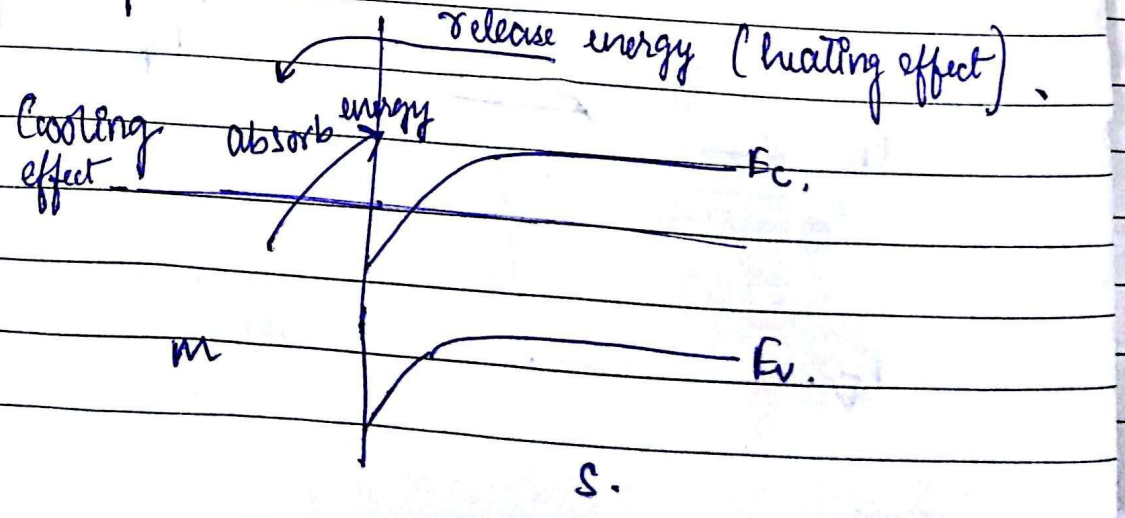
When R.B. increases, E_F further shifts down and thus creating more no. of empty states and more e^- flows due to tunneling.
∴ Current increases.



6/10. Ohmic junction.



keep.



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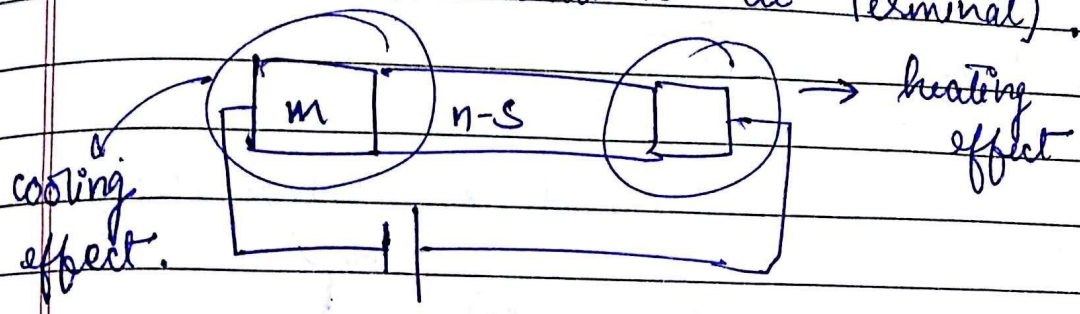
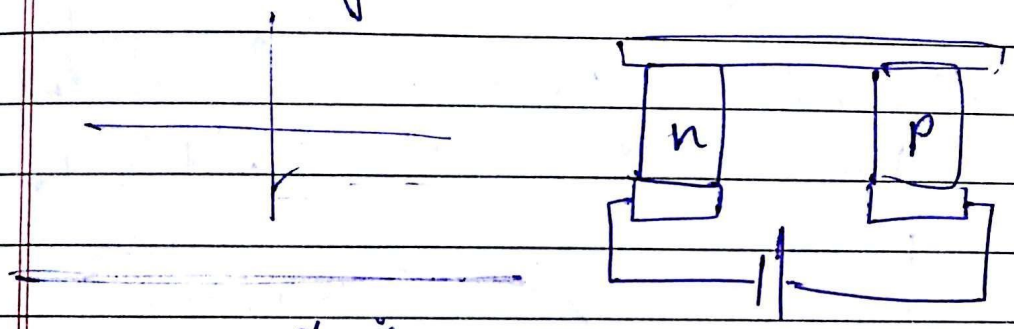
XUPERB

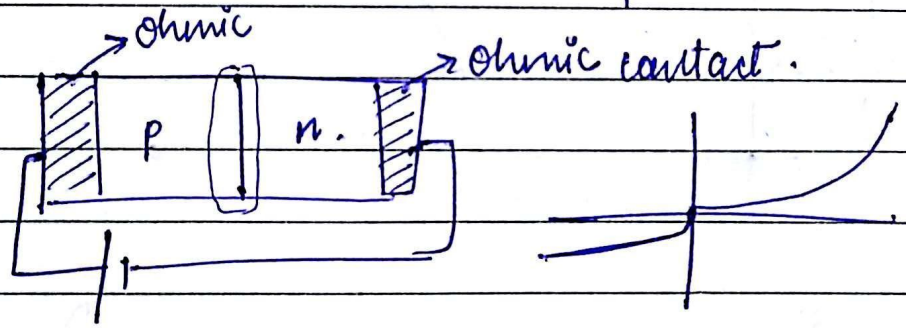
Peltier cooler.

Inject e^- from metal side to s side.
(⇒ connect metal to −ve terminal).

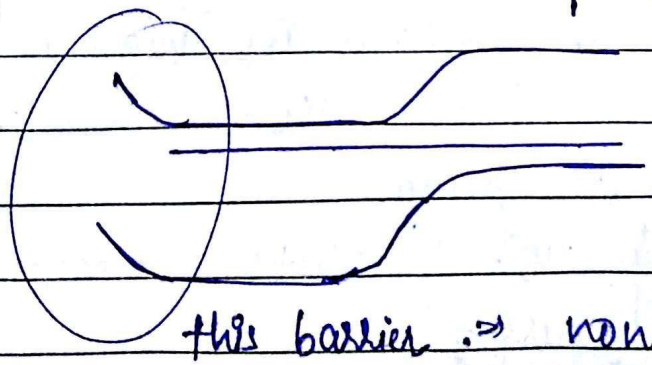


for p-type, case ~~itt~~ is the reverse.





If ohmic contact not present :



this barrier ⇒ non linear characteristic.

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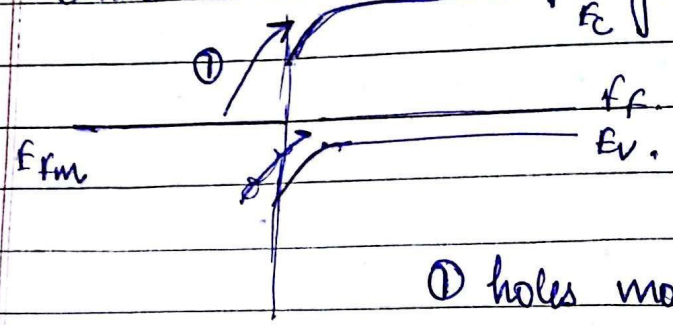
coding ?

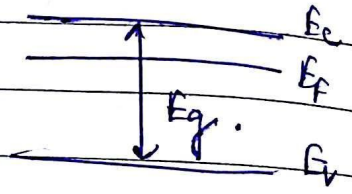
Ohmic contact with p-type.

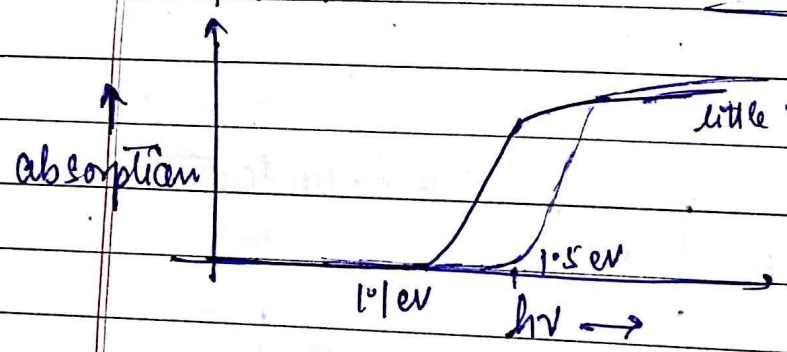


① holes move easily this way.

A → $E_g = 1.1 eV$ (Si)
B → $E_g = 1.5 eV$ (GaAs).

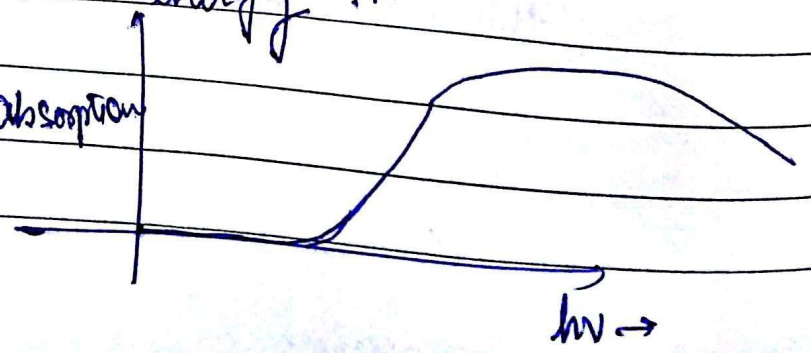
We shine light over them.



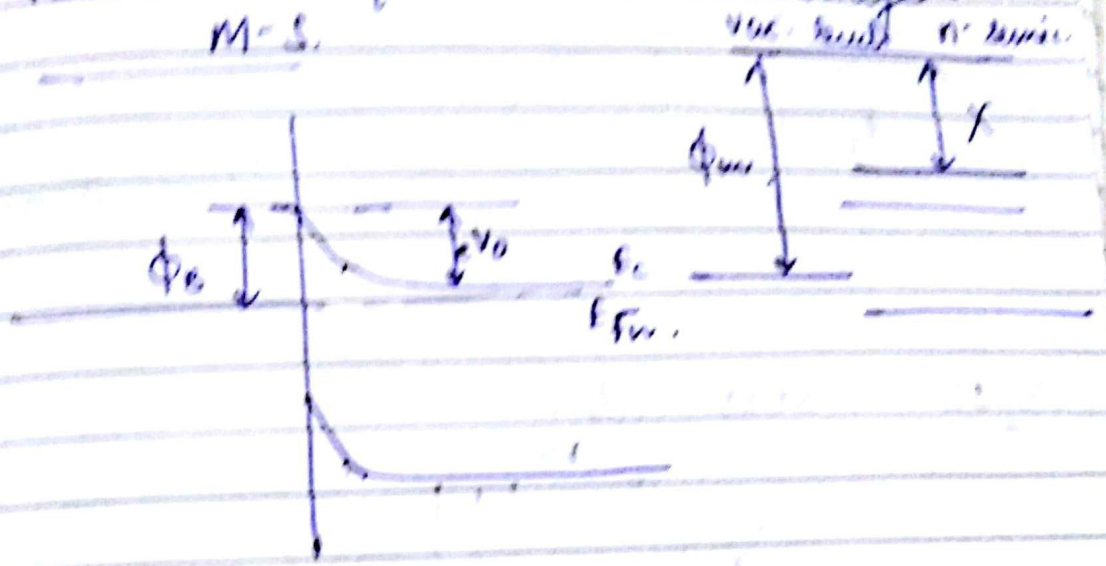


(no absorption till $h\nu = E_g$.)

If no. of photons in high energy region is same then const. curve after jump.
usually no. of photons decreases at high energy.

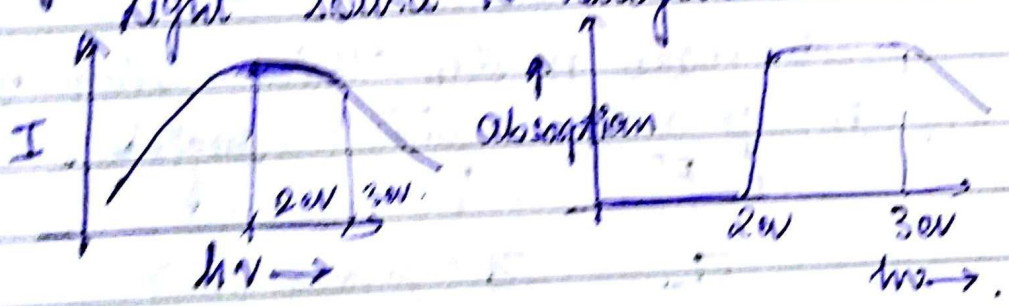


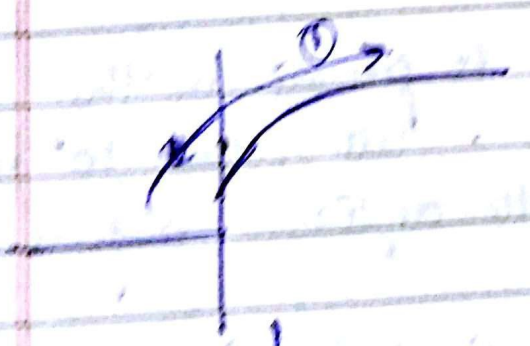
* Relation b/w ϕ_B and built in voltage.



$$\phi_B = \phi_m - \chi$$
$$eV_0 = \phi_B - (E_c - E_{fn})$$

* Eg: $E_g = 2$ eV.
Light source is halogen.





①: barrier for ~~hot~~ e^-
②: easier for hole.

→ not a barrier for hole.
→ barrier for e^-.

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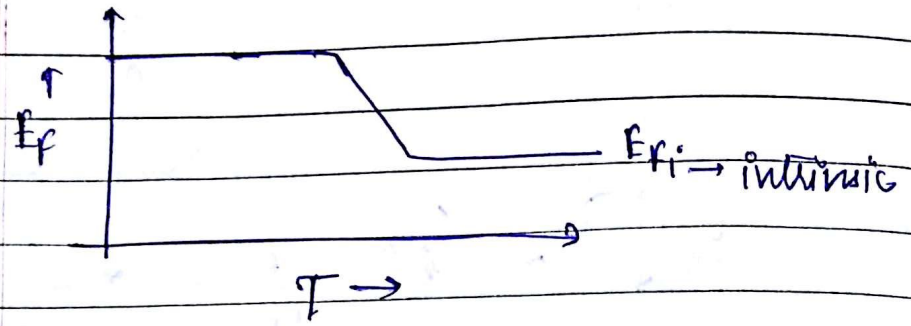
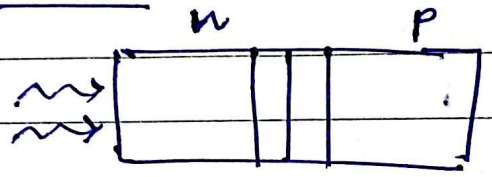
When carrier conc. remains const.



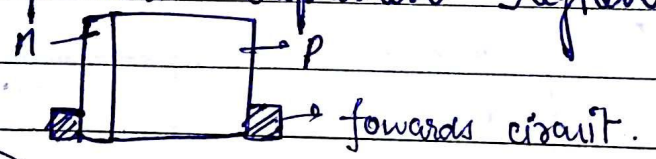
13/10 **Solar cell.**

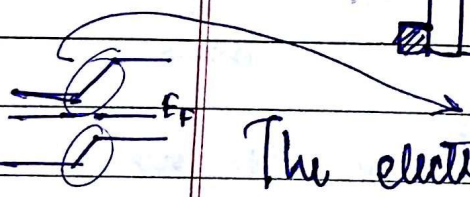


$I = I_0 e^{-\alpha x}$

If n side is thick, holes and e^- will be generated in the ~~p~~ n-side and will recombine.

We make n-side thin. Light reaches n, p and depletion region.



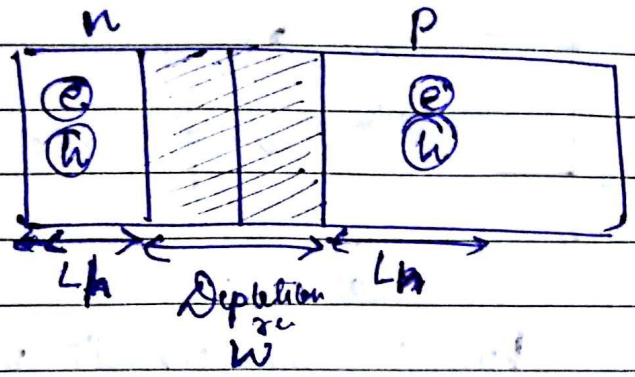


The electric field is present in the depletion region. We want ~~to~~ max. light to reach to depletion region.

Drift current: e^- generated in p-side will easily go to n-side due to the potential diff^n.

Diffusion current:

In forward bias- current is due to majority carriers.

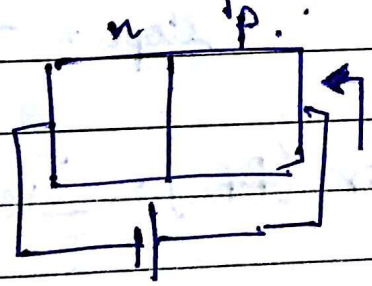


L_n: diffusion length for e^- in p-side.
L_h: diffusion length of hole in n-side

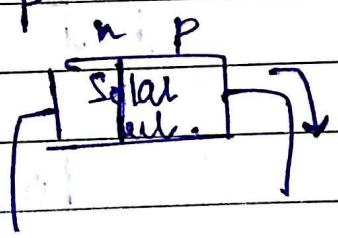
$\boxed{L_h + W + L_n}$ will contribute in working of solar cell.

In solar cell we get current due to minority carriers.

e^- move from p to n.
h move from n to p.



normal.

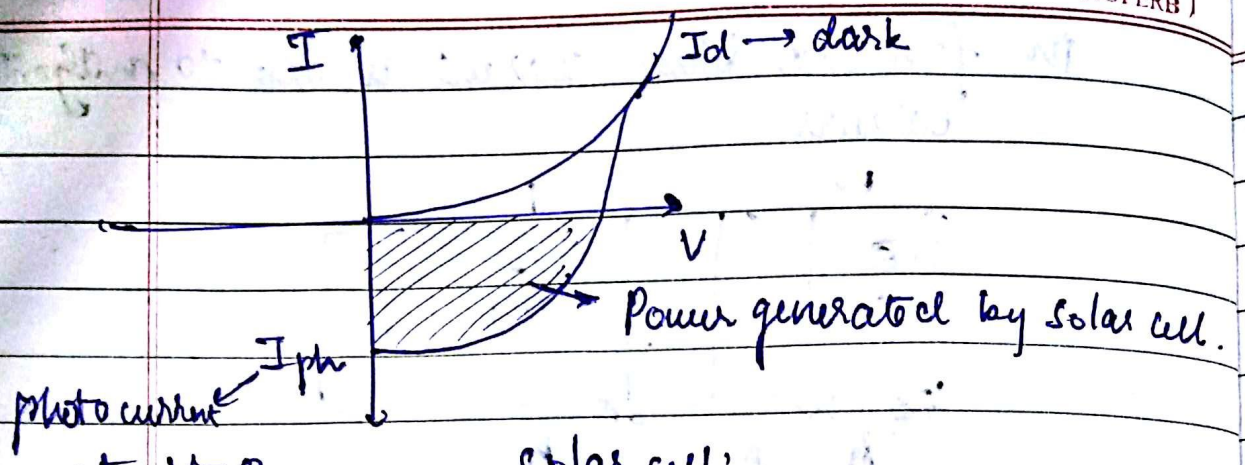


Current in F.B. of a pn junct^n

$$I = I_0 \left[\exp\left(\frac{eV}{\eta kT}\right) - 1 \right]$$

$$I = -I_{ph} + I_0 \left[\exp\left(\frac{qV}{\eta kT}\right) - 1 \right]$$

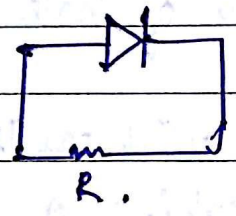
I_{ph} → photo current



Solar cell:

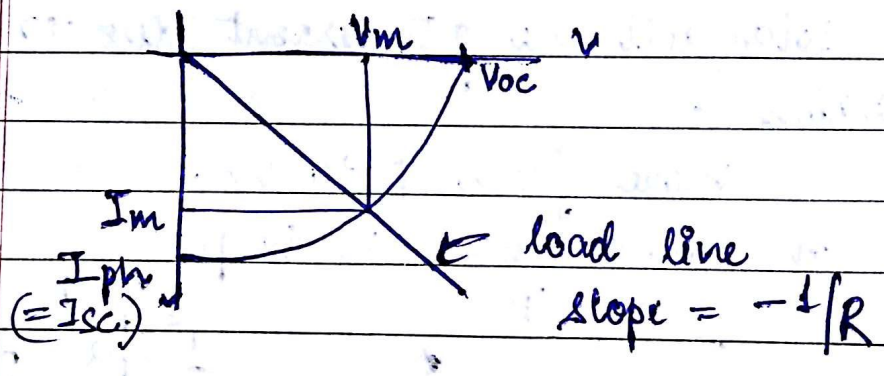
V_{oc} = Open circuit voltage

$I_{ph} = I_{sc}$ (short circuit current)



Efficiency of solar cell:

(How much light energy converted to elec. energy?)



$$\text{fill factor (f.f.)} = \frac{I_m V_m}{I_{sc} \cdot V_{oc}}$$

We want ff to be 1 but ∈ 0.5 - 0.7

$$\text{Efficiency} = \frac{\text{Electrical power generated}}{\text{Incident light power}}$$

$$= \frac{I_m V_m}{P(w)} \times 100\%$$

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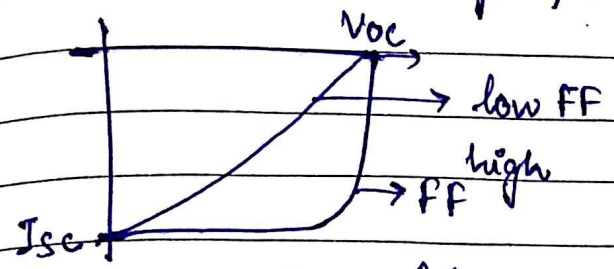
$$= \frac{(FF) \times I_{sc} \cdot V_{oc}}{P(\text{input})} \times 100\%$$



more ff ⇒ more efficiency.

$$I = -I_{ph} + I_0 \left[\exp\left(\frac{eV}{\eta kT}\right) - 1 \right].$$

$V = V_{oc}$, $I = 0$

$$\therefore V_{oc} = \frac{\eta kT}{e} \ln\left(\frac{I_{ph}}{I_0} + 1\right).$$

$\eta \rightarrow$ device parameter.

Eg: $100\ W/m^2 \rightarrow 200\ W/m^2$.

$V_{oc} = 0.53V$

$I_{sc} = 150\ mA$ $(= I_{ph})$

$\eta = 1.5$

find new Voc and Isc.

Same efficiency

$\Rightarrow$ ~~$I_{sc}' V_{oc}' = 2 I_{sc} \cdot V_{oc}$~~

~~$= 2 \times 150 \times 0.53 \times 10^{-3}$~~

~~$= 300 \times 0.53$~~

~~$= 0.159$~~

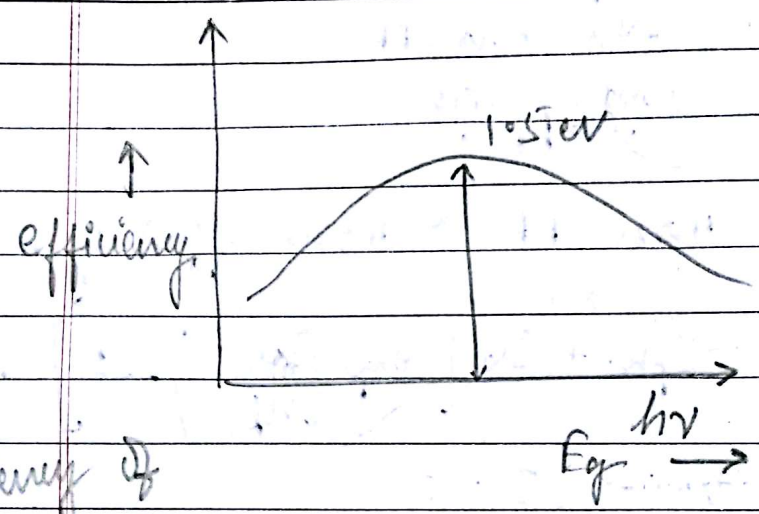
2× light incident.

$\Rightarrow$ 2× e⁻ generates

$\Rightarrow$ 2× current $\Rightarrow I_{sc}' = 2 I_{sc}$

$= 300\ mA$

The band bending can never be larger than the band-gap



Efficiency of solar cell: GaAs ($E_g \sim 1.5$ eV)

① Expensive material used.

→ Si ~ 1.1 eV is used mostly.

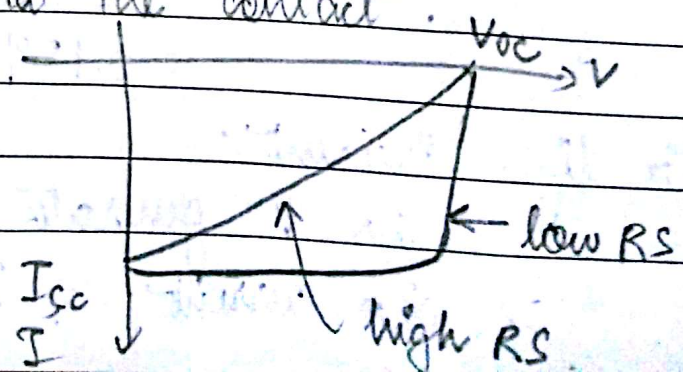
② → L_p, L_e (Diffusion length for hole & e^-) should be sufficiently high.

③ FF (fill-factor) $\eta = \frac{FF \cdot V_{oc} I_{sc}}{P_{in}}$

↓

depends on the series resistance of the cell (R_s)

↳ depends on semiconductor taken and the contact.

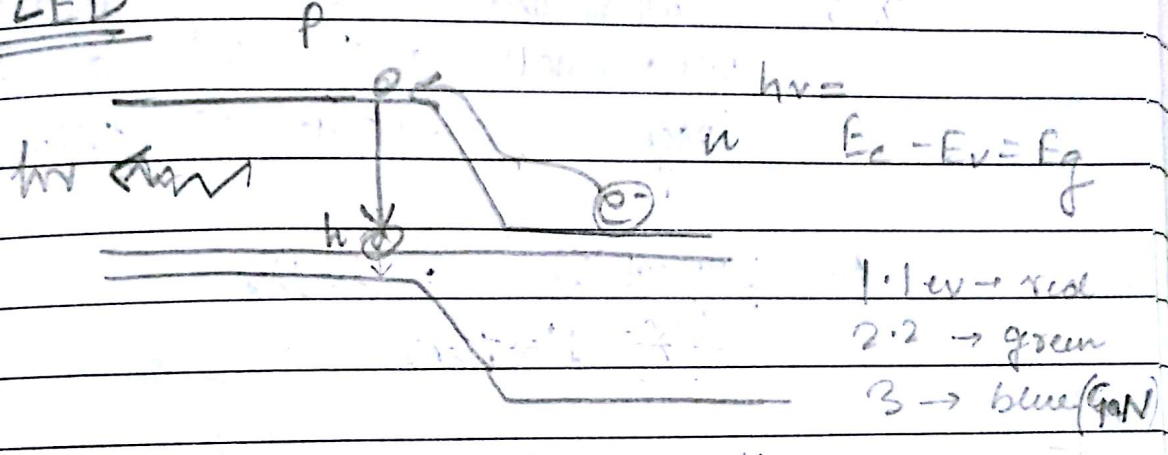


Solar cell ≡ diode

Shunt resistance (associated with depletion region)

Series resistance.

LED



① e⁻ recombines with the hole and light is radiated.

→ * Takes place only when direct recombination ⇒ when there is direct band gap

② No defects should be present.

Very difficult to make p-type material when high band gap

Thus blue LED took time to be discovered.

17/10.

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B-H curve.

$B = \mu H$. $H \to A/m$.

$\chi = \frac{M}{H}$ → magnetisation. (mag. moment / vol) $B \to T$ or Wb/m^2.

↓

Susceptibility.

χ :

very small	+ve	(10^{-6})	paramagnetic
very small	-ve	(-10^{-6})	diamagnetic
large	+ve	$(10^{0}-10^{4})$	ferromagnetic

μ (permeability) $= \mu_0 \mu_r$

~~$\chi = 1 + \mu_r$~~

$B = \mu_0 H + \mu_0 M = \mu_0 H + \mu_0 \chi H$ — (1)

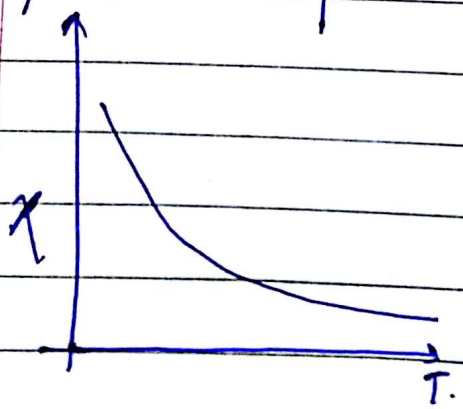
$B = \mu H = \mu_r \mu_0 H$. — (2)

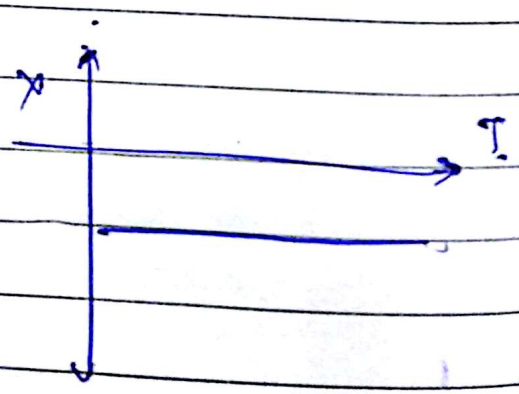
from (1) and (2)

$\mu_r \mu_0 H = \mu_0 H (1+\chi)$

$\Rightarrow \boxed{\mu_r = 1 + \chi}$.

χ with temp :





paramagnetic. diamagnetic.

$\chi \propto \frac{1}{T}$ → Curie Law (almost const. with temp)

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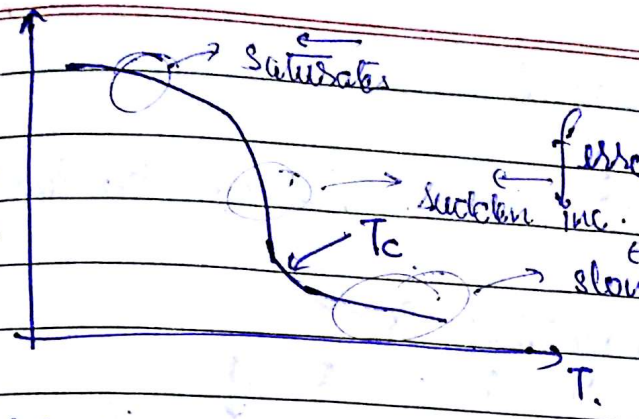
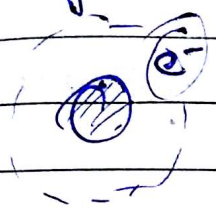


$$\chi = \frac{C}{T - T_c}$$

Curie-Weiss Law

Origin:



orbital magnetic moment
(e^- revolve, → current/area).

μ_B = spin magnetic moment
(Bohr-magneton).

in s-orbital, if two e^-:

Spin moment = 0 (↑↓)

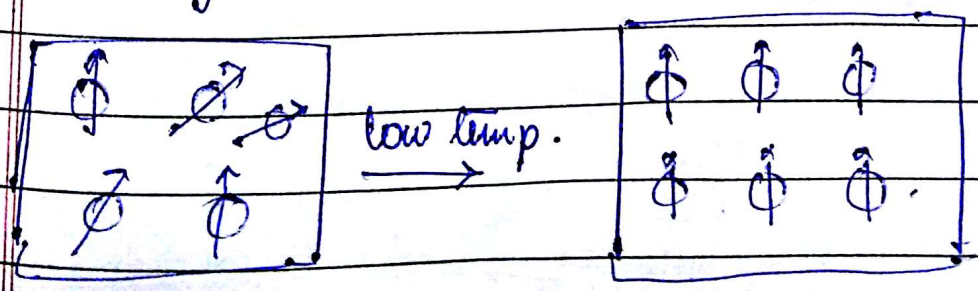
Orbital moment = 0 (all direct^n → symmetrical)
moment cancels out.

consider He,

① when magnetic field is applied, there is some opposition (Lenz law). * diamagnetic behavior.

* Dia. behaviour observed for only those for which spin moment = 0, orbital moment = 0.

② Paramagnetic



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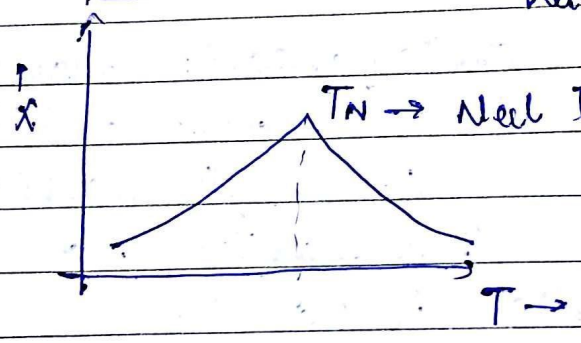
③ Ferromagnetic

$\uparrow \leftarrow \uparrow$ interaction b/w the two atoms

④ Antiferromagnetic : $\chi = 0$.

$\uparrow \downarrow \uparrow \downarrow \uparrow \downarrow$

net moment = 0.



19/10. we get orbital moment when orbital is not symmetric (s).

Eg: # Valence $e^- = 8$. d

↑↓	↑↓	↑↓	↑	↑

2

$$\mu_B = \frac{e\hbar}{2m_e}$$

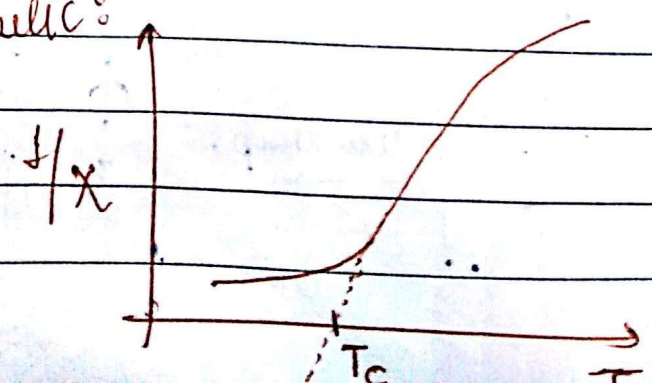
$$= 9.27 \times 10^{-24} \ (J/T).$$

In this eg, $\mu = 2\mu_B$.

$$= 18.54 \times 10^{-24} \ J/T.$$

M = magnetic moment/volume

Ferromagnetic:



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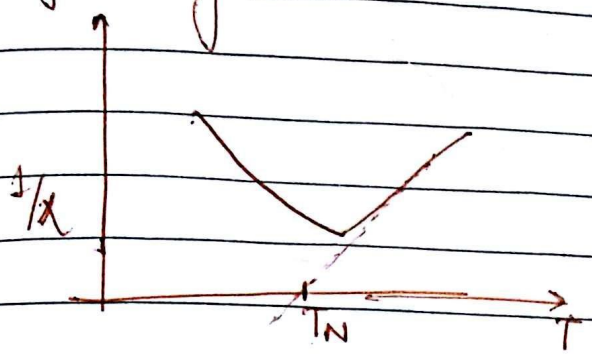
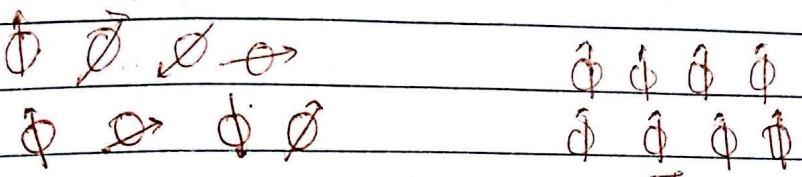
Antiferromagnetic.



$T_N \equiv$ Neil Temp.
$T < T_N \rightarrow$ Antiferro.
$T > T_N \rightarrow$ Paramagnetic

Paramagnetic: no interaction



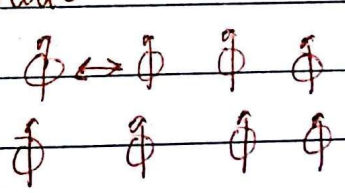
$H = 0$ $\qquad$ $\vec{H} \neq 0$

N atoms per m^3.

$M = N\mu m$.

max. magnetisat^n (depends on T, H).

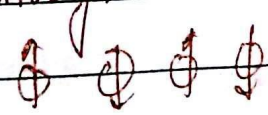
Ferromagnetic:



$H = 0$

Exchange interaction not a dipole-dipole interact^n.
⇓
To be discussed later

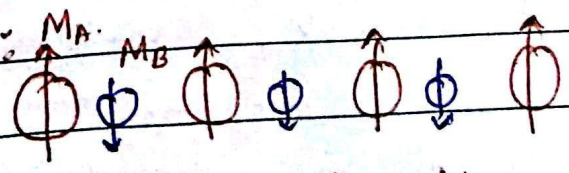
Antiferromagnetic: (Exchange interact^n)



individual moment ✓
overall = 0.

$T > T_N$: AFM → PM

Ferrimagnetic:



$M = M_A - M_B$

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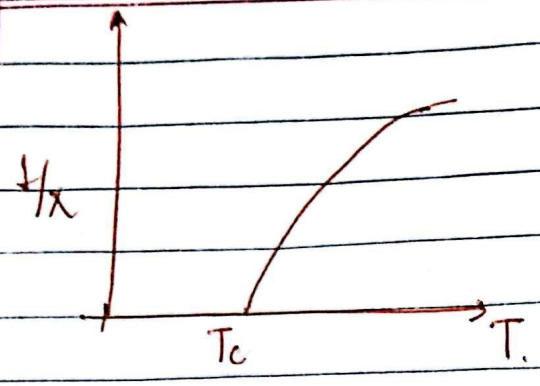
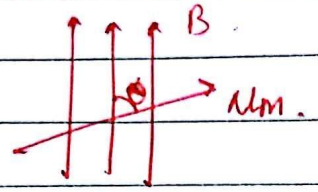


20/10. N atoms → μ_m.



Energy,

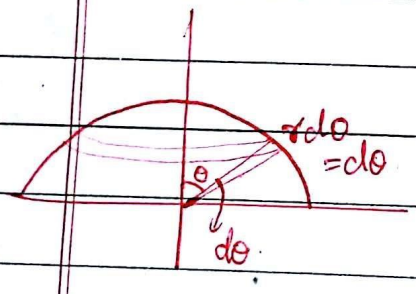
$$E_p = -\vec{\mu_m} \cdot \vec{B}$$
$$= -\mu_m \mu_0 H \cos\theta$$

Probability of magnetic moment, θ.
(μ_m to form an angle θ with $\vec{B}$)

$$= \propto e^{-E_p/kT} \approx C' \exp\left(\frac{\mu_0 \mu_m H \cos\theta}{kT}\right)$$

Boltzman distribution.

No. of atoms whose magnetic moment θ — $\theta + d\theta$.



$$dA = 2\pi \sin\theta \, d\theta. \quad (r = 1)$$

$$dn = C \, dA \, P(\theta) \rightarrow \text{includes density}$$

$$n = C \int_0^{\pi} 2\pi \sin\theta \, d\theta \, . \exp\left(\frac{\mu_0 \mu_m H \cos\theta}{kT}\right) \rightarrow (1)$$

Net magnetisation

$$M = \int \underbrace{\mu_m}_{\mu_m} \cos\theta \, 2\pi \sin\theta \exp\left(\frac{\mu_0 \mu_m H \cos\theta}{kT}\right) d\theta.$$

$$M = n \mu_m \frac{\int_0^{\pi} \cos\theta \sin\theta \exp\left(\frac{\mu_0 \mu_m H \cos\theta}{kT}\right) d\theta}{\int_0^{\pi} \sin\theta \exp\left(\frac{\mu_0 \mu_m H \cos\theta}{kT}\right) d\theta}.$$

replacing c by (1).

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$$\beta = \frac{\mu_0 \mu_m H}{kT}$$

$$= n\mu_m \frac{\int_0^{\pi} \cos\theta \sin\theta \exp(\beta\cos\theta)\, d\theta}{\int_0^{\pi} \sin\theta \exp(\beta\cos\theta)\, d\theta}$$

$$= n\mu_m L(\beta) \quad \text{Langevin funct}^n$$

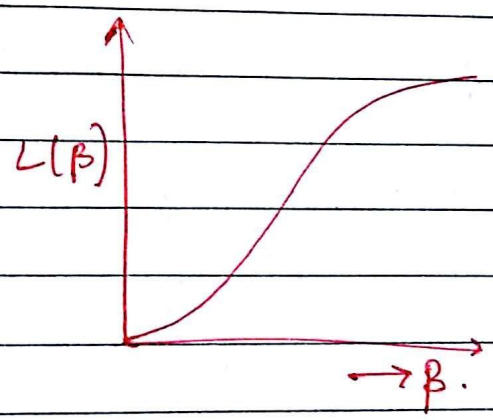
$$= n\mu_m \left(\coth\beta - \frac{1}{\beta}\right)$$

for small field and large temp.

$$\beta = \frac{\mu_0 \mu_m H}{kT}$$

$$L(\beta) \approx \beta/3.$$

$$M = \frac{n\mu_m^2 \mu_0 H}{3kT}.$$



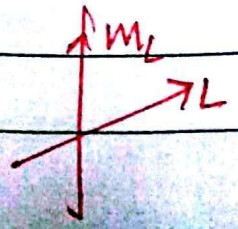
$$\chi = \frac{M}{H} = \frac{n\mu_m^2 \mu_0}{3kT}$$

$$= \frac{A}{T} \qquad A = \frac{n\mu_m^2 \mu_0}{3k}$$

$$\chi \propto \frac{1}{T}$$ Curie's law.

Quantum consideration:

θ will not take all values b/w 0 & π but only discrete values.



$m_L \rightarrow -L, -L+1, \ldots L,$

$(2L+1)$ values.

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Suppose, no contribution of orbital momentum, only spin contr. $\rightarrow -\frac{1}{2}$ or $\frac{1}{2}$

$m_s \rightarrow -\frac{1}{2}, \frac{1}{2}$

$\uparrow\downarrow$

$N_1 \rightarrow$ spin up

$N_2 \rightarrow$ spin down.

$M = (N_1 - N_2)\mu_B$.

$\mu_B \rightarrow$ Bohr magneton.

General formula $\leftarrow$ $\boxed{\bar{\mu} = -g\mu_B \bar{J}}$

$g \rightarrow$ Lande g-factor

$\bar{J} \rightarrow$ Total angular momentum.

$\bar{J} = \bar{L} + \bar{S}$ (orbital, spin)

when $L = 0$

$\bar{J} = \bar{S}$

(when only single e^-)

$\bar{\mu} = -2 \times \mu_B \left(\frac{1}{2}\right)$

$= -\mu_B$.

$\uparrow\downarrow \leftarrow E_1 = \mu_B \cdot B$

Parallel $\leftarrow E_2 = -\mu_B \cdot B$

$M = (N_1 - N_2)\mu_B$

N_1: No. of e^- along dir^n of field

N_2: no. of e^- oppo. to field.

3/11 Ferromagnetic Materials:

$\uparrow\uparrow\uparrow\uparrow$ (field originates on its own)

$H_{int} = H_m = 10^9$ A/m

$H_{int} \rightarrow$ internal, $H_m \rightarrow$ molecular

H_e = External field

$H_T = H_m + H_e$.

$H_T \rightarrow$ Total.

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Molecular field theory:

$$H_m = \gamma M$$

γ → constant, M → magnetisation.

$$\chi = \frac{M}{H_T} = \frac{M}{H_e + H_m} = \frac{M}{H_e + \gamma M}$$

$$= \frac{C}{T} \quad \text{(curie's Law)}$$

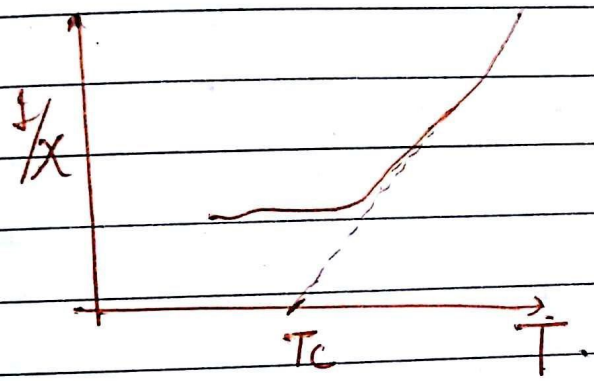
$C \rightarrow$ const.

$$M = \frac{H_e C}{T - \gamma C}$$

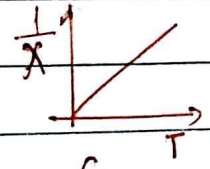
Curie-Weiss Law.

$$\chi_F = \frac{M}{H_e} = \frac{C}{T - \gamma C} = \frac{C}{T - T_c} \quad (T > T_c)$$

$T_c \; (= \gamma C)$ - ferromagnetic critical Temp.



$T > T_c$.



(paramagnetic)

$T > T_c \Rightarrow$ Paramagnetic.

$T \leq T_c \Rightarrow$ ferromagnetic.

When $H = 0$.

Param → $M = 0$

ferro. → $M_s = N \mu_B$

M_s → saturation (max.), N → No. of atoms, μ_B → magnetic moment of each atom

Why T_c comes into picture? (i.e. $\uparrow M$ for $T < T_c$)

$M = n\mu_m L(\beta)$ $\qquad \beta = \frac{\mu_0 \mu_m H_T}{k_B T}$

Langevin fn

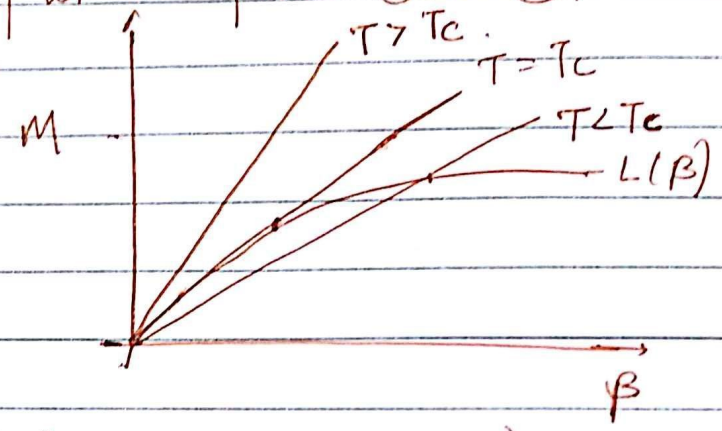
$H_T = H_e + H_m$ $\qquad H_e = 0$

$= H_m$ $\qquad$ (no field applied)

$$M = n\mu_m L(\beta) \quad \text{—(1)}$$

$$\beta = \frac{\mu_0 \mu_m H_m}{k_B T} = \frac{\mu_0 \mu_m \gamma M}{k_B T} \quad \text{—(2)}$$

Plot M from (1) & (2).



$L(\beta) \approx \frac{\beta}{3}$ (Look back).

$M \approx \frac{n\mu_m \beta}{3}$ $\qquad M_s = n\mu_m$.

for $T = T_c$, match the two slopes.

$$\frac{k_B T_c}{\mu_0 \mu_m \gamma} = \frac{n\mu_m}{3}$$

$$M_s = n\mu_m = \frac{3M}{\beta} = \frac{3k_B T}{\mu_0 \mu_m \gamma} \quad \text{(from (2))}$$

$$\Rightarrow \frac{M_s}{3} = \frac{k_B T}{\mu_0 \mu_m \gamma}$$

$$\gamma = \frac{3k_B T}{\mu_0 \mu_m M_s}$$

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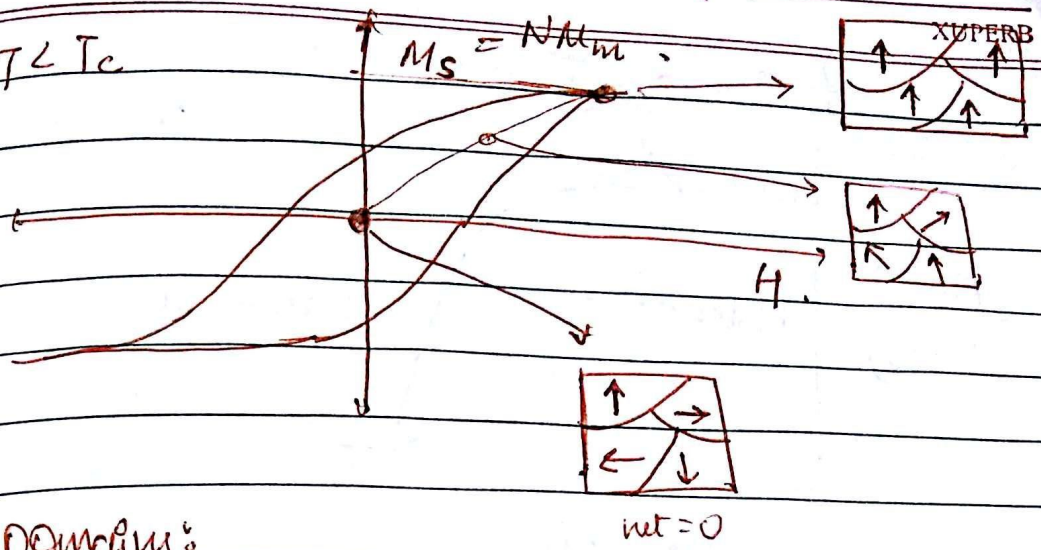
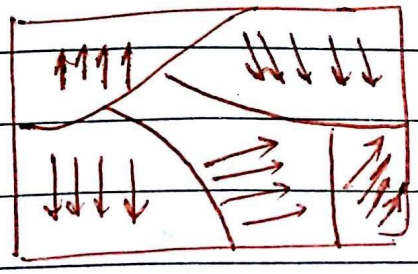
$T < T_c$



Domains:



eg. for iron, $M_s = 1.75 \times 10^6$ A/m, $d = 7.86$ g/cm^3

$M_{atom} = 55.8$

Avg. magnetic moment per atom?

$$N = \frac{d\, N_0}{M_{atom}} = \frac{7.86}{55.8} \times 6.023 \times 10^{23} \ /\text{cm}^3$$

$$M_s = N\mu_m$$

$$\mu_m = \frac{M_s}{N} = \left(\frac{M_s}{N\mu_B}\right)\mu_B$$

$$= \left(\frac{1.75 \times 10^6 \times 55.8}{7.86 \times 6.023 \times 10^{23} \times 9.24 \times 10^{-24}}\right)\mu_B$$

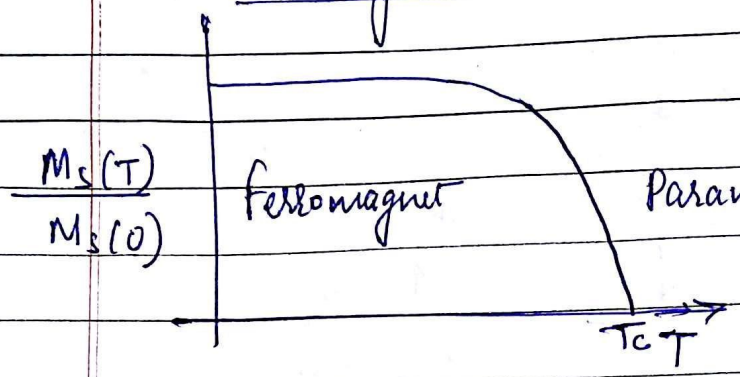
$$= 2.2\ \mu_B$$

fractional? ⇒ Band picture.

7/11

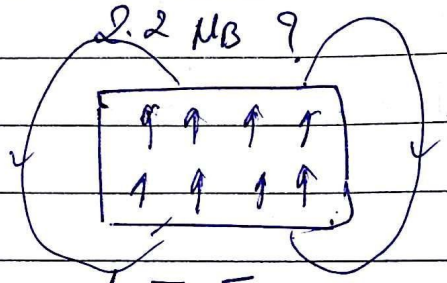
Ferromagnetism :

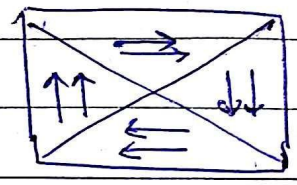
(No ext. field)



$$\mu_B = \frac{e\hbar}{2m}$$

2.2 μ_B ?





$\frac{1}{2}\bar{B}\cdot\bar{H}$: magnetostatic energy.

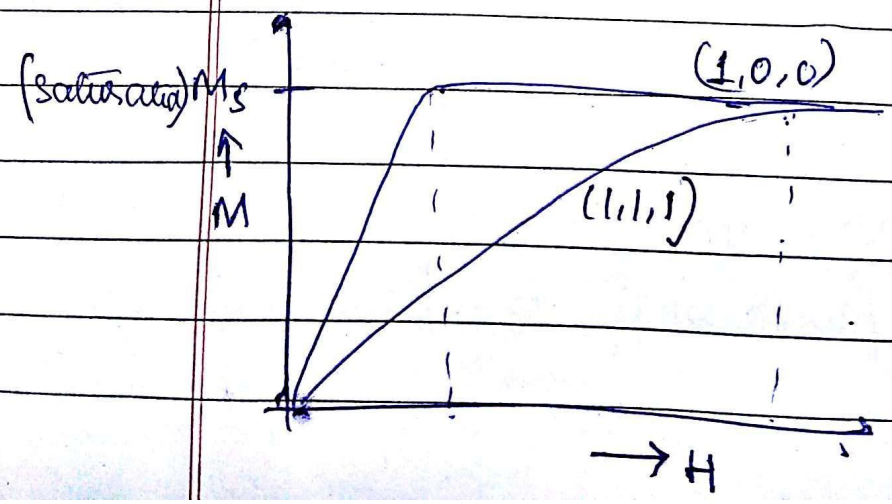
$B = 0$.

Domains are formed to minimize the magnetostatic energy.

Exchange energy
Anisotropic energy.

Magneto anisotropic energy :

diff^n properties in diff^n dir^n.



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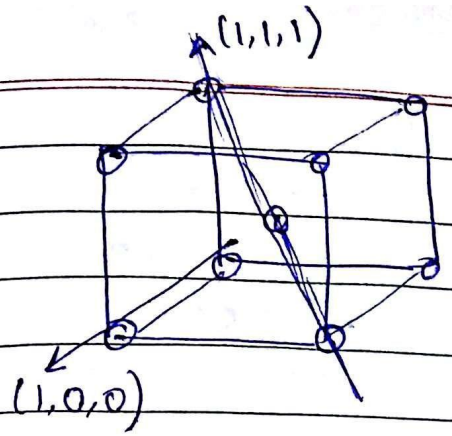
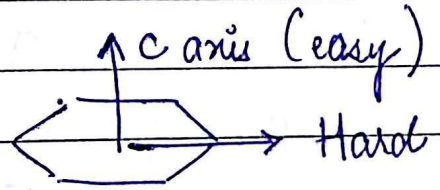


(1,0,0) : Easy dir^n.
(1,1,1) : Hard dir^n
↓
more field req. to saturate

Magneto anisotropic energy = Energy required to shift magnetic moment from easy to hard dir^n.

Fe K (anisotropic energy) = 48 mJ/cm^3
Co K = 480 mJ/cm^3.

Co:



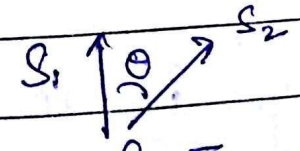
Exchange Energy:

$\bar{S}_1$, $\bar{S}_2$: magnetic moments.

$$\boxed{E = -2J\,\bar{S}_1 \cdot \bar{S}_2}$$

↳ Exchange integral.

E will be min when $\bar{S}_1 \parallel \bar{S}_2$



$|\bar{S}_1| = |\bar{S}_2| = S$

$$E = -2JS^2\cos\theta \quad \theta \text{ is small.}$$

$$= -2JS^2\left(1 - \frac{\theta^2}{2}\right)$$

$$= -2JS^2 + JS^2\theta^2.$$

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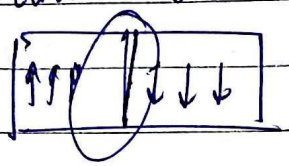
when $S_1 \parallel S_2$

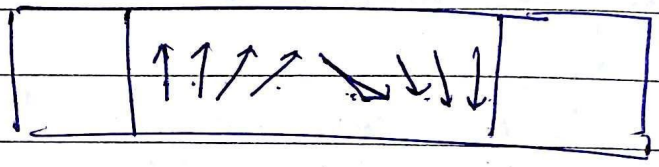
$$E = -2JS^2$$

$\therefore$ Energy for $0^\circ \longrightarrow \theta$

$$= JS^2\theta^2$$

Domain wall:





Suppose N steps (Here $N = 8$)

δ = Thickness of wall $= N \times a$.

$a \rightarrow$ lattice const.

~~w~~ Domain wall Thickness:

- Anisotropic energy $\equiv \sigma_{axis}$.
- Exchange energy $\equiv \sigma_{exchan}$

Total energy $= \sigma_{anis} + \sigma_{exch}$.

$\sigma_{axis} = KNa$ $\quad$ $K \equiv$ anisotropic energy const.

$\rightarrow$ 0° to π.

N steps.

Suppose Each step (π/N).

Exchange energy in each step.

$$= JS^2\frac{\pi^2}{N^2}.$$

N such steps.

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Total exchange energy $= N \times \frac{J s^2 \pi^2}{N^2}$

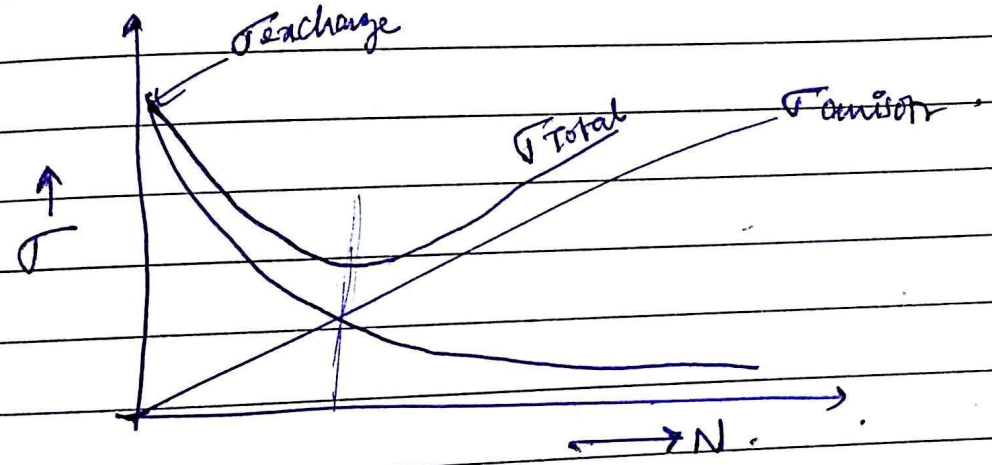
$= \frac{J s^2 \pi^2}{N}$

$\sigma_{exchange} = \frac{\text{Exchange energy}}{\text{Area}}$

Area $= a^2$

$= \frac{J s^2 \pi^2}{N a^2}$

$\sigma(\text{Total}) = \frac{J s^2 \pi^2}{N a^2} + kNa$



$\frac{\partial \sigma}{\partial N} = 0 \Rightarrow \frac{-J s^2 \pi^2}{N^2 a^2} + ka = 0$

$\Rightarrow N = \left(\frac{J \pi^2 s^2}{k a^3}\right)^{1/2}$

thickness $\delta = Na$

$= \left(\frac{J \pi^2 s^2}{ka}\right)^{1/2}$

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To write info., we can use diff^n directions of magnetisation.

Eg:

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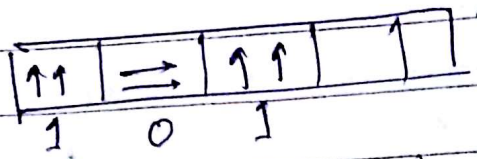


→ Eg: (magnetic clip in cards)

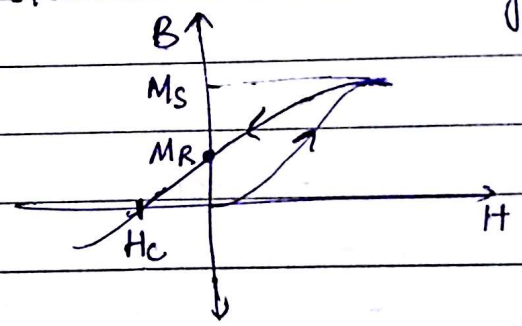
easy ↑ → hard (suppose)

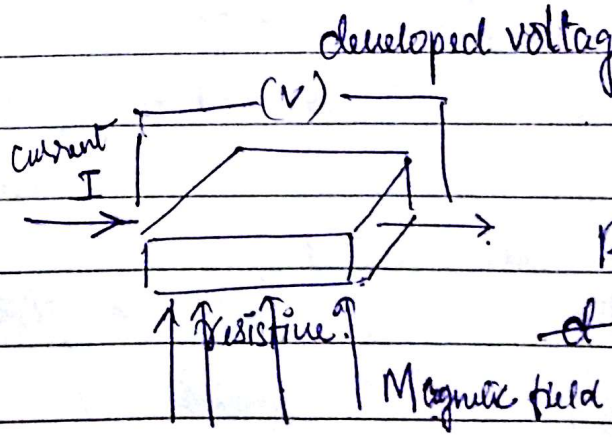
Energy req. for hard → easy too.

* GMR - Giant Magneto resistance (sensors to read info.)
(v. small)

When thermal energy > anisotropic energy, or ext. magnetic field > Hc, the info. stored can be changed or lost.

6/

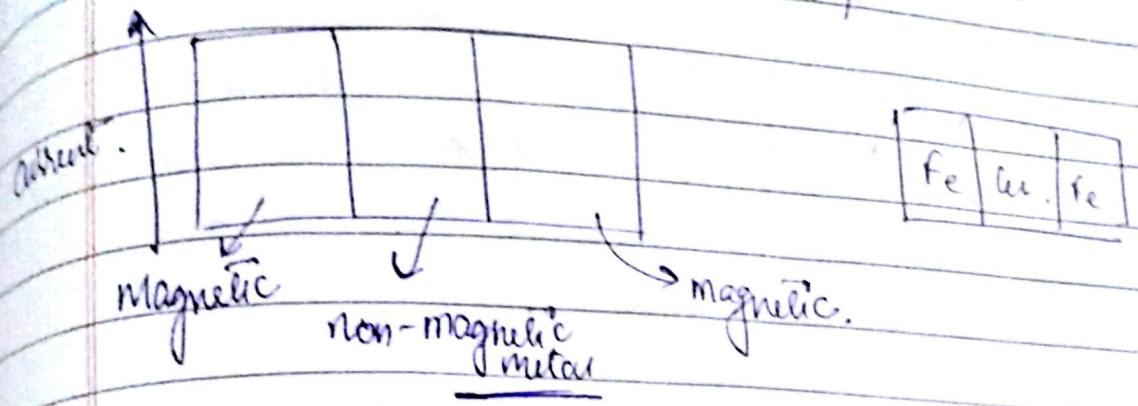


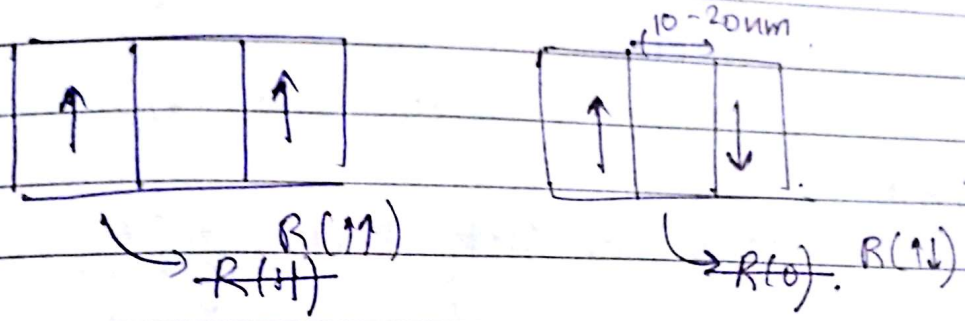


R : R(0) → R(H)
when 0 magnetic field

R(H): Resistance when magnetic field is appli

Resistance ratio $= \frac{R(H) - R(0)}{R(0)} \times 100\%$

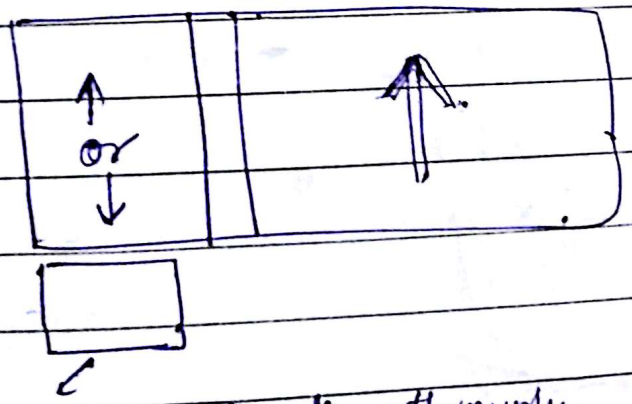


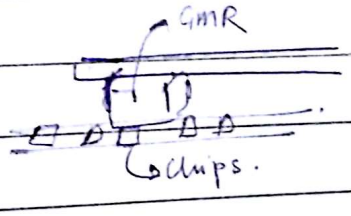


MRR $= \frac{R(\uparrow\uparrow) - R(\uparrow\downarrow)}{R(\uparrow\downarrow)} \rightarrow 99\%$

magneto-resistance ratio

(video)





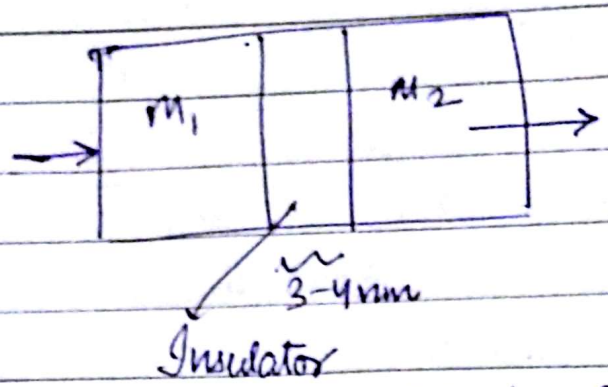
magnetic chips flows through.

either ↑ or ↓.

This changes the value of resistance, hence we can read the info.

Tunneling phenomenon:

TMR: Tunneling magneto resistance.



e^- Tunnelling from M_1 to M_2.

$$TMR = \frac{R(\uparrow\uparrow) - R(\uparrow\downarrow)}{R(\uparrow\downarrow)}$$

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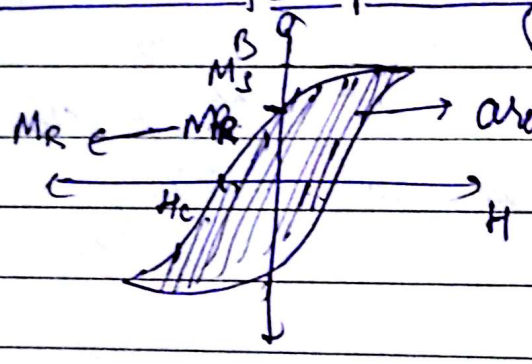
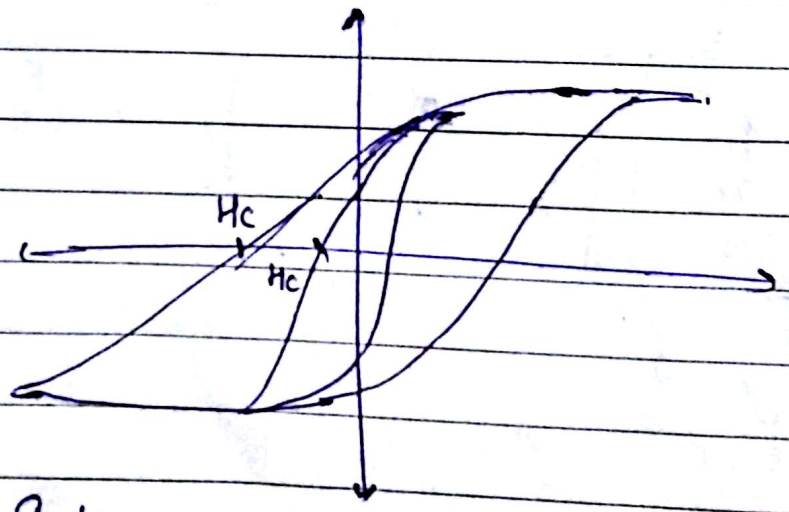
Hard and Soft ferromagnetic Materials:

6/



area: heat energy lost.



Soft magnetic material: Hc is small.
area of BH loop small.
Small magnetic field req. to magnetise and demagnetise the material.
ferrites, Fe, Fe Si.

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- Soft mag. material used for storing ~~to~~ info. (∵ less field req.)
- Also used as core of transformer (∵ less energy loss).

Hard magnetic material: Large Hc (can be order of Tesla).

area of BH loop is large.

We use it as permanent magnets as large field is req. to magnetise / demagnetise hence we do not care about losses.

High freq. transformers use this.

When hammered, shockwaves are generated which change the orientation hence magnetic prop. lost / fades.

How ferromagnetism arise?

↑ ↑ ↑ ↑ $T < T_c$.

$H_{internal}$ ~ few Tesla

Dipole field ~ kiloOersted. kOe.

Exchange Theory:

$\bar{S}_1 \uparrow \quad \nearrow \bar{S}_2$

interaction energy

$$E = -2J\,\bar{S}_1 \cdot \bar{S}_2$$

↳ exchange integral.

System will try to minimize energy.

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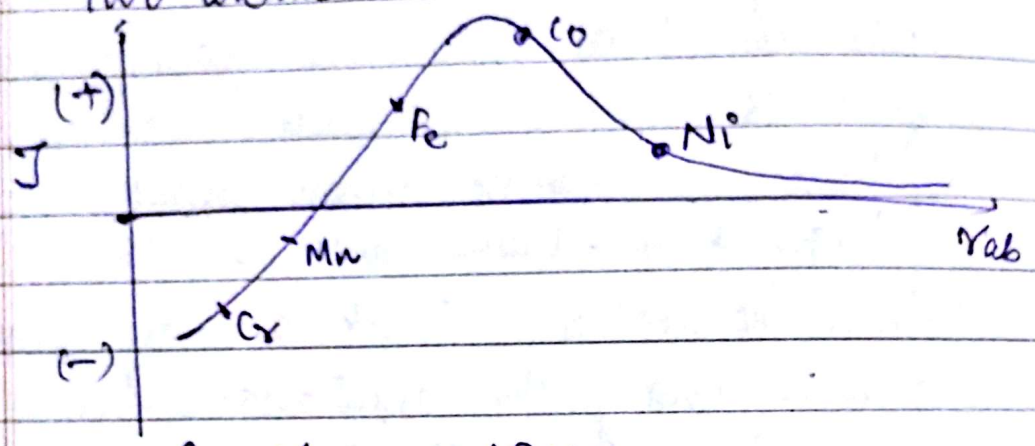
if $J > 0$: $\theta = 0$ ↑↑↑ : Ferromagnetic.

$J < 0$: $\theta = \pi$ ↑↓ : Antiferromagnetic.

One needs to find value of J for a material. It depends on distⁿ b/w two atoms r_{ab}.



Cr, Mn → AFM.
Fe, Co, Ni → FM.

MnO – AFM
Fe_2O_3 – FM
$CoFe_2O_4$ – FM

distⁿ b/w. Mn is large. At large r_{ab}, $J \to 0$.

These cannot be explained based on direct exchange theory.

Paramagnetism:

$$\chi \propto \frac{1}{T}$$

$$M = \frac{N \mu_{eff}^2 \mu_0 H}{3kT}$$

metal: N free e^-s (each e^- has mag. moment = μ_B).

$N\mu_B$.

$$\chi = \frac{M}{H} = \left(\frac{N\mu_B^2\mu_0}{3k}\right)\frac{1}{T}$$

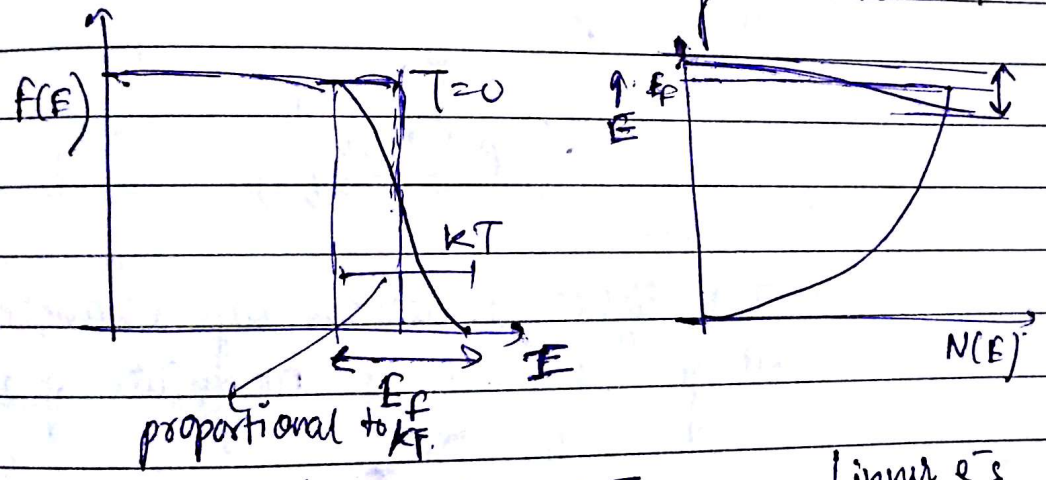
$$\chi \propto \frac{1}{T}$$

But, experimentally,

$\chi_{exp.}$ is independent of temp.

Sommerfield theory: fermi-dirac dist.

e^- near fermi level only contributes.



$$N' = \frac{N}{E_f} \propto kT.$$

e^- near fermi-level participate.

$N'\mu_B$.

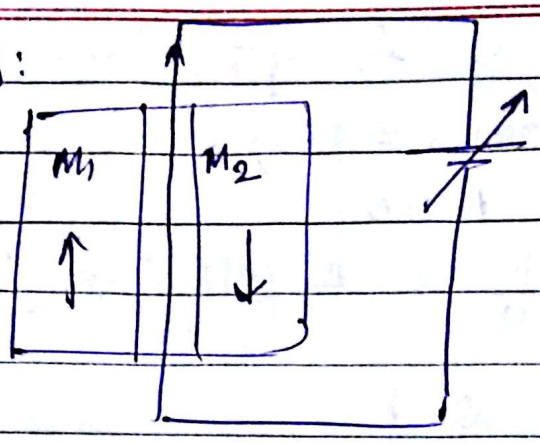
$$M = \frac{N'\mu_B^2\mu_0 H}{kT}$$

$$= \frac{N\mu_B^2\mu_0 H \propto kT}{E_f\, kT}$$

$$\chi = \frac{M}{H} = \frac{N\mu_B^2\mu_0 \propto}{E_f} = \text{const.}$$

inner e^-s are oriented nicely: their fields get cancelled.

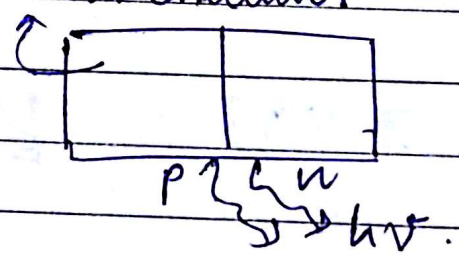
GMR:



flow of current can be controlled by applying a voltage source or by changing magnetic field.

LED:

Magnetic semiconductor



6/

control light emission by changing voltage as well as magnetic field.

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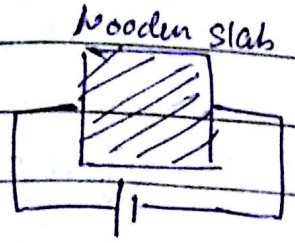
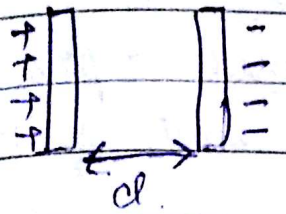
12/11

Dielectric - Insulator.



no current flow.
what happens then?
Polarisation.



$$C_E = \frac{q}{V} = \frac{\varepsilon_0 A}{d}$$

C_E : Capacitance (empty space)

$$C = \frac{\varepsilon_r \varepsilon_0 A}{d}$$

$\varepsilon_r = \frac{C}{C_E}$ = relative permitivity
= dielectric const.

ε_0 = permitivity of empty space.
$= 8.85 \times 10^{-22}$ F/m.

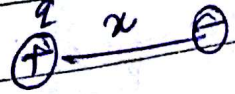
Dielectric displacement:

$$D = \varepsilon E = \varepsilon_r \varepsilon_0 E.$$

ε_r : not large - Si $\varepsilon_r = 11$
Ge $\varepsilon_r = 20$
Air $\varepsilon_r = 1.1$

very large - Ferro~~magnetic~~ electric material, $BaTiO_3$, PZT (lead zirco
(6000 - 10,000)

Bound charges will try to displace →
polarisation

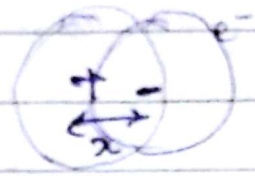


$p = q \cdot x$

1. Electronic polarisation —

centre of gravity of ⊕ and ⊖ coincide when no field.

$p = 0$ when field applied

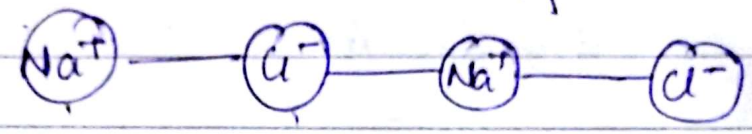


$p = q \cdot x$

Electronic dipole

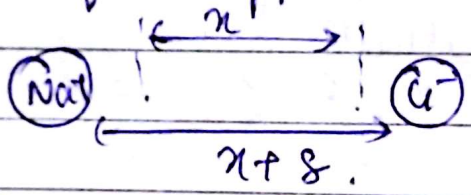
(due to displacement of e^- cloud)

2. Ionic solids — NaCl, KCl.



C.G. of (+) and (−) coincide.

When field applied:



$p_{ion} = \delta q$.

(Small displacement)

$D = \varepsilon E = \varepsilon_r \varepsilon_0 E$.

P = induced dipole moment / vol.

$D = \varepsilon_0 E + P$.

$E = \text{coulm}/m^2$

$$P = \frac{p}{vol}.$$

$D = \varepsilon_0 E + P$

$\varepsilon E = \varepsilon_0 E + P$.

$\varepsilon_r \varepsilon_0 E = \varepsilon_0 E + P$.

$P = \underbrace{(\varepsilon_r - 1)}_{\chi_e} \varepsilon_0 E \Rightarrow P = \chi_e E \varepsilon_0$

χ_e : electric susceptibility.

magnetic — electric.

$H \rightarrow E$.

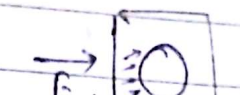
$M \rightarrow P$.

$\chi_m \rightarrow \chi_e$.

$p_{induced} \propto E_{local}$

$p_{ind.} = \alpha E_{local}$.

↳ polarizibility.



$$E_{local} = E + E_{induced}$$

applied — (created by dipoles) — $\frac{P}{3\varepsilon_0}$

$$\boxed{E_{local} = E + \frac{P}{3\varepsilon_0}}$$ → polarisat^n / vol.

$$P = N p_{induced} = \alpha N \left(E + \frac{P}{3\varepsilon_0}\right)$$

N → no. of dipoles per unit vol.

($p_{induced} = \alpha E_{local}$)

$$P = (\varepsilon_r - 1)\varepsilon_0 E.$$

$$(\varepsilon_r - 1)\varepsilon_0 E = \alpha N \left(E + \frac{(\varepsilon_r - 1)\varepsilon_0}{3}\right)$$

$$\boxed{\frac{\varepsilon_r - 1}{\varepsilon_r + 2} = \frac{N\alpha}{3\varepsilon_0}}$$

★ Claussius – Mossitti Relation.

relates this two.

macroscopic property – ε_r

micros– – α

α = polarizability.

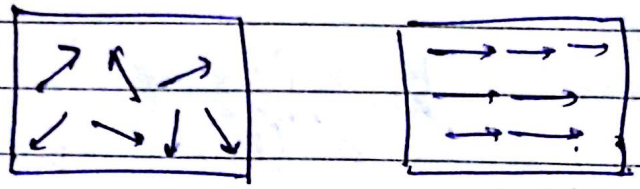
$$= \alpha_e + \alpha_I + \alpha_o$$

electronic ionic orientation.

α_o - ?

Polar materials: (already a dipole exists)

H_2O, HCl.



$p = 0$. electric field orients the dipole.

Eg:

6/ Si → only α_e H_2 - α_e

NaCl → α_e, α_I.

H_2O → α_e, α_o.

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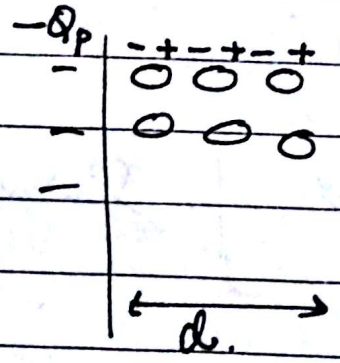
Polarization.

$$P = \frac{\Sigma p_i}{V_\Omega}$$



→ Polarized charge

$$P = \frac{p_{total}}{V_\Omega}.$$

$$= \frac{Q_p \cdot d}{A \cdot d}$$

$$= \frac{Q_p}{A} = \sigma_p$$

Surface polarized charge density

$$P = \chi_e \varepsilon_0 E$$

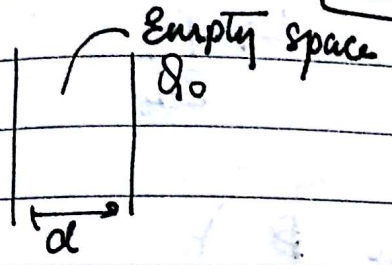
$$p = \alpha_e E$$

polarizibility

$$P = Np$$

$$= N\alpha_e E = \chi_e \varepsilon_0 E$$

$$\Rightarrow \boxed{\chi_e = \frac{N\alpha_e}{\varepsilon_0}} \quad \text{— (1)}$$



$$E = \frac{V}{d} = \frac{Q_0}{\varepsilon d}$$

$$= \frac{Q_0}{\frac{A\varepsilon_0}{d} \times d} = \frac{Q_0}{A\varepsilon_0}$$

$$= \frac{\sigma_0}{\varepsilon_0}$$

$\sigma_0 \equiv$ surface free charge density

$$\sigma = \sigma_0 + \sigma_p$$

$D \equiv$ dielectric displacement

$$D = \varepsilon E = \varepsilon_r \varepsilon_0 E$$

$$D = \varepsilon_0 E + P$$

$$\varepsilon_r \varepsilon_0 E = \varepsilon_0 E + P$$

$$\Rightarrow P = (\varepsilon_r - 1)\varepsilon_0 E = \chi_e \varepsilon_0 E$$

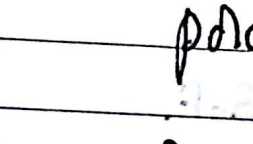
$$\Rightarrow \boxed{\chi_e = \varepsilon_r - 1} \quad \text{— (2)}$$

from (1) and (2),

$$\boxed{\varepsilon_r = 1 + \frac{N\alpha_e}{\varepsilon_0}} \rightarrow (*1)$$

But

Inside of a material, field is not uniform. ($\neq E_{external}$).



Experiences external field as well as field due to polarisation (induced).

$E_{loc} \rightarrow$ E at local point in the dielectric

$$E_{loc} = E + E_{induced}$$

$$\hookrightarrow \frac{P}{3\varepsilon_0}$$

$$p_{ind.} = \alpha_e \cdot E_{local}$$

$$= \alpha_e \left(E + \frac{P}{3\varepsilon_0}\right)$$

$$P = N \cdot p_{ind.}$$

$$\Rightarrow (\varepsilon_r - 1)\varepsilon_0 E = N\alpha_e \left(E + \frac{(\varepsilon_r - 1)\varepsilon_0 E}{3\varepsilon_0}\right)$$

$$\Rightarrow \boxed{\frac{\varepsilon_r - 1}{\varepsilon_r + 2} = \frac{N\alpha_e}{3\varepsilon_0}} \longrightarrow (*2)$$

In (*1) $\rightarrow$ we assumed only external field eg. in gas molecules, less density, hence no pronounced effect.

(*2) $\rightarrow$ Eg: in solid (compact), effect of $E_{ind.}$ is considered.

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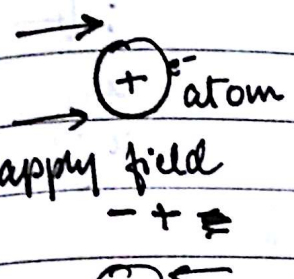
1. Electronic polarization.

$$\alpha_e = \frac{Ze^2}{m_e \omega_0^2}$$

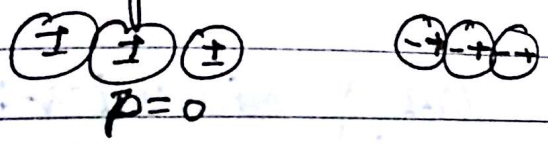
$Z \equiv$ atomic no.
$\omega_0 =$ resonant freq. ($\sim$ uv region)



(if freq > natural freq., dipole won't respond)

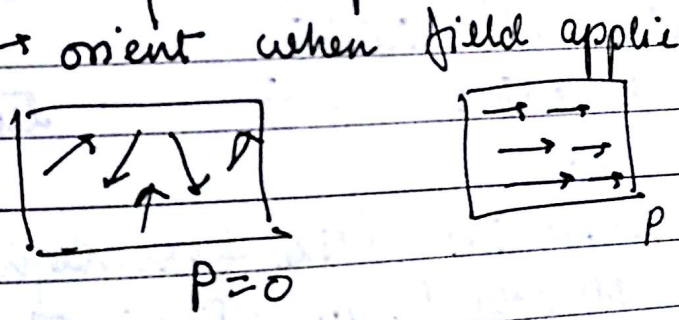
(keep on reversing field dir^n).

2. Ionic polarization.

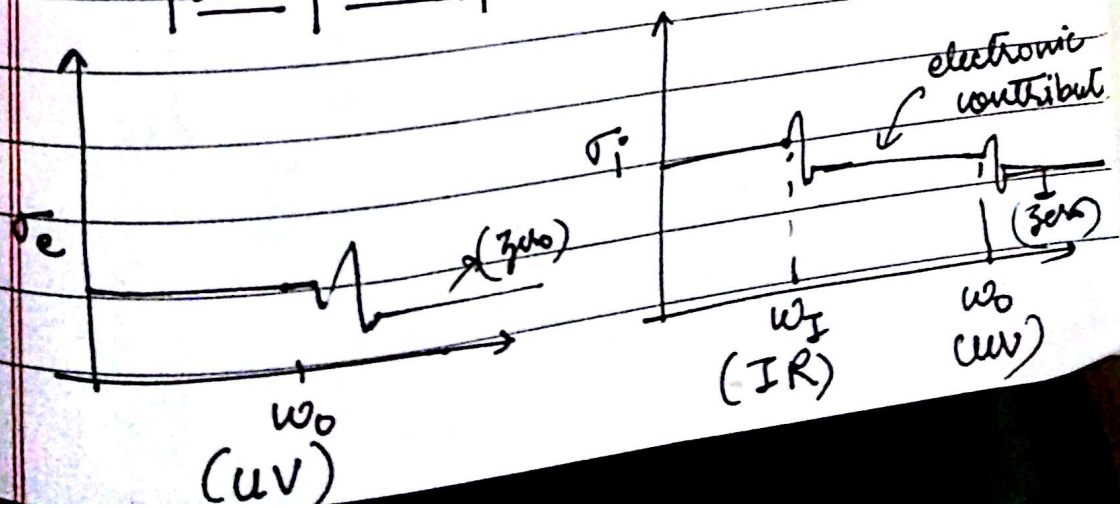


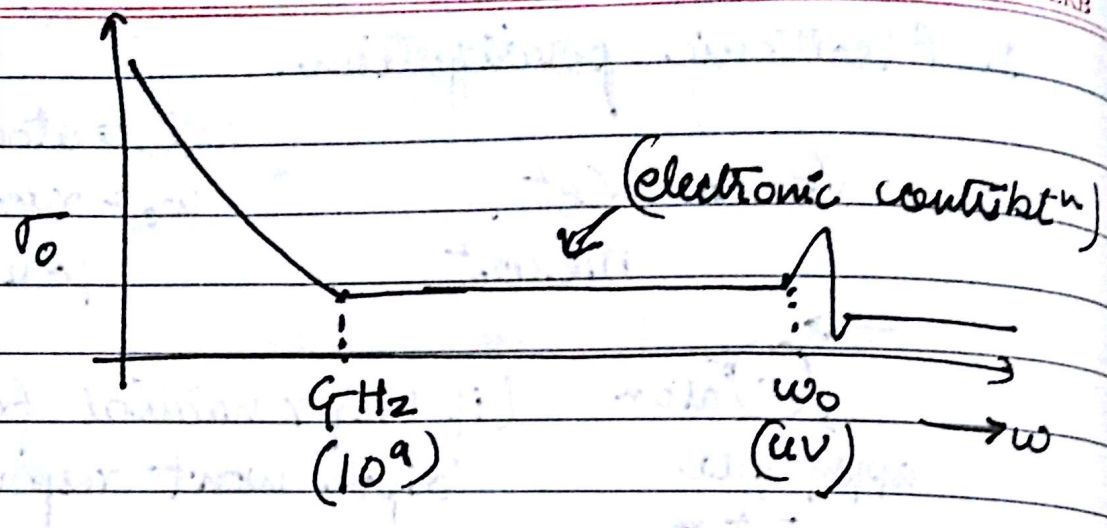
3. Orientation polarization.

already the dipole exists.
→ orient when field applied



Frequency Response of Dielectric





GHz → 10^9
IR → 10^{14}
UV → 10^{16}

Special class of Dielectrics:

6/ 1. Piezoelectric: induced polarization by applying pressure.
(Eg: Gas lighter)

2. Pyroelectric: ——— temperature.
Eg:

3. ferroelectric: (like ferromagnetic).
Already have polarisation.
(memory).

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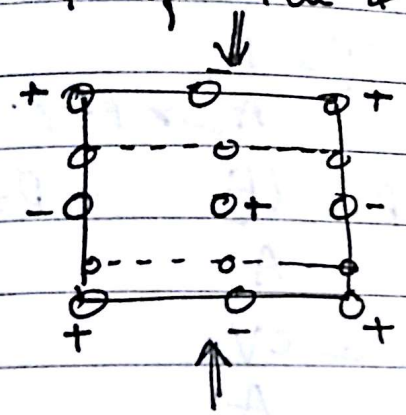
1. **Piezoelectric** : Quartz, $BaTiO_3$, PZT

Stress → polarization

Apply voltage → stress

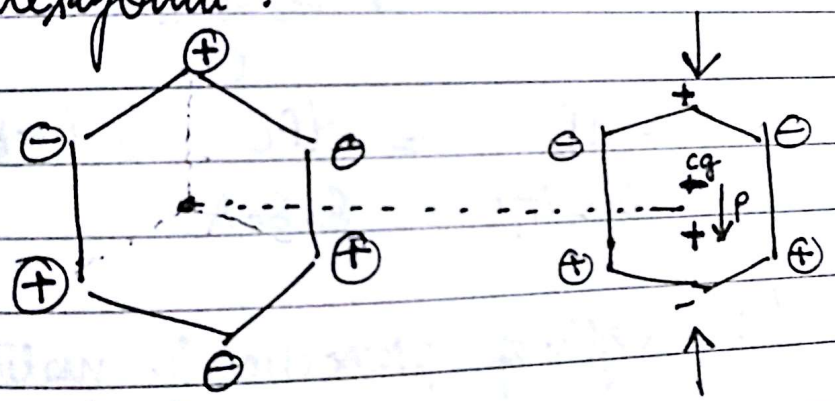
✓ Non centrosymmetric crystal structure

(Centrosymmetric – when stress applied, c.g. of +ve & −ve charge not changed) (cubic)



P = 0

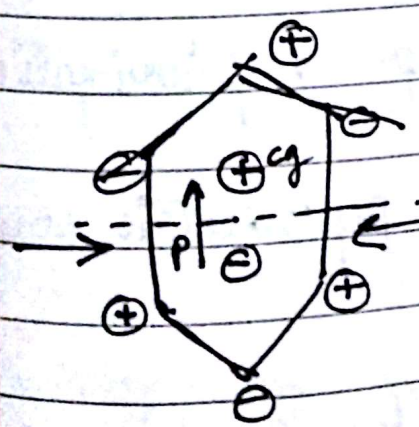
hexagonal :



P ≠ 0

No stress

P = 0



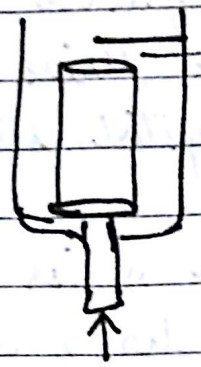
P ≠ 0

$P \propto T$ $T \equiv$ stress

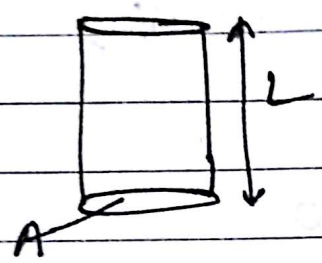
$P = dT$

↑ piezoelectric coefficient

Gas lighter:



→ very less distn.
very high voltage produced.
(corona discharge ?)



$$T = \frac{F}{A}$$

$$P = \frac{dF}{A} \quad \text{(d → piez. constant)} \qquad P = \frac{Q}{A} = \frac{CV}{A}$$

$$\frac{dF}{A} = \frac{CV}{A}$$

$$\Rightarrow V = \frac{dF}{C}$$

6

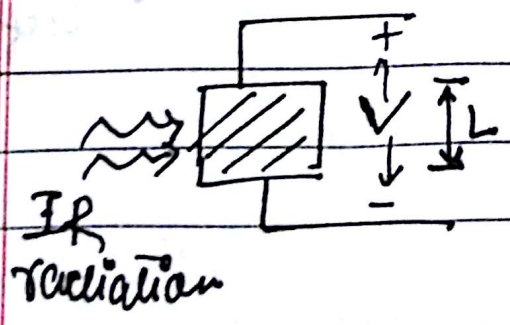
$$= \frac{dF}{\varepsilon_r \varepsilon_0 A/L} = \frac{dFL}{\varepsilon_r \varepsilon_0 A} \quad (\sim kV)$$

L : length of piezoelectric material.

(2) Pyroelectric:

Change in temp. → polarization

$$P_c = \frac{dP}{dT} \rightarrow \text{pyroelectric coeff.}$$



$$P = (\varepsilon_r - 1)\varepsilon_0 E$$

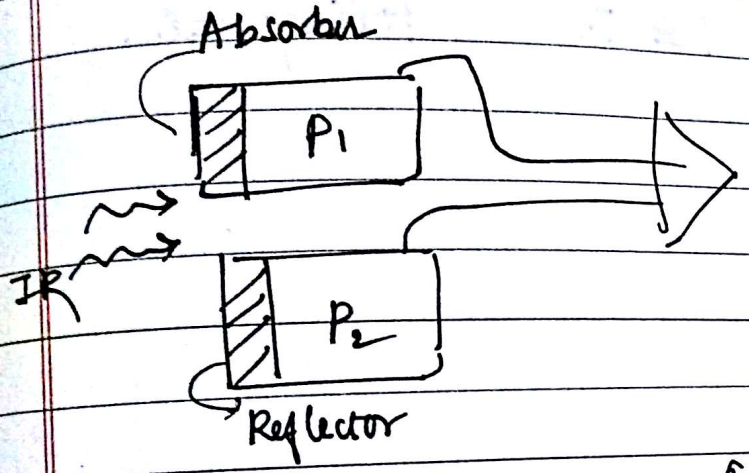
Scanned by CamScanner

$$\delta P = (\varepsilon_r - 1)\varepsilon_0 \delta E$$
$$= (\varepsilon_r - 1)\varepsilon_0 (\delta V / L)$$
$$p_c \cdot \delta T = (\varepsilon_r - 1)\varepsilon_0 \frac{\delta V}{L}$$

Eg. $\delta T = 10^{-3} K$ $\quad p_c = 380 \times 10^{-6} cm^{-2} K^{-1}$

$L = 0.1 mm$

$\delta V = 15 mV$



How is it used in practice?

P_1, P_2 – pyroelectric material

③ Ferroelectric:

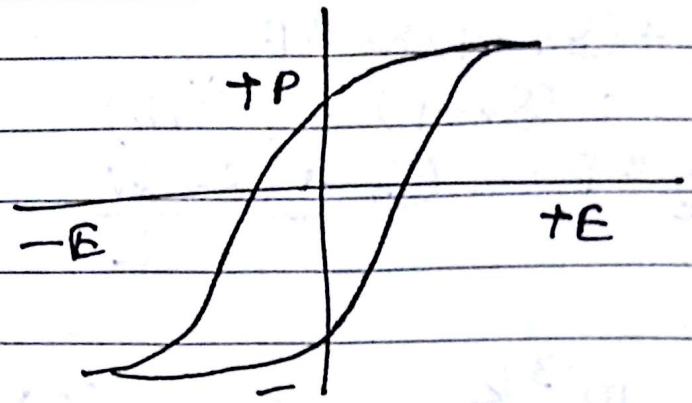
Spontaneous polarization.

T_c (critical Temperature)

$T \leq T_c \rightarrow P \neq 0$

ε_r very large.

By applying field in diff^n dir^n, we can polarize in diff^n dir^n.



$BaTiO_3$

$T > T_c$ cubic structure

$T \leq T_c$ tetragonal.

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