APL 104: Minor 2

Full Marks: 50 Duration: 1 hrs Date: 8^{th} Oct 2016

Problem 1: Combined inflation-torsion of hollow circular tubes: Often a hollow tube is twisted with uniform internal pressure (p) acting on it. Application of pressure leads to increase in radius of the tube which is charecterized as inflation of a tube. Assume that the tube is not allowed to stretch or compress axially.

(a) Explain under what conditions, the displacement function u_r only depends on r and $u_z = 0$ throughout the tube. You have learnt in the class that $u_\theta = r\frac{\theta_0}{L}z$ where θ_0 is the end-to-end relative rotation of the hollow tube of length L. (3)
(b) Find out the corresponding strain and stress matrix in terms of the displacement functions as in part (a). Assume the tube to be isotropic. (5)
(c) Obtain the differential equation for the unknown displacement u_r. (5)
(d) Obtain expressions for variation in σ_{rr} and $\sigma_{\theta\theta}$ as a function of tube's radius. Draw this variation qualitatively from inner radius r_1 to outer radius r_2. You may want to use the boundary condition that $\sigma_{rr} = -p$ at inner radius but vanishes at the outer radius. (6)
(e) Obtain expression for axial force that gets generated in the tube. (3)
(f) Can you find how much pressure would have to be applied if the tube's inner radius has to change from r_1 to λr_1? The constant $(\lambda - 1)$ is usually called the tube's inflation strain. (4)

Problem 2: Think of a composite rectangular beam formed by gluing together aluminium and steel beams. See Fig. 2 in the back for a typical cross-section of this beam and also its dimensions. Assume the Young's moduli of the Aluminium and steeel to be E_a and E_s respectively. Suppose the composite beam is bent into an arc of a circle such that the radius of its neutral line is R.

(a) How would the normal strain ϵ_{xx} and normal stress σ_{xx} vary in the tube's cross-section? Draw two separate graphs for their variations - only qualitatively. (4)
(b) Obtain the location of the neutral axis in the beam. (caution: it does not pass through the geometric centroid of the tube's cross-section). (2)
(c) What is its second moment of area I_{zz} about the neutral axis. (3)
(d) Obtain an expression for the bending moment required to bend the beam as above. (3)

Problem 3: For an I-beam, assume the beam is subjected to transverse load (see Fig. 3 on the back).

(a) Obtain an expression for variation in shear stress τ_{xy} within its cross-section. You can use the formula $\tau_{xy} = \frac{VQ(y)}{I_{zz}T(y)}$. (6)
(b) Using the expression above, draw a graph depicting qualitative variation in shear stress within the cross-section. (3)
(c) Where is the shear stress maximum? Find the ratio of the maximum shear stress to the average shear stress in the cross-section? (3)