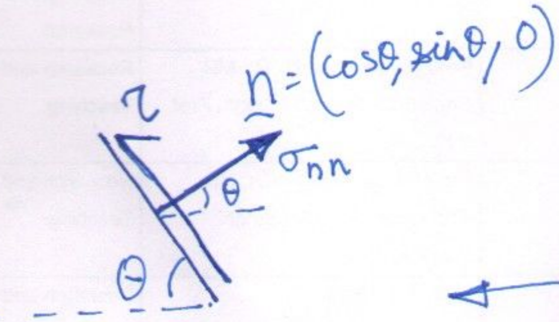
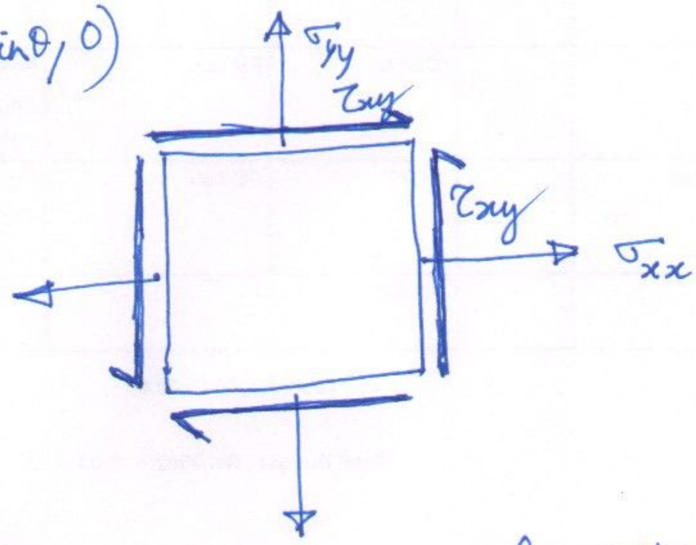
Mohr's circle concept

* Used to determine normal and shear component of traction on an arbitrary plane in case of plane stress condition!

$$\underline{\underline{\sigma}} = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} & 0 \\ \sigma_{xy} & \sigma_{yy} & 0 \\ 0 & 0 & 0 \end{bmatrix} \rightarrow \left(\text{Mohr's circle works even if } \sigma_{zz} \neq 0 \right)$$





What is traction on a plane whose normal makes an angle 'θ' with 'x' axis.

$$\underline{n} = \begin{bmatrix} \cos\theta \\ \sin\theta \\ 0 \end{bmatrix} \Rightarrow \underset{\sim}{t}_n = \underline{\underline{\sigma}}\,\underline{n} =$$

$$\text{So, } \sigma_{nn} = (\underline{\underline{\sigma}}\,\underline{n})\cdot\underline{n} = \sigma_{xx}\cos^2\theta + \sigma_{yy}\sin^2\theta + 2\tau_{xy}\sin\theta\cos\theta$$

$$\tau = (\underline{\underline{\sigma}}\,\underline{n})\cdot\underline{m} = \left(\begin{bmatrix} & & \\ & \underline{\underline{\sigma}} & \\ & & \end{bmatrix} \begin{bmatrix} \cos\theta \\ \sin\theta \\ 0 \end{bmatrix} \right) \cdot \begin{bmatrix} -\sin\theta \\ \cos\theta \\ 0 \end{bmatrix}$$

Using trigonometry, we obtain $\Rightarrow$

$$\sigma_{nn} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\tau = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta$$

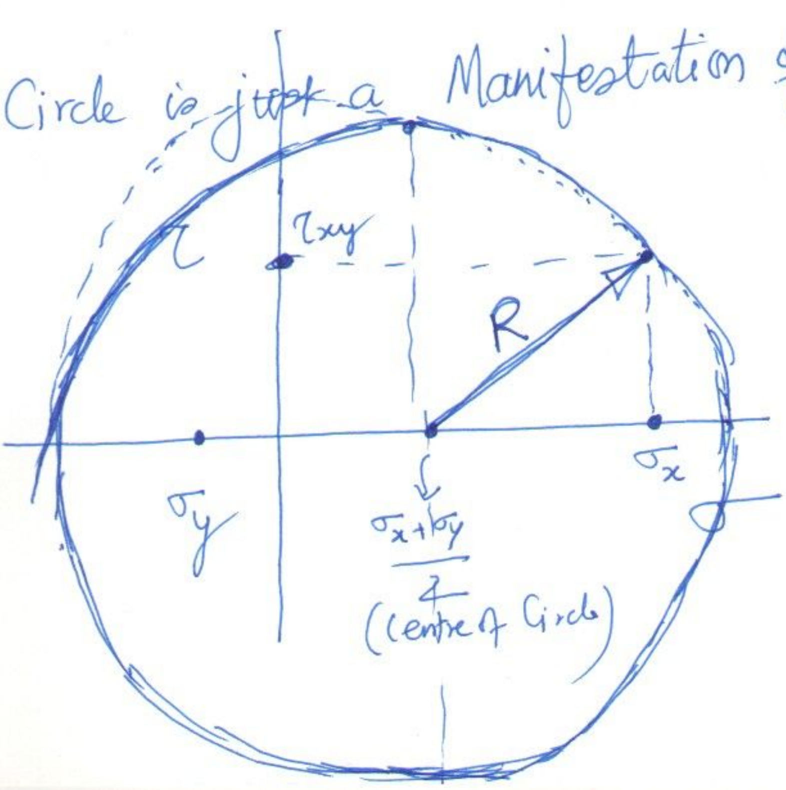
Let, $R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$ and an angle ϕ such that

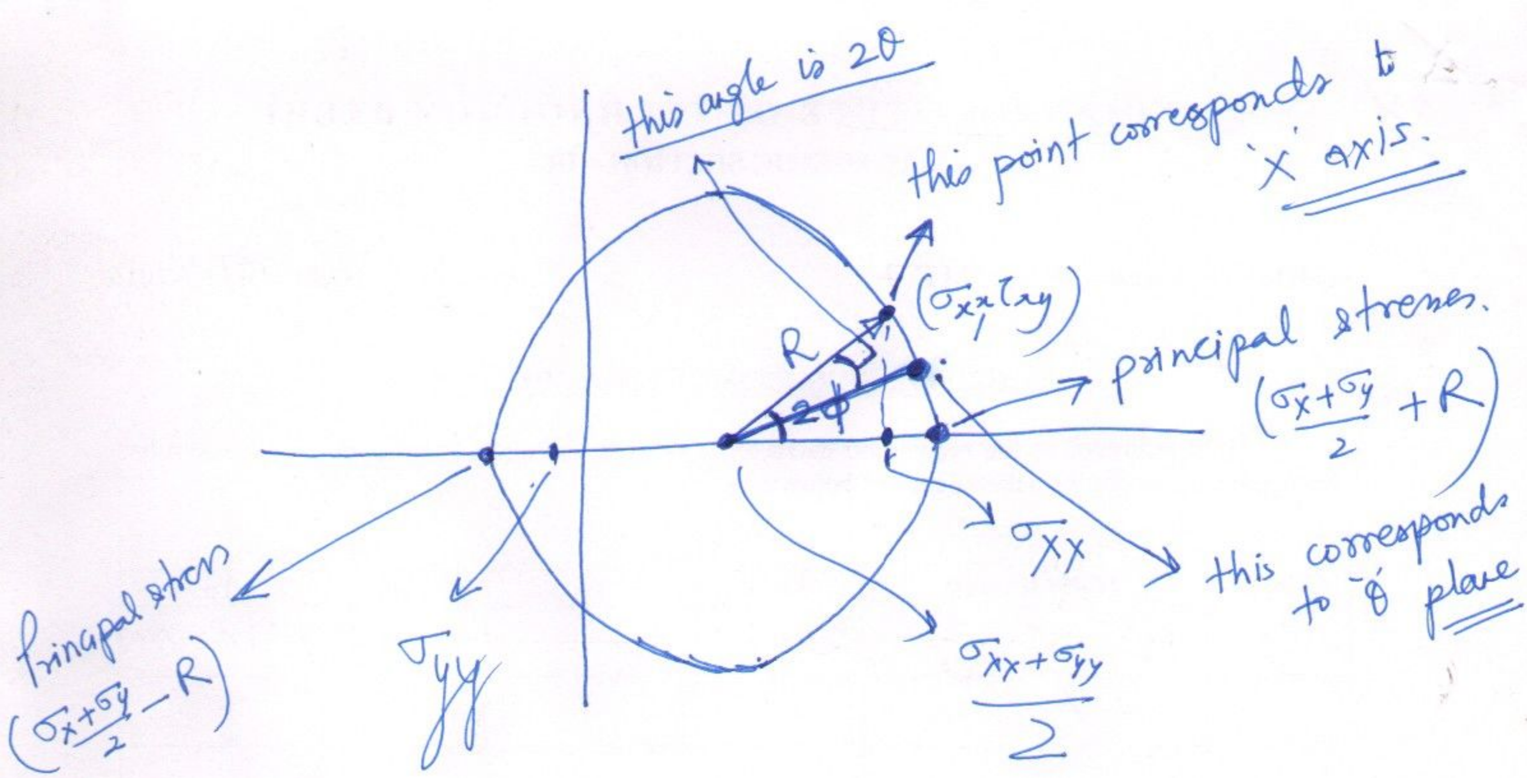
$$R\cos 2\phi = \frac{\sigma_x - \sigma_y}{2}, \quad R\sin 2\phi = \tau_{xy}$$

Substituting them in above eq.

$$\Rightarrow \begin{cases} \sigma_{nn} = \dfrac{\sigma_x + \sigma_y}{2} + R\cos(2\phi - 2\theta) \\ \tau = R\sin(2\phi - 2\theta) \end{cases}$$

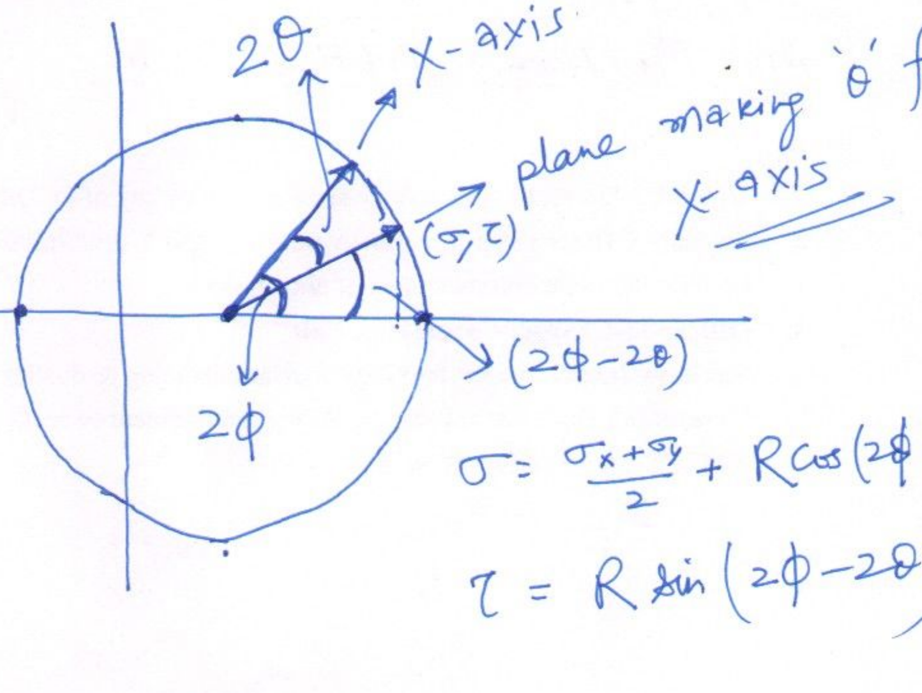
Mohr's Circle is ~~just a~~ Manifestation of these two equations





One can very easily read principal stresses and directions.

To obtain stress on a plane at an angle 'θ' from X axis, one just have to move in the opposite direction but by 2θ.



$$\sigma = \frac{\sigma_x+\sigma_y}{2} + R\cos(2\phi-2\theta)$$

$$\tau = R\sin(2\phi-2\theta)$$