

1 定义

u_n 收敛 v_n 收敛, 则 $u_n + v_n$ 收敛

$\sum_{n=1}^{\infty} u_n$ 收敛, $\sum_{n=1}^{\infty} v_n$ 发散, 则 $\sum_{n=1}^{\infty} (u_n \pm v_n)$ 发散 $\sum_{n=0}^{\infty} \frac{1}{n^a}$ $\begin{cases} a > 1 \text{ 收敛} \\ 0 < a < 1 \text{ 条件收敛} \\ a < 0 \text{ 发散} \end{cases}$

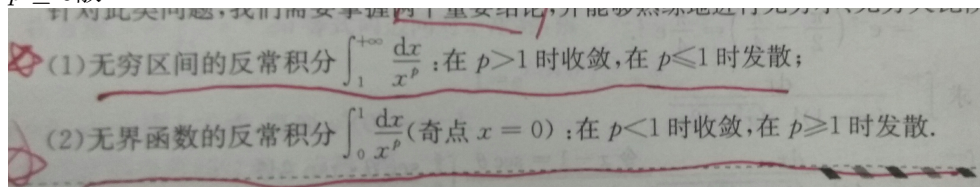
2 反常积分审敛

$$\sum \frac{1}{n^p}$$

$p > 1$ 绝

$0 < p \leq 1$ 条

$p \leq 0$ 散



3 收敛区间

缺项级数 $\lim_{n \rightarrow \infty} \left| \frac{u_{n+1}(x)}{u_n(x)} \right|$ 带上 x

完项级数不带 x

4 级数求和

$$\sum_{n=1}^{\infty} \frac{x^n}{n} = -\ln(1-x), x \in [-1, 1)$$

$$\sum_{n=1}^{\infty} nx^{n-1} = \frac{1}{(1-x)^2}, x \in (-1, 1)$$

5 傅里叶级数

$$f(x) \sim S(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos \frac{n\pi x}{l} + b_n \sin \frac{n\pi x}{l})$$

$$\begin{cases} a_n = \frac{1}{l} \int_{-l}^l f(x) \cos \frac{n\pi x}{l} dx (n = 0, 1, 2, \dots) \\ b_n = \frac{1}{l} \int_{-l}^l f(x) \sin \frac{n\pi x}{l} dx (n = 1, 2, 3, \dots) \end{cases}$$