CS 350: Homework #3

Due 3PM, Tuesday, February 23

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Question 1

Part(a)

We can use the formula that computes E to solve for the standard deviation...

$$E = \frac{\sigma}{\sqrt{n}} * Z_{\alpha/2}$$

$$0.4 = \frac{\sigma}{\sqrt{40}} * 1.96$$

$$2.5298 = 1.96 * \sigma$$

$$\sigma \approx 1.2907$$

Part(b)

The z-value associated with a 99% confidence interval is 2.576, so again we can use the following equation to solve for the interval.

$$2.576 * \frac{1.2907}{\sqrt{40}} = 0.5239$$

$$\textit{Confidence Interval} = [3 - 0.5239, 3 + 0.5239]$$

Part(c)

$$0.1 = 1.96 * \left(\frac{1.2907}{\sqrt{n}}\right)$$

$$0.1\sqrt{n} = 0.25298$$

$$\sqrt{n} = 25.30$$

$$n = 639.976 \approx 640$$

$$640 - 40 = 600 \textit{ more samples}$$

Question 2

Please see the 'hw3.py' file submitted with this write-up to view the Python source code I implemented for this problem.

Within the Python source code, you will find that I generated four different log files for part (a), part (b), part (c), and part(d) of this problem. Whenever you run the 'hw3.py' file, these logs should be regenerated. Please note that the first few times I ran my Python code, I had no issues generating the log files. At some point it unexplainably began generating empty text files though. If you experience this problem, please use the python console to view the logged events. The logs contain every periodic monitoring event I implemented within the queue, but here are sample screenshots of the overall empirical and analytical analyses of the system under different constraints:

Part(a)

For $\lambda = 60$, $T_s = 0.015$, and simulation time = 100...

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      M/M/1 Queue Analysis
--------------------------------

Total # events =  5988

Avg Tw =  0.06300665555993394
Avg Ts =  0.015000000000000504
Avg Tq =  0.07800665555992979

q =  4.680399333595788
w =  3.7803993335960366
p =  0.8981881980922628

--------------------------------
M/M/1 Queue Analytical Analysis
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Avg Tw =  0.13499999999999984
Avg Ts =  0.015
Avg Tq =  0.14999999999999986

q =  8.999999999999991
w =  8.09999999999999
p =  0.8999999999999999
```

Part(b)

For $\lambda = 50$, $T_s = 0.015$, and simulation time = 100...

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      M/M/1 Queue Analysis
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Total # events =  4988

Avg Tw =  0.02124062837183054
Avg Ts =  0.015000000000000492
Avg Tq =  0.03624062837182771

q =  1.8120314185913855
w =  1.062031418591527
p =  0.7480823600833312
```

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 M/M/1 Queue Analytical Analysis
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Avg Tw =  0.045
Avg Ts =  0.015
Avg Tq =  0.06

q =  3.0
w =  2.25
p =  0.75
```

Part(c)

For $\lambda = 65$, $T_s = 0.015$, and simulation time = 100...

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      M/M/1 Queue Analysis
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Total # events =  6596

Avg Tw =  0.3496614702040626
Avg Ts =  0.01500000000000051
Avg Tq =  0.3646614702040598

q =  23.702995563263887
w =  22.727995563264066
p =  0.9893202756132364

-------------------------------
 M/M/1 Queue Analytical Analysis
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Avg Tw =  0.5849999999999994
Avg Ts =  0.015
Avg Tq =  0.5999999999999994

q =  38.999999999999964
w =  38.02499999999996
p =  0.975
```

Part(d)

For $\lambda = 65$, $T_s = 0.020$, and simulation time = 100...

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      M/M/1 Queue Analysis
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Total # events =  6502

Avg Tw =  14.893152268706961
Avg Ts =  0.01999999999999888
Avg Tq =  14.913152268706979

q =  969.3548974659536
w =  968.0548974659525
p =  1.3003292621749225
```

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 M/M/1 Queue Analytical Analysis
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Avg Tw =  -0.08666666666666667
Avg Ts =  0.02
Avg Tq =  -0.06666666666666667

q =  -4.333333333333333
w =  -5.633333333333334
p =  1.3
```

Question 3

Part(a)

I think an appropriate model to represent this gateway is an M/G/1 system. It is given that the inter-arrival time of packets is an exponentially distributed, which means that the packets arrive one by one according to a Poisson stream. We also know that the packet lengths are uniformly distributed, implying that service time must be random and independent. This information along with the outgoing transmission rate gives us some intuition about the ratio of the service time standard deviation and the service time. We can then describe the service time as a "general" distribution.

Part(b)

T_s for packets of 100 bytes $= \dfrac{100\ bytes/\sec}{24{,}000{,}000\ bytes} = (4.0\times 10^{-6}\ s\text{ec})\times 1000 = 0.004\ ms$

T_s for packets of 1,500 bytes $= \dfrac{1{,}500\ bytes}{24{,}000{,}000\ bytes/\sec} = (6.3\times 10^{-5}\ \sec)\times 1000 \approx 0.063\ ms$

mean $T_s = \dfrac{0.004+0.063}{2} = 0.034\ ms$

Part(c)

$$\sigma = \sqrt{\frac{(b-a)^2}{12}} = \sqrt{\frac{(0.063-0.004)^2}{12}} \approx 0.017$$

Part(d)

mean $T_s = \dfrac{0.004+0.063}{2} = 0.034\ ms$

$$\rho = \lambda * T_S = \frac{1{,}200{,}000 \ packets / \min}{60000} * 0.034 = 20 \, packets / ms * 0.034 = 0.68$$

$$A = \frac{1}{2}\left[1 + \left(\frac{\sigma_{Ts}}{T_s}\right)^2\right] = \frac{1}{2}\left[1 + \left(\frac{0.017}{0.034}\right)^2\right] = 0.625$$

$$q = \frac{\rho^2 A}{1-\rho} + \rho = \frac{0.68 * 0.625}{1-0.68} + 0.68 \approx 2.00$$

Part(e)

$$w = \frac{\rho^2 A}{1-\rho} = \frac{0.68^2 * 0.625}{1-0.68} \approx 0.903$$

Part(f)

$$T_q = \frac{q}{\lambda} = \frac{2.00}{20 \, packets / ms} = 0.1 ms$$

Part(g)

$$T_w = \frac{w}{\lambda} = \frac{0.903}{20 \, packets / ms} = 0.046 ms$$

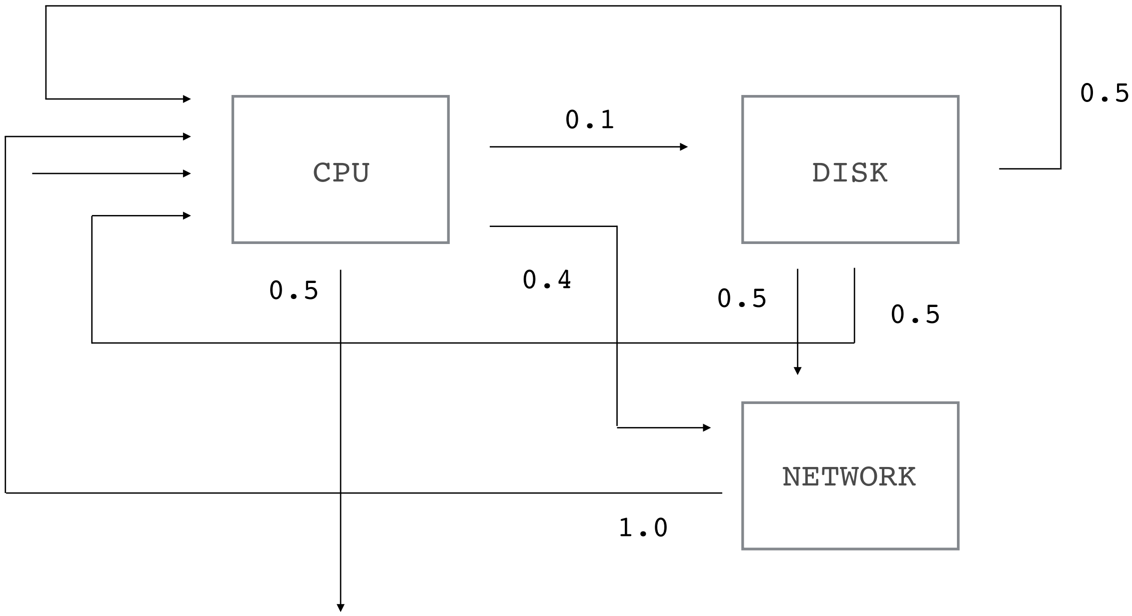
Part(h)

$$P(no\, wait\, in\, queue) = P(0) = 1 - \rho = 1 - 0.68 = 0.32$$

Question 4

Part(a)

Please see next page for diagram.



Part(b)

We can use a system of equations to calculate the arrival rate for each component of the system...

$$\lambda = 40 \ \textit{processes} / \sec$$
$$\lambda_{CPU} = 40 + 0.5\lambda_{Disk} + \lambda_{Net}$$
$$\lambda_{Disk} = 0.1\lambda_{CPU}$$
$$\lambda_{Net} = 0.4\lambda_{CPU} + 0.5\lambda_{Disk}$$

$$\lambda_{CPU} = 40 + 0.5(0.1\lambda_{CPU}) + \lambda_{Net}$$
$$\lambda_{Net} = 0.4\lambda_{CPU} + 0.5(0.1\lambda_{CPU}) = 0.45\lambda_{CPU}$$
$$\lambda_{CPU} = 40 + 0.05\lambda_{CPU} + 0.45\lambda_{CPU}$$
$$0.5\lambda_{CPU} = 40$$

$$\lambda_{CPU} = 80$$
$$\lambda_{Disk} = 0.1\lambda_{CPU} = 0.1 * 80 = 8$$
$$\lambda_{Net} = 0.4\lambda_{CPU} + 0.5\lambda_{Disk} = 0.4(80) + 0.5(8) = 36$$

Now we need to calculate the utilization of each component of the system as seen on the next page...

$$\rho_{CPU} = \frac{\lambda_{CPU} T_S}{N} = \frac{80 * 0.02}{2} = 0.8$$

$$K = \frac{\sum_{i=0}^{2-1} \frac{(2\rho)^i}{i!}}{\sum_{i=0}^{2} \frac{(2\rho)^i}{i!}} = \frac{\frac{(2*0.8)^0}{0!} + \frac{(2*0.8)^1}{1!}}{\frac{(2*0.8)^0}{0!} + \frac{(2*0.8)^1}{1!} + \frac{(2*0.8)^2}{2!}} = 0.6701$$

$$C = \frac{1-K}{1-\rho K} = \frac{1-0.6701}{1-0.8(0.6701)} = 0.711$$

$$q = C\frac{\rho}{1-\rho} + N\rho = 0.711 * \frac{0.8}{1-0.8} + (2*0.8) = 4.444$$

$$T_q = \frac{q}{\lambda_{CPU}} = \frac{4.444}{80} = 0.055$$

$$\rho_{Disk} = \lambda_{Disk} T_S = 8 * 0.1 = 0.8$$

$$T_q = \frac{1/\mu}{1-\rho} = \frac{T_S}{1-\rho} = \frac{0.1}{1-0.8} = 0.5$$

$$\rho_{Net} = \lambda_{Net} T_S = 36 * 0.025 = 0.9$$

$$T_q = \frac{1/\mu}{1-\rho} = \frac{T_S}{1-\rho} = \frac{0.025}{1-0.9} = 0.25$$

$$T_{q-total} = 0.055 + 0.5 + 0.25 = 0.805$$

Part(c)

The bottleneck of the system is the network because it has the greatest utilization of 0.9.

Part(d)

The departure rate of the CPU $= \frac{1}{T_S} = \frac{1}{0.02} = 50$ *processes* / sec. Because this rate is slower than the arrival rate we calculated in part(a) (80 packets/sec), we can conclude that the CPU's arrival rate will render this system unstable.