Complex Data Fitting by Least Squares Analysis via Excel

Laboratory #1, Phy252

Goal

This lab explores curve-fitting procedures for data governed by mathematical expressions more complex than simple straight lines, simple exponential decays, etc.

Equipment

Motion detectors; LabPro units and lab computers; Styrofoam pendulums with support stands and mounting brackets; meter sticks.

Reference

Young and Freedman, Section 13-7 (12th edition), Section 14-7 (13th edition)

Experimental Measurements

To generate some reasonably complex experimental data, we measure the decaying oscillations of a pendulum subject to air resistance. By using a lightweight pendulum bob the decay time is short, allowing the amplitude to decrease significantly in only a couple of minutes. For small initial angular displacements, the horizontal displacement, as measured with the Vernier motion detector, should approximate fairly closely the motion of a damped simple harmonic oscillator (DSHO).

(1) Calibrate the motion detector. All LabPro sensors have built-in calibration. The only thing needed is to input the room temperature, which accounts for the temperature dependence of the speed of the ultrasound signal. This can be done in the set-up window by clicking on the sensor icon at the top left of the data array then right clicking the icon with the device. Use the software Logger Pro as studied in PHY 150-151 (Note: <u>distances from the pendulum ball to the sensors should be greater than 15cm).</u>

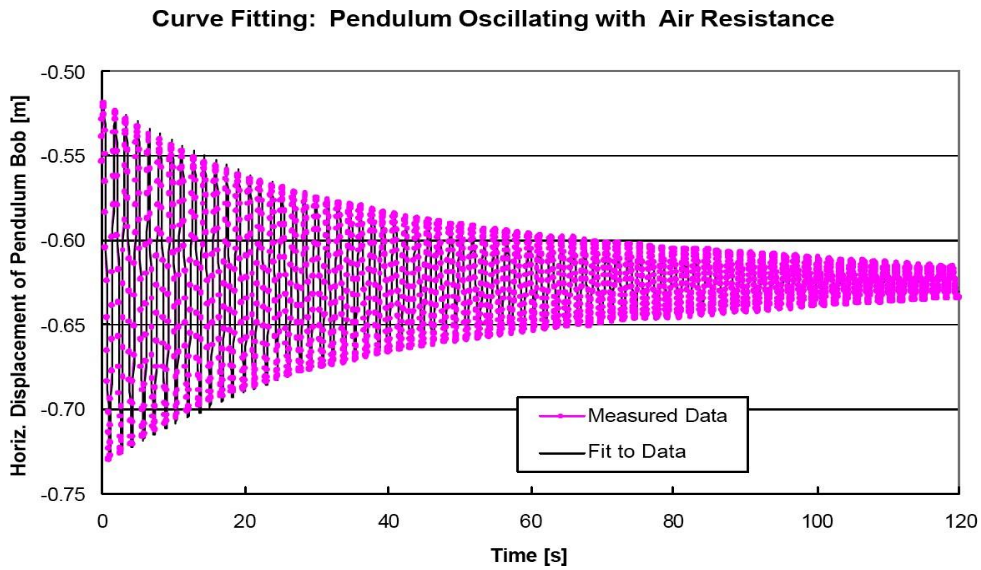

Figure 1 : Example of data collected from motion sensor over 120 seconds.

(2) Record some complex data for fitting. Using the Vernier motion detector, record the displacement of the Styrofoam pendulum ball as a function of time. Set the time resolution such that at least 20 data points are recorded per oscillation of the ball. Set the overall measurement time to record about one order of magnitude of decay in the amplitude of the oscillation (~100 seconds). You should be able to produce a plot similar to that shown above and containing 2,000 – 3,000 data points. Save this Logger Pro data file.

Curve Fitting (Done using Excel)

(1) Transfer the measured $x(t)$ data for the pendulum to an Excel spreadsheet and plot the data as an *x-y* scatter plot using lines. Be sure to label the axes and the plot itself in order to make it an understandable, self-contained unit.

(2) Locate in Young and Freedman the mathematical equation that governs the motion of a damped simple harmonic oscillator (I'll give it here).

$$x(t) = Ae^{-\gamma t}\cos(\omega t + \theta) + \varphi\,, \qquad \gamma = \frac{b}{2m}$$

Note that you will also need to include an additional "offset" (that's φ) parameter related to the distance between the motion sensor and the pendulum. This equation contains several parameters: amplitude, decay constant, angular frequency, phase and offset. Add to your spreadsheet places to store each of these parameters, which will be called 'fitting parameters' and you will manipulate to fit the equation to the data. This should be done in an organized fashion, labeling each parameter and giving its units as well as its value. For example:

Decay Constant	40	s

(3) Examine your plot to get some reasonable initial guesses for your fitting parameters. Now add a column to your spreadsheet to contain this mathematical expression for the pendulum's displacement as a function of time. Enter the expression into the top row of this column, making sure to use absolute cell references for each of the 'fitting parameters' (A1for example). The references to the time column should not be absolute. Copy this expression down for each of the rows acquired through data collection. Plot this new fitted column against

time in the same plot, and compare to the collected data. Play around with the ‘fitting parameters’ to see if you can get the fit to more closely resemble the data.

(4) To see how closely your fit resembles your data, it is useful to display the “residuals”, namely the difference of each fitted value from its corresponding data value. This is easily done by adding yet another column to your spreadsheet that computes the difference. Copy this expression to all rows of data, and plot the residuals as a function of time in a new chart. Now observe the changes in the residuals as you adjust your ‘fitting parameters’; the residuals provide a very graphic display of changes in the closeness of your fit as you change these parameters.

(5) You could spend the next hour painstakingly adjusting and readjusting each of these parameters trying to eyeball graphically the “best” fit to your data—or you could use the root-mean-square of the deviation of the fitted values from the corresponding data values and make Excel solve for it! To compute this, create another column that squares your residuals. Now, label any convenient cell “error signal”, and input an expression to compute the square root of the average of these squared residuals. This cell is the cell you want to minimize by adjusting your parameters. Keep in mind that the parameters interact, such that optimizing one may cause a previously optimized parameter to no longer be ideal. This is the classic difficulty with a multi-dimensional least squares fit (which is exactly what you are doing: trying to optimize all of the ‘fitting parameters’ simultaneously!). Copy/paste the manually input fitting parameters (otherwise you’ll lose them when you use Solver). Next, try using the Excel Solver to see if it can optimize each parameter to minimize the “error signal” value. Try individually (by choosing only one parameter cell to optimize) and collectively (by selecting a range of parameters to optimize). If it seems to get stuck, attempt to get a bit closer by manually changing the parameters, then repeat.

Some Information on Graphing and using “Solver” in Excel

The “Solver” module must first be loaded, which can be done by clicking **File->Options->Add-ins**. In the Manage box, select “Excel Add-ins” and click Go. Select the “Solver Add-in” check box and click OK. The “Solver” will be listed in the **Analysis** group in the **Data** tab.

Report

Document briefly your experimental set-up, equipment calibration, and data measurement. Present and discuss the mathematical expression that you employed to fit the data. Present **the plots of your data with the fitted curve superimposed**. List the **parameter set that minimizes the root-mean square residual**. Give this **minimized RMS value** and present a **plot of the residuals for this optimum fit**. Present **a plot of the relevant first page(s) of your spreadsheet** that contain the parameters and expressions. The mathematical expressions should be shown explicitly in a neat, readable fashion. Provide any explanatory text needed to elucidate these plots, parameter sets, etc.

Would you conclude the DSHO formula provides a good description of the pendulum motion? Support your answer quantitatively. Are there systematic deviations of the fit from the data? How might you modify your expression to improve the fit?

Submission: Report Due September 2, 2016

Lab work is a team effort and only **a single lab report is to be submitted by each team**. However, every member of the team must preserve a copy of every lab report and all data and computer files that are part of the lab report. Each student must save all of this material in a special lab binder. The data files must be easily accessible by each student, either by saving individual copies on memory sticks, or uploading the data to a server that each student can access. The report should specify electronic file names and the directories where these data files are located so that data could be retrieved at some later date if needed**. State the specific contributions of each team member to the lab work. The roles of each student must change from week to week.** The specific style and format of the lab report are up to the team, but the report must be neat, readable, and carefully organized. It must be self-contained, so that an intelligent reader is able to understand what was measured and why without additional information or insight**. An electronic copy of this lab handout is available through Blackboard**. All relevant original data must either be included in the lab report as an appendix, or if it is too copious, specified as an electronic file and its location. Most of any spreadsheet analysis can also go into appendices, with relevant data tables and graphs copied into the main report. **Do not include pages of spreadsheet numbers, but rather a graphical presentation of the data and calculations** along with any mathematical formulas used in the spreadsheet. Include sketches or schematic diagrams of the experimental apparatus to supplement any descriptions.

Here is a quick guide to your lab report for Lab 1. Make sure to read through the actual lab procedure to ensure that your final report is complete.

- Present plots of motion as well as difference between theoretical and experimental results
- Discuss the practicality and difficulty associated with the various methods of fitting (manually adjusting parameters, SOLVER function, visual estimation of parameters from Logger Pro)
- Include parameters before and after solver
- Discuss the equation of motion and its parameters