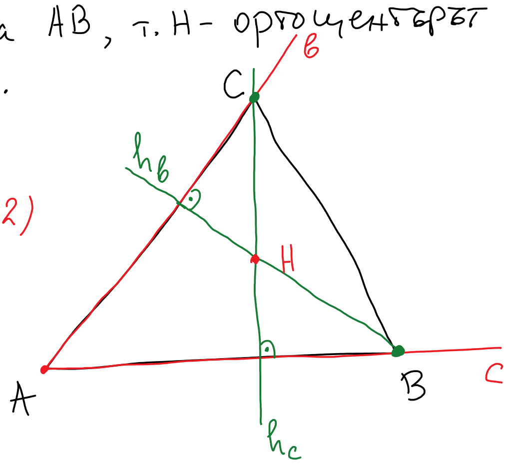
7 зад. ОКС $K = Oxy$

$b: 5x+4y-13=0$

$c: x+2y-5=0$, т. $H(14,15)$

?, коорд. на върховете на $\triangle ABC$, за който $b$ съдържа $AC$, $c$ съдържа $AB$, т. $H$ - ортоцентърът на $\triangle ABC$.




Решение:

1) т. $A = b \cap c \Rightarrow \left| \begin{array}{l} 5x+4y-13=0 \\ x+2y-5=0 \end{array} \right. \Rightarrow A(1,2)$

2) $h_c \begin{cases} \ni H(14,15) \\ \perp c: 1x+2y-5=0 \end{cases}$

$h_c: 2x-y+D=0$

$2.14-15+D=0 \Rightarrow D=-13$

$h_c: 2x-y-13=0$

т. $C = b \cap h_c \Rightarrow \left| \begin{array}{l} 5x+4y-13=0 \\ 2x-y-13=0 \;|.4 \end{array} \right. \stackrel{+}{\Rightarrow} C(5,-3)$

3) $h_b \begin{cases} \ni H(14,15) \\ \perp b: 5x+4y-13=0 \end{cases}$

$h_b: 4x-5y+D=0 \Rightarrow h_b: 4x-5y+19=0$

$4.14-5.15+D=0$

$56-75+D=0$

$D=19$

т. $B = c \cap h_b \Rightarrow \left| \begin{array}{l} x+2y-5=0 \\ 4x-5y+19=0 \end{array} \right. \Rightarrow$ т. $B(-1,3)$

$A(1,2)$
$B(-1,3)$
$C(5,-3)$ $\Rightarrow$ $S_{\triangle ABC} = \frac{1}{2} \cdot \left| \begin{vmatrix} 1 & 2 & 1 \\ -1 & 3 & 1 \\ 5 & -3 & 1 \end{vmatrix} \right|$

ОКС $K = Oxy$

---

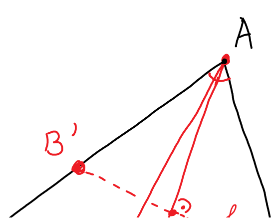
8 зад. $b_A: 2x-3y-5=0$

$m_A: x-8y+4=0$

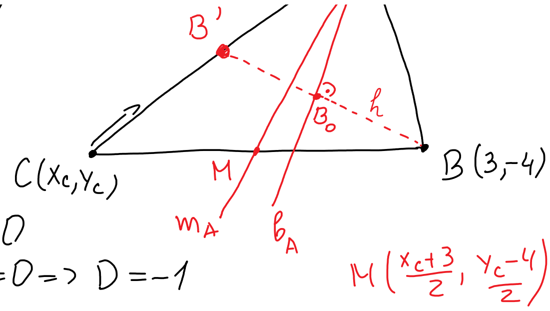
т. $B(3,-4)$

?, коорд. на $A$ и $C$ на $\triangle ABC$, за който $b_A$ - вътр. ъглопол. при $A$, $m_A$ - медиана през $A$

1) $A = b_A \cap m_A \Rightarrow \left| \begin{array}{l} 2x-3y-5=0 \\ x-8y+4=0 \end{array} \right. \Rightarrow A(4,1)$



$x-8y+4=0$
$\;\;4 \quad\; 1$




2) Ако $B \xrightarrow{G_{b_A}} B'$, то $B' \in AC$

$h \begin{cases} \ni B \\ \perp b_A: 2x-3y-5=0 \end{cases} \Rightarrow h: 3x+2y+D=0$

$3.3+2.(-4)+D=0 \Rightarrow D=-1$

$h: 3x+2y-1=0$

т. $B_o = h \cap b_A \Rightarrow \begin{vmatrix} 3x+2y-1=0 \;|.3 \\ 2x-3y-5=0 \;|.2 \end{vmatrix} +$ $\quad 13x-13=0$

$x=1 \Rightarrow 2y = 1-3.1 = -2$

$y=-1$

т. $B_o(1,-1)$ – среда

$B(3,-4)$

$B'(x',y')$ $\Rightarrow$ $\frac{x'+3}{2}=1$, $\frac{y'+(-4)}{2}=-1$ $\quad B'(-1,2)$

3) $AB': \begin{vmatrix} x & y & 1 \\ 4 & 1 & 1 \\ -1 & 2 & 1 \end{vmatrix} = 0$ $\quad AB': x+5y-9=0$

$\;\;4 \quad 1$
$-1 \quad 2$

4) т. $C(x_c,y_c)$

т. $C \in AB' \Rightarrow x_c+5.y_c-9=0$

! $M\left(\underbrace{\frac{x_c+3}{2}}_{x}, \underbrace{\frac{y_c-4}{2}}_{y}\right) \in m_A: x-8y+4=0 \Rightarrow$ ! $\underbrace{\frac{x_c+3}{2} - 8.\left(\frac{y_c-4}{2}\right) + 4 = 0}_{2}$ $\;\; 2 \quad 2$

$x_c - 8.y_c + 43 = 0$

т. C $\begin{vmatrix} x_c+5.y_c-9=0 \\ x_c-8y_c+43=0 \end{vmatrix}$ (–) $\quad 13.y_c - 52 = 0$

$y_c = 4 \Rightarrow x_c = -43+32 = -11$

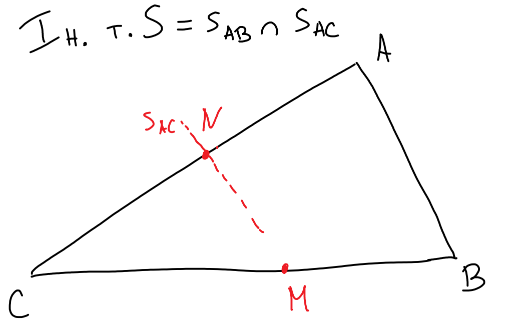
т. $C(-11,4)$

Упр. $S_{\Delta ABC} = ?$

б) $A(4,1)$, $B(3,-4)$, $C(-11,4)$

?, координатите на центъра S и дълж. на радиуса R на описаната около $\Delta ABC$ окръжност

I н. т. $S = s_{AB} \cap s_{AC}$




II н. т. $S(x,y)$: $|\vec{SA}| = |\vec{SB}| = |\vec{SC}|$

$\vec{SA}(x-4, y-1) \Rightarrow |\vec{SA}|^2 = (x-4)^2 + (y-1)^2$

$|\vec{SB}| =$

$|\vec{SC}| =$

AC: $x+5y-9=0$ , $N\left(-\frac{7}{2}, \frac{5}{2}\right)$ – средата на AC

$S_{AC}: 5x-y+D=0$

$5\cdot\left(-\frac{7}{2}\right)-\frac{5}{2}+D=0 \Rightarrow D=20 \Rightarrow \boxed{S_{AC}: 5x-y+20=0}$

$BC: \begin{vmatrix} x & y & 1 \\ 3 & -4 & 1 \\ -11 & 4 & 1 \end{vmatrix} = 0 \Rightarrow BC: 8x+14y+32=0 \mid :2$

$BC: 4x+7y+16=0$

$M(-4,0)$

$S_{BC}: 7x-4y+D=0 \Rightarrow D=28$ ($x=-4$, $y=0$)

$\boxed{S_{BC}: 7x-4y+28=0}$

т. $S$ $\left|\begin{array}{l} 5x-y+20=0 \\ 7x-4y+28=0 \end{array}\right. \Rightarrow$ т. $S(-4,0) \equiv M$ , $R=|\vec{SA}|=\sqrt{65}$

$A(4,1)$

$\vec{SA}(8,1)$

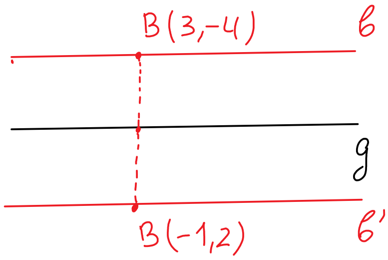
---

9 зад. $g: 2x-3y-5=0$

? образът на права $b$ при $\sigma_g$, ако:

а) $b: 2x-3y-18=0 \Rightarrow b \parallel g$

$b \xrightarrow{\sigma_g} b' \Rightarrow b' \parallel g \parallel b$



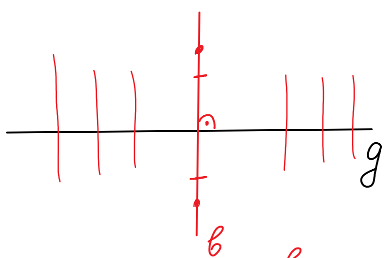


Изб. т. $B \in b$, изб. $B(3,-4) \xrightarrow[\text{от 8 зад.}]{\sigma_g} B'(-1,2)$

$b' \begin{cases} \parallel b \\ \ni B'(-1,2) \end{cases} \Rightarrow b': 2x-3y+8=0$

б) $g: 2x-3y-5=0$

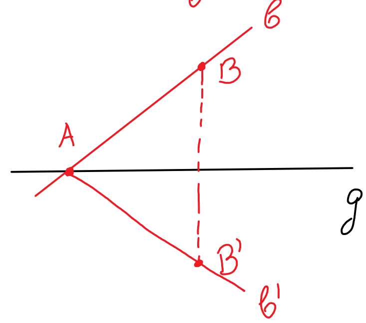
$b: 3x+2y-1=0$ $\Rightarrow$ $b \perp g \Rightarrow b \xrightarrow{\sigma_g} b$





в) $g: 2x-3y-5=0$

$b: 5x-y-19=0$





1) $A = g \cap b \Rightarrow \ldots$ $A(4,1)$ (Упр.)

2) Изб. т. $B \in b$. Изб. т. $B(3,-4) \xrightarrow[\text{от 8 зад.}]{\sigma_g} B'(-1,2)$

3) $b' \begin{cases} \ni A(4,1) \\ \ni B'(-1,2) \end{cases} \Rightarrow \begin{vmatrix} x & y & 1 \\ 4 & 1 & 1 \\ -1 & 2 & 1 \end{vmatrix} = 0 \Rightarrow b': x+5y-9=0$

$$\sigma_g : \begin{pmatrix} x' \\ y' \end{pmatrix} = \frac{1}{A^2+B^2} \cdot \begin{pmatrix} -A^2+B^2 & -2AB \\ -2AB & A^2-B^2 \end{pmatrix} \cdot \begin{pmatrix} x_0 \\ y_0 \end{pmatrix} + \frac{1}{A^2+B^2} \cdot \begin{pmatrix} -2AC \\ -2BC \end{pmatrix}$$

$B'(x', y')$ $\qquad$ $B(x_0, y_0)$

$g: A \cdot x + B \cdot y + C = 0$ , $B \xrightarrow{\sigma_g} B'$

---

10 зад.

$b: 2x - y = 0$

$c: x - 2y + 3 = 0$

$M(3,4)$

а) ? коорд. на върховете на $\triangle ABC$,

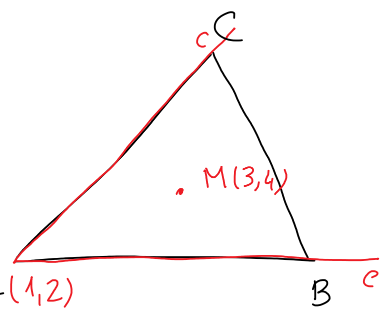
$b$ сбд. $AC$

$c$ сбд. $AB$

$M$ – медицентър на $\triangle ABC$

Решение:

1) т. $A = b \cap c \Rightarrow \begin{cases} 2x - y = 0 \\ x - 2y + 3 = 0 \end{cases} \Rightarrow A(1,2)$




2) $B(x_B, y_B)$ !

$C(x_C, y_C)$ !

$A(1,2)$

$\vec{OM} = \frac{1}{3} \cdot (\vec{OA} + \vec{OB} + \vec{OC})$

$3 = \frac{1}{3} \cdot (1 + x_B + x_C) \Rightarrow \boxed{x_B + x_C = 8}$

$4 = \frac{1}{3} \cdot (2 + y_B + y_C) \Rightarrow \boxed{y_B + y_C = 10}$ !

$B \in c \Rightarrow \boxed{x_B - 2 \cdot y_B + 3 = 0}$

$C \in b \Rightarrow \boxed{2x_C - y_C = 0}$

$$\begin{cases} x_B + x_C = 8 \\ y_B + y_C = 10 \\ x_B - 2y_B + 3 = 0 \\ 2x_C - y_C = 0 \end{cases}$$

Упр. $\quad B(5,4)$, $C(3,6)$, $A(1,2)$

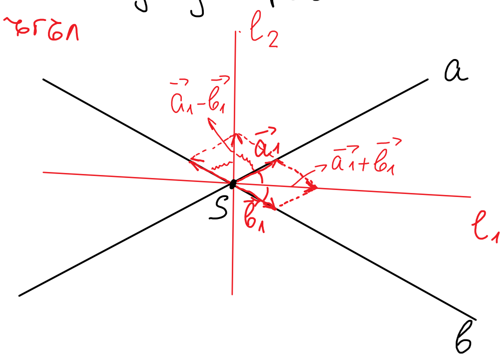
б) Упр.

* $S_{\triangle ABC} = ?$
* $P_{\triangle ABC} = ?$
* Вида на $\triangle ABC$ спр. ъглите
* медиани, медицентър
* височини, ортоцентър

* симетрали, ц-р и рад. на описана окр.

---

Уравнения на ъглополовящи на ъгли между две прави.
ъглополовяща на тъп и остър ъгъл




ОКС $K=Oxy$

$a, b$ - дадени

$a \cap b = т.\ S$

$a \parallel \vec{a} \Rightarrow |\vec{a}| \Rightarrow \vec{a}_1 = \dfrac{\vec{a}}{|\vec{a}|} \Rightarrow |\vec{a}_1| = 1$

$b \parallel \vec{b} \Rightarrow |\vec{b}| \Rightarrow \vec{b}_1 = \dfrac{\vec{b}}{|\vec{b}|} \Rightarrow |\vec{b}_1| = 1$

$\ell_1 \begin{cases} \ni т.S = a\cap b \\ \parallel (\vec{a}_1+\vec{b}_1) \end{cases}$ $\qquad$ $\ell_2 \begin{cases} \ni S \\ \parallel (\vec{a}_1-\vec{b}_1) \end{cases}$

Кога $\ell_1$ е ъглоп. на острия $\angle(a,b)$ и кога - на тъпия?

$\ell_1$ е ъглоп. на $\angle(\vec{a}_1, \vec{b}_1)$

$(\vec{a}_1 . \vec{b}_1) = \cos\angle(\vec{a}_1,\vec{b}_1) > 0 \Rightarrow$ $\ell_1$ – на остър ъгъл
$\ell_2$ – на тъп ъгъл

$(\vec{a}_1 . \vec{b}_1) < 0 \Rightarrow$ $\ell_1$ – на тъп ъгъл
$\ell_2$ – на остър ъгъл

---

1 зад. ОКС $K=Oxy$

$a: 3x-4y+5=0$ $\qquad$ ? уравнения на ъглоп. $\ell_1$ и $\ell_2$ на ъглите м/у $a$ и $b$

$b: 4x-3y-5=0$ $\qquad$ Да се определи коя е ъглоп. на острия ъгъл и коя - на тъпия

1) $т.\ S = a \cap b \Rightarrow S(5,5)$

2) $\boxed{a \parallel \vec{a}(-B,A)}$ $\quad A=3,\ B=-4 \Rightarrow a \parallel \vec{a}(4,3) \Rightarrow |\vec{a}| = 5 \Rightarrow \vec{a}_1 = \dfrac{\vec{a}}{5} \Rightarrow$

$\vec{a}_1\left(\frac{4}{5}, \frac{3}{5}\right)$

$b \parallel \vec{b}(3,4) \Rightarrow |\vec{b}| = 5 \Rightarrow \vec{b}_1 = \dfrac{\vec{b}}{5} \Rightarrow$

$\vec{b}_1\left(\frac{3}{5}, \frac{4}{5}\right)$

$\ell_1 \begin{cases} \ni S(5,5) \\ \parallel (\vec{a}_1+\vec{b}_1)\left(\frac{7}{5}, \frac{7}{5}\right) \\ \qquad\qquad (1, 1) \end{cases} \Rightarrow \ell_1: \begin{cases} x = 5 + 1 . s \\ y = 5 + 1 . s \end{cases}, s \in \mathbb{R} \underset{Oxy}{\Rightarrow} \ell_1: x - y = 0$

$(\ldots)\left(\frac{1}{5}, \frac{1}{5}\right)$ $\quad Y = 5 + 1 \cdot s \quad O_{XY}$

$(1, 1)$

$$l_2 \begin{cases} \ni S(5,5) \\ \parallel (\vec{a}_1 - \vec{b}_1)\left(\frac{1}{5}, -\frac{1}{5}\right) \\ \quad 1, -1 \end{cases} \Rightarrow l_2 : \begin{cases} X = 5 + 1 \cdot p \\ Y = 5 - 1 \cdot p \end{cases}, p \in \mathbb{R} \underset{O_{XY}}{\Rightarrow} l_2 : X + Y - 10 = 0$$

Пресм. $(\vec{a}_1 . \vec{b}_1) = \frac{4}{5} \cdot \frac{3}{5} + \frac{3}{5} \cdot \frac{4}{5} = \frac{24}{25} > 0 \Rightarrow l_1$ – на остър ъгъл

$l_2$ – на тъп ъгъл

2 зад. (Упр.)

$a : X - 3Y = 0 \qquad l_1 ? \quad l_2 ?$

$b : 3X - Y + 8 = 0$

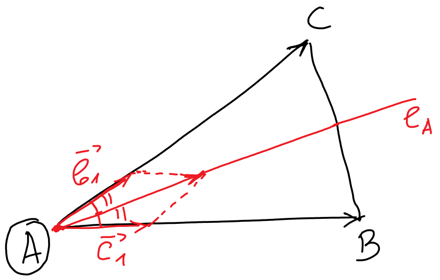
3 зад.

$A(1,2)$, $B(-1,3)$, $C(5,4)$

?, $l_A$ - вътр. ъглополовяща при върха $A$ на $\triangle ABC$

$\vec{c}_1 \uparrow\uparrow \vec{AB}$, $|\vec{c}_1| = 1$

$\vec{b}_1 \uparrow\uparrow \vec{AC}$, $|\vec{b}_1| = 1$ $\Rightarrow (\vec{b}_1 + \vec{c}_1) \parallel l_A$




$\vec{AB}(-2, 1) \Rightarrow |\vec{AB}| = \sqrt{5}$

$\vec{AC}(4, 2) \Rightarrow |\vec{AC}| = \sqrt{20} = 2.\sqrt{5}$

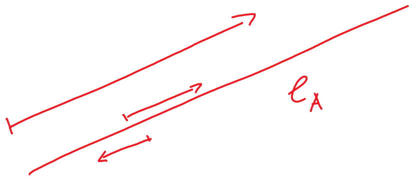
$\vec{c}_1 = \frac{\vec{AB}}{\sqrt{5}} \Rightarrow \vec{c}_1\left(\frac{-2}{\sqrt{5}}, \frac{1}{\sqrt{5}}\right)$

$\vec{b}_1 = \frac{\vec{AC}}{2.\sqrt{5}} \Rightarrow \vec{b}_1\left(\frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}}\right)$ $\Rightarrow (\vec{c}_1 + \vec{b}_1)\left(0, \frac{2}{\sqrt{5}}\right)$

$\quad 0, 1$

$$l_A : \begin{cases} X = 1 + s.0 \\ Y = 2 + s.1 \end{cases}, s \in \mathbb{R}$$

Общо уравнение на $l_A \begin{cases} \ni A(1,2) \\ \parallel \vec{e}_2 \parallel O_Y \end{cases}$




$l_A : X = 1$

4 зад. (Упр.)

$A(1, -2) \quad B(2, 0) \quad C\left(-\frac{2}{3}, \frac{4}{3}\right)$

? коорд. на центъра $I$ на вписаната в $\triangle ABC$ окръжност и $r = ?$