Трансверзали

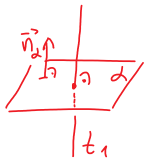
1 зад. ОКС $K=Oxyz$, Ox, Oy, Oz

$$a:\begin{cases} x=1.p \\ y=-2+p \\ z=-1+2p \end{cases},\ p\in\mathbb{R},\quad b:\begin{cases} x+z=0 \\ y+z-2=0 \end{cases},\quad c:\begin{cases} x=1+2q \\ y=-1+6q \\ z=2-1.q \end{cases},\ q\in\mathbb{R}$$

$$\left\{\begin{array}{ll} \pi: x+z=0 \Rightarrow A=1,\ B=0,\ C=1,\ D=0 \\ \pi \parallel \vec{e}_2(0,1,0) \quad A.0+B.1+C.0=0 \end{array}\right\}$$

а) Да се намерят уравнения на онази трансверзала t_1 на кръст. прави a и b, $\boxed{t_1 \parallel c}$.

$\{$? t_1 на кръст. прави a и b, $\boxed{t_1 \perp \alpha}$: $2x+6y-z+2021=0$ $\}$



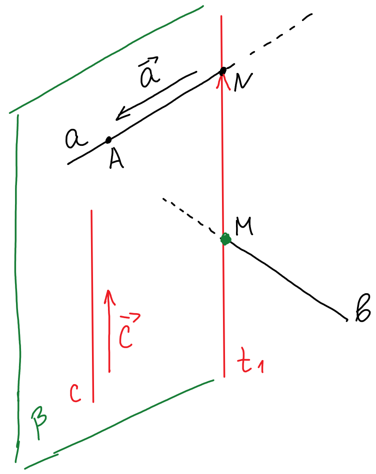
$t_1 \parallel c \parallel \vec{c}(2,6,-1) \Rightarrow t_1 \parallel \vec{c}(2,6,-1)$!

$t_1 \perp \alpha \Rightarrow t_1 \parallel \vec{n}_\alpha(2,6,-1)$!

I н. $N \ni a \Rightarrow N(p,-2+p,-1+2p)$

$M \ni b \Rightarrow M(\quad)$ парам. у-ния на b

$\boxed{\vec{MN} \parallel \vec{c}(2,6,-1)} \Rightarrow M$ и N



II н. Търсим $\beta \begin{cases} \ni a \\ \parallel \vec{c} \end{cases} \Rightarrow t_1 \ni \beta$

1) ✓ $\beta \parallel \vec{c}(2,6,-1)$

✓ $\beta \parallel \vec{a}(1,1,2) \parallel a$

✓ $\beta \ni A(0,-2,-1) \ni a$ при $p=0$ $\Rightarrow$ $\beta: \begin{vmatrix} x-0 & y-(-2) & z-(-1) \\ 2 & 6 & -1 \\ 1 & 1 & 2 \end{vmatrix}=0$

$\beta: x.(12+1)-(y+2).(4+1)+(z+1).(2-6)=0$

$\beta: 13.x-5y-4z-14=0$

$\begin{vmatrix} 2 & 6 & -1 \\ 1 & 1 & 2 \end{vmatrix}$

$A \rightarrow 13.0-5.(-2)-4.(-1)-14=0$

2) ?, коорд. на т. $M = b \cap \beta \Rightarrow \left| \begin{array}{l} x+z=0 \Rightarrow x=-z \\ y+z-2=0 \Rightarrow y=2-z \\ 13x-5y-4z-14=0 \end{array} \right.$ $\} \rightarrow \beta$

$-13z-5.(2-z)-4.z-14=0$

$-12.z-24=0 \Rightarrow z=-2$

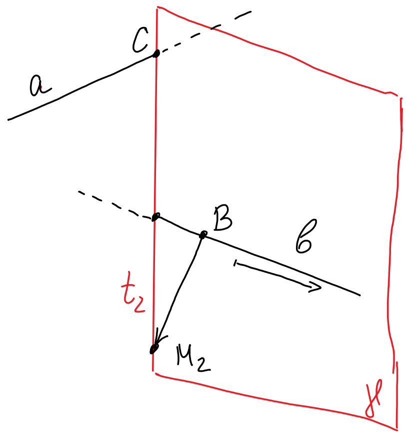
$x=2$

$y=4$ $\quad M(2,4,-2)$

$t_1 \begin{cases} \ni M(2,4,-2) \\ \parallel \vec{c}(2,6,-1) \end{cases} \Rightarrow t_1: \begin{cases} x=2+2.\lambda \\ y=4+6.\lambda \\ z=-2-1.\lambda \end{cases},\ \lambda\in\mathbb{R}$

б) ? уравн. на онази трансв. t_2 на кръст. прави a и b, $t_2 \ni M_2(6,0,4)$

1) Търсим общо уравн на равнината



$$\gamma \begin{cases} \ni M_2(6,0,4) \\ \ni b \end{cases}$$

$$b \begin{cases} x+z=0 \\ y+z-2=0 \end{cases} \Rightarrow b: \begin{cases} x=-1s \\ y=2-1s \\ z=1s \end{cases}, \ s\in\mathbb{R}$$

$\gamma \ni M_2(6,0,4)$ $\qquad$ $b \ni B(0,2,0)$ $s=0$

$\gamma \parallel \vec{b}(1,1,-1)$ $\qquad$ $\vec{BM_2}(6,-2,4)$

$\gamma \ni B(0,2,0)$

$$\gamma \parallel \vec{BM_2}(6,-2,4) \Rightarrow \gamma: \begin{vmatrix} x-6 & y-0 & z-4 \\ 1 & 1 & -1 \\ 6 & -2 & 4 \end{vmatrix} = 0$$

$\gamma: (x-6).(4-2) - y.(4+6) + (z-4).(-2-6) = 0$

$\gamma: 2x - 10.y - 8z + 20 = 0 \ |:2$

$\gamma: x - 5y - 4z + 10 = 0$ → Да

$1 \quad 1 \quad -1 \ |$

$6 \quad -2 \quad 4$

$B \to 0 - 10 - 0 + 10 = 0$

2) ?, коорд. на т. $C = a \cap \gamma \Rightarrow$

$$\begin{cases} x = 1.p \\ y = -2+1p \\ z = -1+2p \\ x-5y-4z+10=0 \end{cases}$$

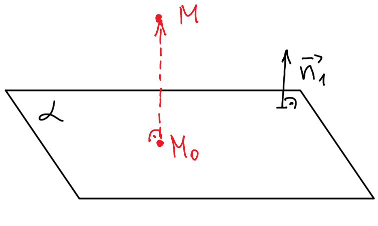
$p - 5.(-2+p) - 4.(-1+2p) + 10 = 0$

$-12p + 24 = 0 \quad p = 2 \Rightarrow C(2,0,3)$

$$t_2 \begin{cases} \ni M_2(6,0,4) \\ \ni C(2,0,3) \end{cases} \Rightarrow \vec{CM_2}(4,0,1) \parallel t_2 \Rightarrow t_2: \begin{cases} x = 6+4.\mu \\ y = 0 \\ z = 4+1.\mu \end{cases}, \ \mu\in\mathbb{R}$$

Разстояние от точка до равнина

ОКС $K = Oxyz$ $\qquad$ $\vec{M_0M} = \delta.\vec{n_1}$



$\alpha: A.x+B.y+C.z+D=0$ → общо

$\vec{n_\alpha}(A,B,C) \Rightarrow |\vec{n_\alpha}| = \sqrt{A^2+B^2+C^2}$

$\vec{n_1} = \dfrac{\vec{n_\alpha}}{|\vec{n_\alpha}|} \Rightarrow |\vec{n_1}| = 1 \quad \vec{n_1}\left(\dfrac{A}{|\vec{n_\alpha}|}, \dfrac{B}{|\vec{n_\alpha}|}, \dfrac{C}{|\vec{n_\alpha}|}\right)$

$$\alpha: \frac{A.x+B.y+C.z+D}{\sqrt{A^2+B^2+C^2}} = 0$$ – нормално уравнение на α

$M(x_M, y_M, z_M)$ – произволна, $M_0 = пр_\alpha M$

$\sqrt{A^2+B^2+C^2}$

$M(x_M, y_M, z_M)$ - произволна, $M_0 = пр_\alpha M$

$$\delta(M,\alpha) = \frac{A.x_M + B.y_M + C.z_M + D}{\sqrt{A^2+B^2+C^2}}$$

$\nearrow < 0 \quad \overrightarrow{M_0M} \uparrow\downarrow \vec{n}_1$

$\to = 0 \quad M \in \alpha$

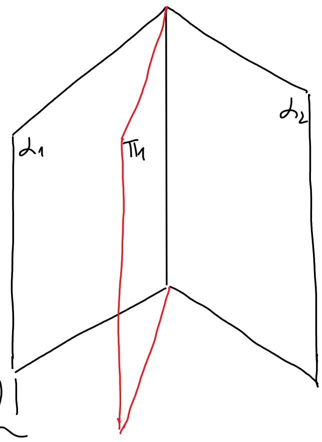
$\searrow > 0 \quad \overrightarrow{M_0M} \uparrow\uparrow \vec{n}_1$

2 зад. ОКС $Oxyz$

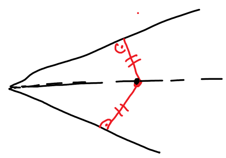
$\alpha_1: 2x - y + 2z + 3 = 0$

$\alpha_2: x - 2y + 2z - 3 = 0$

? общи уравнения на ъглополовящите равнини π_1 и π_2 на двустенните ъгли между α_1 и α_2.



т. $L \in \pi_1\ (\pi_2) \iff |\delta(L,\alpha_1)| = |\delta(L,\alpha_2)|$



$\alpha_1 \to \vec{n}_{\alpha_1}(2,-1,2) \Rightarrow |\vec{n}_{\alpha_1}| = 3$

$\alpha_1: \dfrac{2x - y + 2z + 3}{3} = 0$

$\alpha_2 \to \vec{n}_{\alpha_2}(1,-2,2) \Rightarrow |\vec{n}_{\alpha_2}| = 3$

$\alpha_2: \dfrac{x - 2y + 2z - 3}{3} = 0$

$$|\delta(L,\alpha_1)| = |\delta(L,\alpha_2)|$$

$$\left|\frac{2x - y + 2z + 3}{3}\right| = \left|\frac{x - 2y + 2z - 3}{3}\right| \quad / \cdot 3$$

$$2x - y + 2z + 3 = \pm(x - 2y + 2z - 3)$$

$\pi_1: 2x - y + 2z + 3 = x - 2y + 2z - 3$

$\pi_1: x + y + 6 = 0$

$\pi_2: 2x - y + 2z + 3 = -(x - 2y + 2z - 3)$

$\pi_2: 3x - 3y + 4z = 0$

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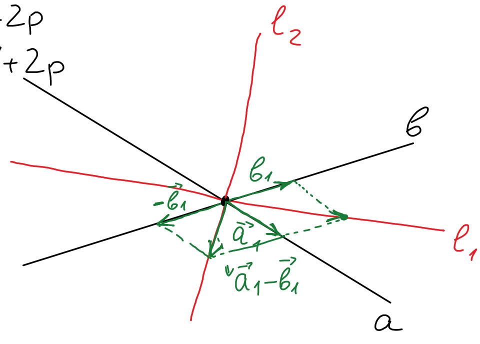
3 зад. ОКС $K = Oxyz$

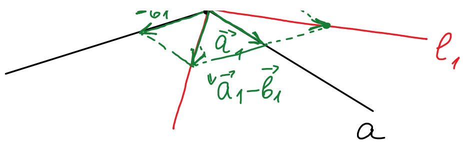
$$a: \begin{cases} x = -1 + 2s \\ y = 3 - s \\ z = 1 + 2s \end{cases} \qquad b: \begin{cases} x = -1 + p \\ y = 6 - 2p \\ z = -1 + 2p \end{cases}$$

1) ?, т. $S = a \cap b \Rightarrow S(1,2,3)$

$$\begin{vmatrix} -1 + 2s = -1 + p \\ 3 - s = 6 - 2p \\ 1 + 2s = -1 + 2p \end{vmatrix} \nearrow$$



$$\begin{cases} -1+2s = -1+p \\ 3-s = 6-2p \\ 1+2s = -1+2p \end{cases} \quad //$$



2)

?, парам. уравн. на l_1 и l_2 - ъглополовящи

$a \parallel \vec{a}(2,-1,2) \Rightarrow |\vec{a}|=3 \Rightarrow \vec{a}_1\left(\frac{2}{3}, -\frac{1}{3}, \frac{2}{3}\right)$

$b \parallel \vec{b}(1,-2,2) \Rightarrow |\vec{b}|=3 \Rightarrow \vec{b}_1\left(\frac{1}{3}, -\frac{2}{3}, \frac{2}{3}\right)$

$$\boxed{A_1 \cdot x_1 + \dots + A_n \cdot x_n + D = 0}$$

хиперравнина

$$l_1 \begin{cases} \parallel \vec{a}_1+\vec{b}_1\left(1,-1,\frac{4}{3}\right) \longrightarrow (3,-3,4) \\ \ni S(1,2,3) \end{cases} \Rightarrow l_1: \begin{cases} x = 1+3.\lambda \\ y = 2-3.\lambda \\ z = 3+4.\lambda \end{cases}, \lambda \in \mathbb{R}$$

$$l_2 \begin{cases} \parallel \vec{a}_1-\vec{b}_1(1,1,0) \\ \ni S(1,2,3) \end{cases} \Rightarrow l_2: \begin{cases} x = 1+\mu \\ y = 2+\mu \\ z = 3 \end{cases}, \mu \in \mathbb{R}$$

Тъп и остър ъгъл : $(\vec{a}_1 . \vec{b}_1) = \frac{8}{9} > 0 \Rightarrow$ $l_1 \to$ остър, $l_2 \to$ тъп