$$\det A = \begin{vmatrix} a_{11} & \cdots & a_{1n} \\ \cdots & \cdots & \cdots \\ a_{n1} & \cdots & a_{nn} \end{vmatrix} = \sum_{\substack{i_1 \ldots i_n \\ \text{перм.}}} (-1)^{[i_1 \ldots i_n]} \, \underline{a_{1i_1} \ldots a_{ni_n}}$$

---

Лема

$$\det A = \begin{vmatrix} a_{11} & \cdots & \cdots & a_{1n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{n-1,1} & \cdots & \cdots & a_{n-1,n} \\ 0 & 0 & \cdots\ 0 & a_{nn} \end{vmatrix} = \underline{\underline{\Delta_{nn}}}\, a_{nn}$$

$$\det A = \sum_{i_1 \ldots i_n} (-1)^{[i_1 \ldots i_n]} a_{1i_1} \ldots a_{ni_n} = \sum_{\substack{i_1 \ldots i_{n-1}, n \\ i_n = n}} (-1)^{[i_1 \ldots i_{n-1}, n] = [i_1 \ldots i_{n-1}]} \, \underline{a_{1i_1} \ldots a_{n-1,i_{n-1}}} \, a_{nn}$$

$i_1 \ldots i_{n-1}$ перм. на $1, \ldots, n-1$

$i_n \neq n$

$a_{ni_n} = 0$

$$= \Bigg( \underbrace{\sum_{i_1 \ldots i_{n-1}} (-1)^{[i_1 \ldots i_{n-1}]} a_{1i_1} \ldots a_{n-1,i_{n-1}}}_{\Delta_{nn}} \Bigg) a_{nn}$$

$$\det A = \left|\begin{array}{ccc} a_{11} \; \mathrm{I} & & \mathrm{II} \quad a_{1n} \\ & a_{ij} & \\ a_{n1} \; \mathrm{III} & & \mathrm{IV} \quad a_{nn} \end{array}\right| \to i$$

$$j$$

$$\Delta_{ij} = \left|\begin{array}{c|c} \mathrm{I} & \mathrm{II} \\ \hline \mathrm{III} & \mathrm{IV} \end{array}\right| \quad \text{ред } n-1$$

$$\Delta_{ij} = \left|\begin{array}{cccc|ccc} a_{11} & a_{12} & \cdots & a_{1,j-1} & a_{1,j+1} & \cdots & a_{1n} \\ & & \mathrm{I} & & & \mathrm{II} & \\ a_{i-1,1} & a_{i-1,2} & \cdots & a_{i-1,j-1} & a_{i-1,j+1} & \cdots & a_{i-1,n} \\ \hline a_{i+1,1} & a_{i+1,2} & \cdots & a_{i+1,j-1} & a_{i+1,j+1} & \cdots & a_{i+1,n} \\ & & \mathrm{III} & & & \mathrm{IV} & \\ a_{n1} & a_{n2} & \cdots & a_{n,j-1} & a_{n,j+1} & \cdots & a_{nn} \end{array}\right|$$

$$n^2$$

$$\begin{vmatrix} 2 & 3 & 1 & 5 \\ 7 & 4 & -2 & 1 \\ 3 & 7 & 1 & 4 \\ 6 & 2 & 7 & 3 \end{vmatrix}$$

$$\Delta_{22} = \begin{vmatrix} 2 & 1 & 5 \\ 3 & 1 & 4 \\ 6 & 7 & 3 \end{vmatrix}$$

$$\Delta_{23} = \begin{vmatrix} 2 & 3 & 5 \\ 3 & 7 & 4 \\ 6 & 2 & 3 \end{vmatrix}$$

$$\Delta_{32} = \begin{vmatrix} 2 & 1 & 5 \\ 7 & -2 & 1 \\ 6 & 7 & 3 \end{vmatrix}$$

Лема 2

$$i \rightarrow \begin{vmatrix} a_{11} & \cdots & & & & \cdots & a_{1n} \\ & & & & & & \\ 0 & \cdots & 0 & a_{ij} & 0 & \cdots & 0 \\ & & & & & & \\ a_{n1} & \cdots & & & & \cdots & a_{nn} \end{vmatrix} = \boxed{(-1)^{i+j}\Delta_{ij}}\, a_{ij} = A_{ij}\, a_{ij}$$

$$= (-1)^{n-i} \begin{vmatrix} a_1 \\ \vdots \\ a_{i-1} \\ a_{i+1} \\ \vdots \\ a_n \\ a_i \end{vmatrix} \underset{\text{размест. стълбове}}{=} (-1)^{n-i}(-1)^{n-j} \begin{vmatrix} & & & \overset{\text{№ } j}{a_{1j}} \\ & \Delta_{ij} & & \vdots \\ & & & a_{i-1,j} \\ & & & a_{i+1,j} \\ & & & \vdots \\ & & & a_{nj} \\ 0 & \cdots & 0 & a_{ij} \end{vmatrix} = (-1)^{2n-i-j}\Delta_{ij}\, a_{ij} =$$

$(-1)^{i+j}\Delta_{ij} = A_{ij}$ адюнгирано количество

**Т**/ $\det A = a_{i1}A_{i1} + a_{i2}A_{i2} + \dots + a_{in}A_{in}$ ($i$ номер на ред)

$A_{n\times n}$

$$A = \begin{pmatrix} a_1 \\ \vdots \\ a_i \\ \vdots \\ a_n \end{pmatrix} \quad a_i = (a_{i1}, a_{i2}, \dots, a_{in}) = \underbrace{(a_{i1}, 0, \dots, 0)}_{b_1} + \underbrace{(0, a_{i2}, 0, \dots, 0)}_{b_2} + \dots + \underbrace{(0, 0, \dots 0, a_{in})}_{b_n}$$

$$a_i = b_1 + b_2 + \dots + b_n$$

$$\det A = \det \begin{pmatrix} a_1 \\ \vdots \\ b_1 + \dots + b_n \\ \vdots \\ a_n \end{pmatrix} = \det \begin{pmatrix} a_1 \\ \vdots \\ b_1 \\ \vdots \\ a_n \end{pmatrix} + \det \begin{pmatrix} a_1 \\ \vdots \\ b_2 \\ \vdots \\ a_n \end{pmatrix} + \dots + \det \begin{pmatrix} a_1 \\ \vdots \\ b_n \\ \vdots \\ a_n \end{pmatrix}$$

прилагаме лема 2

$$= a_{i1}A_{i1} + a_{i2}A_{i2} + \dots + a_{in}A_{in}.$$

**Сл**/ Нека $j$ - номер на стълб

$$\det A = \det A^t = a_{1j}A_{1j} + a_{2j}A_{2j} + \dots + a_{nj}A_{nj}$$