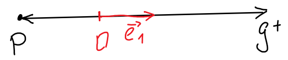
I Ортонормирана к.с. (ОКС)

1)



$K = O\vec{e_1}$

$P \in g^+$

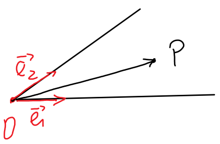
$\vec{OP}$ – радиус-вектор

$\vec{OP} = x.\vec{e_1}$, $x \in \mathbb{R}$ => $\vec{OP}(x)$ спр. K

т. $P(x)$ спр. K

---

2) $K = O\vec{e_1}\vec{e_2}$



$\vec{OP} = x.\vec{e_1} + y.\vec{e_2}$

$\vec{OP}$ – радиус-вектор

$\vec{OP}(x, y)$ спр. K

$P(x, y)$ спр. K

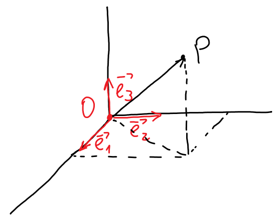
Кога $K = O\vec{e_1}\vec{e_2}$ е ОКС?

$\begin{cases} \vec{e_1} \perp \vec{e_2} \\ |\vec{e_1}| = |\vec{e_2}| = 1 \end{cases}$

---

3) $K = O\vec{e_1}\vec{e_2}\vec{e_3}$ е ОКС $\Leftrightarrow$ $|\vec{e_1}| = |\vec{e_2}| = |\vec{e_3}| = 1$

$\vec{e_1} \perp \vec{e_2} \perp \vec{e_3} \perp \vec{e_1}$



$\begin{cases} \vec{OP} = x.\vec{e_1} + y.\vec{e_2} + z.\vec{e_3} \\ \vec{OP}(x, y, z) \\ \text{т. } P(x, y, z) \end{cases}$

$\vec{e_1}(1, 0, 0)$

$\vec{e_2}(0, 1, 0)$

$\vec{e_3}(0, 0, 1)$

II Скаларно произведение спр. ОКС

$\vec{e_1}^2 = \vec{e_2}^2 = \vec{e_3}^2 = 1$ $\quad (\vec{e_1}.\vec{e_2}) = (\vec{e_2}.\vec{e_1}) = (\vec{e_1}.\vec{e_3}) = 0$

$\vec{a}(a_1, a_2, a_3)$

$\vec{b}(b_1, b_2, b_3)$

$(\vec{a}.\vec{b}) = a_1.b_1 + a_2.b_2 + a_3.b_3$ / ОКС

$\vec{a}^2 = a_1^2 + a_2^2 + a_3^2$ ОКС

$A(x_1, y_1, z_1)$

$B(x_2, y_2, z_2)$

$|\vec{AB}| = ?$ $\quad \vec{AB} = \vec{OB} - \vec{OA}$ => $\vec{AB}(x_2 - x_1, y_2 - y_1, z_2 - z_1)$

$|\vec{AB}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$ ОКС

---

1 зад. ОКС $K = Oxyz = O\vec{e_1}\vec{e_2}\vec{e_3}$

$A(-1, -1, 1)$

$B(2, -7, 4)$

$C(4, -2, 6)$

$M\left(\frac{5}{3}, -\frac{10}{3}, \frac{11}{3}\right)$

а) $P_{\triangle ABC} = ?$

б) Да се определи вида на $\triangle ABC$ спр. ъглите му;

в) $H \begin{cases} \in AB \\ CH \perp AB \end{cases}$, да се намерят коорд. на т H

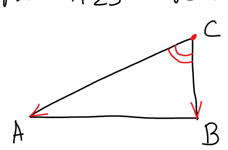
а) $\vec{AB}(2-(-1), -7-(-1), 4-1)$ => $\vec{AB}(3, -6, 3)$ => $|\vec{AB}| = \sqrt{9 + 36 + 9} = \sqrt{54}$

$\vec{BC}(2, 5, 2)$ => $|\vec{BC}| = \sqrt{4 + 25 + 4} = \sqrt{33}$!

! $\vec{AC}(5, -1, 5)$ => $|\vec{AC}| = \sqrt{25 + 1 + 25} = \sqrt{51}$

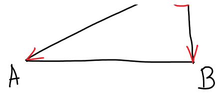
б) $|\vec{AB}| > |\vec{AC}| > |\vec{BC}|$

$\cos \sphericalangle ACB = \cos \gamma$



$\gamma = \sphericalangle(\vec{CA}, \vec{CB})$

$\cos \sphericalangle ACB = \cos \gamma$



$\cos\gamma = \dfrac{(\vec{CA}.\vec{CB})}{|\vec{CA}|.|\vec{CB}|}$

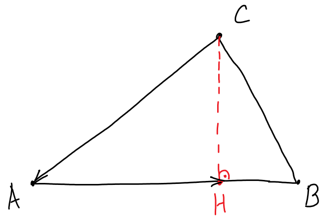
$\vec{CA}(-5, 1, -5)$

$\vec{CB}(-2, -5, -2)$

$\cos\gamma = \dfrac{15}{\sqrt{51}.\sqrt{33}} > 0$

$(\vec{CA}.\vec{CB}) = (-5).(-2) + 1.(-5) + (-5).(-2) = 15$

$\Rightarrow \triangle ABC$ е остроъгълен.



в) $\vec{CH} = \vec{CA} + \vec{AH}$, $\boxed{\vec{AH} = \lambda.\vec{AB}}$ (*)

$\vec{CH} = \vec{CA} + \lambda.\vec{AB}$

$(\vec{CH}.\vec{AB}) = 0 \Rightarrow (\vec{CA} + \lambda.\vec{AB}).\vec{AB} = 0$

$\boxed{(\vec{CA}.\vec{AB}) + \lambda.\vec{AB}^2 = 0}$

$\vec{CA}(-5, 1, -5)$

$\vec{AB}(3, -6, 3)$ $\Rightarrow (\vec{CA}.\vec{AB}) = -15 - 6 - 15 = -36$

$-36 + \lambda.54 = 0$

$\lambda = \frac{2}{3} \longrightarrow$ (*)

$\vec{AH} = \frac{2}{3}.\vec{AB}$

$\vec{OA}(-1, -1, 1)$

$\vec{AB}(3, -6, 3)$

$\vec{OH} - \vec{OA} = \frac{2}{3}.\vec{AB}$

$\boxed{\vec{OH} = \vec{OA} + \frac{2}{3}.\vec{AB}} \Rightarrow$

$x_H = -1 + \frac{2}{3}.3 = 1$

$y_H = -1 + \frac{2}{3}.(-6) = -5$ $\quad H(1, -5, 3)$

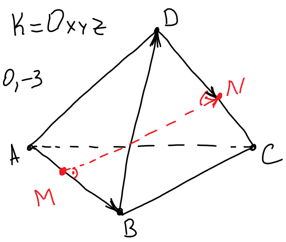
$z_H = 1 + \frac{2}{3}.3 = 3$

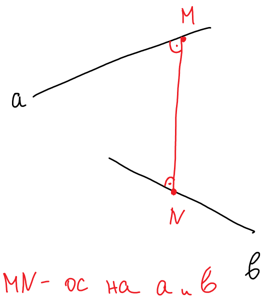
$H(x_H, y_H, z_H)$

---

2 зад. ОКС $K = Oxyz$

$A(0, 0, -2)$
$B(4, 0, -4)$ } $2, 0, -3$
$C(2, 0, 0)$
$D(5, 3, -3)$





а) Да се намерят коорд на т. M и т. N:

$M \in AB$
$N \in CD$
$\vec{MN} \perp \vec{AB}$
$\vec{MN} \perp \vec{CD}$

MN - ос на a и в

$|MN| = d(a, в)$ - разстояние

$\vec{MN} = \vec{MB} + \vec{BD} + \vec{DN}$ $\qquad \vec{MB} = x.\vec{AB}$ $\quad x = ?$

$\vec{MN} = x.\vec{AB} + y.\vec{DC} + \vec{BD}$ $\qquad \vec{DN} = y.\vec{DC}$ $\quad y = ?$

$\begin{cases} (\vec{MN}.\vec{AB}) = 0 \\ (\vec{MN}.\vec{DC}) = 0 \end{cases} \Rightarrow \begin{cases} (x.\vec{AB} + y.\vec{DC} + \vec{BD}).\vec{AB} = 0 \\ (x.\vec{AB} + y.\vec{DC} + \vec{BD}).\vec{DC} = 0 \end{cases}$

$\begin{cases} x.\vec{AB}^2 + y.(\vec{DC}.\vec{AB}) + (\vec{BD}.\vec{AB}) = 0 \\ x.(\vec{AB}.\vec{DC}) + y.\vec{DC}^2 + (\vec{BD}.\vec{DC}) = 0 \end{cases}$

$\vec{AB}(4, 0, -2)$ $\qquad \vec{AB}^2 = 16 + 4 = 20$

$\vec{DC}(-3, -3, 3)$ $\qquad (\vec{DC}.\vec{AB}) = -12 - 6 = -18$

$\vec{BD}(1, 3, 1)$ $\qquad (\vec{BD}.\vec{AB}) = 4 - 2 = 2$

$\vec{DC}^2 = 27$

$(\vec{BD}.\vec{DC}) = -3 - 9 + 3 = -9$

$\begin{cases} 20x - 18.y = -2 & /:2 \\ -18x + 27y = 9 & /:3 \end{cases}$

$\begin{cases} 10x - 9y = -1 \\ -6x + 9.y = +3 \end{cases}$ (+)

$4x = 2 \Rightarrow \boxed{x = \frac{1}{2}}$

$9y = 10.\frac{1}{2} + 1 = 6$

$\boxed{y = \frac{2}{3}}$

$\vec{MB} = \frac{1}{2}.\vec{AB} \Rightarrow$ M е средата на AB $\Rightarrow \boxed{M(2, 0, -3)}$

$\vec{DN} = \frac{2}{3}\cdot\vec{DC} \Rightarrow \vec{ON} - \vec{OD} = \frac{2}{3}\cdot\vec{DC}$

$\vec{ON} = \vec{OD} + \frac{2}{3}\cdot\vec{DC}$ $\quad \vec{OD}(5,3,-3)$, $\vec{DC}(-3,-3,3)$ $\Rightarrow$

$x_N = 5 + \frac{2}{3}\cdot(-3) = 3$

$y_N = 3 + \frac{2}{3}\cdot(-3) = 1$

$z_N = -3 + \frac{2}{3}\cdot 3 = -1$

$N(3,1,-1)$

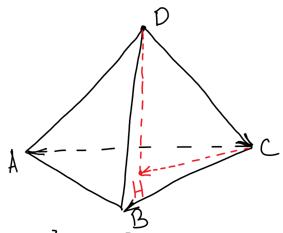
$d(AB,CD) = |\vec{MN}|$

б) (Упр.)

$\vec{CH}$, $\vec{CA}$ и $\vec{CB}$ са компланарни

$\vec{CH} = \alpha.\vec{CA} + \beta.\vec{CB}$

$\vec{DH} = \vec{DC} + \vec{CH} = \vec{DC} + \alpha.\vec{CA} + \beta.\vec{CB}$



$$\left|\begin{array}{l}(\vec{DH}.\vec{CA}) = 0 \\ (\vec{DH}.\vec{CB}) = 0\end{array}\right.$$

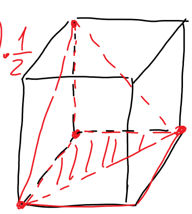
$\alpha = ?$, $\beta = ?$ $\Rightarrow \vec{CH} = \alpha.\vec{CA} + \beta.\vec{CB}$

Отг: $H(5,0,-3)$

$V_{призма} = V_1$ обща основа

$V_{пирамида} = V_2$ равни височини

$V_2 = \frac{1}{3}\cdot V_1 \cdot \frac{1}{2}$



Видове произведения на вектори

Скаларно

$(\vec{a}.\vec{b})$

$|\vec{a}|$

$\cos\sphericalangle(\vec{a},\vec{b})$

$(\vec{a}.\vec{b}) = 0 \Leftrightarrow \vec{a} \perp \vec{b}$

Векторно

$\vec{a} \times \vec{b}$

$S_{усп.} = |\vec{a} \times \vec{b}|$

$\{a,b\} \to \{a, b, a\times b\}$

$\vec{a} \times \vec{b} = \vec{0} \Leftrightarrow \vec{a} \parallel \vec{b}$

$\vec{a} \times \vec{a} = \vec{0}$

Смесено

$(\vec{a}\vec{b}\vec{c}) = (\vec{a} \times \vec{b}).\vec{c}$

$V_{парал.} = |(\vec{a}\vec{b}\vec{c})|$

$V_{тетр.} = \frac{1}{6}\cdot|(\vec{a}\vec{b}\vec{c})|$

Ориентация

$(\vec{a}\vec{b}\vec{c}) = 0 \Leftrightarrow \vec{a}, \vec{b}, \vec{c}$ са л.з.

$(\vec{a}\vec{b}\vec{a}) = 0$

1 зад.

$\vec{a}, \vec{b}: |\vec{a}| = 2, |\vec{b}| = 1, \sphericalangle(\vec{a},\vec{b}) = \frac{\pi}{2}$

Да се определи неизв. вектор $\vec{p}$: $(\vec{a}.\vec{p}) = 4$, $(\vec{b}.\vec{p}) = 2$, $(\vec{a}\vec{b}\vec{p}) = -8 \neq 0$

$\vec{p} = \alpha.\vec{a} + \beta.\vec{b}$, но $\vec{a}, \vec{b}$ и $\vec{p}$ са ЛНЗ $\{$от $(\vec{a}\vec{b}\vec{p}) = -8\}$

$\vec{p} = \alpha.\vec{a} + \beta.\vec{b} + \gamma.(\vec{a}\times\vec{b})$

$(\vec{a}.\vec{p}) = 4 = \vec{a}.(\alpha.\vec{a} + \beta.b + \gamma.a\times b) \Rightarrow \alpha.\underbrace{\vec{a}^2}_{4} + \beta.\underbrace{(\vec{a}\vec{b})}_{0} + \gamma.\underbrace{(\vec{a}\times\vec{b}).\vec{a}}_{0} = 4$

$4\alpha = 4 \Rightarrow \boxed{\alpha = 1}$

$(\vec{b}.\vec{p}) = 2 = \alpha.(\vec{a}.\vec{b}) + \beta.\vec{b}^2 + \gamma((\vec{a}\times\vec{b}).b) \Rightarrow \alpha.0 + \beta.1 + \gamma.0 = 2$

$\boxed{\beta = 2}$

$(a\times b).p = -8 = \alpha.\underbrace{((\vec{a}\times\vec{b}).\vec{a})}_{0} + \beta\underbrace{(\vec{a}\times\vec{b}).\vec{b}}_{0} + \gamma.\underbrace{(\vec{a}\times\vec{b})^2}_{?}$

$(\vec{a}\times\vec{b})^2 = \vec{a}^2.\vec{b}^2 - (\vec{a}.\vec{b})^2 = 4.1 - 0^2 \Rightarrow 4\gamma = -8 \Rightarrow \gamma = -2$

$\vec{p} = 1.\vec{a} + 2.\vec{b} - 2.(\vec{a}\times\vec{b})$

21.04. Сряда от 15:30
Първо контролно по Геометрия

02.06. Сряда от 15:30
Второ контролно по Геометрия