Затвореност относно регулярните операции

Един език L се нарича автоматен, ако има краен автомат A такъв, че $L(A) = L$.

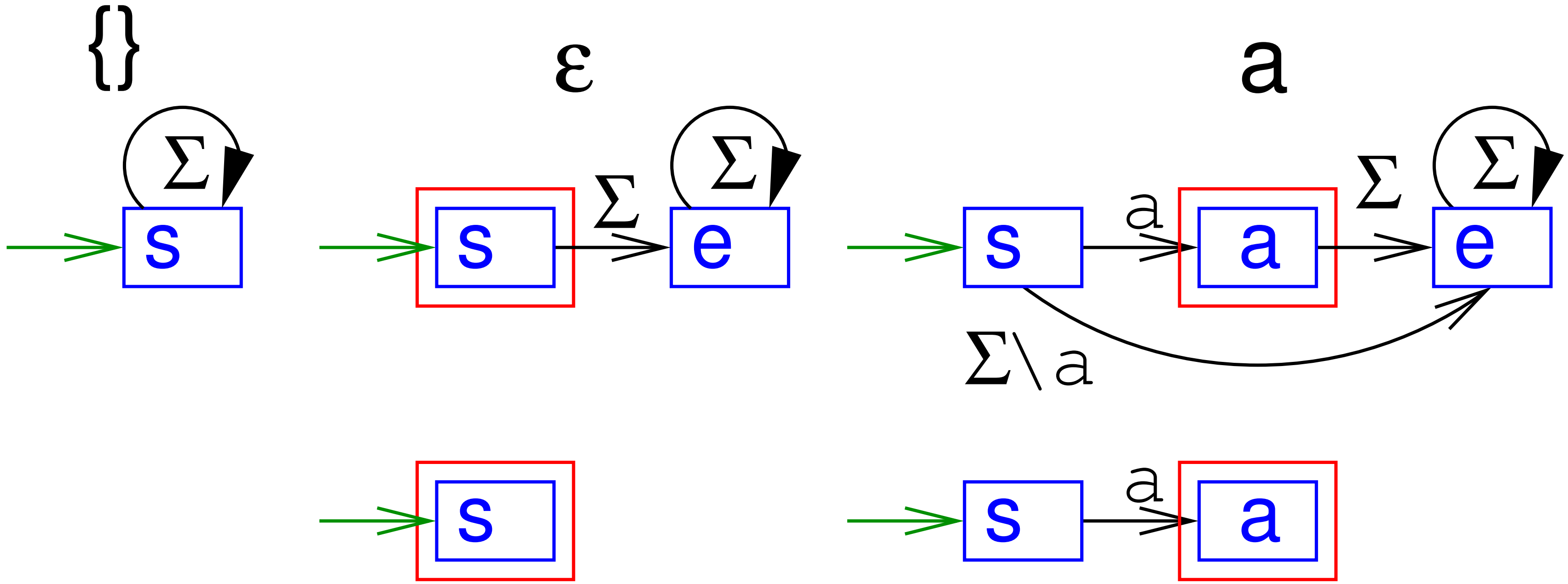
Теорема Всеки регулярен език е автоматен.

Д-во идея:

ще построим автомати, разпознаващи основните езици (основните езици са автоматни)

ще покажем, че регулярните операции запазват автоматността

Базов случай

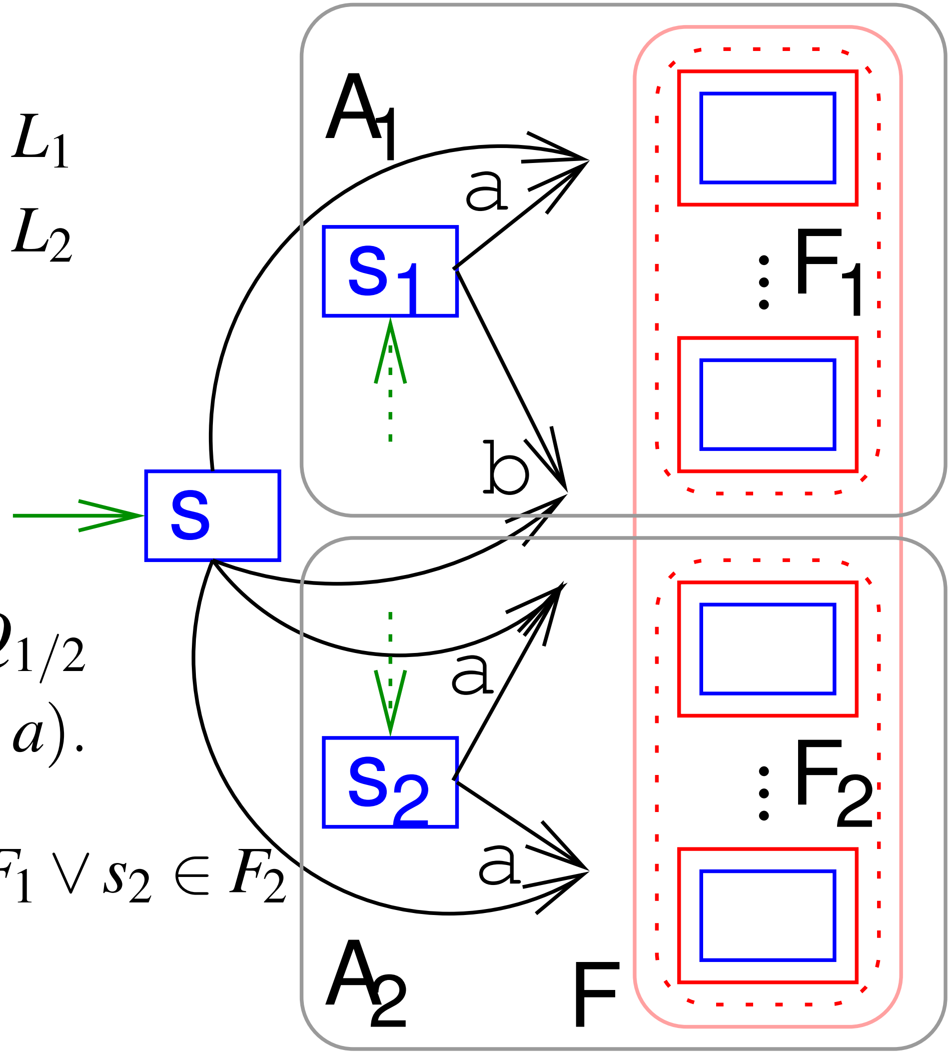


$L_1 \cup L_2$

$A_1 = (Q_1, \Sigma, \delta_1, s_1, F_1)$ и $L(A_1) = L_1$
$A_2 = (Q_2, \Sigma, \delta_2, s_2, F_2)$ и $L(A_2) = L_2$
и БОО $Q_1 \cap Q_2 = \emptyset$

$A := (\{s\} \cup Q_1 \cup Q_2, \Sigma, \delta, s, F)$
δ е дефинирана като $\delta_{1/2}$ за $Q_{1/2}$
$\forall a \in \Sigma : \delta(s,a) := \delta(s_1,a) \cup \delta(s_2,a)$.

$$F := \begin{cases} F_1 \cup F_2 \cup \{s\} & \text{ако } s_1 \in F_1 \vee s_2 \in F_2 \\ F_1 \cup F_2 & \text{иначе} \end{cases}$$



Д-во на $L_1 \cup L_2 \subseteq L(A)$

Нека $w \in L_1 = L(A_1)$ (произволна).

Ако $w = \varepsilon$

$\longrightarrow s_1 \in F_1 \longrightarrow s \in F \longrightarrow w \in L(A)$.

Ако $w = ax$:

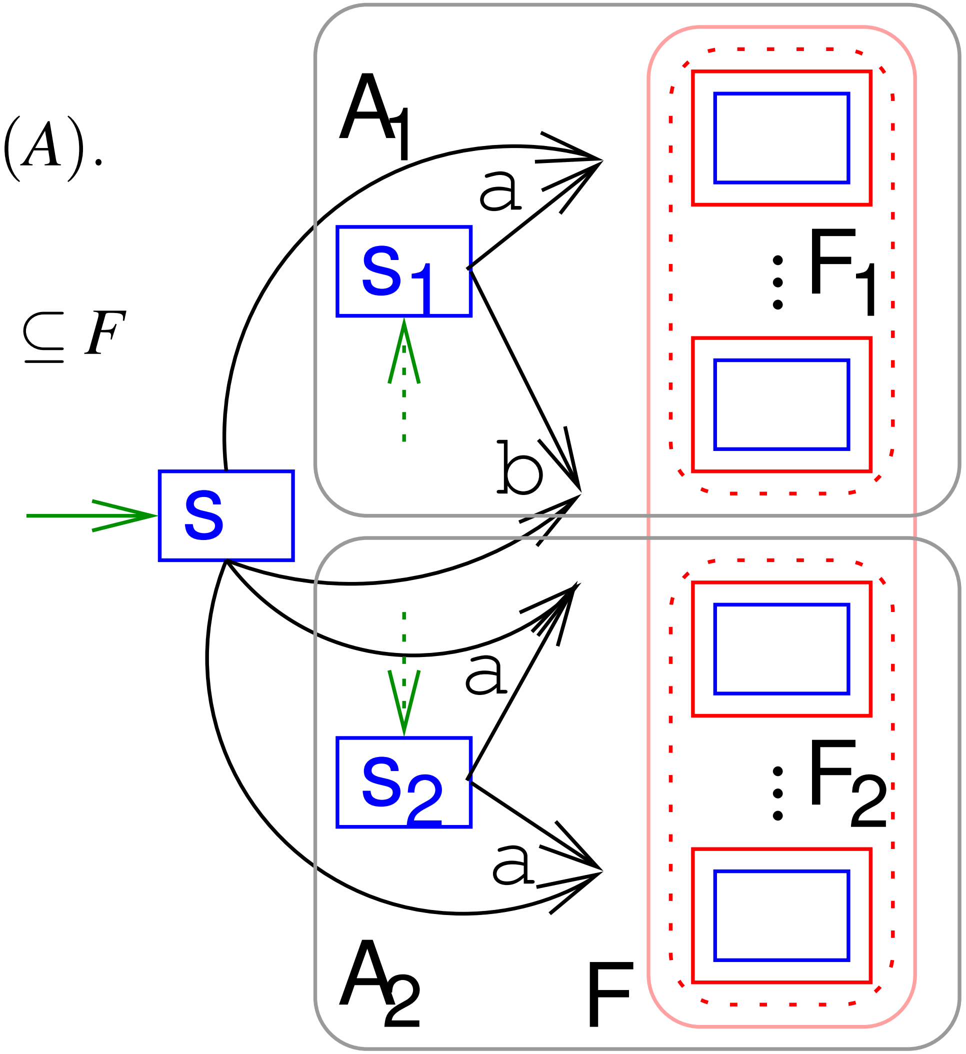
$\longrightarrow \exists$ път $P_1 = s_1 \overset{a}{\Rightarrow} q_1 \overset{x}{\Rightarrow} f_1 \in F_1 \subseteq F$

$\longrightarrow \exists$ път $P = s \overset{a}{\Rightarrow} q_1 \overset{x}{\Rightarrow} f_1 \in F$

$\longrightarrow w \in L(A)$.

$w \in L_2 = L(A_2)$

$\longrightarrow \cdots \to w \in L(A)$.



Д-во на $L(A) \subseteq L_1 \cup L_2$

Нека w е произволна дума $w \in L(A)$.

Ако $w = \varepsilon \longrightarrow s \in F \longrightarrow s_1 \in F_1 \vee s_2 \in F_2$

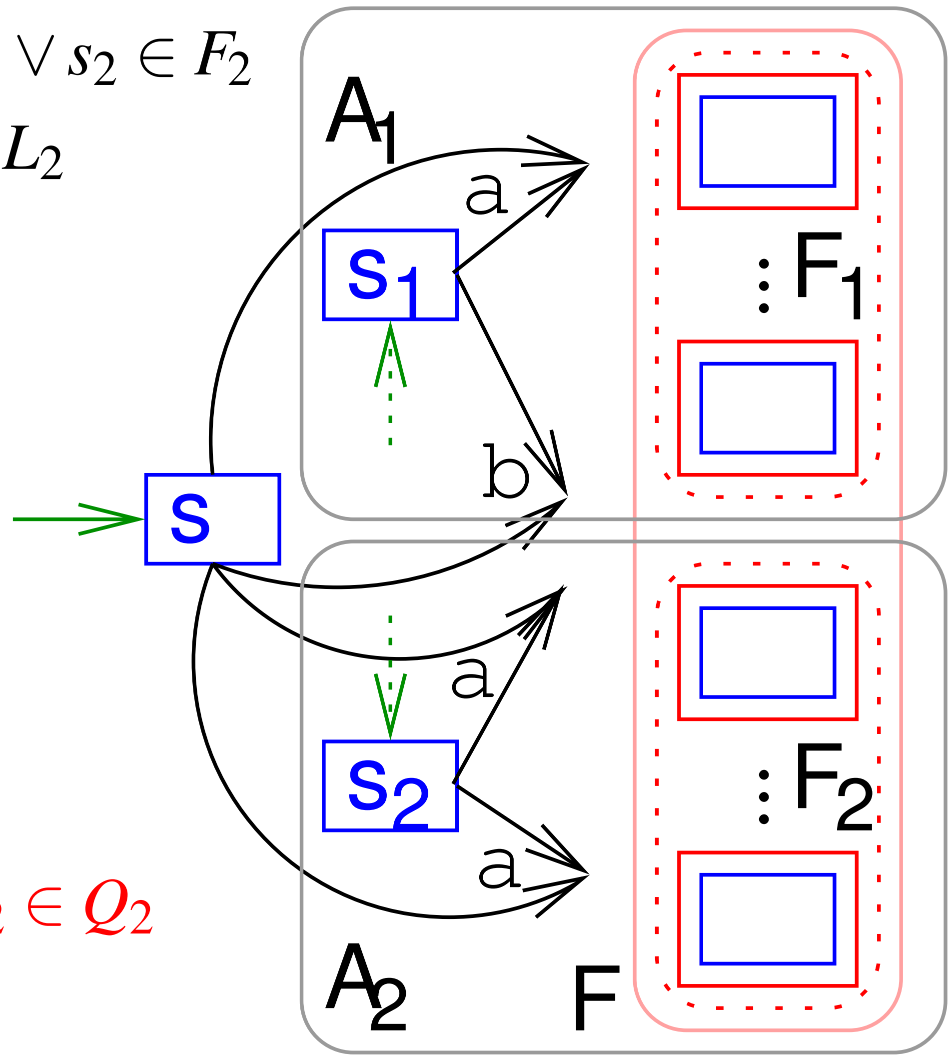
$\longrightarrow \varepsilon \in L_1 \vee \varepsilon \in L_2 \longrightarrow \varepsilon \in L_1 \cup L_2$

Ако $w = ax$:

$\longrightarrow \exists$ път $P = s \overset{a}{\Rightarrow} q \overset{x}{\Rightarrow} f \in F$.

Ако $q = q_1 \in Q_1$:

$\longrightarrow \exists$ път $P_1 = s_1 \overset{a}{\Rightarrow} q_1 \overset{x}{\Rightarrow} f \in F_1$.



(само състояния,

достижими от q_1 са в Q_1.)

$\longrightarrow ax = w \in L_1 \subseteq L_1 \cup L_2$

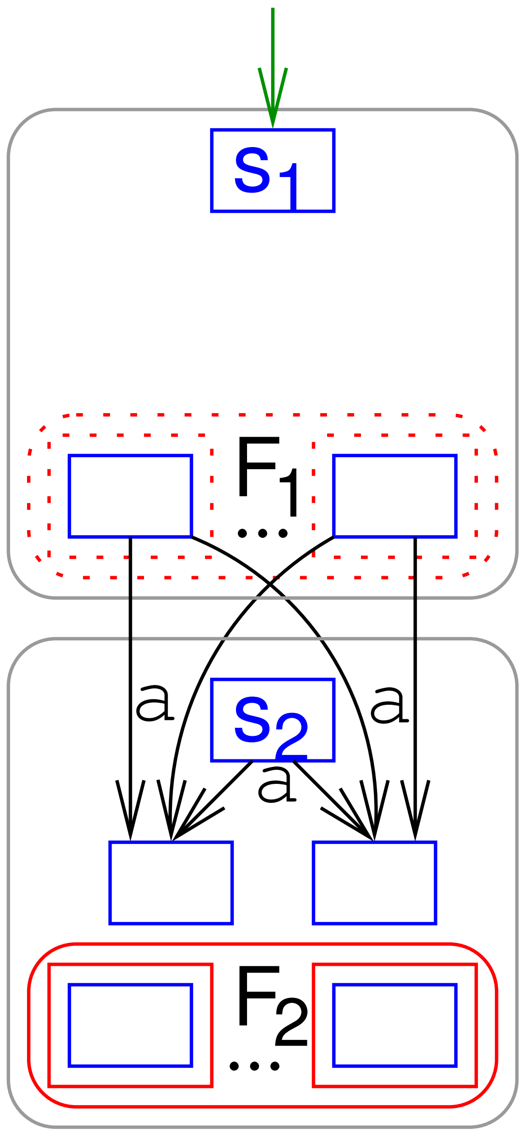
В противен случай: $\longrightarrow q = q_2 \in Q_2$

$\longrightarrow \exists$ път $P_2 = s_2 \overset{a}{\Rightarrow} q_2 \overset{x}{\Rightarrow} f \in F_2$.

$\longrightarrow ax = w \in L_2 \subseteq L_1 \cup L_2$

$\square$

$L_1 \cdot L_2$



$A_1 = (Q_1, \Sigma, \delta_1, s_1, F_1)$ и $L(A_1) = L_1$
$A_2 = (Q_2, \Sigma, \delta_2, s_2, F_2)$ и $L(A_2) = L_2$
и $Q_1 \cap Q_2 = \emptyset$

$A := (Q_1 \cup Q_2, \Sigma, \delta, s_1, F), \forall a \in \Sigma :$

$$\delta(q,a) := \begin{cases} \delta_1(q,a) & \text{ако } q \in Q_1 \setminus F_1 \\ \delta_1(q,a) \cup \delta_2(s_2,a) & \text{ако } q \in F_1 \\ \delta_2(q,a) & \text{иначе} \end{cases}$$

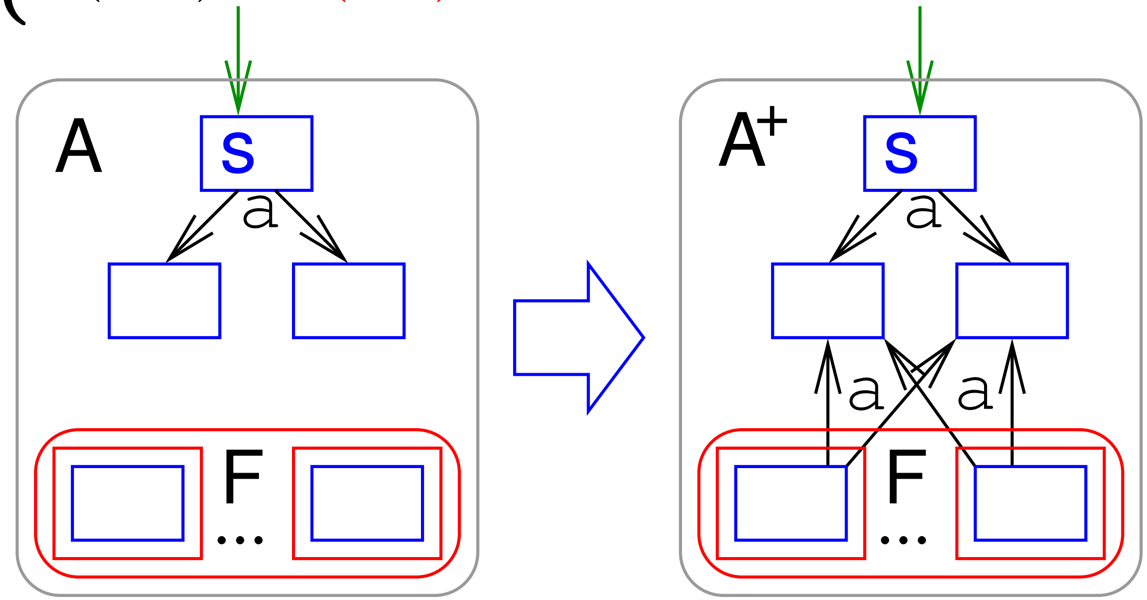
$$F := \begin{cases} F_1 \cup F_2 & \text{ако } s_2 \in F_2 \\ F_2 & \text{иначе} \end{cases}$$

Позитивна обвивка $L^+ = \bigcup_{i \geq 1} L^i$

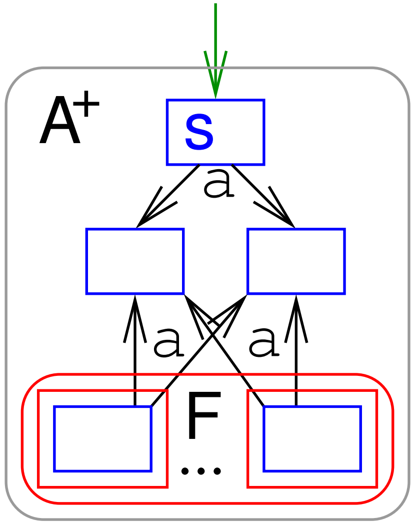
$A = (Q, \Sigma, \delta, s, F)$ и $L(A) = L$

$A^+ := (Q, \Sigma, \delta^+, s, F), \forall a \in \Sigma :$

$$\delta^+(q,a) := \begin{cases} \delta(q,a) & \text{ако } q \in Q \setminus F \\ \delta(q,a) \cup \delta(s,a) & \text{ако } q \in F \end{cases}$$



Д-во на $L(A^+) \subseteq L^+$



Нека $w \in L(A^+)$ е произволна и $w \neq \varepsilon$

Нека $P = s \overset{a_0}{\Rightarrow} q_0 \overset{*}{\Rightarrow} f$ е приемащ път за w.

Декомпозираме P на преходи от вида $f_j \overset{a_j}{\Rightarrow} q_j$

by $q_j \notin \delta(f_j, a_j)$, $j \in 1..i$, $i \geq 0$.

$\longrightarrow f_j \in F$, $q_j \in \delta(s, a_j)$.

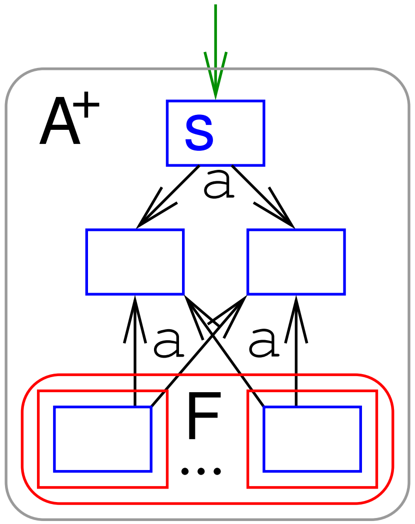
$$P = s \overbrace{\overset{a_0}{\Rightarrow} q_0 \overset{x_0}{\Rightarrow} f_1 \overset{a_1}{\Rightarrow} q_1 \overset{x_1}{\Rightarrow} f_2 \overset{*}{\Rightarrow} f_i \overset{a_i}{\Rightarrow} q_i \overset{x_i}{\Rightarrow}}^{a_0 x_0 a_1 x_1 \cdots a_i x_i = w} f$$

Дефинираме $P_j := s \overset{a_j}{\Rightarrow} q_j \overset{x_j}{\Rightarrow} f_{j+1}$ (с $f_{i+1} := f$).

$\longrightarrow \forall j \in 0..i : P_j$ е един приемащ път A.

$\longrightarrow w \in L^+$ $\square$

Д-во на $L^i \subseteq L(A^+)$ за $i \geq 1$



Нека $w = w_1 \cdots w_i \in L^i$ $(\varepsilon \neq w_i \in L)$.

Да разгледаме $P_j = s \stackrel{a_j}{\Rightarrow} q_j \stackrel{x_j}{\Rightarrow} f_j$, $j \in 1..i$, $f_j \in F$,

които свидетелстват за $w_1 \in L, \ldots, w_i \in L$.

$$\longrightarrow P = s \overbrace{\stackrel{a_1}{\Rightarrow} q_1 \stackrel{x_1}{\Rightarrow} f_1 \stackrel{a_2}{\Rightarrow} q_2 \stackrel{x_2}{\Rightarrow} f_2 \stackrel{*}{\Rightarrow} f_{i-1} \stackrel{a_i}{\Rightarrow} q_i \stackrel{x_i}{\Rightarrow}}^{w} f_i$$

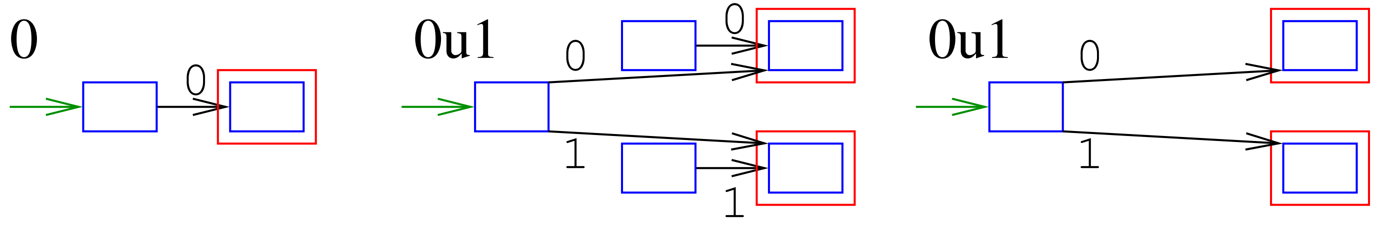
е път в A^+, свидетелстващ за $w \in L(A^+)$.

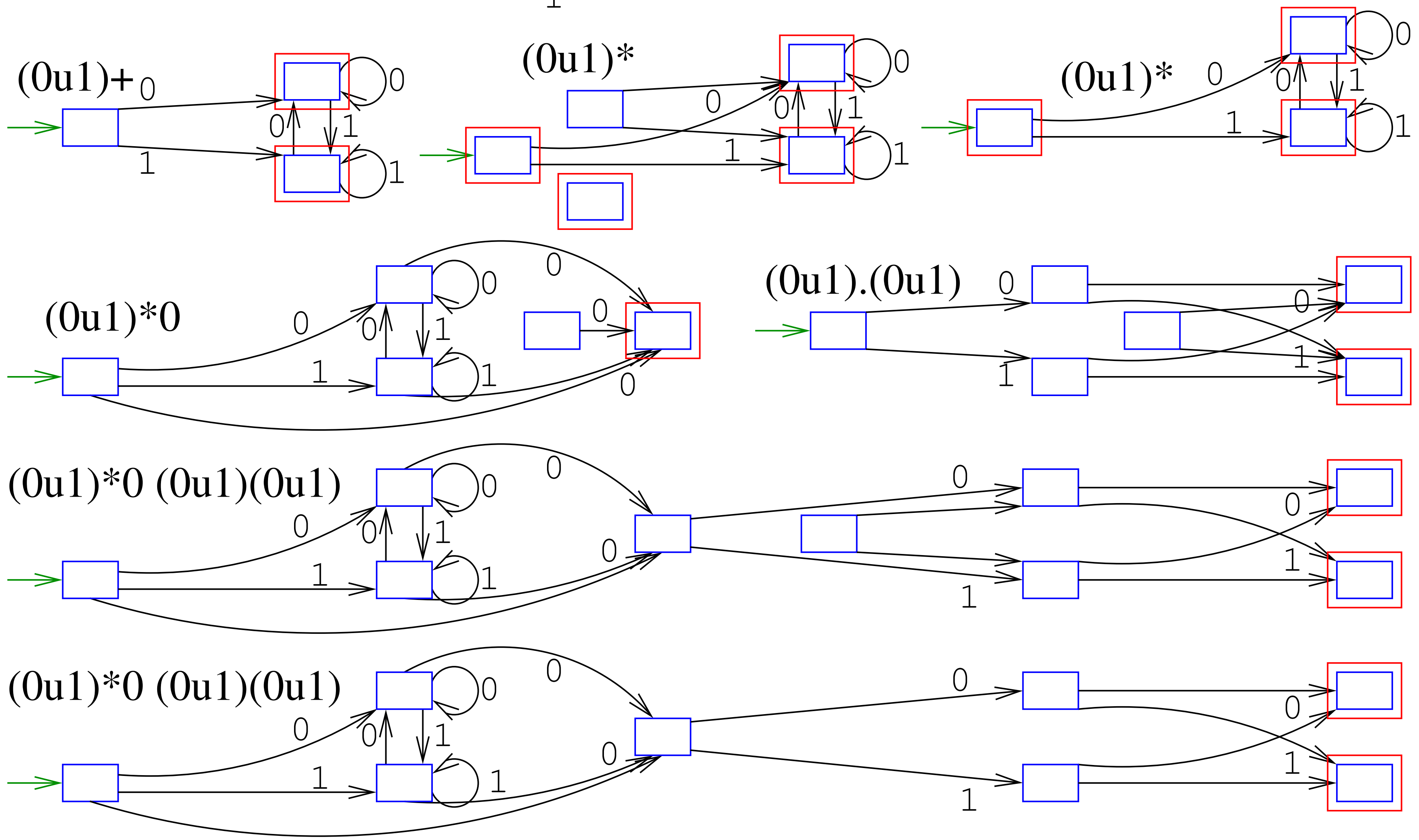
$\longrightarrow w \in L(A^+)$ $\square$

L^* -звезда на Клини

Построяваме автомат за $\varepsilon \cup L^+ = L^*$.

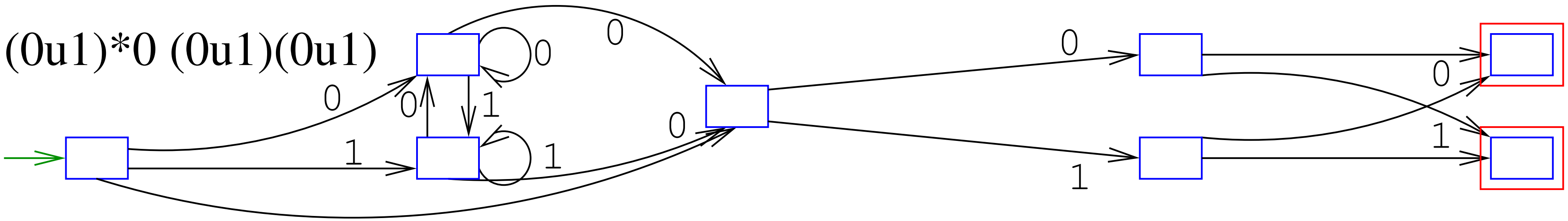
Beispiele





Пример

(0u1)*0 (0u1)(0u1)



(0u1)*0 (0u1)(0u1)