ДР 2

1. Разстояние от точка до права

$K = Oxy$

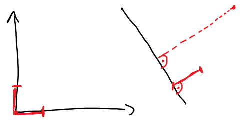
$g: A.x + B.y + C = 0$ – общо

Ако $K = Oxy$ е ОКС

$g: \dfrac{Ax + B.y + C}{\sqrt{A^2+B^2}} = 0$ – нормално

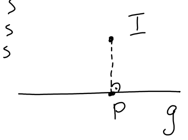
$M_0(x_0, y_0)$

$$\boxed{\delta(M_0, g) = \frac{A.x_0 + B.y_0 + C}{\sqrt{A^2+B^2}}}$$!!!!



ОКС $K = Oxyz$!

$g: \begin{cases} x = x_1 + p_1 . s \\ y = y_1 + p_2 . s \\ z = z_1 + p_3 . s \end{cases}$



$P \in g$ ....

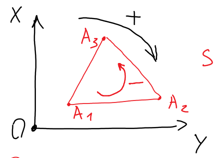
$\alpha: A.x + B.y + C.z + D = 0$

$$\delta(M_0, \alpha) = \frac{A.x_0 + B.y_0 + C.z_0 + D}{\sqrt{A^2+B^2+C^2}}$$

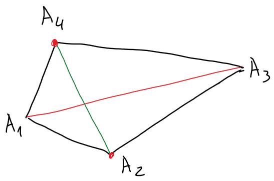
---

2. ОКС $K = \underline{O}xy$

$$S_{\Delta A_1A_2A_3} = \frac{1}{2} \cdot \underbrace{\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}}$$ – ориентирано лице



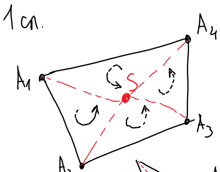
$S_{\Delta A_1A_2A_3} < 0$



$\ell_{A_1A_3}(A_4) . \ell_{A_1A_3}(A_2) < 0$

$\ell_{A_2A_4}(A_1) . \ell_{A_2A_4}(A_3) < 0$

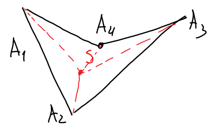
1 сл.



$S_{\Delta A_1A_2S} . S_{\Delta A_2A_3S} . S_{\Delta A_3A_4S} . S_{\Delta A_4A_1S} > 0 \iff$

$\iff S$ е вътрешна за $A_1A_2A_3A_4$

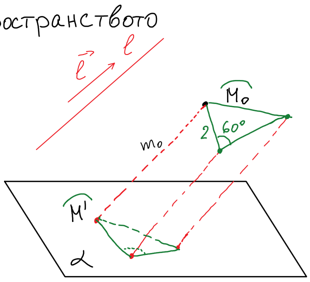
2 сл.



Успоредно проектиране в пространството

α – проекционна равнина

$\ell \nparallel \alpha$, ℓ – проектиращо направление

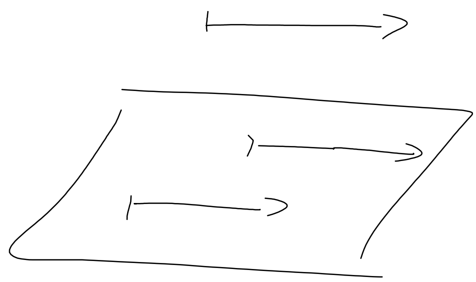


$M_0(x_0, y_0, z_0)$

$\exists!\ m_0 \begin{cases} \ni M_0 \\ \parallel \ell \end{cases}$

$\exists!\ M' = m_0 \cap \alpha$

$M_0 \longrightarrow M'$



1 зад. АКС $Oxyz$

$\underline{\alpha: x + y + 2z + 2 = 0}$, $\vec{e}(1, -2, 0)$

Да се намери аналитично представяне на успоредно проектиране на пространството върху α по направл. на $\vec{e}$.

$\alpha, \vec{e}, M_0(x_0, y_0, z_0) \xrightarrow{?} M'(x', y', z')$

1) проверка дали $\vec{e} \parallel \alpha$?

$\alpha: \underset{A}{1}.x + \underset{B}{1}.y + \underset{C}{2}.z + 2 = 0$

$\vec{e}(1, 1, 0)$

$A.1 + B.(-2) + C.0 = ?$

$1.1 + 1.(-2) + 2.0 = -1 \neq 0 \Rightarrow \vec{e} \nparallel \alpha$

2) $M_0(x_0, y_0, z_0)$ ... $\begin{cases} x = x_0 + 1.s \end{cases}$

$$1.1+1.(-2)+2.0=-1\neq 0 \Rightarrow \vec{\ell}\nparallel\alpha$$

2) $M_0(x_0,y_0,z_0)$, $\vec{\ell}(1,-2,0)$ $\Rightarrow$ $m_0:\begin{cases}x=x_0+1.s\\ y=y_0+(-2).s\ ,\ s\in\mathbb{R}\\ z=z_0+0.s\end{cases}$

$M'=m_0\cap\alpha \Rightarrow \left|\begin{array}{l}x=x_0+s\\ y=y_0-2s\\ z=z_0\\ x+y+2.z+2=0\end{array}\right.$!

$$(x_0+s)+(y_0-2s)+2z_0+2=0$$

$$-1.s+(x_0+y_0+2z_0+2)=0$$

$$s'=x_0+y_0+2z_0+2 \longrightarrow m_0$$

$s' \updownarrow M'$

$$M':\begin{cases}x'=x_0+x_0+y_0+2z_0+2\\ y'=y_0-2.(x_0+y_0+2z_0+2)\\ z'=z_0\end{cases}\qquad \begin{array}{l}x'=2x_0+y_0+2z_0+2\\ y'=-2x_0-y_0-4z_0-4\\ z'=z_0\end{array}$$

! $\begin{pmatrix}x'\\y'\\z'\end{pmatrix}=\begin{pmatrix}2&1&2\\-2&-1&-4\\0&0&1\end{pmatrix}.\begin{pmatrix}x_0\\y_0\\z_0\end{pmatrix}+\begin{pmatrix}2\\-4\\0\end{pmatrix}$ аналит. представяне на успоредно проектиране

---

3 зад. (Ортогонално проектиране)

ОКС $K=Oxyz$

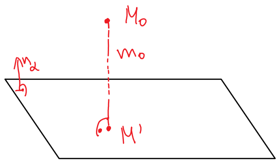
$\alpha: x-y-z+4=0$

?, аналитично представяне на ортогонално проектиране на пространството върху равн. α.

1) $\alpha: x-y-z+4=0$

$\vec{n}_\alpha(1,-1,-1)$

$M_0(x_0,y_0,z_0) \Rightarrow \exists!\ m_0\begin{cases}\ni M_0\\ \parallel\vec{n}_\alpha\end{cases}$



$$m_0:\begin{cases}x=x_0+1.p\\ y=y_0-1.p\ ,\ p\in\mathbb{R}\\ z=z_0-1.p\end{cases}$$

$M'=m_0\cap\alpha \Rightarrow \left|\begin{array}{l}x=x_0+p\\ y=y_0-p\\ z=z_0-p\\ x-y-z+4=0\end{array}\right. \Rightarrow$

$$\Rightarrow x_0+p-(y_0-p)-(z_0-p)+4=0$$

$$3.p+(x_0-y_0-z_0+4)=0$$

$$p=-\frac{x_0-y_0-z_0+4}{3}\longrightarrow m_0 \Rightarrow \begin{array}{l}x'=\overset{3}{x_0}+\left(-\frac{x_0-y_0-z_0+4}{3}\right)\\ y'=\overset{3}{y_0}-\left(-\frac{x_0-y_0-z_0+4}{3}\right)\\ z'=\overset{3}{z_0}-\left(-\frac{x_0-y_0-z_0+4}{3}\right)\end{array}$$

$$\begin{array}{l}x'=\frac{1}{3}.(2x_0+y_0+z_0-4)\\ y'=\frac{1}{3}.(x_0+2y_0-z_0+4)\\ z'=\frac{1}{3}.(x_0-y_0+2z_0+4)\end{array}\Rightarrow \boxed{\begin{pmatrix}x'\\y'\\z'\end{pmatrix}=\frac{1}{3}.\begin{pmatrix}2&1&1\\1&2&-1\\1&-1&2\end{pmatrix}.\begin{pmatrix}x_0\\y_0\\z_0\end{pmatrix}+\frac{1}{3}.\begin{pmatrix}-4\\4\\4\end{pmatrix}}$$

Централно проектиране

α – проекционна равнина

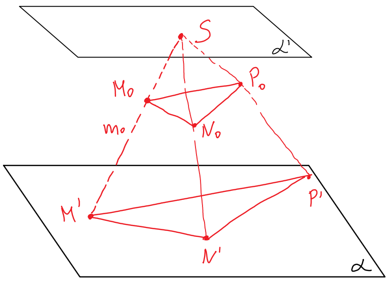
Централно проектиране

α - проекционна равнина

S - проекционен център

$S \notin \alpha$!

$\alpha' \begin{cases} \ni S \\ \parallel \alpha \end{cases}$ точките от α' нямат образи при централното проектиране с ц-р S върху α



4 зад. АКС $Oxyz$

$\alpha: x - y + 2z - 1 = 0$, $S(2,0,1)$

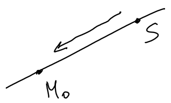
? аналитично представяне на централно проектиране на пространството с ц-р S върху α.

1) Дали $S \in \alpha$?

$2 - 0 + 2.1 - 1 = 3 \neq 0 \Rightarrow S \notin \alpha$

2) $M_0(x_0, y_0, z_0) \notin \alpha'$

$\exists!\, m_0 \begin{cases} \ni M_0(x_0, y_0, z_0) \\ \ni S(2,0,1) \end{cases}$



$m_0 \parallel \overrightarrow{SM_0}(x_0 - 2, y_0, z_0 - 1)$

$$m_0 \begin{cases} x = 2 + (x_0 - 2).t \\ y = 0 + y_0.t \\ z = 1 + (z_0 - 1).t \end{cases}$$

$$M' = m_0 \cap \alpha \Rightarrow \begin{cases} x = 2 + (x_0 - 2).t \\ y = 0 + y_0.t \\ z = 1 + (z_0 - 1).t \\ x - y + 2z - 1 = 0 \end{cases}$$

$$2 + (x_0 - 2).t - y_0.t + 2.(1 + (z_0 - 1).t) - 1 = 0$$

$$t.(x_0 - 2 - y_0 + 2z_0 - 2) + 3 = 0$$

$$t.\underbrace{(x_0 - y_0 + 2.z_0 - 4)}_{\alpha'|_{M_0}} + 3 = 0 \Rightarrow t = -\frac{3}{x_0 - y_0 + 2z_0 - 4} \to m_0$$

$$M': \begin{cases} x' = 2 - \dfrac{3.(x_0 - 2)}{x_0 - y_0 + 2z_0 - 4} = \dfrac{-x_0 - 2y_0 + 4z_0 - 2}{x_0 - y_0 + 2z_0 - 4} \\ y' = 0 - \dfrac{3.y_0}{x_0 - y_0 + 2z_0 - 4} = \dfrac{-3y_0}{x_0 - y_0 + 2z_0 - 4} \\ z' = 1 - \dfrac{3.(z_0 - 1)}{x_0 - y_0 + 2z_0 - 4} = \dfrac{x_0 - y_0 - z_0 - 1}{x_0 - y_0 + 2z_0 - 4} \end{cases}$$

Контролно по Геометрия: 02.06, Сряда, 16:30

Освобождаване от писмен изпит с оценка $\geq 4{,}50$