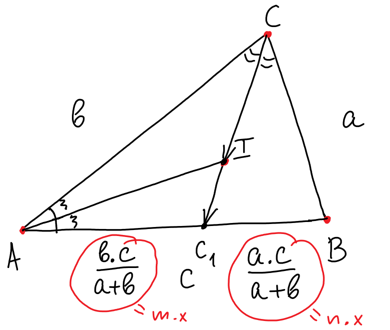
Основна задача за изразяване на център на вписана окръжност

1) CC_1 – ъглополовяща $\Rightarrow \left| \begin{array}{l} \dfrac{AC_1}{C_1B} = \dfrac{b}{a} \\ AC_1 + C_1B = c \end{array} \right.$



$\Rightarrow AC_1 = \dfrac{b.c}{a+b}, \quad C_1B = \dfrac{a.c}{a+b}$

2) $\triangle ACC_1$, AI – ъглополовяща

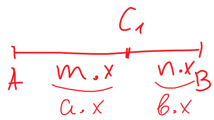
$\left| \dfrac{CI}{IC_1} = \dfrac{CA}{AC_1} = \dfrac{b}{\frac{b.c}{a+b}} = \dfrac{a+b}{c} \right. \Rightarrow \begin{array}{l} CI = (a+b).x \\ IC_1 = c.x \\ CC_1 = (a+b+c).x \end{array}$

$CI : CC_1 = \boxed{\dfrac{a+b}{a+b+c}}$! $\Rightarrow \vec{CI} = \dfrac{a+b}{a+b+c} \cdot \vec{CC_1}$!

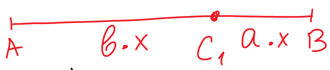
$\vec{CI} = \vec{OI} - \vec{OC}$, $\quad \vec{CC_1} = \vec{OC_1} - \vec{OC}$, т. O – произволна

$\vec{OI} = \underbrace{\vec{OC}}_{a+b+c} + \dfrac{a+b}{a+b+c} \cdot \vec{OC_1} - \dfrac{a+b}{a+b+c} \cdot \vec{OC} = \dfrac{c}{a+b+c} \cdot \vec{OC} + \dfrac{a+b}{a+b+c} \cdot \vec{OC_1} \; (*)$?

За $\vec{OC_1}$ от осн. зад $\vec{OC_1} = \dfrac{n}{m+n} \cdot \vec{OA} + \dfrac{m}{m+n} \cdot \vec{OB}$



$\vec{OC_1} = \dfrac{a}{a+b} \cdot \vec{OA} + \dfrac{b}{a+b} \cdot \vec{OB} \longrightarrow (*)$



$\vec{OI} = \dfrac{c}{a+b+c} \cdot \vec{OC} + \dfrac{a+b}{a+b+c} \cdot \left(\dfrac{a}{a+b} \cdot \vec{OA} + \dfrac{b}{a+b} \cdot \vec{OB} \right)$

$\vec{OI} = \dfrac{c}{a+b+c} \cdot \vec{OC} + \dfrac{a}{a+b+c} \cdot \vec{OA} + \dfrac{b}{a+b+c} \cdot \vec{OB}$

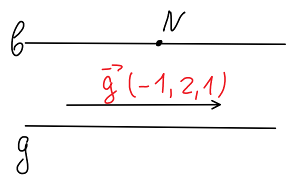
$$\boxed{\vec{OI} = \dfrac{a.\vec{OA} + b.\vec{OB} + c.\vec{OC}}{a+b+c}}$$

Права и равнина в пространството

2 зад. ОКС $K = Oxyz$

$g: \begin{cases} 2x + y - 3 = 0 \\ x + z + 2 = 0 \end{cases}$ т. $M(1,2,3)$
т. $N(5,-1,1) \notin g$ (проверка)

а) ?, парам. уравнения на правата $b \begin{cases} \ni N \\ \parallel g \end{cases}$



1) Параметрични уравнения на g

Изб. $x = s \Rightarrow y = 3 - 2s$, $z = -2 - s$ $\Rightarrow g: \begin{cases} x = 1.s \\ y = 3 - 2.s \\ z = -2 - 1s \end{cases}, s \in \mathbb{R} \Rightarrow g \parallel \vec{g}(-1,2,1)$
$(1,-2,-1)$

2) $b \parallel \vec{g}(-1,2,1)$
$b \ni N(5,-1,1)$ $\Rightarrow b: \begin{cases} x = 5 - p \\ y = -1 + 2p \\ z = 1 + p \end{cases}, p \in \mathbb{R}$

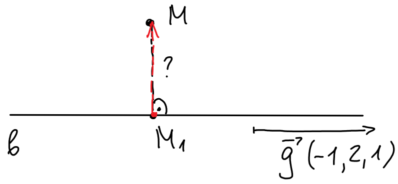
б) ?, разстоянието от т. M до правата в

$d(M,b)=?$ $M(1,2,3)$

Дали $M \in b$? $5-p=1 \Rightarrow p=4$
$-1+2p=2 \Rightarrow p=3/2$ $\Rightarrow M \notin b$
$1+p=3 \Rightarrow p=2$

$d(M,b)=|\overrightarrow{M_1M}|$

$$b:\begin{cases} x=5-p \\ y=-1+2p \\ z=1+p \end{cases}$$



Търсим т. $M_1 \begin{cases} \in b \\ \overrightarrow{M_1M} \perp \vec{g}(-1,2,1) \end{cases}$

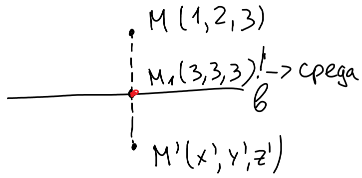
$M_1 \in b \Rightarrow M_1(5-p,\ -1+2.p,\ 1+p)$
$M\ (1,\ 2,\ 3)$
$\Rightarrow \overrightarrow{M_1M}(1-(5-p),\ 2-(-1+2p),\ 3-(1+p))$

$\overrightarrow{M_1M}(-4+p,\ 3-2p,\ 2-p)$
$\vec{g}\ (-1,\ 2,\ 1)$
$(-4+p).(-1)+2(3-2p)+1.(2-p)=0$
$p=2 \rightarrow M_1 \Rightarrow$

$\Rightarrow M_1(3,3,3)$

$\overrightarrow{M_1M}(-2,-1,0) \Rightarrow d(M,b)=|\overrightarrow{M_1M}|=\sqrt{5}$

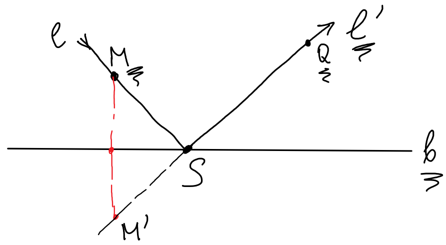
$M \xrightarrow{\sigma_b} M'$, търсим коорд. на т. M'



$M'(5,4,3)$

в) Светл. лъч $\vec{l} \ni M$, отразява се от b и отразеният лъч $\vec{l'} \ni Q(10,-1,0)$
?, параметр. уравнения на l и l'

1) Ако $M \xrightarrow{\sigma_b} M'$, то $M' \in l'$



$\begin{cases} M'(5,4,3) \text{ от б)} \\ Q(10,-1,0) \end{cases}$

$\overrightarrow{M'Q}(5,-5,-3) \parallel l' \Rightarrow l':\begin{cases} x=5+5.t \\ y=4-5.t \\ z=3-3.t \end{cases}$
$t \in \mathbb{R}$

2) ?, т. $S = b \cap l' \Rightarrow$

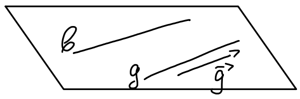
$$b:\begin{cases} x=5-p \\ y=-1+2p \\ z=1+p \end{cases}$$

$\begin{cases} 5-p=5+5.t \\ -1+2p=4-5.t \\ 1+p=3-3t \end{cases} \Rightarrow \boxed{\begin{matrix} p=5 \\ t=-1 \end{matrix}} \Rightarrow S(0,9,6)$

3) $l \begin{cases} \ni M(1,2,3) \\ \ni S(0,9,6) \end{cases} \Rightarrow \overrightarrow{MS}(-1,7,3) \Rightarrow l:\begin{cases} x=1-1.\lambda \\ y=2+7.\lambda \end{cases}$, $\lambda \in \mathbb{R}$

3) $\ell \begin{cases} \ni M(1,2,3) \\ \ni S(0,9,6) \end{cases} \Rightarrow \vec{MS}(-1,7,3) \Rightarrow \ell: \begin{cases} x = 1 - 1.\lambda \\ y = 2 + 7.\lambda \\ z = 3 + 3.\lambda \end{cases}, \lambda \in \mathbb{R}$

г) ?, общо уравнение на р-та $\beta \begin{cases} \ni g \\ \ni b \end{cases}$



Отг: $\beta: 5x + 4y - 3z - 18 = 0$

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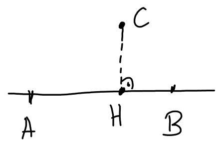
3 зад. (Упр.) ОКС $K = Oxyz$

$A(0,2,4)$ $C(-4,2,1)$

$B(1,0,2)$ $D(-3,0,-3)$

а) ? коорд. на т. $H \begin{cases} \in AB \\ CH \perp AB \end{cases}$



б) $S_{\Delta ABC} = ?$ $S = \dfrac{|\vec{AB} \times \vec{AC}|}{2}$

в) $V_{ABCD} = ?$ $V = \dfrac{1}{6} \cdot |(\vec{AB}\ \vec{AC}\ \vec{AD})|$

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5 зад. ОКС $Oxyz$ $\alpha: x + 2y - z - 2 = 0$

Да се определи взаимното положение на правата a и р-та α.

?, парам. уравнения на $a' = \sigma_\alpha(a)$, $a \xrightarrow{\sigma_\alpha} a'$, ако:

а) $a: \begin{cases} x = 2 + 3.s \\ y = 1 - 1.s, \ s \in \mathbb{R} \\ z = 2 + 1.s \end{cases}$

1) $a \parallel \vec{a}(3, -1, 1)$ $2+3s+2(1-s)-(2+s)-2 \equiv 0$

$\alpha: x + 2y - z - 2 = 0$

$A = 1$ $B = 2$ $C = -1$

$\vec{a} \overset{?}{\parallel} \alpha$

$A.3 + B.(-1) + C.1 \overset{?}{=} 0$

$1.3 + 2.(-1) + (-1).1 = 0$

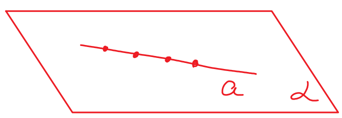
$\Downarrow$

$\vec{a} \parallel \alpha \Rightarrow a \subset \alpha$ или $a \parallel \alpha$

2) т. $P(2,1,2) \to \alpha$ $2 + 2.1 - 2 - 2 = 0$

$P \in \alpha$

$\left.\begin{matrix} \vec{a} \parallel \alpha \\ P \in \alpha \end{matrix}\right\} \Rightarrow a \subset \alpha \Rightarrow a \xrightarrow{\sigma_\alpha} a$ неподвижна



II н. $\begin{matrix} x = 2 + 3s \\ y = 1 - s \\ z = 2 + s \end{matrix} \to \alpha: x + 2y - z - 2 = 0$

$2 + 3s + 2(1 - s) - (2 + s) - 2 \overset{?}{=} 0$

$0 \equiv 0 \Rightarrow \forall P \in a \Rightarrow P \in \alpha \Rightarrow a \subset \alpha$

Обобщение: a и α $\left| \begin{matrix} x = x(s) \\ y = y(s) \\ z = z(s) \\ A.x + B.y + C.z + D = 0 \end{matrix} \right.$

$A.x(s) + B.y(s) + C.z(s) + D = 0$

$\lambda.s = \mu$ 1 сл. $\lambda = 0, \mu = 0$

$0 \equiv 0 \Rightarrow a \subset \alpha$

2 сл. $\lambda = 0, \mu \neq 0 \Rightarrow a \parallel \alpha$

3 сл. $\lambda \neq 0, \mu \in \mathbb{R} \Rightarrow a \cap \alpha =$ т. $S(x(s_0), y(s_0), z(s_0))$

има ! реш. $s_0 = \frac{\mu}{\lambda}$

3 сл. $\lambda \neq 0, \mu \in \mathbb{R} \Rightarrow a \cap \alpha =$ т. $S(x(s_0), y(s_0), z(s_0))$
има ! реш. $s_0 = \frac{\mu}{\lambda}$

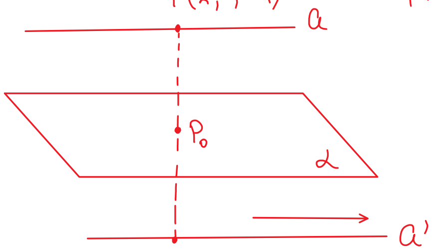
б) $a \begin{cases} x = 2+1p \\ y = 1-1p \\ z = -4-1p \end{cases}$ $\alpha: x+2y-z-2=0$

$2+p+2(1-p)-(-4-p)-2=0$

$0.p+8=0$ – н.р. $\Rightarrow a \parallel \alpha \Rightarrow a \xrightarrow{\sigma_\alpha} a'$

$A.p_1+B.p_2+C.p_3=0$

$P(2,1,-4)$



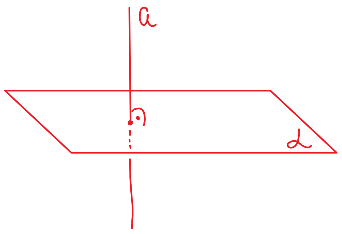
$a \parallel \alpha \parallel a'$
$a \parallel a'$

$P'(0,-3,-2)$ (Упр.) $\Rightarrow a' \begin{cases} x = 0+1q \\ y = -3-1q \\ z = -2-1q \end{cases}, q \in \mathbb{R}$

в) $a: \begin{cases} x = 1+1.t \\ y = 2+2.t \\ z = 3-1.t \end{cases}$ $\alpha: 1x+2y-1z-2=0 \Rightarrow \vec{n}_\alpha(1,2,-1)$ $\Big\}$ $\vec{a} \parallel \vec{n}_\alpha$

$a \parallel \vec{a}(1,2,-1)$

$a \perp \alpha$



$\forall a \perp \alpha \Rightarrow a \xrightarrow{\sigma_\alpha} a$

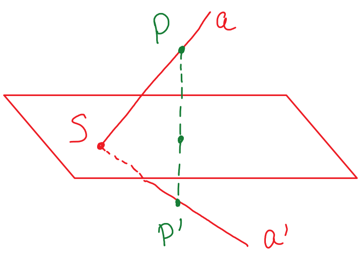
г) $a: \begin{cases} x = 2+1.\mu \\ y = 1-1.\mu \\ z = -4+5.\mu \end{cases}, \mu \in \mathbb{R}$ $\alpha: x+2y-z-2=0$

$2+\mu+2(1-\mu)-(-4+5\mu)-2=0$

$\mu = +1 \rightarrow a$ $\qquad -6\mu+6=0$

$S(3,0,1) = a \cap \alpha$

$a \parallel \vec{a}(1,-1,5)$
$\alpha \perp \vec{n}_\alpha(1,2,-1)$ л.н.з. $\Rightarrow a \not\perp \alpha$



$P(2,1,-4) \xrightarrow{\sigma_\alpha} P'$?

$a' \begin{cases} \ni S \\ \ni P' \end{cases}$

Трансверзали и ос на кръстосани прави

1 зад. (Ос на кръстосани прави)

ОКС $K = Oxyz$ $\quad a: \begin{cases} x = 5+s \\ y = -1+2.s, s \in \mathbb{R} \end{cases}$ $\quad b: \begin{cases} x = -4-7p \\ y = 3+2p, p \in \mathbb{R} \end{cases}$

ОКС $K = Oxyz$

$$a: \begin{cases} x = 5 + s \\ y = -1 + 2.s, \ s \in \mathbb{R} \\ z = 11 - s \end{cases} \qquad b: \begin{cases} x = -4 - 7p \\ y = 3 + 2p, \ p \in \mathbb{R} \\ z = 4 + 3p \end{cases}$$

$a \equiv b$, $a \parallel b$, $a \cap b = $ т. S, a и b – кръстосани

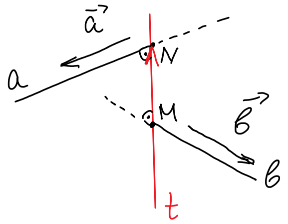
1) $a \parallel \vec{a}(1, 2, -1)$ ЛНЗ $(\vec{a} \times \vec{b} \neq \vec{0}) \Rightarrow a \not\equiv b, \ a \nparallel b$

$b \parallel \vec{b}(-7, 2, 3)$

2) $a \cap b = ? \Rightarrow \begin{vmatrix} 5 + s = -4 - 7p \\ -1 + 2.s = 3 + 2p \\ 11 - s = 4 + 3p \end{vmatrix}$ НЕ $\Rightarrow a \cap b = \emptyset$

a и b са кръстосани

Ос: $\begin{vmatrix} t \cap a = N \\ t \cap b = M \\ t \perp a \\ t \perp b \end{vmatrix}$ MN – ос-отсечка, $|\overrightarrow{MN}| = d(a, b)$



$M \in b \Rightarrow M(-4 - \underset{3}{7p}, 3 + \underset{3}{2p}, 4 + \underset{3}{3p})$ $p = -1$

$N \in a \Rightarrow N(5 + s, -1 + 2s, 11 - s)$ $s = 2$

$\overrightarrow{MN}(9 + 7p + s, -4 - 2p + 2s, 7 - 3p - s)$ ✓

$a \parallel \vec{a}(1, 2, -1)$ ✓

$b \parallel \vec{b}(-7, 2, 3)$ ✓

$\begin{vmatrix} \overrightarrow{MN} \perp \vec{a} \\ \overrightarrow{MN} \perp \vec{b} \end{vmatrix}$

$p + s - 1 = 0$

$$\begin{vmatrix} (\overrightarrow{MN} \cdot \vec{a}) = 0 \\ (\overrightarrow{MN} \cdot \vec{b}) = 0 \end{vmatrix} \Rightarrow \begin{vmatrix} 9 + 7p + s - 8 - 4p + 4s - 7 + 3p + s = 0 \\ -63 - 49p - 7s - 8 - 4p + 4s + 21 - 9p - 3s = 0 \end{vmatrix}$$

$$\begin{vmatrix} p + s = 1 \ / \ 6 \\ -62p - 6s = +50 \end{vmatrix} \quad \begin{vmatrix} 6p + 6s = 6 \\ -62p - 6s = +50 \end{vmatrix} \Big\} + \quad p + s = 1$$

$-56p = 56$ $\quad p = -1 \to M(3, 1, 1)$

$s = 2 \to N(7, 3, 9)$

$$t \begin{cases} \in M(3, 1, 1) \\ \parallel \overrightarrow{MN}(4, 2, 8) \end{cases} \Rightarrow t: \begin{cases} x = 3 + 4.\lambda \\ y = 1 + 2.\lambda, \ \lambda \in \mathbb{R} \\ z = 1 + 8.\lambda \end{cases}$$

$d(a, b) = |\overrightarrow{MN}| = \sqrt{16 + 4 + 64} = \sqrt{84} = 2.\sqrt{21}$