

Formulario matemáticas

Versión 3.0

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1 EXPONENTES

$$\begin{aligned} a^0 &= 1 & (1) \\ a^1 &= a & (2) \\ a^m a^n &= a^{m+n} & (3) \\ (a^m)^n &= a^{mn} & (4) \\ (ab)^n &= a^n b^n & (5) \\ \frac{1}{a^n} &= a^{-n} & (6) \\ \frac{a^m}{a^n} &= a^{m-n} & (7) \\ \left(\frac{a}{b}\right)^n &= \frac{a^n}{b^n} & (8) \end{aligned}$$

2 RADICALES

$$\begin{aligned} \sqrt[n]{ab} &= \sqrt[n]{a} \sqrt[n]{b} & (9) \\ \sqrt[m]{\sqrt[n]{a}} &= \sqrt[mn]{a} & (10) \\ \sqrt[n]{a} &= a^{\frac{1}{n}} & (11) \\ \sqrt[n]{a^m} &= (\sqrt[n]{a})^m = a^{\frac{m}{n}} & (12) \\ \frac{\sqrt[n]{a}}{\sqrt[n]{b}} &= \sqrt[n]{\frac{a}{b}} = \left(\frac{a}{b}\right)^{\frac{1}{n}} = \frac{a^{\frac{1}{n}}}{b^{\frac{1}{n}}} & (13) \\ a \sqrt[n]{d} + b \sqrt[n]{d} - c \sqrt[n]{d} &= (a + b - c) \sqrt[n]{d} & (14) \\ \sqrt[m]{a} \cdot \sqrt[n]{b} &= \sqrt[m \cdot n]{a^n \cdot b^m} & (15) \\ \frac{\sqrt[n]{a^m}}{c} &= \frac{a}{c \cdot \sqrt[n]{a^{n-m}}} & (16) \\ \frac{a}{\sqrt[n]{b^m}} &= \frac{a \cdot \sqrt[n]{b^{n-m}}}{b} & (17) \\ a^m \cdot \sqrt[n]{b} &= \sqrt[n]{(a^m)^n \cdot b} & (18) \end{aligned}$$

3 PRODUCTOS NOTABLES

$$\begin{aligned} (x + y)^2 &= x^2 + 2xy + y^2 & (19) \\ (x - y)^2 &= x^2 - 2xy + y^2 & (20) \\ (a + b + c)^2 &= a^2 + b^2 + c^2 + 2ab + 2ac + 2bc & (21) \\ (x + y)^3 &= x^3 + 3x^2y + 3xy^2 + y^3 & (22) \\ (x - y)^3 &= x^3 - 3x^2y + 3xy^2 - y^3 & (23) \\ (x + y)^4 &= x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4 & (24) \\ (x - y)^4 &= x^4 - 4x^3y + 6x^2y^2 - 4xy^3 + y^4 & (25) \end{aligned}$$

4 FACTORES NOTABLES

$$\begin{aligned} x^2 - y^2 &= (x + y)(x - y) & (26) \\ x^3 - y^3 &= (x - y)(x^2 + xy + y^2) & (27) \\ x^3 + y^3 &= (x + y)(x^2 - xy + y^2) & (28) \\ x^4 - y^4 &= (x + y)(x - y)(x^2 + y^2) & (29) \\ x^5 - y^5 &= (x - y)(x^4 + x^3y + x^2y^2 + xy^3 + y^4) & (30) \\ x^5 + y^5 &= (x + y)(x^4 - x^3y + x^2y^2 - xy^3 + y^4) & (31) \end{aligned}$$

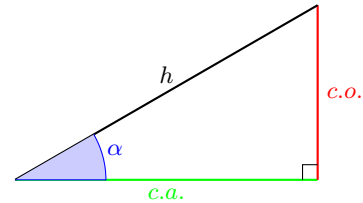
5 FACTORES NOTABLES ESPECIALES

$$\begin{aligned} a^2 + 2ab + b^2 &= (a + b)^2 & (32) \\ a^2 - 2ab + b^2 &= (a - b)^2 & (33) \\ am + bm + a^2 + ab &= (a + b)(m + a) & (34) \\ x^2 + (a + b)x + ab &= (x + a)(x + b) & (35) \\ m^2 + 2mn + n^2 - p^2 &= (m + n - p)(m + n + p) & (36) \\ \frac{a \pm b}{c} &= \frac{a^2 - b^2}{c \cdot (a \mp b)} & (37) \\ \sqrt[3]{a} - \sqrt[3]{b} &= \frac{a - b}{a^{\frac{2}{3}} + a^{\frac{1}{3}}b^{\frac{1}{3}} + b^{\frac{2}{3}}} & (38) \\ \sqrt[3]{a} + \sqrt[3]{b} &= \frac{a + b}{a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}} + b^{\frac{2}{3}}} & (39) \end{aligned}$$

6 LEYES DE LOS LOGARÍTMOS

$$\begin{aligned} x &= b^y & (40) \\ y &= \log_b x & (41) \\ \log_b 1 &= 0 & (42) \\ \log_b b &= 1 & (43) \\ \log_b (u \cdot v) &= \log_b (u) + \log_b (v) & (44) \\ \log_b \left(\frac{u}{v}\right) &= \log_b (u) - \log_b (v) & (45) \\ \log_b (u^n) &= n \cdot \log_b (u) & (46) \end{aligned}$$

7 TRIGONOMETRÍA



$$\begin{aligned} \sin(\alpha) &= \frac{c.o.}{h} = \frac{1}{\csc(\alpha)} & (47) \\ \cos(\alpha) &= \frac{c.a.}{h} = \frac{1}{\sec(\alpha)} & (48) \\ \tan(\alpha) &= \frac{c.o.}{c.a.} = \frac{1}{\cot(\alpha)} & (49) \\ \cot(\alpha) &= \frac{c.a.}{c.o.} = \frac{1}{\tan(\alpha)} & (50) \\ \sec(\alpha) &= \frac{h}{c.a.} = \frac{1}{\cos(\alpha)} & (51) \\ \csc(\alpha) &= \frac{h}{c.o.} = \frac{1}{\sin(\alpha)} & (52) \\ \tan(\alpha) &= \frac{\sin(\alpha)}{\cos(\alpha)} & (53) \\ \cot(\alpha) &= \frac{\cos(\alpha)}{\sin(\alpha)} & (54) \end{aligned}$$

8 RECÍPROCAS

$$\begin{aligned} \sin(\alpha) \csc(\alpha) &= 1 & (55) \\ \cos(\alpha) \sec(\alpha) &= 1 & (56) \\ \tan(\alpha) \cot(\alpha) &= 1 & (57) \end{aligned}$$

9 IDENTIDADES PITAGÓRICAS

$$\begin{aligned} \sin^2(\alpha) + \cos^2(\alpha) &= 1 & (58) \\ \sec^2(\alpha) - \tan^2(\alpha) &= 1 & (59) \\ \csc^2(\alpha) - \cot^2(\alpha) &= 1 & (60) \end{aligned}$$

10 SUMA Y RESTA DE ÁNGULOS

$$\begin{aligned} \sin(a + b) &= \sin(a) \cos(b) + \cos(a) \sin(b) & (61) \\ \sin(a - b) &= \sin(a) \cos(b) - \cos(a) \sin(b) & (62) \\ \cos(a + b) &= \cos(a) \cos(b) - \sin(a) \sin(b) & (63) \\ \cos(a - b) &= \cos(a) \cos(b) + \sin(a) \sin(b) & (64) \\ \tan(a + b) &= \frac{\tan(a) + \tan(b)}{1 - \tan(a) \tan(b)} & (65) \\ \tan(a - b) &= \frac{\tan(a) - \tan(b)}{1 + \tan(a) \tan(b)} & (66) \\ \cot(a + b) &= \frac{\cot(a) \cot(b) - 1}{\cot(a) + \cot(b)} & (67) \\ \cot(a - b) &= \frac{\cot(a) \cot(b) + 1}{\cot(a) - \cot(b)} & (68) \end{aligned}$$

11 DOBLE ÁNGULO

$$\sin(2\alpha) = 2\sin(\alpha)\cos(\alpha) \quad (69)$$

$$\cos(2\alpha) = \cos^2(\alpha) - \sin^2(\alpha) \quad (70)$$

$$\tan(2\alpha) = \frac{2\tan(\alpha)}{1 - \tan^2(\alpha)} \quad (71)$$

12 MITAD DE UN ÁNGULO

$$\sin\left(\frac{1}{2}\right) = \sqrt{\frac{1 - \cos(\alpha)}{2}} \quad (72)$$

$$\cos\left(\frac{1}{2}\right) = \sqrt{\frac{1 + \cos(\alpha)}{2}} \quad (73)$$

$$\tan\left(\frac{1}{2}\right) = \sqrt{\frac{1 - \cos(\alpha)}{1 + \cos(\alpha)}} = \frac{\sin(\alpha)}{1 + \cos(\alpha)} \quad (74)$$

13 MÚLTIPLO DE UN ÁNGULO

$$\sin(3\alpha) = 3\sin(\alpha) - 4\sin^3(\alpha) \quad (75)$$

$$\cos(3\alpha) = 4\cos^3(\alpha) - 3\cos(\alpha) \quad (76)$$

$$\tan(3\alpha) = \frac{3\tan(\alpha) - \tan^3(\alpha)}{1 - 3\tan^2(\alpha)} \quad (77)$$

14 POTENCIAS DE LAS FUNCIONES TRIGONOMÉTRICAS

$$\sin(\alpha)\cos(\alpha) = \frac{1}{2}\sin(2\alpha) \quad (78)$$

$$\sin^2(\alpha) = \frac{1}{2} - \frac{1}{2}\cos(2\alpha) \quad (79)$$

$$\cos^2(\alpha) = \frac{1}{2} + \frac{1}{2}\cos(2\alpha) \quad (80)$$

$$\sin^3(\alpha) = \frac{3}{4}\sin(\alpha) - \frac{1}{4}\sin(3\alpha) \quad (81)$$

$$\cos^3(\alpha) = \frac{3}{4}\cos(\alpha) + \frac{1}{4}\cos(3\alpha) \quad (82)$$

15 PARIDAD

$$\sin(-\alpha) = -\sin(\alpha) \quad (83)$$

$$\cos(-\alpha) = \cos(\alpha) \quad (84)$$

$$\tan(-\alpha) = -\tan(\alpha) \quad (85)$$

$$\cot(-\alpha) = -\cot(\alpha) \quad (86)$$

$$\sec(-\alpha) = \sec(\alpha) \quad (87)$$

$$\csc(-\alpha) = -\csc(\alpha) \quad (88)$$

16 ALGUNAS FÓRMULAS DE REDUCCIÓN

$$\sin\left(\frac{\pi}{2} - x\right) = \cos(x) \quad (89)$$

$$\sin\left(\frac{\pi}{2} + x\right) = \cos(x) \quad (90)$$

$$\sin(\pi - x) = \sin(x) \quad (91)$$

$$\cos\left(\frac{\pi}{2} - x\right) = \sin(x) \quad (92)$$

$$\cos\left(\frac{\pi}{2} + x\right) = -\sin(x) \quad (93)$$

$$\cos(\pi - x) = -\cos(x) \quad (94)$$

$$\tan\left(\frac{\pi}{2} - x\right) = \cot(x) \quad (95)$$

$$\tan\left(\frac{\pi}{2} + x\right) = -\cot(x) \quad (96)$$

$$\tan(\pi - x) = -\tan(x) \quad (97)$$

17 TABLA RADIANTES-GRADOS

| π rad | grados | $\sin(\theta)$ | $\cos(\theta)$ | $\tan(\theta)$ |
|------------------|--------|----------------------|----------------------|----------------------|
| 0 | 0° | 0 | 1 | 0 |
| $\frac{\pi}{6}$ | 30° | $\frac{1}{2}$ | $\frac{\sqrt{3}}{2}$ | $\frac{\sqrt{3}}{3}$ |
| $\frac{\pi}{4}$ | 45° | $\frac{\sqrt{2}}{2}$ | $\frac{\sqrt{2}}{2}$ | 1 |
| $\frac{\pi}{3}$ | 60° | $\frac{\sqrt{3}}{2}$ | $\frac{1}{2}$ | $\sqrt{3}$ |
| $\frac{\pi}{2}$ | 90° | 1 | 0 | indefinido |
| π | 180° | 0 | -1 | 0 |
| $\frac{3\pi}{2}$ | 270° | -1 | 0 | indefinido |

18 RELACIÓN ENTRE LADOS Y ÁNGULOS

18.1. Ley de Senos

$$\frac{a}{\sin a} = \frac{b}{\sin b} = \frac{c}{\sin c} \quad (98)$$

18.2. Ley de Cosenos

$$c^2 = a^2 + b^2 - 2ab \cos c \quad (99)$$

18.3. Ley de Tangentes

$$\frac{a+b}{a-b} = \frac{\tan \frac{1}{2}(a+b)}{\tan \frac{1}{2}(a-b)} \quad (100)$$

19 GEOMETRÍA ANALÍTICA

19.1. Distancia entre dos puntos

$$d = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2} \quad (101)$$

19.2. Punto medio

$$x = \frac{x_1 + x_2}{2} \quad y = \frac{y_1 + y_2}{2} \quad (102)$$

19.3. División de un segmento en una razón dada

$$x = \frac{x_1 + rx_2}{1+r} \quad y = \frac{y_1 + ry_2}{1+r} \quad (103)$$

19.4. Área de un triángulo

$$A = \frac{1}{2} \begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \end{vmatrix} = \frac{1}{2} |x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)| \quad (104)$$

20.1. Pendiente

$$m = \frac{y_2 - y_1}{x_2 - x_1} \quad (105)$$

$$x^2 + y^2 = r^2 \quad (117)$$

20.2. Forma de los dos puntos

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1} (x - x_1) \quad (106)$$

20.3. Forma punto pendiente

$$y - y_1 = m (x - x_1) \quad (107)$$

20.4. Forma simétrica

$$\frac{x}{a} + \frac{y}{b} = 1 \quad (108)$$

20.5. Forma pendiente-intercepción

$$y = mx + b \quad (109)$$

20.6. Forma general

$$Ax + By + C = 0 \quad (110)$$

20.7. Transformación general a ordinaria

$$y = -\frac{A}{B}x - \frac{C}{B} \quad (111)$$

20.8. Transformación general a simétrica

$$\frac{x}{-\frac{C}{A}} + \frac{y}{-\frac{C}{B}} = 1 \quad (112)$$

20.9. Forma normal

$$x \cos(\theta) + y \sin(\theta - p) = 0 \quad (113)$$

20.10. Ángulo entre dos rectas

$$\tan(\alpha) = \frac{m_2 - m_1}{1 + m_1 m_2} \quad (114)$$

21 CIRCUNFERENCIA

21.1. Ecuación ordinaria

$$(x - h)^2 + (y - k)^2 = r^2 \quad (115)$$

21.2. Ecuación general

$$Ax^2 + Cy^2 + Dx + Ey + F = 0; \quad A = C \quad (116)$$

21.4. Transformación general a ordinaria

$$\left(x + \frac{D}{2A}\right)^2 + \left(y + \frac{E}{2A}\right)^2 = \frac{D^2 + E^2 - 4AF}{4A^2} \quad (118)$$

$$Centro = \left(-\frac{D}{2A}, -\frac{E}{2A}\right) \quad (119)$$

$$radio = \frac{1}{2A} \sqrt{D^2 + E^2 - 4AF} \quad (120)$$

22 PARÁBOLA

22.1. Horizontal

$$(y - k)^2 = 4p(x - h) \quad (121)$$

$$Cy^2 + Dx + Ey + F = 0 \quad (122)$$

$$V(h, k) \quad (123)$$

$$F(h + p, k) \quad (124)$$

$$x = h - p \quad (125)$$

$$LR = |4p| \quad (126)$$

22.2. Vertical

$$(x - h)^2 = 4p(y - k) \quad (127)$$

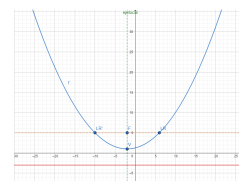
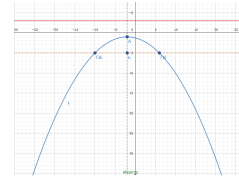
$$Ax^2 + Dx + Ey + F = 0 \quad (128)$$

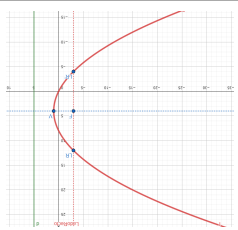
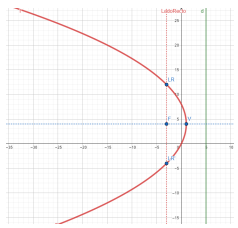
$$V(h, k) \quad (129)$$

$$F(h, k + p) \quad (130)$$

$$y = k - p \quad (131)$$

$$LR = |4p| \quad (132)$$

| ECUACIÓN | GRÁFICA |
|--------------|---|
| $x^2 = 4Py$ |  |
| $x^2 = -4Py$ |  |

| ECUACIÓN | GRÁFICA |
|--------------|---|
| $y^2 = 4Px$ |  |
| $y^2 = -4Px$ |  |

23 ELIPSE

23.1. Ecuación general

$$Ax^2 + Cy^2 + Dx + Ey + F = 0 \quad (133)$$

$$a^2 = b^2 + c^2; a > b \quad (134)$$

$$e = \frac{c}{a} \quad (135)$$

23.2. Horizontal

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1 \quad (136)$$

$$V_1V_2 = 2a \text{ eje mayor} \quad (137)$$

$$F_1F_2 = 2c \text{ eje focal} \quad (138)$$

$$B_1B_2 = 2b \text{ eje menor} \quad (139)$$

$$LR = \frac{2b^2}{a} \quad (140)$$

$$V(h \pm a, k) \quad (141)$$

$$F(h \pm c, k) \quad (142)$$

$$B(h, k \pm b) \quad (143)$$

23.3. Vertical

$$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1 \quad (144)$$

$$V(h, k \pm a) \quad (145)$$

$$F(h, k \pm c) \quad (146)$$

$$B(h \pm b, k) \quad (147)$$

24 HIPÉRBOLA

24.1. Ecuación general

$$Ax^2 + Cy^2 + Dx + Ey + F = 0 \quad (148)$$

$$c^2 = a^2 + b^2 \quad (149)$$

$$LR = \frac{2b^2}{a} \quad (150)$$

24.2. Horizontal

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1 \quad (151)$$

$$V(h \pm a, k) \quad (152)$$

$$F(h \pm c, k) \quad (153)$$

$$B(h, k \pm b) \quad (154)$$

$$l_1: y - k = \frac{b}{a}(x - h) \quad (155)$$

$$l_2: y - k = -\frac{b}{a}(x - h) \quad (156)$$

24.3. Vertical

$$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1 \quad (157)$$

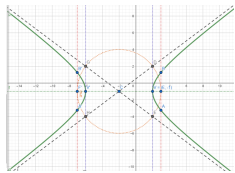
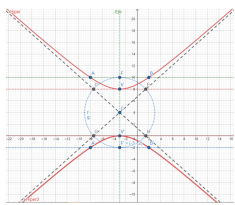
$$V(h, k \pm a) \quad (158)$$

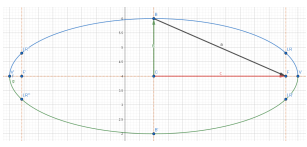
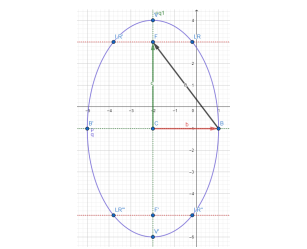
$$F(h, k \pm c) \quad (159)$$

$$B(h \pm b, k) \quad (160)$$

$$l_1: y - k = \frac{a}{b}(x - h) \quad (161)$$

$$l_2: y - k = -\frac{a}{b}(x - h) \quad (162)$$

| ECUACIÓN | GRÁFICA |
|---|---|
| $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$ |  |
| $\frac{(y-k)^2}{b^2} - \frac{(x-h)^2}{a^2} = 1$ |  |

| ECUACIÓN | GRÁFICA |
|---|---|
| $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ |  |
| $\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$ |  |

25.1. Límites

$$\lim_{x \rightarrow a} c = c \quad (163)$$

$$\lim_{x \rightarrow a} x = a \quad (164)$$

$$\lim_{x \rightarrow a} c \cdot f(x) = c \cdot \lim_{x \rightarrow a} f(x) \quad (165)$$

$$\lim_{x \rightarrow a} [f(x) \pm g(x)] = \lim_{x \rightarrow a} f(x) \pm \lim_{x \rightarrow a} g(x) \quad (166)$$

$$\lim_{x \rightarrow a} [f(x) \cdot g(x)] = \lim_{x \rightarrow a} f(x) \cdot \lim_{x \rightarrow a} g(x) \quad (167)$$

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{\lim_{x \rightarrow a} f(x)}{\lim_{x \rightarrow a} g(x)} \quad (168)$$

$$\lim_{x \rightarrow a} [f(x)]^n = \left[\lim_{x \rightarrow a} f(x) \right]^n \quad (169)$$

25.2. Límites indeterminados

$$\lim_{x \rightarrow \infty} \frac{c}{x^n} = 0 \quad (170)$$

$$\lim_{v \rightarrow 0} \frac{\sin v}{v} = 1 \quad (171)$$

$$\lim_{v \rightarrow 0} \frac{1 - \cos v}{v} = 0 \quad (172)$$

$$\lim_{v \rightarrow 0} \frac{\cos v - 1}{v} = 0 \quad (173)$$

25.3. Derivada vía límite

$$\frac{dy}{dx} = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{\Delta h} \quad (174)$$

25.4. Reglas de derivación

$$\frac{d}{dx} (c) = 0 \quad (175)$$

$$\frac{d}{dx} (cx) = c \quad (176)$$

$$\frac{d}{dx} (cx^n) = cnx^{n-1} \quad (177)$$

$$\frac{d}{dx} [cu(x)] = c \frac{du}{dx} \quad (178)$$

$$\frac{d}{dx} [v(x)^n] = nv^{n-1} \frac{dv}{dx} \quad (179)$$

$$\frac{d}{dx} [u(x) + v(x)] = \frac{du}{dx} + \frac{dv}{dx} \quad (180)$$

$$\frac{d}{dx} [u(x) \pm v(x) \pm w(x)] = \frac{du}{dx} \pm \frac{dv}{dx} \pm \frac{dw}{dx} \quad (181)$$

$$\frac{d}{dx} [u(x)v(x)] = u \frac{dv}{dx} + v \frac{du}{dx} \quad (182)$$

$$\frac{d}{dx} [u(x)v(x)w(x)] = uv \frac{dw}{dx} + uw \frac{dv}{dx} + vw \frac{du}{dx} \quad (183)$$

$$\frac{d}{dx} \left[\frac{u(x)}{v(x)} \right] = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2} \quad (184)$$

25.5. Regla de la cadena

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} \quad (185)$$

25.6. Derivadas de funciones trigonométricas

$$\frac{d}{dx} [\sin(v)] = \cos(v) \cdot \frac{dv}{dx} \quad (186)$$

$$\frac{d}{dx} [\cos(v)] = -\sin(v) \cdot \frac{dv}{dx} \quad (187)$$

$$\frac{d}{dx} [\tan(v)] = \sec^2(v) \cdot \frac{dv}{dx} \quad (188)$$

$$\frac{d}{dx} [\csc(v)] = -\csc(v) \cdot \cot(v) \cdot \frac{dv}{dx} \quad (189)$$

$$\frac{d}{dx} [\sec(v)] = \sec(v) \cdot \tan(v) \cdot \frac{dv}{dx} \quad (190)$$

$$\frac{d}{dx} [\cot(v)] = -\csc^2(v) \cdot \frac{dv}{dx} \quad (191)$$

26 DERIVADAS DE FUNCIONES TRIGONOMÉTRICAS INVERSAS

$$\frac{d}{dx} [\arcsin(v)] = \frac{1}{\sqrt{1-v^2}} \cdot \frac{dv}{dx} \quad (192)$$

$$\frac{d}{dx} [\arccos(v)] = -\frac{1}{\sqrt{1-v^2}} \cdot \frac{dv}{dx} \quad (193)$$

$$\frac{d}{dx} [\arctan(v)] = \frac{1}{1+v^2} \cdot \frac{dv}{dx} \quad (194)$$

$$\frac{d}{dx} [\operatorname{arccot}(v)] = -\frac{1}{1+v^2} \cdot \frac{dv}{dx} \quad (195)$$

$$\frac{d}{dx} [\operatorname{arcsec}(v)] = \frac{1}{v\sqrt{v^2-1}} \cdot \frac{dv}{dx} \quad (196)$$

$$\frac{d}{dx} [\operatorname{arccsc}(v)] = -\frac{1}{v\sqrt{v^2-1}} \cdot \frac{dv}{dx} \quad (197)$$

26.1. Derivadas de funciones logarítmicas y exponenciales

$$\frac{d}{dx} [\log_a(v)] = \frac{1}{v} \cdot \log_a(e) \cdot \frac{dv}{dx} \quad (198)$$

$$\frac{d}{dx} [\ln(v)] = \frac{1}{v} \cdot \frac{dv}{dx} \quad (199)$$

$$\frac{d}{dx} [a^{(v)}] = a^v \cdot \ln(a) \cdot \frac{dv}{dx} \quad (200)$$

$$\frac{d}{dx} [e^{(v)}] = e^v \cdot \frac{dv}{dx} \quad (201)$$

$$\frac{d}{dx} [u^{(v)}] = vu^{v-1} \cdot \frac{du}{dx} + u^v \cdot \ln(u) \cdot \frac{dv}{dx} \quad (202)$$

27.1. Reglas de integración

$$\int (u \pm v) dx = \int (u) dx \pm \int (v) dx \quad (203)$$

27.2. Integrales inmediatas

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C \quad (204)$$

$$\int v^n dx = \frac{v^{n+1}}{n+1} + C \quad (205)$$

$$\int \frac{dv}{v} = \ln |v| + C \quad (206)$$

$$\int e^v dv = e^v + C \quad (207)$$

$$\int a^v dv = \frac{a^v}{\ln(a)} + C \quad (208)$$

$$\int \ln(v) dv = v \ln(v) - v + C \quad (209)$$

$$\int \sin(v) dv = -\cos(v) + C \quad (210)$$

$$\int \cos(v) dv = \sin(v) + C \quad (211)$$

$$\int \tan(v) dv = -\ln |\cos(v)| + C \quad (212)$$

$$\int \csc(v) \cot(v) dv = -\csc(v) + C \quad (213)$$

$$\int \sec(v) \tan(v) dv = \sec(v) + C \quad (214)$$

$$\int \cot(v) dv = \ln |\sin(v)| + C \quad (215)$$

$$\int \csc(v) dv = \ln |\csc(v) - \cot(v)| + C \quad (216)$$

$$\int \sec(v) dv = \ln |\sec(v) + \tan(v)| + C \quad (217)$$

$$\int \sec^2(v) dv = \tan(v) + C \quad (218)$$

$$\int \csc^2(v) dv = -\cot(v) + C \quad (219)$$

$$\int \arcsin(ax) dx = x \arcsin(ax) + \frac{1}{a} \sqrt{1 - a^2 x^2} + C \quad (220)$$

$$\int \arccos(ax) dx = x \arccos(ax) - \frac{1}{a} \sqrt{1 - a^2 x^2} + C \quad (221)$$

$$\int \arctan(ax) dx = x \arctan(ax) - \frac{1}{2a} \ln(1 + a^2 x^2) + C \quad (222)$$

27.3. Integrales: $\sqrt{v^2 \pm a^2}$, $\sqrt{a^2 - v^2}$, $v^2 \pm a^2$, $a^2 - v^2$

$$\int \frac{dv}{v^2 + a^2} = \frac{1}{a} \arctan\left(\frac{v}{a}\right) + C \quad (223)$$

$$\int \frac{dv}{v^2 - a^2} = \frac{1}{2a} \ln \left| \frac{v-a}{v+a} \right| + C \quad (224)$$

$$\int \frac{dv}{a^2 - v^2} = \frac{1}{2a} \ln \left| \frac{a+v}{a-v} \right| + C \quad (225)$$

$$\int \frac{dv}{\sqrt{a^2 - v^2}} = \arcsin\left(\frac{v}{a}\right) + C \quad (226)$$

$$\int \frac{dv}{\sqrt{v^2 \pm a^2}} = \ln \left(v + \sqrt{v^2 \pm a^2} \right) + C \quad (227)$$

$$\int \frac{dv}{v\sqrt{v^2 - a^2}} = \frac{1}{a} \operatorname{arcsec}\left(\frac{v}{a}\right) + C \quad (228)$$

$$\int \sqrt{a^2 - v^2} dv = \frac{v}{2} \sqrt{a^2 - v^2} + \frac{a^2}{2} \arcsin\left(\frac{v}{a}\right) + C \quad (229)$$

$$\int \sqrt{v^2 \pm a^2} dv = \frac{v}{2} \sqrt{v^2 \pm a^2} \pm \frac{a^2}{2} \ln \left(v + \sqrt{v^2 \pm a^2} \right) \quad (230)$$

28 SUMATORIAS

28.1. Propiedades

$$\sum_{i=a}^n k = (n-a+1)k \quad (231)$$

$$\sum_{i=a}^n [f(i) + g(i)] = \sum_{i=a}^n f(i) + \sum_{i=a}^n g(i) \quad (232)$$

$$\sum_{i=a}^n c \cdot f(i) = c \sum_{i=a}^n f(i) \quad (233)$$

$$\sum_{i=a}^n [f(i) - f(i-1)] = f(n) - f(0) \quad (234)$$

$$(235)$$

28.2. Sumas básicas

$$\sum_{i=1}^n k = kn \quad (236)$$

$$\sum_{i=1}^n i = \frac{n(n+1)}{2} \quad (237)$$

$$\sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6} \quad (238)$$

$$\sum_{i=1}^n i^3 = \frac{n^2(n+1)^2}{4} \quad (239)$$

$$\sum_{i=1}^n i^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30} \quad (240)$$

28.3. Sumas de Riemann

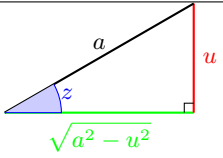
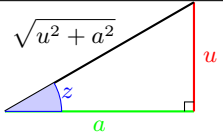
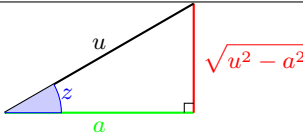
28.3.1. Rectángulos inscritos

$$A = \lim_{n \rightarrow \infty} \sum_{i=1}^n \left(\frac{b-a}{n} \right) f(a + (i-1)\Delta x)$$

28.3.2. Rectángulos circunscritos

$$A = \lim_{n \rightarrow \infty} \sum_{i=1}^n \left(\frac{b-a}{n} \right) f(a + i\Delta x)$$

28.4. Integración por sustitución trigonométrica

| Caso | Cambio | Diferencial | Transformación | Triángulo |
|--------------------|-----------------|-----------------------------------|--------------------------------|---|
| $\sqrt{a^2 - u^2}$ | $u = a \sin(z)$ | $du = a \cos(z) dz$ | $\sqrt{a^2 - u^2} = a \cos(z)$ |  |
| $\sqrt{u^2 + a^2}$ | $u = a \tan(z)$ | $du = a \sec^2(z) dz$ | $\sqrt{u^2 + a^2} = a \sec(z)$ |  |
| $\sqrt{u^2 - a^2}$ | $u = a \sec(z)$ | $du = a \sec(z) \cdot \tan(z) dz$ | $\sqrt{u^2 - a^2} = a \tan(z)$ |  |

REFERENCIAS

- [1] Hahn, J. \LaTeX for everyone. Prentice Hall, New Jersey, 1993.
- [2] Pineda, A. Cálculo diferencial e integral. Prentice Hall, México, 2010.
- [3] Alan & Hui. Handbook of Mathematical Formulas and Integrals. Helsevier, USA, 2008.
- [4] Murray, R. Manual de fórmulas y tablas matemáticas. McGraw-Hill, México, 1991.
- [5] Verónica, G. Representación algebraica y gráfica de relaciones, Ediciones BIR, Estado de México, 2020.