% ⇒ Modulus → Remainder

$n \% a$ ⇒ Remainder when we divide n by a.

$10 \% 4 = 2$

$13 \% 5 = 3$

$$\begin{array}{l} \text{Divisor} \quad \overset{D}{\text{Dividend}} \quad \overset{q}{\text{Quotient}} \\ d \;\; 5 \,)\, 13 \,(\, 2 \\ \quad\;\; -10 \\ \quad\;\; \overline{\;\;3\;\;} \end{array}$$

Dividend = Quotient × Divisor + Remainder

$D = qd + r$

Remainder = Dividend − $\underbrace{\text{(Quotient × Divisor)}}_{\text{greatest multiple of divisor} <= \text{Dividend}}$

Quiz 1 : $150 \% 11$

$150 - 143$

$$\begin{array}{l} 11 \,)\, 150 \,(\, 13 \\ \quad\;\; \underline{11} \\ \quad\;\;\; 40 \\ \quad\;\;\; \underline{33} \\ \quad\;\;\;\; 7 \end{array}$$

Quiz 2 : $100 \% 7$

$100 - 98 = 2$

$-100 > -\underline{200}$

Quiz 3

KBC
(Kids v)

$-43, \quad -42, \quad -76, \quad -35$

$-76 < -43 < -42 < -35$

Quiz 4

$-40 \% 7$

$-40 - (\text{Greatest multiple of } 7 <= -40)$

$-40 - (-42)$

$\underline{\underline{2}}$

Quiz 7

$-60 \% 9$

$-60 - (-63)$

3

$\boxed{a \% m \Rightarrow [0, m-1]}$

(C/C++/Java/JS)		Python
$-40 \% 7$	$= -5 + 7$	$\longrightarrow 2$
$-60 \% 9$	$= -6 + 9$	$\longrightarrow 3$
$-30 \% 4$	$= -2 + 4$	$\longrightarrow 2$

$\underbrace{a \% b} + b$

1
2
3
4
5
6
7
8
9
10

$\% 5 \rightarrow$ 0, 1, 2, 3, 4

$-\infty$ ... $+\infty$ $\Big\}$ $\% 5 \Rightarrow [0, 4]$

Applications of %

- HashMap / HashTable / dict
  map / unordered_map
- Consistent hashing
- Encryption

// Modular Arithmetic

$+ \quad - \quad \times \quad \div$

(Maths)

$$\boxed{(a+b) \% M = (a \% M + b \% M) \% M}$$

$(a+b) \% M \xrightarrow{\text{Range}} [0, M-1]$

$M-1$ → Max

$a = 6, \quad b = 8$

$M = 10$

$(6+8) \% 10 = 4$

$(6 \% 10 + 8 \% 10) \% 10$

$= (14) \% 10$

$= 4$

$$\boxed{(a \times b) \% M = ((a \% M) \times (b \% M)) \% M}$$

Q. Implement power function that takes 3 arguments. $\underbrace{a, n, p}_{\text{Integers}}$

(Not call Math.Pow) $\text{power}(a, n, p) = a^n \% p$

$a = 2,\ n = 5,\ p = 7 \longrightarrow 2^5 \% 7 \Rightarrow 32 \% 7 = 4$

$a = 3,\ n = 4,\ p = 6 \longrightarrow 3^4 \% 6 \Rightarrow 81 \% 6 = 3$

```
int power (a, n ){
    for (i= 1; i<= n; i++){
        a = a * a;
    }
    ret a ;
}
```

i	a_{before}	a_{after}
1	a	$a = a^2$
2	a^2	$a = a^4$
3	a^4	$a = a^8$
4	a^8	$a = a^{16}$

```
int power (a, n, p){
long ~~int~~ ans = 1;
    for (i= 1; i<= n; i++){      n iteration
        // ans = ans x a
        ans = ((ans % p) x (a % p)) % p;
    }
    ret ans;
}
```

$(ans \% p) \to (P-1)$, $(a \% p) \to (P-1)$

$(P-1)^2 \approx 10^{18}$ long

$a = 10,\ n = 40$

$ans = 10^{40} \to$ long ✗

TC: $O(n)$
SC: $O(1)$

$x \% p \Rightarrow [0, P-1]$, $P-1 \to \approx 10^9$

Divisibility Rules

Rule for 3 → Sum of the digits has to be divisible by 3.

$(4372) \% 3 \Rightarrow (4 \times 10^3 + 3 \times 10^2 + 7 \times 10^1 + 2 \times 10^0) \% 3$

$= (a+b) \% M = (a \% M + b \% M) \% M$

$((4 \times 10^3) \% 3 + (3 \times 10^2) \% 3 + (7 \times 10) \% 3 + (2 \times 10^0) \% 3) \% 3$

$(a \times b) \% M = (a \% M \times b \% M) \% M$

$10 \% 3 \rightarrow 1$

$10^2 \% 3 \rightarrow 1$

$10^3 \% 3 \rightarrow 1$

$\vdots$

$10^n \% 3 \rightarrow 1$

$((4 \% 3) \times 1) \% 3 + ((3 \% 3) \times 1) \% 3 + ((7 \% 3) \times 1) \% 3 + (2 \% 3 \times 1) \% 3) \% 3$

$(a \% b) \% b = a \% b$

$((\overset{a}{4} \% \overset{M}{3}) + (\overset{b}{3} \% \overset{m}{3}) + (\overset{c}{7} \% \overset{M}{3}) + (\overset{d}{2} \% \overset{m}{3})) \% 3$

$(a \% M + b \% M) \% M = (a+b) \% M$

Sum of digits of the no.

$(4 + 3 + 7 + 2) \% 3$

Rule for 4 $\Rightarrow$ No. formed by the last two digits should be divisible by 4.

$$(3484)\%4 \Rightarrow (3\times10^3 + 4\times10^2 + 8\times10 + 4)\%4$$

$$\left(\underbrace{(3\times10^3)\%4}_{} + \underbrace{(4\times10^2)\%4}_{0} + \underbrace{(8\times10)\%4 + 4\%4}_{}\right)\%4$$

$$\left((3\%4)\times\cancel{(10^3\%4)}^{0}\right)\%4$$

$100\%4 \Rightarrow 0$

$10^3\%4 \Rightarrow 0$

$10^4\%4 \Rightarrow 0$

$\vdots$

$10^n\%4 \Rightarrow 0$

$(n \geq 2)$

$$\Big(0 + 0 + \overbrace{(8\times10)}^{a}\%\overset{M}{4} + \overset{b}{4}\%\overset{m}{4}\Big)\%\overset{M}{4}$$

$$(a\%M + b\%M)\%M = (a+b)\%M$$

$$((8\times10) + 4)\%4$$

$$\underbrace{(84)}\%4$$

$\hookrightarrow$ No formed by the last two digits of the no.

HW: 9, 5, 11

Google

Q Given a no. in the form of an array (A) (size N)
Given a no. P.

Return $A \% P$

$1 \le P \le 10^9$
$1 \le N \le 10^5$ ← size of the array

Ex

A: | 1 | 2 | 3 | 4 | 4 | ⇒ 12344

P = 4

$A \% P \Rightarrow 0$

int $\boxed{10^9}$ 10^9

long $\boxed{10^{18}}$ → $\boxed{\text{19 digits}}$

$P \Rightarrow 2, 3, 4, 8, 9, 7, 10,$

10^{20} 10^{40}, 10^{100}

The array size can be 10^5
i.e 10^5 digits in no.
$|A| \approx 10^{100000}$ Can't be stored in long.

$N: (\overset{0}{3}\ \overset{1}{8}\ \overset{2}{4}\ \overset{3}{3}\ \overset{4}{6}\ \overset{5}{8}\ \overset{6}{9}) \% P$

$((3 \times 10^6) \% P$ $10^6 \% P$

$(8 \times 10^5) \% P$ $10^5 \% P$

$(4 \times 10^4) \% P$ $10^4 \% P$

$(3 \times 10^3) \% P$

$(6 \times 10^2) \% P$

$(8 \times P)$

$(9)) \% P$

N

$\left[\underbrace{A[0]}_{a} \times \underbrace{10^{N-1}}_{b}\right] \% P$

$(a \% P \times b \% P) \% P$

$\boxed{(10^{N-1}) \% P}$

$N_{max} = 10^5$

$10^{N-1} = 10^{100000}$

$$A: \overset{0}{a_0},\ \overset{1}{a_1},\ \overset{2}{a_2},\ \overset{3}{a_3},\ \overset{4}{a_4},\ \overset{5}{a_5}\ \text{-----}\ \overset{N-1}{a_{n-1}}$$

$$\left((a_0 \times 10^{n-1}) + (a_1 \times 10^{n-2}) + (a_2 \times 10^{n-3}) \text{-----} + a_{n-1}\right) \% p$$

$$\left((a_0 \times 10^{n-1})\% p + (a_1 \times 10^{n-2})\% p + (a_2 \times 10^{n-3})\% p \text{....}\right) \% p$$

$$\underbrace{(a_0 \% p \times 10^{n-1} \% p) \% p}_{\text{Power}(10,\, n-1,\, p)} + (\underbrace{a_1 \% p \times 10^{n-2} \% p}_{\text{Power}(10,\, n-2,\, p)}) \% p + \text{----}) \% p$$

ans = 0;

for (i = 0; i < N; i++) { ⟹ N iterations

  ans = (ans + A[i] % p × power(10, n-1-i, p)) % p;

  ↳ N iterations

}

TC : $O(N^2)$

$$a^x \times a^y = a^{x+y}$$

i			
0 ⟶	$10^{N-1} \% p$	⟹	$(10 \times 10^{N-2}) \% p$
1 ⟶	$10^{N-2} \% p$	⟹	$(10 \times 10^{N-3}) \% p$
2 ⟶	$10^{N-3} \% p$	⟹	$(10 \times 10^{N-4}) \% p$
3 ⟶	$10^{N-4} \% p$	⟹	$(10 \times 10^{N-5}) \% p$
⋮	⋮		

$n-5 \longrightarrow 10^4 \% p \Rightarrow (10 \times 10^3) \% p \Rightarrow (10 \% p \times 10^3 \% p) \% p$

$n-4 \longrightarrow 10^3 \% p \Rightarrow (10 \times 10^2) \% p \Rightarrow (10 \% p \times 10^2 \% p) \% p$

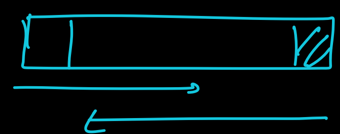
$n-3 \longrightarrow 10^2 \% p \Rightarrow (10 \times 10^1) \% p \Rightarrow (10 \% p \times 10^1 \% p) \% p$

$\hookrightarrow O(1)$

$n-2 \longrightarrow 10^1 \% p \longrightarrow 10 \% p \rightarrow 10$

$n-1 \longrightarrow 10^0 \% p \longrightarrow 1 \% p \rightarrow 1$

r



```
r = 1
for (i = N-1 ; i >= 0 ; i--) {
    ans = (ans + (A[i] % p × r)) % p;
    r = (r × 10) % p;
}
ret ans;
```

TC : O(N)

SC : O(1)

Carry Forward