

Hi,

I have computed histograms for MMD under various settings, see the Figure below.

Let

$$\Sigma = \begin{pmatrix} 15.5 & 14.5 \\ 14.5 & 15.5 \end{pmatrix},$$

$P_0 = N([0 \ 0], \Sigma)$ ,  $P_1 = N([0 \ 1], \Sigma)$  and  $P_6 = N([0 \ 6], \Sigma)$ .

The top graph compares MMD estimator distribution calculated using samples from  $(P_0, P_0)$  and  $(P_0, P_6)$  i.e.  $MMD(P_0, P_0)$  and  $MMD(P_0, P_6)$ . **The samples were obtained from Gibbs sampler, size of each sample was 200.** We see that the histograms are disjoint so we would expect test procedures to have 5% type one error and 0% type two error.

The wild bootstrap reports 17% type one error (I believe this is due to 'artificial degeneration' which does not converge very well) and 0% type two error.

Middle graph compares MMD estimator distribution calculated using samples from  $(P_0, P_0)$  and  $(P_0, P_1)$  i.e.  $MMD(P_0, P_0)$  and  $MMD(P_0, P_1)$ . The samples were also obtained from the Gibbs sampler. 95 percentile of the  $MMD(P_0, P_0)$  distribution is 25.55. Empirical probability that  $MMD(P_0, P_1)$  estimator is smaller than 25.55 is 0.93. Therefore we expect type two error to be around 93%.

Finally the bottom graph compares MMD estimator distribution calculated using samples from  $(P_0, P_0)$  and  $(P_0, P_1)$ , but the samples have no temporal dependence (they are IID, generated using Matlab function for multivariate Gaussian). **In this setting the expected type two error is 14% !.** This is inconsistent with the result obtained by Dino - he reported type two error around 1%. That means we have different MMD procedures or there is something going on with random variables generation (e.g. mistake in  $\Sigma$ ) or some other bug.

All histograms were obtained from 2000 samples.

