List 1

Michał Balcerek, Krzysztof Burnecki
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Theorem 1 (Law of Large Numbers (LLN))**.** *What are the assumptions, what is the statement?*

Exercise 1 (LLN for exponential and Pareto distributions)**.** *Please check LLN for the exponential and Parero distributions ($\alpha < 1$ and $\alpha > 1$).*

Pareto distribution has the cumulative distribution function (CDF) of the form:

$$1 - F(t) = \left(\frac{\lambda}{\lambda + t}\right)^{\alpha}, \quad t \geq 0,$$

for $\lambda > 0$, $\alpha > 0$.

Theorem 2 (Central Limit Theorem)**.** *What are the assumptions, what is the statement?*

For the following exercises, to calculate CDF, probability density function (PDF) and characteristic function (CF) use the Monte Carlo simulations.

Exercise 2 (Rule of 12)**.** *Compare CDF's, PDF's and CF's of*

$$Z = \frac{\sum_{k=1}^{12} U_k - 12\mathbb{E}(U_1)}{\sqrt{12}\sqrt{\mathrm{Var} U_1}},$$

where $\{U_k\}_{k=1}^{12}$ have standard uniform distribution, with the corresponding quantities for the standard normal distribution.

Exercise 3 (CLT for exponential distribution)**.** *Compare CDF's, PDF's and CF's of*

$$Z = \frac{\sum_{k=1}^{n} X_k - n\mathbb{E}(X_1)}{\sqrt{n}\sqrt{\mathrm{Var} X_1}},$$

where $\{X_k\}_{k=1}^{n}$ are i.i.d exponential random numbers (with some $\lambda > 0$), with the corresponding quantities for the standard normal distribution. Use different n's.

Exercise 4 (CLT (or lack thereof) for Pareto distribution)**.** *Compare CDFs, PDFs and CF's of*

$$Z = \frac{\sum_{k=1}^{n} X_k - n\mathbb{E}(X_1)}{\sqrt{n}\sqrt{\mathrm{Var} X_1}},$$

where $\{X_k\}_{k=1}^n$ have Pareto distribution (with $\lambda = 1$ and some $\alpha > 2$) with the corresponding quantities for the standard normal distribution. Use different n's.
Furthermore, check the behavior of the distribution of

$$Z = \frac{\sum_{k=1}^{n} X_k - n\mathbb{E}(X_1)}{\sqrt{n}},$$

where $\{X_k\}_{k=1}^n$ have Pareto distribution (with $\lambda = 1$ and some $1 < \alpha < 2$).
Hint: *To simulate Pareto (or any other distribution with easily invertible CDF) you can use the inverse transform method: to generate random variable X with CDF F_X you can simulate standard uniform random variable and plug it into the inverse of F_X:*

$$F_X^{-1}(U) \sim X \quad \text{for } U \sim \mathcal{U}(0,1).$$