(6) $A(x) = a_0 + a_1 x + a_2 x^2 + \dots$

$B(x) = 0 \cdot 1 + 0 \cdot x + \dots + 0 \cdot x^{k-1} + a_0 x^k + a_1 x^{k+1} + \dots + a_n x^{k+n} + \dots$

$B(x) = x^k \underbrace{(a_0 + a_1 x + \dots + a_n x^n + \dots)}_{A(x)}$

$B(x) = x^k \cdot A(x)$ ✓

$C(x) = a_k \cdot x^0 + a_{k+1} x^1 + \dots + a_{k+n} x^n + \dots$

$C(x) = ?$

(5) $(0, 0, 0, 1, 3, 7, 15, 31, \dots)$

$$\begin{cases} a_n = 3a_{n-1} - 2a_{n-2} \\ a_0 = a_1 = a_2 = 0 \quad a_3 = 1 \end{cases}$$

Spethia

↓

$A = 0\cdot 1 + 0\cdot x + 0\cdot x^2 + x^3 + 3x^4 + 7x^5 + 15x^6 + \dots + a_n x^n + \dots \quad / \cdot(-3x)$

$-3x\,A = 0 + 0 + 0 + 0 - 3x^4 - 9x^5 - 21x^6 - \dots - 3a_{n-1}x^n - \dots$

$2x^2 A = 0 + 0 + 0 + 0 + 0 + 2x^5 + 6x^6 + \dots + 2a_{n-2}x^n$

$+$

$A(1 - 3x + 2x^2) = x^3$

$$A = \frac{x^3}{1 - 3x + 2x^2}$$

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$a^3 \mid b^2 \overset{?}{\Rightarrow} a \mid b$

$p_i \in \mathbb{P} \qquad a = p_1^{\alpha_1} \cdot \dots \cdot p_n^{\alpha_n}$

$b = p_1^{\beta_1} \cdot \dots \cdot p_n^{\beta_n} \qquad \alpha_i, \beta_i \geq 0$

$a^3 \mid b^2 \Rightarrow p_1^{3\alpha_1} \cdot \dots \cdot p_n^{3\alpha_n} \mid p_1^{\beta_1} \cdot \dots \cdot p_n^{\beta_n} \Rightarrow$

$\forall i \quad 3\alpha_i \leq \beta_i \iff \alpha_i \leq \frac{\beta_i}{3}$

$a \mid b \iff p_1^{\alpha_1} \cdot \dots \cdot p_n^{\alpha_n} \mid p_1^{\beta_1} \cdot \dots \cdot p_n^{\beta_n}$

$\overset{\forall i}{\alpha_i \leq \beta_i}$ ale $\alpha_i \leq \frac{\beta_i}{3} \iff \alpha_i \leq \beta_i$ ✓