

5.8. Niech  $f(x) - L_n(x) = R_n(x)$

$$R_n(x) = \frac{f^{(n+1)}(\xi)}{(n+1)!} \cdot p_{n+1}(x)$$

$$p_{n+1}(x) = \prod_{i=0}^n (x - x_i) ; x_0, \dots, x_n - \text{zera } T_{n+1}(x)$$

zauważamy, że  $p_{n+1}(x) = \frac{1}{2^n} T_{n+1}(x)$

wtedy  $R_n(x) = \frac{f^{(n+1)}(\xi)}{(n+1)!} \cdot \frac{1}{2^n} T_{n+1}(x)$

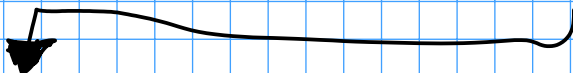
$$f(x) = e^x$$

$$\|R_n(x)\|_{[-1,1]} = \left\| \frac{f^{(n+1)}(\xi)}{(n+1)!} \cdot \frac{1}{2^n} \cdot T_{n+1}(x) \right\|_{[-1,1]}$$

zauważamy, że  $\|f^{(n+1)}(\xi)\|_{[-1,1]} = e^1 = e$

oraz  $T_{n+1}(x) \leq 1$

wtedy:  $\|R_n(x)\| \leq \frac{e}{2^n (n+1)!} \stackrel{?}{\leq} 10^{-5}$



$$\frac{1}{2^n \cdot (n+1)!} \cdot e \leq 10^{-5}$$

$$e \leq 10 \text{ więc}$$

$$\frac{1}{2^n (n+1)!} \cdot e \leq \left[ \frac{1}{2^n (n+1)!} \cdot 10 \leq 10^{-5} \right] / :10$$

$$\frac{1}{2^n (n+1)!} \leq 10^{-6}$$

$$2^n (n+1)! \geq 1000000$$

dla  $n \geq 7$  zachodzi