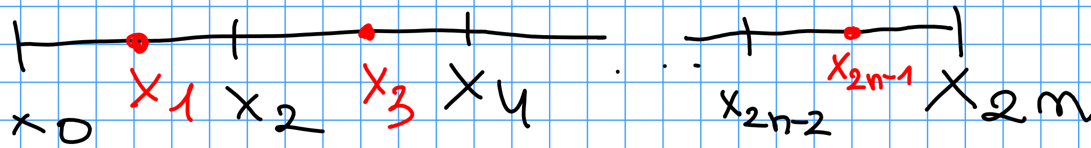
10.9

$$h_n = \frac{b-a}{n} \; ; \; h_{2n} = \frac{b-a}{2n}$$



$$T_n(f) = \sum_{i=0}^{n}{}'' h_n f(x_{2i})$$

$$T_{2n}(f) = \sum_{i=0}^{2n}{}'' h_{2n} f(x_i) =$$

$$4 \cdot / \underbrace{h_{2n}} \left(\frac{1}{2} f(x_0) + f(x_1) + \dots + f(x_{2n-1}) + \frac{1}{2} f(x_{2n}) \right)$$

$$\cancel{4}^2 \cdot \frac{h_n}{\cancel{2}_1} \left(\frac{1}{2} f(x_0) + f(x_1) + \dots + f(x_{2n-1}) + \frac{1}{2} f(x_{2n}) \right)$$

$$(1) \; h_n \left(f(x_0) + 2f(x_1) + \dots + 2f(x_{2n-1}) + f(x_{2n}) \right)$$

$T_n(f) =$

$$(2) \; h_n \left(\frac{1}{2} f(x_0) + f(x_2) + \dots + f(x_{2n-2}) + \frac{1}{2} f(x_{2n}) \right)$$

$(1) - (2)$:

$$h_n \left(\frac{1}{2} f(x_0) + 2f(x_1) + f(x_2) + 2f(x_3) + \dots + 2f(x_{2n-1}) + \frac{1}{2} f(x_{2n}) \right)$$

(3)

Rozpiszmy $S_{2n}(f)$:

$$S_{2n}(f) = \underbrace{\frac{h_{2n}}{3}\left(f(x_1) + 4f(x_2) + 2f(x_3) + \ldots + 4f(x_{2n-1})\right.}+f(x_{2n})$$

$h_{2n} = \frac{h_n}{2}$ więc

(4)

$$\frac{h_n}{3}\left(\frac{1}{2}f(x_1) + 2f(x_2) + f(x_3) + \ldots + 2f(x_{2n-1}) + \frac{1}{2}f(x_{2n})\right)$$

zatem $\frac{1}{3}\cdot(3) = (4)$

więc $S_{2n}(f) = \frac{1}{3}\left(4T_{2n}(f) - T_n(f)\right)$

podstawiając $n := \frac{n}{2}$ otrzymujemy

$$S_n(f) = \frac{1}{3}\left(4T_n(f) - T_{\frac{n}{2}}(f)\right)$$