$\boxed{11.4}$

$$w_n(x) = {\sum_{k=0}^{n}}' a_k T_k(x)$$

$$\int_{-1}^{1} w_n(x)\,dx = {\sum_{k=0}^{n}}' a_k \int_{-1}^{1} T_k(x)\,dx \quad (1)$$

zauważmy, że T_{2i+1} jest nieparzyste $(i \geq 0)$

zatem $\int T_{2i}(x)\,dx = 0$ więc

$$(1) = {\sum_{k=0}^{\lfloor \frac{n}{2} \rfloor}}' a_{2k} \int_{-1}^{1} T_{2k}(x)\,dx$$

$$\int_{-1}^{1} T_{2k}(x)\,dx = \int_{-1}^{1} \cos(2k \arccos x)\,dx =$$

$$= \left\{ \begin{array}{l} x = \cos t \\ dx = -\sin t\,dt \end{array} \right\} = \int_{-\pi}^{0} \cos(2kt)\cdot(-1)\sin t\,dt$$

$$= \int_{0}^{\pi} \cos(2kt)\sin t\,dt = \frac{1}{2}\int_{0}^{\pi} 2\cos(2kt)\sin t\,dt$$

$$= \frac{1}{2}\int_{0}^{\pi} \sin(2kt+t) + \sin(2kt-t)\,dt =$$

$$= \frac{1}{2}\int_{0}^{\pi} \sin(t(2k+1)) - \sin(t(2k-1))\,dt$$

$$= \frac{1}{2}\left(\frac{\cos(2k-1)t}{2k-1} - \frac{\cos(2k+1)t}{2k+1}\right)\Bigg|_0^{\pi} =$$

$$= \frac{1}{2}\left(-\frac{1}{2k-1} + \frac{1}{2k+1} - \frac{1}{2k-1} + \frac{1}{2k+1}\right) =$$

$$= \frac{1}{2k+1} - \frac{1}{2k-1} = \frac{2}{1-4k^2}$$

więc:

$$(1) = \sum_{k=0}^{n}{}' a_k \cdot \frac{2}{1-4k^2} = \underline{2\sum_{k=0}^{n}{}' \frac{a_k}{1-4k^2}}$$