L03Z07 KACPER

3.7.

$$\Phi(x) = x + f(x) \cdot g(x)$$

1) $\Phi(\alpha) = \alpha + \underbrace{f(\alpha) \cdot g(\alpha)}_{0} = \alpha$

$$\Phi'(\alpha) = 1 + f'(\alpha) \cdot g(\alpha) + \underbrace{f(\alpha) \cdot g'(\alpha)}_{0} =$$

$$= 1 + f'(\alpha) \cdot g(\alpha) = 0$$

$$g(\alpha) = \frac{-1}{f'(\alpha)}$$

$$\Phi''(\alpha) = f''(\alpha) \cdot g(\alpha) + f'(\alpha) \cdot g'(\alpha) +$$

$$+ f'(\alpha) \cdot g'(\alpha) + \underbrace{f(\alpha) \cdot g''(\alpha)}_{0} = 0$$

$$= 2f'(\alpha) \cdot g'(\alpha) + \frac{f''(\alpha)}{-f'(\alpha)} = 0$$

$$\boxed{g'(\alpha) = \frac{f''(\alpha)}{2(f'(\alpha))^2}}$$

$$\Phi''(x) = f''(x)\cdot g(x) + 2f'(x)\cdot g'(x) + f(x)\cdot g''(x)$$

$$\Phi'''(\alpha) \neq 0$$

$$= f'''(\alpha)\cdot g(\alpha) + f''(\alpha)\cdot g'(\alpha) + 2f''(\alpha)\cdot g'(\alpha) + 2f'(\alpha)g''(\alpha)$$

$$+ f'(\alpha)\cdot g''(\alpha) + \underbrace{f(\alpha)\cdot g'''(\alpha)}_{0} =$$

$$= 3f''(\alpha)\cdot g'(\alpha) + 3f'(\alpha)\cdot g''(\alpha) + f'''(\alpha)\,g(\alpha)$$

$$\neq 0$$

$$3f''(\alpha)\cdot\frac{f''(\alpha)}{2(f'(\alpha))^2} + 3f'(\alpha)\cdot g''(\alpha) + f'''(\alpha)\cdot\frac{-1}{f'(\alpha)}$$

$$g''(\alpha)\cdot 3f'(\alpha) \neq \frac{f'''(\alpha)}{f'(\alpha)} - \frac{3(f''(\alpha))^2}{2(f'(\alpha))^2}$$

$$\boxed{g''(\alpha) \neq \frac{2f'''(\alpha)f'(\alpha) - 3(f''(\alpha))^2}{6(f'(\alpha))^3}}$$