7.1 $k \neq l$, $k,l = 0, \dots$

i) $\int_{-1}^{1} (1-x^2)^{-\frac{1}{2}} T_k(x) T_l(x) \, dx = 0$

niech $x = \cos\alpha$ wtedy

$$T_n(x) = T_n(\cos\alpha) = \cos(n \arccos(\cos\alpha))$$

$$= \cos(n\alpha)$$

więc:

$$x = \cos\alpha$$

$$\frac{1}{\sqrt{\sin^2\alpha}} \qquad dx = -\sin\alpha \, d\alpha$$

$$\int_{-1}^{1} (1-x^2)^{-\frac{1}{2}} T_k(x) T_l(x) \, dx = -\int_{\pi}^{0} \cos(k\alpha) \cdot \cos(l\alpha) \, d\alpha$$

$$= \int_0^{\pi} \cos(k\alpha) \cos(l\alpha) \, d\alpha =$$

$$= \frac{1}{2} \int_0^{\pi} \cos(k+l)\alpha + \cos(k-l)\alpha \, d\alpha =$$

$$= \frac{1}{2} \left[\frac{\sin(k+l)\alpha}{k+l} + \frac{\sin(k-l)\alpha}{k-l} \right] \Bigg|_0^{\pi} =$$

$$= \frac{1}{2} \left[\frac{\sin(k+l)\pi}{k+l} + \frac{\sin(k-l)\pi}{k-l} \right] =$$

$$= \frac{1}{2} [0 + 0] = \underline{0} \quad \checkmark$$

ii) $\int_{-1}^{1} (1-x^2)^{-\frac{1}{2}} [T_k(x)]^2 \, dx$

2 i) mamy $(n \neq 0)$

$$\int_0^{\pi} \cos^2(\alpha n) \, d\alpha =$$

$$= \left[\frac{\sin(2nx) + 2nx}{4n} \right]_0^{\pi} = \frac{\pi}{2} - 0 = \underline{\frac{\pi}{2}}$$

$n = 0$ $\quad \int_0^{\pi} 1 \, d\alpha = \alpha \Big|_0^{\pi} = \underline{\pi}$ $\quad$ ✓