Zad 2.7

a) $f(x) = \frac{1}{x^2+c}$ ; $f'(x) = \frac{-2x}{(x^2+c)^2}$

$$wsk = \left|\frac{x \cdot f'(x)}{f(x)}\right| = \left|\frac{x \cdot \frac{-2x}{(x^2+c)^2}}{\frac{1}{x^2+c}}\right| = \left|\frac{-2x^2}{(x^2+c)^{\cancel{2}}} \cdot \frac{\cancel{x^2+c}}{1}\right| =$$

$$= \left|\frac{-2x^2}{x^2+c}\right| = \left|\frac{-2}{1+\frac{c}{x^2}}\right| = \frac{2}{\left|1+\frac{c}{x^2}\right|}$$

$1^0$ $c \geq 0$ wtedy $\frac{2}{\left|1+\frac{c}{x^2}\right|} \leq 2$ zawsze bo

$$1 \leq \left|1 \oplus \frac{c}{x^2}\right|$$

$$\cancel{1} \leq \cancel{1} + \frac{c}{x^2}$$

$$0 \leq \frac{c}{x^2} \quad (bo\ c, x^2 \geq 0).$$

$2^0$ $c < 0$ wtedy

$\frac{2}{\left|1+\frac{c}{x^2}\right|} \to \infty$ gdy $\frac{c}{x^2} \approx -1$ a to zajdzie gdy $x \approx \sqrt{-c}$

wtedy funkcja jest źle uwarunkowana.

b) $f(x) = \dfrac{1-\cos x}{x^2}$ ; $x \neq 0$ ; $f'(x) = \dfrac{\sin x \cdot x^2 - (1-\cos x)\cdot 2x}{x^4} =$

$$= \frac{x\sin x + 2\cos x - 2}{x^3}$$

$$\text{wsp} = \left|\frac{x\cdot f'(x)}{f(x)}\right| = \left|\frac{x\cdot \frac{x\sin x + 2\cos x - 2}{x^3}}{\frac{1-\cos x}{x^2}}\right| = \left|\frac{x\cdot \sin x + 2\cos x - 2}{\cancel{x^2}}\cdot\frac{\cancel{x^2}}{1-\cos x}\right| =$$

$$= \left|\frac{x\sin x - 2(1-\cos x)}{1-\cos x}\right| = \left|\frac{x\sin x}{1-\cos x} - 2\right| =$$

$$= \left|x\cdot \mathrm{ctg}\left(\frac{x}{2}\right) - 2\right|$$

wsk $\to \infty_+$ gdy $\left|x\,\mathrm{ctg}\left(\frac{x}{2}\right)\right| \to \infty_+$ ;

a to zachodzi gdy $x \approx 2k\pi$. (wtedy $\left|\mathrm{ctg}\left(\frac{x}{2}\right)\right| \to \infty_+$)

Zatem wskaźnik $\to \infty_+$ okresowo, więc funkcja jest źle uwarunkowana.