3.1 Obliczamy $\frac{1}{c}$ $\quad \frac{1}{c} = \alpha$

$$f(x) = \frac{1}{x} - c$$

$$f'(x) = -\frac{1}{x^2}$$

$$x_{n+1} = x_n - \frac{\frac{1}{x_n} - c}{-\frac{1}{x_n^2}}$$

Newton

$$x_{n+1} = x_n - \frac{f(x)}{f'(x)}$$

$$x_{n+1} = x_n + (x_n - x_n^2 c)$$

$$x_{n+1} = 2x_n - x_n^2 c = \underline{x_n(2 - x_n c)}$$

$$\Phi(x_n) = x_n(2 - x_n c)$$

Zbieżność, gdy $|\Phi'(\eta)| < 1$

$$\Phi'(x) = 2 - 2xc$$

$$|\Phi'(\eta)| < 1$$

$$|2 - 2\eta c| < 1$$

$1-\eta c < \frac{1}{2} \wedge 1-\eta c > -\frac{1}{2}$

$\eta c > \frac{1}{2} \wedge \eta c < \frac{3}{2}$

$\frac{1}{2\cancel{c}} < \frac{3}{2\cancel{c}}$

$1 < 3$ ✓

1° $c > 0$ wtedy

$\eta > \frac{1}{2c} \wedge \eta < \frac{3}{2c} \Rightarrow \underline{\eta \in \left(\frac{1}{2c}; \frac{3}{2c}\right)}$

2° $c < 0$

$\eta < \frac{1}{2c} \wedge \eta > \frac{3}{2c}$

wtedy $\frac{1}{2c} > \frac{3}{2c} \quad / 2c$

$1 < 3$

$\eta \in \left(\frac{3}{2c}; \frac{1}{2c}\right)$

$1^\circ \wedge 2^\circ \Rightarrow \eta \in \text{interv}\left(\frac{1}{2c}; \frac{3}{2c}\right)$

więc metoda ta jest zbieżna na obszarze

interv $\left(\frac{1}{2c}; \frac{3}{2c}\right)$

więc $|\Phi'(\eta)| < 1 \iff \underline{\eta \in \text{interw}\left(\frac{1}{2c}; \frac{3}{2c}\right)}$

$$e_n = x_n - \alpha$$

$$e_{n+1} = x_{n+1} - \alpha = \Phi(x_n) - \Phi(\alpha)$$

$$e_{n+1} = \Phi(x_n) - \Phi(\alpha) = \underbrace{\Phi'(\eta)(x_n - \alpha)}$$

$$\underline{e_{n+1} = \Phi'(\eta)\, e_n}$$

skoro $|\Phi'(\eta)| < 1$

$\boxed{\alpha = \frac{1}{c}}$ to $e_n \to 0$ więc $x_n \to \alpha$

- Gdy weźmiemy dowolne $x_0 \in \underline{\text{interw}\left(\frac{1}{2c}; \frac{3}{2 \cdot c}\right)} = \underline{\text{interw}\left(\frac{\alpha}{2}; \frac{3\alpha}{2}\right)}$

to każde kolejne $x_i \to \alpha$ oraz będzie coraz

"bliżej" α więc niż wyraz poprzedni z przedziału.