Zadanie 6

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(1) $\int_{-1}^{1} f(x)\,dx = \int_0^{\pi} f(\cos\theta)\sin\theta\,d\theta$ zmieniamy postać całki

$x = \cos\theta \quad dx = -\sin\theta\,d\theta$

Rozwijamy w szereg Czebyszewa

$f(\cos\theta) = \frac{a_0}{2} + \sum_{k=1}^{\infty} a_k \cos(k\theta)$ i całkujemy

(2) $\int_0^{\pi} f(\cos\theta)\sin\theta\,d\theta = \underbrace{\int_0^{\pi}\frac{a_0}{2}\sin\theta\,d\theta}_{(a)} + \sum_{k=1}^{\infty} a_k \underbrace{\int_0^{\pi} 2\cos(k\theta)\sin\theta\,d\theta}_{(b)}$

$\Downarrow$

$2\cos\alpha\sin\beta = \sin(\alpha+\beta) - \sin(\alpha-\beta)$

(a) $\int_0^{\pi}\sin\theta\,d\theta = -\cos\theta\Big|_0^{\pi} = 2$

(b) $\frac{1}{2}\int_0^{\pi} 2\cos k\theta\sin\theta\,d\theta = \frac{1}{2}\int_0^{\pi} \sin(k+1)\theta - \sin(k-1)\theta\,d\theta = \begin{cases} \emptyset & k \text{ niepar.} \\ \frac{2}{1-k^2} & k \text{ parzyste} \end{cases}$

dla $k=1$

$\int_0^{\pi} 2\cos k\theta\sin\theta\,d\theta = \int_0^{\pi} \sin 2\theta - \underbrace{\sin\emptyset}_{\text{zero}}\,d\theta = -\frac{\cos 2\theta}{2}\Big|_0^{\pi} = \emptyset$

$\int_0^{\pi} f(\cos\theta)\sin\theta\,d\theta \approx a_0 + \sum_{k=1}^{\infty}\frac{2a_{2k}}{1-4k^2} = \sum_{k=0}^{\infty}{}' \frac{2a_{2k}}{1-4k^2}$

$a_k = \frac{2}{\pi}\int_0^{\pi} f(\cos\theta)\cos(k\theta)\,d\theta$

Jako punkty kwadratury przyjmujemy $t_j = \frac{j\pi}{n}$ $j=0,\dots n$

$a_k \approx \frac{2}{n}\left[\frac{f(1)}{2} + \frac{f(-1)}{2}(-1)^k + \sum_{j=1}^{n-1} f(\cos\frac{j\pi}{n})\cos\frac{jk\pi}{n}\right]$

Zakładamy, że n jest parzyste

chcemy policzyć tylko $a_0, a_2, \dots, a_{n-2}, a_n$

$a_0 = a_n \quad a_2 = a_{n-2}$ etc.

$a_{2k} \approx \frac{2}{n}\left[\frac{f(1)}{2} + \frac{f(-1)}{2} + \sum_{j=1}^{n-1} f(\cos\frac{j\pi}{n})\cos\frac{jk\pi}{n/2}\right]$

$\cos\frac{j\,2k\pi}{n} = \cos\frac{jk\pi}{n/2}$

Np. $n = 8$

$f(\cos\frac{\pi}{8})\cos\frac{2k\pi}{n}$ $\quad$ $f(\cos\frac{2\pi}{8})\cos\frac{4k\pi}{n}$ $\quad$ $f(\cos\frac{3\pi}{8})\cos\frac{6k\pi}{8}$

$\boxed{\begin{matrix} f(-\cos\frac{\pi}{8}) \\ f(-\cos\frac{2\pi}{8}) \\ f(-\cos\frac{3\pi}{8}) \end{matrix}} \equiv \boxed{\begin{matrix} f(\cos\frac{7}{8}\pi) \\ f(\cos\frac{6\pi}{8}) \\ f(\cos\frac{5\pi}{8}) \end{matrix}}$ $\begin{matrix} \cos\frac{14k\pi}{8} \\ \cos\frac{12k\pi}{8} \\ \cos\frac{10k}{8}\pi \end{matrix}$

$f(\cos\frac{4\pi}{8})\cos\frac{8k\pi}{8} = f(0)(-1)^k$