④ $(a+b)^n = \sum_{i=0}^{n} \binom{n}{i} a^i b^{n-i}$

1^0 baza ind

$n=1$

$(a+b) = \binom{1}{0} a^0 b^1 + \binom{1}{1} a^1 b^0 = b+a$ ✓

2^0 krok indukcyjny:

założmy, że $(a+b)^n = \sum_{i=0}^{n} \binom{n}{i} a^i b^{n-i}$

$$(a+b)^{n+1} = (a+b)(a+b)^n = (a+b)\sum_{i=0}^{n}\binom{n}{i} a^i b^{n-i} =$$

$$= \sum_{i=0}^{n} \binom{n}{i} a^{i+1} b^{n-i} + \sum_{i=0}^{n} \binom{n}{i} a^i b^{n-i+1} =$$

$$= \underbrace{\binom{n}{n} a^{n+1}}_{=} + \sum_{i=1}^{n} \binom{n}{i-1} a^i b^{n-i+1} + \underbrace{\binom{n}{0} b^{n+1}}_{=} + \sum_{i=1}^{n} \binom{n}{i} a^i b^{n-i+1}$$

$$= \binom{n+1}{n+1} a^{n+1} + \sum_{i=1}^{n} \left(\Big[\underbrace{\binom{n}{i-1} + \binom{n}{i}}_{\binom{n+1}{i}} \Big] a^i b^{n-i+1} \right) + \binom{n+1}{0} b^{n+1} =$$

$$= \binom{n+1}{n+1} a^{n+1} + \sum_{i=1}^{n} \binom{n+1}{i} a^i \cdot b^{n-i+1} + \binom{n+1}{0} b^{n+1} =$$

$$= \sum_{i=0}^{n+1} \binom{n+1}{i} a^i b^{(n+1)-i}$$

□