5.8. Niech $f(x) - L_n(x) = R_n(x)$

$$R_n(x) = \frac{f^{(n+1)}(\xi)}{(n+1)!} \cdot p_{n+1}(x)$$

$$p_{n+1}(x) = \prod_{i=0}^{n} (x - x_i) \; ; \; x_0; \dots; x_n - \text{zera } T_{n+1}(x)$$

zauważmy, że $p_{n+1}(x) = \frac{1}{2^n} T_{n+1}(x)$

więc $$R_n(x) = \frac{f^{(n+1)}(\xi)}{(n+1)!} \cdot \frac{1}{2^n} T_{n+1}(x)$$

$f(x) = e^x$

$$\|R_n(x)\|_{[-1,1]} = \left\| \left| \frac{f^{(n+1)}(\xi)}{(n+1)!} \right| \cdot \frac{1}{2^n} \cdot |T_{n+1}(x)| \right\|_{[-1,1]}$$

zauważmy, że $\| f^{(n+1)}(\xi) \|_{[-1,1]} = e^1 = e$

oraz $T_{n+1}(x) \leq 1$

więc: $$\|R_n(x)\| \leq \underbrace{\frac{e}{2^n (n+1)!} \overset{?}{\leq} 10^{-5}}$$

$$\frac{1}{2^n \cdot (n+1)!} \cdot e \leqslant 10^{-5}$$

$e \leqslant 10$ więc

$$\frac{1}{2^n(n+1)!} \cdot e \leqslant \frac{1}{2^n(n+1)!} \cdot 10 \leqslant 10^{-5} \quad / : 10$$

$$\frac{1}{2^n(n+1)!} \leqslant 10^{-6}$$

$$2^n(n+1)! \geqslant 1000000$$

dla $n \geqslant 7$ zachodzi