10.6

weźmy wielomian Hermite'a (oraz węzły x_0, x_1, x_1, x_2)

$$H_3(x) = L_2(x) + f[x_0, x_1, x_1, x_2](x-x_0)(x-x_1)(x-x_2)$$

$$\int_a^b H_3(x) = \int_a^b L_2(x) + \int_a^b f[x_0, x_1, x_1, x_2](x-x_0)(x-x_1)(x-x_2)$$

$$f[x_0, x_1, x_1, x_2] \int_a^b (x-x_0)(x-x_1)(x-x_2) =$$

$$= \left\{ \begin{array}{l} x = a + th \\ dx = dt \cdot h \end{array} \right\} = f[x_0, x_1, x_1, x_2] \underbrace{\int_0^2 t(t-1)(t-2) \cdot h^4 \cdot h}_{=0}$$

$$\text{więc} \int_0^2 H_3(x) = \int_0^2 L_2(x)$$

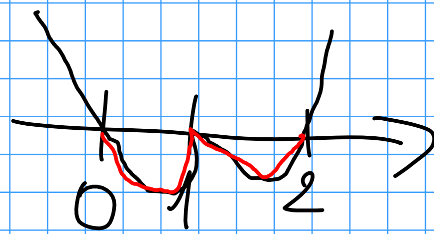
$$I(f) - Q_2^{NC}(f) = \int_a^b f(x) - \int L_2(x) =$$

$$= \int_a^b f(x) - H_3(x) = \int_a^b \frac{f^{(4)}(\xi_x)}{4!} \cdot (x-x_0)(x-x_1)^2(x-x_2) =$$

$$= \left\{ \begin{array}{l} x = a + th \\ dx = h \cdot dt \end{array} \right\} \quad \boxed{x_i = a + ih} \quad = \int_0^2 \frac{f^{(4)}(\xi_x)}{4!} \cdot t(t-1)^2(t-2) \cdot h^4 \cdot h =$$

$$= h^5 \int_0^2 \frac{f^{(4)}(\xi_x)}{4!} t(t-1)^2(t-2)$$

zauważmy, że $t(t-1)^2(t-2)$ ma stały znak na $[0;2]$



więc z tw. o wartości średniej dla całek mamy:

$$= h^5 \cdot \frac{f^{(4)}(\eta)}{4!} \int_0^2 t(t-1)^2(t-2) = h^5 \cdot \frac{f^{(4)}(\eta)}{4!} \cdot \left(-\frac{4}{15}\right) =$$

$$= \boxed{h^5 \cdot \frac{f^{(4)}(\eta)}{-90}}$$