1.

$$A = \begin{bmatrix} -1 & 1 & -4 \\ 2 & 2 & 0 \\ 3 & 3 & 2 \end{bmatrix} \qquad b = \begin{bmatrix} 0 \\ 1 \\ 0{,}5 \end{bmatrix}$$

$$Ax = b$$

$$\left[\begin{array}{ccc|c} -1 & 1 & -4 & 0 \\ 2 & 2 & 0 & 1 \\ 3 & 3 & 2 & 0{,}5 \end{array}\right] \xrightarrow{w_2 = w_2 - 2 \cdot w_1} \left[\begin{array}{ccc|c} -1 & 1 & -4 & 0 \\ 0 & 4 & -8 & 1 \\ 3 & 3 & 2 & 0{,}5 \end{array}\right]$$

$$\xrightarrow{w_3 = w_3 - 3w_1} \left[\begin{array}{ccc|c} -1 & 1 & -4 & 0 \\ 0 & 4 & -8 & 1 \\ 0 & 6 & -10 & 0{,}5 \end{array}\right] \xrightarrow{w_3 = w_3 - \frac{3}{2} w_2} \left[\begin{array}{ccc|c} -1 & 1 & -4 & 0 \\ 0 & 4 & -8 & 1 \\ 0 & 0 & 2 & -1 \end{array}\right]$$

$$\begin{cases} -x_1 + x_2 - 4x_3 = 0 \\ 4x_2 - 8x_3 = 1 \\ 2x_3 = -1 \end{cases}$$

$$x_3 = -0{,}5$$

$$x_2 = -0{,}75$$

$$x_1 = 1{,}25$$

$$x = \begin{bmatrix} 1{,}25 \\ -0{,}75 \\ -0{,}5 \end{bmatrix}$$

$LUx = b$

$Ly = b$

$$\begin{cases} 1y_1 = 1 \\ \frac{3}{2}y_1 + y_2 = -1 \\ \frac{1}{2}y_1 + \frac{11}{13}y_2 + y_3 = 2 \end{cases}$$

$y_1 = 1$

$y_2 = -1 - \frac{3}{2} = -\frac{5}{2}$

$y_3 = 2 - \frac{1}{2} - \frac{11}{13} \cdot \left(-\frac{5}{2}\right) = \frac{47}{13}$

$$y = \begin{bmatrix} 1 \\ -\frac{5}{2} \\ \frac{47}{13} \end{bmatrix}$$

$Ux = y$

$$U = \begin{matrix} 2 & -3 & -1 \\ 0 & \frac{13}{2} & -\frac{7}{2} \\ 0 & 0 & \frac{32}{13} \end{matrix}$$

$$\begin{cases} 2x_1 - 3x_2 - x_3 = 1 \\ 0x_1 + \frac{13}{2}x_2 - \frac{7}{2}x_3 = -\frac{5}{2} \\ \frac{32}{13}x_3 = \frac{47}{13} \end{cases} \Rightarrow$$

$x_3 = \frac{47}{32}$

$x_2 = \frac{13}{32}$

$x_1 = \frac{59}{32}$

$$X = \begin{bmatrix} \frac{59}{32} \\ \frac{13}{32} \\ \frac{47}{32} \end{bmatrix}$$