**Universitat Rovira i Virgili**

# Task 2 - Aggregation Operators for Numerical Data

## Application of Weighted Sum Model and Ordered Weighting Average

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# Contents

# 1 Introduction

The purpose of this task is to practice applying two methods of aggregation presented in class: WSM (Weighted Sum Model) which is equivalent to a weighted average, and OWA (Ordered Weighted Average). The task is presented as a notebook containing multiple sub-tasks, with each tackling a different problem to be solved and analysed.

# 2 Methodology

As mentioned in Section 1, the two aggregation operators covered in this notebook are Weighted Average, and Ordered Weighted Average. The following provides a brief overview of what each operator entails.

- **Weighted Average:** Weights are assigned to each of the criterion as part of the alternative evaluation. These weights are multiplied by the corresponding partial utility scores, and the results are summed to provide an overall utility score.
- **Ordered Weighted Average:** Similar in operation to Weighted Average, with the crucial difference that the input criterion are ordered in descending order at the initial phase. Importantly, the weights are not applied to the criterion themselves, but rather to the positions in the ordered set.

# 3 Task Analysis and Results

## 3.1 Task 1: Weighted Average or Weighted Sum

In this task, a weighted average is computed using a list of alternatives, a list of criteria, and corresponding weights for the criteria. The weights sum up to 1, and represent the relative importance of each criterion.

We consider the following alternatives and criterion:

| **Alternatives** | **Criteria** |
|---|---|
| Terrat | Food |
| RacoAbat | Personnel |
| Dominos | Atmosphere |
| Ancora | Category |
| Frida | Location |
| Barhaus | Terrace |

The criterion are used to evaluate each of the alternatives.

Following this, a performance table is constructed. Each row in the table corresponds to one of the listed restaurants and each column contains the partial utility score for the set of criteria, within the range [0, 1]. Figure 1 displays this performance table.

| | Food | Personnel | Atmosphere | Category | Location | Terrace |
|---|---|---|---|---|---|---|
| Terrat | 0.7 | 0.9 | 0.6 | 0.9 | 0.0 | 0.4 |
| RacoAbat | 0.5 | 0.7 | 0.6 | 0.7 | 0.2 | 0.0 |
| Dominos | 0.2 | 0.3 | 0.1 | 0.2 | 0.8 | 0.0 |
| Ancora | 0.9 | 0.5 | 0.4 | 0.3 | 1.0 | 0.7 |
| Frida | 0.8 | 0.7 | 0.1 | 0.5 | 0.1 | 0.4 |
| Barhaus | 0.9 | 1.0 | 0.3 | 0.8 | 0.2 | 1.0 |

Figure 1: Performance Scores of the Various Alternatives.

A radar projection visualisation is created to provide a clearer comparison of results. Figure 2 displays this radar projection.

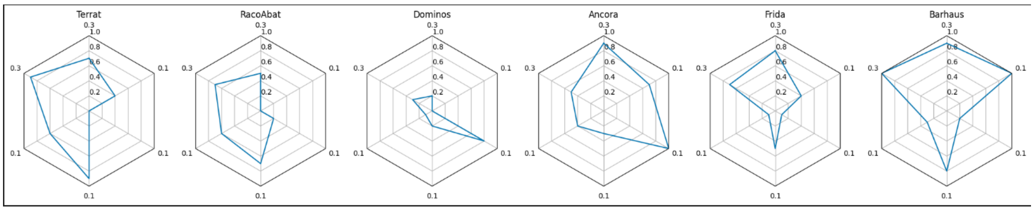


Figure 2: Radar Projection for the Alternatives.

Following on from this, weights are applied to the corresponding utility score. Figure 3 displays this weighted table.

| | Food | Personnel | Atmosphere | Category | Location | Terrace |
|---|---|---|---|---|---|---|
| Terrat | 0.21 | 0.27 | 0.06 | 0.09 | 0.00 | 0.04 |
| RacoAbat | 0.15 | 0.21 | 0.06 | 0.07 | 0.02 | 0.00 |
| Dominos | 0.06 | 0.09 | 0.01 | 0.02 | 0.08 | 0.00 |
| Ancora | 0.27 | 0.15 | 0.04 | 0.03 | 0.10 | 0.07 |
| Frida | 0.24 | 0.21 | 0.01 | 0.05 | 0.01 | 0.04 |
| Barhaus | 0.27 | 0.30 | 0.03 | 0.08 | 0.02 | 0.10 |

Figure 3: Weighted Criterion Scores for Each Alternative.

Finally, the partial weighted utility scores are summed for each alternative to provide an overall utility score. Figure 4 displays these overall scores.

```
Terrat      0.67
RacoAbat    0.51
Dominos     0.26
Ancora      0.66
Frida       0.56
Barhaus     0.80
dtype: float64
```

Figure 4: Overall Utility Scores.

Figure 5 depicts a barplot of each of the alternatives ranked against each other using their overall utility scores.

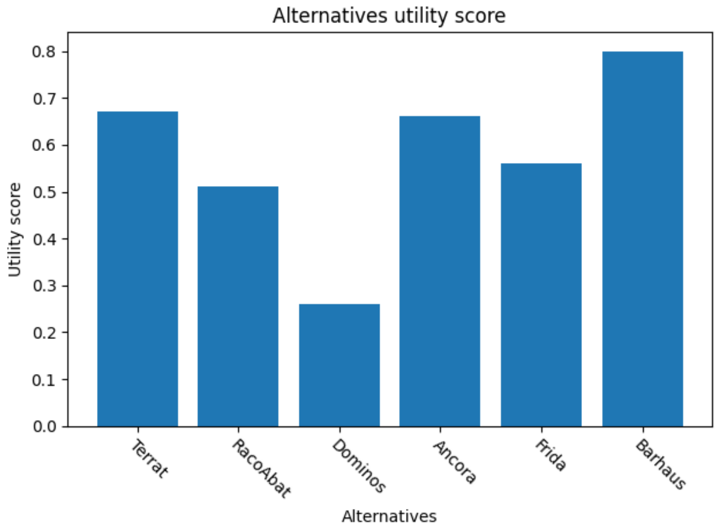


Figure 5: Alternatives Bar Plot.

*According to this plot, which alternative do you think is the best? Why?*

According to the plot, Barhaus is the best alternative, achieving a score of 0.8. The next best option is Terrat, with a score of 0.67, closely followed by Ancora at 0.66. Frida ranks next with 0.56. The second-lowest alternative is RacoAbat, scoring 0.51, while Dominos ranks the lowest with 0.26. These scores reflect the subjective requirements set at the beginning of the task, meaning the ranking depends on the predefined evaluation criteria.

### 3.2 Task 2: Add New Alternatives

This task builds on the previous task by adding two new additional alternatives with the following characteristics:

- 1. A restaurant that has the worst evaluations in all the criteria except in Personnel. (As per correction posted on Moodle.)
- 2. A restaurant that has the worst evaluations in all the criteria except in Location.

Figure 6 displays the code which creates the new alternatives and the scores for each criterion.

```python
# Alternatives names
new_alternatives = ["New_Alternative1", "New_Alternative2"]
# Alternatives scores for each criterion
alternatives_values = {
    "Food": [0.0, 0.0],
    "Personnel": [1.0, 0.0],
    "Atmosphere": [0.0, 0.0],
    "Category": [0.0, 0.0],
    "Location": [0.0, 1.0],
    "Terrace": [0.0, 0.0]
}
```

Figure 6: New Alternatives and their Scores.

Figure 7 displays the new performance table, including the two new alternatives at the bottom. As expected, the new alternatives achieve a maximum score in their respective criterion which scored a 1.0.

| | Food | Personnel | Atmosphere | Category | Location | Terrace |
|---|---|---|---|---|---|---|
| Terrat | 0.7 | 0.9 | 0.6 | 0.9 | 0.0 | 0.4 |
| RacoAbat | 0.5 | 0.7 | 0.6 | 0.7 | 0.2 | 0.0 |
| Dominos | 0.2 | 0.3 | 0.1 | 0.2 | 0.8 | 0.0 |
| Ancora | 0.9 | 0.5 | 0.4 | 0.3 | 1.0 | 0.7 |
| Frida | 0.8 | 0.7 | 0.1 | 0.5 | 0.1 | 0.4 |
| Barhaus | 0.9 | 1.0 | 0.3 | 0.8 | 0.2 | 1.0 |
| New_Alternative1 | 0.0 | 1.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| New_Alternative2 | 0.0 | 0.0 | 0.0 | 0.0 | 1.0 | 0.0 |

Figure 7: Performance Table with New Alternatives

Figure 8 displays the updated radar projection graph which includes the two new alternatives at the bottom. The radars resemble straight lines because of the minimum scoring in all but one criteria.

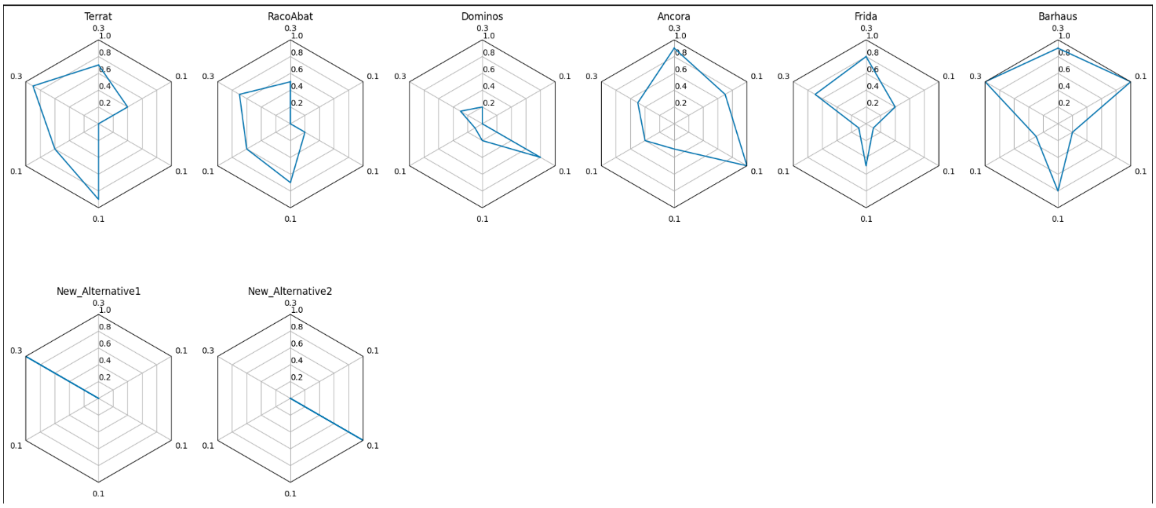


Figure 8: Radar Projection with New Alternatives

Figure 9 depicts the partial utility scores after the criteria weights are applied to the corresponding criterion.

| | Food | Personnel | Atmosphere | Category | Location | Terrace |
|---|---|---|---|---|---|---|
| Terrat | 0.21 | 0.27 | 0.06 | 0.09 | 0.00 | 0.04 |
| RacoAbat | 0.15 | 0.21 | 0.06 | 0.07 | 0.02 | 0.00 |
| Dominos | 0.06 | 0.09 | 0.01 | 0.02 | 0.08 | 0.00 |
| Ancora | 0.27 | 0.15 | 0.04 | 0.03 | 0.10 | 0.07 |
| Frida | 0.24 | 0.21 | 0.01 | 0.05 | 0.01 | 0.04 |
| Barhaus | 0.27 | 0.30 | 0.03 | 0.08 | 0.02 | 0.10 |
| New_Alternative1 | 0.00 | 0.30 | 0.00 | 0.00 | 0.00 | 0.00 |
| New_Alternative2 | 0.00 | 0.00 | 0.00 | 0.00 | 0.10 | 0.00 |

Figure 9: New Weighted Performance Table.

Figure 10 and Figure 11 both display the overall utility scores for all of the alternatives, including the two new ones.

```
Terrat               0.67
RacoAbat             0.51
Dominos              0.26
Ancora               0.66
Frida                0.56
Barhaus              0.80
New_Alternative1     0.30
New_Alternative2     0.10
dtype: float64
```

Figure 10: New Overall Utility Scores.

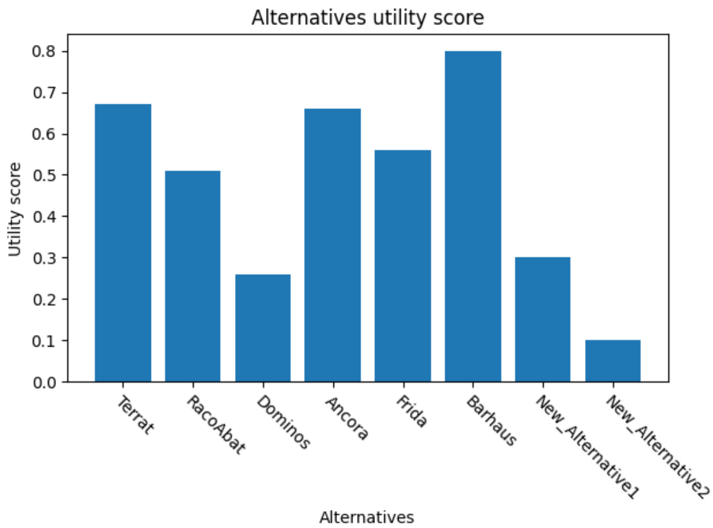


Figure 11: New Alternative Barplot.

New_Alternative1 has a very high score in Personnel but minimal scores elsewhere. Because Personnel has a weight of 0.3, it lifts its overall utility somewhat, but not enough to surpass the majority of the restaurants. New_Alternative2 has a very high score in Location but minimal scores elsewhere. Given that the Location is weighted 0.1, its overall utility remains low. Comparing the new alternatives to the original ones reveals that Barhaus still remains the top choice. Dominos is no longer the lowest scoring alternative, with New_Alternative2 achieving a worse score.

### 3.3 Task 3: Arithmetic Non-Weighted Average

In this task, the criterion weights from previous steps are made equal for all criteria in order to compute a non-weighted arithmetic average.

Figure 12 depicts the new weightings of the criterion, making them all equal.

```
{'Food': 0.16666666666666666, 'Personnel': 0.16666666666666666, 'Atmosphere': 0.16666666666666666, 'Category': 0.16666666666666666, 'Location': 0.16666666666666666, 'Terrace': 0.16666666666666666}
```

Figure 12: New Equal Weights.

Figure 13 depicts the updated performance table using equal weighting for all criterion.

| | Food | Personnel | Atmosphere | Category | Location | Terrace |
|---|---|---|---|---|---|---|
| Terrat | 0.116667 | 0.150000 | 0.100000 | 0.150000 | 0.000000 | 0.066667 |
| RacoAbat | 0.083333 | 0.116667 | 0.100000 | 0.116667 | 0.033333 | 0.000000 |
| Dominos | 0.033333 | 0.050000 | 0.016667 | 0.033333 | 0.133333 | 0.000000 |
| Ancora | 0.150000 | 0.083333 | 0.066667 | 0.050000 | 0.166667 | 0.116667 |
| Frida | 0.133333 | 0.116667 | 0.016667 | 0.083333 | 0.016667 | 0.066667 |
| Barhaus | 0.150000 | 0.166667 | 0.050000 | 0.133333 | 0.033333 | 0.166667 |
| New_Alternative1 | 0.000000 | 0.166667 | 0.000000 | 0.000000 | 0.000000 | 0.000000 |
| New_Alternative2 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.166667 | 0.000000 |

Figure 13: New Updated Performance Table.

Figure 14 displays the new overall utility scores for each of the alternatives, after equal weights are applied to each of the criterion.

```
Terrat              0.583333
RacoAbat            0.450000
Dominos             0.266667
Ancora              0.633333
Frida               0.433333
Barhaus             0.700000
New_Alternative1    0.166667
New_Alternative2    0.166667
dtype: float64
```

Figure 14: New Overall Utility Scores.

Figure 15 displays a graphical depiction of the overall utility scores in the form of a barplot.

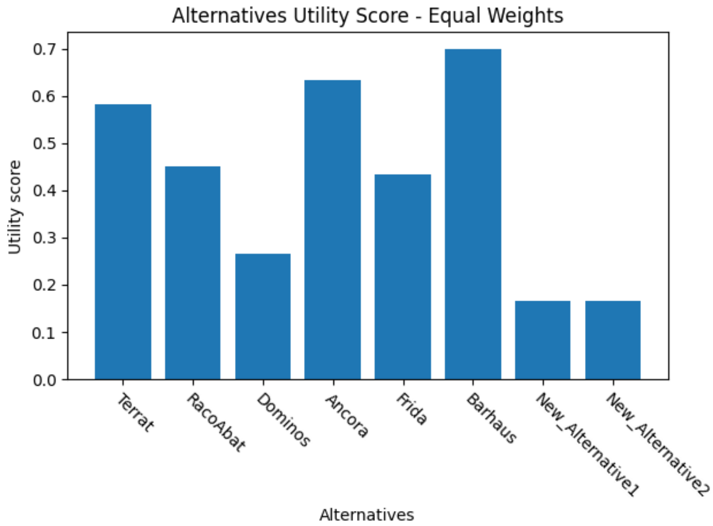


Figure 15: Alternatives Utility Scores - Equal Weights.

With equal weights, each of the criterion has the same importance (1/6). Despite achieving a lower score than previously, Barhaus remains the top scoring alternative with a score of 0.7. Ancora drops marginally from 0.66 to 0.63 recurring, but is now the second best alternative, overtaking Terrat in the process. Terrat drops from second to third overall, also losing score from 0.67 to 0.583. RacoAbat is next with a score of 0.45 which is also a reduction from 0.51. 0.433 recurring for Frida is also a significant reduction from 0.56. Tied for the worst performing alternative are the two new alternatives added in Task 2, both with a score of 0.166667.

Its apparent in our case applying equal weighting has reduced the overall performance for the alternatives.

### 3.4 Task 4: OWA Operator

In this task we studied the Ordered Weighted Average. Noteworthy, the weights are not applied to the criterion directly but rather to the position in the set therefore all criteria assume the same importance. The weights used are as follows: [0.1, 0.3, 0.6, 0.0, 0.0, 0.0]. This are the weights provided within the notebook.

These weights represent a disjunctive aggregation policy, albeit only moderately disjunctive. Using the orness degree formula with these weights and positions, we achieve a score of 0.7. An arithmetic mean is achieved at 0.5, and any score above 0.5 is considered disjunctive. We say this policy is only moderately disjunctive because a score of 0.7 (out of 1) is near the centre. This can also be seen within the weights array as higher emphasis is placed on the weights near the middle.

Figure 16 displays the performance table for each of the criterion using OWA.

```
Terrat               0.78
RacoAbat             0.64
Dominos              0.29
Ancora               0.79
Frida                0.59
Barhaus              0.94
New_Alternative1     0.10
New_Alternative2     0.10
dtype: float64
```

Figure 16: Alternatives Performance Table - OWA

Figure 17 displays a bar plot representation of the table depicted in Figure 16.

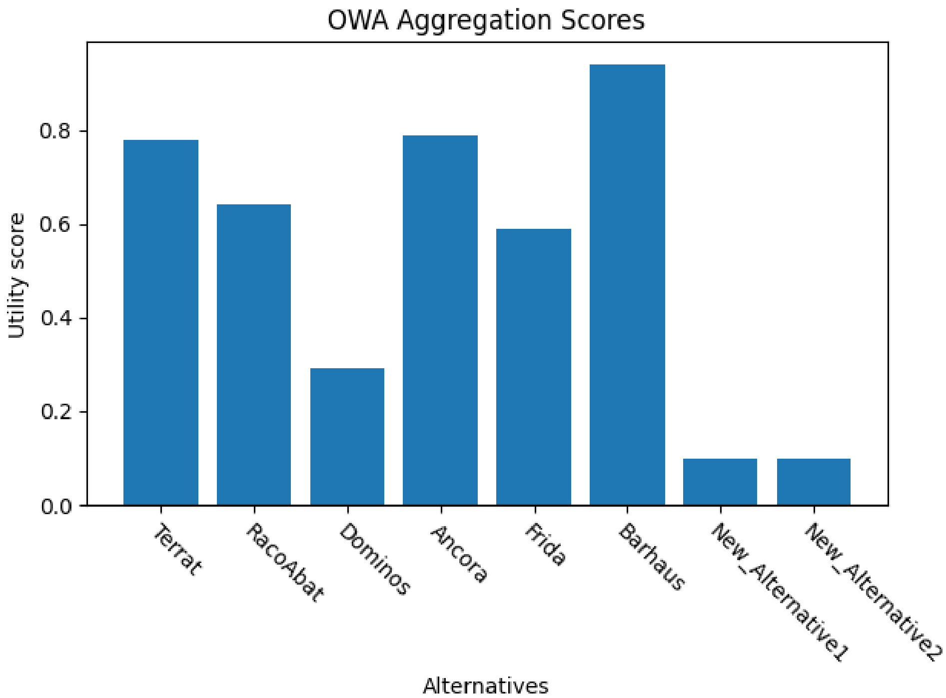


Figure 17: Alternatives Aggregation Scores - OWA

In the next sub-task, another disjunctive policy is defined which assigns zero weight to the two worst values and this is then applied to the restaurants. The weights assigned are as follows: [0.25, 0.25, 0.25, 0.25, 0, 0].

Figure 18 and Figure 19 display the performance scores applying the above weights to the restaurants using OWA in a tabular form and bar plot.

```
Disjunctive OWA utilities:
 Terrat               0.775
RacoAbat              0.625
Dominos               0.375
Ancora                0.775
Frida                 0.600
Barhaus               0.925
New_Alternative1      0.250
New_Alternative2      0.250
dtype: float64
```

Figure 18: Alternatives Aggregation Scores Table - Disjunctive Policy

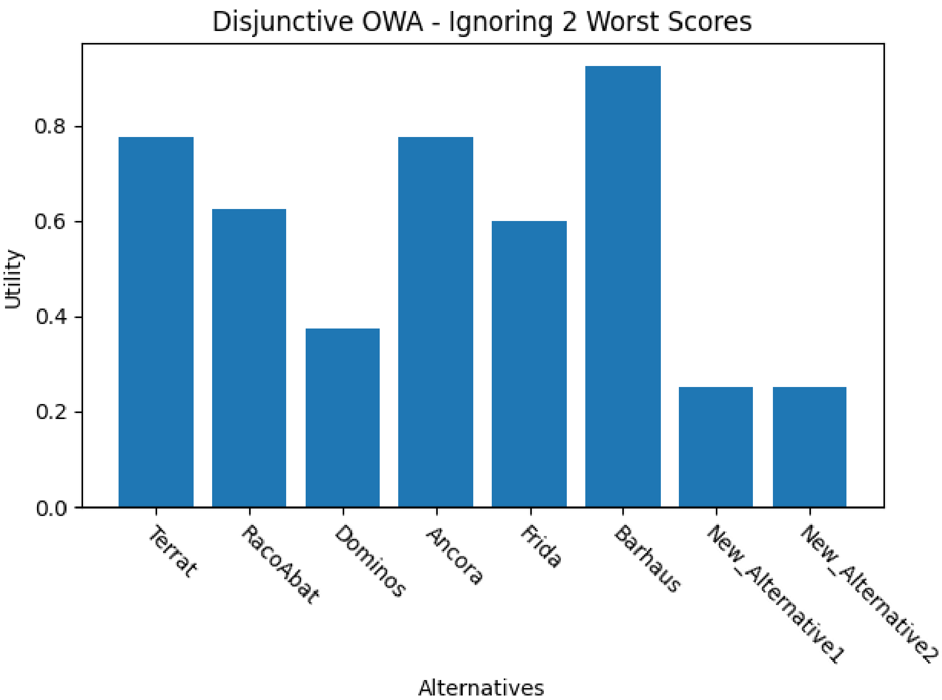


Figure 19: Alternatives Aggregation Scores Plot - Disjunctive Policy

In subtask 10, a conjunctive policy is established and the same experiment is performed. In this case, the weights applied are as follows: [0, 0, 0, 0.2, 0.3, 0.5].

Figure 20 and Figure 21 display the performance scores applying the above weights to the restaurants using OWA in a tabular form and bar plot.

```
Conjunctive OWA utilities:
 Terrat               0.24
RacoAbat             0.16
Dominos              0.07
Ancora               0.37
Frida                0.16
Barhaus              0.35
New_Alternative1     0.00
New_Alternative2     0.00
dtype: float64
```

Figure 20: Alternatives Aggregation Scores Table - Conjunctive Policy

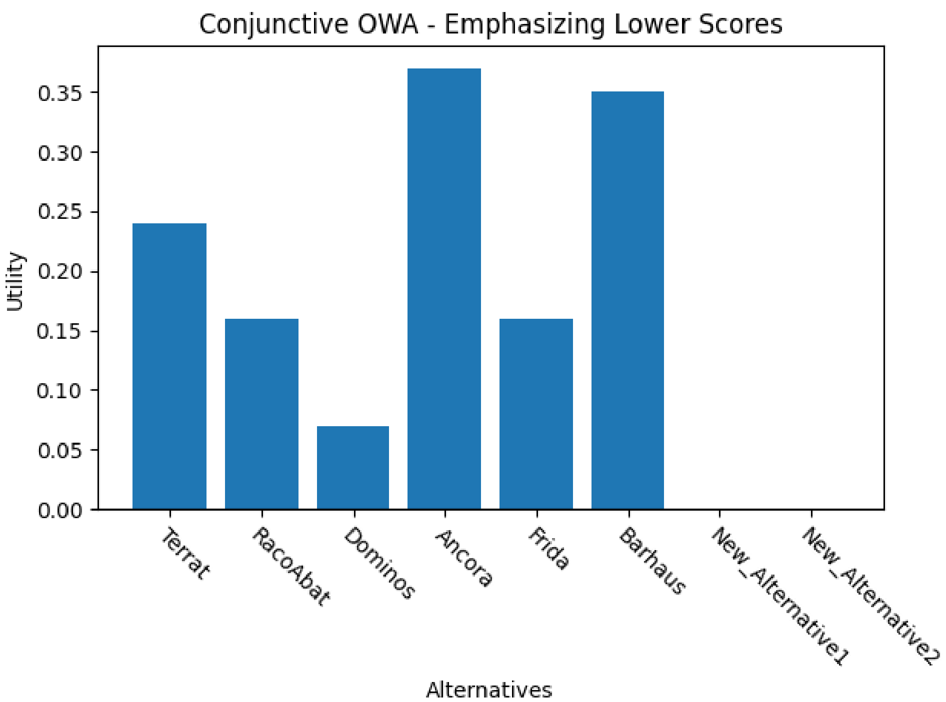


Figure 21: Alternatives Aggregation Scores Plot - Conjunctive Policy

In subtask 11, the results of task 9 and task 10 are compared, amongst themselves and also in respect to the arithmetic mean.

```
Arithmetic average utilities:
 Terrat                0.583333
RacoAbat              0.450000
Dominos               0.266667
Ancora                0.633333
Frida                 0.433333
Barhaus               0.700000
New_Alternative1      0.166667
New_Alternative2      0.166667
dtype: float64
```

Figure 22: Alternatives Aggregation Scores Table - Arithmetic Mean

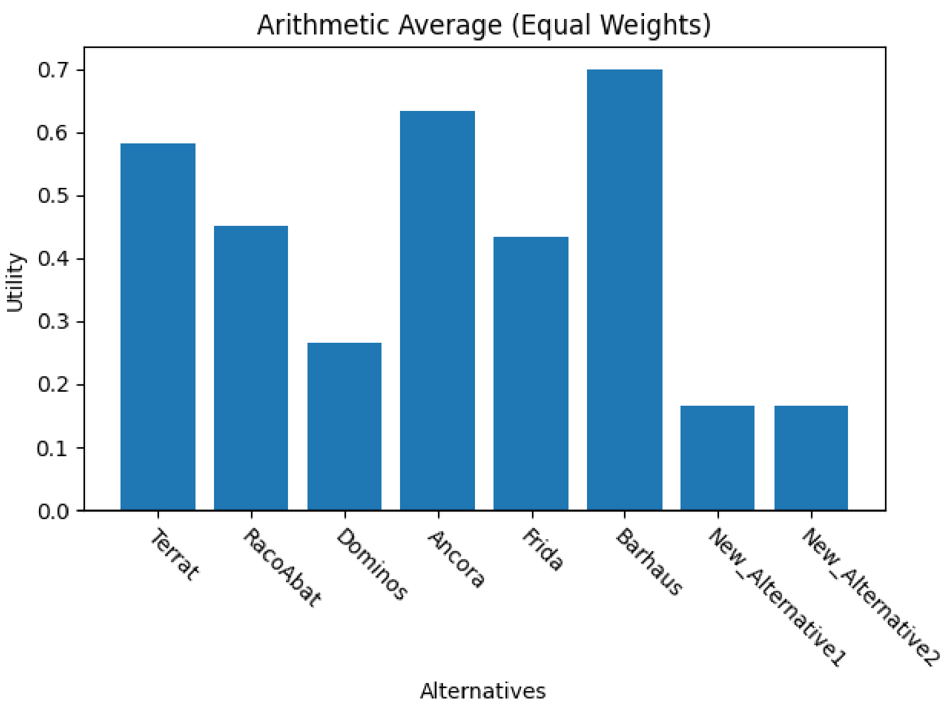


Figure 23: Alternatives Aggregation Scores Plot - Arithmetic Mean

By applying a disjunctive policy as seen in task 9, the performance scores of each of the alternatives are greater higher than when a conjunctive policy is applied as seen in task 10. This is expected as by definition, a disjunctive policy allows for compensation in poor performing criterion. Whereas with a conjunctive policy, all criterion must satisfy the requirements to some degree to achieve a high score - it is apparent no alternative did so as the highest score achieved is 0.37 by Ancora.

The arithmetic mean on the other hand is the harmonic balance between conjunctivity and disjunctivity - it allows for some compensation in poor scoring.

### 3.5 Task 5: Aggregation Policies: Descriptors of the OWA Weights

In this task different measures are captured which characterise the vector of weights such as balance, divergence, entropy, and orness.

Code is provided to calculate these four metrics using the weights provided in the table.

Figure 24 displays the resulting table of metrics that were calculated from the code provided within the notebook.

| Weights | Balance | Divergence | Entropy | Orness |
|---|---|---|---|---|
| 0.0, 0.3, 0.4, 0.3, 0.0, 0.0 | 0.2 | 0.024 | 1.089 | 0.6 |
| 0.0, 0.0, 0.0, 0.2, 0.3, 0.5 | -0.72 | 0.024 | 1.030 | 0.14 |
| 1.0, 0.0, 0.0, 0.0, 0.0, 0.0 | 1.0 | 0.0 | -0.0 | 1.0 |
| 0.1, 0.1, 0.3, 0.3, 0.1, 0.1 | 0.0 | 0.074 | 1.643 | 0.5 |
| 0.1, 0.3, 0.6, 0.0, 0.0, 0.0 | 0.4 | 0.018 | 0.898 | 0.7 |

Figure 24: Table of Metrics

The following describes the metrics themselves and the scores achieved.

- **Balance:** Indicates the skew or symmetry of the weight distribution. A value near 0 suggests a balanced set of weights, whereas a larger positive or negative value indicates skewness.
- **Divergence:** Shows how far the weights are from a uniform distribution. Higher values mean the weights are more uneven.
- **Entropy:** Measures how spread out the weights are. Larger entropy means the weights are more evenly distributed.
- **Orness:** Ranges from 0 (fully conjunctive/min-like) to 1 (fully disjunctive/max-like). A higher orness favors the best scores, while a lower orness favors the worst scores.

**Weight Vector:** `0.0, 0.3, 0.4, 0.3, 0.0, 0.0`

- *Balance (0.2):* The slight positive balance indicates a modest bias toward the better (higher-ranked) positions.
- *Orness (0.6):* With an orness of 0.6 (above the neutral 0.5), this vector favors the top scores but still considers the middle values.
- *Entropy (1.089) & Divergence (0.024):* The spread is moderate and the weights deviate only slightly from a uniform distribution, confirming a balanced yet somewhat optimistic approach.

**Weight Vector:** `0.0, 0.0, 0.0, 0.2, 0.3, 0.5`

- *Balance (-0.72):* The strongly negative balance shows a heavy bias toward the lower positions, meaning poor scores have a large impact.
- *Orness (0.14):* This very low orness confirms a highly conjunctive policy, where the worst scores dominate the outcome.
- *Entropy (1.030) & Divergence (0.024):* Although the spread is similar to the first vector, the focus is shifted to the worst criteria, reinforcing the conjunctive nature.

**Weight Vector:** `1.0, 0.0, 0.0, 0.0, 0.0, 0.0`

- *Balance (1.0):* The entire weight is on the best criterion, showing maximum skewness toward the top.
- *Orness (1.0):* An orness of 1.0 reflects a pure disjunctive (max-like) operator—only the best performance counts.
- *Entropy (-0.0) & Divergence (0.0):* There is no spread since all weight is concentrated in one position.

**Weight Vector:** `0.1, 0.1, 0.3, 0.3, 0.1, 0.1`

- *Balance (0.0):* The zero balance indicates a perfectly symmetric distribution around the middle.
- *Orness (0.5):* An orness of 0.5 shows that this aggregation is neutral—neither favoring the best nor the worst scores.
- *Entropy (1.643) & Divergence (0.074):* The high entropy indicates a very even spread of weights, resulting in an aggregation similar to an arithmetic mean.

**Weight Vector:** `0.1, 0.3, 0.6, 0.0, 0.0, 0.0`

- *Balance (0.4):* A moderate positive balance shows that the weight distribution is skewed toward the top, emphasizing the better criteria.
- *Orness (0.7):* With an orness of 0.7, this vector is quite disjunctive.
- *Entropy (0.898) & Divergence (0.018):* The lower entropy indicates a more concentrated weighting on the top positions, further confirming the strong emphasis on high performance.

# 4 Conclusion

The analysis of the OWA weight descriptors has provided a valuable insight into how different weighting schemes influence the aggregation of criteria. The computed orness values distinguish between disjunctive policies and conjunctive policies. As well as this, the balance, divergence, and entropy metrics provide a deeper understanding of each weight vector's skewness and spread.

Overall, these descriptors are essential for selecting an aggregation strategy that aligns with the decision maker's priorities and ensures that the evaluation of alternatives is reflective of the desired emphasis on criteria performance.