PMCDSS Task 6: ELECTRE Exercises

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6.1 ELECTRE for Choice Problems

Q1. Data is defined below. Weights indicate the relative importance of each criterion in the concordance calculation. What can you say about the weights with respect to the interests of the user?

Weights, with respect to the interests of the user, indicate the importance of the criteria that is associated with the weight value. For example, if criteria A has weight 0.8, and criteria B has weight 0.2, criteria A has a greater subjective importance to the user in the calculation of the overall decision.

Q2. The ELECTRE-1 uses binary concordance for each criterion and then applies the weights. Reproduce manually the concordance between MED and BALT.

1. Identify the performance scores for **MED** and **BALT** for each criterion from the performance table.

	Capacity	Company	Facilities	Duration	Reviews
0.MED	900	8	5	5	3
1.BALT	1200	9	4	10	5

2. Identify the weights of each criterion.

Capacity = 0.2

Company = 0.2

Facilities = 0.1

Duration = 0.3

Reviews = 0.2

3. Determine the criteria where MED performs at least as well as BALT.

Performance(MED) >= Performance(BALT):

- Facilities (5 > 4)

4. Sum the weights of the criteria identified in Step 3.

Only 1 criterion is identified, therefore the sum is just the weight of Facilities: 0.1.

5. Divide the sum from Step 4 by the total sum of all weights to get the concordance index C(MED, BALT).

C(MED, BALT) = (Sum of concordant weights) / (Total weight)

C(MED, BALT) = 0.1 / 1.0

C(MED, BALT) = 0.1.

The validity of the result above can be verified against the concordance matrix created in the provided notebook:

	0.MED	1.BALT	2.CARIB	3.GRE	4.NIL	5.DAN
0.MED	1.0	0.1	0.5	0.3	0.5	0.3
1.BALT	0.9	1.0	0.5	0.5	0.7	0.3
2.CARIB	0.5	0.5	1.0	0.3	0.5	0.8
3.GRE	0.7	0.5	0.7	1.0	0.9	0.5
4.NIL	0.6	0.3	0.5	0.3	1.0	0.3
5.DAN	0.7	0.7	0.8	0.5	0.7	1.0

Q3. After the concordance, the method calculates the discordance in a boolean way too. In this implementation, the discordance is calculated as a proportion of the maximum difference between two values on each criterion. A negative value means no discordance at all. Find a case with maximum discordance and explain why it happens.

The maximum discordance present in the matrix is 1.0. This value is attributed to row CARIB and column DAN.

➔ D(CARIB, DAN) = 1.0

Q3. After the concordance, the method calculates the discordance in a b
A negative value means no discordance at all. Find a case with maximum

```
discordance_matrix = electre1.discordance(performance_table)
discordance_matrix
```

[10] ✓ 0.1s

...

	0.MED	1.BALT	2.CARIB	3.GRE	4.NIL	5.DAN
0.MED	0.000	0.300	0.009	0.100	0.006	0.5
1.BALT	0.001	0.000	0.004	0.005	0.004	0.2
2.CARIB	0.500	0.800	0.000	0.600	0.300	1.0
3.GRE	0.004	0.200	0.003	0.000	0.003	0.4
4.NIL	0.200	0.500	0.006	0.300	0.000	0.7
5.DAN	0.002	0.002	0.001	0.003	0.003	0.0

A maximum discordance of 1.0 implies heavily that a veto has been triggered by at least one of the evaluated criteria.

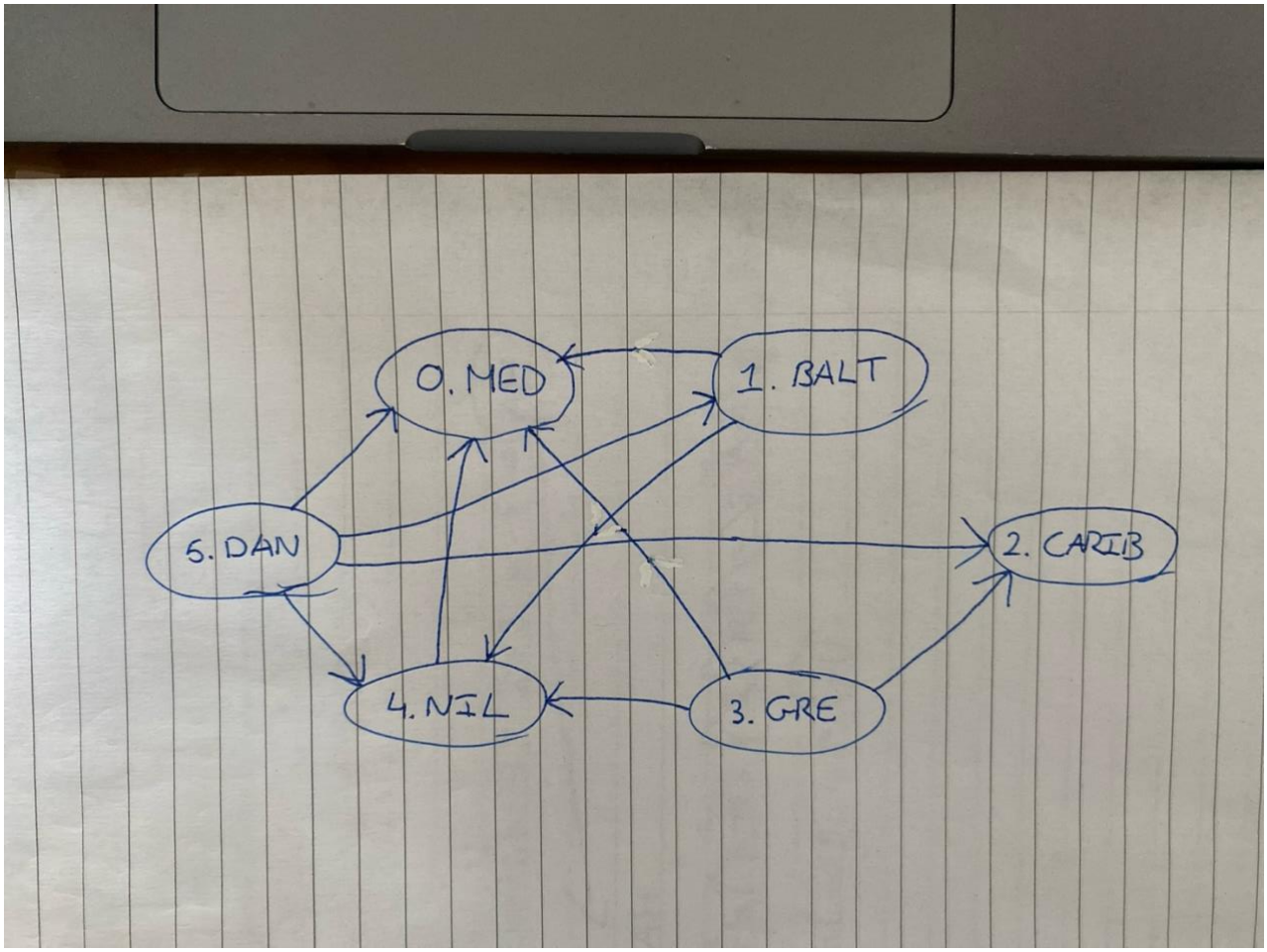
Looking at the performance scores for both CARIB and DAN:

2.CARIB	400	7	3	14	7
3.GRE	1000	4	2	15	9
4.NIL	700	2	5	8	9
5.DAN	1400	7	3	14	6

We can see that the only criteria in which DAN outperforms CARIB is Capacity (1400 to 400 in a 200 to 1500 scale), with the remaining criterion scores are tied, except Reviews where CARIB outperforms DAN marginally (7 to 6 in a 1-10 scale). Due to DAN outperforming CARIB on Capacity in such a great manner (1000 difference), the discordance is automatically set to 1.0 (maximum).

Q4. Draw the graph of credibility and find the kernel manually. Check the results with the ones given by the software.

To begin, we must draw the graph of credibility manually based on the credibility matrix:



From this graph, we can find the kernels. The kernel K of a graph is a subset of nodes such that:

- No node inside the kernel outranks another node inside the kernel.
- Every node outside of the kernel is outranked by at least one node inside the kernel.

Using this definition, we can conclude **the kernel for this graph is GRE and DAN.** This aligns with the programmatic answer provided in the notebook.

6.2 ELECTRE for Choice Problems

Q1. The ELECTRE method needs some discrimination thresholds to calculate the concordance. Explain the meaning of the values of thresholds in this case study.

1. Indifference: Defines the maximum performance difference considered negligible. For example, q[“Capacity”] = 100 means cruises differing by up to 100 passengers are considered indifferent on this criterion. For Company and Facilities (q=0), any difference DOES matter.
2. Preference: Defines the minimum performance difference needed to establish a strict preference. For example, p[“Capacity”] = 200 means a cruise needs 200+ more passengers to

be strictly preferred on capacity alone when compared to another cruise. Differences between q and p indicate weak preference.

3. Veto: Defines an unacceptable performance difference where alternative b significantly outperforms a, outright blocking the conclusion that a outranks b. For example, v=None means there is no veto for that criterion. For v["Duration"] = 6, if b is 6+ days longer than a, it's a veto.

Q2. To calculate the concordance with pseudo-criteria, we must use the version III of ELECTRE. Look at the values in the matrix and the plot and explain them. You can increase the cut value in the plotting function, to just see a subset of the arcs.

Each value C(a, b) in the matrix represents the degree of evidence supporting the statement "alternative a is at least as good as alternative b". It's an index between 0 and 1. As opposed to ELECTRE I, ELECTRE III employs the use indifferences and preference thresholds to output a more accurate score matrix.

For each criterion j a partial concordance index c_j(a, b) is calculated:

- It's 1 if a is significantly better than b (performance difference >= p_j) or if the difference is negligible (within q_j).
- It's 0 if b is significantly better than a (performance difference favouring b is >= p_j).
- It's between 0 and 1 if the difference falls between the indifference and preference thresholds, indicating weak preference.

The final C(a, b) is the weighted sum of these partial concordances:

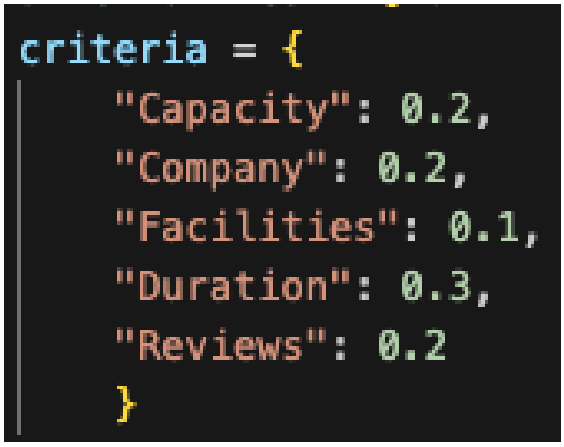
The plot visualises the concordance relationships as a directed graph. The plot simplifies the matrix by focusing on pairs where there's a reasonably high degree of evidence that the source alternative is at least as good as the target. Relationships with concordance below 0.6 are considered too weak to be displayed at this threshold level. Increasing the cut_threshold (e.g., to 0.8) would show only the very strongest relationships, while decreasing it would show more, including weaker ones.

Q3. Reproduce manually the calculation of concordance between the alternatives MED and BALT.

The performance scores of each of the criteria:

	Capacity	Company	Facilities	Duration	Reviews
0.MED	900	8	5	5	3
1.BALT	1200	9	4	10	5

The weights associated to the criteria:



Going through each criterion, we get the following:

1. Capacity

G(A) = 900, G(B) = 1200

Q = 100, P = 200

Checking strict preference: G(B) > G(A) + P

1200 > 900 + 200

1200 > 1100? **Yes**

➔ Assign a score of 0.

2. Company

G(A) = 8, G(B) = 9

Q = 0, P = 1

Checking strict preference: G(B) > G(A) + P

9 > 8 + 1? **No**

Checking indifference: G(B) <= G(A) + Q

9 <= 8 + 0? **No**

Need to calculate partial preference:

((G(A) + P) - G(B)) / (P - Q)

((8 + 1) - 9) / (1 - 0)

(9 - 9) / 1

0 / 1 = 0

➔ Assign a score of 0.

3. Facilities

G(A) = 5, G(B) = 4

Q = 0, P = 2

Checking strict preference: G(B) > G(A) + P

5 > 4 + 2? **No**

Checking indifference: G(B) <= G(A) + Q

4 <= 5+ 0? **Yes**

➔ Assign a score of 1.

4. Duration

G(A) = 5, G(B) = 10

Q = 2, P = 4

Checking strict preference: G(B) > G(A) + P

10 > 5 + 4? **Yes**

➔ Assign a score of 0.

5. Reviews

G(A) = 3, G(B) = 5

Q = 1, P = 3

Checking strict preference: G(B) > G(A) + P

5 > 3 + 3? **No**

Checking indifference: G(B) <= G(A) + Q

5 <= 3 + 1? **No**

Need to calculate partial preference:

((G(A) + P) - G(B)) / (P - Q)

((3 + 3) – 5) / 3 – 1

6 – 5 / 3 – 1

1 / 2

- Assign a score of 0.5

Now we need to multiply each score to the corresponding weight for that criterion:

Facilities = 1 * 0.1 = 0.1

Reviews = 0.5 * 0.2 = 0.1

And finally sum the results:

0.1 + 0.1 = 0.2

Final answer: 0.2. This aligns with the score seen in the matrix within the notebook.

Q4. After the concordance, the method calculates the discordance (using the veto thresholds). Explain the result obtained.

The displayed discordance_matrix shows the results of the ELECTRE III discordance calculation, which uses veto thresholds (v). Each cell (a, b) contains a breakdown showing the partial discordance index (d_j) for each criterion j. This index d_j(a, b) measures the degree (from 0 to 1) to which criterion j opposes the statement "alternative a is at least as good as alternative b".

- A value of 0 means criterion j offers no opposition.
- A value of 1 signifies that the performance difference favouring b over a on criterion j was large enough to exceed the veto threshold (v_j), effectively registering a strong objection ("veto") from that criterion.
- Values between 0 and 1 indicate partial opposition, calculated based on the performance difference relative to the preference (p_j) and veto (v_j) thresholds.

Q5. Reproduce manually the discordance between the alternatives MED and BALT.

Required data is as follows:

a = MED, b = BALT

G(MED): [Cap=900, Comp=8, Fac=5, Dur=5, Rev=3]

G(BALT): [Cap=1200, Comp=9, Fac=4, Dur=10, Rev=5]

P: [Cap=200, Comp=1, Fac=2, Dur=4, Rev=3]

V: [Cap=None, Comp=3, Fac=None, Dur=6, Rev=None]

All criteria are MAX preference direction.

Formulas we will be following:

1. If V is None, assign value 0.

2. If G(B) <= G(A) + P, assign value 0.
3. If G(B) >= G(A) + V, assign value 1.
4. Otherwise, (G(B) – P – G(A)) / (V – P)

1. Capacity

G(A) = 900, G(B) = 1200

P = 200, V = 0

➔ Assign value 0 (veto is None)

2. Company

G(A) = 8, G(B) = 9

P = 1, V = 3

Checking no opposition: G(B) <= G(A) + P

9 <= 8 + 1 ? **Yes**

➔ Assign value 0.

3. Facilities

G(A) = 5, G(B) = 4

P = 2, V = 0

➔ Assign value 0 (veto is None)

4. Duration

G(A) = 5, G(B) = 10

P = 4, V = 6

Checking no opposition: G(B) <= G(A) + P

10 <= 5 + 4 ? **No**

Checking full veto: G(B) >= G(A) + V

10 >= 5 + 6? **No**

Calculate partial discordance:

(G(B) - P - G(A)) / (V - P)

(10 – 4 – 5) / (6 – 4) -> 0.5

➔ Assign score 0.5

5. Reviews

G(A) = 3, G(B) = 5

P = 3, V = 0

➔ Assignment score 0 (veto Is None).

This leaves us with the following results:

- Capacity 0.0, Company 0.0, Facilities 0.0, Duration 0.5, Reviews 0.0.

This aligns with the values calculated in the notebook.

Q6. What changes can you detect between the matrix of concordance and the matrix of credibility? Why do we have those changes? How many alternatives increase their concordance level? Why?

Credibility Matrix

	0.MED	1.BALT	2.CARIB	3.GRE	4.NIL	5.DAN
0.MED	1.00	0.125	0.0	0.0	0.65	0.0
1.BALT	0.95	1.000	0.6	0.5	0.75	0.5
2.CARIB	0.50	0.550	1.0	0.7	0.60	0.8
3.GRE	0.00	0.000	0.0	1.0	0.90	0.0
4.NIL	0.00	0.000	0.0	0.0	1.00	0.0
5.DAN	0.70	0.750	1.0	0.8	0.70	1.0

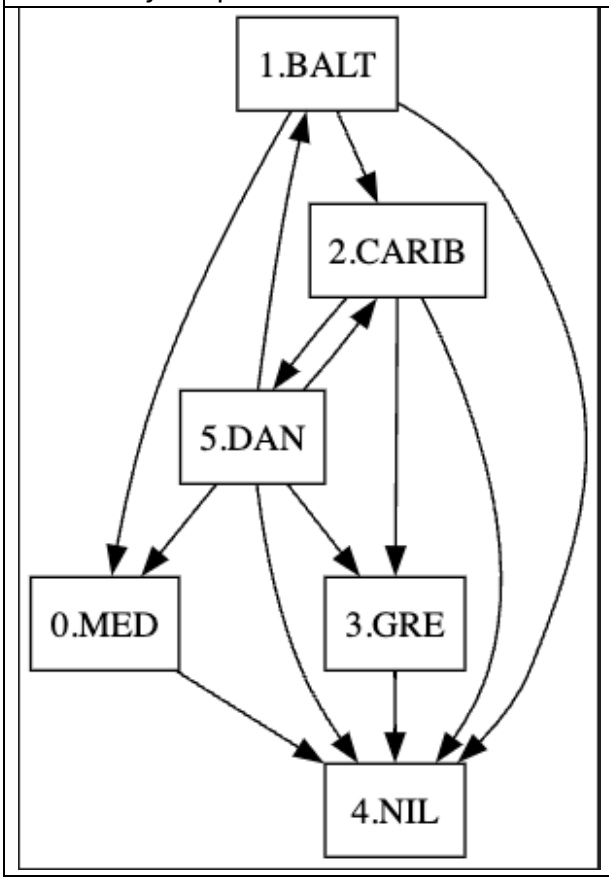
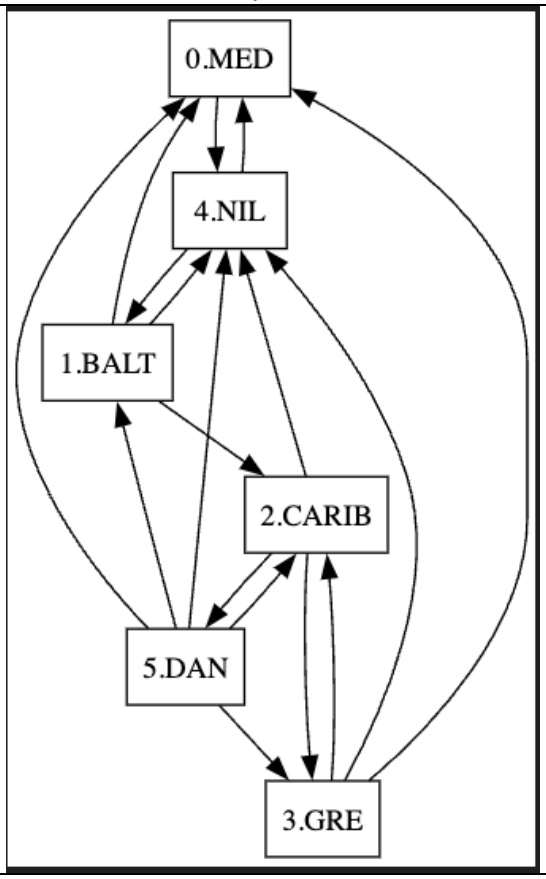
Concordance Matrix

	0.MED	1.BALT	2.CARIB	3.GRE	4.NIL	5.DAN
0.MED	1.00	0.20	0.50	0.5	0.65	0.30
1.BALT	0.95	1.00	0.60	0.5	0.75	0.50
2.CARIB	0.50	0.55	1.00	0.7	0.60	0.80
3.GRE	0.70	0.50	0.75	1.0	0.90	0.55
4.NIL	0.60	0.60	0.50	0.3	1.00	0.30
5.DAN	0.70	0.75	1.00	0.8	0.70	1.00

Many values in the Credibility Matrix are lower than the corresponding values in the Concordance Matrix. For example, Concordance(0, 1) = 0.20 vs Credibility(0, 1) = 0.125.

The Credibility S(a, b) is calculated by starting with the Concordance C(a, b) and then reducing it if there is significant disagreement (discordance) on any criterion. Specifically, if any partial discordance index d_j(a, b) is greater than C(a, b), the credibility is weakened according to the ELECTRE III formula. Strong opposition (high d_j) on even one criterion can significantly lower the credibility score.

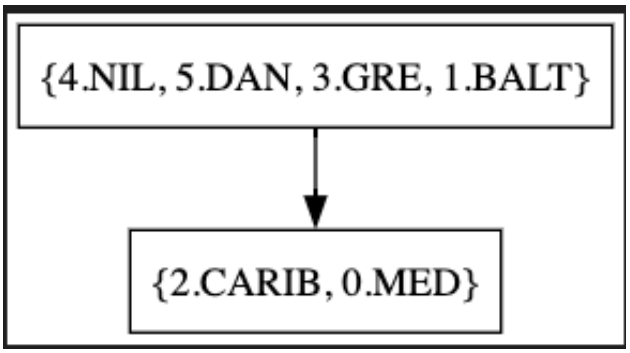
No alternatives increase their value from the Concordance to the Credibility matrix. This is because the calculation only subtracts or maintains the concordance value based on the discordance.

Credibility Graph	Concordance Graph
	

We can observe from the graphs for the corresponding matrices that this validates my answer from previously. There are less connections to each of the nodes. For example, MED has 1 outgoing arrow, and 2 incoming arrows in the credibility graph but has 4 incoming and 1 outgoing in the concordance graph.

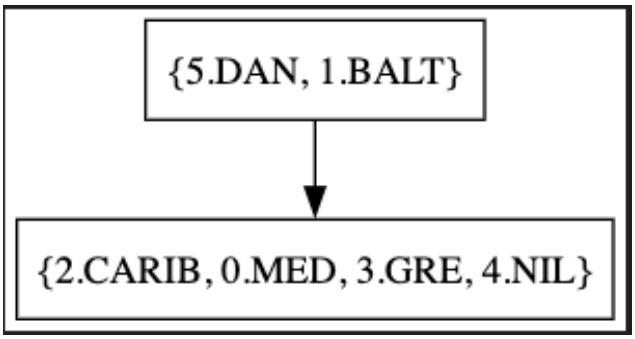
Q7. Execute the code and explain the graphics.

1. Distillation (ascending)



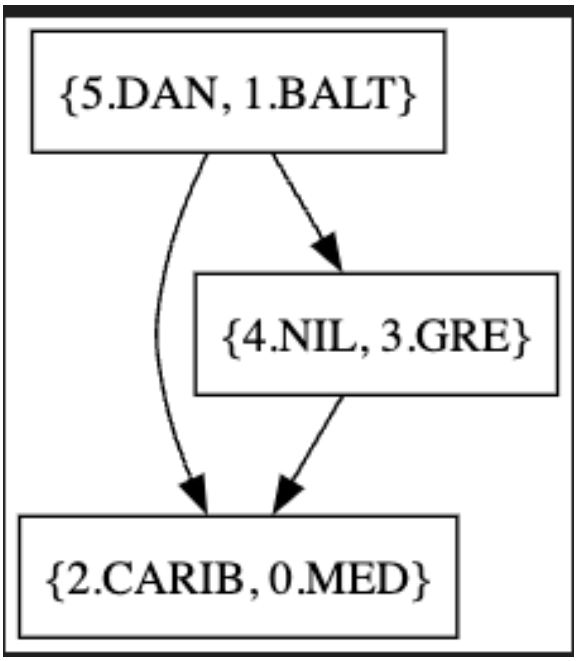
This shows the result of ranking alternatives from worst to best. Alternatives {2.CARIB, 0.MED} are identified as the lowest-ranked group. The group {4.NIL, 5.DAN, 3.GRE, 1.BALT} is ranked higher. Alternatives within the same box are considered equivalent in this ranking pass.

2. Distillation (descending)



This shows the result of ranking alternatives from best to worst. Alternatives {5.DAN, 1.BALT} are identified as the highest-ranked group. The group {2.CARIB, 0.MED, 3.GRE, 4.NIL} is ranked lower. Alternatives within the same box are considered equivalent in this ranking pass.

3. Ranked exploitation



This is the final ranking, combining the results of both distillations. It shows a partial preorder:

- {5.DAN, 1.BALT} is the top-ranked group.
- {4.NIL, 3.GRE} is ranked below the top group.
- {2.CARIB, 0.MED} is the bottom-ranked group.

The arrows indicate preference: the top group is preferred to the middle and bottom groups, and the middle group is preferred to the bottom group. Alternatives within the same box are considered equivalent or incomparable in the final ranking.

Q8. Introduce a new alternative in the dataset above (the one defined in question 1). It must be very good in the all criteria except in the duration, were it has a bad value of performance.

Repeat the exercise and compare the results. Explain its relations in the credibility matrix and its position in the final ranking.

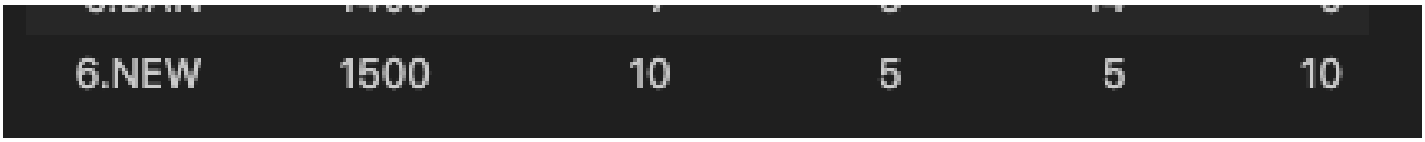
Below is the code including the new alternative, along with the performance table.

```
alternatives = ["0.MED", "1.BALT", "2.CARIB", "3.GRE", "4.NIL", "5.DAN", "6.NEW"]

new_performance_table = PerformanceTable(
                        [[900, 8, 5, 5, 3],
                         [1200, 9, 4, 10, 5],
                         [400, 7, 3, 14, 7],
                         [1000, 4, 2, 15, 9],
                         [700, 2, 5, 8, 9],
                         [1400, 7, 3, 14, 6],
                         [1500, 10, 5, 5, 10]],
                        alternatives = alternatives,
                        criteria = criteria,
                        scales = scales)

new_performance_table.df
```

✓ 0.0s

	Capacity	Company	Facilities	Duration	Reviews
0.MED	900	8	5	5	3
1.BALT	1200	9	4	10	5
2.CARIB	400	7	3	14	7
3.GRE	1000	4	2	15	9
4.NIL	700	2	5	8	9
5.DAN	1400	7	3	14	6
6.NEW	1500	10	5	5	10

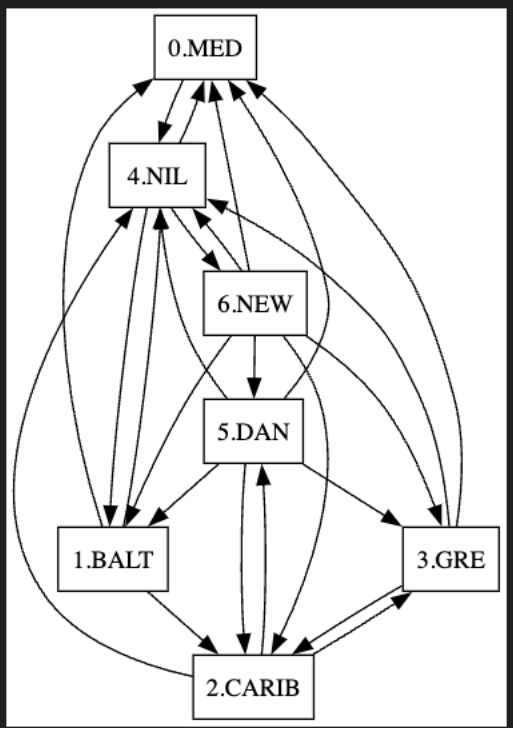
The newly created alternative is aptly named ‘NEW’. It has been given max scores in every criterion except ‘Duration’, where it’s given the worst score.



```
new_concordance_matrix = electre3.concordance(new_performance_table)

display(new_concordance_matrix)

#Now we plot the graph of concordance with relations above the cut threshold
cut_threshold = 0.6
new_conc_mat_cut = new_concordance_matrix.where(new_concordance_matrix >= cut_threshold, other=0)
AdjacencyMatrix(new_conc_mat_cut).plot()
```

✓ 0.2s

	0.MED	1.BALT	2.CARIB	3.GRE	4.NIL	5.DAN	6.NEW
0.MED	1.00	0.20	0.50	0.5	0.65	0.30	0.40
1.BALT	0.95	1.00	0.60	0.5	0.75	0.50	0.35
2.CARIB	0.50	0.55	1.00	0.7	0.60	0.80	0.30
3.GRE	0.70	0.50	0.75	1.0	0.90	0.55	0.50
4.NIL	0.60	0.60	0.50	0.3	1.00	0.30	0.60
5.DAN	0.70	0.75	1.00	0.8	0.70	1.00	0.50
6.NEW	1.00	0.70	0.70	0.7	0.85	0.70	1.00



We create a new concordance matrix and corresponding graph using this new performance table. The results of this are shown above. We utilise the same cut threshold as previously (0.6) to create a direct comparison with the old concordance graph.

As expected, NEW is considered at least as good as the other criterion in most cases due to its high performance on all criteria except Duration. We witness some lower concordance scores against some of the alternatives (for example, 0.7 with BALT, CARIB, GRE, and DAN) because of the poor Duration performance.

The concordance plot visualises NEW having outgoing arrows to most others, reflecting its general high performance.

```
new_discordance_matrix = electre3.discordance(new_performance_table)
new_discordance_matrix
```

✓ 0.0s Python

	0.MED	1.BALT	2.CARIB	3.GRE	4.NIL	5.DAN	6.NEW
0.MED	Capacity 0 Company 0 Facilities 0 Duration 0 Reviews 0 dtype: int64	Capacity 0.0 Company 0.0 Facilities 0.0 Duration 0.5 Reviews 0.0 dtype: float64	Capacity 0 Company 0 Facilities 0 Duration 1 Reviews 0 dtype: int64	Capacity 0 Company 0 Facilities 0 Duration 1 Reviews 0 dtype: int64	Capacity 0 Company 0 Facilities 0 Duration 0 Reviews 0 dtype: int64	Capacity 0 Company 0 Facilities 0 Duration 1 Reviews 0 dtype: int64	Capacity 0.0 Company 0.5 Facilities 0.0 Duration 0.0 Reviews 0.0 dtype: float64
1.BALT	Capacity 0 Company 0 Facilities 0 Duration 0 Reviews 0 dtype: int64	Capacity 0 Company 0 Facilities 0 Duration 0 Reviews 0 dtype: int64	Capacity 0 Company 0 Facilities 0 Duration 0 Reviews 0 dtype: int64	Capacity 0.0 Company 0.0 Facilities 0.0 Duration 0.5 Reviews 0.0 dtype: float64	Capacity 0 Company 0 Facilities 0 Duration 0 Reviews 0 dtype: int64	Capacity 0 Company 0 Facilities 0 Duration 0 Reviews 0 dtype: int64	Capacity 0 Company 0 Facilities 0 Duration 0 Reviews 0 dtype: int64
2.CARIB	Capacity 0 Company 0 Facilities 0 Duration 0 Reviews 0 dtype: int64	Capacity 0.0 Company 0.5 Facilities 0.0 Duration 0.0 Reviews 0.0 dtype: float64	Capacity 0 Company 0 Facilities 0 Duration 0 Reviews 0 dtype: int64	Capacity 0 Company 0 Facilities 0 Duration 0 Reviews 0 dtype: int64	Capacity 0 Company 0 Facilities 0 Duration 0 Reviews 0 dtype: int64	Capacity 0 Company 0 Facilities 0 Duration 0 Reviews 0 dtype: int64	Capacity 0.0 Company 1.0 Facilities 0.0 Duration 0.0 Reviews 0.0 dtype: float64
3.GRE	Capacity 0 Company 1 Facilities 0 Duration 0 Reviews 0 dtype: int64	Capacity 0 Company 1 Facilities 0 Duration 0 Reviews 0 dtype: int64	Capacity 0.0 Company 1.0 Facilities 0.0 Duration 0.0 Reviews 0.0 dtype: float64	Capacity 0 Company 0 Facilities 0 Duration 0 Reviews 0 dtype: int64	Capacity 0 Company 0 Facilities 0 Duration 0 Reviews 0 dtype: int64	Capacity 0.0 Company 1.0 Facilities 0.0 Duration 0.0 Reviews 0.0 dtype: float64	Capacity 0 Company 1 Facilities 0 Duration 0 Reviews 0 dtype: int64
4.NIL	Capacity 0 Company 1 Facilities 0 Duration 0 Reviews 0 dtype: int64	Capacity 0 Company 1 Facilities 0 Duration 0 Reviews 0 dtype: int64	Capacity 0.0 Company 1.0 Facilities 0.0 Duration 1.0 Reviews 0.0 dtype: float64	Capacity 0.0 Company 0.5 Facilities 0.0 Duration 1.0 Reviews 0.0 dtype: float64	Capacity 0 Company 0 Facilities 0 Duration 0 Reviews 0 dtype: int64	Capacity 0.0 Company 1.0 Facilities 0.0 Duration 1.0 Reviews 0.0 dtype: float64	Capacity 0 Company 1 Facilities 0 Duration 0 Reviews 0 dtype: int64
5.DAN	Capacity 0 Company 0 Facilities 0 Duration 0 Reviews 0 dtype: int64	Capacity 0.0 Company 0.5 Facilities 0.0 Duration 0.0 Reviews 0.0 dtype: float64	Capacity 0 Company 0 Facilities 0 Duration 0 Reviews 0 dtype: int64	Capacity 0 Company 0 Facilities 0 Duration 0 Reviews 0 dtype: int64	Capacity 0 Company 0 Facilities 0 Duration 0 Reviews 0 dtype: int64	Capacity 0 Company 0 Facilities 0 Duration 0 Reviews 0 dtype: int64	Capacity 0.0 Company 1.0 Facilities 0.0 Duration 0.0 Reviews 0.0 dtype: float64
6.NEW	Capacity 0 Company 0 Facilities 0 Duration 0 Reviews 0 dtype: int64	Capacity 0.0 Company 0.0 Facilities 0.0 Duration 0.5 Reviews 0.0 dtype: float64	Capacity 0 Company 0 Facilities 0 Duration 1 Reviews 0 dtype: int64	Capacity 0 Company 0 Facilities 0 Duration 1 Reviews 0 dtype: int64	Capacity 0 Company 0 Facilities 0 Duration 0 Reviews 0 dtype: int64	Capacity 0 Company 0 Facilities 0 Duration 1 Reviews 0 dtype: int64	Capacity 0 Company 0 Facilities 0 Duration 0 Reviews 0 dtype: int64

We then re-calculate the discordance matrix, which is shown above. Notably, the very poor Duration performance that NEW exhibits result in partial discordance against CARIB, GRE, and DAN (for this criterion). This has a profound effect on the re-calculation of the credibility matrix, which is shown below:

```
new_credibility_matrix = electre3.construct(new_performance_table)
new_credibility_matrix
```

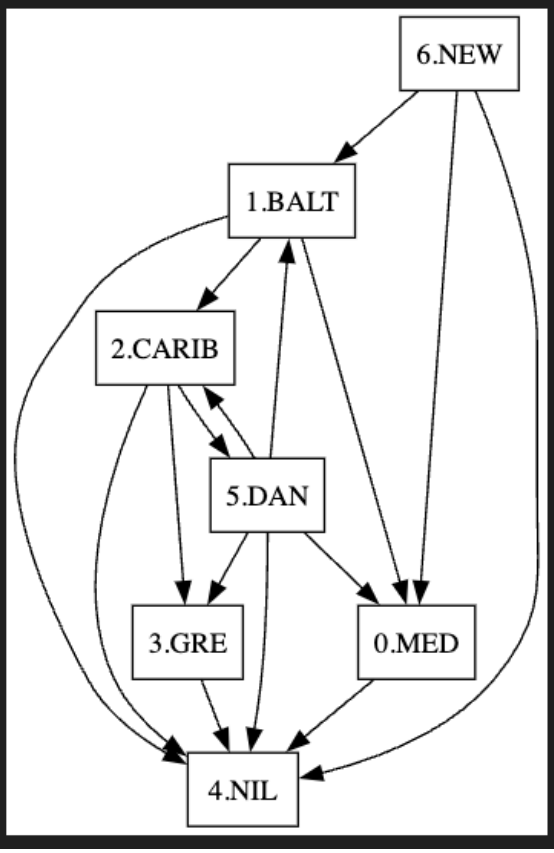
✓ 0.0s

	0.MED	1.BALT	2.CARIB	3.GRE	4.NIL	5.DAN	6.NEW
0.MED	1.00	0.125	0.0	0.0	0.65	0.0	0.333333
1.BALT	0.95	1.000	0.6	0.5	0.75	0.5	0.350000
2.CARIB	0.50	0.550	1.0	0.7	0.60	0.8	0.000000
3.GRE	0.00	0.000	0.0	1.0	0.90	0.0	0.000000
4.NIL	0.00	0.000	0.0	0.0	1.00	0.0	0.000000
5.DAN	0.70	0.750	1.0	0.8	0.70	1.0	0.000000
6.NEW	1.00	0.700	0.0	0.0	0.85	0.0	1.000000

As stated above, despite relatively high concordance, the partial discordance caused by poor Duration performance results in 0 credibility scores for NEW against CARIB, GRE, and DAN. Additional verification of this comes in the form of the graph, seen below:

```
cut_threshold = 0.6

new_cred_mat_cut = new_credibility_matrix.where(new_credibility_matrix >= cut_threshold, other=0)

AdjacencyMatrix(new_cred_mat_cut).plot()
```

✓ 0.1s



The credibility plot (again filtered at >= 0.6) correctly reflects the above statement. Fewer arrows originate from NEW compared to the concordance plot, because the vetoes destroyed its credibility against several alternatives.

Q9. What happens if we change the thresholds as indicated below? Run again and compare the credibility matrix and the ranking obtained with the ones of question 10. Explain the results and the motivation of the changes.

The code correctly defines the new, generally stricter dictionaries for q (indifference), p (preference), and v (veto) thresholds.

Because we define new thresholds, we must create a new ELECTRE3 object to utilise these thresholds:

```
q9_electre3 = Electre3(criteria_weights=criteria, preference_thresholds=p, indifference_thresholds=q, veto_thresholds=v, alpha=0.2, beta=-0.1)
```

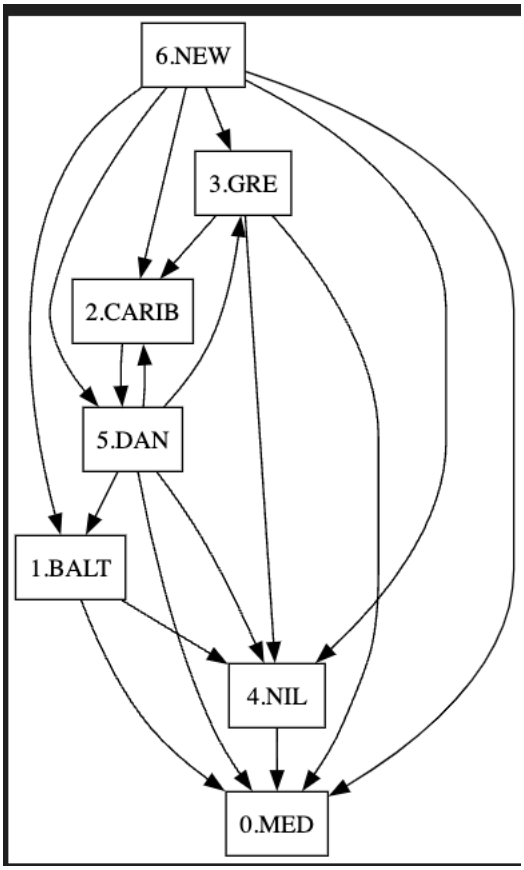
With this we can re-calculate the concordance matrix:

```
q9_concordance_matrix = q9_electre3.concordance(new_performance_table)

display(q9_concordance_matrix)

#Now we plot the graph of concordance with relations above the cut threshold
cut_threshold = 0.6
q9_conc_mat_cut = q9_concordance_matrix.where(q9_concordance_matrix >= cut_threshold, other=0)
AdjacencyMatrix(q9_conc_mat_cut).plot()
```

✓ 0.2s

	0.MED	1.BALT	2.CARIB	3.GRE	4.NIL	5.DAN	6.NEW
0.MED	1.0	0.1	0.5	0.30	0.5	0.3	0.4
1.BALT	0.9	1.0	0.5	0.50	0.7	0.3	0.3
2.CARIB	0.5	0.5	1.0	0.45	0.5	0.8	0.3
3.GRE	0.7	0.5	0.7	1.00	0.9	0.5	0.3
4.NIL	0.6	0.3	0.5	0.30	1.0	0.3	0.4
5.DAN	0.7	0.7	0.8	0.65	0.7	1.0	0.3
6.NEW	1.0	0.7	0.7	0.70	0.7	0.7	1.0

The q9_concordance_matrix is calculated. The values are different from the previous concordance matrix in Question 8 due to the changed q and p thresholds, mainly due to the elimination of indifference zones with q=0. The corresponding graph follows:



The plot reflects these new concordance relationships (once again maintaining a threshold of 0.6).

Next, we re-calculate the discordance matrix:

```
q9_discordance_matrix = q9_electre3.discordance(new_performance_table)
q9_discordance_matrix
```

✓ 0.0s Python

	0.MED	1.BALT	2.CARIB	3.GRE	4.NIL	5.DAN	6.NEW
0.MED	Capacity 0 Company 0 Facilities 0 Duration 0 Reviews 0 dtype: int64	Capacity 1.0 Company 0.5 Facilities 0.0 Duration 1.0 Reviews 1.0 dtype: float64	Capacity 0 Company 0 Facilities 0 Duration 1 Reviews 1 dtype: int64	Capacity 0 Company 0 Facilities 0 Duration 1 Reviews 1 dtype: int64	Capacity 0.0 Company 0.0 Facilities 0.0 Duration 0.5 Reviews 1.0 dtype: float64	Capacity 1 Company 0 Facilities 0 Duration 1 Reviews 1 dtype: int64	Capacity 1.0 Company 1.0 Facilities 0.0 Duration 0.0 Reviews 1.0 dtype: float64
1.BALT	Capacity 0 Company 0 Facilities 0 Duration 0 Reviews 0 dtype: int64	Capacity 0 Company 0 Facilities 0 Duration 0 Reviews 0 dtype: int64	Capacity 0.0 Company 0.0 Facilities 0.0 Duration 1.0 Reviews 1.0 dtype: float64	Capacity 0 Company 0 Facilities 0 Duration 1 Reviews 1 dtype: int64	Capacity 0 Company 0 Facilities 0 Duration 0 Reviews 1 dtype: int64	Capacity 0.5 Company 0.0 Facilities 0.0 Duration 1.0 Reviews 0.5 dtype: float64	Capacity 1.0 Company 0.5 Facilities 0.0 Duration 0.0 Reviews 1.0 dtype: float64
2.CARIB	Capacity 1.0 Company 0.5 Facilities 1.0 Duration 0.0 Reviews 0.0 dtype: float64	Capacity 1.0 Company 1.0 Facilities 0.0 Duration 0.0 Reviews 0.0 dtype: float64	Capacity 0 Company 0 Facilities 0 Duration 0 Reviews 0 dtype: int64	Capacity 1.0 Company 0.0 Facilities 0.0 Duration 0.0 Reviews 1.0 dtype: float64	Capacity 1.0 Company 0.0 Facilities 1.0 Duration 0.0 Reviews 1.0 dtype: float64	Capacity 1 Company 0 Facilities 0 Duration 0 Reviews 0 dtype: int64	Capacity 1 Company 1 Facilities 1 Duration 0 Reviews 1 dtype: int64
3.GRE	Capacity 0 Company 1 Facilities 1 Duration 0 Reviews 0 dtype: int64	Capacity 0.5 Company 1.0 Facilities 1.0 Duration 0.0 Reviews 0.0 dtype: float64	Capacity 0 Company 1 Facilities 0 Duration 0 Reviews 0 dtype: int64	Capacity 0 Company 0 Facilities 0 Duration 0 Reviews 0 dtype: int64	Capacity 0 Company 0 Facilities 1 Duration 0 Reviews 0 dtype: int64	Capacity 1 Company 1 Facilities 0 Duration 0 Reviews 0 dtype: int64	Capacity 1.0 Company 1.0 Facilities 1.0 Duration 0.0 Reviews 0.5 dtype: float64
4.NIL	Capacity 0.5 Company 1.0 Facilities 0.0 Duration 0.0 Reviews 0.0 dtype: float64	Capacity 1 Company 1 Facilities 0 Duration 0 Reviews 0 dtype: int64	Capacity 0 Company 1 Facilities 0 Duration 1 Reviews 0 dtype: int64	Capacity 1.0 Company 1.0 Facilities 0.0 Duration 1.0 Reviews 0.0 dtype: float64	Capacity 0 Company 0 Facilities 0 Duration 0 Reviews 0 dtype: int64	Capacity 1 Company 1 Facilities 0 Duration 1 Reviews 0 dtype: int64	Capacity 1.0 Company 1.0 Facilities 0.0 Duration 0.0 Reviews 0.5 dtype: float64
5.DAN	Capacity 0.0 Company 0.5 Facilities 1.0 Duration 0.0 Reviews 0.0 dtype: float64	Capacity 0.0 Company 1.0 Facilities 0.0 Duration 0.0 Reviews 0.0 dtype: float64	Capacity 0.0 Company 0.0 Facilities 0.0 Duration 0.0 Reviews 0.5 dtype: float64	Capacity 0 Company 0 Facilities 0 Duration 0 Reviews 1 dtype: int64	Capacity 0 Company 0 Facilities 1 Duration 0 Reviews 1 dtype: int64	Capacity 0 Company 0 Facilities 0 Duration 0 Reviews 0 dtype: int64	Capacity 0 Company 1 Facilities 1 Duration 0 Reviews 1 dtype: int64
6.NEW	Capacity 0 Company 0 Facilities 0 Duration 0 Reviews 0 dtype: int64	Capacity 0 Company 0 Facilities 0 Duration 1 Reviews 0 dtype: int64	Capacity 0 Company 0 Facilities 0 Duration 1 Reviews 0 dtype: int64	Capacity 0 Company 0 Facilities 0 Duration 1 Reviews 0 dtype: int64	Capacity 0.0 Company 0.0 Facilities 0.0 Duration 0.5 Reviews 0.0 dtype: float64	Capacity 0 Company 0 Facilities 0 Duration 1 Reviews 0 dtype: int64	Capacity 0 Company 0 Facilities 0 Duration 0 Reviews 0 dtype: int64

As expected, with lower and newly introduced veto thresholds, there are visibly more non-zero partial discordance values and more instances of 1.0 (veto triggered) compared to the previous discordance matrix in Question 8.

Finally, we re-calculate the credibility matrix:

```
q9_credibility_matrix = q9_electre3.construct(new_performance_table)
q9_credibility_matrix
```

✓ 0.0s

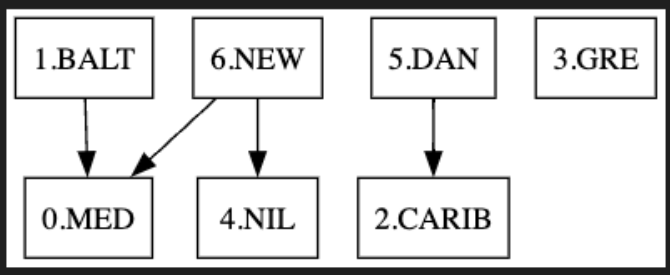
	0.MED	1.BALT	2.CARIB	3.GRE	4.NIL	5.DAN	6.NEW
0.MED	1.0	0.0	0.0	0.0	0.0	0.0	0.0
1.BALT	0.9	1.0	0.0	0.0	0.0	0.0	0.0
2.CARIB	0.0	0.0	1.0	0.0	0.0	0.0	0.0
3.GRE	0.0	0.0	0.0	1.0	0.0	0.0	0.0
4.NIL	0.0	0.0	0.0	0.0	1.0	0.0	0.0
5.DAN	0.0	0.0	0.8	0.0	0.0	1.0	0.0
6.NEW	1.0	0.0	0.0	0.0	0.7	0.0	1.0

The q9_credibility_matrix is calculated using the new concordance and much higher discordance. The resulting credibility values are significantly lower overall compared to Question 8, with many relationships dropping to 0.0 due to increased discordance/vetoes.

The plot verifies this claim:

```
cut_threshold = 0.6

q9_cred_mat_cut = q9_credibility_matrix.where(q9_credibility_matrix >= cut_threshold, other=0)

AdjacencyMatrix(q9_cred_mat_cut).plot()
```

✓ 0.2s



The plot verifies this claim as there are much sparser set of relationships where credibility remains using a threshold of 0.6.