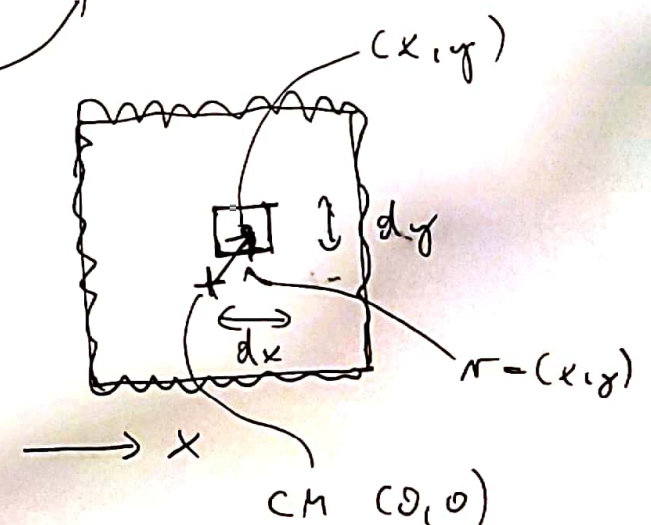


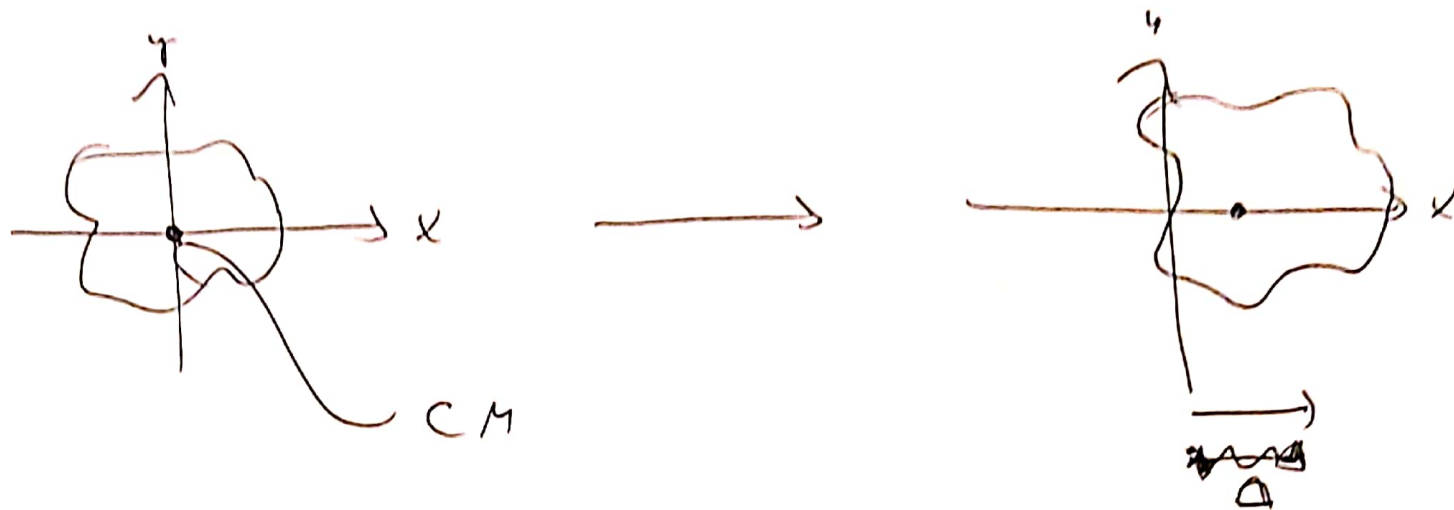
(1)

LC Twierdzenie Steinera

$$I_{CM} = \int dx \int dy \rho(x, y) \underbrace{(x^2 + y^2)}_{\substack{\text{odległość od} \\ \text{środku masy}}} \quad \substack{\uparrow \\ \text{środek masy } = (0,0)}$$

$$I_{CM} = \sum_i m_i \underbrace{\vec{r}_i^2}_{\substack{\uparrow \\ \rho(x, y) \cdot dx dy}}$$


The diagram illustrates a rectangular area element in a coordinate system. The origin is labeled $CM (0,0)$. A point (x,y) is marked on the rectangle. A small square element of width dx and height dy is shown within the rectangle. The distance from the origin to the point (x,y) is labeled $r = (x,y)$. The area element is labeled $\rho(x,y) \cdot dx dy$.



$$\underline{I} = \int dx dy \, \delta(x, y) \underbrace{\left((x+\Delta)^2 + y^2 \right)}_{x^2 + y^2 + 2x \cdot \Delta + \Delta^2} =$$

$$= \underbrace{\int dx dy \, \delta(x, y) \cdot (x^2 + y^2)}_{\underline{I}_{CM}} + \dots$$

$$\dots + \int dx dy g(x, y) \Delta^2 + \int dx dy g(x, y) \cdot 2x \Delta =$$

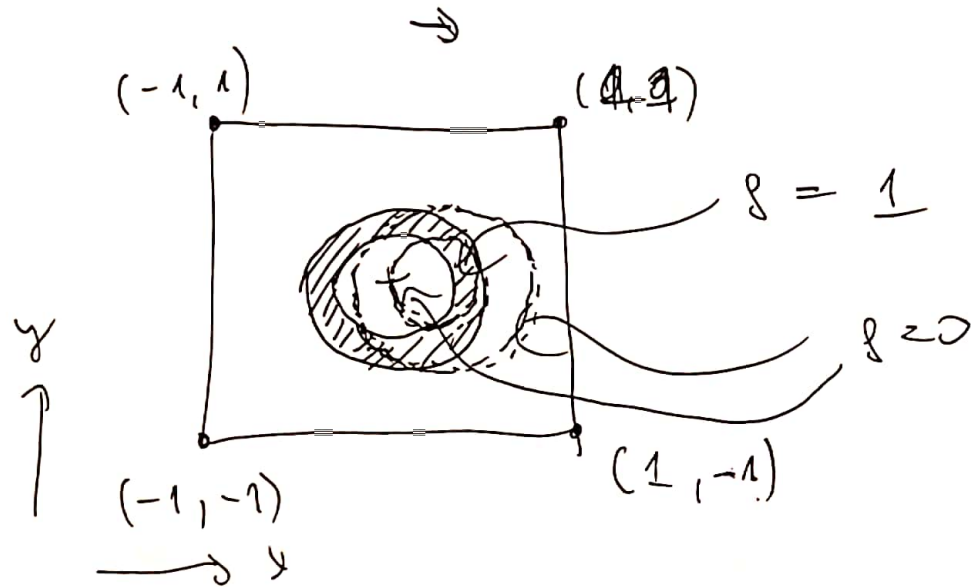
$$= I_{CM} + \Delta^2 \underbrace{\int dx dy g(x, y)}_m + 2\Delta \underbrace{\int dx dy g(x, y) \cdot x}_0 =$$

"tot. jaksy" $\sum_i m_i$

$$\vec{\pi}_{CM} = \underbrace{\int dx dy g(x, y) \left(\frac{x}{y} \right)}_{\text{"tot. jaksy" } \sum_i m_i \cdot \vec{\pi}_i} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$I_{CM} + \Delta^2 m$$

$\int dx dy \dots \rightarrow \text{NIntegrate}[\dots, \{x, y\}]$



W macielem momentu bierzemy pod uwagę:

$$(x, y, z) \Leftrightarrow (1, 2, 3)$$

$$\vec{a} = \begin{pmatrix} a_x \\ a_y \\ a_z \end{pmatrix}$$

$$\vec{b} = \begin{pmatrix} b_x \\ b_y \\ b_z \end{pmatrix}$$

$$\epsilon_{123} = 1$$

$$\epsilon_{ijk} = \begin{cases} 0 & \text{gdy } i, j, k \text{ nie s\u0105 wszystkie} \\ 1 & \text{gdy p\u00f3r\u00f3\u017ani\u0119 per.} \\ -1 & \text{gdy nie p\u00f3r\u00f3\u017ani\u0119 per.} \end{cases}$$

$$i, j, k \rightarrow 1, 2, 3$$

$$\vec{a} \times \vec{b} = \begin{pmatrix} -a_z b_y + a_y b_z \\ a_z b_x - a_x b_z \\ -a_y b_x + a_x b_y \end{pmatrix}$$

$$(\vec{a} \times \vec{b})_i = \sum_{j,k} \epsilon_{ijk} a_j b_k$$

\uparrow
 symbol Levi Civita

$$(\vec{a} \times \vec{b})_i = \sum_{j,k} \epsilon_{ijk} a_j b_k =$$

$$= \sum_k \left(\underbrace{\sum_j \epsilon_{ijk} a_j}_{\text{matrix}} \right) b_k =$$

$$= \sum_k \underbrace{[\tilde{\vec{a}}]}_{\text{matrix}} i_k \underbrace{b_k}_{\text{vector}}$$

$$(\vec{a} \times \vec{b}) = [\tilde{\vec{a}}] \cdot \vec{b}$$

zestawny w CM.

$$L = \sum_{i=1}^N m_i (\mathbf{r}_i \times \mathbf{v}_i) =$$

\uparrow moment pędu.
 \uparrow masa punktu "i"
 \uparrow położenie punktu "i"
 \uparrow prędkość punktu "i"
 $\omega = |\omega| \hat{u}$

$$= \sum_{i=1}^N m_i (\mathbf{r}_i \times (\boldsymbol{\omega} \times \mathbf{r}_i)) = \dots$$

\uparrow prędkość kątowa

$$= - \sum_{i=1}^W m_i \left(\mathbf{r}_i \times (\mathbf{r}_i \times \boldsymbol{\omega}) \right) =$$

$$= - \sum_{i=1}^W m_i \quad [\hat{\mathbf{r}}_i] \cdot [\hat{\mathbf{r}}_i] \cdot \boldsymbol{\omega} =$$

$$= \left(- \sum_{i=1}^W m_i [\hat{\mathbf{r}}_i] \cdot [\hat{\mathbf{r}}_i] \right) \cdot \boldsymbol{\omega}$$

$$[\mathbf{I}]_{CM} = - \sum_{i=1}^W m_i [\hat{\mathbf{r}}_i] \cdot [\hat{\mathbf{r}}_i] -$$

$$= \begin{pmatrix} I_{xx} & -I_{xy} & -I_{xz} \\ -I_{xy} & I_{yy} & -I_{yz} \\ -I_{xz} & -I_{yz} & I_{zz} \end{pmatrix}$$