Równanie Cauchiego Riemanna

– pochodna liczby zespolonej

$$f(z) = f(x, y)$$

z – l. zespolona

f – funkcja

$$z = \underbrace{x + iy}_{\mathbb{R}}$$

pochodna

$$f'(z) = \lim_{h \to 0} \frac{f(z+h) - f(z)}{h}$$

$z \in \mathbb{C}$, $h \in \mathbb{C}$

$$f'(z) = \lim_{h \to 0} \frac{f(z+h) - f(z)}{h}$$

$$h = \underset{\mathbb{R}}{\delta} + i \underset{\mathbb{R}}{\varsigma}$$

$\delta = 0$

$\varsigma = 0$

$$f'(x+iy) = \lim_{\delta \to 0} \frac{\overbrace{f(x+\delta+iy)}^{f(x+\delta,\,y)} - \overbrace{f(x+iy)}^{f(x,y)}}{\delta} = \frac{\partial f}{\partial x}$$

$f(x+iy) = f(x,y) = f$

$$f'(x+iy) = \lim_{\varsigma \to 0} \frac{\overbrace{f(x+i(y+\varsigma))}^{f(x,\,y+\varsigma)} - \overbrace{f(x+iy)}^{f(x,y)}}{i\varsigma} = \frac{1}{i} \frac{\partial f}{\partial y}$$

$f(x+iy) = f(x,y) = f$

granica po osi rzeczywistej oraz urojonej powinna być taka sama!

$$\frac{\partial f}{\partial x} = \frac{1}{i}\frac{\partial f}{\partial y}$$

$$\underbrace{f(x,y)}_{f(x+iy)} = u(x,y) + i\,v(x,y)$$

$$i\left(\frac{\partial u(x,y)}{\partial x} + i\,\frac{\partial v(x,y)}{\partial x}\right) =$$

$$= \frac{\partial u(x,y)}{\partial y} + i\,\frac{\partial v(x,y)}{\partial y}$$

$$-\frac{\partial v(x,y)}{\partial x} + i\,\frac{\partial u(x,y)}{\partial x} = \frac{\partial u(x,y)}{\partial y} + i\,\frac{\partial v(x,y)}{\partial y}$$

$$
\begin{cases}
-\dfrac{\partial v(x,y)}{\partial x} = \dfrac{\partial u(x,y)}{\partial y} \\[2ex]
\dfrac{\partial v(x,y)}{\partial y} = \dfrac{\partial u(x,y)}{\partial x}
\end{cases}
$$

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