Równania Lagrangea dla 2D brył sztywnych

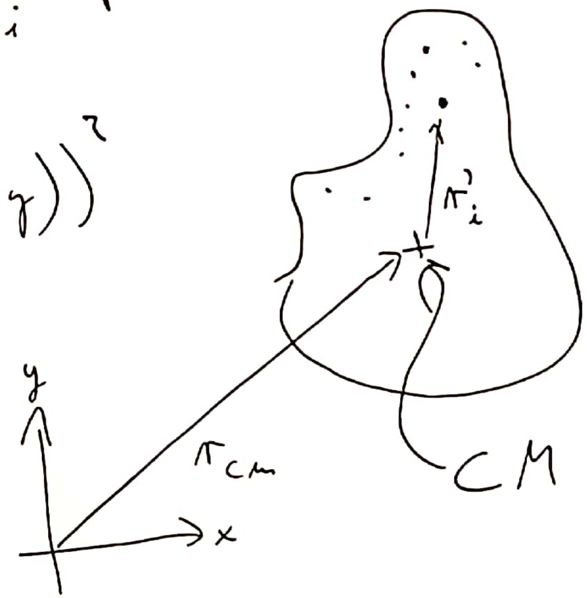
l. punktów

$$T = \sum_{i=1}^{N} \frac{1}{2} m_i \dot{r}_i^2 = \dots$$

prędkość punktu i

masa punktu i

$$\underbrace{\int dx\, dy\, \rho(x,y)}_{\text{masa fragmentu}} \frac{1}{2}\left(v(x,y)\right)^2$$

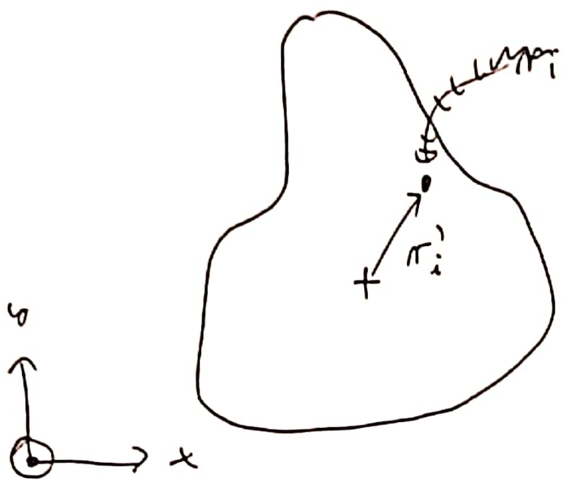


$$\ldots = \sum_{i=1}^{N} \frac{1}{2} m_i \left(\dot{r}_{CM} + \dot{r}'_i\right)\cdot\left(\dot{r}_{CM} + \dot{r}'_i\right) =$$

$$= \sum_{i=1}^{N} \frac{1}{2} m_i \left(\dot{r}_{CM}^2 + \dot{r}'^2_i + 2\,\dot{r}_{CM}\cdot\dot{r}'_i\right) =$$

$$= \dot{r}_{CM} \underbrace{\sum_{i=1}^{N} m_i \dot{r}'_i}_{0} + \frac{1}{2}\underbrace{\left(\sum_{i=1}^{N} m_i\right)}_{M} \dot{r}_{CM}^2 + \sum_{i=1}^{N} \frac{1}{2} m_i \dot{r}'^2_i =$$

$$= \frac{1}{2} M \dot{r}_{CM}^2 + \sum_{i=1}^{N} \frac{1}{2} m_i \dot{r}'^2_i = \ldots$$



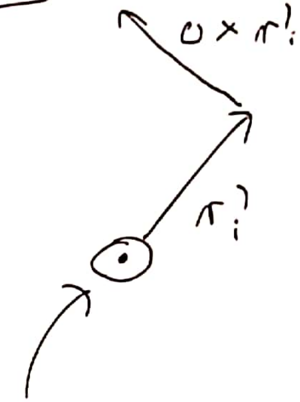
$$\underbrace{(\dot r_i')^2} = \underbrace{\omega^2 r_i'^2}$$

prędkość kątowa

$$\dot r_i' = \omega \times r_i'$$

$$\omega = \begin{pmatrix} 0 \\ 0 \\ \frac{2\pi}{T} \end{pmatrix}$$

okres obrotu



$$\ldots = \frac{1}{2} M \dot{r}_{cm}^{2} + \sum_{i=1}^{N} \frac{1}{2} m_i \underline{\dot{r}_i'}^{2} =$$

$$= \frac{1}{2} M \dot{r}_{cm}^{2} + \sum_{i=1}^{N} \frac{1}{2} m_i \omega^2 (r_i')^2 =$$

$$= \frac{1}{2} M \dot{r}_{cm}^{2} + \frac{1}{2} \underbrace{\left(\sum_{i=1}^{N} m_i (r_i')^2 \right)}_{I_{cm} \Leftrightarrow \int dx\,dy\, \rho(x,y)(x^2+y^2)} \omega^2 =$$

$$= \frac{1}{2} M \dot{r}_{cm}^{2} + \frac{1}{2} I_{cm} \omega^2$$

[zasada minimalizacji działania

$$S[r] = \int dt\, L(r(t), \dot{r}(t))$$

$\underbrace{r = r(t)}_{\text{trajektoria}}$

działanie

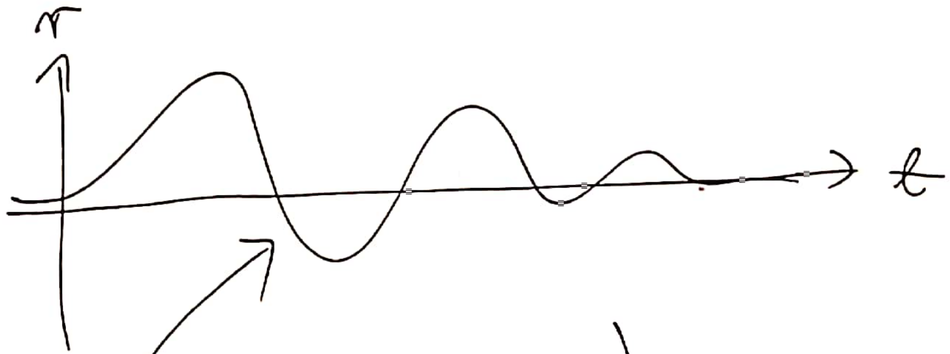
- chcemy znaleźć taką trajektorię $r(t)$ min. działanie

r. Eulera Lagrangea:

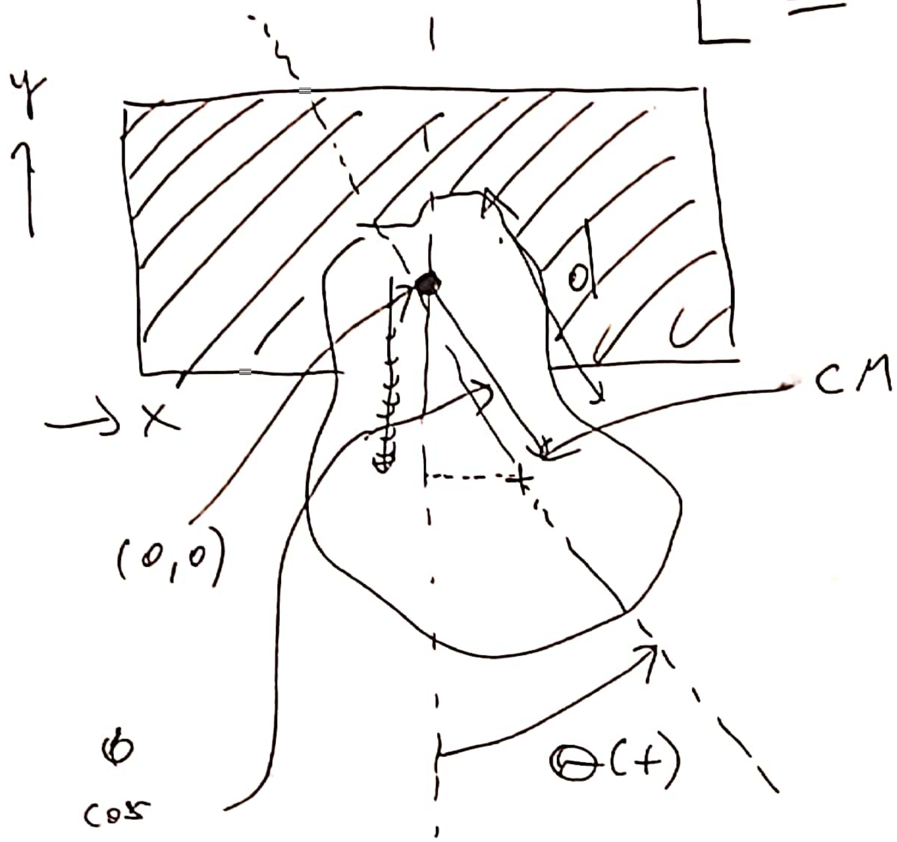
$$\frac{\partial L(r, \dot r)}{\partial r} = \frac{d}{dt}\frac{\partial L(r, \dot r)}{\partial \dot r}$$

liczba charakteryzująca system z 1 stopień swobody

$\downarrow$ rozwiń



rozwiązanie fizyczne



$$L = T - V$$

T – en. kinetická

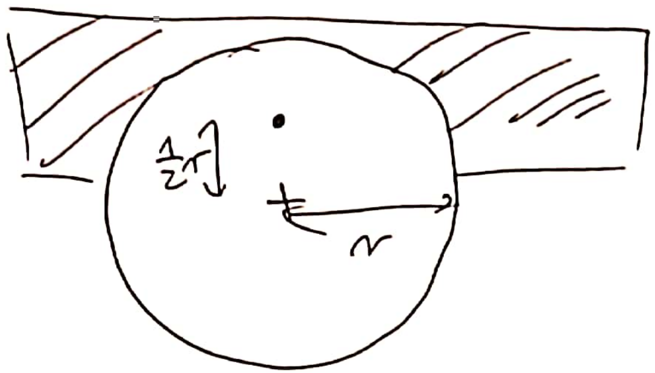
V – en. potenciálna

$$d\begin{pmatrix} \sin(\Theta(t)) \\ -\cos(\Theta(t)) \end{pmatrix} = r_{cm}(t)$$

$$T = \frac{1}{2} M\, d^2 (\dot{\Theta}(t))^2 + \frac{1}{2} (\dot{\Theta}(t))^2 \, I_{cm}$$

predchozí kapitole

$$V = M \cdot g\, d\, (-\cos\Theta(t))$$



$M = 1$ $\quad$ $r = \underline{1}$