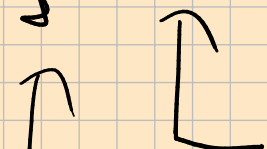


# Równanie Cauchyego Riemana

- pochodna liczby zespolonej

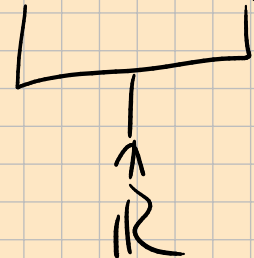
$$f(z) = f(x, y)$$



l. zespolona

funkcje

$$z = x + iy$$



pochodne

$$f'(z) = \lim_{h \rightarrow 0} \frac{f(z+h) - f(z)}{h}$$

$$\frac{f(z+h) - f(z)}{h}$$

$$f'(z) = \lim_{h \rightarrow 0} \frac{f(z+h) - f(z)}{h}$$

$$h = \underset{\substack{\uparrow \\ \mathbb{R}}}{\delta} + i \underset{\substack{\uparrow \\ \mathbb{R}}}{\eta}$$

$\delta = 0$

$\eta = 0$

$$f'(x+iy) = \lim_{\delta \rightarrow 0} \frac{f(x+\delta+iy) - f(x+iy)}{\delta}$$

$$f'(x+iy) = \lim_{\eta \rightarrow 0} \frac{f(x+i(y+\eta)) - f(x+iy)}{i\eta}$$

$$\frac{\overbrace{f(x+\delta, y)}^{f(x,y)} - \overbrace{f(x,y)}^{f(x,y)}}{\delta} =$$

$$\frac{\overbrace{f(x, y+\eta)}^{f(x,y)} - \overbrace{f(x,y)}^{f(x,y)}}{i\eta} =$$

$$\frac{\partial f}{\partial x}(x,y)$$

$$\frac{1}{i} \frac{\partial f}{\partial y}(x,y)$$

granica po osi rzeczywistej oraz urojonej powinna  
być taka sama!

$$\frac{\partial f}{\partial x} = \frac{1}{i} \frac{\partial f}{\partial y}$$

$$\underbrace{f(x, y)}_{f(x+iy)} = u(x, y) + i v(x, y)$$

$$f(x+iy)$$

$$i \left( \frac{\partial u(x,y)}{\partial x} + i \frac{\partial v(x,y)}{\partial x} \right) =$$

$$= \frac{\partial u(x,y)}{\partial y} + i \frac{\partial v(x,y)}{\partial y}$$

$$- \frac{\partial v(x,y)}{\partial x} + i \frac{\partial u(x,y)}{\partial x} = \frac{\partial u(x,y)}{\partial y} + i \frac{\partial v(x,y)}{\partial y}$$

$$-\frac{\partial v(x,y)}{\partial x} = \frac{\partial u(x,y)}{\partial y}$$

$$\frac{\partial v(x,y)}{\partial y} = \frac{\partial u(x,y)}{\partial x}$$

formule Cauchy-Riemann