Reprezentacje liczb zespolonych

i – jednostka urojona $\quad i \cdot i = -1$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \qquad \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \quad - \quad a + bi \longleftrightarrow \underline{a \cdot \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + b \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}}$$

$$\begin{matrix} \updownarrow & & \updownarrow & & \cap & & \cap \\ 1 & & i & & \mathbb{R} & & \mathbb{R} \end{matrix}$$

$$C_{ij} = \sum_k A_{ik} B_{kj}$$

$$\left(x \cdot (\ ) + y(\ )\right)\left(x'(\ ) + y'(\ )\right)$$

$$\|$$

$$\left(x'(\ ) + y'(\ )\right)\left(x(\ ) + y(\ )\right)$$

$(a, b) \Leftrightarrow a + i b$

$\uparrow \quad \uparrow$

$(\; , \;) + (\; , \;)$

MatrixExp $\left[\phi \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \right]$

$(\; , \;) \cdot (\; , \;)$

$\mathbb{R}$

$\uparrow$

$$\boxed{e^{i\phi} = \cos\phi + i \sin\phi}$$

$$e^{x} = \sum_n \frac{x^n}{n!} \leftarrow \leftarrow$$

$$\boxed{a \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + b \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}}$$

MatrixExp

$$\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} \longleftrightarrow \underline{1} \qquad \begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix} \longleftrightarrow i$$

$$\underbrace{\mathrm{Exp}(i\phi) = \sum_n \frac{(i\phi)^n}{n!}}$$

MatrixExp[ ]   Solve[ ]

$$\underbrace{e^{i\phi} = \cos\phi + i\sin\phi}$$

$$\frac{df}{dx} = f'(x) = \lim_{\substack{h \to 0 \\ \uparrow \\ \mathbb{C}}} \frac{f(x+h) - f(x)}{h}$$

$$\lim_{\substack{h \to 0 \\ \uparrow \\ \mathbb{R}}} \frac{f(z+h) - f(z)}{h} = \lim_{\substack{h \to 0 \\ \uparrow \\ \mathbb{R}}} \frac{f(z+ih) - f(z)}{ih}$$

$$\underset{\substack{\uparrow \\ \mathbb{C}}}{f}(x+iy) = \underset{\substack{\uparrow \\ \mathbb{R}}}{u}(x+iy) + i\,\underset{\substack{\uparrow \\ \mathbb{R}}}{v}(x+iy)$$

$$f(x+iy),\ u(x+iy),\ v(x+iy) \Leftrightarrow$$

$$\Leftrightarrow f(x,y),\ \underset{\mathbb{R}}{\underset{\Uparrow}{u}}(x,y),\ \underset{\mathbb{R}}{\underset{\Uparrow}{v}}(x,y)$$

$$\frac{\partial f(x,y)}{\partial x} = \frac{1}{i}\frac{\partial f(x,y)}{\partial y}$$

$$\frac{\partial}{\partial x}\left(u(x,y)+iv(x,y)\right) = \frac{1}{i}\frac{\partial}{\partial y}\left(u(x,y)+iv(x,y)\right)$$

$$\partial_x u(x,y) + \underline{i\,\partial_x v(x,y)} = \underline{-i\,\partial_y u(x,y)} + \partial_y v(x,y)$$

$$\begin{cases} \partial_x u(x,y) = \partial_y v(x,y) \\ \partial_x v(x,y) = -\partial_y u(x,y) \end{cases}$$

równania Cauchy – Riem

$$f(x+iy) = \underset{\substack{\uparrow \\ \mathbb{R}}}{u}\overset{(x,y)}{(x+iy)} + i\,\underset{\substack{\uparrow \\ \mathbb{R}}}{v}\overset{(x,y)}{(x+iy)}$$

$$\underbrace{e^{x}}_{\substack{\text{Liczba} \\ \text{macierz}}} = \underbrace{\sum_{n=0}^{\infty} \frac{(x^n)}{n!}}$$

$$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \longleftrightarrow i$$

$$\text{MatrixExp}[\phi \cdot \{\{0,-1\},\{1,0\}\}] =$$

$$= \underbrace{\text{wzór Eulera}} = \cos\phi \overset{1}{\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}} + \sin\phi \underset{i}{\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}}$$