LC Twierdzenie Steinera

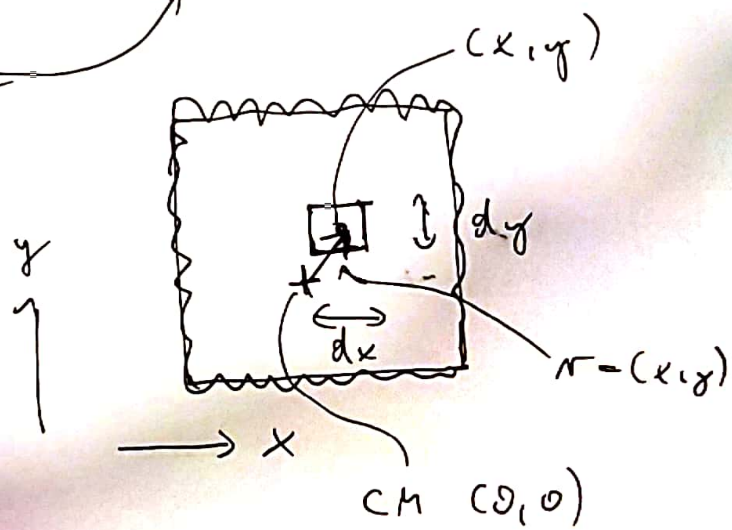
$$I_{CM} = \int dx \int dy \; \rho(x,y) \, \underbrace{(x^2+y^2)}$$

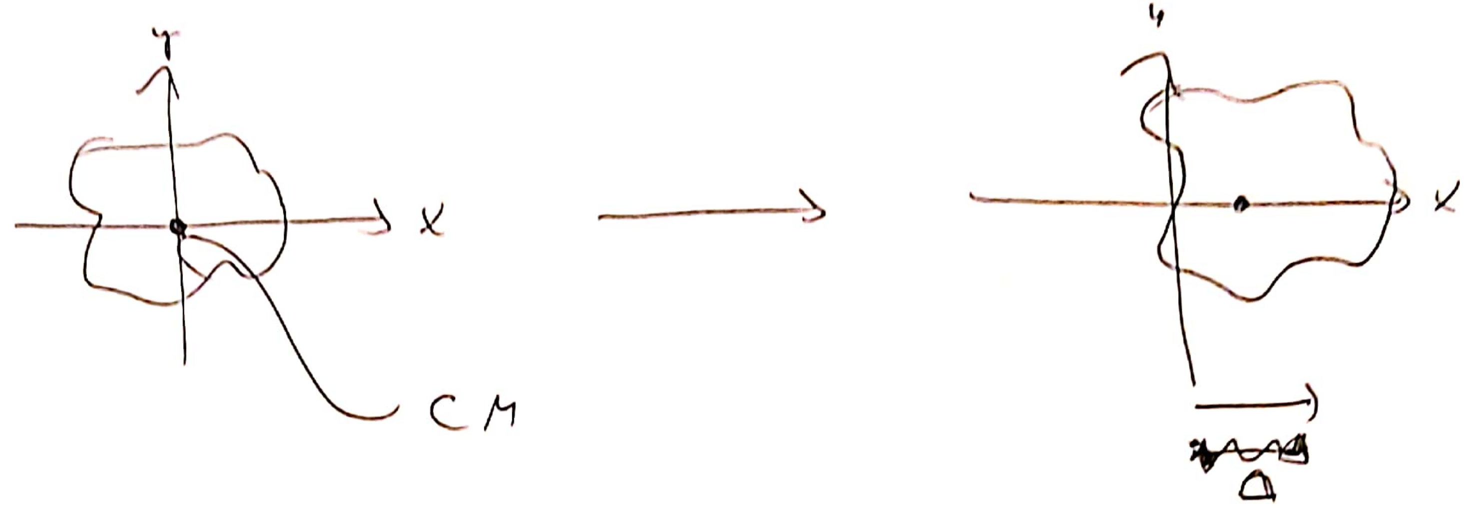
środek masy $=(0,0)$

$(\text{odległość od środka masy})^2$

$$I_{CM} = \sum_i m_i \, \vec{r}_i^{\,2}$$

$$\rho(x,y) \cdot dx\,dy$$





$$I = \int dx\, dy\, \rho(x,y) \underbrace{\left((x+\Delta)^2 + y^2 \right)}_{\underbrace{x^2+y^2}+2x\cdot\Delta+\Delta^2} =$$

$$= \underbrace{\int dx\, dy\, \rho(x,y)\cdot(x^2+y^2)}_{I_{CM}} + \ldots$$

$$\ldots + \int dx\,dy\,\rho(x,y)\,\Delta^2 + \int dx\,dy\,\rho(x,y)\cdot 2x\Delta =$$

$$= I_{CM} + \Delta^2 \underbrace{\int dx\,dy\,\rho(x,y)}_{m} + 2\Delta \overbrace{\int dx\,dy\,\rho(x,y)\cdot x}^{0} =$$

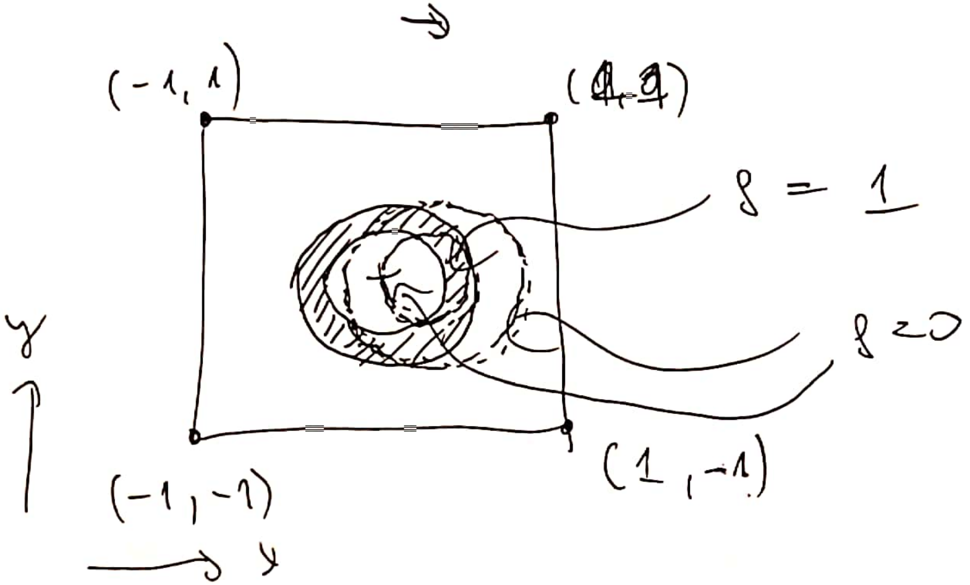
„tak jakby" $\sum_i m_i$

$$I_{CM} + \Delta^2 m$$

$$\vec{r}_{CM} = \underbrace{\int dx\,dy\,\rho(x,y)\begin{pmatrix} x \\ y \end{pmatrix}}_{} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

„tak jakby" $\sum_i m_i \cdot \vec{r}_i$

$\int dx\,dy \ldots$ — NIntegrate[, {}, {}]



II macierz momentów bezwładności

$(x, y, z) \Leftrightarrow (1, 2, 3)$

$\epsilon_{123} = 1$

$$\vec{a} = \begin{pmatrix} a_x \\ a_y \\ a_z \end{pmatrix} \qquad \vec{b} = \begin{pmatrix} b_x \\ b_y \\ b_z \end{pmatrix}$$

$$\epsilon_{ijk} = \begin{cases} 0 & \text{gdy } ijk \text{ się powtarzają} \\ 1 & \text{gdy parzysta per.} \\ -1 & \text{gdy nieparzysta per.} \end{cases}$$

$ijk \to 123$

$$\vec{a} \times \vec{b} = \begin{pmatrix} -a_z b_y + a_y b_z \\ a_z b_x - a_x b_z \\ -a_y b_x + a_x b_y \end{pmatrix}$$

$$(\vec{a} \times \vec{b})_i = \sum_{j,k} \epsilon_{ijk} a_j b_k$$

symbol Levi Civita

$$(a \times b)_i = \sum_{jk} \epsilon_{ijk} a_j b_k =$$

$$= \sum_k \underbrace{\left(\sum_j \epsilon_{ijk} a_j \right)} b_k =$$

$$= \sum_k \underbrace{[\tilde{a}]_{ik}}_{\text{macierz}} \; \underbrace{b_k}_{\text{wektor}}$$

$$(\vec{a} \times \vec{b}) = [\tilde{a}] . \vec{b}$$

Jesteśmy w CM.

położenie punktu „i”

$$L = \sum_{i=1}^{N} m_i \left(r_i \times v_i \right) =$$

prędkość punktu „i”

masa punktu „i”

moment pędu.

$\omega = |\omega| \, \hat{u}$

$$= \sum_{i=1}^{N} m_i \Big(r_i \times \left(\omega \times r_i \right) \Big) = \ldots$$

prędkość kątowa

$$= -\sum_{i=1}^{W} m_i \left(r_i \times (r_i \times \omega) \right) =$$

$$= -\sum_{i=1}^{W} m_i \quad [\tilde{r}_i] \cdot [\tilde{r}_i] \cdot \omega =$$

$$= \underbrace{\left(-\sum_{i=1}^{W} m_i [\tilde{r}_i] \cdot [\tilde{r}_i] \right)} \cdot \omega$$

$$\left[\underline{I}_{CM} \right] = -\sum_{i=1}^{W} m_i [\tilde{r}_i] \cdot [\tilde{r}_i] -$$

$$= \begin{pmatrix} I_{xx} & -I_{xy} & -I_{xz} \\ -I_{xy} & I_{\cancel{x}y} & -I_{xz} \\ -I_{xz} & -I_{yz} & I_{zz} \end{pmatrix}$$