

### Uniqueness of the Principal Components

While in theory the principal components need not be unique, in almost all practical settings they are (up to sign flips). This means that two different software packages will yield the same principal component loading vectors, although the signs of those loading vectors may differ. The signs may differ because each principal component loading vector specifies a direction in  $p$ -dimensional space: flipping the sign has no effect as the direction does not change. (Consider Figure 6.14—the principal component loading vector is a line that extends in either direction, and flipping its sign would have no effect.) Similarly, the score vectors are unique up to a sign flip, since the variance of  $Z$  is the same as the variance of  $-Z$ . It is worth noting that when we use (12.5) to approximate  $x_{ij}$  we multiply  $z_{im}$  by  $\phi_{jm}$ . Hence, if the sign is flipped on both the loading and score vectors, the final product of the two quantities is unchanged.

### Deciding How Many Principal Components to Use

In general, an  $n \times p$  data matrix  $\mathbf{X}$  has  $\min(n - 1, p)$  distinct principal components. However, we usually are not interested in all of them; rather, we would like to use just the first few principal components in order to visualize or interpret the data. In fact, we would like to use the smallest number of principal components required to get a *good* understanding of the data. How many principal components are needed? Unfortunately, there is no single (or simple!) answer to this question.

We typically decide on the number of principal components required to visualize the data by examining a *scree plot*, such as the one shown in the left-hand panel of Figure 12.3. We choose the smallest number of principal components that are required in order to explain a sizable amount of the variation in the data. This is done by eyeballing the scree plot, and looking for a point at which the proportion of variance explained by each subsequent principal component drops off. This drop is often referred to as an *elbow* in the scree plot. For instance, by inspection of Figure 12.3, one might conclude that a fair amount of variance is explained by the first two principal components, and that there is an elbow after the second component. After all, the third principal component explains less than ten percent of the variance in the data, and the fourth principal component explains less than half that and so is essentially worthless.

However, this type of visual analysis is inherently *ad hoc*. Unfortunately, there is no well-accepted objective way to decide how many principal components are *enough*. In fact, the question of how many principal components are enough is inherently ill-defined, and will depend on the specific area of application and the specific data set. In practice, we tend to look at the first few principal components in order to find interesting patterns in the data. If no interesting patterns are found in the first few principal components, then further principal components are unlikely to be of interest. Conversely, if the first few principal components are interesting, then we typically continue to look at subsequent principal components until no further interesting patterns are found. This is admittedly a subjective ap-