Prolog Site

Search this site

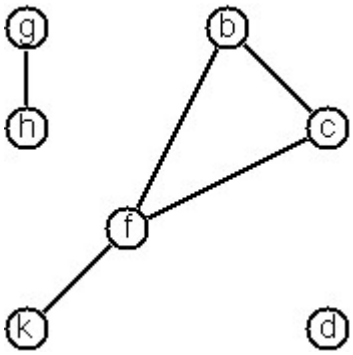
Prolog Problems >

6. Graphs

Solutions can be found here.

A preliminary remark: The vocabulary in graph theory varies considerably. Some authors use the same word with different meanings. Some authors use different words to mean the same thing. I hope that our definitions are free of contradictions.

A graph is defined as a set of *nodes* and a set of *edges*, where each edge is a pair of nodes.



There are several ways to represent graphs in Prolog.

One method is to represent each edge separately as one clause (fact). In this form, the graph depicted opposite is represented as the following predicate:

```
edge(h,g).
edge(k,f).
edge(f,b).
...
```

We call this *edge-clause form*.

Obviously, isolated nodes cannot be represented. Another method is to represent the whole graph as one data object. According to the definition of the graph as a pair of two sets (nodes and edges), we may use the following Prolog term to represent the above example graph:

```
graph([b,c,d,f,g,h,k],[e(b,c),e(b,f),e(c,f),e(f,k),e(g,h)])
```

We call this *graph-term form*. Note, that the lists are kept sorted, they are really *sets*, without duplicated elements. Each edge appears only once in the edge list; i.e. an edge from a node x to another node y is represented as e(x,y), the term e(y,x) is not present. The graph-term form is our default representation. In SWI-Prolog there are

predefined predicates to work with sets.

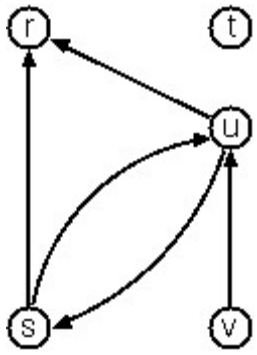
A third representation method is to associate with each node the set of nodes that are adjacent to that node. We call this the *adjacency-list form*. In our example:

```
[n(b,[c,f]), n(c,[b,f]), n(d,[]), n(f,[b,c,k]), ...]
```

The representations we introduced so far are Prolog terms and therefore well suited for automated processing, but their syntax is not very user-friendly. Typing the terms by hand is cumbersome and error-prone. We can define a more compact and "human-friendly" notation as follows: A graph is represented by a list of atoms and terms of the type X-Y (i.e. functor '-' and arity 2). The atoms stand for isolated nodes, the X-Y terms describe edges. If an X appears as an endpoint of an edge, it is automatically defined as a node. Our example could be written as:

```
[b-c, f-c, g-h, d, f-b, k-f, h-g]
```

We call this the *human-friendly form*. As the example shows, the list does not have to be sorted and may even contain the same edge multiple times. Notice the isolated node d. (Actually, isolated nodes do not even have to be atoms in the Prolog sense, they can be compound terms, as in `d(3.75,blue)` instead of d in the example).



When the edges are *directed* we call them *arcs*. These are represented by *ordered* pairs. Such a graph is called directed graph (or digraph, for short). To represent a directed graph, the forms discussed above are slightly modified. The example graph opposite is represented as follows:

Arc-clause form

```
arc(s,u).
arc(u,r).
...
```

Graph-term form

```
digraph([r,s,t,u,v],
[a(s,r),a(s,u),a(u,r),a(u,s),a(v,u)])
```

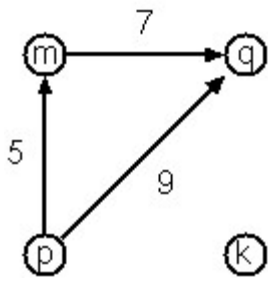
Adjacency-list form

```
[n(r,[]),n(s,[r,u]),n(t,[]),n(u,[r]),n(v,[u])]
```

Note that the adjacency-list does not have the information on whether it is a graph or a digraph.

Human-friendly form

```
[s > r, t, u > r, s > u, u > s, v > u]
```



Finally, graphs and digraphs may have additional information attached to nodes and edges (arcs). For the nodes, this is no problem, as we can easily replace the single character identifiers with arbitrary compound terms, such as `city('London',4711)`. On the other hand, for edges we have to extend our notation. Graphs with additional information attached to edges are called labeled graphs.

Arc-clause form

```
arc(m,q,7).
arc(p,q,9).
arc(p,m,5).
```

Graph-term form

```
digraph([k,m,p,q],[a(m,p,7),a(p,m,5),a(p,q,9)])
```

Adjacency-list form

```
[n(k,[]),n(m,[q/7]),n(p,[m/5,q/9]),n(q,[])]
```

Notice how the edge information has been packed into a term with functor '/' and arity 2, together with the corresponding node.

Human-friendly form

```
[p>q/9, m>q/7, k, p>m/5]
```

The notation for labeled graphs can also be used for so-called multi-graphs, where more than one edge (or arc) are allowed between two given nodes.

6.01 (***) Conversions

Write predicates to convert between the different graph representations. With these predicates, all representations are equivalent; i.e. for the following problems you can always freely pick the most convenient form. The reason this problem is rated (***) is not because it's particularly difficult, but because it's a lot of work to deal with all the special cases.

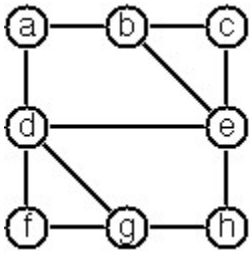
6.02 (**) Path from one node to another one

Write a predicate path(G,A,B,P) to find an acyclic path P from node A to node B in the graph G. The predicate should return all paths via backtracking.

6.03 (*) Cycle from a given node

Write a predicate cycle(G,A,P) to find a closed path (cycle) P starting at a given node A in the graph G. The predicate should return all cycles via backtracking.

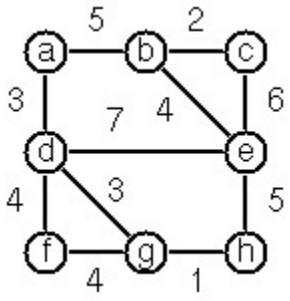
6.04 (**) Construct all spanning trees



Write a predicate s_tree(Graph,Tree) to construct (by backtracking) all spanning trees of a given graph. With this predicate, find out how many spanning trees there are for the graph depicted to the left. The data of this example graph can be found in the file p6_04.dat. When you have a correct solution for the s_tree/2 predicate, use it to define two other useful predicates: is_tree(Graph) and is_connected(Graph). Both are five-minutes tasks!

6.05 (**) Construct the minimal spanning tree

Write a predicate ms_tree(Graph,Tree,Sum) to construct the minimal spanning tree of a given labelled graph. Hint: Use the algorithm of Prim. A small modification of the solution of 6.04 does the trick. The data of the example graph to the right can be found in the file p6_05.dat.



6.06 (**) Graph isomorphism

Two graphs G1(N1,E1) and G2(N2,E2) are isomorphic if there is a bijection f: N1 -> N2 such that for any nodes X,Y of N1, X and Y are adjacent if and only if f(X) and f(Y) are adjacent.

Write a predicate that determines whether two graphs are

isomorphic. Hint: Use an open-ended list to represent the function f.

6.07 (**) Node degree and graph coloration

a) Write a predicate degree(Graph,Node,Deg) that determines the degree of a given node.

b) Write a predicate that generates a list of all nodes of a graph sorted according to decreasing degree.

c) Use Welch-Powell's algorithm to paint the nodes of a graph in such a way that adjacent nodes have different colors.

6.08 (**) Depth-first order graph traversal

Write a predicate that generates a depth-first order graph traversal sequence. The starting point should be specified, and the output should be a list of nodes that are reachable from this starting point (in depth-first order).

6.09 (**) Connected components

Write a predicate that splits a graph into its connected components.

6.10 (**) Bipartite graphs

Write a predicate that finds out whether a given graph is bipartite.

6.11 (***) Generate K-regular simple graphs with N nodes

In a K-regular graph all nodes have a degree of K; i.e. the number of edges incident in each node is K. How many (non-isomorphic!) 3-regular graphs with 6 nodes are there?

See also the table of results in p6_11.txt.

Subpages (1): Solutions-6