Northwestern University **Name:** _____

Math 224-0 Final Exam

Summer 2014

Thursday, August 14, 2014

Problem	Points	Score
1	12	
2	6	
3	10	
4	12	
5	16	
6	10	
7	8	
8	10	
9	10	
10	6	
Total	100	

Instructions

- Read each problem carefully.
- Write neatly and make sure your final answers are clearly indicated.
- SHOW ALL OF YOUR WORK!
- The exam consists of 10 problems.
- You may not use books, notes, or calculators.
- You have 2 hours to complete this exam.
- GOOD LUCK!!

1. (*12 points*) Find both the radius and interval of convergence of the power series

$$\sum_{n=2}^{\infty} \frac{2^n(x+1)^n}{n-1}$$

Clearly identify which test(s) you use.

2. Consider the definite integral

$$\int_1^7 \frac{1}{x}\,dx$$

(i) (*3 points*) Use Simpson's rule with $n = 6$ to estimate the integral. (You do not have to simplify your answer.)

(ii) (*3 points*) Use the trapezoidal rule with $n = 6$ to estimate the integral. (Again, you don't need to simplify your answer.)

3. (*10 points*) Find the Taylor series centered at 1 of the function

$$f(x) = \frac{1}{x^2}$$

4. (*12 points*) Determine whether the following series is absolutely convergent, conditionally convergent, or divergent. Justify your answer, and identify which test(s) you use.

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}+10}$$

5. Evaluate the following integrals:
   (i) (*4 points*)

$$\int \sin^{2014}\theta \cos^3\theta \, d\theta$$

   (ii) (*4 points*)

$$\int_{1/7}^{e/7} \frac{\ln(7x)}{x} \, dx$$

(iii) (*8 points*)

$$\int \sqrt{9 - x^2}\, dx$$

You might find useful the trig identities

$$\sin(2\theta) = 2\sin\theta\cos\theta \qquad\qquad \cos^2\theta = \frac{1 + \cos(2\theta)}{2}$$

6. (*10 points*) Determine whether the following integral is convergent or divergent. Justify your answer.

$$\int_0^5 \frac{dx}{x-2}$$

7. (i) (*4 points*) Set up, *but do not evaluate*, an integral to find the area of the region enclosed by the curves $y = x^2$ and $x = y$.

(ii) (*4 points*) Set up, *but do not evaluate*, the integral to find the volume of the solid obtained by revolving the region bounded by $y = x^2$ and $x = y$ around the line $x = -1$.

8. (*10 points*) Find the 16^{th} derivative of the function

$$f(x) = \frac{x}{1 - 2x^5}$$

at $x = 0$ (i.e. find $f^{(16)}(0)$).
Hint: Don't attempt to compute 16 derivatives of f. Please don't. Instead, think about the formula for the coefficients c_n of a Maclaurin series.

9. (*10 points*) Solve the differential equation

$$\frac{dy}{dx} + 2xy = y$$

subject to the initial condition $y(0) = 2$. Be sure to solve for y in terms of x. For simplicity, assume that y is always positive.

10. (*6 points*) Find the sum of the convergent series

$$\sum_{n=1}^{\infty} n \left(\frac{1}{4}\right)^{n-1}$$