**Northwestern University** Name: _____

# Math 224-0 Midterm Exam

## Summer 2014

## Thursday, July 31, 2014

### Instructions

- Read each problem carefully.
- Write legibly and make sure your final answers are clearly indicated.
- Show all of your work on these sheets.
- The exam consists of 5 problems.
- You may not use books, notes or calculators.
- You have 1 hour to complete this exam.
- Good luck!!

| Problem | Points | Score |
|---|---|---|
| 1 | 16 | |
| 2 | 20 | |
| 3 | 20 | |
| 4 | 24 | |
| 5 | 20 | |
| Total | 100 | |

You may use the following formulas:

$$\sum_{i=1}^{n} 1 = n$$

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{i=1}^{n} i^3 = \left[\frac{n(n+1)}{2}\right]^2$$

1. (*16 points*) Evaluate the indefinite integral

$$\int \frac{8x+1}{x^2+x-2}\,dx$$

2. (*20 points*) Evaluate the indefinite integral

$$\int \sqrt{4 - x^2}\, dx$$

You might find useful the trig identities

$$\begin{aligned} \sin(2\theta) &= 2\sin\theta\cos\theta \\ \cos^2\theta &= \frac{1+\cos(2\theta)}{2} \\ \sin^2\theta &= \frac{1-\cos(2\theta)}{2} \end{aligned}$$

3. (i) (*10 points*) Determine whether the improper integral

$$\int_1^\infty \frac{4}{e^x}\,dx$$

is convergent or divergent.

(ii) (*10 points*) Determine whether the improper integral

$$\int_1^\infty \frac{4}{e^x + x}\,dx$$

is convergent or divergent. *Hint:* Use the comparison theorem, and compare it to the integral in part (i).

4. Evaluate the following integrals:
   (i) (*8 points*)

$$\int \frac{x}{\sqrt{x+10}}\,dx$$

   (ii) (*6 points*)

$$\int \cos(3\theta)\sin(\sin(3\theta))\,d\theta$$

(iii) (*10 points*)

$$\int_1^e x^2 \ln x \, dx$$

5. (*20 points*) Evaluate the definite integral

$$\int_0^3 (x^2 - 1)\, dx$$

using the definition of the integral with equal subdivisions and sample points at right endpoints.