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diary on
format compact
%Johnny Li
%EEL3135 Fall 2018
%Lab 6 Part 1

%1.1.1

%Show by hand the frequency response for the 4-point running average
%operator.
```

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P-3

Lab 6 Part 1

1.1.1) Euler formulas

$$y[n] = \frac{1}{L}\sum_{k=0}^{L-1} x[n-k] = \frac{1}{4}\sum_{k=0}^{4-1} x[n-k] = \frac{1}{4}\sum_{k=0}^{3} x[n-k]$$

$$= \frac{1}{4}\left[x[n]+x[n-1]+x[n-2]+x[n-3]\right]$$

$$Y(e^{j\hat\omega}) = \frac{1}{4}\left[X(e^{j\hat\omega}) + X(e^{j\hat\omega})e^{-j\hat\omega} + X(e^{j\hat\omega})e^{-j2\hat\omega} + X(e^{j\hat\omega})e^{-j3\hat\omega}\right]$$

$$= \frac{1}{4}X(e^{j\hat\omega})\left[1+e^{-j\hat\omega}+e^{-j2\hat\omega}+e^{-j3\hat\omega}\right]$$

$$H(e^{j\hat\omega}) = \frac{Y(e^{j\hat\omega})}{X(e^{j\hat\omega})} = \frac{1}{4}\left[1+e^{-j\hat\omega}+e^{-j2\hat\omega}+e^{-j3\hat\omega}\right]$$

$$= \frac{1}{4}\left[e^{j1.5\hat\omega}+e^{j0.5\hat\omega}+e^{-j0.5\hat\omega}+e^{-j1.5\hat\omega}\right]$$

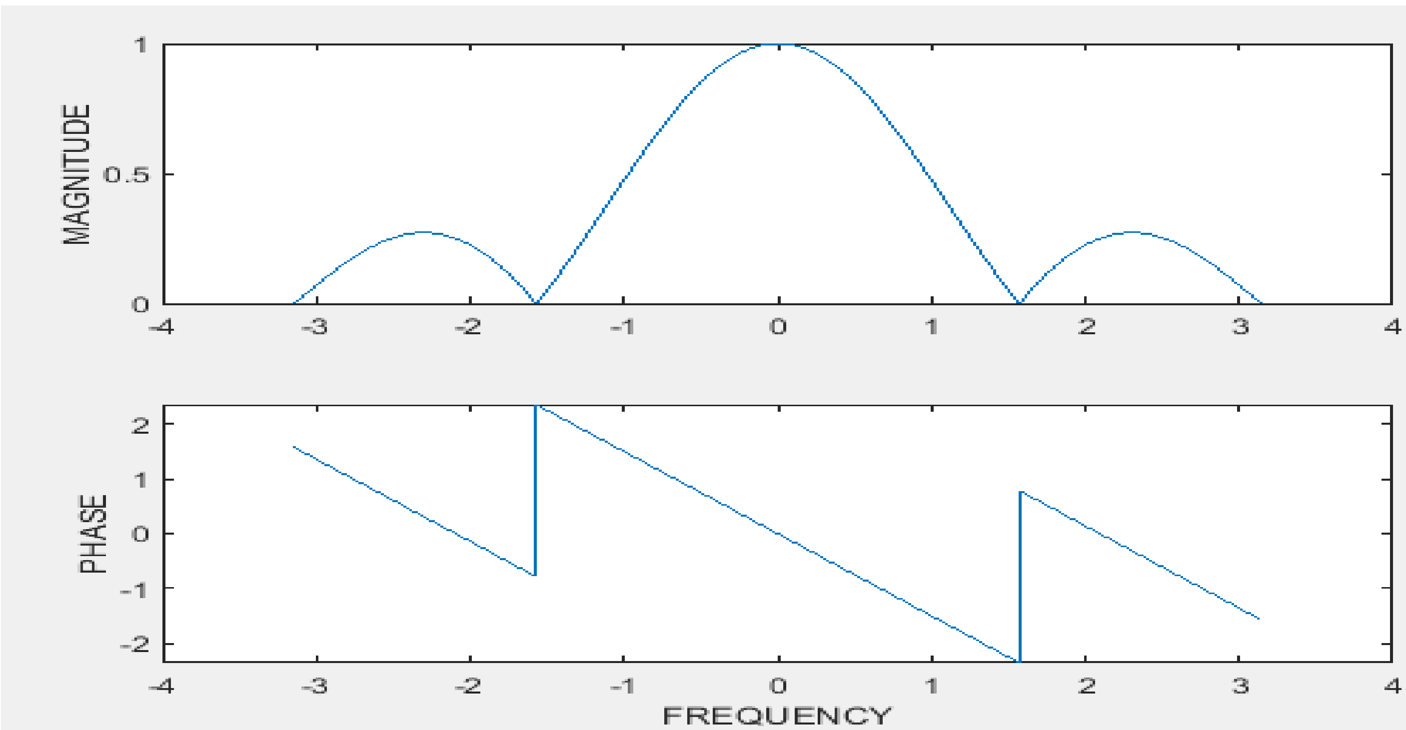
$$= \frac{1}{4}\left[2\left(\frac{e^{j1.5\hat\omega}+e^{-j1.5\hat\omega}}{2}\right)+2\left(\frac{e^{j0.5\hat\omega}+e^{-j0.5\hat\omega}}{2}\right)\right]$$

$$\boxed{H(e^{j\hat\omega}) = \frac{e^{-j1.5\hat\omega}}{4}\left[2\cos(0.5\hat\omega)+2\cos(1.5\hat\omega)\right]}$$

```
%1.1.2

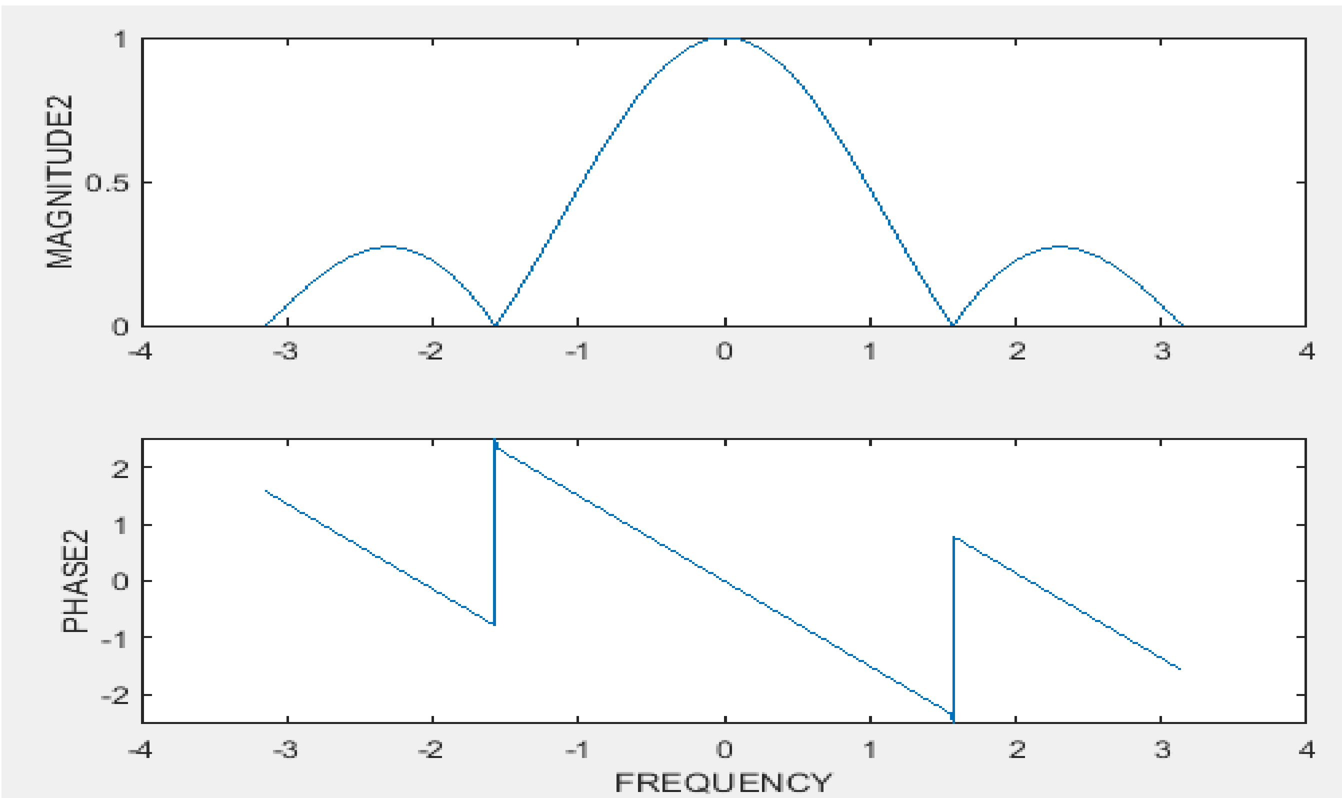
%Implement 4-point running average operator directly in code and plot the
%magnitude and phase response of this filter, w is a vector that includes
%400 samples between -pi and pi.
%Summation
bb = 1/4.*ones(1,4);
%Sample frequency
fs=400;
%Time Interval between -pi and pi
ww = -pi:(pi/fs):pi;
%Signal Function
H=((2*cos(0.5*ww)+2*cos(1.5*ww))/4).*exp(-j*1.5*ww);
```

```
%Plot
%Magnitude
subplot(2,1,1)
plot( ww, abs(H) )
ylabel("MAGNITUDE")
%Phase
subplot(2,1,2)
plot( ww, angle(H) )
ylabel("PHASE")
xlabel("FREQUENCY")
```



```
%1.1.3

%Plot the magnitude and phase response of this filter with freqz.
H = freqz( bb, 1, ww );
subplot(2,1,1)
plot( ww, abs(H) )
ylabel("MAGNITUDE2")
%Phase
subplot(2,1,2)
plot( ww, angle(H) )
ylabel("PHASE2")
xlabel("FREQUENCY")
```



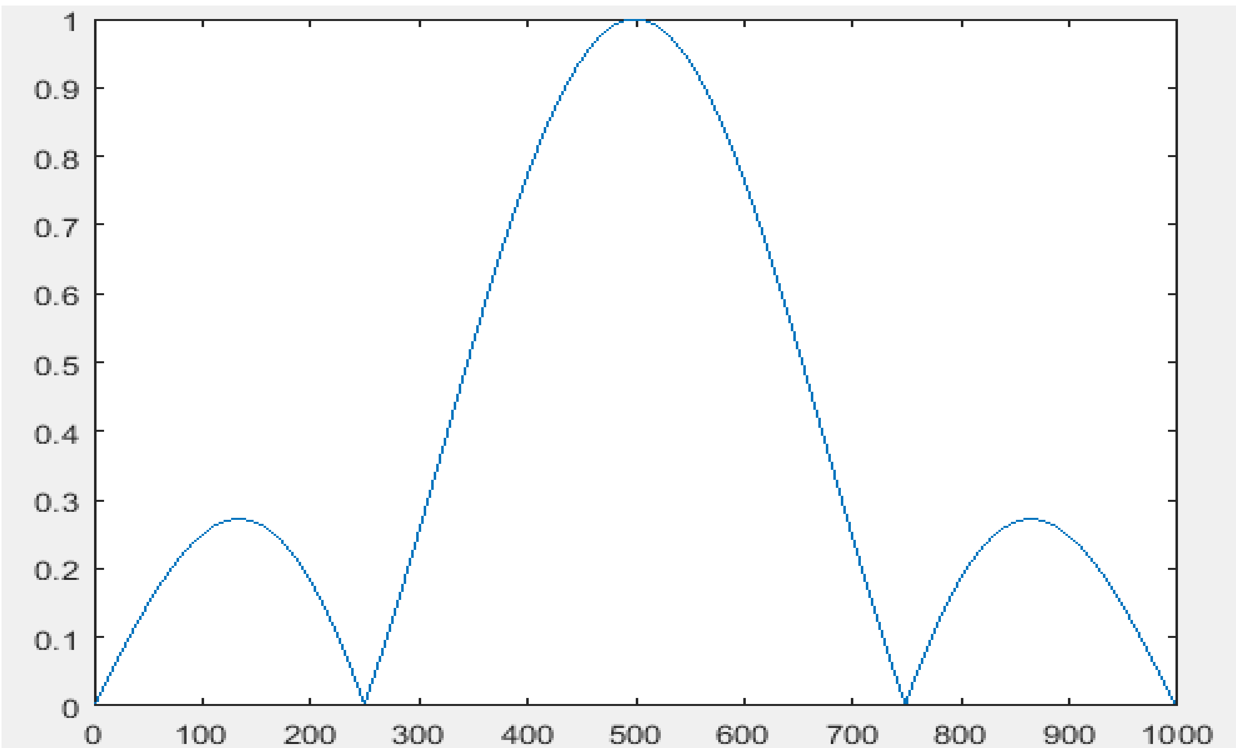

```
%1.2

%Display the list of frequencies where H is approximately zero.
%Summation
bb = ones(1,4) / 4;
%Time Interval between -pi and pi with sampling frequency of 500 Hz.
ww = -pi:pi/500:pi;
%Signal Function with freqz function.
H = freqz(bb,1,ww);

%Logical Condiction
index = find(abs(H) > 0.001);
H(index);
```

```
%Approximately 997 indexs equals to zeros.


%Question: Does this match the frequency response that you plotted for the
%4-point average?
%Yes, this frequency response matches the frequency response for the 4-
%point average as seen in the plot, same shape.
plot(abs(H(index)));
```




```
%1.3.1

%Design a filtering system that consists of the cascade of two FIR nulling
%filters that will eliminate the following input frequencies: w1=0.44pi,
%and w2=0.7pi.
%Input frequencies.
w1=0.44*pi;
w2=0.7*pi;

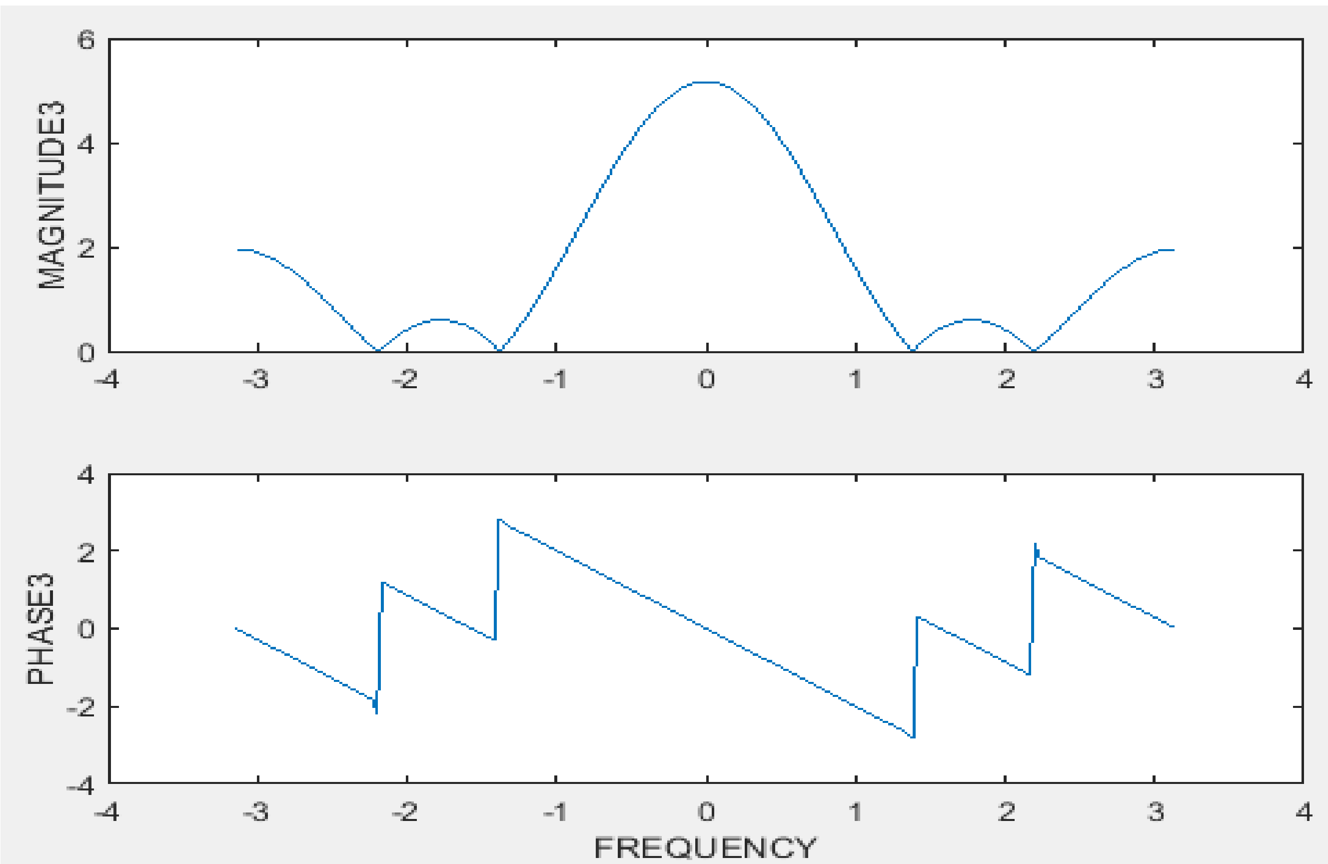
%Nulling Filter
bb1 = [1 -2*cos(w1) 1];
bb2 = [1 -2*cos(w2) 1];

%Cascade by convolution
bc=conv(bb1,bb2);

%Time intervale with sample frequency of 100 Hz
ww=-pi:pi/100:pi;
```

```
%Frequency Response of convolution
H=freqz(bc,1,ww);

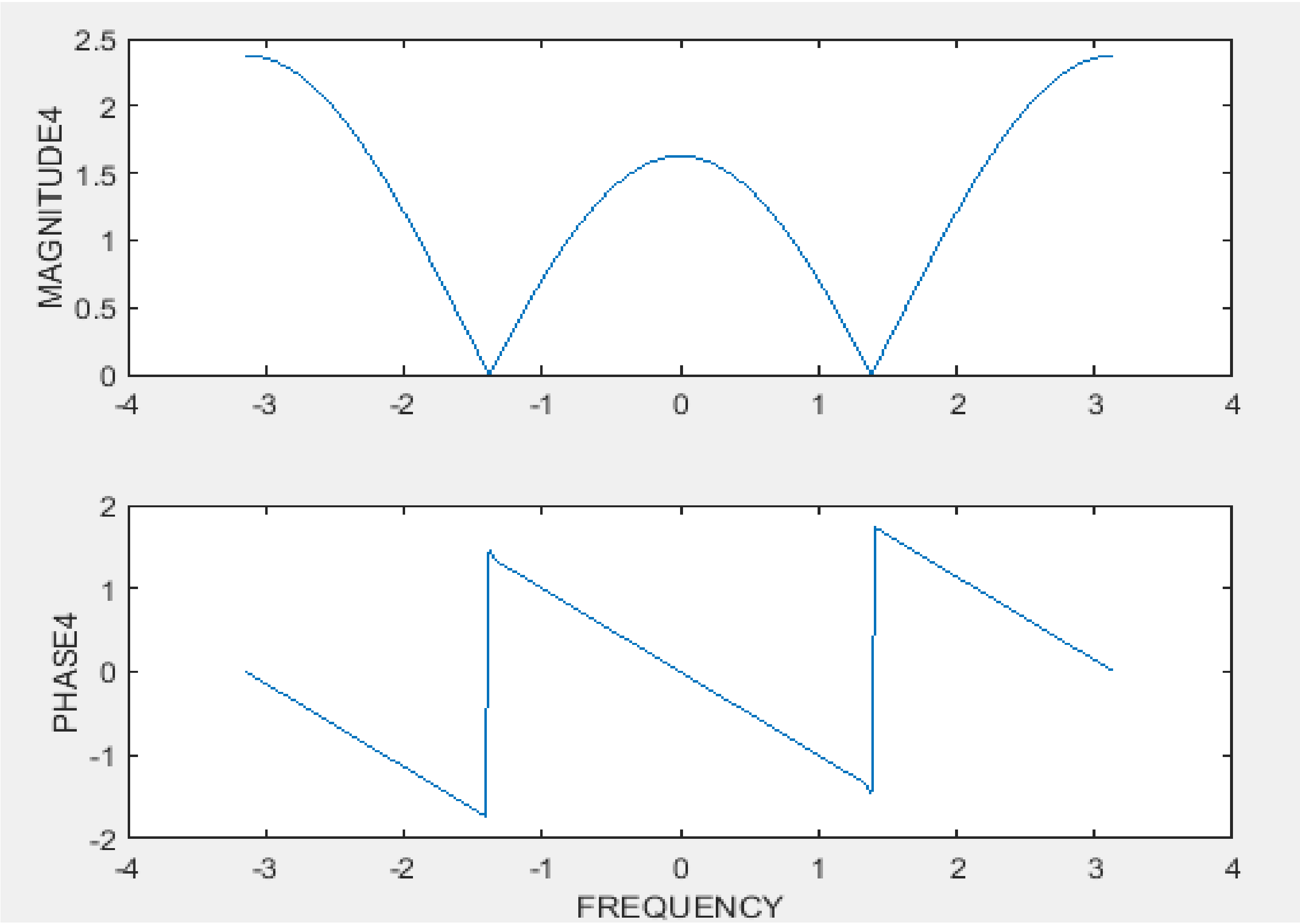
%Plot convolution magnitude and phase response of this filter.
subplot(2,1,1)
plot( ww, abs(H) )
ylabel("MAGNITUDE3")
%Phase
subplot(2,1,2)
plot( ww, angle(H) )
ylabel("PHASE3")
xlabel("FREQUENCY")
```




```
%Frequency Response of nulling filter 1 (0.44*pi)
H=freqz(bb1,1,ww);

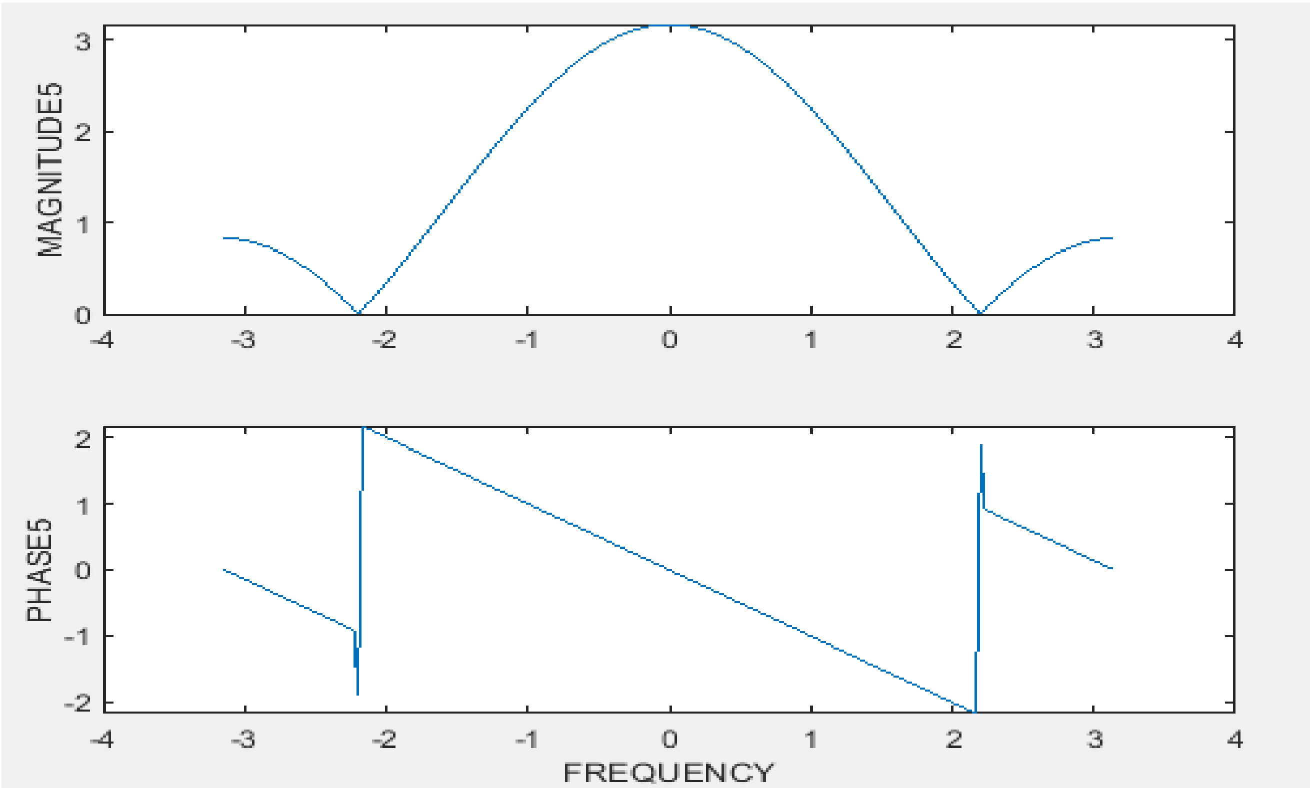
%Plot nulling filter 1 (0.44*pi) magnitude and phase response of this
%filter.
subplot(2,1,1)
plot( ww, abs(H) )
ylabel("MAGNITUDE4")
%Phase
subplot(2,1,2)
plot( ww, angle(H) )
ylabel("PHASE4")
```

```
xlabel("FREQUENCY")
```




```
%Frequency Response of nulling filter 1 (0.7*pi)
H=freqz(bb2,1,ww);

%Plot nulling filter 1 (0.7*pi) magnitude and phase response of this
%filter.
subplot(2,1,1)
plot( ww, abs(H) )
ylabel("MAGNITUDE5")
%Phase
subplot(2,1,2)
plot( ww, angle(H) )
ylabel("PHASE5")
xlabel("FREQUENCY")
```




```
%1.3.2

%Generate an input signal x[n] that is the sum of three sinusoids:
%x[n]=5cos(0.3pin)+22cos(0.44pin-pi/3)+22cos(0.7pin-pi/4) when 150 samples
%long over the range 0<=pi<=149.
%Range
n=0:149;
%Sum of three sinusoids
xn = 5*cos(0.3*pi*n)+22*cos((0.44*pi*n)-(pi/3))+22*cos((0.7*pi*n)-(pi/4));

%1.3.3

%Filter the sum-of-three-sinusoids signal x[n] through the filters designed
%previously.
%Input frequencies.
w1=0.44*pi;
w2=0.7*pi;

%Nulling Filter
bb1 = [1 -2*cos(w1) 1];
bb2 = [1 -2*cos(w2) 1];

%Cascade by convolution
bc=conv(bb1,bb2);

%Time intervale
n=0:149;
```

```
%Frequency Response of convolution
H=freqz(bc,1,n);

%Sum of three sinusoids
xn = 5*cos(0.3*pi*n)+22*cos((0.44*pi*n)-(pi/3))+22*cos((0.7*pi*n)-(pi/4));

%Result of filtering xn through the filter.
yy=H.*xn;

%1.3.4

%Make a plot of the first 50 points of the output signal. Determine by hand
%the exact mathematical formula for the output signal for n>=5.
%Plot nulling filter 1 (0.7*pi) magnitude and phase response of this
%filter.
%Input frequencies.
w1=0.44*pi;
w2=0.7*pi;

%Nulling Filter
bb1 = [1 -2*cos(w1) 1];
bb2 = [1 -2*cos(w2) 1];

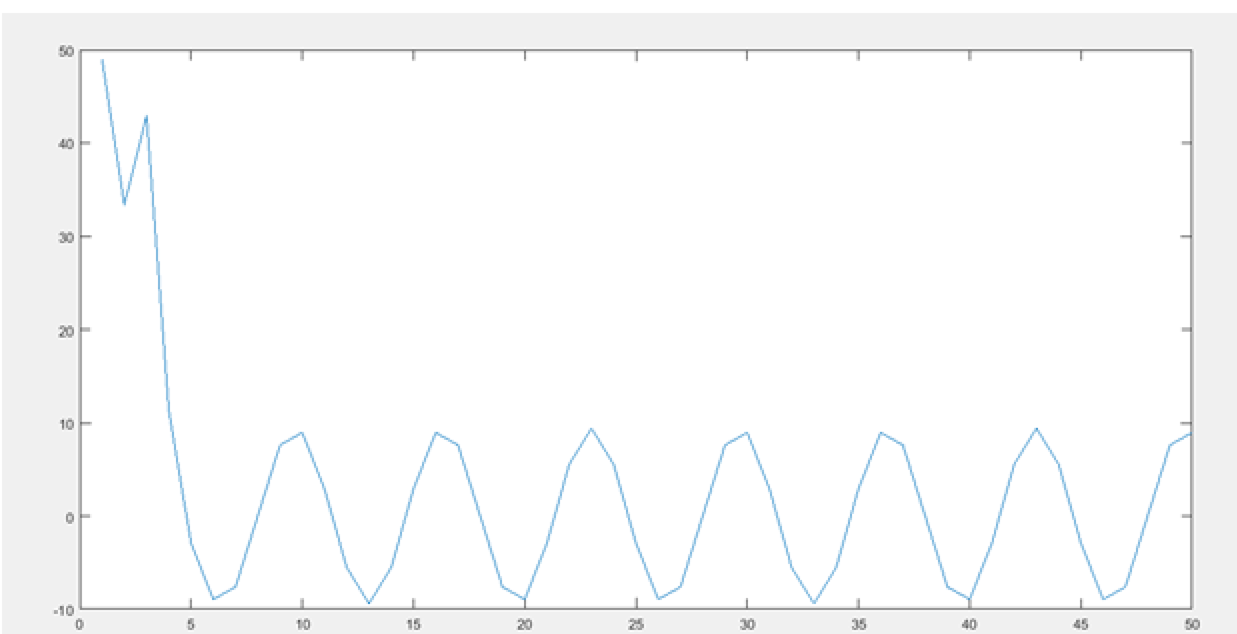
%Cascade by convolution
bc=conv(bb1,bb2);

%Time interval
n=5:50;

%Frequency Response of convolution
H=freqz(bc,1,n);

%Sum of three sinusoids
xn = 5*cos(0.3*pi*n)+22*cos((0.44*pi*n)-(pi/3))+22*cos((0.7*pi*n)-(pi/4));

%Result of filtering xn through the filter.
yy=conv(H,xn);
plot( n, yy )
```



1.3.4) The convolution of bb_1 and bb_2.

$$bb_1 = x[n] - 2\cos(0.44\pi n)\,x[n-1] + x[n-2]$$
$$bb_2 = x[n] - 2\cos(0.7\pi n)\,x[n-1] + x[n-2]$$
$$bc = bb_1 * bb_2$$
$$= (x[n] - 2\cos(0.44\pi n)x[n-1] + x[n-2])(x[n] - 2\cos(0.7\pi)x[n-1] + x[n-2])$$
$$= x[n] - 2\cos(0.44\pi)x[n-1] + x[n-2] - 2\cos(0.7\pi)x[n-1]$$
$$+ 4\cos\left(\tfrac{7\pi}{10}\right)\cos\left(\tfrac{11\pi}{25}\right)x[n-2] - 2\cos(0.7\pi)x[n-3] + x[n-2]$$
$$- 2\cos(0.44\pi)x[n-3] + x[n-4]$$
$$bc = x[n] + \left(-2\cos\left(\tfrac{11\pi}{25}\right) + \sqrt{\tfrac{5-\sqrt{5}}{2}}\,x[n-1]\right) + \left(2 - \sqrt{2}\cos\left(\tfrac{11\pi}{50}\right)\sqrt{5-\sqrt{5}}\right)x[n-2]$$
$$- 2\cos\left(\tfrac{11\pi}{25}\right) + \sqrt{\tfrac{5-\sqrt{5}}{2}}\,x[n-3] + x[n-4]$$

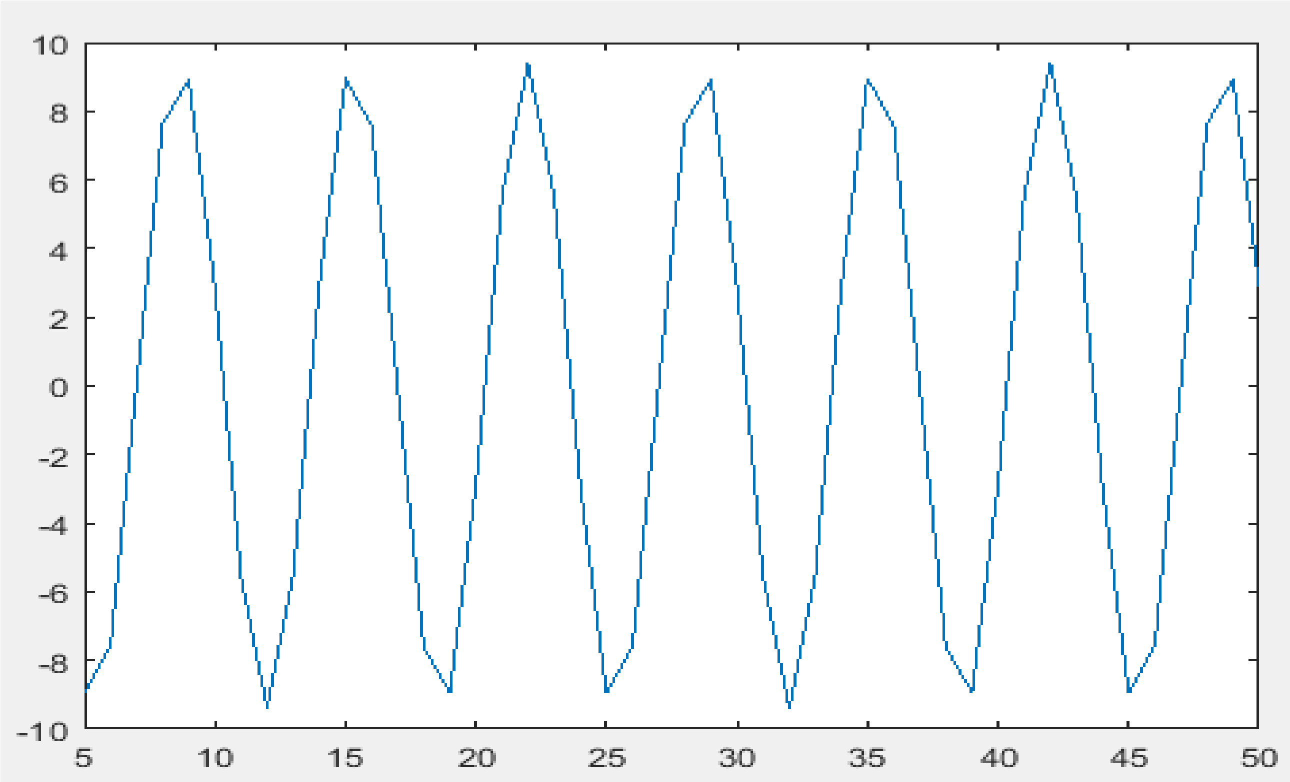
The convultion of bc and xn.

$$xn = 5\cos(0.3\pi n) + 22\cos((0.44\pi n) - \pi/3) + 22\cos(0.7\pi n - \pi/4));$$

$$yy = bc * xn = \left(x[n] + \left(-2\cos\left(\tfrac{11\pi}{25}\right) + \sqrt{\tfrac{5-\sqrt{5}}{2}}\,x[n-1]\right) + 2 - \sqrt{2}\cos\left(\tfrac{11\pi}{50}\right)\right.$$
$$\left.\sqrt{5-\sqrt{5}}\,x[n-2] - 2\cos\left(\tfrac{11\pi}{25}\right) + \sqrt{\tfrac{5-\sqrt{5}}{2}}\,x[n-3] + x[n-4]\right)$$
$$\left(5\cos(0.3\pi n) + 22\cos\left((0.44\pi n - \tfrac{\pi}{3}\right) + 22\cos\left(0.7\pi n - \tfrac{\pi}{4}\right)\right)$$
$$\boxed{= 9.4\cos(0.3\pi n - 1.88)}$$

```
%1.3.5

%Plot the mathematical formula determined compare it to the plot obtained
%previously to show that it matches the filter output over the range
%5<=n<=50.
%Time interval
n=5:50;
yy=9.4*cos(0.3*pi*n-1.88);
plot(n,yy);
```




```
%The plot from the formula does not match the plot produced previously.

%1.3.6

%Explain why the output signal is different for the first few points.
%The output signal is different for the first few points because the
%nulling filter takes in samples of 5 points where the values of these
%points can skew the initials results of the signal.

%How many "start-up" points are found?
%As seen in the beginning, there are 5 "start-up" points where the
%differences in voltages are more noticeable/ less stable.

%How is this number related to the lengths of the filters designed
%previously?
%This length is the length of the two filters designed summed together and
%subtracting one.
```