Computer Project 3

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1) $\frac{dy}{dx} = x^2 + y \quad y(-2) = 2$

a) Picard's Theorem

$f(x,y) = x^2 + y$

$f(-2,-2) = (-2)^2 + (-2) = 4 + (-2) = 2 \rightarrow$ No discontinuous / asymptotes

If $f(t,y)$ is continuous near (t_0, y_0) a solution exists. ✓

$\left|\frac{df}{dy}\right| = |1| = 1 \leq$ bound by a constant for all (x,y) $\{(x,y) ; -2 \leq x \leq 2 \text{ and } -\infty < y < \infty\}$

If $\frac{df}{dy}$ is also continuous near (t_0, y_0), solution is unique. ✓

$\left.\frac{df}{dy}\right|_{(-2,-2)} 1 \rightarrow$ continuous near (t_0, y_0)

$\therefore$ The IVP has a unique solution.

b) $y(x) = -2 - 2x - x^2$ is a solution to the IVP.

$y'(x) = x^2 + y(x) \rightarrow -2 - 2x = \cancel{x^2} + (-2 - 2x - \cancel{x^2}) = -2 - 2x$ ✓ RHS matches LHS.

$y(-2) = -2 - 2(-2) - (-2)^2 = -2 \cancel{+4} \cancel{-4} = -2$ ✓

Match Initial condition of $y(-2) = -2$

$\therefore y(x) = -2 - 2x - x^2$ is a solution to the IVP.

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