



COMILLAS

UNIVERSIDAD PONTIFICIA

ICAI

**GRADO EN INGENIERÍA DE TECNOLOGÍAS
INDUSTRIALES (GITI)**

TRABAJO FIN DE GRADO

**MODE SEPARATED SUSPENSION SYSTEM FOR A
FORMULA SAE CAR**

Autor: Álvaro Martínez Cuenca

Director: Leonard J. Hamilton

Madrid

Julio de 2019

AUTHORIZATION FOR DIGITALIZATION, STORAGE AND DISSEMINATION IN THE NETWORK OF END-OF-DEGREE PROJECTS, MASTER PROJECTS, DISSERTATIONS OR BACHILLERATO REPORTS

1. Declaration of authorship and accreditation thereof.

The author Mr. /Ms. Álvaro Martínez Cuenca

HEREBY DECLARES that he/she owns the intellectual property rights regarding the piece of work:
MODE SEPARATED SUSPENSION SYSTEM FOR A FORMULA SAE CAR
that this is an original piece of work, and that he/she holds the status of author, in the sense granted by the Intellectual Property Law.

2. Subject matter and purpose of this assignment.

With the aim of disseminating the aforementioned piece of work as widely as possible using the University's Institutional Repository the author hereby **GRANTS** Comillas Pontifical University, on a royalty-free and non-exclusive basis, for the maximum legal term and with universal scope, the digitization, archiving, reproduction, distribution and public communication rights, including the right to make it electronically available, as described in the Intellectual Property Law. Transformation rights are assigned solely for the purposes described in a) of the following section.

3. Transfer and access terms

Without prejudice to the ownership of the work, which remains with its author, the transfer of rights covered by this license enables:

- a) Transform it in order to adapt it to any technology suitable for sharing it online, as well as including metadata to register the piece of work and include "watermarks" or any other security or protection system.
- b) Reproduce it in any digital medium in order to be included on an electronic database, including the right to reproduce and store the work on servers for the purposes of guaranteeing its security, maintaining it and preserving its format.
- c) Communicate it, by default, by means of an institutional open archive, which has open and cost-free online access.
- d) Any other way of access (restricted, embargoed, closed) shall be explicitly requested and requires that good cause be demonstrated.
- e) Assign these pieces of work a Creative Commons license by default.
- f) Assign these pieces of work a HANDLE (*persistent URL*). by default.

4. Copyright.

The author, as the owner of a piece of work, has the right to:

- a) Have his/her name clearly identified by the University as the author
- b) Communicate and publish the work in the version assigned and in other subsequent versions using any medium.
- c) Request that the work be withdrawn from the repository for just cause.
- d) Receive reliable communication of any claims third parties may make in relation to the work and, in particular, any claims relating to its intellectual property rights.

5. Duties of the author.

The author agrees to:

- a) Guarantee that the commitment undertaken by means of this official document does not infringe any third party rights, regardless of whether they relate to industrial or intellectual property or any other type.

- b) Guarantee that the content of the work does not infringe any third party honor, privacy or image rights.
- c) Take responsibility for all claims and liability, including compensation for any damages, which may be brought against the University by third parties who believe that their rights and interests have been infringed by the assignment.
- d) Take responsibility in the event that the institutions are found guilty of a rights infringement regarding the work subject to assignment.

6. Institutional Repository purposes and functioning.

The work shall be made available to the users so that they may use it in a fair and respectful way with regards to the copyright, according to the allowances given in the relevant legislation, and for study or research purposes, or any other legal use. With this aim in mind, the University undertakes the following duties and reserves the following powers:

- a) The University shall inform the archive users of the permitted uses; however, it shall not guarantee or take any responsibility for any other subsequent ways the work may be used by users, which are non-compliant with the legislation in force. Any subsequent use, beyond private copying, shall require the source to be cited and authorship to be recognized, as well as the guarantee not to use it to gain commercial profit or carry out any derivative works.
- b) The University shall not review the content of the works, which shall at all times fall under the exclusive responsibility of the author and it shall not be obligated to take part in lawsuits on behalf of the author in the event of any infringement of intellectual property rights deriving from storing and archiving the works. The author hereby waives any claim against the University due to any way the users may use the works that is not in keeping with the legislation in force.
- c) The University shall adopt the necessary measures to safeguard the work in the future.
- d) The University reserves the right to withdraw the work, after notifying the author, in sufficiently justified cases, or in the event of third party claims.

Madrid, on 11..... of July....., 2019.

HEREBY ACCEPTS

Signed..... Álvaro Martínez Cuenca



Reasons for requesting the restricted, closed or embargoed access to the work in the Institution's Repository

--

I, hereby, declare that I am the only author of the project report with title:

MODE SEPARATED SUSPENSION SYSTEM FOR A
FORMULA SAE CAR

which has been submitted to ICAI School of Engineering of Comillas Pontifical University in the academic year 2019/07. This project is original, has not been submitted before for any other purpose and has not been copied from any other source either fully or partially. All information sources used have been rightly acknowledged.



Fdo.: Álvaro Martínez Cuenca Date: 10/07/2019

I authorize the submission of this project

PROJECT SUPERVISOR



Fdo.: Leonard J. Hamilton Date: 12/07/2019



COMILLAS

UNIVERSIDAD PONTIFICIA

ICAI

**GRADO EN INGENIERÍA DE TECNOLOGÍAS
INDUSTRIALES (GITI)**

TRABAJO FIN DE GRADO

**MODE SEPARATED SUSPENSION SYSTEM FOR A
FORMULA SAE CAR**

Autor: Alvaro Martinez Cuenca

Director: Leonard J. Hamilton

Madrid

Julio de 2019

SUSPENSIÓN SEPARADORA DE MODOS PARA UN VEHÍCULO DE FORMULA SAE

Autor: Martínez Cuenca, Álvaro

Director: Hamilton, Leonard J.

Entidad colaboradora: Universidad de Maryland

RESUMEN DEL PROYECTO

Introducción

Para mejorar el rendimiento del coche “TR19” desarrollado por el equipo TERPS Racing de la Universidad de Maryland, el desarrollo de un nuevo sistema de suspensiones fue considerado. El sistema de suspensión denominado como Separador de Modos tiene como objetivo conseguir diferentes características de amortiguación y rigidez en los muelles dependiendo del movimiento del vehículo. Cualquier vehículo tiene tres principales movimientos: Cabeceo (Pitch), Balanceo (Roll) y Giro (Yaw). Estos dos últimos se pueden relacionar con el balanceo, ya que el giro del coche provoca que el cuerpo se balancee sobre su eje longitudinal. Este movimiento se produce cuando el coche toma una curva, lo que produce que el chasis del vehículo gire con respecto a su eje longitudinal. El Cabeceo es el movimiento hacia arriba o hacia debajo de cualquiera de los ejes delantero o trasero del vehículo. Esta situación se da sobre todo cuando el coche acelera o frena. Otro movimiento a destacar se produce cuando el vehículo se encuentra con una imperfección en la superficie que afecta a una de las ruedas del eje únicamente, que debe ser absorbido por el sistema de suspensión.

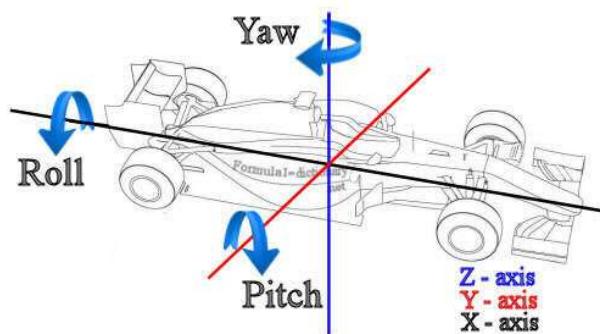


Figure 4.2.1: Basic motions of a car, from source FORM19

Para un rendimiento óptimo del vehículo, estos tres posibles escenarios han de tenerse en cuenta en el diseño del sistema de suspensión. Idealmente, las características óptimas de rigidez de los muelles y amortiguamiento son diferentes para estos tres movimientos. Un sistema de suspensión convencional, al tener un muelle y amortiguador por cada rueda que actúan sin distinguir el movimiento del coche, necesita hacer un balance entre las diferentes características requeridas en cada movimiento. Además,

el Sistema Separador de Modos permite una mayor capacidad de ajuste de los diferentes reglajes de la suspensión, adaptándose mejor a las características de cada circuito y las preferencias del piloto.



Figure 3.1: TR19 from TERPS Racing, front view (TERP19)

Para el diseño de un sistema de suspensión adecuado a las características del “TR19”, debía considerarse el hecho de que el vehículo tiene un chasis monocasco es su parte delantera y uno tubular en la parte posterior. Además, se debía analizar el espacio disponible para el emplazamiento del mismo. El monocasco se había diseñado con una hendidura a la altura del eje delantero para colocar el anterior sistema de suspensión, que debía aprovecharse para no comprometer la visibilidad del piloto y la eficiencia aerodinámica. En el caso de la parte posterior, la conexión del sistema de suspensión al chasis debía ser mediante soldadura, y evitando la interferencia con el diferencial, motor y elementos aerodinámicos.

El diseño está basado en el sistema de suspensión presente en el AMG Mercedes Project One, desarrollado por Mercedes Benz y presentado a finales de 2017, el cual puede observarse en la Figura 1.1 y tomado de la fuente PERK17, que puede encontrarse en el Apéndice I.

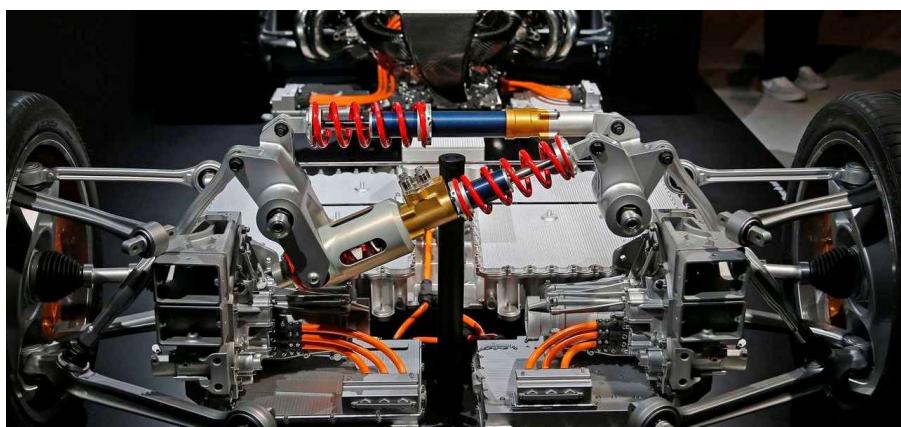


Figure 1.1: Mercedes Project One suspension system (PERK17)

Para explicar su funcionamiento bajo diferentes movimientos se utilizará el esquema desarrollado para adecuar esta suspensión a la parte delantera del TR19, que será explicado más adelante. En la Figura 4.1.1 se muestra el anteriormente mencionado esquema en su posición de equilibrio, con sus diferentes partes nombradas.

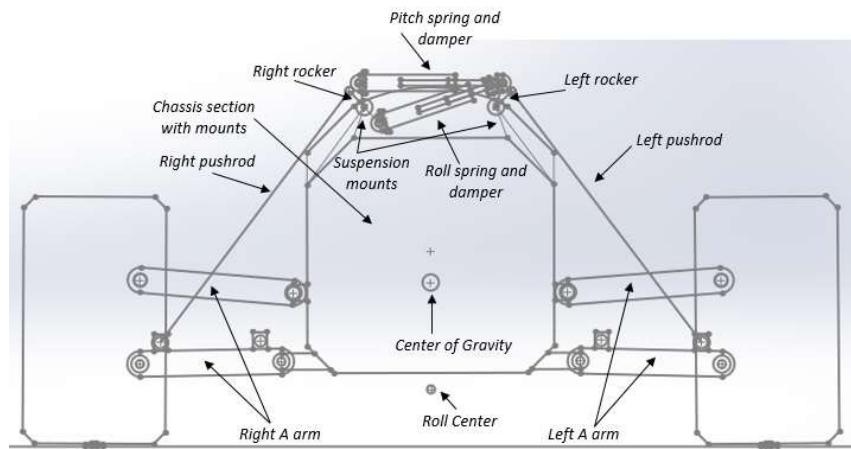


Figure 4.1.1: Sketch of the front view of the front suspension system with its main components

- El primer movimiento analizado es el Cabeceo (pitch o ride). Como ya se ha introducido, este movimiento es provocado por la transferencia de peso entre el eje delantero y trasero durante la aceleración o frenada, lo que provoca que el Centro de gravedad en ese eje se desplace verticalmente. En la Figura 4.2.2, siguiendo el movimiento de las flechas se puede observar cómo, debido a la geometría asimétrica de las palancas que conectan los dos amortiguadores con los apoyos del chasis, el muelle colocado horizontalmente es comprimido mientras que el diagonal no sufre ninguna compresión, puesto que los dos extremos de este muelle se mueven en la misma dirección en una ratio muy similar. Esto significa que el muelle diagonal no producirá ninguna fuerza, mientras que el horizontal reaccionará todo el movimiento.

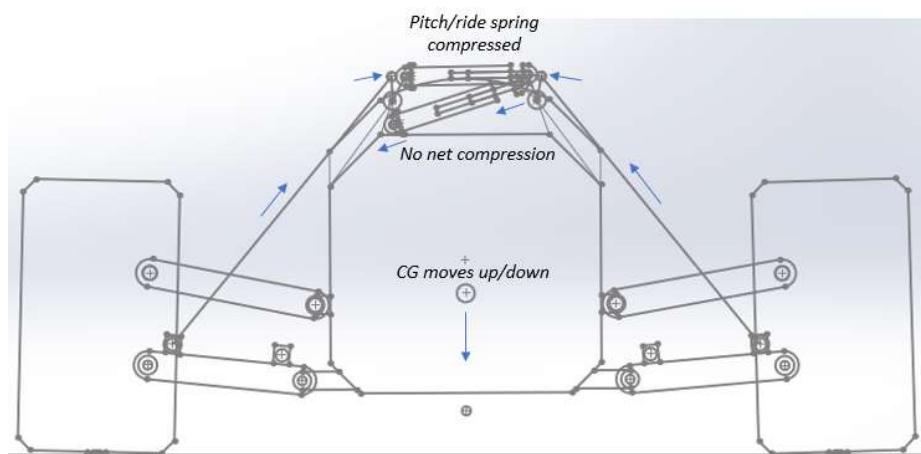


Figure 4.2.2: Suspension system motion under pitch

- En la Figura 4.2.3 se puede observar el sistema de suspensión reaccionando al movimiento de Balanceo. Al tomar una curva, el cuerpo del vehículo gira en torno al Centro de Giro, diferente para cada eje. Ya que los “pushrods” (barras que transmiten el movimiento de la rueda a los muelles) se mantienen estáticos ante este movimiento, el sistema de suspensión reacciona de la manera expresada en las flechas. Como se puede observar, el muelle horizontal no sufre ninguna compresión, mientras que el diagonal reacciona todo el movimiento, estirándose en este caso.

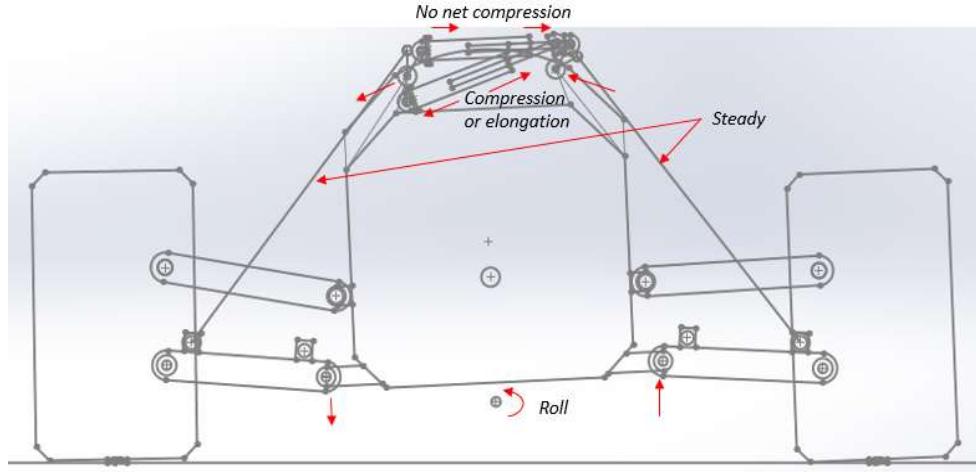


Figure 4.2.3: Suspension system motion while the vehicle is cornering to the left

- Por último, cuando el vehículo afronta un bache que hace levantar una de sus ruedas, el sistema de suspensión actúa conforme a lo reflejado en la Figura 4.2.4. En este caso, los dos muelles reaccionan conjuntamente, por lo que no se logrará aislar este movimiento, siendo dependiente de las características seleccionadas para los movimientos previamente mencionados.

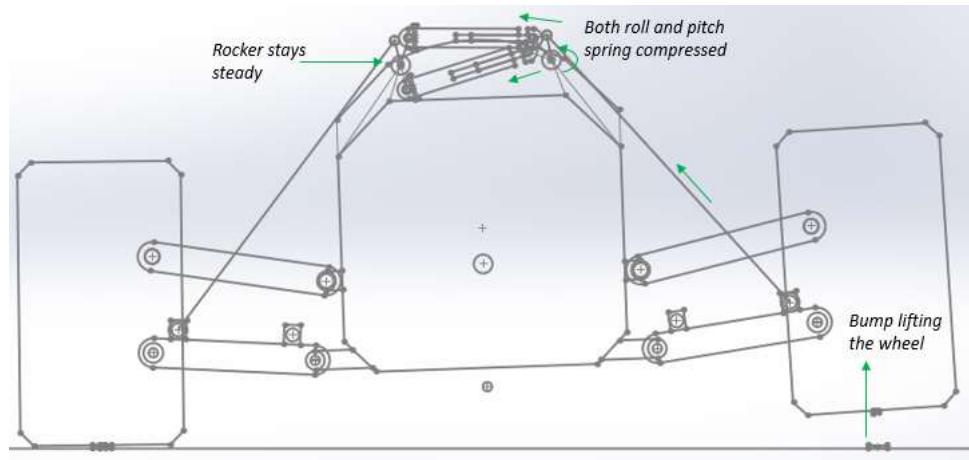


Figure 4.2.4: Suspension system motion over a single wheel bump

El objetivo del proyecto es diseñar un sistema de suspensión capaz de tener diferentes características dependiendo del movimiento del vehículo. Para lograr esto, se han de analizar los parámetros óptimos del sistema de suspensión y las fuerzas presentes para cada movimiento. Posteriormente, se usará un software adecuado que permita el diseño, simulación y análisis del modelo.

Metodología

El siguiente paso en el análisis del funcionamiento de este sistema de suspensión fue desarrollar las ecuaciones necesarias para obtener las fuerzas que actuaban en el sistema bajo diferentes movimientos, y cómo se veían afectadas por los parámetros de dureza de los muelles o amortiguamiento. Puesto que el sistema de suspensión desarrollado no es simétrico, dichas ecuaciones diferían de las de un sistema convencional (consultadas en la fuente GILL92 y OPTG19), y debían ser analizadas y recalcadas. Las fuerzas analizadas fueron: la transferencia de masa lateral, el ángulo de giro del vehículo y la transferencia de masa longitudinal.

La transferencia de carga lateral se produce cuando el vehículo toma una curva, y es debida a la reacción del vehículo a la fuerza ficticia que aparece en el CG debido a la aceleración normal que está experimentando. Esto hace que las ruedas del lado exterior de la curva aumenten su fuerza vertical, mientras que las del interior la verán disminuida. Esta fuerza se puede calcular mediante la Ecuación 1 (m: masa del vehículo, v: velocidad de paso por curva, R: radio de la curva, h: altura del CG, t: distancia entre las ruedas del mismo eje):

$$\text{Ecuación 1: } \Delta W = \frac{m \cdot \frac{v^2}{R} \cdot h}{t}$$

La distribución de dicha carga en los ejes traseros y delanteros se rige por las ecuaciones 5 y 6:

$$\text{Ecuación 5: } \Delta F_{zf} = \frac{\frac{Wf}{g} \cdot \frac{v^2}{R} \cdot hf}{tf} + \frac{k\varphi f \cdot \varphi}{tf}$$

$$\text{Ecuación 6: } \Delta F_{zr} = \frac{\frac{Wr}{g} \cdot \frac{v^2}{R} \cdot hr}{tr} + \frac{k\varphi r \cdot \varphi}{tr}$$

Siendo φ el ángulo que se desplaza el CG respecto a la vertical (Calculado en la Ecuación 3), tr y tf distancia entre las ruedas del eje trasero y delantero respectivamente, h1 la distancia del centro de giro al CG y k φ f y k φ r la rigidez de los ejes delantero y trasero respectivamente.

$$\text{Ecuación 3: } \varphi = \frac{m \cdot \frac{v^2}{R} h1}{k\varphi f + k\varphi r - mgh1}$$

Analizando las Ecuaciones 5 y 6 se puede observar cómo, cambiando la rigidez de la suspensión en un eje, se puede conseguir que dicho eje soporte más o menos carga, pudiendo variar la tendencia del vehículo a subvirar o sobrevirar. Un análisis y explicación detallados de este proceso se recoge en el proyecto. En cuanto a la carga longitudinal, se rige por la Ecuación 8 y no puede ser modificada variando los parámetros de la suspensión.

$$\text{Ecuación 8: } \Delta F_{long} = \frac{h}{L} \cdot \frac{W}{g} \cdot ax$$

A continuación, y para más tarde simular el sistema de suspensión, se realizó el cálculo de los parámetros óptimos de la rigidez de los muelles y amortiguamiento para cada movimiento. El primer

paso es calcular el Ratio de Movimiento entre el muelle y la rueda ($MR = \frac{\Delta X_{muelle}}{\Delta X_{rueda}}$) para ambos ejes y tanto para los muelles horizontales como los diagonales.

Para el muelle horizontal, para obtener la K del muelle adecuada es necesario especificar un valor de frecuencia natural. Esta frecuencia determinará el comportamiento general del coche: su rigidez, respuesta a irregularidades o altura del centro de gravedad. Para el vehículo analizado, dicha frecuencia se seleccionó como 4 Hz para la parte frontal y 3.7 Hz para la parte trasera. Dichos valores se eligieron tomando como referencia vehículos de características parecidas, y el hecho de ser diferentes para la parte trasera y delantera se debe a que, teniendo una suspensión menos rígida en la parte trasera, la eficiencia aerodinámica aumentaba (debido a que a gran velocidad en rectas el “drag” generado por el alerón trasero se reduce al estar más cercano al suelo) y se lograba una mayor aceleración.

Dicha frecuencia natural se puede relacionar con la rigidez del muelle mediante: $\omega = \frac{1}{2\pi} \cdot \sqrt{\frac{kr}{M}}$

Siendo kr la K equivalente correspondiente al rincón analizado del coche y M la masa soportada por esa rueda. Esta K equivalente representa la rigidez conjunta del neumático, kt, y la rigidez del muelle trasladada a la rueda (usando el Ratio de Movimiento), kw. Desarrollando las ecuaciones pertinentes, se puede demostrar que:

$$kr = \frac{kw \cdot kt}{kw + kt}$$

$$kw = ks \cdot MR^2$$

Por lo tanto, la rigidez para las suspensiones delantera y trasera obtenidas son: Ks delantero (ride) = 48809.3 N/m; Ks trasero (ride) = 87952.5 N/m

En cuanto al muelle diagonal, se siguió un procedimiento similar. Se debe seleccionar un parámetro de “Grado de giro (Roll Gradient, $\frac{\phi}{Ay}$)”, que expresa cuanto gira el cuerpo por cada 1G de aceleración normal. Para el coche analizado, este valor se tomó como 0.45. Se puede relacionar con la dureza de la suspensión mediante:

$$\frac{\phi}{Ay} = -W * \frac{h1}{K\phi_{total}}$$

Al igual que el caso anterior, la dureza de la suspensión para cada eje es diferente, tomando el delantero un 52.5% de la dureza total y el trasero el 47.5% restante. K ϕ se puede expresar como (siendo t la distancia entre ruedas del mismo eje):

$$K\phi = Kr \cdot \frac{\pi}{180} \cdot \frac{t^2}{2} \text{ (tanto para el eje delantero como para el trasero)}$$

Una vez obtenida Kr para cada eje, el procedimiento para obtener Ks (constante del muelle) es el mismo que el caso anterior.

Los valores obtenidos son: Ks delantero (roll) = 44588.3 N/m; Ks trasero (roll) = 81424.9 N/m. Como se puede observar, estos parámetros difieren del caso de Cabeceo. En los sistemas de suspensión

convencionales, para conseguir durezas distintas en estos dos movimientos se usan barras estabilizadoras, pero estos elementos añaden peso y tienen muy poca capacidad de ajuste.

El otro parámetro a calcular es la C de los amortiguadores. Para ello, el primer paso es seleccionar un coeficiente de amortiguamiento adecuado. Para el vehículo analizado, se seleccionó $\zeta = 0.68$ para el amortiguador horizontal y $\zeta = 0.65$ para el diagonal. Esto fue así ya que para coches de competición no es deseable que el cuerpo oscile muchas veces con respecto a su valor de referencia, manteniendo un tiempo de alcance adecuado. Una vez elegido ζ , la obtención de la C de los amortiguadores se realiza mediante: $C = \zeta \cdot 2 \cdot \sqrt{K_r \cdot M}$. Resolviendo para cada eje y tanto para los amortiguadores horizontales como los diagonales, se tiene:

C delantero (pitch and ride): 2195.96 N/(m/s)

C trasero (pitch and ride): 3191.64 N/(m/s)

C delantero (roll): 3397.11 N/(m/s)

C trasero (roll): 3316.56 N/(m/s)

El hecho de disponer de factores de amortiguamiento distintos para dichos movimientos permite ajustar la respuesta del vehículo en todo momento. Mediante este sistema se puede lograr, por ejemplo, un amortiguamiento superior en el muelle horizontal para limitar la variación de altura del CG, para así lograr reducir su distancia al suelo (con las ventajas que esto conlleva), manteniendo una configuración más blanda para el muelle diagonal, evitando que el coche sea demasiado rígido y pierda adherencia en las curvas.

Tras realizar un esquema aproximado del sistema de suspensión, era necesario buscar un software capaz de diseñar, desarrollar y analizar un modelo del mismo. Se comenzó con uno de los complementos o “Add-ins” de Matlab, llamado “Simscape Multibody” o “Simmechanics”. Este programa permitía un diseño y análisis de estructuras 3D mediante el uso de bloques. Estos bloques representaban cuerpos, uniones, fuerzas y momentos, sensores, o restricciones físicas que podían ser unidos para crear y simular un conjunto. Puesto que era un programa muy completo, se comenzó el modelado del sistema. El problema era que, puesto que la forma y dimensiones de los elementos de la suspensión no eran conocidos, y debían diseñarse, muchas de las dimensiones debían ponerse en forma de parámetros, lo que ralentizaba y dificultaba realizar las uniones entre componentes. Puesto que el diseño a través de este programa se volvió muy complicado, se debía buscar una nueva alternativa.

Muchos de los elementos del sistema de suspensión, como los brazos que conectan la rueda al chasis (A-arms), el eje central de la rueda, con los apoyos para los brazos, el “pushrod” (barra que transmite el movimiento de la rueda a los muelles) o las llantas y neumáticos iban a mantenerse con respecto al coche anterior, además del monocasco y el chasis trasero. Todos estos elementos estaban ya diseñados en CAD en Solidworks, por lo que ahorraría trabajo utilizar esta plataforma.

Este programa tiene una herramienta, llamado Solidworks Motion Study que permite simular los ensamblajes realizados, añadiendo fuerzas, muelles y amortiguadores, restricciones de movimiento y salidas de resultados, por lo que resultaba atractiva para su utilización. Puesto que aún se debía experimentar con los diferentes elementos, y conocer sus dimensiones óptimas para su adecuado funcionamiento, un modelo 2D de la suspensión delantera y trasera fue realizado. Este modelo utiliza

en conjunto de bloques con la estructura de un croquis que pueden luego ser ensamblados como un sistema 3D de piezas.

Las medidas de los amortiguadores debían ser adecuadas para poder reutilizar los usados en el TR19 y así ahorrar en el coste total, y las dimensiones de las palancas y conexiones con el chasis debían asegurar el correcto funcionamiento del sistema y la máxima compactación posible dentro del espacio facilitado. Las medidas definitivas se recogen en las Figuras 7.2.1 y 7.2.2.

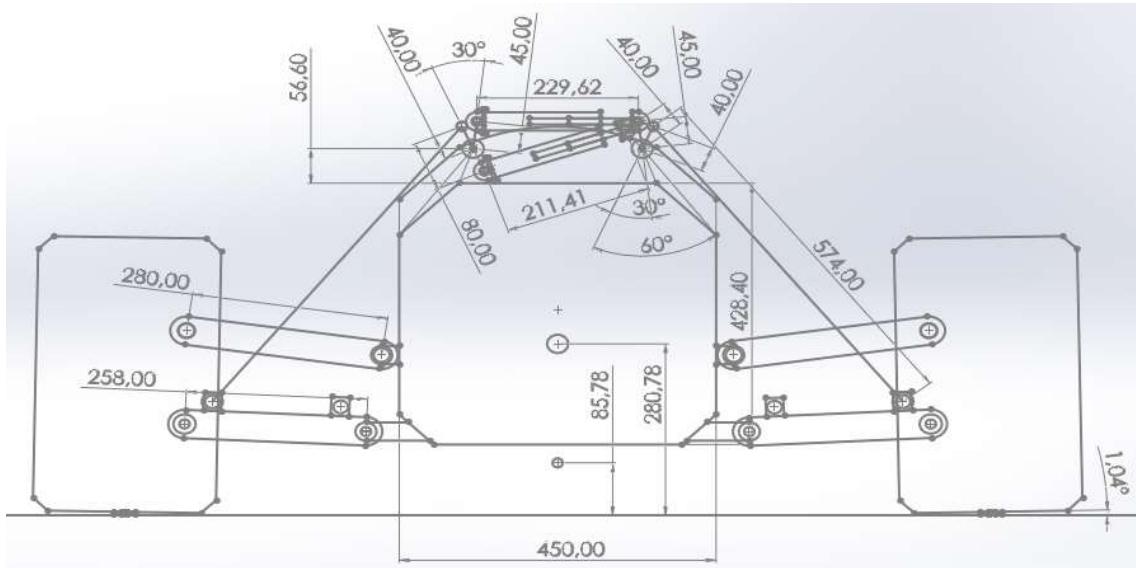


Figure 7.2.1: Final sketch of the front axle of the vehicle's suspension

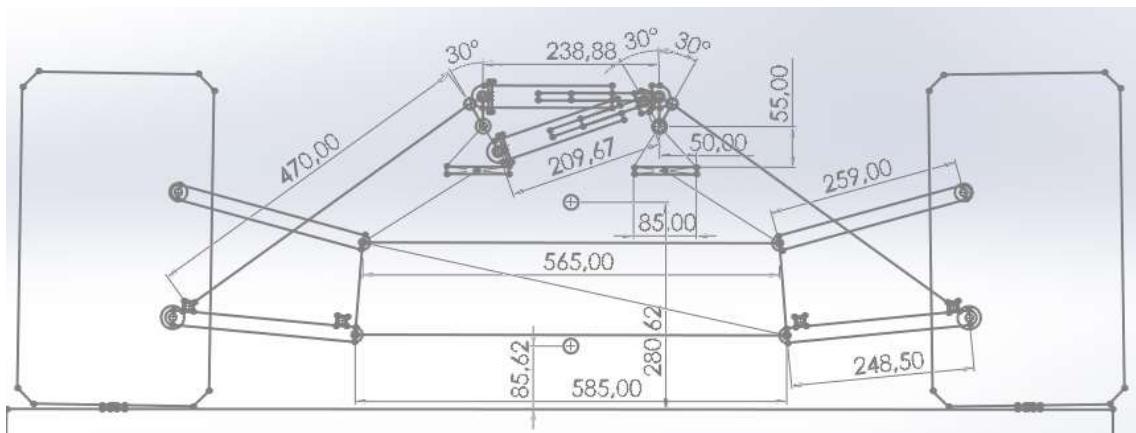


Figure 7.2.2: Final sketch of the rear axle of the vehicle's suspension

Estos modelos serían posteriormente simulados junto con otro modelo 2D de la anterior suspensión delantera.

Una vez desarrollado el esquema, se procedió al modelado en 3D de las piezas necesarias, y a su ensamblaje para comprobar la viabilidad del diseño. Para ello, se reensamblaron y simplificaron algunas de las piezas del anterior coche (recogidas en la fuente XDRI), y se crearon las piezas necesarias (montajes de la suspensión, palancas, “pushrods” y muelles). Las formas y dimensiones detalladas de estas piezas se pueden observar en los planos adjuntados. En las figuras 7.3.1 y 7.3.2 se

recoge el ensamblaje completo del coche, y en las 7.3.12 y 7.3.17 se muestra el detalle de cada una de ellas.

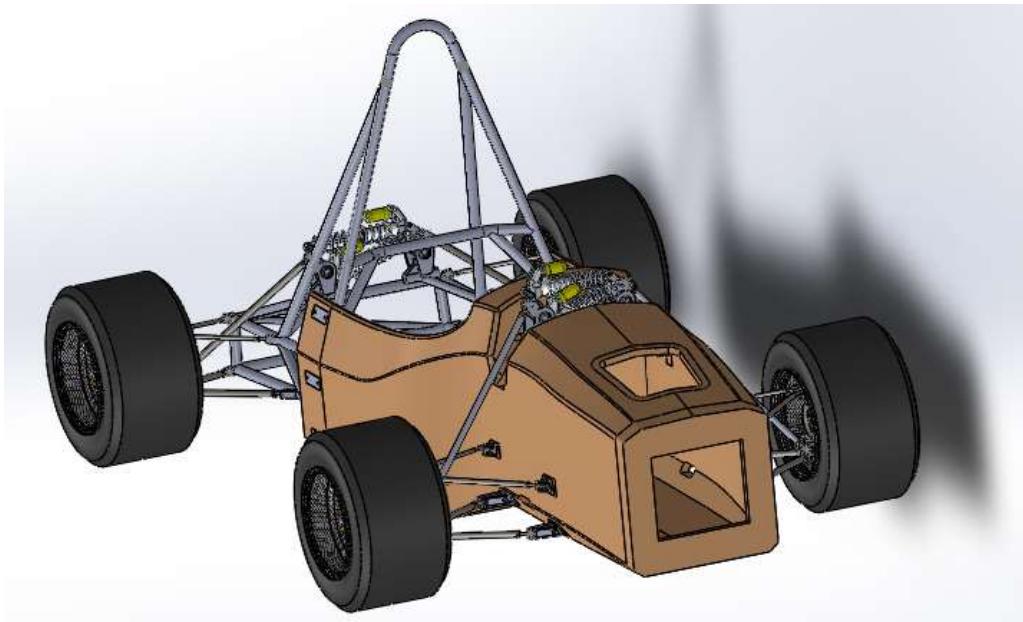


Figure 7.3.1: Whole vehicle assembly, front view

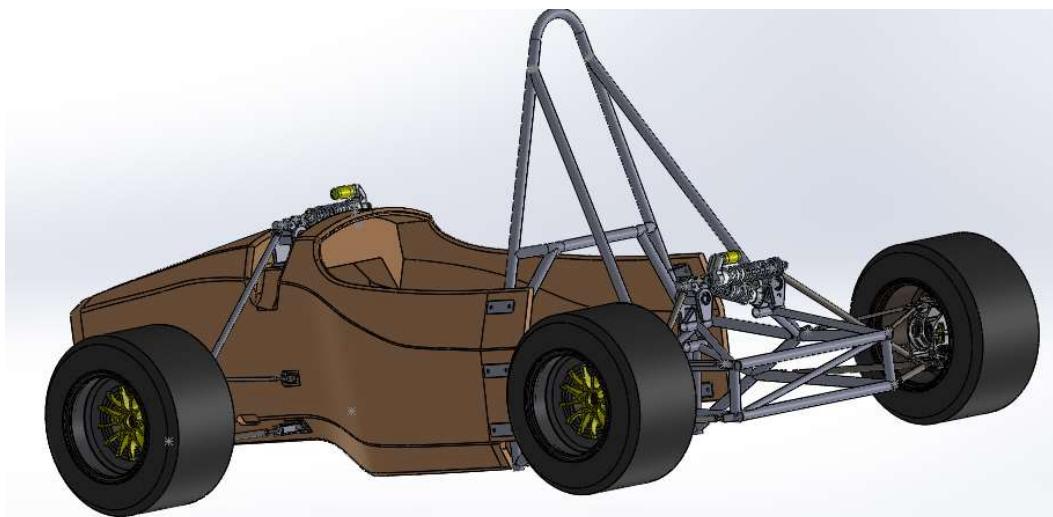


Figure 7.3.2: Whole vehicle assembly, rear-side view

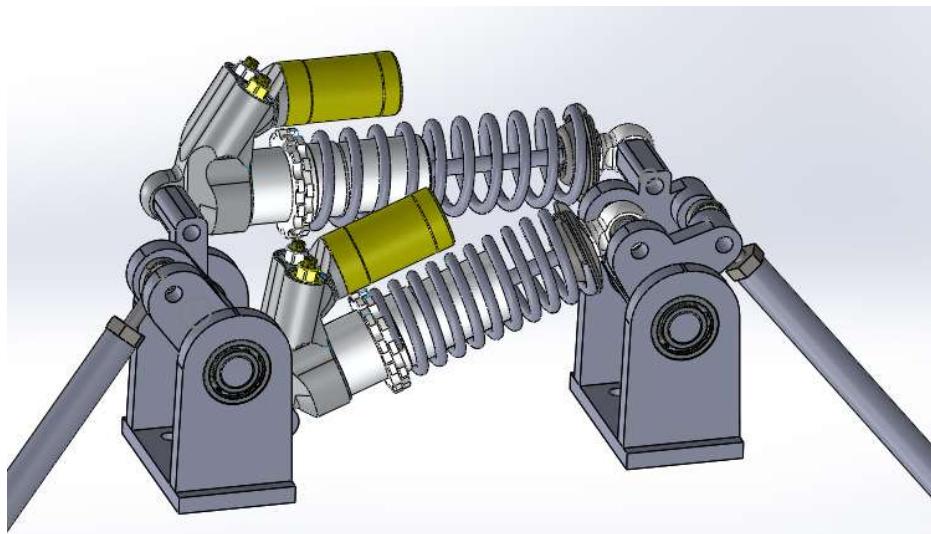


Figure 7.3.13: Front suspension assembly

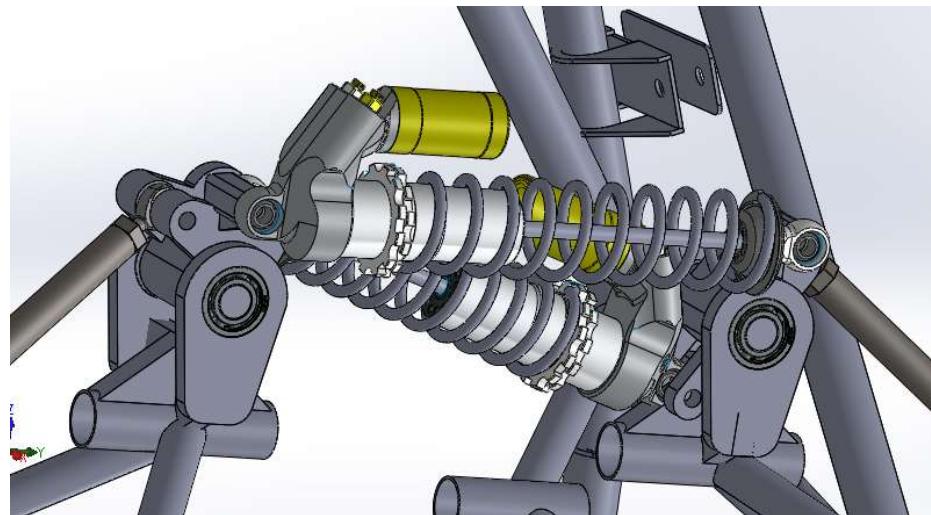


Figure 7.3.17: Rear suspension assembly

Resultados

Posteriormente, se comenzó la simulación. Para ello se utilizaron las simplificaciones 2D del sistema, que, aunque no tenían tanto detalle como la versión 3D, ofrecían un resultado muy similar sin la complejidad que requería la simulación 3D.

Esta simulación consistía en el análisis del comportamiento del vehículo al tomar una curva. En la Figura 8.1 se observan las diferentes distancias y velocidades en las secciones de dicha curva. En la primera sección, de frenado, el coche decelera de 100 mph (161 km/h) a 50 mph (80 km/h) en 20m. Conociendo estos datos, se puede calcular la aceleración y por tanto (usando la ecuación (antes explicada), la fuerza que experimentará cada eje (en este caso 1850 N aplicados en el CG que

comprimirán el eje delantero y elongarán el trasero). En la siguiente sección, el coche mantiene parte de la fuerza de frenado y comienza el giro. En este caso utilizando la ecuación antes mencionada junto con la ecuación 3, y sabiendo la K del vehículo (tanto delantera como trasera) cuando el coche está girando, se obtienen una fuerza de 900 N vertical, aplicada en el CG y un momento de giro aplicado en el centro de giro, tal que:

$$\text{Ecuación 10: } M\varphi \text{ delantero} = \varphi \cdot k\varphi f = 1301.2 \text{ N/m}$$

$$\text{Ecuación 11: } M\varphi \text{ trasero} = \varphi \cdot k\varphi r = 1123.76 \text{ N/m}$$

En la siguiente sección, el coche mantendrá solo el momento debido al giro. Al llegar la aceleración, y siguiendo el mismo procedimiento que en la frenada, el coche experimentará una transferencia de carga de 1044.5 N al salir de la curva y de 522.5 N en el último tramo del giro.

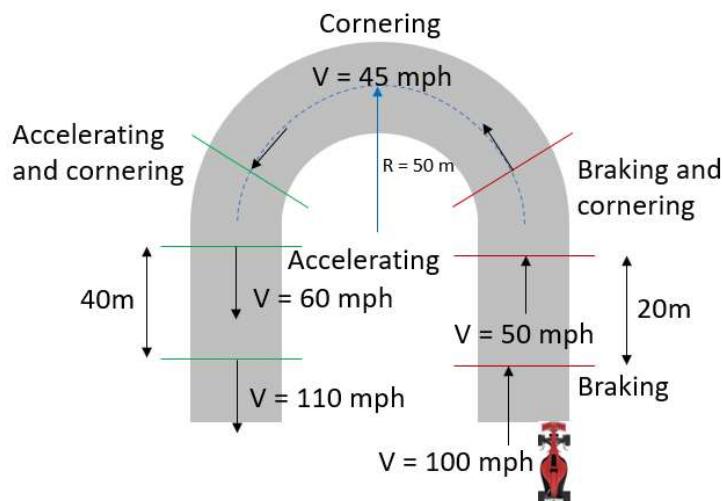


Figure 8.1: Cornering situation analyzed

Una vez obtenidas todas las fuerzas, se introdujeron en el modelo y se realizó la simulación.

Los resultados medidos fueron: la compresión de los muelles horizontal (cabeceo) y diagonal (balanceo), el ángulo que forman las ruedas con el suelo (camber), la variación de altura del centro de gravedad y el ángulo de giro del cuerpo. En las Figuras 9.1.3 y 9.2.1 se muestran respectivamente los muelles horizontal (cabeceo o pitch) y diagonal (balanceo o roll) de la parte delantera. Aunque también se obtuvieron los del eje trasero, los resultados no se muestran en este resumen al ser muy similares a los de la parte frontal, y al no aportar tanto al análisis del sistema.

Front Pitch Spring

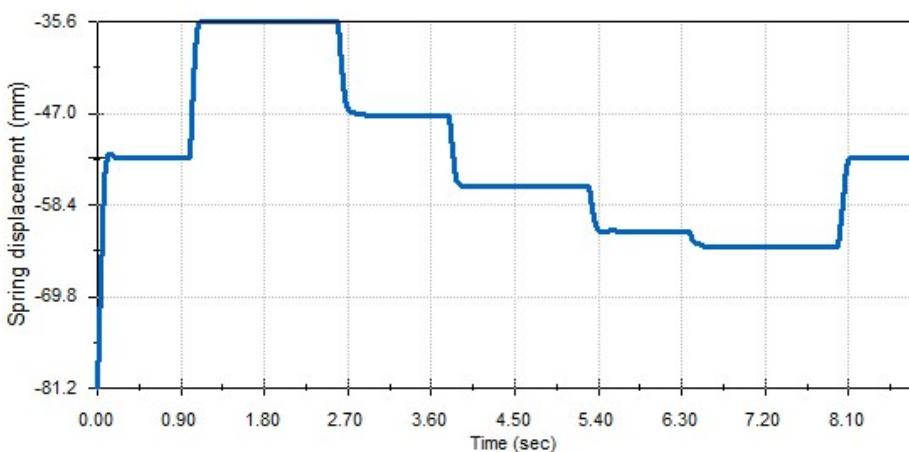


Figure 9.1.3: Front pitch spring compression throughout the simulation

Front Roll Spring

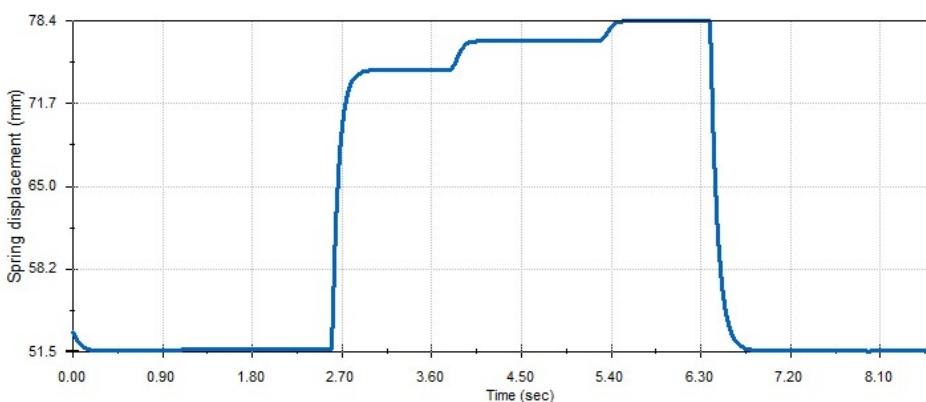


Figure 9.2.1: Front roll spring compression throughout the simulation

Como se esperaba, el muelle de cabeceo absorbe toda la fuerza generada por la frenada inicial, permaneciendo la compresión del muelle de balanceo en su valor estático. Cuando surge la acción de giro, y a medida que desaparece la fuerza de frenado inicial y aparece más tarde la de aceleración, el muelle de balanceo (roll) se va comprimiendo más. Observando la compresión del muelle de cabeceo (pitch), se aprecia como al aparecer las fuerzas verticales de aceleración y frenada, su compresión varía, permaneciendo en un valor similar al inicial cuando el movimiento es giro puro. Finalmente, se observa cómo, cuando el giro desaparece, el valor de compresión del muelle de balanceo vuelve a su valor inicial, siendo el muelle de cabeceo el que reacciona al movimiento. Otro aspecto destacable es la reacción del sistema de suspensión, conforme a lo teóricamente esperado. Los valores de dureza y amortiguamiento seleccionados se reflejan en la respuesta de los muelles, con lo que se consigue una respuesta adecuada en tiempo de reacción y valores finales. Para el eje trasero, al ser la suspensión más blanda, las compresiones de los muelles son ligeramente mayores, pero siempre dentro del margen esperado.

El ángulo que forman las ruedas con el suelo delimita la adherencia del coche, siendo esta máxima cuando las ruedas tienen toda su superficie en contacto con el suelo. Para maximizar el agarre en las curvas, es deseado que las ruedas exteriores (las que soportarán mayor peso) estén lo más planas posibles. El sistema separador de modos permite ajustar este valor mediante el muelle de balanceo, regulando el giro del vehículo y por tanto la variación de la geometría de la suspensión. Los resultados obtenidos reflejan que se consigue un ángulo muy próximo a 0° en las ruedas exteriores para esta situación, por lo que el agarre será mayor que en el anterior sistema.

En cuanto la variación de la altura del CG con respecto al suelo, para el sistema separador de modos será regulada mediante el muelle de cabeceo. Cuanto más estático permanezca dicha altura, el cuerpo del vehículo se podrá bajar más próximo al suelo, mejorando las transferencias de carga y la eficiencia aerodinámica. Sin embargo, al disminuir esta variación se endurece el comportamiento del vehículo y su reacción a las irregularidades, pudiendo perder tracción si la superficie presenta irregularidades. Este balance se tiene en cuenta mediante la frecuencia natural antes calculada, y los resultados obtenidos concuerdan con lo esperado. El CG del eje frontal varía entre 27 cm y 28.7 cm en su punto máximo, y el trasero, al tener una suspensión más blanda, entre 26.5 y 29. Estos valores son adecuados, puesto que el fondo del coche no contactará con el suelo en ningún momento.

Por último, el ángulo de giro del coche alcanza un máximo de 1.9° , similar al calculado en la teoría y adecuado a las características del coche y las fuerzas a las que está sometido.

Conclusión

El proyecto realizado consigue calcular los valores teóricos necesarios para un estudio adecuado del Sistema Separador de Modos, desarrollando un modelo de simulación preciso y ofreciendo resultados válidos e interesantes. Además, el modelo 3D desarrollado ofrece una posible implementación de dicho sistema en el vehículo conforme a las medidas adecuadas y reduciendo el uso de material y espacio. Además, las imágenes aportadas demuestran que los diferentes elementos del sistema no interferirán entre ellos ni con otros elementos del coche.

El sistema presenta numerosas ventajas con respecto a una suspensión convencional. El más importante es la capacidad de ajuste, ya que permite tener diferentes valores de dureza de muelles y amortiguación dependiendo del movimiento. En un sistema de suspensión como el presente en el TR19, al ajustar alguno de estos parámetros, se condicionaba el comportamiento del coche ante otros movimientos. Esta mayor libertad de ajuste permite regular la capacidad de sobreviraje o subviraje del coche (ajustando el muelle de balanceo) sin comprometer la altura del CG o la frecuencia natural del coche. Como se ha podido demostrar, esto tiene numerosas ventajas en el rendimiento general del vehículo. La capacidad de ajuste de la suspensión es un aspecto muy importante, ya que, dependiendo de las condiciones y características del circuito, o de las preferencias y sensaciones del piloto, serán necesario hacer muchos cambios con respecto a los valores teóricos.

Además, se logra reducir peso al no ser necesaria una barra estabilizadora, y se aumenta el comportamiento en curva del coche al poder ajustar el amortiguamiento para este movimiento, cosa que no se puede lograr con las barras estabilizadoras. Sin embargo, al usar este sistema se compromete la respuesta del vehículo a los baches que afectan a una sola rueda, ya que a actuar ambos muelles a la

vez la dureza será mayor que la ideal. Al ser un vehículo de competición, y debido a que las barras estabilizadoras anteriormente presentes en el coche también añaden dureza, este aspecto no comprometerá tanto el rendimiento.

Si el sistema decide llevarse a cabo, se ha de analizar la capacidad de los amortiguadores para regular su posición inicial y su capacidad para funcionar tanto a compresión como a extensión, sobre todo para el muelle de balanceo.

NOTA: las referencias se pueden encontrar al final del proyecto (Apéndice III)

MODE SEPARATED SUSPENSION SYSTEM FOR A FORMULA SAE CAR

Author: Martínez Cuenca, Álvaro

Director: Hamilton, Leonard J.

In collaboration with: University of Maryland

ABSTRACT

Introduction

In order to improve the design of the “TR19” car developed by the TERPS Racing Team from the University of Maryland, a new suspension system design was considered. Named as Mode Separated, this system aims to have different characteristics for the spring and dampers depending on the motion of the car. Every vehicle has three main motions: Roll, Pitch and Yaw. These last two are related, as when the car is turning, the body tends to roll along the longitudinal roll axis. Pitch is the up and down motion of the front or rear parts of the vehicle, which happens specially when the vehicle is accelerating or braking. It is important to mention that there is also another important movement to consider, when the vehicle faces a single wheel bump.

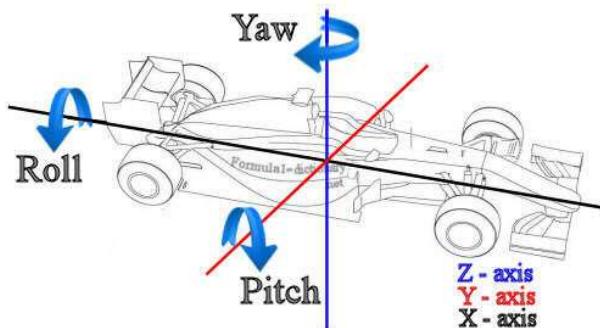


Figure 4.2.1: Basic motions of a car (FORM19)

For an optimal performance of the vehicle, these three possible scenarios should be considered when designing the suspension system. Ideally, the spring and damping rates are different for each movement. A conventional suspension system has to do a trade-off between these optimal values, since they have spring and dampers acting indistinctively for every movement. Also, the Mode Separated System allows for a better tuning of the system depending on the track conditions and driver preferences.



Figure 3.1: TR19 from TERPS Racing, front view (TERP19)

To design a suspension system according to the characteristics of “TR19”, it is important to note that this vehicle has a monocoque frame in the front and a steel tubular one in the rear. Also, the space available to fit the suspension system was limited, as it needed to be fitted in a gap in the monocoque for the front (to improve the aerodynamic efficiency and help with driver’s visibility), and welded to the tubular chassis in the rear without interfering other components like the engine, differential or aero parts.

The design was based in the suspension system that can be found in the AMG Project One car, developed by Mercedes Benz and launched at the end of 2017. This system is shown in Figure 1.1.

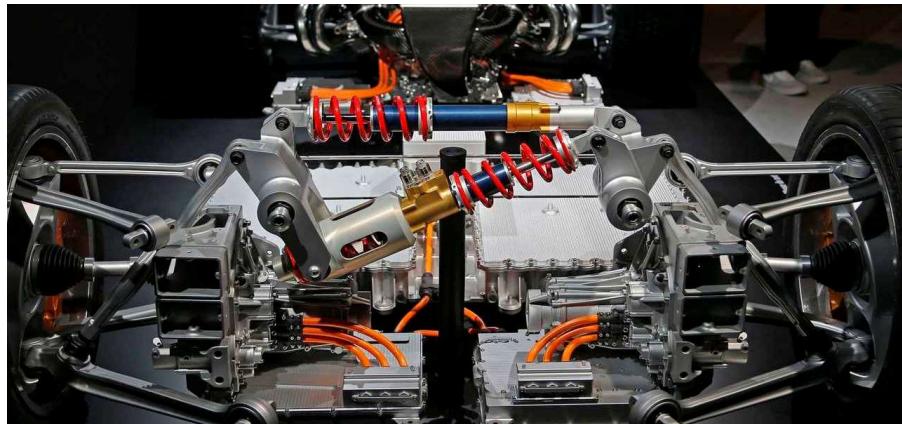


Figure 1.1: Mercedes Project One suspension system (PERK17)

To explain how the system works under different motions, the sketch developed for the front suspension system in TR19 will be used, which will be explained below. In Figure 4.1.1 it can be observed this sketch on its unstretched position, with the name of all its components.

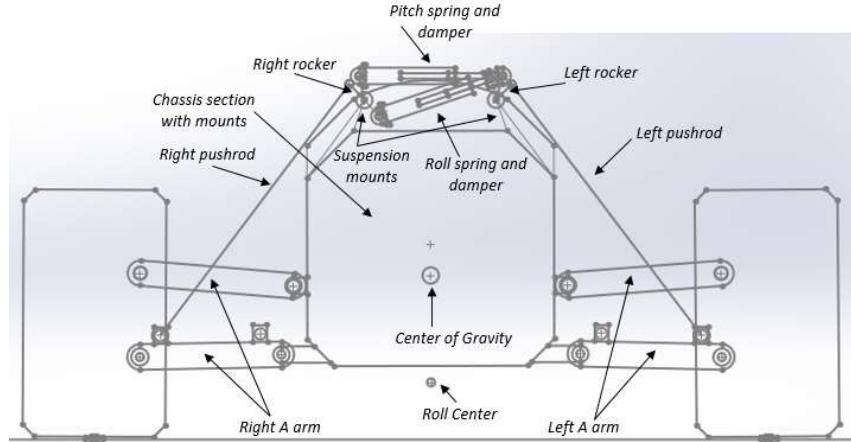


Figure 4.1.1: Sketch of the front view of the front suspension system with its main components

- The first motion analyzed is pitch. As it was said, this motion is the result of the longitudinal load transfer between the front and rear axles that happens between the front and the rear, which make the CG on each axle to be compressed or stretched, and therefore moved vertically. On Figure 4.2.2, if the motion of the arrows is followed, it can be observed how due to the asymmetric geometry of the suspension levers, the horizontal spring is compressed, while the diagonal one sees no net compression since both sides of the spring move in the same direction at the same rate. For this reason, the diagonal spring won't be acting during this motion, while the horizontal one will be reacting the whole movement.

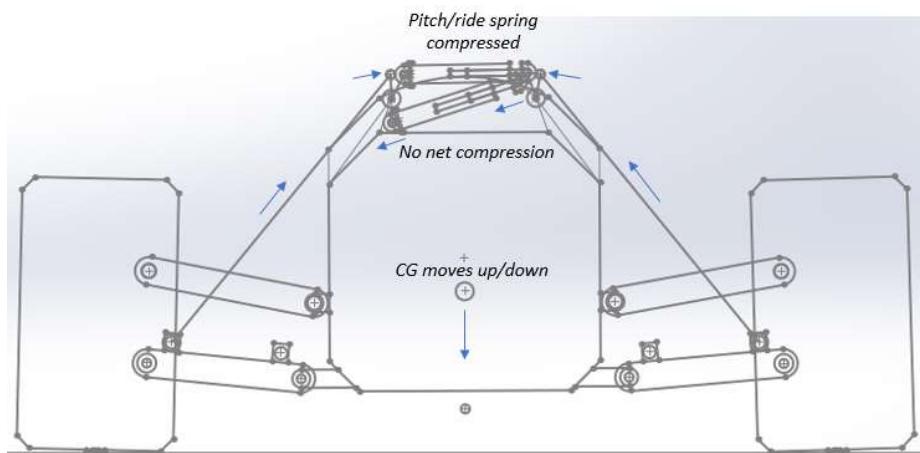


Figure 4.2.2: Suspension system motion under pitch

- On Figure 4.2.3, the system reacting to the roll movement can be observed. While the vehicle is taking a corner, its body tends to roll along the Roll Center, which is different for the two axles and depends on the suspension geometry. Since the pushrods stay steady during this motion, the suspension will react as shown in the arrows. The horizontal spring has no net compression, while the diagonal one reacts the whole motion, stretching in this case.

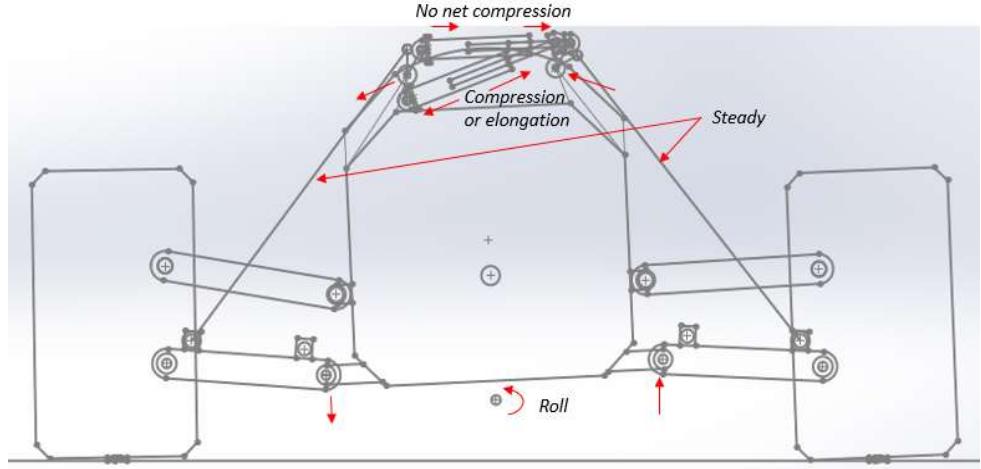


Figure 4.2.3: Suspension system motion while the vehicle is cornering to the left

- The last motion analyzed is the single wheel bump. For this motion, shown in Figure 4.2.4, the two springs will be compressed and will act together. For this reason, the spring and damping rates cannot be selected for this motion and will be a result of the other two suspension parameters previously calculated.

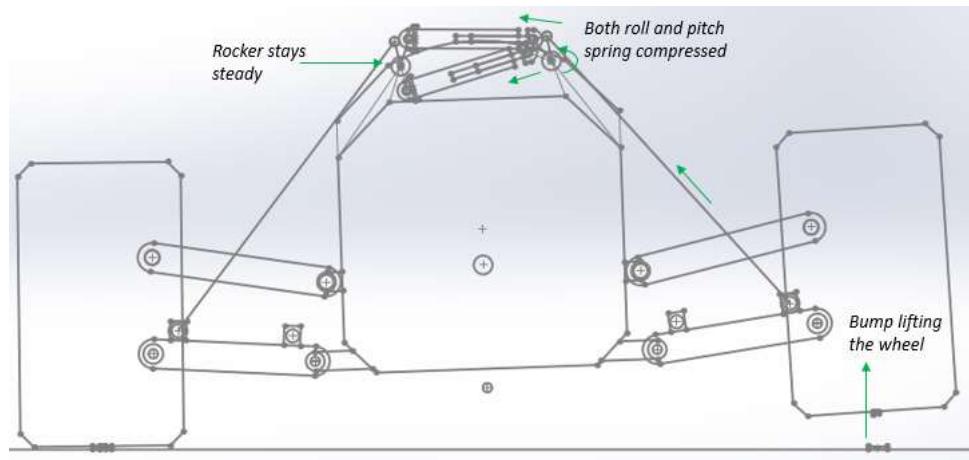


Figure 4.2.4: Suspension system motion over a single wheel bump

The goal of this project is to design a suspension system that has different characteristics and responses depending on the motion of the vehicle. To do so, the optimal suspension parameters and forces acting on the system for each motion need to be analyzed. Then, a computer program will be used to design, simulate and analyze the model.

Methodology

Next step on the suspension analysis is develop the equations to obtain the forces acting on the system under the motions previously introduced, and how they are affected by the spring and damping rates. Since the suspension system is not symmetric, these equations will differ from a conventional system (found on sources GILL92 and OPTG19) and should be recalculated. These forces are the lateral load transfer, roll angle of the vehicle, and longitudinal load transfer.

Lateral load transfer is produced when the vehicle is taking a corner and is the result of the fictitious force acting on the CG due to the normal acceleration it is experiencing. This loads the outside tires, while the vertical force acting on the inner ones are reduced. This force is calculated by Equation 1 (V: cornering speed, R: corner radius, h: CG height, t: trackwidth):

$$Equation\ 1: \Delta W = \frac{m \cdot \frac{v^2}{R} \cdot h}{t}$$

The load distribution for both axles is shown in equation 5 and 6:

$$Equation\ 5: \Delta F_{zf} = \frac{\frac{Wf}{g} \cdot \frac{v^2}{R} \cdot hf}{tf} + \frac{k\varphi f \cdot \varphi}{tf}$$

$$Equation\ 6: \Delta F_{zr} = \frac{\frac{Wr}{g} \cdot \frac{v^2}{R} \cdot hr}{tr} + \frac{k\varphi r \cdot \varphi}{tr}$$

Being φ the roll angle of the CG with respect to its initial position (Equation 3), tr and tf the front and rear trackwidths, h1 the distance between the roll center and the CG and k φ f and k φ r the front and rear roll rates.

$$Equation\ 3: \varphi = \frac{m \cdot \frac{v^2}{R} h_1}{k\varphi f + k\varphi r - mgh_1}$$

Taking a closer look at equations 5 and 6 it can be seen how changing the roll rates on each axle, the load distribution between these axles could be modified, loading or unloading the correspondent axle. A more complete analysis on this aspect could be found in the project memory.

Longitudinal load transfer can be calculated using equation 8 and could not be modified by the spring rates.

$$Equation\ 8: \Delta F_{long} = \frac{h}{L} \cdot \frac{W}{g} \cdot ax$$

Next, and to obtain the parameters needed to simulate the model, the spring and damping rates were obtained for each movement. First step was obtaining the motion ratios between the wheel and the spring ($MR = \frac{\Delta X_{spring}}{\Delta X_{wheel}}$) for the front and rear axles and for both the horizontal and vertical springs.

For the horizontal spring, the Pitch and Ride one, in order to obtain the spring rate first is necessary to specify a natural frequency. This parameter will determine the general behavior of the car: its rigidity, CG height or response to irregularities. For the vehicle analyzed, this frequency was chosen to be 4 Hz for the front and 3.7 Hz for the rear. These values were chosen taking as a reference vehicle with similar characteristics. The rear has a softer suspension system in order to improve aerodynamic efficiency (at high speeds the rear will come closer to the ground reducing the drag generated by the rear wing) and achieving a better acceleration.

This natural frequency can be expressed as follows: $\omega = \frac{1}{2\pi} \cdot \sqrt{\frac{kr}{M}}$

Being kr the equivalent spring rate in the corner of the vehicle analyzed and M the weight on that corner. This kr represents the equivalent spring rate of the tire and the wheel rate (string rate in the wheel) combined. It can be stated that:

$$kr = \frac{kw \cdot kt}{kw + kt}$$

$$kw = ks \cdot MR^2$$

Solving for each corner: ks front (ride) = 48809.3 N/m; ks rear (ride) = 87952.5 N/m

For the Roll spring, the diagonal spring, a similar procedure was followed. First step is choosing the Roll Gradient parameter ($\frac{\phi}{Ay}$), that express how much the vehicle roll for every 1G of normal acceleration. For a racing car like TR19, the value selected was 0.45. This can be related with the suspension's spring rate:

$$\frac{\phi}{Ay} = -W * \frac{h1}{K\phi_{total}}$$

Similar to the previous case, each axle has different parameters for $K\phi$. The front will take 52.2% of the total stiffness and the rear 47.5%. $K\phi$ for both front and rear can be expressed as:

$$K\phi = Kr \cdot \frac{\pi}{180} \cdot \frac{t^2}{2}$$

Once Kr is obtained, the procedure to obtain ks (spring rate) is the same as before.

The values obtained are: ks front (roll) = 44588.3 N/m; ks rear (roll) = 81424.9 N/m. It can be observed that these values are different from the ride case. Conventional pushrod systems, in order to

get different spring rates for these two motions, use Anti-Roll Bars, which add more unsprung weight and have a limited range of adjustment.

The damping rates should be now obtained. To do so, first the damping coefficient should be specified. Again, considering TR19's characteristics, $\zeta = 0.68$ was selected for the pitch spring and $\zeta = 0.65$ for the roll one. For racing cars, the body's oscillation when it approaches the steady state should be reduced but having a decent time of response. Once ζ is selected, C can be obtained as:

$$C = \zeta \cdot 2 \cdot \sqrt{K_r \cdot M}$$

Solving for each axle, and for both roll and pitch springs, the following values are obtained:

C front (pitch and ride): 2195.96 N/(m/s)

C rear (pitch and ride): 3191.64 N/(m/s)

C front (roll): 3397.11 N/(m/s)

C rear (roll): 3316.56 N/(m/s)

Being able to have different damping rates mean that the vehicle's response can be adjusted in every instance. Having a higher damping rate in the pitch spring will mean reducing the rebound of the car while braking and accelerating, so the car could be lowered with the advantages that this have while maintaining a lower damping rate for the roll one for better grip in corners.

After creating the initial sketch of the possible design for both front and rear, finding a software capable of designing, simulating and analyzing the system was the next step. One of Matlab "Add-ins" called "Simscape Multibody" or "Simmechanics" was chosen as the best option. Simscape Multibody is a 3D design and analysis software for several mechanical systems. It allows to model multibody structures using blocks. These blocks represent solid bodies, joints, forces and moments, physical constraints, sensors or motors and actuators. However, as the dimensions and shape of the suspension components wasn't defined, and needed to be created, many of the parameters needed to be introduced as parameters, which made assembling the system slow and difficult. For this reason, another alternative needed to be studied.

Some elements, such as the suspension links, wheels and spindles or pushrods, were going to be the same as last year, including the monocoque and the rear frame. These elements were already design in CAD using Solidworks, so using this computer program would save time and work.

Solidworks has a tool named "Solidworks Motion Study", which allows to simulate the assemblies created, adding forces, spring and dampers or movement constraints. Since the dimensions weren't definitive and the design still needed to be optimized, the 2D block assembly of the front and rear was used. These block 2D sketches can later be joined together as a normal 3D assembly.

The unstretched lengths of the spring and dampers needed to be adequate in order to use the same as the ones used in TR19, which will save the cost of buying new ones. The different mounts and suspension levers had to assure an optimal operation and minimum space, to fit into the space available. The final dimensions of both front and rear can be observed in Figures 7.2.1 and 7.2.2.

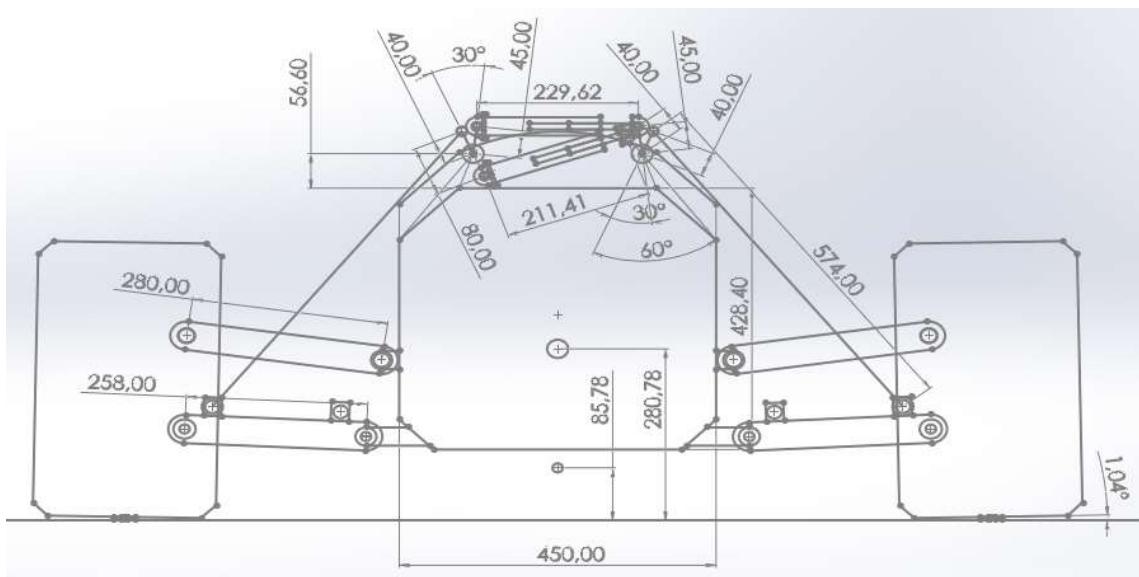


Figure 7.2.1: Final sketch of the front axle of the vehicle's suspension

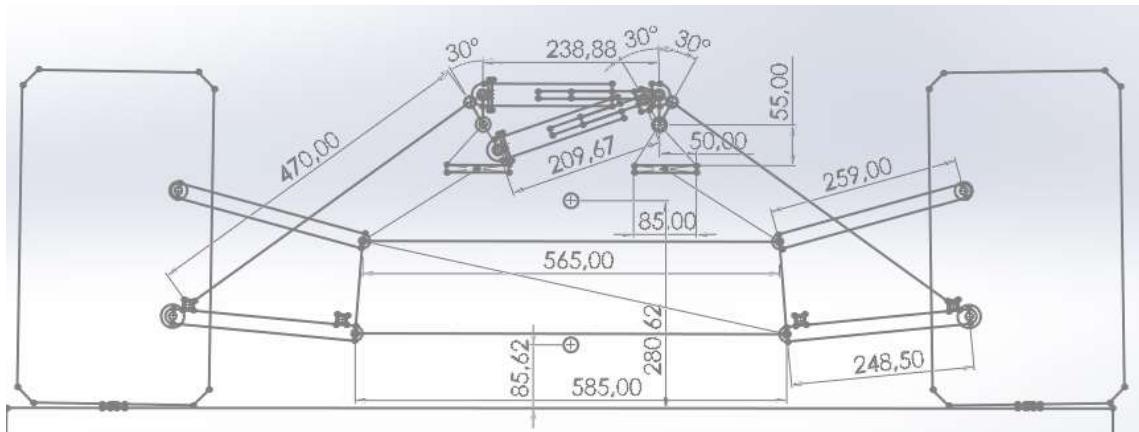


Figure 7.2.2: Final sketch of the rear axle of the vehicle's suspension

These models were later simulated along with a 2D model of TR19's front suspension.

After creating the sketches, the 3D model needed to be done in order to check the design. To do so, some of the parts from TR19 were reassembled and simplified (found on XDRI source), and others were created (found on the technical drawings attached). These parts where the mounts, levers, springs and pushrod bars. On Figures 7.3.1 and 7.3.2 a complete assembly of the car and the suspension can be found, and in 7.3.12 and 7.3.17 there is a detailed view of each of them.

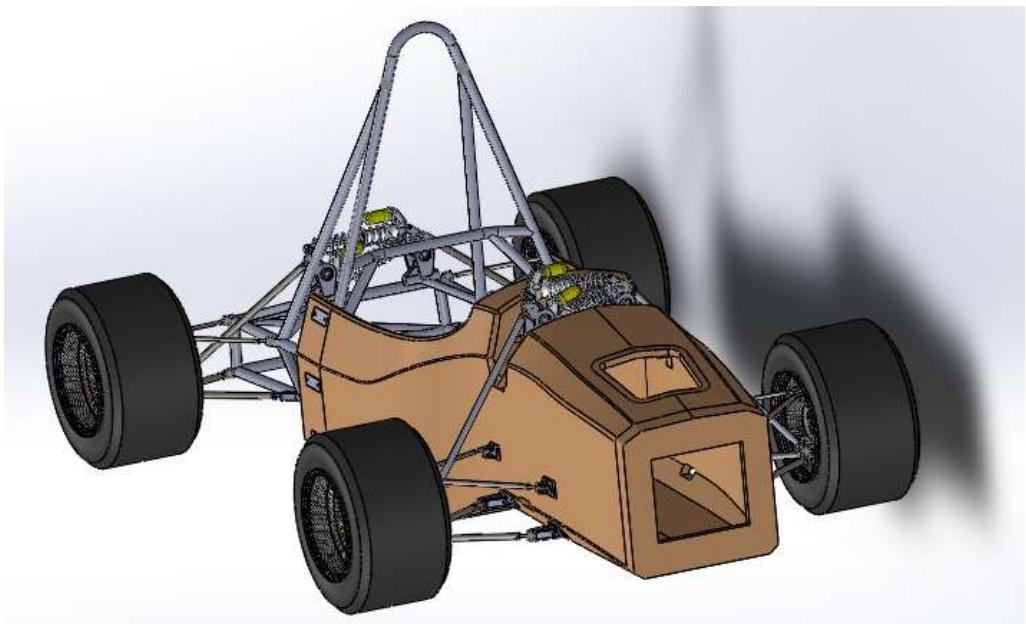


Figure 7.3.1: Whole vehicle assembly, front view

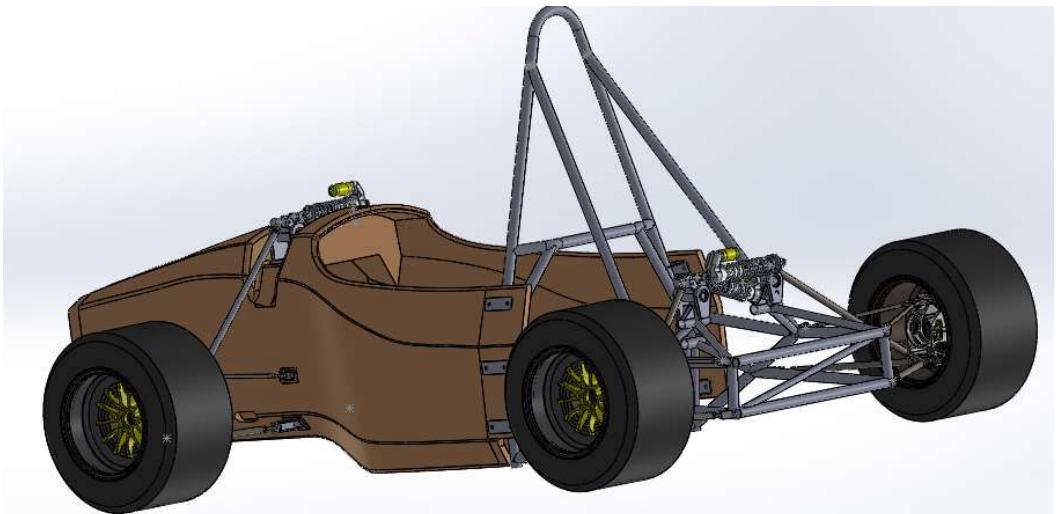


Figure 7.3.2: Whole vehicle assembly, rear-side view

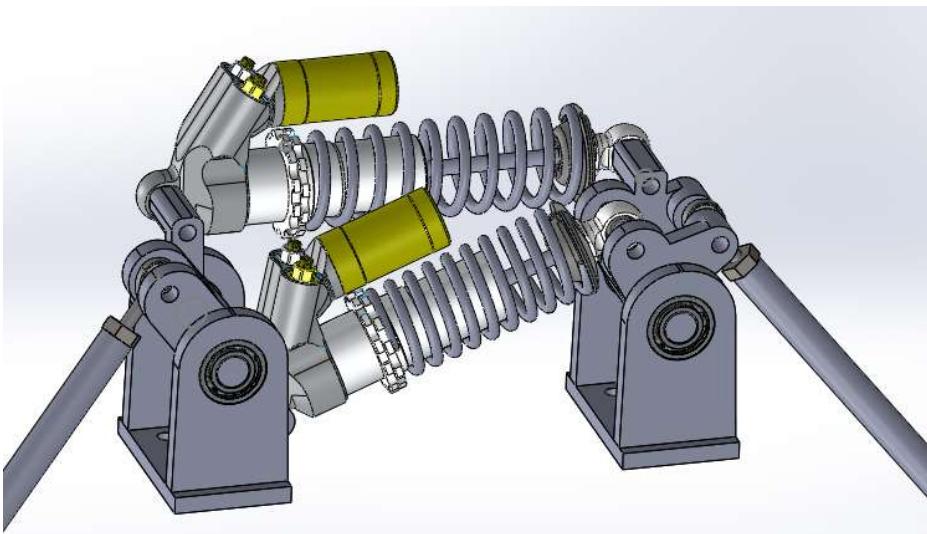


Figure 7.3.13: Front suspension assembly

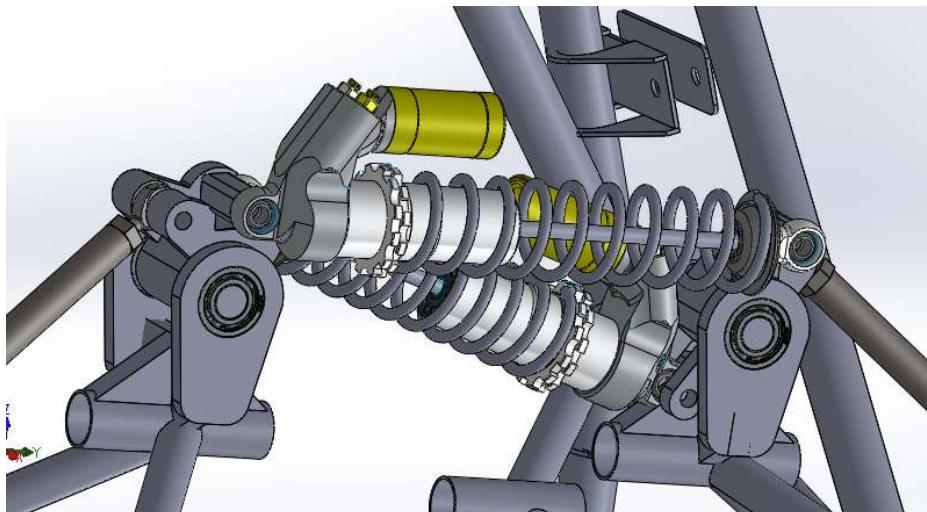


Figure 7.3.17: Rear suspension assembly

Results

After creating the sketches, the simulation was done. The 2D assembly was used for it, and even though the 3D version was more detailed, it was complex to simulate as it has multiple pieces.

This simulation consisted on analyzing the behavior of the car when taking a corner. On Figure 8.1 all the different stages analyzed can be observed. In the first section, the car brakes from 100 mph to 50 mph in 20 meters. The acceleration for this period can be calculated, and therefore (using the Longitudinal load transfer previously explained), the force in each axle will be 1850 N, acting on the CG (compression in the front axle and elongation in the rear). In the next section, the car is still having some braking force, and starts steering. Using now equation 3 previously introduced, and knowing both front and rear spring rates, the steering moment can be calculated as follows:

$$\text{Ecuación 10: } M\varphi \text{ delantero} = \varphi \cdot k\varphi f = 1301.2 \text{ N/m}$$

$$\text{Ecuación 11: } M\varphi \text{ trasero} = \varphi \cdot k\varphi r = 1123.76 \text{ N/m}$$

This rolling moment will be applied in the front and rear roll centers, and the braking force (calculated following the same procedure as in the first stage and assuming it to be 900 N), will be applied in the CG.

Next section, the car will only keep the rolling moment. When the acceleration section starts and following the same procedure as in the braking section, the car will experiment a longitudinal load transfer of 1044.5 N exiting the curve and of 52.5 N at the end of the steering section.

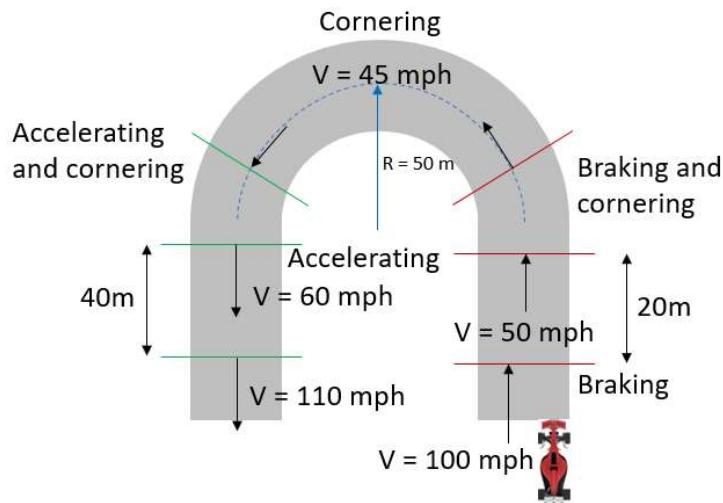


Figure 8.1: Cornering situation analyzed

After all the forces were obtained, they were introduced in the model and the simulation was performed.

The results plotted were: compression on roll and pitch springs, wheel camber for the four tires, CG vertical displacement and roll angle. On Figures 9.1.3 and 9.2.1 the pitch and roll compression results are shown, respectively. Only the front axle is shown, as the rear results were very similar, and they were not as relevant for the result analysis.

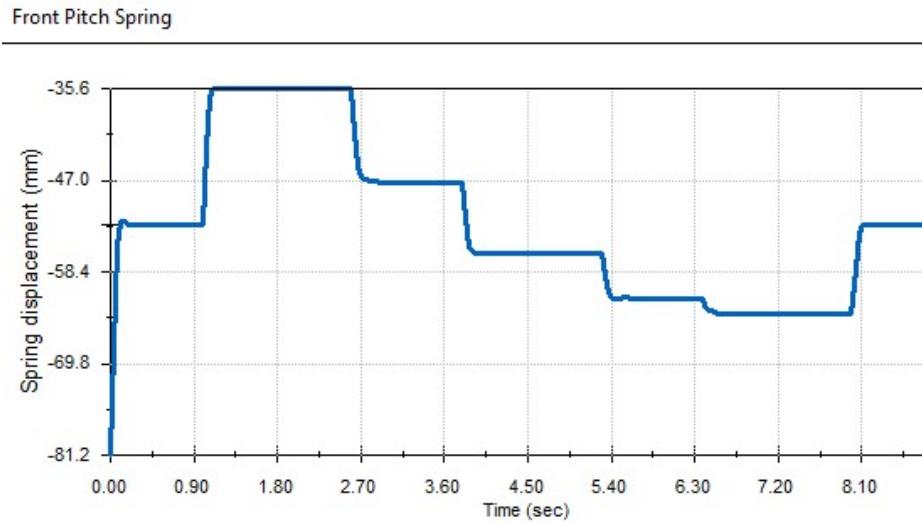


Figure 9.1.3: Front pitch spring compression throughout the simulation

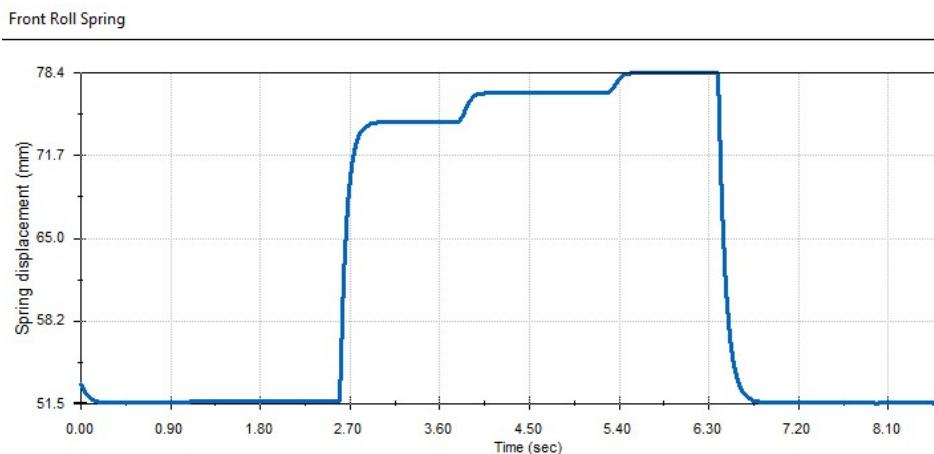


Figure 9.2.1: Front roll spring compression throughout the simulation

As expected, the pitch spring absorbs all the force generated during the first braking section, and the roll compression is kept almost on its initial value. When the turning section begins, and as the minor braking force disappears and the acceleration section begins, the roll spring is further compressed. Taking a closer look at the pitch spring, as the acceleration and braking forces appear, its compression value change, returning to the steady state value when there is only a turning action. When the turning situation finishes, the roll compression goes back to its initial value, so again the pitch spring is the one reacting these motions. It is also important to note that the suspension response is very similar with what was expected in theory. The spring and damping rates selected were adequate to obtain a good reaction time and final values. For the rear axle, since the suspension is softer, the spring compression values are higher, but always under the margins.

The angle between the wheels and the ground define the grip in every situation, being it maximus when this angle is 0 and the contact patch is maximum. For that reason, it is desired that, during the cornering

situation, the camber on the outside tires is as flat as possible. The Mode Separated System allows to adjust this value changing the roll spring rate, and therefore the suspension geometry. The results obtained show that the outside camber is very close to 0 during the cornering situation.

The CG height variation can be regulated using the pitch spring. The more static this value is, the closer the vehicle can be placed with respect to the ground, improving aerodynamic efficiency and weight transfer. However, the vehicle will be more harsh and rigid, which can reduce grip if the surface has irregularities. A good balance between being too hard or too soft is considered with the natural frequency. CG height varies between 27cm and 28.7 cm in the front and 26.5 and 29 in the rear, since the natural frequency is lower and therefore the suspension will be softer.

Last, the roll angle has a maximum of 1.9°, similar with the one obtained in the theoretical part and adequate to the vehicle characteristics.

Conclusion

The project successfully shows how to obtain the theoretical parameters needed for an adequate study of the Mode Separated System, developing a precise simulation model and creating a 3D assembly of a possible suspension design, minimizing the material and space used. The images attached show that the different elements will not interfere with each other and with other elements on the car.

This system has several advantages with respect to a conventional suspension design. The most important one is its tuning capacity, since it allows to have different suspension settings depending on the motion. In the TR19's suspension system, if one of the spring or damping rates were changed, the behavior of the car will be affected. The car can now be tuned to adjust the oversteer/understeer tendency of the car without affecting the ride natural frequency or CG height variation, which will have several performance improvements on the vehicle.

Suspension tuning is a very important aspect, as the conditions of the track or driver preferences will mean making a lot of changes on the suspension settings.

Also, this system reduces weight compared to the previous one, as an Anti-Roll Bar is not necessary. Also, the car's behavior during a cornering situation is increased as the damping can be controlled in this motion. However, this system compromises the vehicle's response when a single wheel bump is hit, as both pitch and roll spring will act together reacting the motion.

If the system is finally elected to be used, the dampers capacity to work under compression and extension modes should be considered, especially for the roll motion. The unstretched length of the dampers may be also considered, and its capacity to change this parameter.

TABLE OF CONTENT

1.	Motivation	39
2.	Goals.....	41
3.	The Car	43
3.1	Characteristics	43
4.	Introduction	47
4.1	Basic Design.....	47
4.2	Motion Analysis	50
5.	Analysis of the Forces Acting on the Car.....	53
5.1	Suspension Geometry.....	53
5.2	Lateral Load Transfer	54
5.2.1	Body Roll.....	59
5.3	Longitudinal Load Transfer.....	60
5.4	Possible Theoretical Advantages Compared to the Previous System.....	61
6.	Spring and Damping Rates Analysis	63
6.1	Spring Rate Analysis	63
6.1.1	Ride and Pitch.....	65
6.1.2	Roll	68
6.1.3	Single Wheel Bump.....	71
6.2	Damping Rates Analysis	72
6.2.1	Ride and Pitch.....	75
6.2.2	Roll	77
6.3	Springs and Shock Absorbers Used.....	78
7.	Computer Modeling the Suspension System.....	79
7.1	Matlab and Simscape Multibody	79
7.2	Solidworks 2D Model.....	84
7.3	Solidworks 3D Model.....	88
8.	Calculations for the Software Simulation.....	97
9.	Results and Analysis.....	101
9.1	Pitch Spring	101
9.2	Roll Spring.....	105
9.3	Wheel Camber.....	106
9.4	Body Roll.....	113
9.5	CG Vertical Displacement.....	116
10.	Estimated Cost.....	119
11.	Comments and Conclusion.....	121

12.	Future Steps.....	123
14.	Appendix I: References	125
15.	Appendix II: List of Drawings.....	127
16.	Appendix III: Suspension assembly: List of parts.....	129
17.	Appendix IV: List of Figures	131

1. Motivation

In order to improve the performance of the Formula SAE car developed by the TERPS Racing Team of the University of Maryland, a new suspension system configuration will be analyzed. The car currently has a pushrod setup with Anti-Roll Bars. To give a little background about the previous suspension system, its basic geometry will be explained. This system consists on a pushrod that connects the lower A arm with the bell crank lever (or rocker), compressing the spring and damper, mounted between this lever and the monocoque as the wheels go upwards. The system is very common in racing cars, as it allows to locate the components towards the center of the car for better weight distribution. It also has several aerodynamic advantages, as the spring and dampers are hidden in a cavity specifically designed in the monocoque for this purpose. Another important aspect of every suspension system is the suspension and wheel links, or A-Arms. They connect the wheel with the monocoque and the way they are placed determine the toe and camber curves, and therefore, the overall grip of the car. For this project, the same A arms as the TR19 car will be used, with the same connection points in the monocoque. This will help to obtain adequate toe and camber curves, as its design was proven to be very efficient.

The team is using Anti-Roll Bars to connect the left and right side of the suspension system, which can help to reduce body roll, and transfer forces during single wheel bumps or cornering. When both wheels move up or down at the same time, there is no force transfer between the two sides.

Conventional pushrod suspension systems cannot isolate the two main motions of the car: roll during cornering and pitch during braking and accelerating. For this reason, it is not possible to have different tuning setups for these two movements, which can limit the overall grip of the car. The Mode Separated System seem to solve this problem, as it uses different spring and dampers for these two main motions of the car. Being able to isolate these motions can make a huge difference in terms of performance, as different tuning setups could be made depending on the motions affecting each spring and damper. This can improve the grip during cornering, accelerating or braking, and the overall stability of the car.

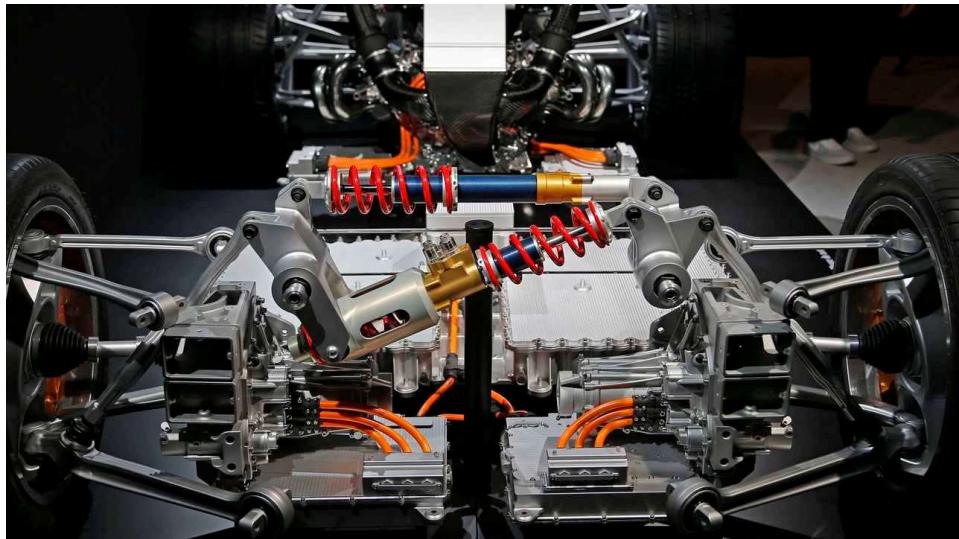


Figure 1.1: Mercedes Project One suspension system (PERK17)

For this reason, during the last few years, several racing teams and car manufacturers are starting to develop this suspension setup for their racing cars.

This is a relatively new suspension system, so a lot of research can be done in order to determine its advantages, different ways of operation, tuning or possible implementation in the TERPS Racing Formula SAE car.

To begin, I would like to thank the whole TERPS Racing team, and Captain Mr. Leonard Hamilton in particular, for its help and advices, sharing their knowledge and passion.



Figure 1.2: TERPS Racing Logo (TERP19)

2. Goals

The main goal of this project is to design and analyze a suspension system capable of isolate the two main motions of the formula SAE car (longitudinal and lateral motions) under different spring and dampers configurations. If done properly, it will allow to have different tunings for each motion, increasing the vehicle's performance. Analyze the possible advantages this configuration will have compared to the current suspension system.

In order to achieve this goal, some steps were followed:

- Analyze all the possible spring and damper configurations sketches, considering the physical and budget constraints of the vehicle, and its parameters (weight, wheelbase, trackwidth, downforce or weigh distribution). Front and rear configurations may be different due to the possible mount locations on the chassis. Once a first sketch is done, find the way to optimize it in terms of aerodynamic efficiency, number of components, packaging, tuning options and simplicity of operation.
- Create a suspension model in a computer program. This software must be able to create a model of both front and rear suspension systems and simulate them under different working conditions, analyzing the motion of the different parts. It has to be possible to change the parameters of the suspension components (spring rates, damping coefficient, overall length of the components, etc.) to see how they affect the vehicle performance.
- Create a 3D model of the final version to illustrate how to pack the different components and its final characteristics.
- Do a force and frequency analysis to obtain the optimal parameters for our vehicle. In order to do that, it is first necessary to study the different load transfers in the car and how they can be controlled by the suspension. For the frequency analysis, the optimal spring and damper parameter need to be obtained. Look into how changing these parameters may affect the performance of the car.
- Compare the results obtained with the previous suspension system. A model of the previous car may be required to obtain the necessary values.
- Analyze the results and create a research paper to explain the final design and conclusions.

3. The Car

3.1 Characteristics

The car in which this project is based is the TR19 from TERPS Racing, the Formula SAE Team from the University of Maryland.

To develop the suspension, the first step is to analyze the car we will be working with. Parameters like the overall dimensions, the wheelbase and track, mass and center of gravity, tire size, roll center, or the previous suspension characteristics are crucial to develop the suspension properly. All this data was already provided and can be found in the FSAE Design Spec Sheet for TR19 car (Figure 3.3).

However, some suspension parameters will change as the spring and damper characteristics and orientation are changing.

As an additional information, this car has a monocoque frame in the front and a tubular structure in the rear, which will improve the overall stiffness from last edition's TR18 car. It has some aerodynamic features to provide more downforce like a front and rear wings and ground effects.

The monocoque has a section where the suspension system has to be placed. Since it is made of carbon fiber, we have more freedom on where to locate the suspension mounts, since the holes to secure them onto the chassis can be easily drilled. However, it has to be considered that some steel plates have to be attached to the drilled zone (above and under the composite section) as a reinforcement. For the rear, the suspension mounts must be welded to some of the tubes of the frame. If necessary, additional structures can be added to the existing chassis for a better operation of the suspension.

In Figures 3.1 and 3.2, the vehicle being analyzed is shown (TR19 from TERPS Racing Formula SAE Team, from the University of Maryland).



Figure 3.1: TR19 from TERPS Racing, front view (TERP19)



Figure 3.2: TR19 from TERPS Racing, rear view (TERP19)

The FSAE Design Spec Sheet for TR19 car will be now provided. Many references will be made to it throughout this project, as it has all the general information of the vehicle.

School	University of Maryland College Park						
Dimensions	Units						
Overall Dimensions	mm	Length:	3022	Width:	1422	Height:	1193
Wheelbase & Track	mm	Wheelbase:	1549	Front Track:	1219	Rear Track:	1219
Center of Gravity Design Height	mm	CG Height:	280,0	Confirmed Via:	Static Weight Transfer Tests		
Mass without driver	kg	Front:	97,0	Rear:	107,0	Total:	204,0
Weight Distribution with 68kg driver		% Front:	47,5	% Left:	50,0		
Suspension Parameters	Units	Front			Rear		
Tire Size, Compound and Make		18 x 7,5-10	R25B	Hoosier	18 x 7,5-10	R25B	Hoosier
Wheels (diameter, width, material)	inch	Diameter (col D): Width (col E):	10,0	9,0	Diameter (col G): Width (col H):	10,0	9,0
Wheel material and construction		3 piece (Carbon Fiber/Aluminum)			3 piece (Carbon Fiber/Aluminum)		
Suspension Type		Double unequal length a-arm. Push-rod actuated horizontally oriented coil-over, adjustable ARB			Double unequal length a-arm. Push-rod actuated horizontally oriented coil-over		
Suspension design travel	mm	Jounce (col D): Rebound (col E):	25,4	25,4	Jounce (col G): Rebound (col H):	25,4	25,4
Wheel rate (chassis to wheel center)	N/mm	23				35	
Roll rate (chassis to wheel center)	Nm/deg	0,8				0,8	
Sprung mass natural frequency	Hz	3,06				3,64	
Jounce Damping	% critical	60	at __ mm/sec:	30	60,00	at __ mm/sec:	30
Rebound Damping	% critical	100	at __ mm/sec:	60	110,00	at __ mm/sec:	60
Motion ratio	-__:_1	1,00	Type:	Linear	1,00	Type:	Linear
Ride Camber (Rate of Camber Change)	deg/m	36,0				76	
Roll Camber	deg/deg	0,6				0,19	
Static Toe (- out, + in)	deg	0,13				.25	
Static camber	deg	-0,25				-.5	
Static camber adjustment method		Outboard A-Arm Shim Plates			Outboard A-Arm Shim Plates		
Anti dive / Anti Squat	%	34				34/50	
Roll center height above ground, static	mm	54,6				92,5	
Roll center position at 1g lateral acc	mm	Height (col D): Lateral (col E):	52,7	-113,3	Height (col G): Lateral (col H):	91,4	-66,9
Front Caster, Trail, and Scrub Radius		Caster (deg):	4,4	Kin Trail (mm):	2,8	Scrub Rad (mm):	75,0
Front Kingpin Axis		Inclination (deg):	0,0	Offset (mm):	75,0		
Static Ackermann	%	85	Adjustable?	No			
Suspension Adjustment Methods		A-Arm leg length changes via threaded Rod-ends, Camber via shims on upright, Rear Bump Toe adjustable					
Steer Ratio, C-Factor, Steer Arm Length		Steer Ratio (x:1)	3,2	c-factor (mm)	130,6	Steer Arm Length	86,0

Figure 3.3: FSAE Design Spec Sheet for TR19 (XDR1)

4. Introduction

4.1 Basic Design

To start designing the system, the first think noticed is that having a conventional symmetric suspension system, the goal of decoupling the two main motions under different suspension characteristics won't be possible.

As it is not a common suspension system, there weren't many examples on how to develop it. Some other Formula SAE teams have already introduced some additional dampers joining the two wheels, but not in a way to fully been able to tune each spring and damper to modify the roll and pitch stiffness. However, and based on the suspension system from AMG Mercedes Project One, found on source PERK17 (Figure 1.1), a first possible sketch of how the system should look like was created.

However, the AMG Mercedes Project One was a streetcar, and even though its suspension was a good starting point, all the components must be re-designed to fit TR19's car. As it was said before, there is a gap in the front monocoque specifically designed to fit the spring and dampers for the suspension system. The Mode Separated configuration requires more space than the previous pushrod system, so the goal will be to place the two spring and shocks in the most optimal way in that available space. For the back, the space is limited by the rear wing and the engine and differential components. The suspension mounts will be welded onto the rear frame.

After developing several possible configurations, a final sketch of the front and rear suspension was created, which can be seen in Figures 4.1.1 and 4.1.2. These sketches show a front view of the section of the center of the axle.

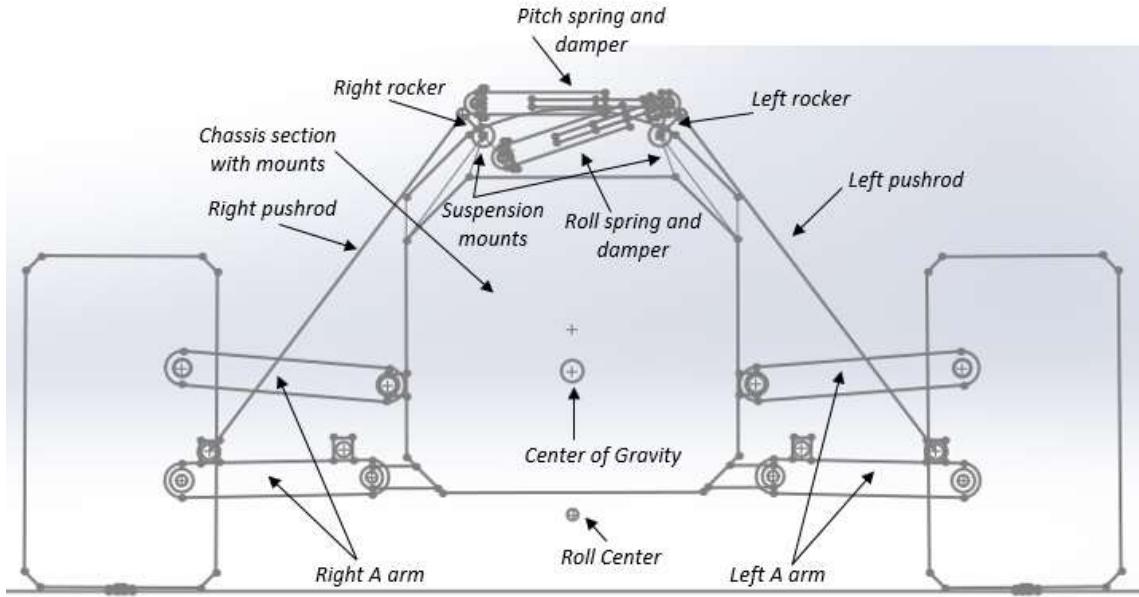


Figure 4.1.1: Sketch of the front view of the front suspension system with its main components

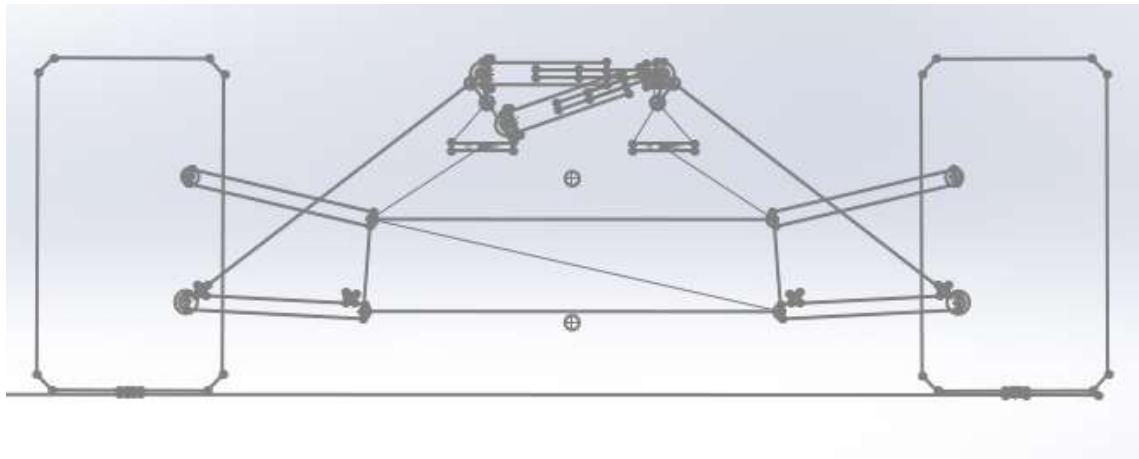


Figure 4.1.2: Sketch of the front view of the rear suspension system with its main components

All the details about the length and shape of the different components will be discussed further in Sections 6 and 7.2.

As it can be seen in Figures 4.1.1 and 4.1.2, there are two springs and dampers connecting each side of the car. The A arms, monocoque connection points, wheels and tires remain the same as in TR19, so the point where the pushrod is connected to the lower A arm is still the same. However, the pushrod length has increased for a better connection with the bell crank lever (rockers). These rockers are now different from the right and left, as they need to transmit the motion of the pushrod to the spring and dampers. The rockers can pivot from a point attached to the frame (this point needs to be elevated from the frame, as the rockers, spring and dampers cannot interfere with the monocoque).

Now there are two springs that connect both sides of the car, for the front and rear axle. As the wheels move up and down, they transfer the motion to the right and left rockers via the pushrods. These rockers are connected to the chassis through the mounts, which allow them to rotate thanks to some ball bearings situated in the connections with the rockers. The rockers are connected to one another with the springs and dampers, so they will be actuated with the different motions of the vehicle.

4.2 Motion Analysis

It will now be explained how the system works under certain conditions. There are four main motions in every vehicle: ride and pitch, roll, yaw and single wheel bump. Some of them can be observed in Figure 4.2.1, from source FORM19.

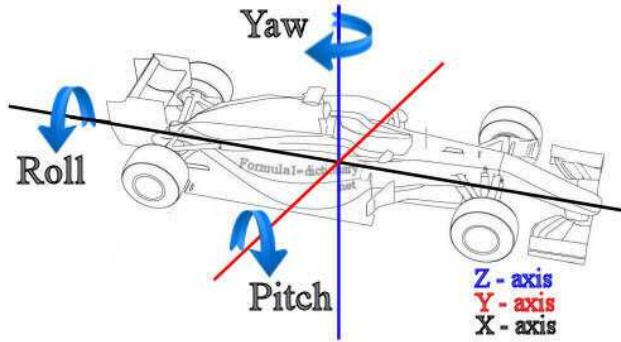


Figure 4.2.1: Basic motions of a car (FORM19)

- Pitch can be defined as the rotation of the vehicle along the lateral axis. In the Mode Separated System developed, Ride and Pitch are reacted by the same spring and damper. Ride is the main motion of the car. It defines the vehicle's overall behavior, as it is the one controlling the car's response to the road surface. During these motions, either the two wheel of an axle or the front and the back of the car move up and down, compressing the wheels in that axle at the same rate, so both pushrods act on each rocker with the same motion. The top part of the rockers come closer to each other, compressing the upper spring. However, and because of the motion of the rockers, the lower spring is not compressing as it sees no relative motion. This happens because each end of the spring (connected to the rocker of its corresponding side) is moving in the same direction at almost the same time (producing no net compression or stretching). On Figure 4.2.2 it can be seen how this motion affects the different parts of the front suspension.

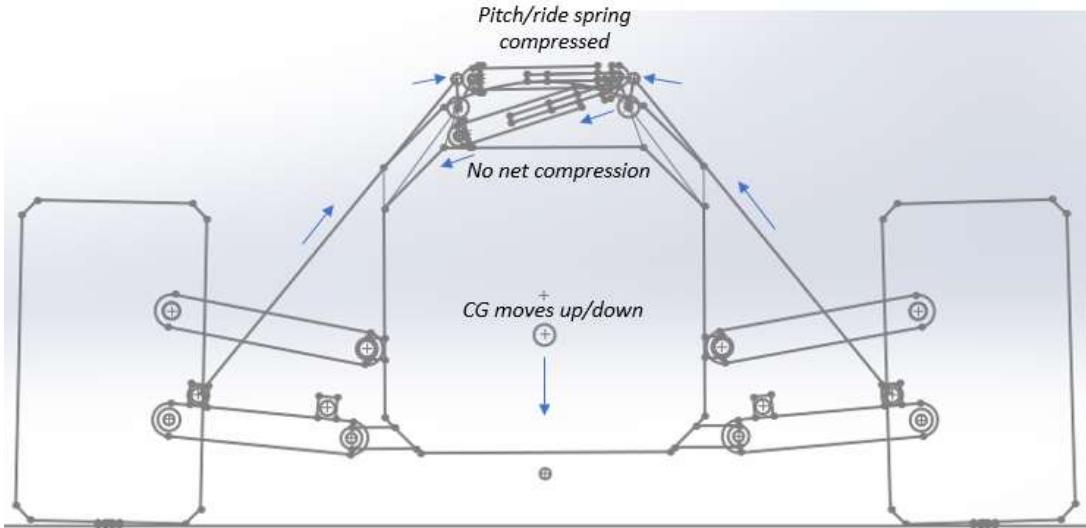


Figure 4.2.2: Suspension system motion under pitch

- During roll, the CG of the car tends to roll round the roll center, tilting the body. This motion is also related to yaw, as roll is a result of the car turning in a corner. As the rockers are connected to the frame, this body roll is transferred to the pushrods and therefore the springs, which will counteract this movement. In this case, the upper spring sees no net compressing force, but the lower one compresses or stretches depending on the direction of the body roll. In the case shown in Figure 4.2.3, the car is turning to the left (Important note: this sketch provides a front view of the car), making the car roll towards the right. This makes the lower spring to compress, as the rockers are moving as shown. If the vehicle was turning to the right, all the components will move in the opposite direction as shown by the arrows, so the lower spring will stretch.

An important design consideration is that, if our spring and dampers can only act in either tension or compression, we will need two of them for the lower part of our suspension system, one acting in compression for the right corners, and one capable of working in tension for the left ones. These kind of spring and dampers can be found easily, and it won't compromise the feasibility of our system.

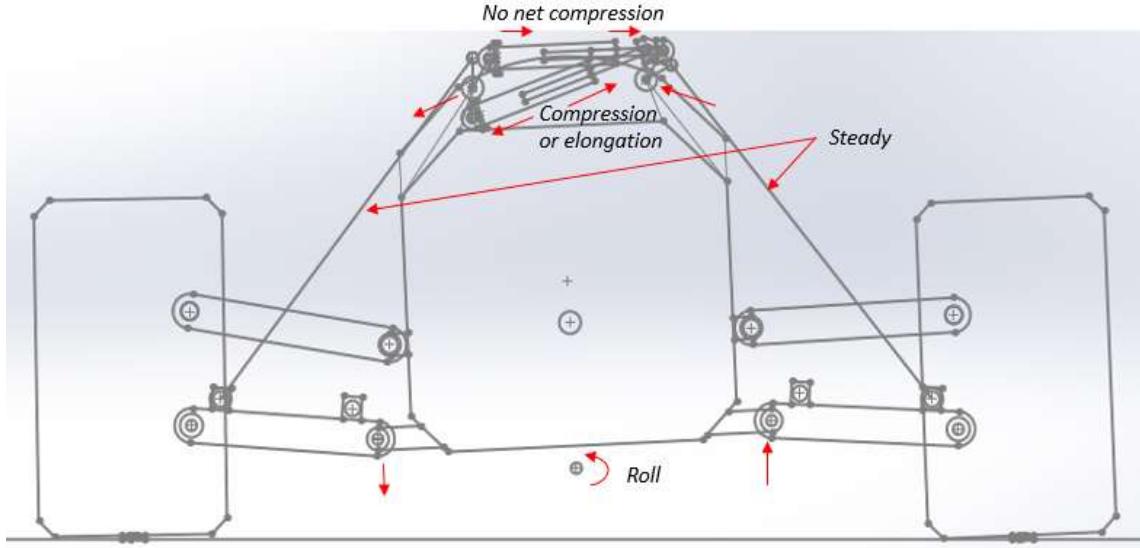


Figure 4.2.3: Suspension system motion while the vehicle is cornering to the left

- When the vehicle encounters a single wheel bump, both springs react together. This situation cannot be tuned separately, and it will depend on the spring and damping rates that we have for roll and pitch. Single wheel bumps are not a situation as frequent as the previous cases, therefore having a specific tuning for them won't improve that much the performance. On figure 4.2.4 we can see how the system will react to a single wheel bump, in this case the left one (again, this sketch shows a front view of the vehicle).

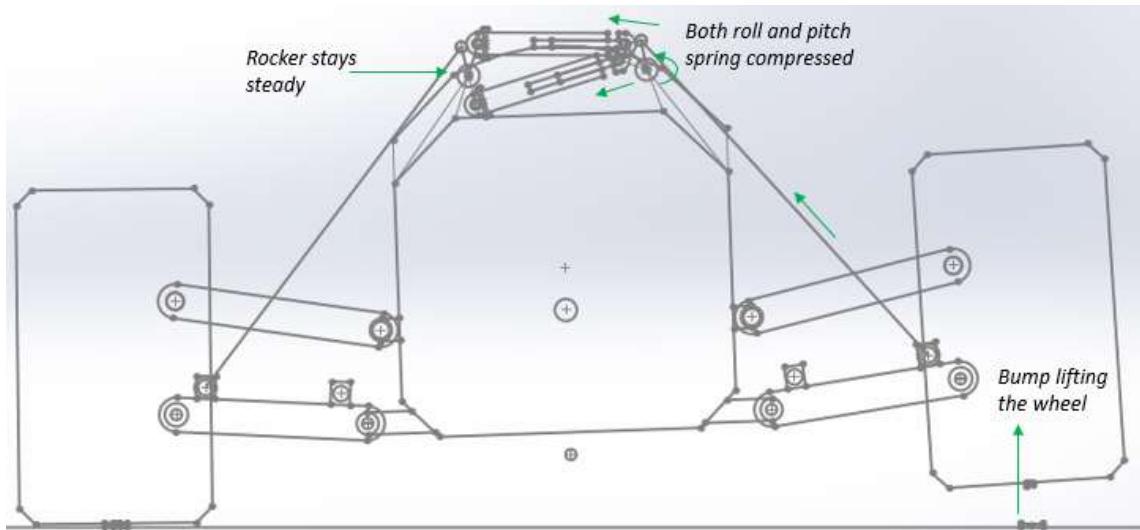


Figure 4.2.4: Suspension system motion over a single wheel bump

5. Analysis of the Forces Acting on the Car

5.1 Suspension Geometry

For a better understanding on how to tune the suspension properly, the different motions of the vehicle and how they are affected by different spring rates need to be analyzed.

Spring rates can be tuned along with the suspension links (A arms) to change the behavior of the car. Since the same A arms from last year's car are going to be used, the roll center and instant centers for the steady state situation are going to be the same. The instant center is the point of the body that has no relative motion with respect to the unsprung mass (wheels, brake system and wheel hub). It can be obtained by lengthen the projection of the A arms on the center of the wheel and see where they meet.

The roll center is the point of the chassis that reacts the force from the suspension. Although it is being assumed to be equal when running the simulation, it changes for every instance, as the suspension geometry varies. It is the point where the imaginary line that joins the IC with the contact patch of the wheel crosses the center line of the car. It will be different for the front and rear as the suspension geometry varies.

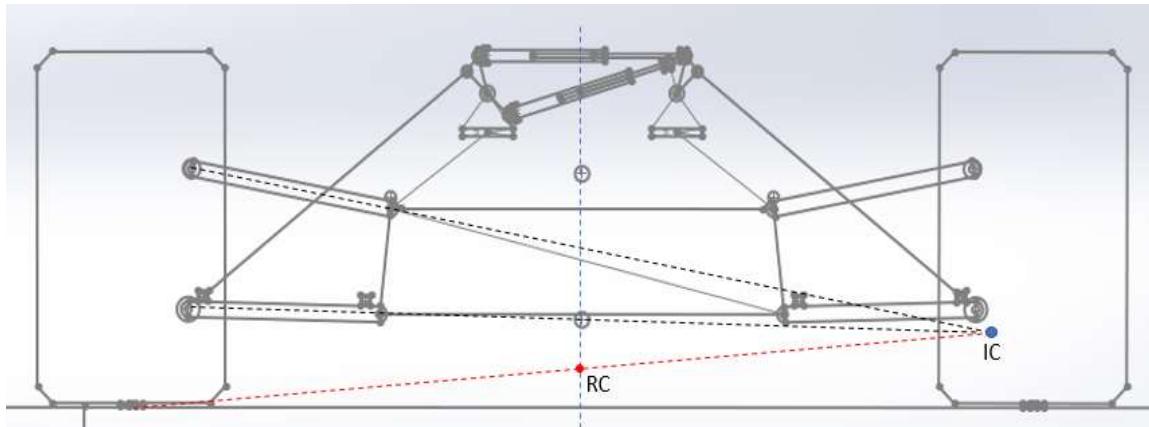


Figure 5.1.1: Roll Center and Instant Center

The suspension links can be designed with different initial geometries, which will affect the way the vehicle reacts to braking or accelerating. The different design options are: anti-squat (keeps the rear suspension of a rear wheel drive from compressing while accelerating), anti-pitch (keep the pitch angle stable while accelerating), anti-dive (keeps the front suspension from compressing under braking) or anti-lift (keeps the car's rear suspension from extending under braking). Since the suspension links (A arms) are the same as last year's car, this suspension components will not be re-designed. In our vehicle configuration, the car is designed to have some anti-pitch and anti-lift.

As it was introduced, there are some vehicle's motions that need to be analyzed to obtain the optimal parameters for the suspension system. The most important ones are Longitudinal and Lateral Load Transfer. For Lateral Load Transfer, the vehicle's body roll will be analyzed as well.

5.2 Lateral Load Transfer

As the vehicle accelerates and brakes, and while it is cornering, some weight transfer between the front and the rear axles and both sides of the vehicle will happen. Understanding how this weight transfer affects the car and being able to analyze all the aspects involved will be very useful for a better tuning of the suspension system, which will react these two major motions.

When the vehicle is cornering, some weight transfer will occur between the inner and the outer tire of the vehicle. In Figure 5.2.1 from OLIV15 we can see that the lateral force that can be outputted by a tire is not linear with respect to the vertical load, so when lateral load transfer happens, the total lateral force the wheels will resist will decrease. In the second picture it can be observed that, as more lateral load transfer happens, the average lateral force is decreased, decreasing grip.

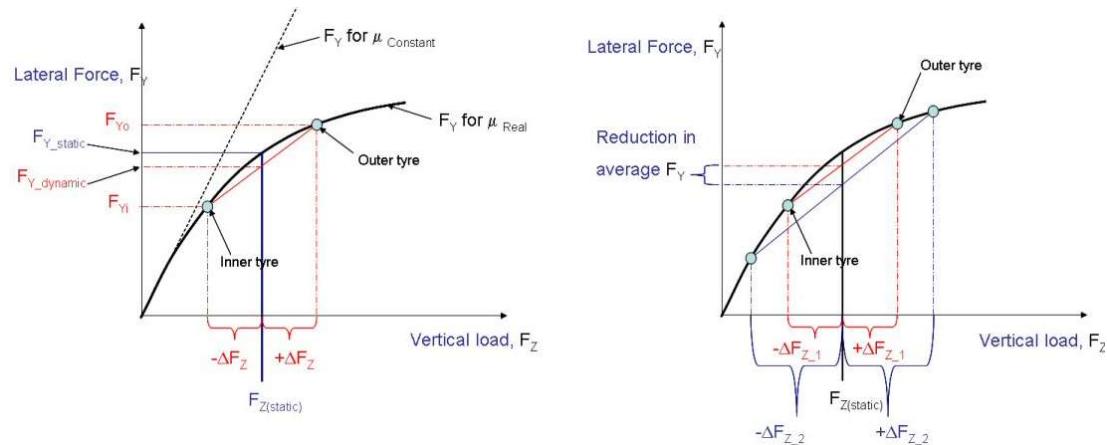


Figure 5.2.1: Vertical load and lateral load transfer (OLIV15)

The total lateral load transfer can be obtained solving the free body diagram in Figure 5.2.2.

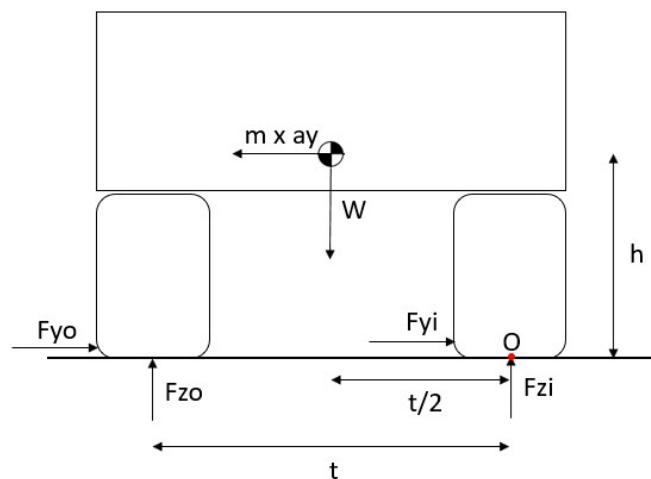


Figure 5.2.2: FBD of the vehicle taking a corner

Taking moments about the contact point of the right tire (point O),

$$m \cdot ay \cdot h - F_{zo} \cdot t + W \cdot \frac{t}{2} = 0 ; F_{zo} \cdot t = W \cdot \left(\frac{t}{2}\right) + m \cdot ay \cdot h$$

F_{zo} can be expressed as: $F_{zo} = \frac{W}{2} + \Delta W ; W = m \cdot g ; ay = v^2/R$

So,

$$\text{Equation 1: } \Delta W = \frac{m \cdot \frac{v^2}{R} \cdot h}{t}$$

It must be said that total lateral load transfer only depends on h , t , the weight of the vehicle and the speed and characteristics of the corner, and the suspension geometry doesn't influence the amount of weight that is transferred.

It will now be analyzed how this weight transfer is split between the front and rear axles, and how the suspension geometry absorbs it. To do so, a FBD (Free Body Diagram) of a vehicle turning to the right will be analyzed.

In Figure 5.2.3, some important parameters that will be used to perform the calculations are shown. In red, we have front and rear Roll Centers, and joining them together we have the roll axis. This is the virtual line about the vehicle will seem to be turning. ϵ refers to the angle the roll axis has with the horizontal line, and h_1 the vertical distance from the CG to the roll axis.

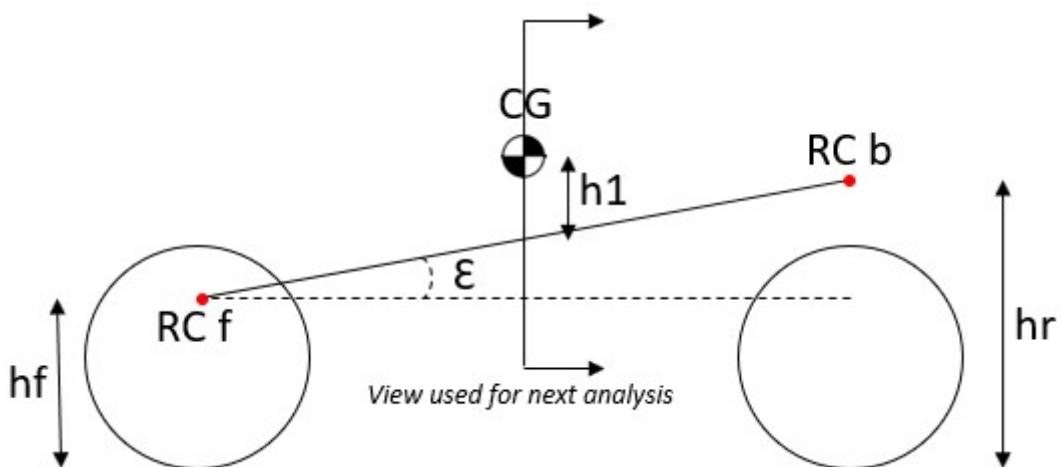


Figure 5.2.3: FBD with the Roll Axis, Roll Centers and Center of Gravity

With that said, Figure 5.2.4 will now be introduced to obtain the roll angle φ about the parameters from the suspension system. We will consider the steady state case, where

$$d\varphi/dt = 0 \text{ and } d\varphi/(dt)^2 = 0.$$

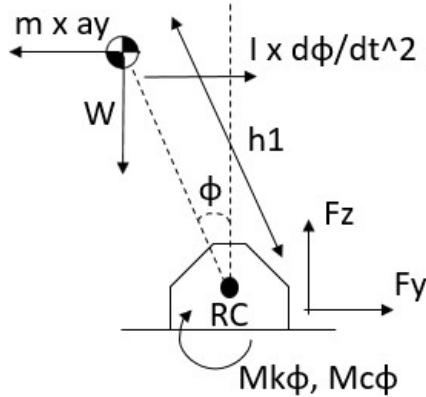


Figure 5.2.4: Forces acting in the Roll Center as the body rolls during cornering

As the vehicle is turning to the right, the CG rolls about the roll axis an angle φ as shown. The suspension system (both spring and damper) reacts this moment, creating the moments $Mk\varphi$ and $Mc\varphi$. Taking moments about the Roll Axis (RC):

$$\Sigma M_{rc} = -I \cdot \frac{d\varphi}{dt^2} + (m \cdot a_y \cdot \cos \varphi + mg \cdot h_1 \cdot \sin \varphi) \cdot \cos \varepsilon - Mk\varphi - Mc\varphi = 0$$

$$\text{Equation 2: } Mk\varphi = (k\varphi f + k\varphi r) \cdot \varphi \cdot \cos \varepsilon$$

Assuming small angles ($\cos \varphi = 1$; $\sin \varphi = 0$):

$$(m \cdot a_y \cdot \cos \varphi + mg \cdot h_1 \cdot \sin \varphi) \cdot \cos \varepsilon - (k\varphi f + k\varphi r) \cdot \varphi \cdot \cos \varepsilon = 0$$

$$m \cdot \frac{v^2}{R} \cdot h_1 = (k\varphi f + k\varphi r - mg \cdot h_1) \cdot \varphi$$

$$\text{Equation 3: } \varphi = \frac{m \cdot \frac{v^2}{R} h_1}{k\varphi f + k\varphi r - mgh_1}$$

This roll angle φ will be the same for front and rear, but the increment on vertical load due to the weight transfer will not be the same. To obtain both ΔF_z , we use the FBD shown in Figure 5.2.5. The front axle, and therefore ΔF_{zf} will be analyzed.

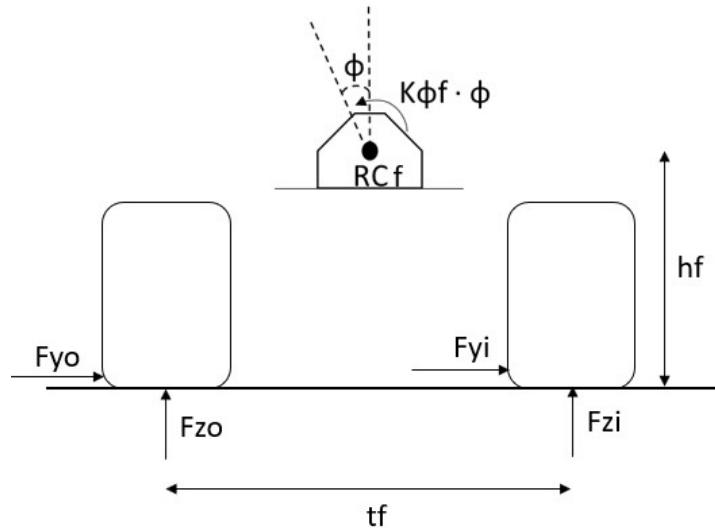


Figure 5.2.5: FBD of the rolling moment and lateral forces on the wheels

$$Fzo = Fzs + \Delta Fzf$$

$$Fzi = Fzs - \Delta Fzf$$

$$Fyo + Fyi = Fyf = \frac{Wf}{g} \cdot ay = \frac{Wf}{g} \cdot \frac{v^2}{R}$$

$$\text{Equation 4: } \Sigma M_{rc} = \Delta Fzf \cdot tf - (Fyo + Fyi) \cdot hf - k\varphi f \cdot \varphi = 0$$

Now, ΔFzf can be obtained:

$$\text{Equation 5: } \Delta Fzf = \frac{\frac{Wf}{g} \cdot \frac{v^2}{R} \cdot hf}{tf} + \frac{k\varphi f \cdot \varphi}{tf}$$

Following the same procedure, ΔFzr can be obtained:

$$\text{Equation 6: } \Delta Fzr = \frac{\frac{Wr}{g} \cdot \frac{v^2}{R} \cdot hr}{tr} + \frac{k\varphi r \cdot \varphi}{tr}$$

Analyzing the results, it can be determined how much of that load transfer happens through the elastic components (springs) of the suspension and how much through the rigid parts (A arms, suspension links between the wheel and the chassis). Since the roll angle φ has also been obtained, by tuning the spring rate of the front and rear, we can determine how much of that weight transfer happens in each axle. This is very important to achieve an optimal tuning of the suspension, since by changing the

spring rates of the front and rear, we can change how the car handles through the corners and can correct the car's tendency to understeer or oversteer.

As an example, let's consider that our car has a tendency to oversteer that needs to be corrected. If the car is oversteering, this means that the rear end is less capable than the front, and that we are going to need less steer angle as we take the turn. To explain this, lets analyze the components in the steer angle equation.

Equation 7, from GILL92:

$$\delta = 57.3 \cdot \frac{L}{R} + \left(\frac{Wf}{C\alpha f} - \frac{Wr}{C\alpha r} \right) \cdot \left(\frac{V^2}{R \cdot g} \right)$$

Were $C\alpha$ is the cornering stiffness, which is the relation between the lateral force and the slip angle of the wheel (and can't be changed by changing the spring rate). If $\frac{Wf}{C\alpha f} > \frac{Wr}{C\alpha r}$, the car will tend to understeer, and the opposite will happen when the car oversteers. Going back to the ΔF_z (front and rear), it can be noticed that if we increase the spring rate in one axle, that axle is going to support more lateral weight transfer. For the example discussed, if the car is oversteering, we can either increase the front spring rate or decrease the rear, decreasing the total weight that is supported by that axle (Wr decreases).

5.2.1 Body Roll

Body roll and its effects on cornering speed will be now analyzed. Even though it is thought that body roll compromises the grip of the car when it is taking a corner, the effects of the CG (center of gravity) displacement towards the outside of the corner is very small compared to the total lateral load transfer. We can obtain this load transfer computing the new distance of the CG to each side. For our car, using the parameters from the Design Spec Sheet (Trackwidth = 1.219 m, Weight = 274 kg, CG height = 0.28 m), and assuming 5 degrees of roll, we can compute the weight at the outside wheels as:

$$W \times (1.219/2 + \text{CG height} \times \sin 5^\circ)/1.219 = 274 \times (1.219/2 + 0.28 \times \sin 5)/1.219 = 142.49 \text{ kg}$$

If we compare this value with the initial weight supported by each side ($274/2 = 137 \text{ kg}$), we only have 5.49 kg of load transfer due to the CG lateral displacement, much smaller than the total Lateral Load transfer, calculated in Sections 6 and 8.

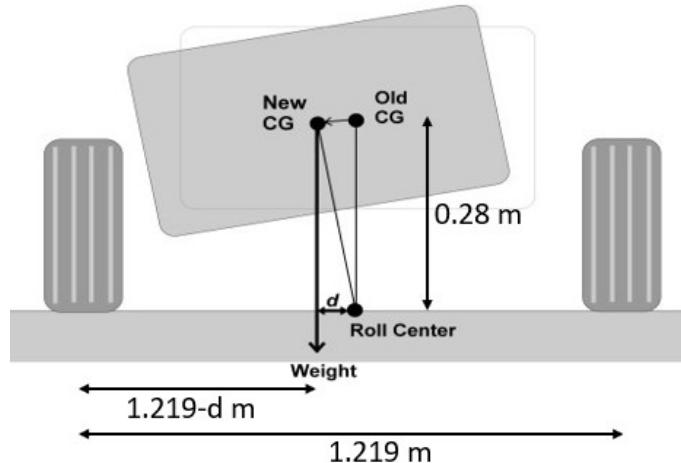


Figure 5.2.1.1: Weight transfer due to body roll (OLIV15)

As the body roll doesn't have a significant effect on load transfer, we want to keep it to an adequate level to avoid contact between the ground and our chassis. It can be a good signal to warn the driver about how close to the grip limit he is during a cornering situation and can allow to change the outside wheel's camber to maximize grip. This is the most important aspect that needs to be studied, because as the chassis of our car is rolling, the geometry of the suspension changes (the A arms or links between the wheel and the chassis change from its initial position), changing the wheel camber and therefore the contact between the wheel and the ground.

Body roll can also affect how nimble the car feels, as it can compromise how fast the vehicle changes direction.

5.3 Longitudinal Load Transfer

Longitudinal load transfer, which will lift or compress both wheels at the same rate, can be analyzed as a fictitious force acting at the CG, due to the acceleration or deceleration rate of the mass of the vehicle. This force is represented in Figure 5.3.1, and the resulting load transfer is calculated in Equation 8.

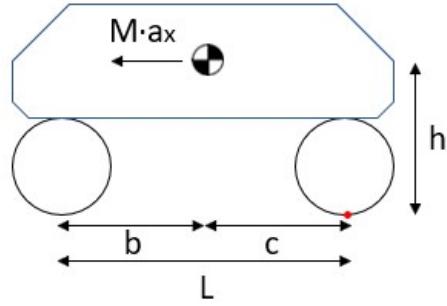


Figure 5.3.1: Longitudinal load transfer

$$\text{Equation 8: Long. Load Transfer} = \frac{h}{L} \cdot \frac{W}{g} \cdot ax$$

Static weight in each axle (without load transfer):

$$\text{Front: } W_f = \frac{c}{L} \cdot W; \text{ Rear: } W_r = \frac{b}{L} \cdot W$$

5.4 Possible Theoretical Advantages Compared to the Previous System

The suspension system that is being developed has one spring and damper for each axle that can be tuned independently just to this purpose. As the diagonal spring only operates during roll, it can be tuned to adequate the understeer gradient without affecting the car's response to pitch. This aspect will increase the performance, as the CG can be kept in its optimal position, which is determined by the pitch spring. This will increase the aerodynamic performance as well, because changing the roll spring rates won't affect the spring compression due to aerodynamic forces, regulated with the pitch spring. With conventional pushrod systems, as we change the spring rates to adequate the understeer gradient (something critical to increase the mechanical grip of our car) other parameters can be compromised.

Also, something similar can be done with longitudinal load transfer. Being able to tune the pitch spring to optimize the response of the car to acceleration or braking, or the aerodynamic efficiency without altering the cornering capability is a huge advantage compared to conventional pushrod systems.

It may also be interesting to have different spring rates for the rear and the front. For example, having a softer rear spring will reduce drag for circuits with long straights (as the rear spoiler has less undisturbed air) and increase acceleration (as more weight is transferred to the rear), but will compromise the vehicle's consistency through the corners for different speeds some of this adjustments were obtain from OLSE18 source.

Once all the equations were obtained, the optimal spring and damper rates need to be analyzed. After having all the necessary parameters, the final values will be introduced in the equation and used to plug the right values in the simulation.

6. Spring and Damping Rates Analysis

The main purpose of developing the mode separated suspension for our car is being able to have more tuning options than the ones we have in a pushrod system. Suspension tuning can make a huge difference in the performance of the car, as it can maximize the grip or aerodynamic efficiency, as well as the driver's feeling.

The goal of the suspension systems is to keep the tires in contact with the road, connect the wheels to the chassis, react the forces generated from the tires and limit the allowable wheel hub with respect to the body, controlling steering and camber orientation.

Choosing the right spring and damping rate can be very difficult. Depending on the characteristics of the track we are racing (whether if there are more fast or slow corners, the length of the straights or the bumpiness of the surface), different suspension tunings will be necessary. Even though we can estimate some adequate values for all the suspension parameters, we need to test our vehicle to determine the optimal values. Some general cases will be now explained, which can help for a better tuning of the car after testing it on the track.

6.1 Spring Rate Analysis

Ideally, different spring rates are going to be needed for all the different conditions that can affect the car. The optimal suspension characteristics for the vehicle running on the straights, during braking and accelerating, or while cornering in fast or slow corners will be different. For this reason, in order to achieve the fastest lap time, a tradeoff between all the different conditions needs to be done. Since conventional pushrod or pullrod systems, the most common suspension system on racing cars, only have one spring and damper connecting each wheel to the chassis, we have less freedom to tune our car, and some aspects may be compromised.

The Mode Separated System developed for this vehicle works in a completely different way compared to conventional suspension systems. For each axle, we have two spring and dampers connecting both wheels and the chassis, that can be tuned separately. The innovative aspect about this system is that, for each axle, different spring and dampers are going to react the motion of the vehicle. This can improve the performance of the vehicle as we can tune each spring depending on the motion, without compromising the vehicle's reaction to other situation.

Spring rates affect not only mechanical grip, but also aerodynamic performance and the driver's feeling. Higher spring rates will have more resistance to be compressed, and softer ones will compress more easily.

Racing cars tend to have higher spring rates than passenger cars, as the driver's comfort is not a priority. The main advantage of higher spring rates is that the vehicle will have less suspension travel, which allows to lower the car to an optimal position in terms of performance. As the center of gravity sits lower with respect to the ground, the lateral and longitudinal load transfer are reduced, improving the performance of the vehicle while cornering and braking or accelerating.

Racing cars are designed to have an optimal ride height where the aerodynamics work more efficiently. Lifting or lowering the car from this point can make a huge difference in the downforce and drag generated and compromise the overall performance. A higher spring rate will decrease this CG motion along the vertical axis, allowing for a more consistent aerodynamic behavior regardless of speed. This is not always true, as sometimes having a lower rate in the rear can decrease drag in straights (since the rear spoiler sits lower it has less air resistance) and increase downforce in slow corners. Having an excessive stiff spring rate will result in a harsh ride, compromising the grip during cornering, braking and accelerating.

On the other hand, a softer spring rate will allow for a better vehicle response to the track imperfections. As the vehicle has more suspension travel, the wheels will keep contact with the ground, increasing the grip (if the suspension is too stiff, the vehicle may go airborne when a bump is hit, which can result in spinning and having an accident).

Analyzing the mode separated system's optimal spring and damping rates differs slightly from the conventional suspension system's analysis, found on source OPTG19 and GILL92. It has two spring and dampers connecting both sides of the car for each axle (acting for different motions), and this aspect needs to be considered in the calculations.

The optimal spring rates for the different motions of the vehicle will be now explained

6.1.1 Ride and Pitch

The first step in choosing the adequate spring stiffness is to specify the desired ride frequency. This frequency will depend on the type of vehicle and will affect directly the suspension design. Ride motion defines how the car will behave under the general road irregularities, as well as react the downforce forces or longitudinal load transfer (as it will be the same as the pitch spring). How the vehicle's motion under these movement is shown in Figure 4.2.2. For our project, we are using similar ride frequencies as the ones we had in TR19. The normalized values for racecars are between 3 and 5 Hz, so for our vehicle a natural frequency of 4 Hz will be chosen. Higher frequencies produce harder (stiffer) suspensions with faster responses to irregularities but less mechanical grip, since the wheel stays less time in contact with the ground. Higher frequencies, since they are stiffer, create lower suspension travel, improving aerodynamics and overall efficiency. The suspension travel will be twice the one in the previous suspension, as both wheels acts the same spring instead of having one spring for each wheel.

It is also important to note that frequencies for front and rear are normally different. For road cars, and to improve comfort, the rear has a higher frequency in order to 'catch up' the front when a bump is hit. As the frequency is higher, the response of the rear axle will be faster, to counter the time interval between the bump hitting both axles, which will depend on the wheelbase. Therefore, as comfort is not important in racing cars, this situation can differ a lot. As racecars have higher damping ratios, the car will oscillate less when a bump is hit, so other aspect will be prioritized. In contrast, for our vehicle it is interesting to run higher frequencies for the front, as they improve transient response at the beginning of the cornering situation, less CG height variation on the front, improving aerodynamics as the front of the car is usually affected in a bigger way. Also, having a lower frequency in the rear can improve traction when accelerating and reduce drag in the straights, as the CG will be lowered more than with a stiffer spring rate.

Before starting the calculations, it is necessary to obtain the motion ratios for both the roll and pitch springs. The motion ratio, or installation ratio, indicate how much the spring moves with respect to the wheel for a certain wheel travel. For the calculations, the equivalent spring of the system in the wheel is needed, and the installation ratio (or motion ratio) allows to find the equivalent spring rate for the corner or axle of the vehicle supported by the springs. The Mode Separated System was designed to have the same motion ratios for the left and right tires. Even though the geometry is not symmetric, this was achieved by playing special attention to the geometry of the rockers and the mounts of the chassis.

These are the motion ratios for just one wheel. To calculate it, we let the wheel free in the sketch model developed in SolidWorks and measure the distances before and after the movement for each spring.

Front:

$$MR \text{ pitch} = \frac{\Delta X \text{ spring}}{\Delta X \text{ wheel}} = \frac{57.5 - 27.4}{30} = 1$$

$$MR \text{ roll} = \frac{\Delta X \text{ spring}}{\Delta X \text{ wheel}} = \frac{84.5 - 51.5}{30} = 1.1$$

Rear:

$$MR \text{ pitch} = \frac{\Delta X \text{ spring}}{\Delta X \text{ wheel}} = \frac{47.7 - 26.2}{30} = 0.72$$

$$MR \text{ roll} = \frac{\Delta X \text{ spring}}{\Delta X \text{ wheel}} = \frac{67.9 - 45.7}{30} = 0.74$$

An adequate ride motion will determine the behavior of the car, since the ride spring rates are determined by the natural frequencies. In the mode separated system, the ride spring (horizontal one) can be freely tuned to meet the requirements without affecting the car behavior during roll.

To obtain the ride spring rate, only the horizontal (pitch and ride) spring motion ratio will be considered, as the roll spring doesn't contribute to react this motion. Spring rates will be different for front and rear since different spring rates will be needed.

The equation that relates the axle natural frequency is the following:

$$\omega = \frac{1}{2\pi} \cdot \sqrt{\frac{kr}{M}}$$

Where ω is the natural frequency, kr is the equivalent spring in ride, and M is the weight supported by one wheel (the mass in that axle divided by 2).

Plugging in the values for our vehicle ($\omega_{\text{front}} = 4 \text{ Hz}$; $\omega_{\text{rear}} = 3.7 \text{ Hz}$; Mass wheel front = $129.3/2 = 64.65 \text{ kg}$; Mass wheel rear = $142.7/2 = 71.35 \text{ kg}$), the values for $k_{r \text{ front}}$ and $k_{r \text{ rear}}$ are obtained:

$$k_{r \text{ front}} = 40836.5 \text{ N/m}$$

$$k_{r \text{ rear}} = 38594.6 \text{ N/m}$$

These ride frequencies are not the ones we will need for our springs. They are obtained from the corner vehicle model, and they are the resultant of the combined tire spring rate (kt) and equivalent wheel rate of our spring (kw). The following diagram shows this simplification:

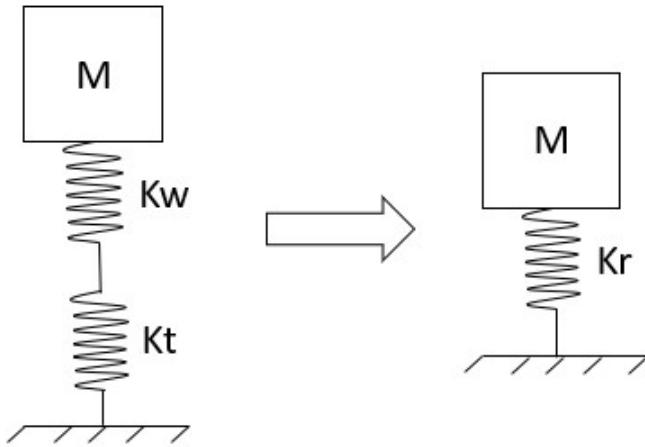


Figure 6.1.1: Equivalent spring of a system in series

Since we have two springs in series, it can be stated that $kr = \frac{kw \cdot kt}{kw + kt}$

For kt , we can assume it to be $kt = 250000 \text{ N/m}$. As the tire is compressed, it acts like a spring reacting this movement. This value is a general case for racing tires.

Now that all the values are known, the equivalent wheel spring rate for the front and rear can be obtained:

$$Kw \text{ front} = 48809.3 \text{ N/m}$$

$$Kw \text{ rear} = 45594.6 \text{ N/m}$$

To finally obtain the spring rate, motion ratios need to be considered. An energy analysis will be made to obtain the spring stiffness in terms of the spring rate at the wheel and the motion ratio.

$$U = \frac{1}{2} \cdot kr \cdot (\Delta Xw)^2 = \frac{1}{2} \cdot ks \cdot (\Delta Xs)^2 ; \frac{1}{2} \cdot (MR \cdot \Delta Xw)^2 = \frac{1}{2} \cdot ks \cdot MR^2 \cdot (\Delta Xw)^2$$

From this previous equation, it can be seen that $kw = ks \cdot MR^2$. Therefore, and plugging the values of our vehicle:

$$Ks \text{ front} = 48809.3 \text{ N/m}$$

$$Ks \text{ rear} = 87952.5 \text{ N/m}$$

From the previous results it can be seen how important the motion ratio is for the final spring rate, as it can sometimes differ a lot from the equivalent ride wheel rate.

6.1.2 Roll

The roll spring rate will be now analyzed. The vehicle's behavior during roll can be seen in figure 4.2.3. To begin and following a similar procedure as with the ride spring, a roll stiffness must be chosen. This roll stiffness indicates how much the body will roll per 1G of lateral acceleration. The higher the roll gradient is, the more body roll the car will have. For racing cars, it is interesting to have lower roll gradient, which allows for faster direction changes, but can reduce mechanical grip if a bump is hit during cornering. The roll gradient (ϕ/A_y) takes values from 0.2 to 0.7 for racecars with high stiffness springs. For our vehicle, we will select a roll gradient of 0.45. This roll gradient is not split equally between both axles, and a common distribution is to use a 5% higher division than the static front weight distribution. Using this criterion, the front will take $47.5\% + 5\% = 52.5\%$, and the rear 47.5% . To relate the roll gradient ($\frac{\phi}{A_y}$) with the total roll stiffness ($K\phi$), the following formula is used, were W is the total weight of the vehicle and h_1 the distance from the roll center to the CG:

$$\frac{\phi}{A_y} = -W * \frac{h_1}{K\phi_{\text{total}}}$$

Solving for the values of our vehicle ($W = 272 * 9.8 \text{ N}$, $h_1 = 0.206 \text{ m}$, $\frac{\phi}{A_y} = 0.45$), $K\phi_{\text{total}} = 1245.16 \text{ Nm/deg roll}$. The front, as it takes 52.5%, will have $K\phi_{\text{front}} = 684.838 \text{ Nm/deg roll}$ and the rear, $K\phi_{\text{rear}} = 591.451 \text{ Nm/deg roll}$.

Now, using moments equation, the ride spring rate needs to be expressed in terms of the roll stiffness and the parameters of the vehicle. To do so, a moment M is applied in the roll center as shown. K_r is the equivalent spring rate for each tire, and even though we only have one spring for both tires, K_r considers the contribution of this spring and the tire stiffness to each tire.

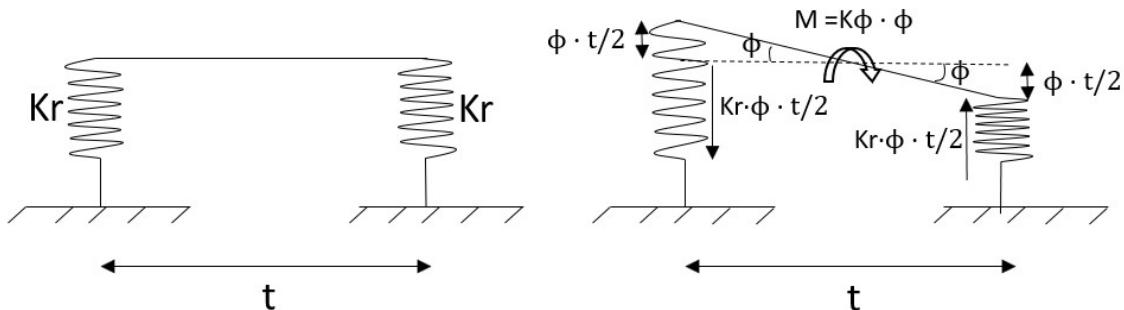


Figure 6.1.2.1: FBD of the roll moment affecting the equivalent spring in each corner

Taking moments about the roll center, and considering ϕ in radians: $M = K\phi \cdot \phi = Kr \cdot \phi \cdot t/2 \cdot t/2 + Kr \cdot \phi \cdot t/2 \cdot t/2$

$$K\phi = Kr \cdot \frac{\pi}{180} \cdot \frac{t^2}{2}$$

As for the ride spring, $Kr = \frac{kw \cdot kt}{kw + kt}$ (being kw the wheel spring rate ($kw = ks(roll) \cdot MR^2$) and Kt the tire stiffness = 250000 N/m).

Solving, we have ($t_{front} = t_{rear} = 1.219$ m):

$$K\phi_{front} = Kr_{front} \cdot \frac{\pi}{180} \cdot \frac{t_{front}^2}{2}; Kr_{front} = 52812.1 \text{ N/m}$$

$$K\phi_{rear} = Kr_{rear} \cdot \frac{\pi}{180} \cdot \frac{t_{rear}^2}{2}; Kr_{rear} = 45610.4 \text{ N/m}$$

$$Kw_{front} = 53951.8 \text{ N/m}$$

$$Kw_{rear} = 46458.1 \text{ N/m}$$

Knowing the motion ratios for the roll spring ($MR_{front\ roll} = 1.1$; $MR_{rear\ roll} = 0.74$)

$$Ks_{front\ (roll)} = 44588.3 \text{ N/m}$$

$$Ks_{rear\ (roll)} = 81424.9 \text{ N/m}$$

As it can be observed, the ideal values for the pitch and roll spring rates are a bit different. In conventional suspension systems, to achieve the desired spring rate for a roll rate given, it is necessary the use of Anti Roll Bars (Figure 6.1.2.2). These devices connect springs from both sides of the axle, and they act during roll motion to add additional stiffness to that axle. The problem with these ARB is that they are designed to supply a limited range of stiffness, and they have a limited number of tuning options. Also, they add unsprung weight to the vehicle, which can compromise its performance.

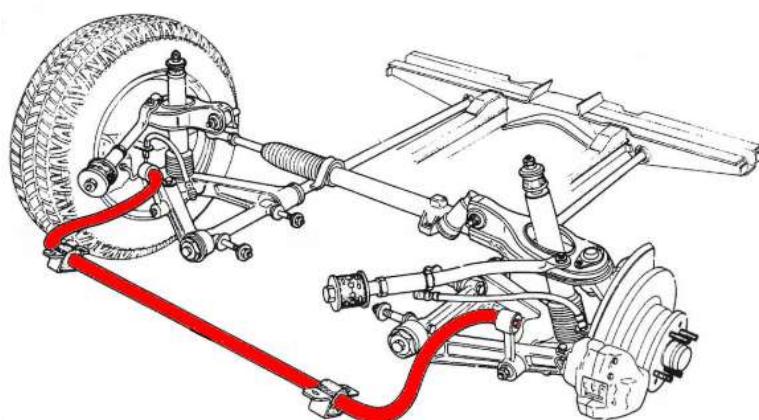


Figure 6.1.2.2: Drawing of an Anti-Roll Bar (ROBI18)

In the mode separated suspension system, as the roll spring can be tuned to a desired rate, we avoid using ARBs, saving weight and having many more tuning options compared to the conventional system. Also, it is possible to tune the roll rate without affecting the ride and pitch performance, as they are limited by a different spring. It is also important to notice that having a spring that only limits roll will allow to tune the understeer – oversteer tendency of the car just by changing these spring rates. For conventional pushrod systems, as the stiffness of an axle is changed for this purpose, the ride and pitch performance are altered.

6.1.3 Single Wheel Bump

Finally, the motion that needs to be analyzed is the single wheel bump. As the wheel goes up, it compresses both ride and roll springs (Figure 4.2.4), so the combined spring rate for this motion will be the sum of K_s roll + K_s pitch, as they are in parallel. The resultant spring rate will be much larger than the optimal one for this motion, because it is interesting to have small natural frequencies, and therefore small spring rates for the wheel to not lose contact with the ground. This is the main disadvantage of the Mode Separated System compared to a conventional System with one spring per wheel, as the total spring rate per wheel for this motion will be much bigger. This can result of the car going airborne when a bump is hit by a wheel, but since most of the racetrack have smooth surface and the most important motions to properly tune the car are roll and ride, it won't have that much impact on the overall performance. Even though, if this suspension System is chosen, it is very important to warn the drivers about this aspect, for them to avoid cutting corners and hitting bumps.

The resulting frequency for this motion will be:

$$\omega = \frac{1}{2\pi} \cdot \sqrt{\frac{k_{s.w.bump}}{M_{corner}}}$$

Being k (single wheel bump) = k ride + k roll

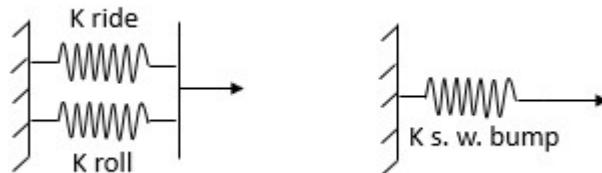


Figure 6.1.3: Equivalent spring of a system in parallel

Plugging in the parameters of our vehicle, the natural frequencies for front and rear are:

$$\omega_{front} = \frac{1}{2\pi} \cdot \sqrt{\frac{46117.7}{64.65}} = 4.25 \text{ Hz}$$

$$\omega_{rear} = \frac{1}{2\pi} \cdot \sqrt{\frac{43155.6}{71.35}} = 3.91 \text{ Hz}$$

These frequencies are greater than the ones obtained for ride. As we said, this aspect can limit the performance of the vehicle when hitting a single wheel bump compared to the current suspension system of TR19. However, as the previous suspension system has Anti-roll Bars, the ride frequency of this motion was compromised as well. ARBs add an additional stiffness to have the desired roll frequency, but also affect when the vehicle hits a bump.

6.2 Damping Rates Analysis

Next step in the suspension design is analyzing the dampers and choosing the adequate damping ratios (ζ) for every motion. Dampers produce force proportional to the speed of compression or elongation, which means that they only produce force when their internal parts are in motion. The higher the velocity, the greater is the reaction force. Their main purpose is to damp the vehicle from oscillating at its natural frequency. Without the dampers, the vehicle will have no resistance to oscillation at any road irregularities. Choosing the proper amount of damping can increase the performance of the vehicle, as having too much underdamping or overdamping can reduce grip and increase tire wear variation (having changing tire forces).

The first aspect that needs to be determined is the critical damping coefficient, defined as the necessary damping to return an oscillating mass to its steady state with no overshoot as quickly as possible. It can be obtained as: $C_{cr} = 2 \cdot \sqrt{K_r \cdot M}$. The damping ratio shows the relation between the actual damping of the system and the critical damping. It can be obtained as: $\zeta = C/C_{cr}$. Figure 6.2.1 shows the response of the system to different damping rates, found on source BALA09.

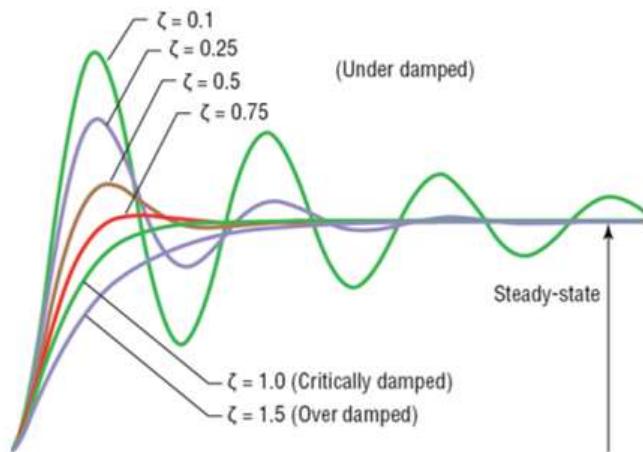


Figure 6.2.1: Generic Damping Ratios (BALA09)

Dampers are very important for suspension tuning. Since they act like a dynamic spring (providing force depending on the velocity), they can split longitudinal and lateral load transfer at the right rate and moment from one corner of the vehicle to another. This feature is important in corner transitions. As the Mode Separated system has a spring acting just during roll and another one for pitch and ride, there are many more tuning options, and it is much easier to see the reactions of the car when changing one of them. To illustrate how important this is, and completing the information on source KASP19, a cornering situation will be explained with the optimal damper adjustments for every section. As we can see in Figure 6.2.2, there are 6 different sections to analyze during a cornering situation.

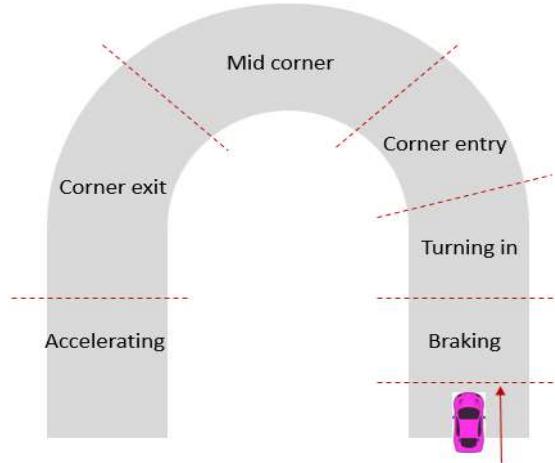


Figure 6.2.2: Different phases on a cornering situation

During braking, there is some longitudinal load transfer that needs to be controlled by the dampers and springs. Analyzing the damping, if this weight is transferred too slow (too much rear rebound or front compression), braking force will not be optimal, increasing braking distance. On the other hand, if this transfer occurs too fast (small compression for both axles), the front tires may lock up because they won't be able to absorb such a rapid load transfer. A right tuning of the ride damper can reduce braking distance, so apart from the said before, we must watch for a wrong amount of damping in the rear, which can make the car unstable (too much and the rear axle will tend to lift, and too little and the rear will pop up).

As the car approaches to the turning operation, almost all the braking must be done. When the initial turn in occurs, we want the vehicle to respond as fast as possible. The most important corner of the vehicle to control this steering is the front right corner. Even though the roll spring is controlled to load this tire as the vehicle rolls, it requires an amount of displacement in the spring to load it. Since we have a damper controlling roll as well (unlike common suspension systems) we can respond faster, since the dampers will start applying force as soon as the vehicle starts rolling. This will increase the vehicle turning response, but it is important to apply the correct amount of damping, as too much can make the wheel loose capability producing understeer. Like the springs, they can also control lateral weight transfer, especially rear rebound.

As the next cornering section approaches, dampers play an important part on how this lateral load transfer is delivered, timing the rate of load transfer. Analyzing the force vs velocity curve, as we have less compression to speed rate, the load transfer happens more slowly. An interesting thing that can be done if the car starts losing grip in this section is add more rebound to the inside wheels (we can control compression and rebound of the dampers separately).

During the mid-part of the corner, the vehicle has reached steady state and will move based on the front and rear stiffness, so the dampers won't play an important part in this section.

During the corner exit, the vehicle transitions from turning in 'steady state' into accelerating. Some longitudinal and lateral load transfer will happen at the same time, since the inside front tire will start

to lose weight to the outside rear one. Setting the right amount of rear compression can avoid oversteering, and the proper front rebound will keep the front tires in contact with the ground. If the car begins to oversteer, the best option will be to soften the rear roll and ride damper compression, reducing the timing of the weight transfer to the rear and making this weight transfer more gradual. On the other hand, if the car understeers, we should increase the rear compression or front rebound, as the weight transfer is happening too slow. Increasing the front rebound too much can lift the front tires, so we must be cautious with that.

Finally, in the acceleration phase, and similarly to the braking phase, there is some longitudinal load transfer from the front to the rear. The faster the weight is transferred to the rear wheels, the better traction you get. However, transferring the weight too fast can make the rear wheels to start spinning. Adjusting the pitch damper's rear compression and front rebound will improve the timing of this weight transfer.

Being able to have different compression and rebound characteristics for ride and pitch dampers will increase the grip through the corner enormously. As we showed, conventional suspension systems have to do a tradeoff between the optimal configurations for the different motions, losing grip in some sections. The Mode Separated System allows to obtain a lot more grip as the vehicle can be tuned for different damping rates for pitch (braking and accelerating) and roll (corner entry and corner exit).

As for spring rates, ideal damping rates will be different for ride and roll and will depend on the spring rates previously calculated.

6.2.1 Ride and Pitch

Some typical values for racecar damping ration in ride are between 0.65 and 0.70, as comfort is not a priority, which allows to maximize body control and faster responses. For our vehicle, a damping ratio of 0.68 is selected, for both rear and front. Using the values previously obtained, we can now get the parameters needed:

$$C_{cr\ ride\ (front)} = 2 \cdot \sqrt{K_{r\ front} \cdot M_{front}} = 3229.36 \text{ N/(m/s)}$$

$$\zeta = C/C_{cr\ (front)}; C_{ride\ (front)} = 2195.96 \text{ N/(m/s)}$$

$$C_{cr\ ride\ (rear)} = 2 \cdot \sqrt{K_{r\ rear} \cdot M_{rear}} = 4693.59 \text{ N/(m/s)}$$

$$\zeta = C/C_{cr\ (rear)}; C_{ride\ (rear)} = 3191.64 \text{ N/(m/s)}$$

However, as the damping force is not linear, in order to tune for maximum grip, we have to design the force vs velocity curve for our damper. Once that is done, selecting the adequate tuning option from the ones available will be much easier. For every vehicle, there is a crossover point which divides the low frequencies from the high ones. This point can be found at $\sqrt{2 \cdot \omega_{natural}}$. At low frequencies, a higher damping ratio is preferred since transmissibility (ratio between the output and input amplitude for a given response) is reduced. On the other hand, for high frequencies, a lower damping ratio will be preferred. The first thing that needs to be calculated is the initial slope for the Force vs velocity curve.

For the ride and pitch motion, this slope (named as ideal slope in Figure 6.2.1.1) can be calculated as: $4 \cdot \pi \cdot \zeta(\text{ride}) \cdot \omega \cdot M$. Plugging in the parameters of our vehicle, the following results are obtained for front and rear:

Front damper initial slope: 4419.4 N/(m/s)

Rear damper initial slope: 4633.68 N/(m/s)

The F vs V curve provided by the manufacturer takes another aspect into account, and that is that for high speeds, it is preferred to reduce damping ratio to avoid harshness. For this reason, the force vs velocity slope is reduced to $\frac{1}{2}$ of its value in low speed.

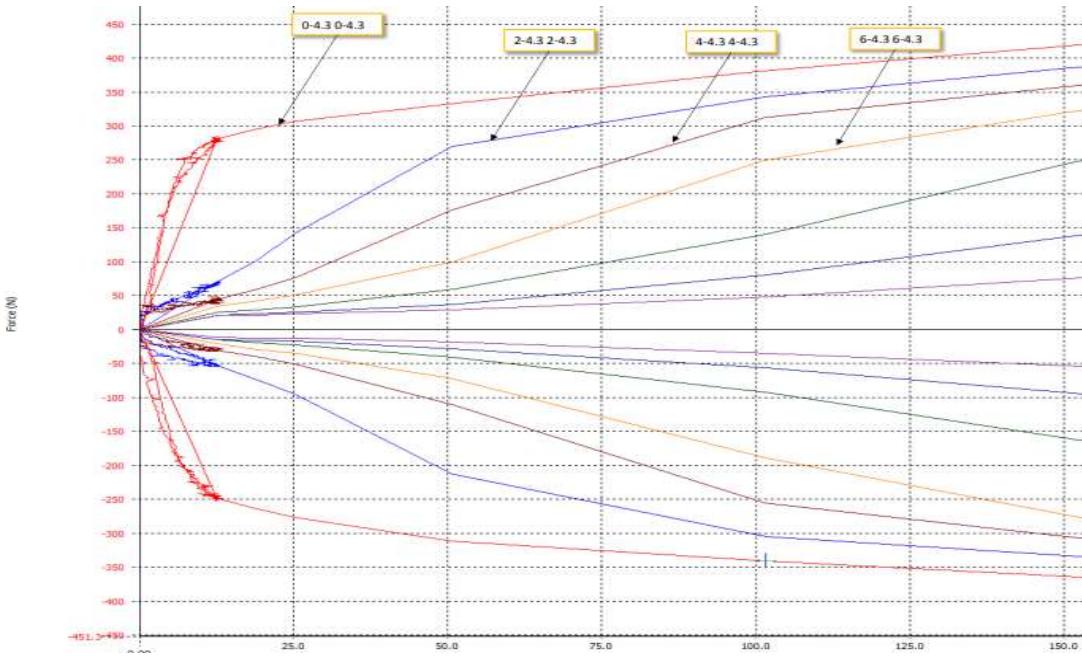


Figure 6.2.1.1: Force vs Velocity plot of the Ohlins TTX25 MkII damper (OHLI19)

We will therefore start with the dark blue configuration for both front and rear (numbers refer for adjustments made in the damper parameters).

However, most of the time the theoretical values differ from the adequate configuration. Some assumptions are being made relating the calculation of the optimal damping coefficient of the vehicle or spring rates and can depend on the circuit or preferences of the driver. For this reason, testing is very important to achieve better lap times and sometimes the final values are different from the theoretical ones.

6.2.2 Roll

Analyzing roll, it is important to say that conventional suspension systems don't use a specific damper for roll, as the Anti Roll Bars used to achieve the desired roll stiffness doesn't contribute in damping at all. Having a damper for roll will hugely increase the grip going over corner and reduce tire wear. Following a similar procedure as in roll, an initial damping ratio must be chosen. An adequate damping will be 0.65, which maximizes grip and makes the car nimble for changes in direction. Using the same equations as in pitch, and the parameters for the roll motion, we obtain:

$$C_{cr\ roll\ (front)} = 2 \cdot \sqrt{K_{roll\ front} \cdot M_{front}} = 5226.32 \text{ N/(m/s)}$$

$$\zeta = C/C_{cr\ (front)}; C_{(front)} = 3397.11 \text{ N/(m/s)}$$

$$C_{cr\ roll\ (rear)} = 2 \cdot \sqrt{K_{roll\ rear} \cdot M_{rear}} = 5102.39 \text{ N/(m/s)}$$

$$\zeta = C/C_{cr\ (rear)}; C_{(rear)} = 3316.56 \text{ N/(m/s)}$$

It is important to note that the spring and dampers used in roll have to act in the same way for compression and elongation, as they must have the same characteristics for both right and left corners. For this reason, the roll damping plot (developed for the previous motion) must be as symmetrical as possible.

6.3 Springs and Shock Absorbers Used

The Shock absorbers from last edition's TR19 will be used. The model name is Ohlins TTX25 MkII, shown in Figure 17. The available spring rates for this spring are: 150 lb/in, 175 lb/in, 200 lb/in, 225 lb/in, 250 lb/in, 300 lb/in, 350 lb/in, 400 lb/in, 450 lb/in, 500 lb/in, 550 lb/in, 600 lb/in and 650 lb/in. (NOTE: 1lb/in = 0,1751 N/mm). They are available in two lengths: 200 mm and 267 mm, so they will fit the designed proposed. All the information about other characteristics or more damping curves can be found in Ohlins website (OHLI19).



Figure 6.3: Ohlins TTX25 MkII (OHLI19)

7. Computer Modeling the Suspension System

After sketching how the suspension system looks like, the next step was investigating what computer programs can be useful to develop the system, from designing the components to analyzing the different parameters and results needed.

For previous editions, the Team used a software called Lotus Shark, which allows to sketch the wheels, connection arms, springs and other suspension components to analyze its motion under different working conditions. The Team used it to obtain the motion ratios for the suspension, as well as static roll center and other parameters. The problem with this software is that, for the version available (and the one used by the team), the suspension system has to be symmetrical. This means that we could not use it, as the Mode Separated configuration doesn't meet the requirements.

Another important thing to consider is that some of the parts needed, like the monocoque, wheels and spindles or A arms were already designed in SolidWorks as they are going to be the same as the ones we can find in TR19. For the 3D assembly of the car with the suspension system, these parts will be reused. This will save time, as some of the parts like the monocoque or the rear frame are complex to design.

Because Lotus Shark was not an option, some research was done in order to find possible computer programs that will meet the requirements.

7.1 Matlab and Simscape Multibody

After analyzing different alternatives, it was decided that one of Matlab Add-Ins, called Simscape Multibody (or Simmechanics), would be the best software to begin analyzing the system, as we would be able to model the whole suspension system, obtain all the graphics needed and use a Matlab script to work with the different parameters.

According to the information that can be found in source MATL19, Simscape Multibody is a 3D design and analysis software for several mechanical systems. It allows to model multibody structures using blocks. These blocks represent solid bodies, joints, forces and moments, physical constraints, sensors or motors and actuators. Unlike Simulink, where it is necessary to create equations of motion for the system to be solved, Simscape solves the resultant equations of motion as the block are joined together. It is also possible to import complete assemblies from Computer Aided Design programs such as SolidWorks, including all the joints, shapes or constraints. When the simulation is completed, a 3d animation is generated in order to show the dynamics of the completed system.

According to the description, this program allowed to do every aspect of the suspension analysis, but since nobody in the team had previous experience, all the basic commands and how to work with the imported parts and files from SolidWorks needed to be learnt.

The way the program works is by connecting different blocks together (each of them with different features that will be explained) and then running the model to see a 3D animation of the system. In Figure 7.1.1, some of the main blocks that can find in Simmechanics are shown.

The ones on the left provide all the initialization parameters, such as the origin and coordinate system, parameters like gravity and the direction it will be acting, and other variables needed to run the simulation.

To its right, we have the three main blocks we can find in the program.

- The first one is the *Solid Block*, in which you can provide the shape and mass properties of the parts you will be working with. You can also specify where to place the coordinate system for that part, what can help you for future steps. If the shape of your solid is complex, it can be imported from Solidworks, saving the file in this program as a STEP file. This was done to import the monocoque, back part of the frame and A arms.
- To its right, the *Transform Block*, which allows to rotate and translate the parts. This is a crucial block, as it allows to specify the connection between all your components and place them in the right position in the system. There are multiple choices on how to rotate and translate the parts, but it is important to keep in mind how and where you placed the coordinate systems in the parts you will be connecting.
- The third one on the right is a *Joint block*, which allows motion between two parts. There are multiple kind of joints, depending on the operation you want to perform.

Even though we have only included the three main ones, there are many more blocks that allow you to perform any operation you need. You can also group different blocks as a subsystem, which can be very useful when you are creating a large model.

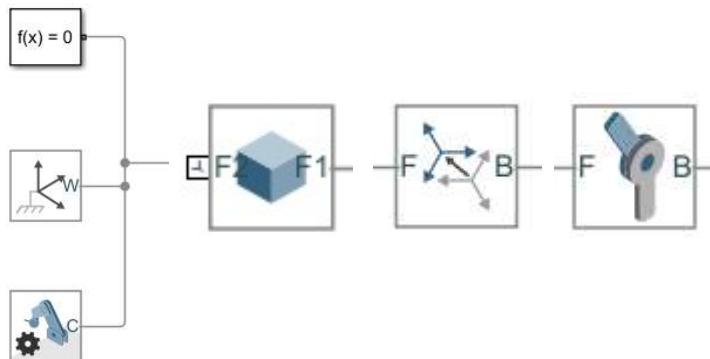


Figure 7.1.1: Main blocks in Simscape Multibody or Simmechanics (MATL19)

In order to start designing the suspension system, a very useful file was found in the MATL19 source, which showed the design of the front part of a generic pushrod system. This file helped to understand how the program works and how to perform more complex operations. The whole block diagram is shown in Figure 7.1.2. The program has many Subsystems, so the display of all the parts and connections where easier to understand. The main Subsystem, which include the frame, rockers, pushrods, connection arms and wheels subsystems is shown in Figure 7.1.3.

To see the motion of the different components and the results of the different scopes you have added, the simulation needs to be run.

In the example, the wheels are interacting with the platforms that provide the input motion for each wheel. These platforms move according to a road profile that was already uploaded and can be changed depending on the working conditions of our vehicle. The wheels were under the main assembly subsystem, as well as the other parts of the vehicle and the connections between them. An example of the model running is shown in Figure 7.1.4, where the platforms are oscillating with a Mid frequency sinusoid with the left and right road profiles out of phase.

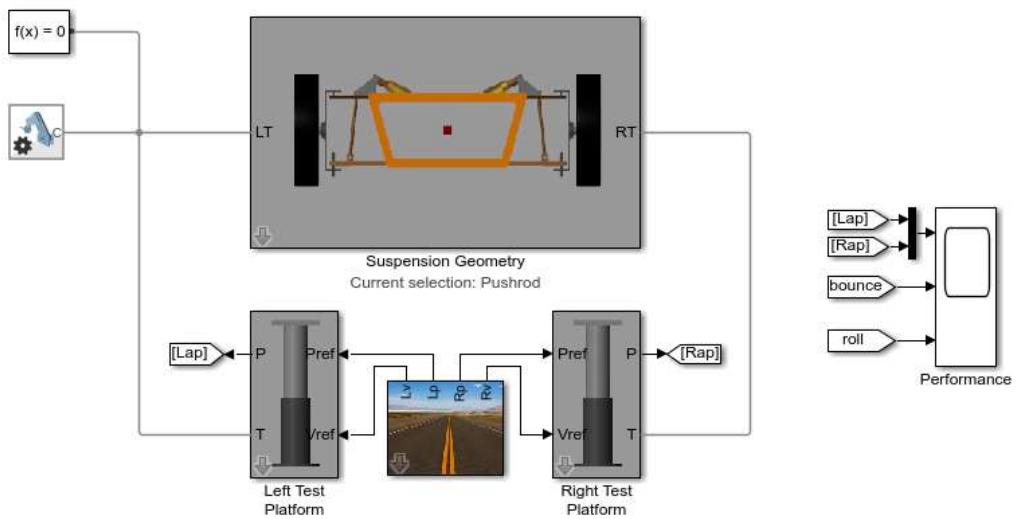


Figure 7.1.2: Main subsystems for the pushrod suspension example (MATL19)

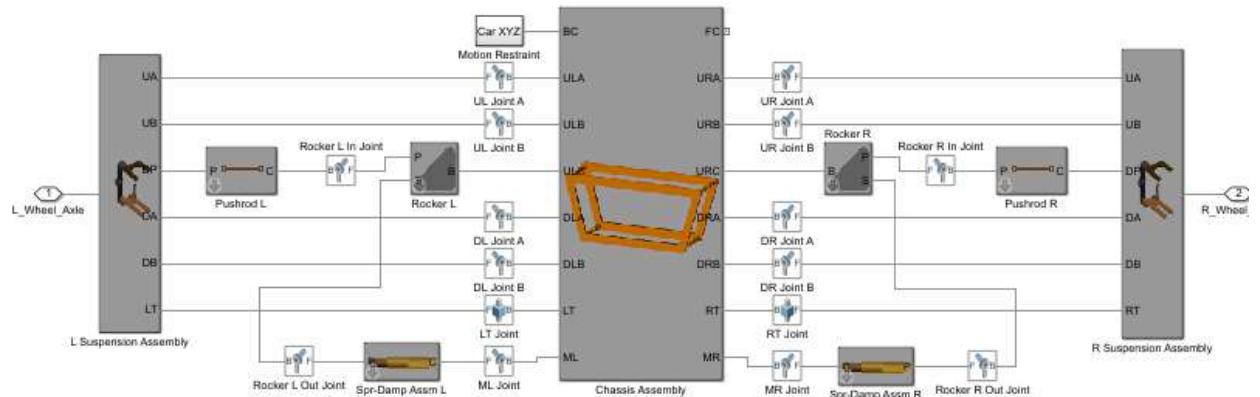


Figure 7.1.3: Different blocks and subsystems under the main assembly (MATL19)

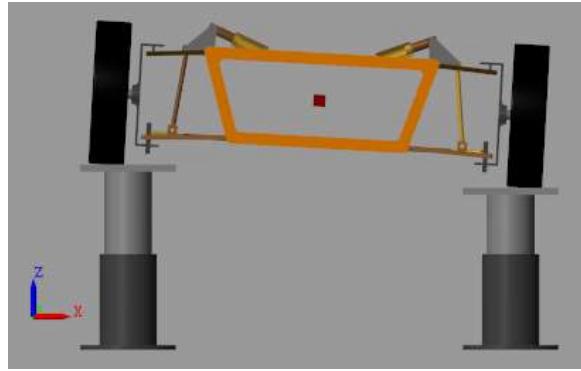


Figure 7.1.4: Results of running the simulation for the example (MATL19)

To start modeling the system, we need to consider that the whole model of the car needs to be created, with the four wheels interfacing with platforms. To do that, we first imported all the complex solids needed from SolidWorks. These files included the monocoque, back frame wheels and all the tubes that will be forming the A Arms. There is an option to transform a SolidWorks assembly to Simscape (by using the Simmechanics link you can find in the Matlab webpage), but the resultant files were very difficult to work with, and weren't working properly, so it was decided to start from scratch. On Figure 7.1.5 it can be seen all the blocks corresponding to the monocoque, the A Arms tubes and the connection and transform blocks between them. The nomenclature of the different blocks were according to the following criteria: F (front), R (right), L (left), UCA (upper control arm), LCA (lower control arm), 1 (the bar in the A arm closer to the front), 2 (the bar in the A arm far from the front).

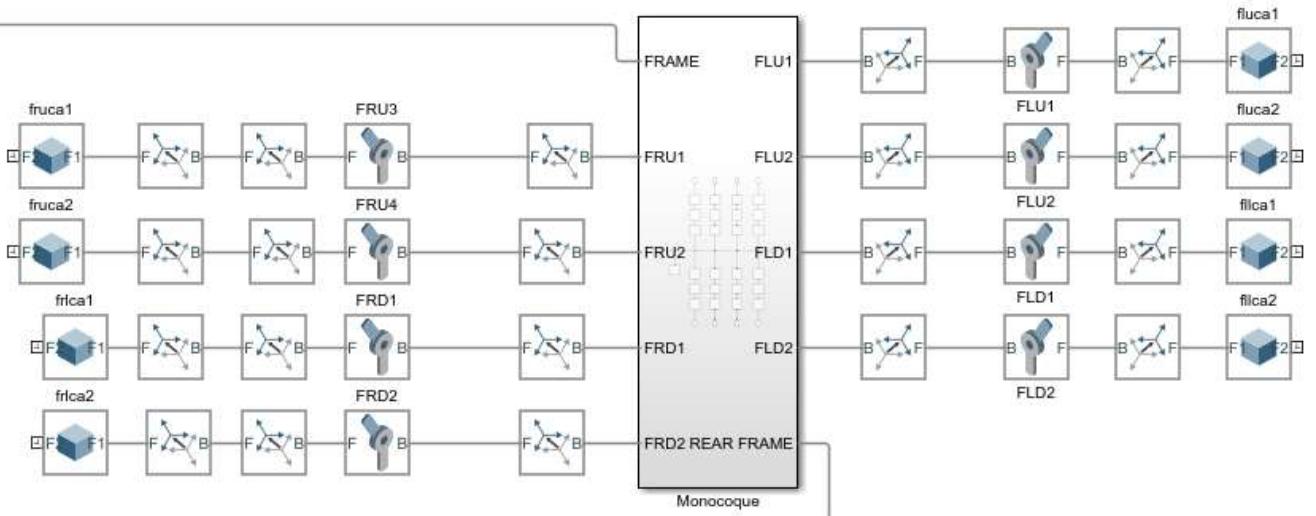


Figure 7.1.5: Simscape model developed for the Mode Separated System

In Figure 7.1.6 the result of the simulation is shown. The tubes oscillate due to their own weight from the connection point in the monocoque, even though they are shown steady in their initial position. As

they have no interference or viscous element in their connection with the chassis, this oscillation was perpetual.

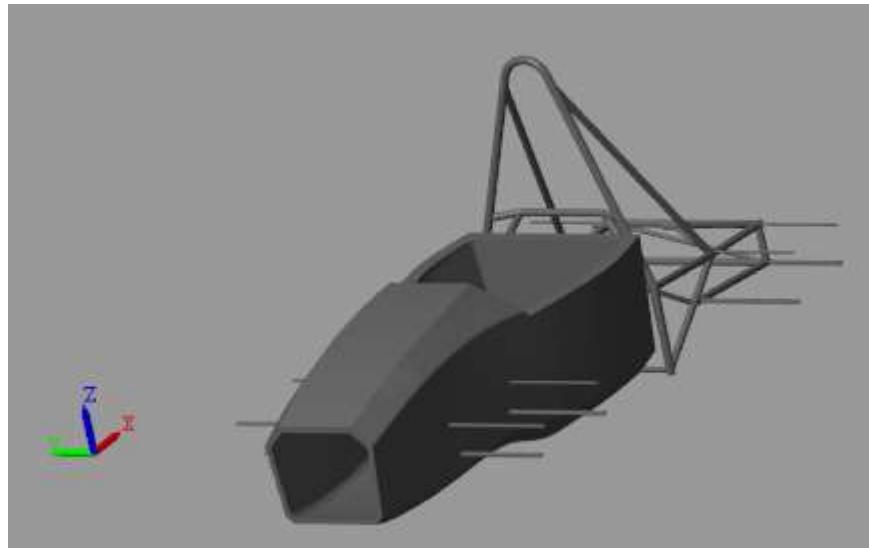


Figure 7.1.6: Results of the simulation using Simscape

This was the final version of the model developed in Simscape. The reason why the model was not finished is because developing the hole model using Simscape Multibody was very complex and would take much more time and effort than using other computer programs. The problem with this Matlab Add-In is that, in order to specify where to place all the components in the system, you need to use transformation blocks, which most of the time require to specify measures you need to figure out in advance, and for systems that are not already designed, it can be complicated. Even though a Matlab script was created to change these parameters easier, some of them didn't match with some of the requirements of the blocks, like rolling orientation or connections to other blocks. For this reason, other program needs to be considered.

7.2 Solidworks 2D Model

After evaluating the capabilities of SolidWorks and its motion analysis, the decision of changing to this program was made, as it is easier to work with all the components and its dimensions. To simulate the model, SolidWorks Motion Study was used, a Complement that allows the simulation of your assembly.

Even though some of the components were already designed (because they remain the same as the ones the team used for TR19 car), their assemblies didn't allow the simulation as some parts were fixed or over defined, so some of them were made from scratch.

In order to improve the simulation, the assemblies of the different components were simplified to the minimum geometry that allows to define completely its motion. Elements like bolts or bearings, engine, aerodynamic package or differentials were removed as they are not relevant to define the suspension motion.

Since start from scratch a 3D model can be complex, a 2D version of the front and rear suspension systems was developed, with a simplified sketch of all the components.

To do so, the monocoque section corresponding to the center of the wheel hub was measured and drawn as sketch in SolidWorks. After that, it was saved as a block, which can be operated as a 'part' and therefore create an assembly with the rest of the blocks. The same was done with the wheel and A-arms, and after that, we begin sketching the rockers, pushrods and springs.

In Figures 7.2.1 and 7.2.1, the final lengths of these components for both the front and rear suspension systems are shown. To design them, the gap in the monocoque where the springs and rockers will be mounted was considered, trying to place them as close to the lower part as possible (for better aerodynamic and weight distribution). As we can appreciate in Figure 7.2.1, the upper spring couldn't be fitted entirely inside this gap, as some space and angle between the upper and the lower springs is needed for the system to work effectively. Even though they may seem to collide in their connection with the rocker, in the 3D model they won't, as both springs and shocks have some horizontal spacing between them to allow their movement. This distance is limited by the width of the monocoque gap, which is 120 mm in its narrowest point.

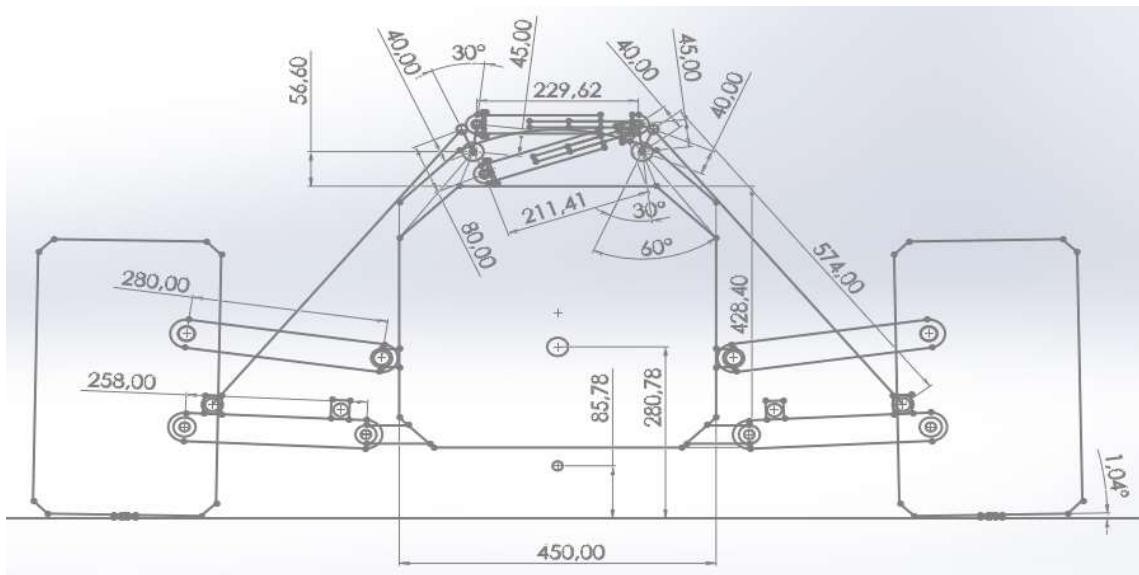


Figure 7.2.1: Final sketch of the front axle of the vehicle's suspension

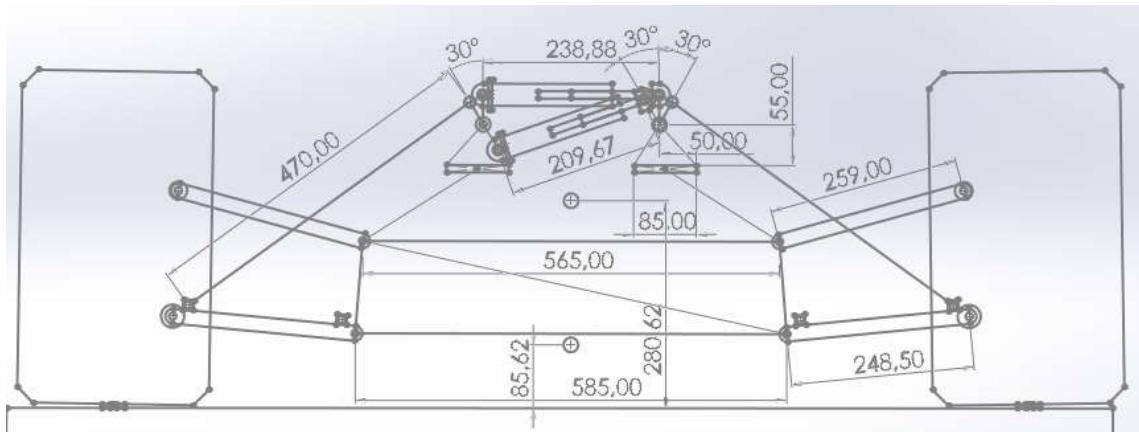


Figure 7.2.2: Final sketch of the rear axle of the vehicle's suspension

Once all the blocks were assembled, the simulation could be performed. These was done like a regular Solidworks assembly, but since the blocks are 2D parts, it is simpler to operate with. A line block was used as the ground, which always stay fixed. The weight of both axles, applied at the CG (at 280 mm, as it is shown in the Data Sheet), as well as the weight of some components like the wheels were introduced.

To compare the results with a conventional pushrod system, a 2D block assembly of the previous TR19's suspension was made. In Figures 7.2.3 and 7.2.4 there is a 3D model of one of the corners of the car, and in Figure 7.2.5 the correspondent 2D version.

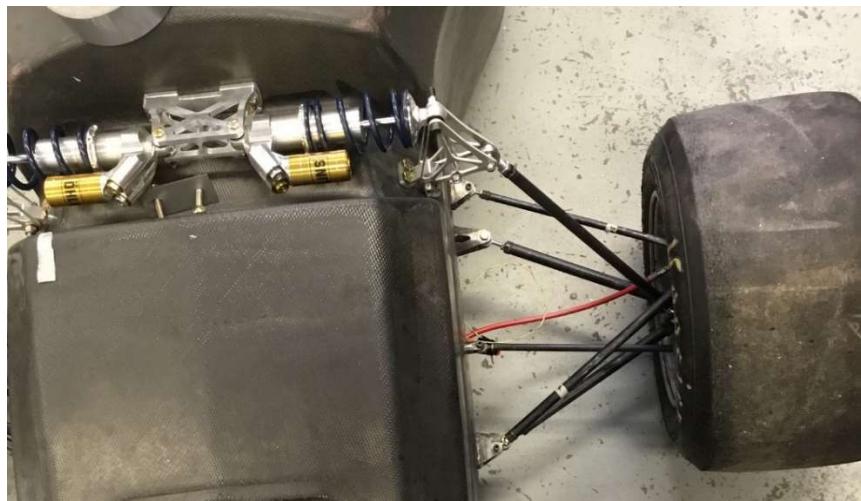


Figure 7.2.3: TR19's front suspension

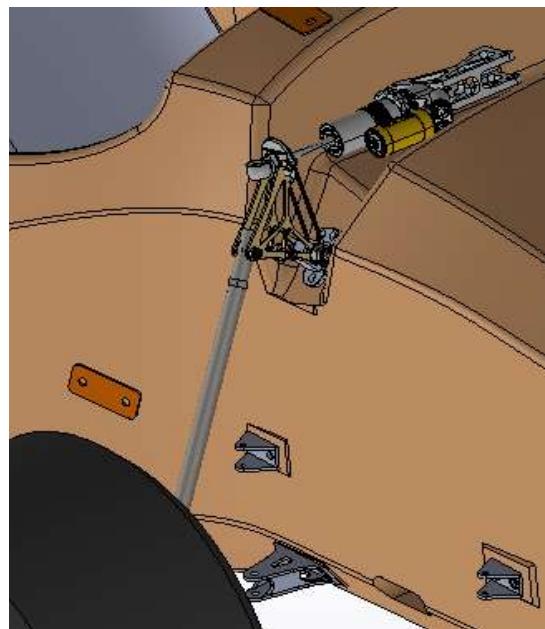


Figure 7.2.4: 3D assembly of the front suspension on TR19 (XDRI)

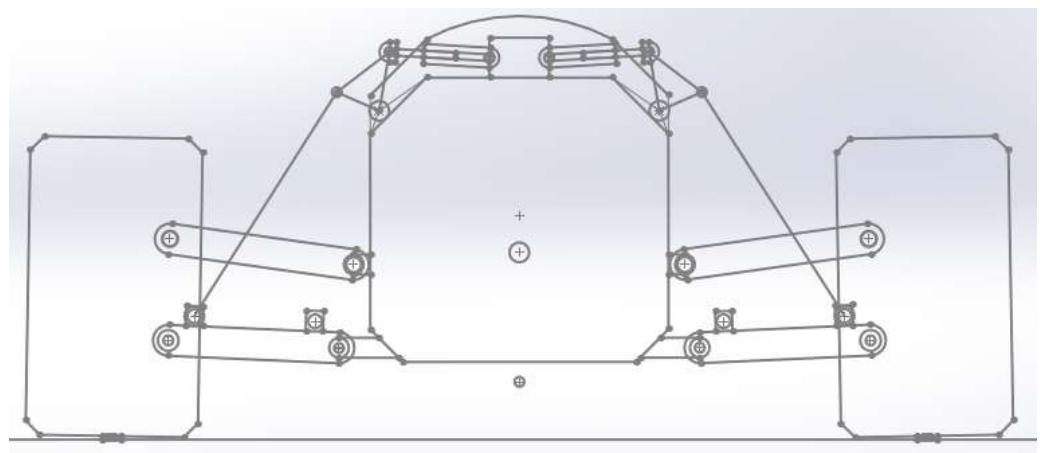


Figure 7.2.5: Sketch of the front suspension on TR19

7.3 Solidworks 3D Model

In order to show how the mode separated system could be fitted in the car, a 3D assembly was made using Solidworks. Some parts, like the monocoque, tubular back frame, suspension A arms, wheels and hubs or spindles were already made by the team for the previous TR19 car. The Figures with some relevant measures are provided below.

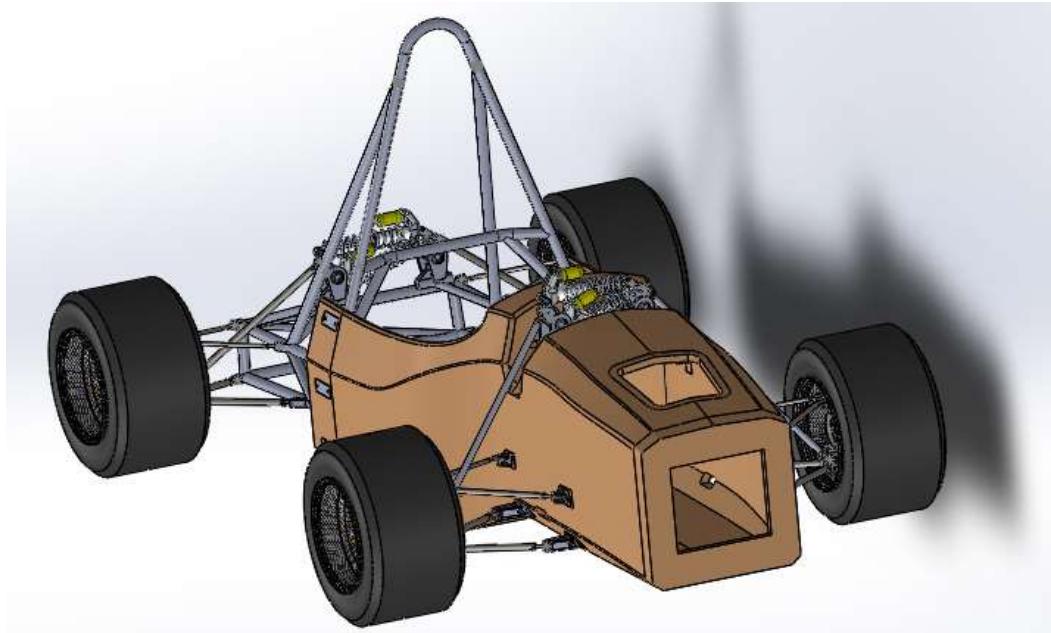


Figure 7.3.1: Whole vehicle assembly, front view

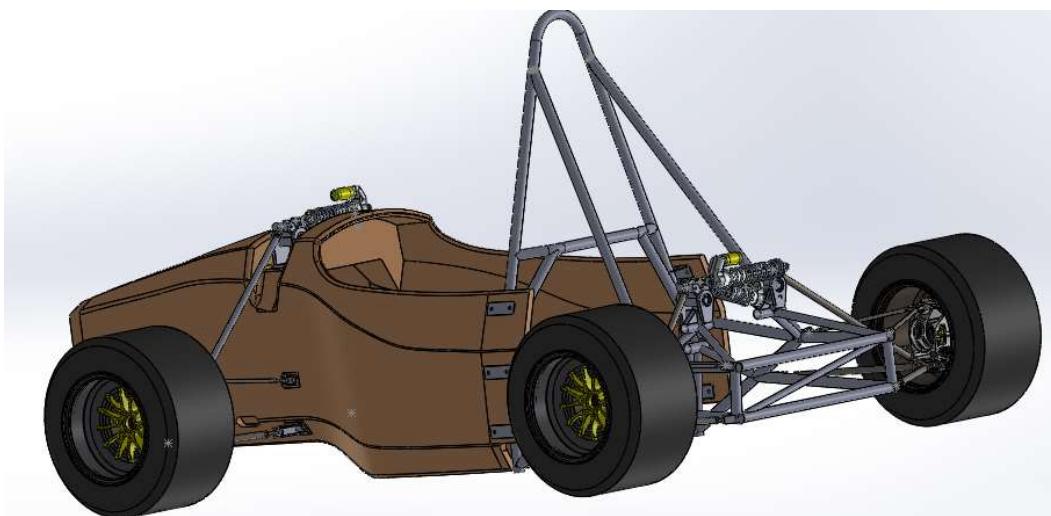


Figure 7.3.2: Whole vehicle assembly, rear-side view

For developing the new parts, the 2D sketch was used as a starting point. The distances between the suspension connections were the same, as they were developed for the system to work as effectively as possible. The final dimensions of these parts can be observed in Drawings 1 to 7. The lever's shape was created to allow the motions of the springs without colliding with other suspension parts like the mounts, monocoque, back frame or even the springs themselves. For that reason, they have a very rounded shape with different extensions to keep the springs with colliding. Also, the extensions have some reinforcement pieces to increase its resistance. The reason why they have 8 mm holes to fit the spring shafts is because the spring connections (bearings in both sides of the spring and damper) have a bore size of 8mm. In the case of the lever-mount hole, it is 15 mm wide to match with the bearings placed in the suspension mounts. Some of the Figures from the complete assembly are listed below, some obtained from XDRI. In Appendix III the complete list of sources is provided.



Figure 7.3.3: Front Lower Control Arm assembly (XDRI)



Figure 7.3.4: Front Upper Control Arm assembly (XDRI)

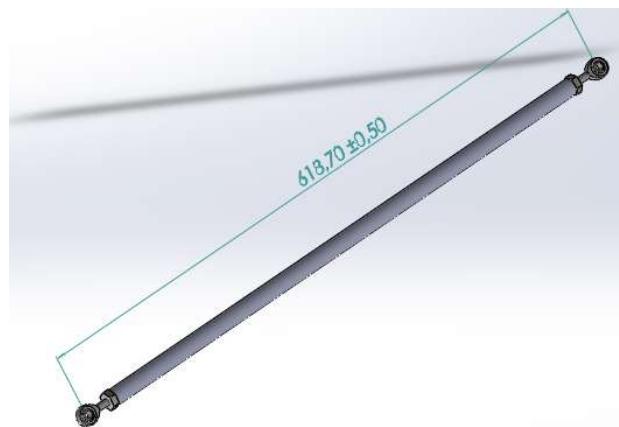


Figure 7.3.5: Front pushrod assembly



Figure 7.3.6: Wheel and spindle assembly



Figure 7.3.7: Rear Lower Control Arm assembly (XDRI)



Figure 7.3.8: Rear Upper Control Arm assembly (XDRI)

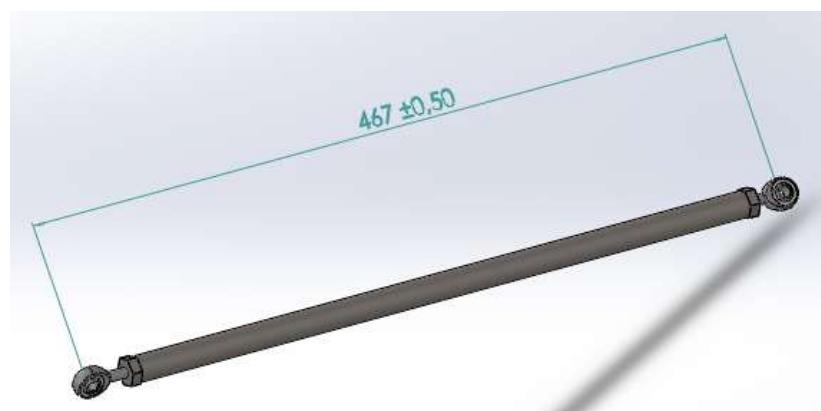


Figure 7.3.9: Rear pushrod assembly

In Figures 7.3.11 to 7.3.18 the final assembly of the front and rear suspensions are shown. The spring used and some relevant measures are shown in Figure 7.3.10. The spring model was obtained from source OHLI19. Even though its initial unstretched distance can be either 200 mm or 267 mm, for the final assembly the unstretched distances were different from the standard, but all under that range.

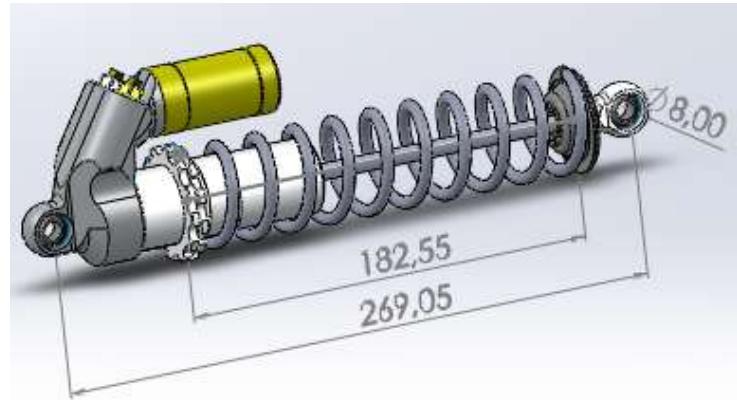


Figure 7.3.10: Ohlins TTX25 MkII CAD Model (OHLI19)

The suspension mounts (Drawings 1 and 2), are the parts that join the suspension and the chassis. They are made of steel plates and have two 15x32x9 mm series 6002 ball bearings in each side for the lever to rotate freely. The ones in the front are attached to the monocoque with four M8 bolts, and the ones in the rear are welded to the back frame.

Some details about the front and rear suspension assemblies are shown in Figures 7.3.11 to 7.3.18, as well as a front and rear view of the whole car (Figures 7.3.1 and 7.3.2).

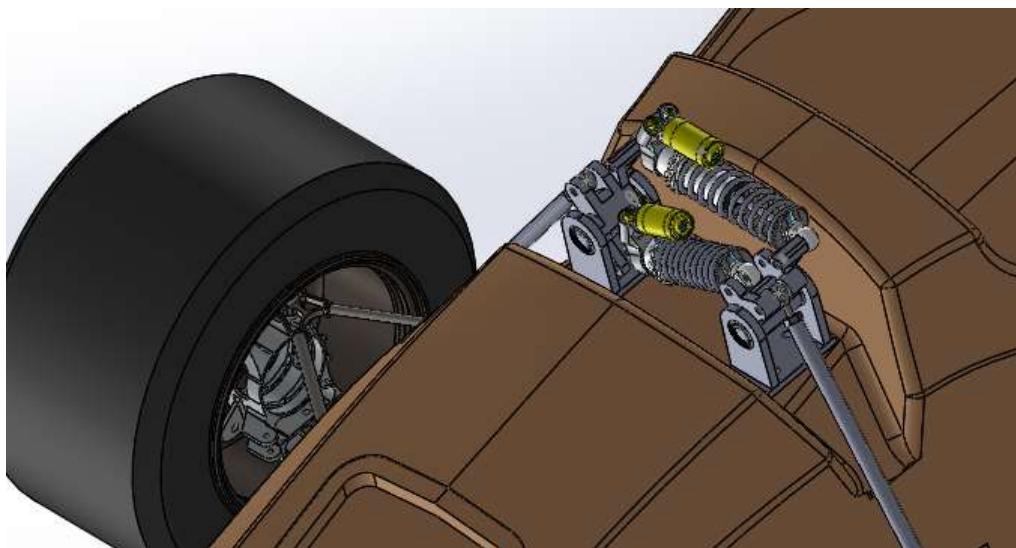


Figure 7.3.11: Front suspension Assembly, side view

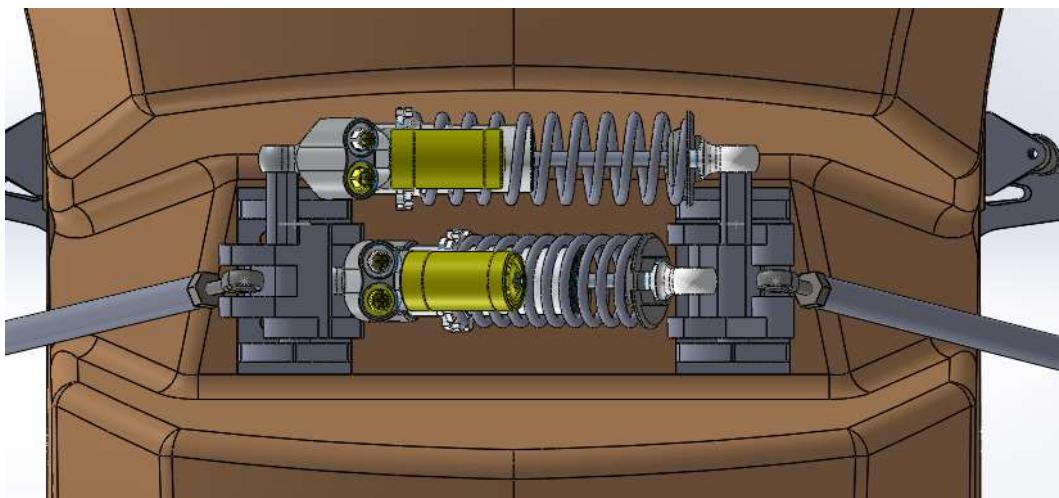


Figure 7.3.12: Front suspension Assembly, upper view

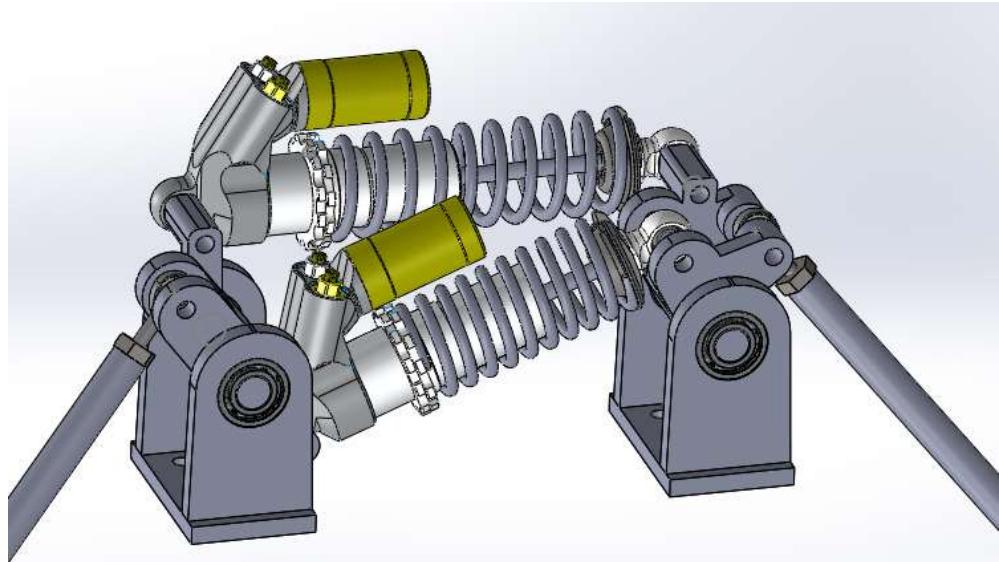


Figure 7.3.13: Front suspension assembly

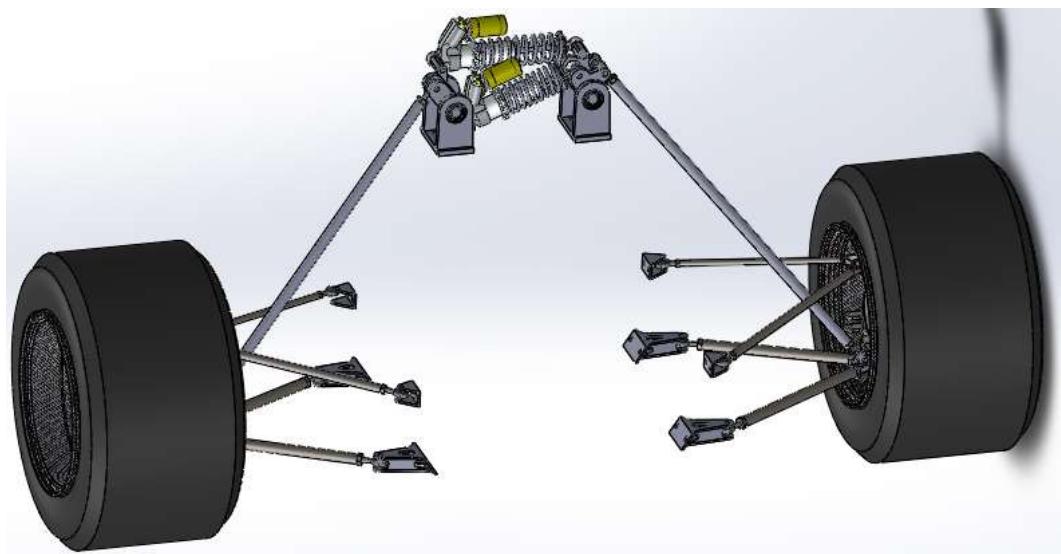


Figure 7.3.14: Front suspension assembly with control arms, pushrods and wheels

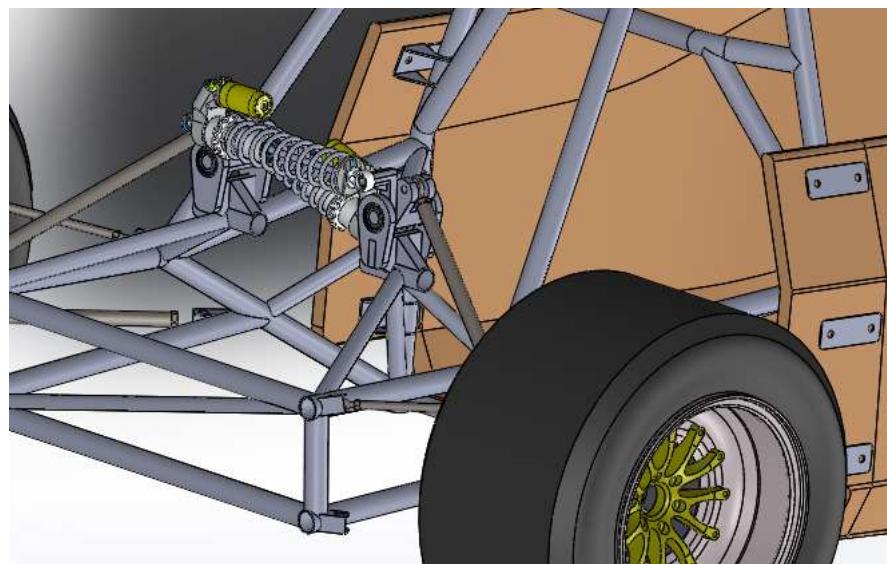


Figure 7.3.14: Rear suspension assembly, side view

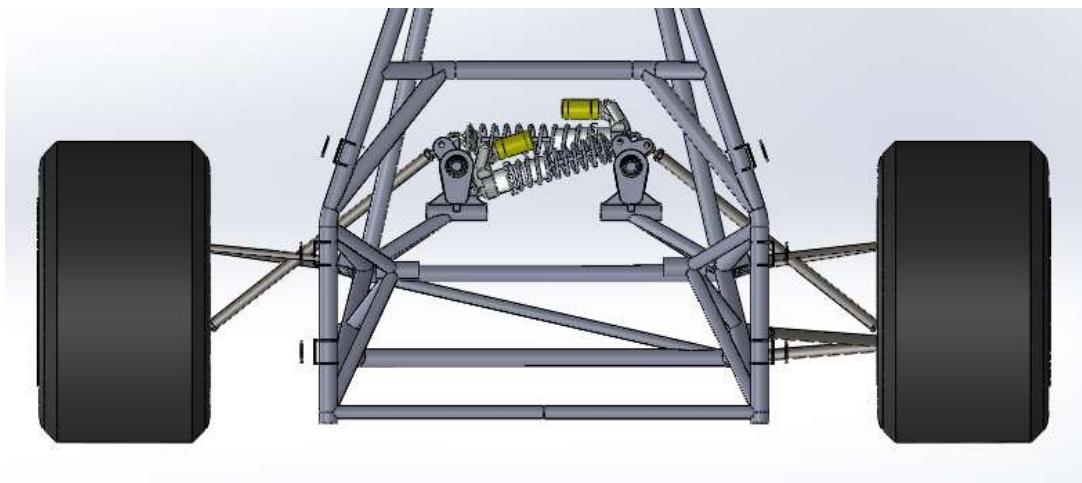


Figure 7.3.15: Rear suspension assembly, rear view

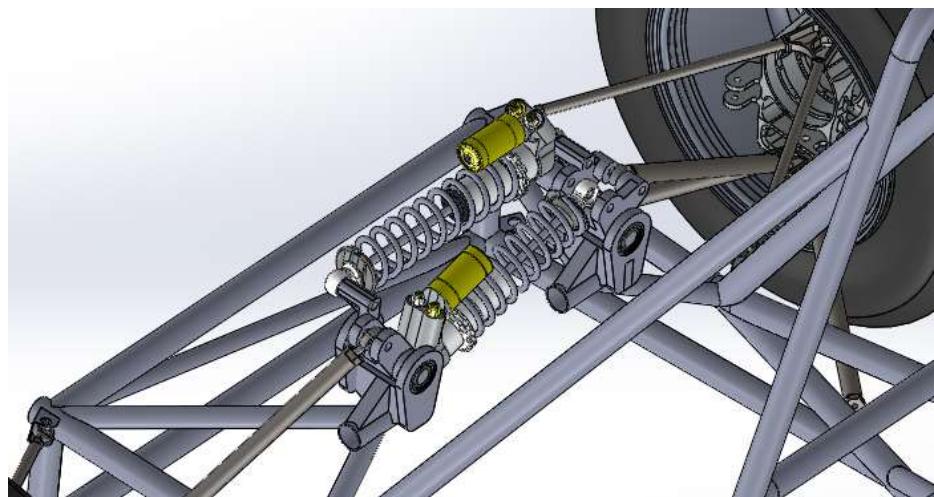


Figure 7.3.16: Rear suspension assembly, upper side view

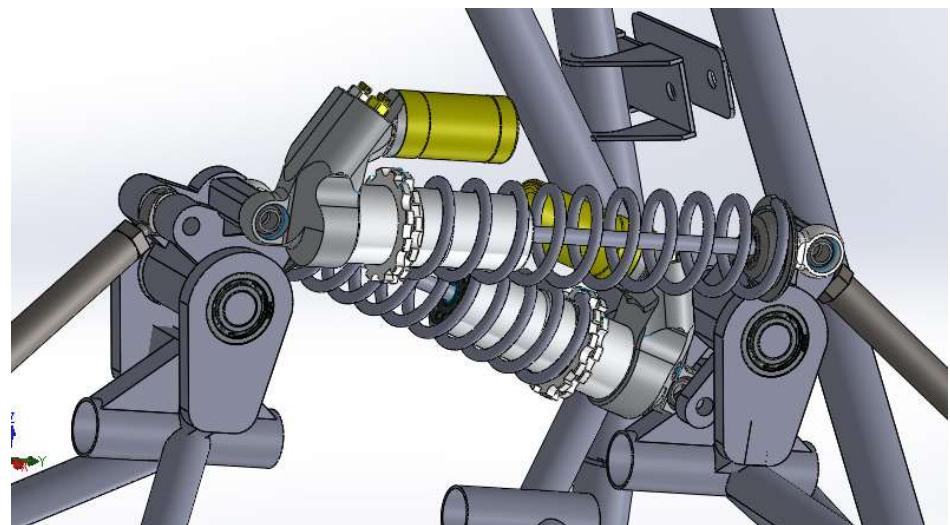


Figure 7.3.17: Rear suspension assembly

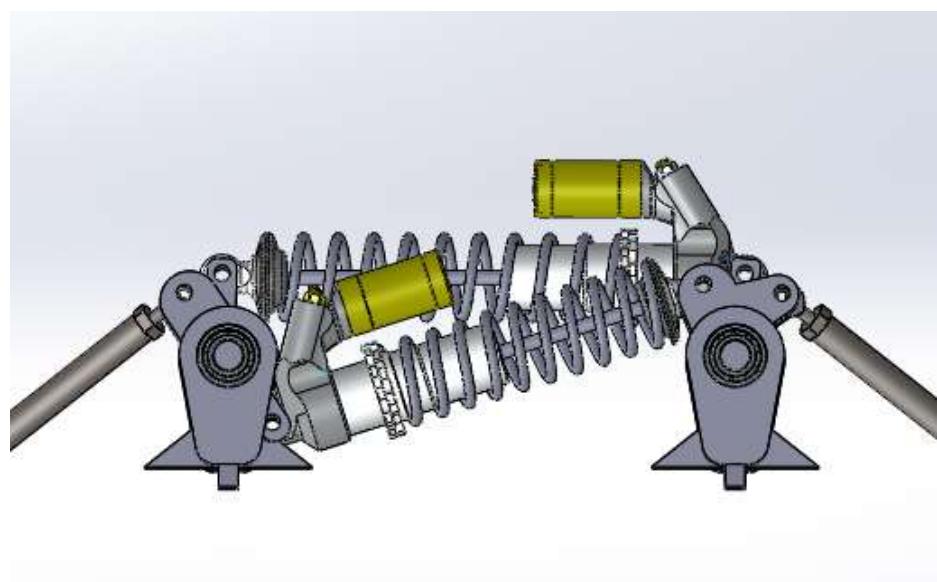


Figure 7.3.18: Rear suspension assembly, front view

8. Calculations for the Software Simulation

In order to simulate the model, a common cornering situation will be analyzed.

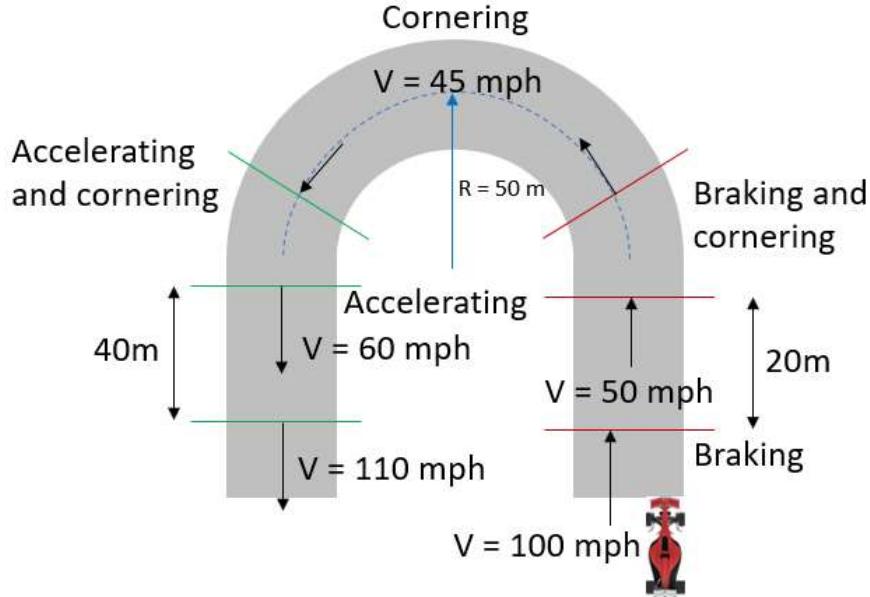


Figure 8.1: Cornering situation analyzed

The vehicle is moving at 100 mph (161 km/h or 44.7 m/s), and brakes to 50 mph (80 km/h or 22.2 m/s) in 20 meters, before taking a left corner. To simulate this motion, the correspondent longitudinal load transfer calculated was applied in the CG of both axles (as a compression force in the front axle and as a lifting force in the rear one). This load transfer was calculated using Equation 1. Knowing the values of all parameters ($h = 0.28 \text{ m}$, $L = 1.549 \text{ m}$, $W = 272 \times 9.8 \text{ N}$, $ax = ax = \frac{v_f^2 - v_i^2}{2 \cdot \Delta x} = 37.63 \text{ m/s}$

$$\text{Braking force} = \frac{h}{L} \cdot \frac{W}{g} \cdot ax = 1850 \text{ N}$$

As we said, this force will compress the front axle and lift the rear

As it starts to take the corner, a rolling moment (applied in the roll center) appears, consequence of the lateral load transfer produced by the cornering situation. This rolling moment is different for the front and the rear. A small braking force was also introduced in the beginning of steering part.

This braking force was considered as half the one calculated for the previous section, 900N. For the rolling moment, Equations 2 and 3 were used. The roll angle φ was obtained as follows ($m = 272 \text{ kg}$, $V = 22.2 \text{ m/s}$ or 80 km/h , $R = 20 \text{ m}$, $h_1 = 0.206 \text{ m}$, $k\varphi f = 684.838 \frac{\text{Nm}}{\text{deg}}$; $k\varphi r = 591.454 \frac{\text{Nm}}{\text{deg}}$):

$$\text{Roll angle } \varphi = \frac{m \cdot \frac{v^2}{R} h_1}{k\varphi f + k\varphi r - mgh_1} = 1.9022 \text{ degrees}$$

Now that we know the roll angle, and using Equations 5 and 6, the lateral load transfer for front and rear can be obtained ($W_f/g = 129.3 \text{ kg}$, $W_r/g = 142.7 \text{ kg}$, $h_f = 0.0546 \text{ m}$; $h_r = 0.0925 \text{ m}$; $t_f = t_r = 1.219 \text{ m}$, $k\varphi f = 684.838 \frac{\text{Nm}}{\text{deg}}$; $k\varphi r = 591.454 \frac{\text{Nm}}{\text{deg}}$; $\varphi = 1.9022 \text{ degrees}$):

$$\Delta F_{zf} = \frac{\frac{W_f}{g} \cdot \frac{v^2}{R} \cdot h_f}{t_f} + \frac{k\varphi f \cdot \varphi}{t_f} = 1210.4 \text{ N}$$

$$\Delta F_{zr} = \frac{\frac{W_r}{g} \cdot \frac{v^2}{R} \cdot h_r}{t_r} + \frac{k\varphi r \cdot \varphi}{t_r} = 1189.2 \text{ N}$$

Even though the lateral load transfer for front and rear wasn't necessary to obtain the rolling moment, it was calculated to show the weight transfer produced in each axle for the conditions stated. As it was said before, the amount of load transfer produced in each axle can be changed by changing the roll spring rate. This can help to solve possible understeer/oversteer problems (see previous sections to have more details on this).

The rolling moment produced in each axle can be calculated as:

$$\text{Equation 10: } M\varphi f = \varphi \cdot k\varphi f = 1301.2 \text{ N/m}$$

$$\text{Equation 11: } M\varphi r = \varphi \cdot k\varphi r = 1123.76 \text{ N/m}$$

An assumption will be made about this moment staying steady throughout the central part of the corner.

As the end of the curve approaches, the driver starts to accelerate, maintaining the steering. This accelerating force compresses the rear axle and lifts the front one, because of the longitudinal load transfer. For this reason, during this part a rolling moment (applied at the roll center) and a force (applied at the CG) were combined.

The accelerating force is considered half of the one produced when all the steering is finished. The vehicle exits the corner at 60 mph and accelerates at 110 mph, with its maximum capability. It takes 40 meters for it to accelerate ($a_x = 21.23 \text{ m/s}^2$). This force is calculated as (following the same procedure as the braking situation):

$$\text{Accelerating force} = \frac{h}{L} \cdot \frac{W}{g} \cdot ax = 1044.5 \text{ N}$$

Therefore, the acceleration when the car is exiting the curve creates a force of $1044.5/2 = 522.25 \text{ N}$

The entire simulation is shown in Figures 8.2 (front) and 8.3 (rear). Solidworks Motion Study allows to plot some of the results, like forces, angular and vertical displacements or moments acting in the components selected. The results collected were pitch spring displacement, roll spring displacement, wheel camber (for both left and right), CG height and body roll.

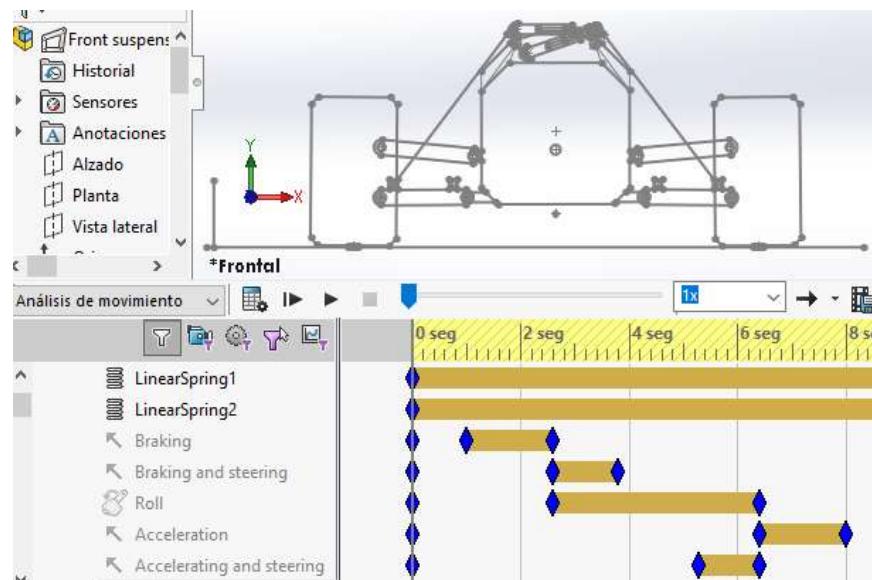


Figure 8.2: Front suspension simulation using Solidworks Motion study

The same simulation was made for the rear, with its correspondent forces, spring and damping rates.

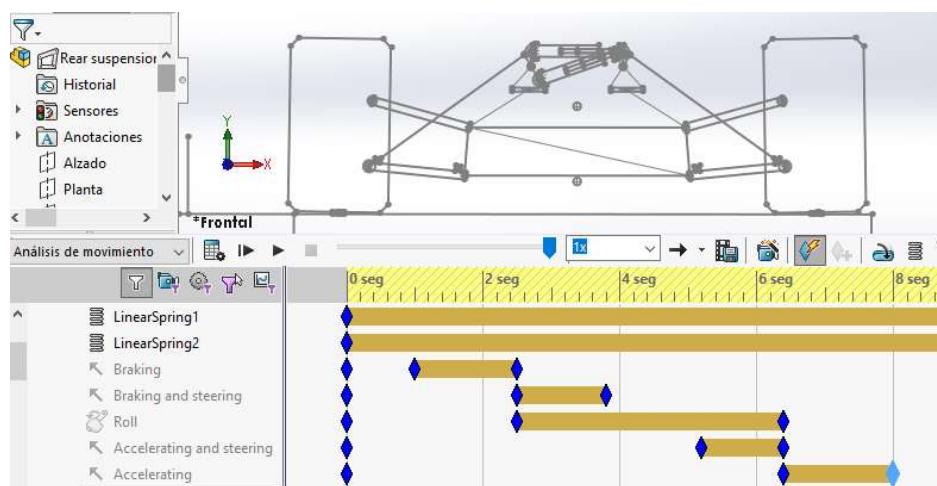


Figure 8.3: Rear suspension simulation using Solidworks Motion study

In order to compare the performance of the Mode Separated System, an assembly of TR19's suspension was created (Figure 8.4). Only the front axle's suspension was developed, as developing also the rear wasn't necessary to compare the system (the rear behavior will be very similar). All the adequate forces and moments were calculated and introduced in the Motion Study simulation, with the vehicle performing the same operation. The parameters plotted were the same, which will allow to compare the performance of both systems.

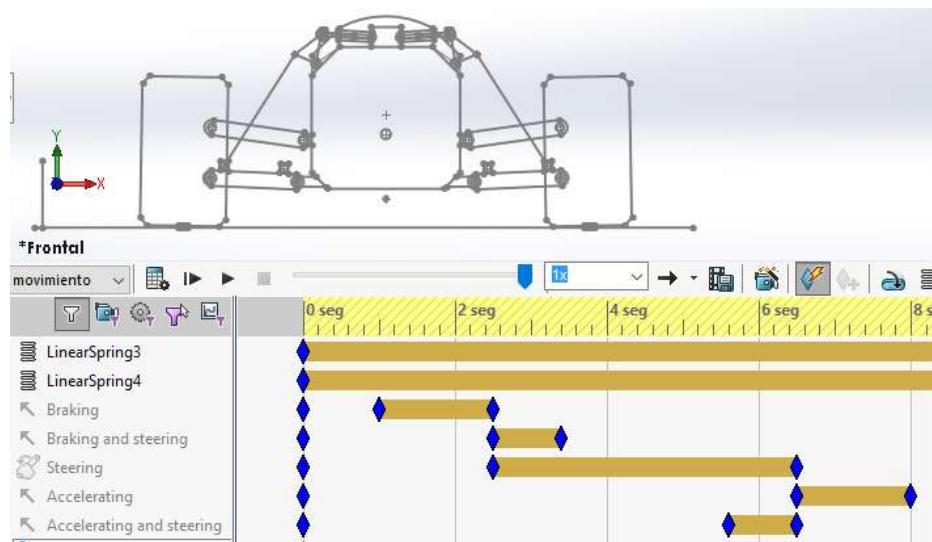


Figure 8.4: TR19's Front suspension simulation using Solidworks Motion study

The results will be analyzed in the following section.



Figure 8.5: Results being plotted in the simulation

9. Results and Analysis

As it was introduced in the previous section, a simulation of the vehicle taking a corner to the left was performed. The plots being analyzed are the pitch spring displacement, roll spring displacement, wheel camber (for both left and right), CG height and body roll. There are many more results that can be analyzed, but these basic ones allow to have a general idea about how the system reacts to every motion during the operation. During the results analysis, a comparison between the theoretical values obtained in the calculation and the results from the simulation will be made, to see if the simulation was performed adequately and if the results are what it was expected. Also, they will be compared with the ones obtained from the TR19's suspension simulation.

The forces introduced in the simulation (braking and accelerating vertical forces and rolling moment) will be entirely reacted by the suspension system. The spring's compression can be directly correlated by the force that spring is reacting ($F = k \cdot x$), and the sum of the forces compressing (or elongating) both springs are going to be always the same as the ones acting on that precise moment.

9.1 Pitch Spring

The pitch spring compression and elongation for both front and rear can be found in Figures 9.1.3 and 9.1.4. As it can be seen, the spring compresses initially due to the action of the weight of the axle. This weight considers a 68 kg driver, with the weight distribution from TR19's data sheet.

Figures 9.1.1 and 9.1.2 show in detail the spring reaction to the initial compression due to the force.

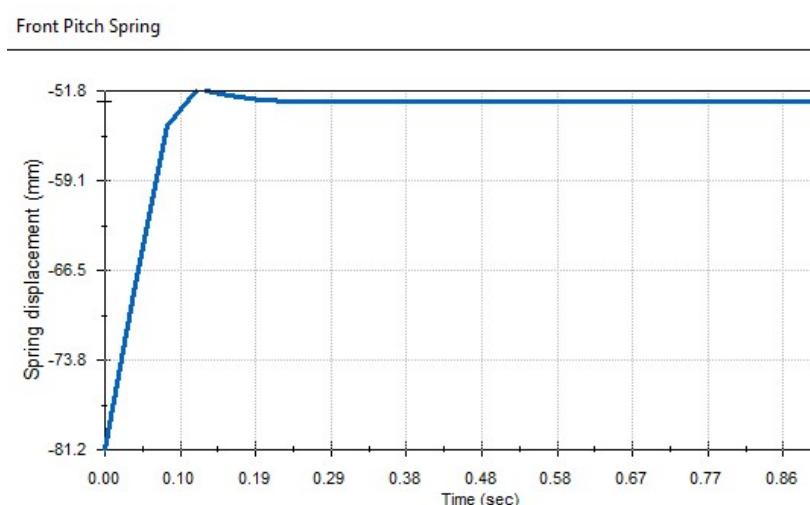


Figure 9.1.1: Front suspension initial reaction to the weight of the car

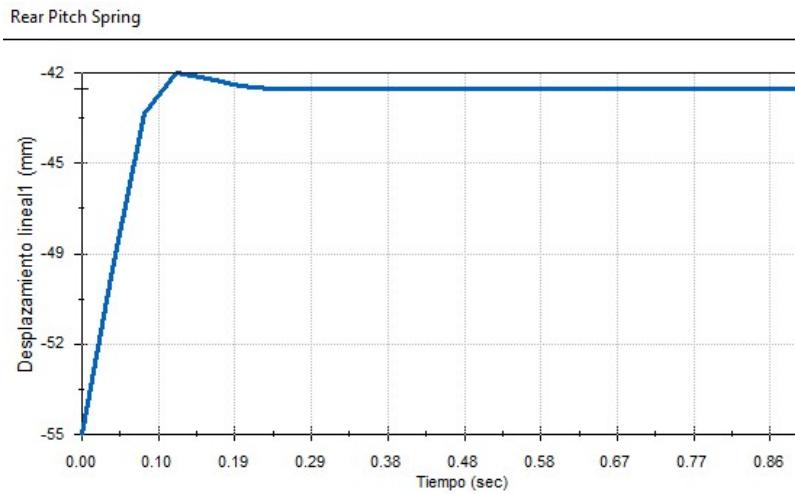


Figure 9.1.2: Rear suspension initial reaction to the weight of the car

The results were very similar as the ones expected, since the compression from both axles follow a curve that depends on the damping ratio. The damping ratio for both axles is 0.68, and if these curves are compared with Figure 6.2.1 (Response of a system to different damping ratios), they are very similar. Another thing worth noticing is the total compression as the compression of the front spring is larger than the one in the rear. This is because of the Motion Ratios. As the front has a MR of 1.1 and the rear of 0.7, the wheel displacement will produce a higher compression in the front. Also, the slope of these curves, that depend on the spring and damping rates, are almost identical. The system behaves exactly as it was expected and shows that the values selected for the spring and damping rates in both axles were adequate. It can be observed that the system reaches steady state in almost 0.2 seconds with almost any rebound, which was exactly the characteristic it was aimed. These shows that the vehicle will behave as it was expected from the natural frequencies during ride. This behavior could be changed, if necessary, by recalculating all the spring and damping rates for another natural frequency more adequate to the circuit or condition the car is racing on.

The behavior of the pitch spring along the whole simulation will be now analyzed.

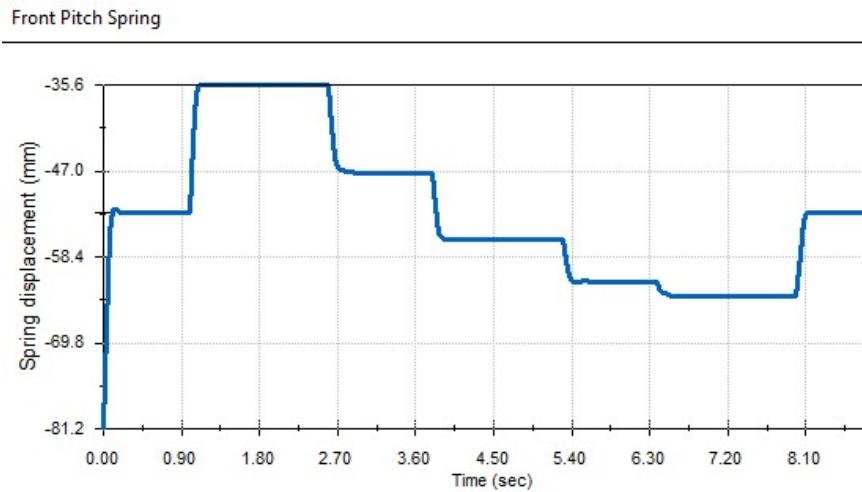


Figure 9.1.3: Front pitch spring compression throughout the simulation

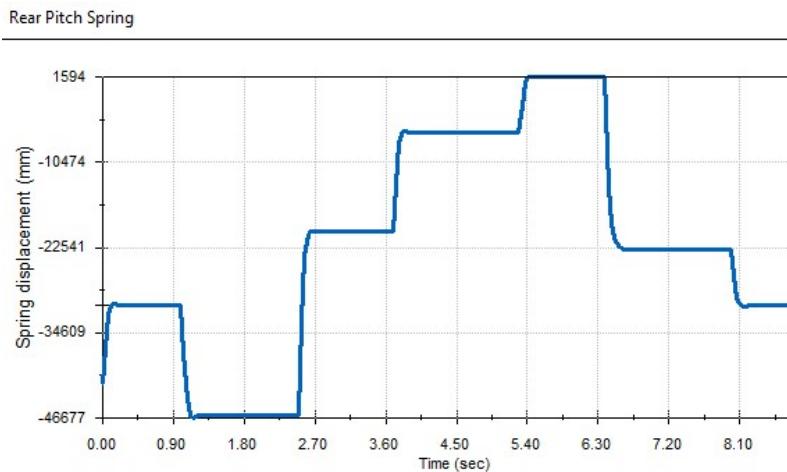


Figure 9.1.4: Rear pitch spring compression throughout the simulation

As it was explained, the pitch spring was designed to only react the rolling moments along the lateral axis, which tend to compress or lift both wheels in the axle at the same rate. Forces like braking or accelerating produce this motion. The first force acting on the system is the main stopping force, which compresses the front axle and elongates the rear. The rate of compression follows the same pattern analyzed before, and since the damping rate keeps this load transfer from happening very sudden, the vehicle can be kept at its maximum braking and acceleration force. It can also be seen that the rate of compression follows a linear pattern depending on the force, as the minor braking and accelerating forces that happen while the vehicle is steering (which are half its correspondent main braking or accelerating forces) produce half the spring compression. Another very important aspect that can be

highlighted is that, when the vehicle is only under a rolling moment, the pitch spring's compression is kept at almost the value of the steady state situation. This shows that the system is performing as expected, as this rolling moment is being absorbed almost entirely by the roll and damper spring.

9.2 Roll Spring

The roll spring will be now analyzed. The amount of rolling of the vehicle will depend on the speed and the radius of the corner, as well as the rolling stiffness of the vehicle. The rolling stiffness chosen for the vehicle derives from the optimal stiffness value for a racing car of similar characteristics as TR19. Since the Mode Separated System allows to freely tune this parameter without affecting aspects like the ride high or ride natural frequency, the performance will be hugely improved. Figures 9.2.1 and 9.2.2 show the roll spring compression along the whole simulation.

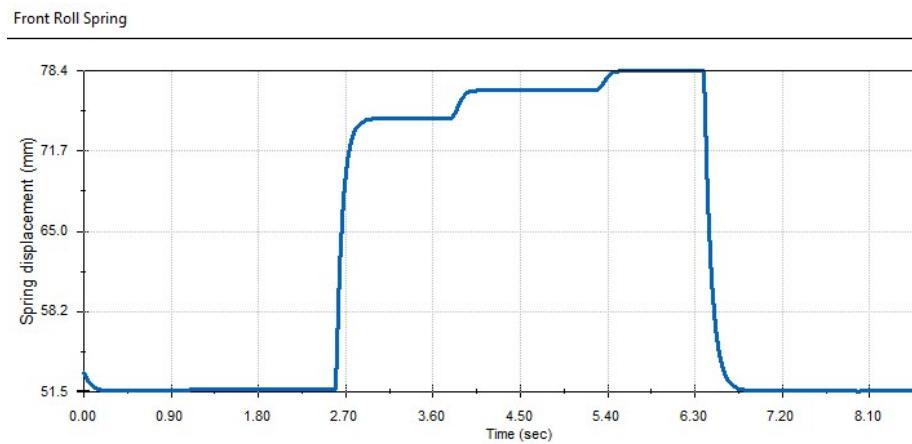


Figure 9.2.1: Front roll spring compression throughout the simulation

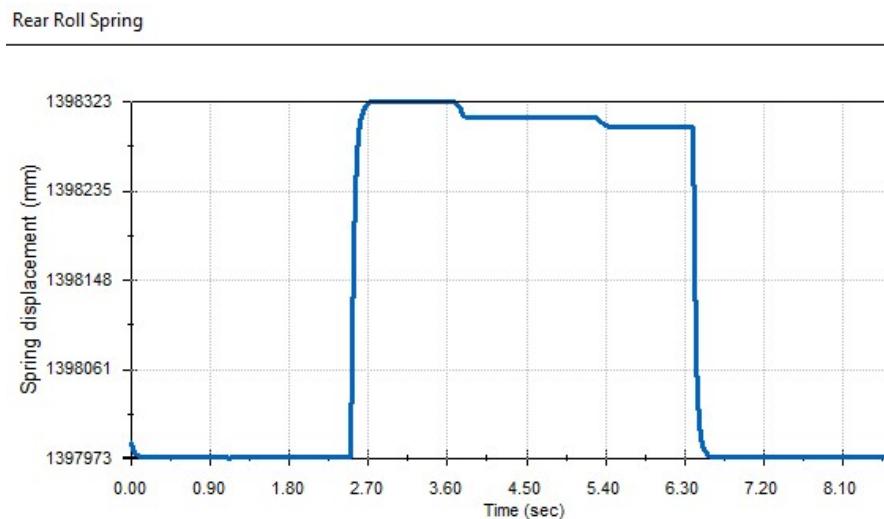


Figure 9.2.2: Rear roll spring compression throughout the simulation

The first thing that can be observed from the charts is the fact that both roll spring's elongation stay steady under the main braking and accelerating events. As it happens with the pitch spring during roll, this movement is entirely reacted by the pitch spring, because of the geometry of the system. However, when the vehicle is turning and a vertical force acts along the axle, the roll spring compression changes slightly, an amount that can be neglected if we consider the pitch spring displacement. As it was said, spring displacement can be directly related with a force, and the sum of both pitch and spring compression forces will be always the total force acting on the system (as we are not considering horizontal forces like tractive effort). Again, the displacement of the springs will depend on the Motion Ratio, and since the front has a bigger one, its elongation will be larger.

When the vehicle corners to the opposite direction, to the right, the roll spring will compress instead of elongate, as the suspension geometry is designed for that purpose. This aspect can be observed in Figure 4.2.3.

The right spring displacement from TR19's car is shown in Figure 9.2.3. As it was said, conventional pushrod systems don't have different springs acting for different motions, so the tuning options are hugely reduced. The slope and rebound of the initial reaction to the vehicle's weight is determined by the damping ratio and the spring rate. In the Mode Separated System, the values for ride, pitch and roll could be different, optimizing the vehicle's response to the body motion. In the TR19's case, a trade off must be done between the different motions, so the overall performance will be reduced.

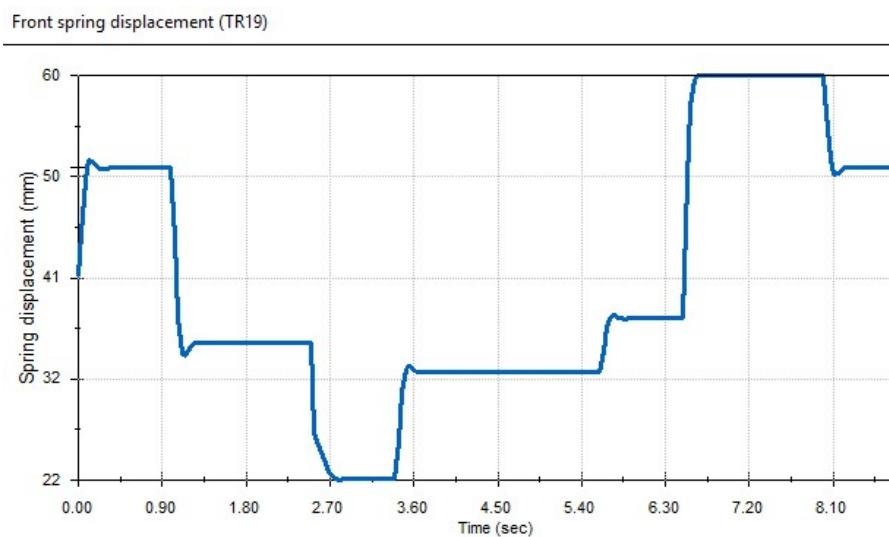


Figure 9.2.3: Front spring displacement of TR19's suspension system throughout the simulation

9.3 Wheel Camber

Wheel camber determine the angular displacement between the wheel and the ground. It is an important aspect to consider in order to maximize the performance of the vehicle, as it is related with the amount of grip. In figure 9.3.1 it can be seen how camber is measured and its positive criteria.

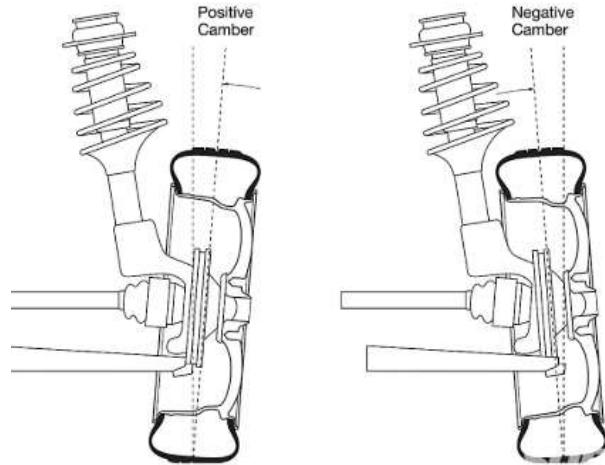


Figure 9.3.1: Positive and negative wheel camber (PARR19)

The flatter the wheel is with respect to the ground, the greater the contact patch between the wheel and the surface, and the bigger the tractive force we can get from that wheel is. Wheel camber depend on the suspension links geometry, and as these move due to the body movement, their values change. It will be ideal to keep the wheel camber always at an optimal position, but it will compromise the performance as it will require extremely big spring rates, which will result in a harsh ride and the car loosing contact with the ground (going airborne) for every small road irregularity. It can also produce an overturn situation as the rolling moment is not absorbed by the springs.

During cornering, it is desired to have a 0-degree camber in the outside tires, as they are the ones reacting the lateral weigh transfer. Some cars are designed to have the optimal camber at the end of the curve, when the vehicle is accelerating. This maximizes the speed of the car at the end of the cornering situation.

The camber curves obtained will be now analyzed.

Front Left Wheel Camber

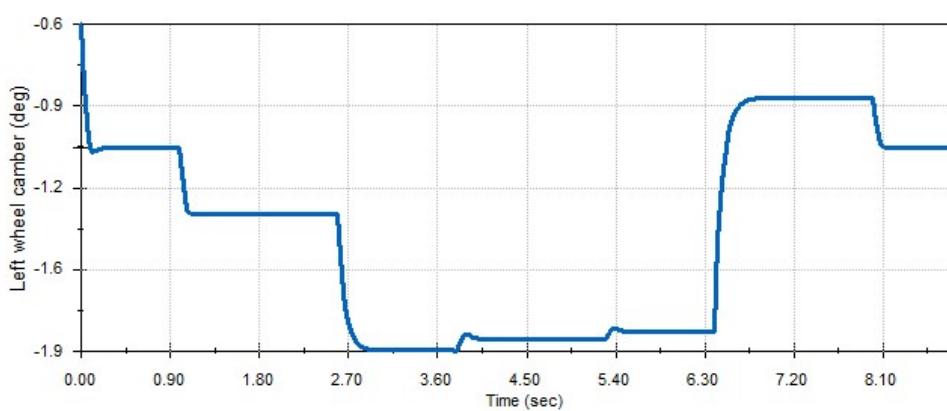


Figure 9.3.2: Front inner wheel camber throughout the simulation

Front left wheel camber (TR19)

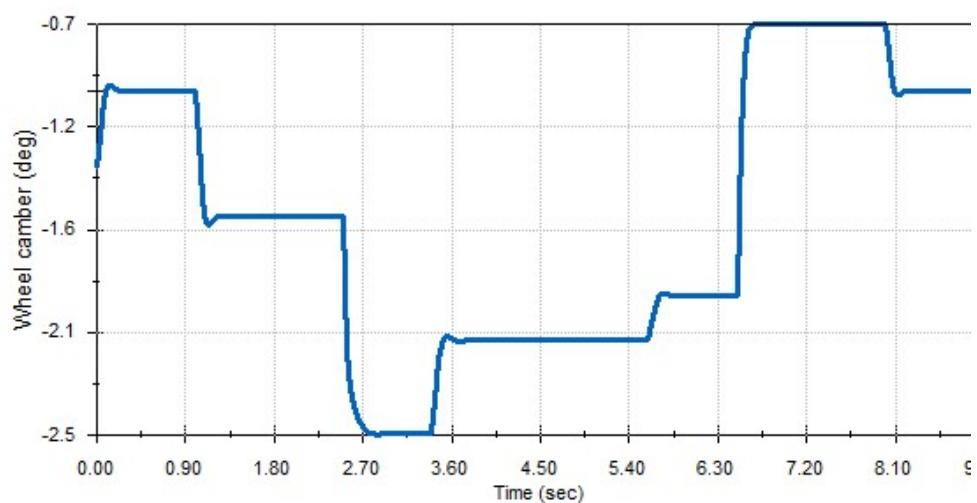


Figure 9.3.3: Front inner TR19's wheel camber throughout the simulation

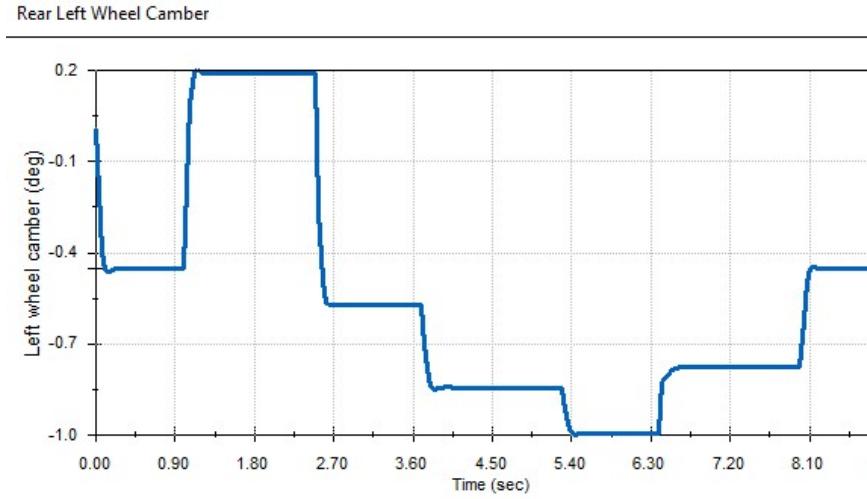


Figure 9.3.4: Rear inner wheel camber throughout the simulation

The suspension links were designed to, as the axle come closer to the ground, increase the camber (as a negative angle). However, this behavior can be changed by changing the A arms total length. As it can be seen in Figure 9.3.5, these links have a threaded mechanism in both ends to increase or decrease the overall length as desired.

In Figures 9.3.2, 9.3.3 and 9.3.4 it can be seen the camber of the inner wheel of the corner for both TR19's and the new system. The camber of these wheel is sacrifice in favor of the outer ones, as they will react more weight during cornering. However, the camber is kept to a very small value, so the wheels will still produce a decent amount of grip (very close to the optimal situation). Because the links are symmetrical (left and right ones are the same), the behavior of the car turning to the opposite direction will be the same.

Comparing the front axle of both suspension systems, it can be observed that the camber change in TR19's car is greater, and since a tradeoff must be done for the damping ratio, the rate of change for this camber is less smooth, which can result in the car loosing grip as the camber change is produced too fast.



Figure 9.3.5: Front left corner of TR19's car showing the adjustable control arms

The outer wheels behavior will be now analyzed.

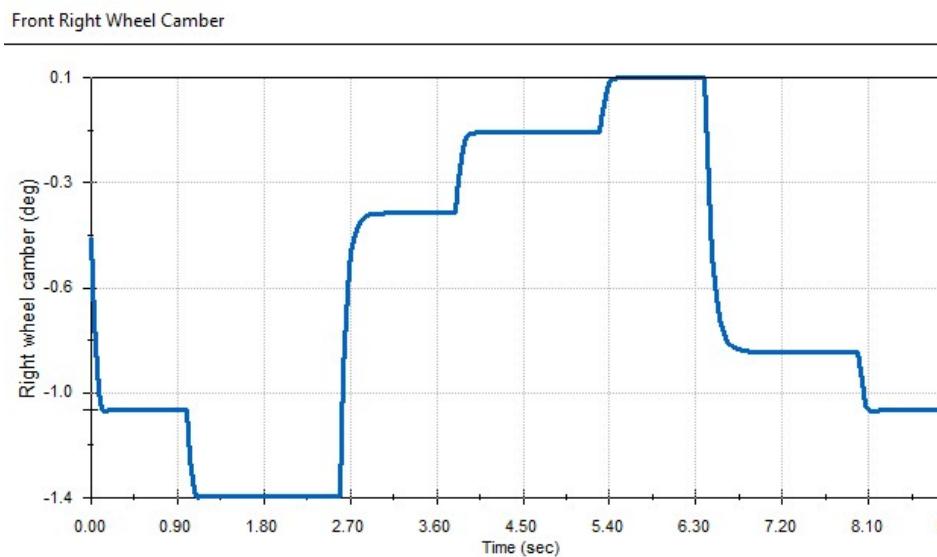


Figure 9.3.6: Front outer wheel camber throughout the simulation

Front right wheel camber (TR19)

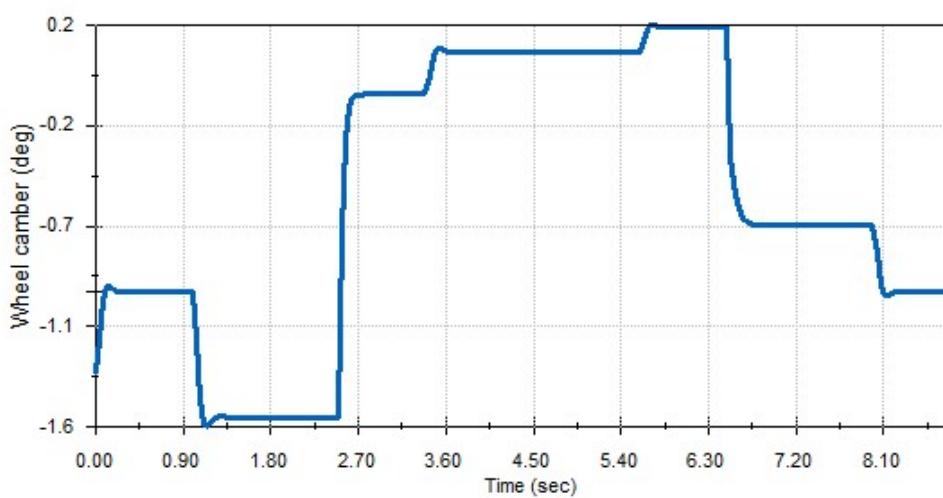


Figure 9.3.7: Front TR19's outer wheel camber throughout the simulation

Rear Right Wheel Camber

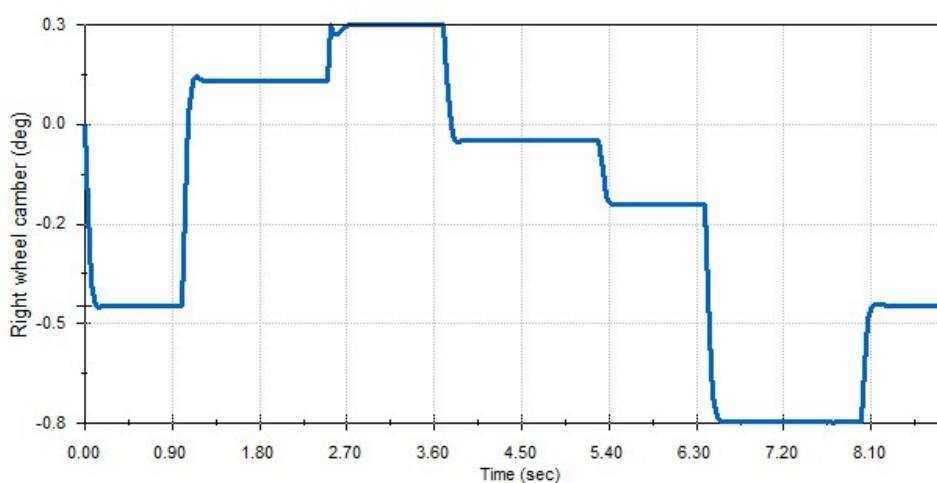


Figure 9.3.8: Rear outer wheel camber throughout the simulation

In Figures 9.3.6 and 9.3.8, the aspect explained before can be again observed. During the cornering situation, the camber is kept in values very close to 0. For the front, the optimal value (0-degree angle) happens when only a rolling force is applied, and for the rear, when the vehicle exits the curve. The rear-outer wheel will be the one reacting more forces as the vehicle exits the turn, so it should have the best wheel camber under this situation.

The rate of change of the four tires depend on the spring and damping rate, and how fast they react the forces on the car. The damping force has been chosen wisely as they control the timing between when the force is applied and its effect on the wheels in an appropriate way.

The irregularity on the second braking situation is due to the fact that the wheels also move horizontally, and a sudden movement may make the angle change that way.

The wheel camber change for the outer wheel for both Mode Separated and conventional pushrod systems is very similar. As it was said for the inner wheel, the camber change rate is faster for the TR19's suspension (Figure 9.3.7), which can compromise grip during transitions. The overall camber change is a bit greater for the pushrod system, but since both have the same suspension links and the same forces are acting, the shape of the camber plot will be very similar.

9.4 Body Roll

Body roll and its effects on cornering were analyzed in previous sections. As it was said, the perfect amount of roll angle is difficult to determine, as rolling affects many different aspects on the vehicle's performance. As the body rolls, the geometry of the suspension is altered, affecting wheel camber and the roll center position. Also, body roll can be a good indicative of the capability of the vehicle throughout a corner and can warn the driver when the car is going to start to lose grip. It produces a small load transfer as well, as the car's CG moves in a horizontal line.

In Figures 9.4.1 and 9.4.3 it can be observed the front and rear body roll. As expected, the car's angle stays steady during braking and acceleration, and as the speed is increased during cornering, the roll angle increases. Rear roll angle is slightly smaller than the one in the front. This is due to the fact that the front and rear suspension links, and therefore roll angles, are different. The Motion Ratio is greater in the rear than in the front, so the body rolling will be greater in the front axle.

This is not a problem, as having smaller roll angle variations in the rear end will help to have a steadier acceleration at the end of the curve.

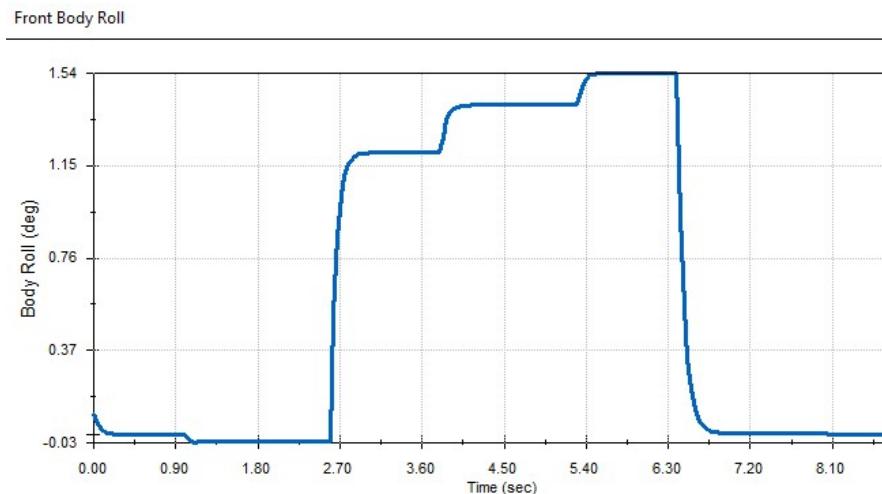


Figure 9.4.1: Front body roll throughout the simulation

Front body roll (TR19)

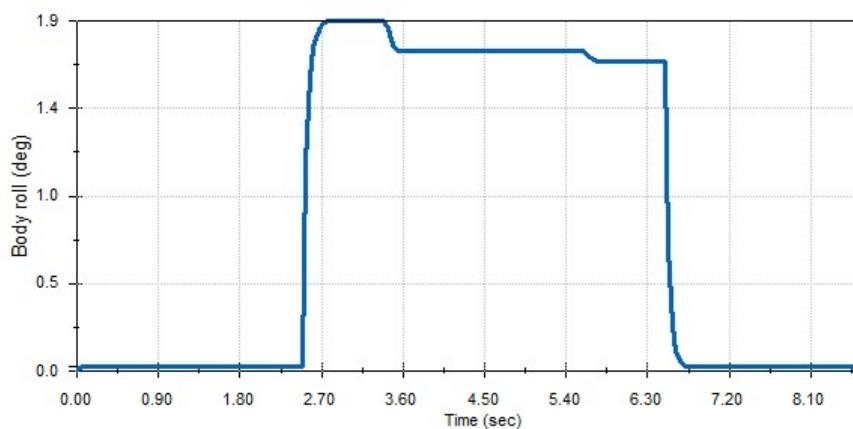


Figure 9.4.2: TR19's body roll throughout the simulation

Rear Body Roll

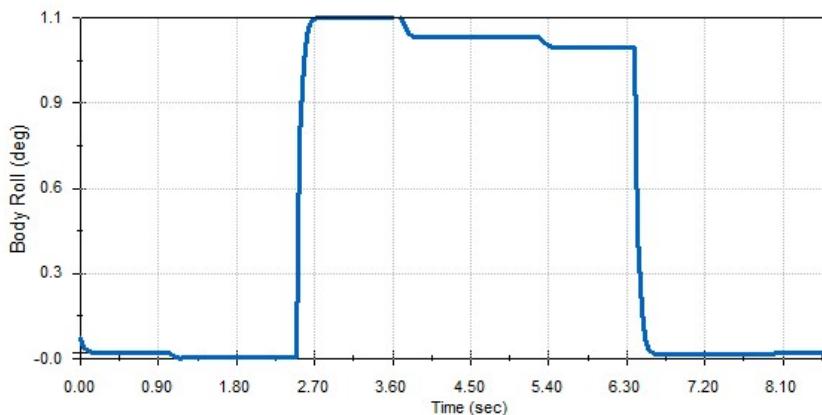


Figure 9.4.3: Rear body roll throughout the simulation

Taking a closer look at the roll angle transitions it can be observed that the timing between when the force is applied, and the reaction of the body corresponds with what it was expected when the damping ratio was selected. Being able to modify this is a big step forward compared to the previous suspension system, as the damping ratio for rolling couldn't be modified. The body roll was more uncontrolled, as the Anti Roll Bars, in charge of selecting the proper spring rate for roll, doesn't have any damping.

Something very interesting can be observed in Figure 9.4.2. The Body Roll plot for the front is very different for the two suspensions analyzed, as in the pushrod system, the body roll angle is reduced as the speed is increased. This is due to the fact that, in the Mode Separated System, roll and pitch are

reacted by different springs, so as the vehicle accelerates, the pitch spring is decompressed and the roll spring is kept the same, so the vehicle tends to tilt more as the speed is increased during a corner. However, for the conventional pushrod system, as the vehicle accelerates both springs in the front axle tend to be decompressed, which results in a decrease of the overall body angle.

It is important to note that this behavior occurs only for transitions. As the vehicle increases its cornering speed, the body angle will be greater for both cases.

9.5 CG Vertical Displacement

The steady state CG height is set at 280 mm above ground. The same CG height will be considered for both front and rear, each axle with its own weight, following the weight distribution stated in the TR19 Data Sheet.

Keeping the CG between small vertical variations will mean having a more predictable aerodynamic behavior, which will help the driver to know the amount of grip the car is going to have. Running different ride frequencies for front and rear can have several advantages (analyzed in previous sections). For our vehicle, the rear has a slightly lower ride frequency, which results in a softer suspension and more height variation. This was done to reduce drag on straights (as the CG sits lower when the aerodynamic force pushes down, the air resistance is smaller) and better acceleration. This can be observed in Figures 9.5.1 and 9.5.3. The rolling movement of the body also affects the CG height, increasing it as the roll center sits lower than the CG. Some teams design the links to have the RC closer to the CG, but this have some secondary effects that won't be analyzed in this project.

The maximum wheel travel allowed in the Formula SAE regulations is 2 inches, which in our simulation can be related to the deviation of the CG from its steady state position. As it can be seen, there is no problem as the maximum variation for the rear is less than 2 cm.

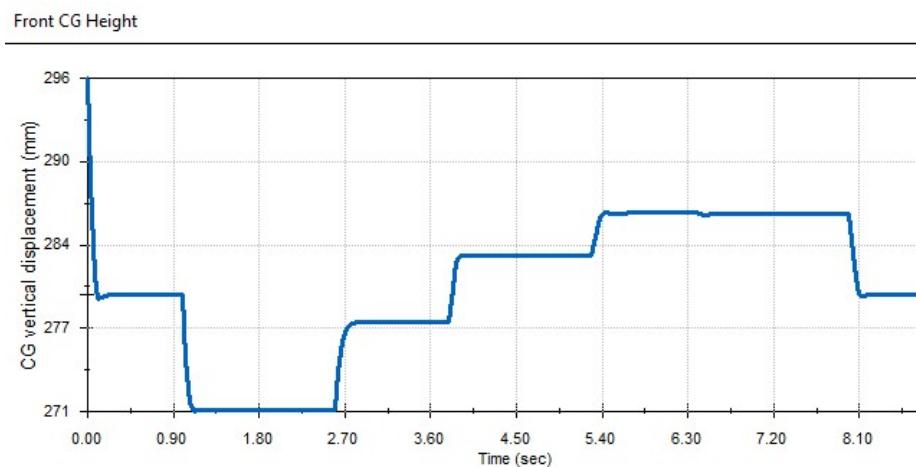


Figure 9.5.1: Front CG height throughout the simulation

Front CG height (TR19)

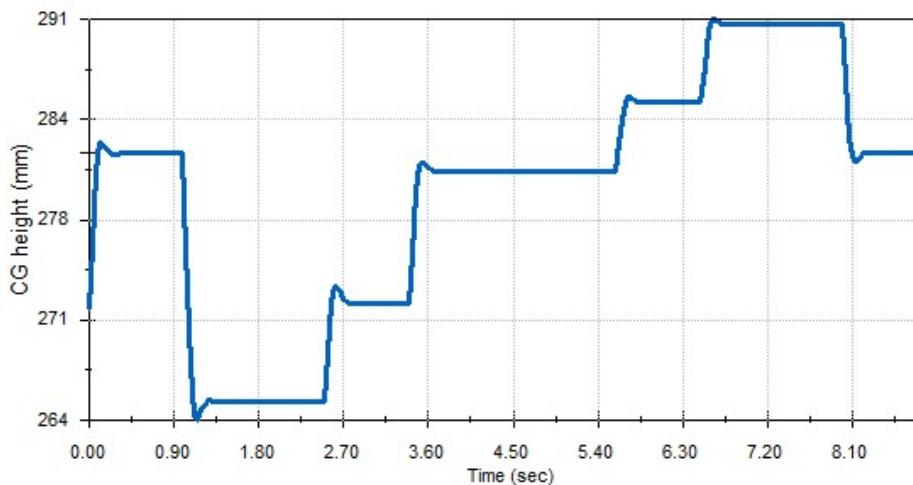


Figure 9.5.2: TR19's front CG height throughout the simulation

Rear CG Height

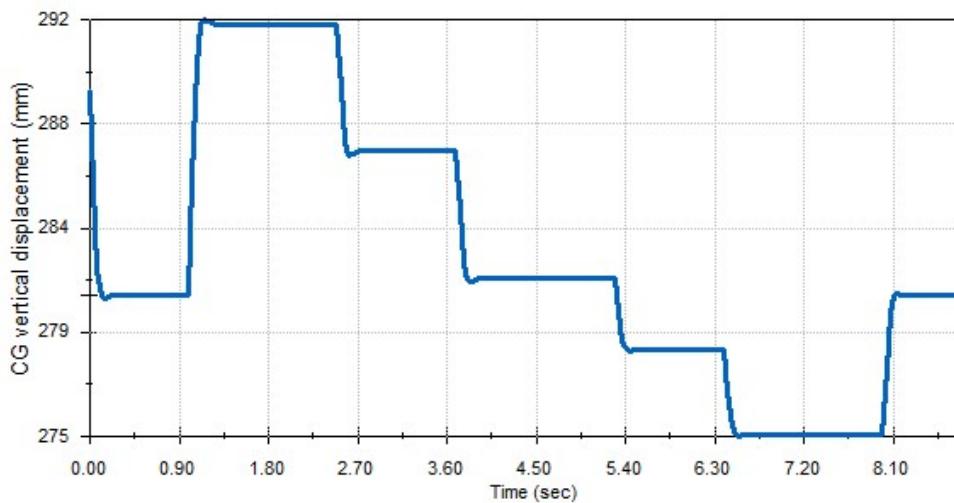


Figure 9.5.3: Rear CG height throughout the simulation

The biggest difference in terms of performance can be related with the CG height variation. As it was said, keeping the CG vertical displacement as steady as possible has several aerodynamic advantages, as well as others related with lateral and longitudinal load transfer. In Figure 9.5.2 the overall CG variation is greater for the conventional pushrod system, as the optimal ride spring rate was modified slightly in order to not compromise the vehicle's behavior during roll. Also, and keeping the tendency with the previous aspects analyzed, the transitions from the different motions are vary abrupt, which

can be noticed in the timing between the start of the motion and the body reacting to it (in this case changing the CG height).

To conclude, it is important to say that the main advantage of the Mode Separated System over conventional pushrods systems is its tuning capacity. This aspect wasn't analyzed throughout this simulation, but during testing in the circuit it will make tuning much easier and intuitive, as well as increase the performance when some parameters need to be changed.

Aspects like the understeer-oversteer tendency could be corrected without affecting the vehicle's response in ride and pitch. To illustrate this, if the vehicle has a tendency to understeer and the front spring rate is softened, the ride behavior will be the same. The vehicle will get to the steady state when a vertical load is applied in exactly the same time and will still have the same response. If this was done in any conventional suspension system, the front CG height variation will change, as well as the overall ride behavior and the steady state.

Also, as an example, if the damping coefficient in ride is decreased, the vehicle will get to the steady state faster, but will oscillate more after reaching it. If the damping coefficient for roll is kept the same, the vehicle's response to roll is not altered. However, if the damping coefficient is modified in the TR19's car, the overall behavior of the car will be altered.

10. Estimated Cost

In the table below the estimated cost of the project is shown. In blue, the four spring and damper could be re-used from last year's TR19 car, as they will remain the same with some length differences that needs to be studied. As these springs are the most expensive part of the project, if they are re-used the total cost will be very low.

It is important to note that the building cost were not considered. Total man hours were neglected as the project will be built by the team, and no salaries were considered. For the building cost, the team already has all the necessary tools to build the suspension system. These tools are a metallic lathe, a metal-cutting tool a welder and some basic ones like a screwdriver or similar.

ITEM	NUMBER	UNIT PRICE	TOTAL PRICE (US \$)
Ohlins TTX25 MkII	4	987,7	3950,8
Bearing SKF 6002 15x35x9	8	3,6	28,8
Alluminum Block	4	20	80
8mm steel shaft	2	5	10
15mm steel shaft	2	10	20
9mm steel plates	1	100	100
M8 bots	8	5	40
16mm steel tubes	4	10	40
TOTAL			4269,6

Figure 10: Estimated cost of the project

11. Comments and Conclusion

Throughout this project, a Mode Separated Suspension System was developed for the Formula SAE car from the University of Maryland Team TERPS Racing. To achieve all the goals that were initially proposed, and a few more that were later added, many aspects were considered.

The first and most important one was analyzing which was the best option for developing an effective system given the geometry of the car. Since it is a rare configuration, there weren't many examples in which the system could be based. When the Mercedes Project One suspension was found to be the best base to develop the system, sketching a possible geometry given the characteristics of TR19's car was the next step. Having a monocoque structure for the front and a tubular frame for the rear will mean that the suspension mounts will be different for front and rear. Since the space in both axles was limited, it was necessary to study how to place all the elements to keep them from colliding with other parts of the suspension and the frame itself. Since the geometry of the front and rear frames were different, the Motion Ratios of both systems was different, which means that both axles require different geometries and therefore different simulations. When designing the sketches of how the system should look like, all the elements were kept as low and centered as possible, to improve aerodynamic and balance performances. For the front, placing the suspension system as compacted in the monocoque gap as possible increased the driver's visibility as well. The suspension levers for both front and rear were the same, to keep manufacturing cost as low as possible.

Analyzing how spring and damping ratios affect the overall behavior of the car and developing the optimal configurations for both front and rear was crucial to create an adequate simulation. Some typical configurations and its effects in the behavior of the car were also included to help with tuning. All the necessary equations were developed for a complete analysis of this suspension system, including spring and damping rates, forces under different motions or load transfers. Since the suspension system was not symmetrical and therefore like other common systems, the equations were not trivial, and some of them needed to be created for the Mode Separated System.

After doing some research on possible computer programs to create and analyze the suspension model, Matlab Simscape Multibody (or Simmechanics) was chosen as the optimal software. After learning how to work with it and starting the creation of the model, this program was left because it would have taken too much time to create an appropriate working model, as the suspension has many unknown dimensions that need to be studied and optimized. For that reason, the new program chosen was Solidworks. Using a 2D block assembly, a model was created for both front and rear. Using Solidworks

Motion Study, it was possible to add the calculated forces and parameters to the model to run the simulation, and test the vehicle behavior under different suspension parameters, forces and conditions. A 2D block assembly model of TR19's front suspension was also developed to compare the results with the new suspension system. After having the 2D model of all the parts, a 3D assembly with all the components, for both front and rear, was created. These models showed that the system could be fitted in the available space without compromising other important performance requirements, like the aerodynamic efficiency.

As a conclusion, and analyzing the results from the simulation, it can be observed that the vehicle's performance will be increased. The results from the simulation were as expected, matching the equations developed and the theoretical values obtained.

The model simulation and the parameters analyzed gave a clear vision on how the system works and its possible advantages over the previous suspension system. The results showed that the system was able to decouple ride, pitch and roll motions under different spring and damper systems, allowing to have different configuration for each motion. As it was explained, these motions have some ideal parameters for the spring and damping rates for the vehicle's optimal performance, that were impossible to achieve with conventional suspension systems. Being able to decouple these two main motions also meant being able to tune the car differently for both. The roll spring could be tuned to correct the understeer/oversteer gradient, or the desired roll gradient (roll stiffness) without affecting the vehicle ride frequency or CG height. On the other hand, being able to adjust the ride spring and damping rate meant having a more stable CG height, an adequate ride natural frequency depending on the track and also different ride rates for the front and rear (with the advantages explained throughout the project) without changing lateral load transfer distribution or roll stiffness.

However, it is important to highlight that the vehicle's performance going over a single wheel bump will be compromised, as the two springs will react to the motion together. Comparing it to the previous suspension system, which has Anti-roll Bars, the ride frequency of this motion won't be as compromised. ARBs add an additional stiffness to have the desired roll frequency, but also affect when the vehicle hits a bump. Optimal ride frequencies for single wheel bumps are lower than the ones for ride and roll, which is neither achieved with the previous TR19's car.

It will also reduce weight, since the two heavy ARBs won't be necessary.

Finally, the increased tuning capability of the car will help the Team to get the most out of the car, adjusting the vehicle to different road surfaces, track characteristics, driver preferences or different events or runs.

12. Future Steps

To continue developing the Mode Separated System, there are some aspects that need to be considered. First, it is important to note that the unstretched length for the four springs needed is not the same. Even though they are in the range provided by the manufacturer (source OHLI19), it is possible that the only available lengths are 200 and 267 mm. This distance could be modified by reducing the spring's overall length thanks to the nut shown in Figures 6.3 and 7.3.10 (CAD Model). If this is not possible, or the range of lengths is lower than necessary, the distance between the suspension mounts, as well as the overall parameters for the suspension levers, will need to be slightly modified. Also, this modification will be necessary if there are other springs available for this project, as its dimensions may not be the same.

It is important to consider the fact that both springs, specially the roll one, must be capable of working in either compression or extension mode. If the actual spring only allows compression, another spring working for the extension motion must be added. It is very important to analyze this, as the space available to place the additional spring in the diagonal bar is limited. If this happens

Another aspect to consider is that the parts that were developed for this project were not simulated to proof if they resist the forces of the system. Even though the parts have reinforcement pieces and are wide enough considering the forces acting on them, there may be some weak points that could compromise the structural integrity of that part. Also, some material could be removed from the parts of the piece that are less stressed, helping to reduce weight. The suspension lever was chosen to be made of aluminum to reduce weight, but if necessary, this material could be changed. The feasibility of the manufacture of the suspension system was considered, but some of the parts may be a bit complex for the team to make, and some specialist could be necessary to manufacture them. This shape could be simplified, but always considering that all the parts don't have to collide with other suspension pieces, and that the system has to be able to work as stated.

Aspects like the tolerances or the fits between all the parts were not extensively studied. Since it was only a study project, the final design was provided to show a 3D drawing of how the system could be addressed. These aspects must be considered if the team decides to use this suspension system for their future cars.

It is also important to note that the suspension simulation was only made using a 2D model. For this reason, some assumptions were made in order to obtain an adequate simplification of the original model. Even though the simulation gives a general idea of how the behavior of the system will be, to

properly simulate the model the 3D assembly needs to be used. This assembly is already developed, but due to time restrictions, the motions were not introduced into the model. Since the goal of this project was understanding how the system works, compare it to the previous system and provide a possible design for the suspension, the 2D simulation was a valid tool.

Finally, for the estimated cost of the project, aspects like the fabrication man hours or the hours needed to make this project were not considered. Also, some the prices of the items needed (like aluminum blocks, steel plates or bolts) were an estimation based on the average prices found in different specialized websites but may differ from the real price of the specific material needed.

14. Appendix I: References

- [MATL19] Matlab Support Team. “Documentation - Simsacpe Multibody”. January 2019. <https://www.mathworks.com/help/physmod/sm/index.html>
- [OHLI19] Öhlins USA. “TTX25 MkII: Motorsport – Automotive”. January 2019. <https://www.ohlinsusa.com/suspension-products/ttx25-mkii>
- [OPTG19] Optimum G Engineering Team. “Optimum G Technical Papers - Tech Tips – Spring and Dampers”. January 2019. <http://www.optimumg.com/technical/technical-papers/>
- [TERP19] TERPS Racing Team. “Formula Team”. January 2019. <http://racing.umd.edu/teams/formula>
- [PERK17] Perkins, Chris. “How the Mercedes-AMG Project One’s Wacky Pushrod Suspension Works”. September 25, 2017. <https://www.roadandtrack.com/new-cars/future-cars/a12465870/how-the-mercedes-amg-project-ones-wacky-pushrod-suspension-works/>
- [SOLW19] SOLIDWORKS Help Team. “2016 SOLIDWORKS Help - Making Blocks”. 2016. https://help.solidworks.com/2016/english/SolidWorks/sldworks/t_Making_Blocks.htm
- [OLIV15] Oliver Santos, Rodrigo de. “A discussion on Steady-State Lateral Weight Transfer and how to use it in setup”. June 9, 2015. <http://racingcardynamics.com/weight-transfer/>
- [GILL92] Gillespie, Tom. “Fundamentals of Vehicle Dynamics”. Society of Automotive Engineers. 1992.
- [OLSE18] Olsen Motorsports. “Soft does not equal grip”. August 07, 2018. <https://www.olsenmotorsports.net/tech-blog/2018/8/7/soft-does-not-equal-grip-part-two>
- [PARR19] Parry, Tommy. “Camber: What is it and how can it make you faster”. April 27, 2019. <https://www.turnology.com/tech-stories/brakes-suspension/camber-what-is-it-and-how-can-it-make-you-faster/>
- [BALA09] Balachandran, Balakumar and Magrab, Edward B. “Vibrations Second Edition”. Cengage Learning. 2009.
- [ROBI18] Robinson, Matt. “How do Anti Roll Bars actually work?” July 2018. <https://www.carthrottle.com/post/how-do-anti-roll-bars-actually-work/>

- [KASP19] Kasprzak, Jim. “A guide to your dampers, Kaz Technologies Tech Tips” 2014.
<https://www.kaztechnologies.com/>
- [FORM19] Formula 1 Dictionary. “Motions on a Formula 1 car”. January 2019.
http://www.formula1-dictionary.net/motions_of_f1_car.html
- [XDRI] X Drive. Private online site (TERPS Racing Team access only) with documents and files from past edition’s cars and overall information about TR19’s car.
- [SKF19] SKF. “6002-2RSH Deep Groove Ball Bearings”. January 2019.
<https://www.skf.com/es/products/bearings-units-housings/ball-bearings/deep-groove-ball-bearings/>

15. Appendix II: List of Drawings

DRAWING NAME	NUMBER
Front Suspension Mounts	1
Rear Suspension Mounts	2
Right Suspension Lever	3
Left Suspension lever	4
Pushrod Bar Front	5
Pushrod Bar Rear	6
Full Car Assembly	7

16. Appendix III: Suspension assembly: List of parts

The complete list of parts of the assembly is provided below. Some of the parts were reassembled to meet the new dimensions and to simplified them leaving only the necessary parts related with the suspension system.

Part name	Author
Front monocoque	TERPS Racing Team
Rear tubular frame	TERPS Racing Team
Chassis mounts	TERPS Racing Team
Suspension mounts to the monocoque (Drawing 1)	Álvaro Martínez
Suspension mounts to the rear frame (Drawing 2)	Álvaro Martínez
Front Lower Control Arm (Figure 7.3.3)	TERPS Racing Team, reassembled and simplified by Álvaro Martínez
Front Upper Control Arm (Figure 7.3.4)	TERPS Racing Team, reassembled and simplified by Álvaro Martínez
Rear Lower Control Arm (Figure 7.3.7)	TERPS Racing Team, reassembled and simplified by Álvaro Martínez
Rear Upper Control Arm (Figure 7.3.8)	TERPS Racing Team, reassembled and simplified by Álvaro Martínez
Wheel and Spindle assembly (Figure 7.3.6)	TERPS Racing Team, reassembled and simplified by Álvaro Martínez
Front pushrod assembly (Figure 7.3.5)	TERPS Racing Team, reassembled and simplified by Álvaro Martínez
Front pushrod bar (Drawing 5)	Álvaro Martínez
Rear pushrod bar (Drawing 6)	Álvaro Martínez
Rear pushrod assembly (Figure 7.3.9)	TERPS Racing Team, reassembled and simplified by Álvaro Martínez
Right suspension lever (Drawing 3)	Álvaro Martínez
Left suspension lever (Drawing 4)	Álvaro Martínez
Ohlins TTX25 MkII	OHLI19
6002 SKF ball bearings	SKF19
Suspension shaft (mounts to lever)	Álvaro Martínez

17. Appendix IV: List of Figures

Note: If the source is not specified, the author is the same as the author of the project:
 Álvaro Martínez (2019)

Figure number and name	Source
Figure 1.1: Mercedes Project One suspension system	PERK17
Figure 1.2: TERPS Racing Logo	TERP19
Figure 3.1: TR19 from TERPS Racing, front view	TERP19
Figure 3.2: TR19 from TERPS Racing, rear view	TERP19
Figure 3.3: FSAE Design Spec Sheet for TR19	XDRI
Figure 4.1.1: Sketch of the front view of the front suspension system with its main components	
Figure 4.1.2: Sketch of the front view of the rear suspension system with its main components	
Figure 4.2.1: Basic motions of a car	FORM19
Figure 4.2.2: Suspension system motion under pitch	
Figure 4.2.3: Suspension system motion while the vehicle is cornering to the left	
Figure 4.2.4: Suspension system motion over a single wheel bump	
Figure 5.1.1: Roll Center and Instant Center	
Figure 5.2.1: Vertical load and lateral load transfer	OLIV15
Figure 5.2.2: FBD of the vehicle taking a corner	
Figure 5.2.3: FBD with the Roll Axis, Roll Centers and Center of Gravity	
Figure 5.2.4: Forces acting in the Roll Center as the body rolls during cornering	
Figure 5.2.5: FBD of the rolling moment and lateral forces on the wheels	
Figure 5.2.1.1: Weight transfer due to body roll	OLIV15
Figure 5.3.1: Longitudinal load transfer	

Figure 6.1.1: Equivalent spring of a system in series	
Figure 6.1.2.1: FBD of the roll moment affecting the equivalent spring in each corner	
Figure 6.1.2.2: Drawing of an Anti-Roll Bar	ROBI18
Figure 6.1.3: Equivalent spring of a system in parallel	
Figure 6.2.1: Generic Damping Ratios	BALA09
Figure 6.2.2: Different phases on a cornering situation	
Figure 6.2.1.1: Force vs Velocity plot of the Ohlins TTX25 MkII damper	OHLI19
Figure 6.3: Ohlins TTX25 MkII	OHLI19
Figure 7.1.1: Main blocks in Simscape Multibody or Simmechanics	MATL19
Figure 7.1.2: Main subsystems for the pushrod suspension example	MATL19
Figure 7.1.3: Different blocks and subsystems under the main assembly	MATL19
Figure 7.1.4: Results of running the simulation for the example	MATL19
Figure 7.1.5: Simscape model developed for the Mode Separated System	
Figure 7.1.6: Results of the simulation using Simscape	
Figure 7.2.1: Final sketch of the front axle of the vehicle's suspension	
Figure 7.2.2: Final sketch of the rear axle of the vehicle's suspension	
Figure 7.2.3: TR19's front suspension	
Figure 7.2.4: 3D assembly of the front suspension on TR19	XDRI
Figure 7.2.5: Sketch of the front suspension on TR19	
Figure 7.3.1: Whole vehicle assembly, front view	
Figure 7.3.2: Whole vehicle assembly, rear-side view	
Figure 7.3.3: Front Lower Control Arm assembly	XDRI
Figure 7.3.4: Front Upper Control Arm assembly	XDRI
Figure 7.3.5: Front pushrod assembly	
Figure 7.3.6: Wheel and spindle assembly	
Figure 7.3.7: Rear Lower Control Arm assembly	XDRI

Figure 7.3.8: Rear Upper Control Arm assembly	XDRI
Figure 7.3.9: Rear pushrod assembly	
Figure 7.3.10: Ohlins TTX25 MkII CAD Model	OHLI19
Figure 7.3.11: Front suspension Assembly, side view	
Figure 7.3.12: Front suspension Assembly, upper view	
Figure 7.3.13: Front suspension assembly	
Figure 7.3.14: Front suspension assembly with control arms, pushrods and wheels	
Figure 7.3.14: Rear suspension assembly, side view	
Figure 7.3.15: Rear suspension assembly, rear view	
Figure 7.3.16: Rear suspension assembly, upper side view	
Figure 7.3.17: Rear suspension assembly	
Figure 7.3.18: Rear suspension assembly, front view	
Figure 8.1: Cornering situation analyzed	
Figure 8.2: Front suspension simulation using Solidworks Motion study	
Figure 8.3: Rear suspension simulation using Solidworks Motion study	
Figure 8.4: TR19's Front suspension simulation using Solidworks Motion study	
Figure 8.5: Results being plotted in the simulation	
Figure 9.1.1: Front suspension initial reaction to the weight of the car	
Figure 9.1.2: Rear suspension initial reaction to the weight of the car	
Figure 9.1.3: Front pitch spring compression throughout the simulation	
Figure 9.1.4: Rear pitch spring compression throughout the simulation	
Figure 9.2.1: Front roll spring compression throughout the simulation	
Figure 9.2.2: Rear roll spring compression throughout the simulation	
Figure 9.2.3: Front spring displacement of TR19's suspension system throughout the simulation	

Figure 9.3.1: Positive and negative wheel camber	PARR19
Figure 9.3.2: Front inner wheel camber throughout the simulation	
Figure 9.3.3: Front inner TR19's wheel camber throughout the simulation	
Figure 9.3.4: Rear inner wheel camber throughout the simulation	
Figure 9.3.5: Front left corner of TR19's car showing the adjustable control arms	
Figure 9.3.6: Front outer wheel camber throughout the simulation	
Figure 9.3.7: Front TR19's outer wheel camber throughout the simulation	
Figure 9.3.8: Rear outer wheel camber throughout the simulation	
Figure 9.4.1: Front body roll throughout the simulation	
Figure 9.4.2: TR19's body roll throughout the simulation	
Figure 9.4.3: Rear body roll throughout the simulation	
Figure 9.5.1: Front CG height throughout the simulation	
Figure 9.5.2: TR19's front CG height throughout the simulation	
Figure 9.5.3: Rear CG height throughout the simulation	
Figure 10: Estimated cost of the project	