

## Appendix A: The physics of the Direct $T_e$ method

UV photons from young stars in an H II region keep the interstellar gas partially ionized. In an H II region, a photon with an energy  $E_\gamma \gtrsim 13.6$  eV ionizes a hydrogen atom, producing an  $H^+$  ion and an electron with some kinetic energy  $K_{e-} = E_\gamma - 13.6$  eV. As the free electron moves around in the gas, it loses some of its kinetic energy as it collisionally excites other ions in the gas. Because we are in a Strömgren sphere (an ionized region or an H II region), the cross-section for this electron scattering is much larger than the photoionization cross section, so the free electrons will thermalize quickly (De Robertis et al., 1987). Eventually, the free electron recombines with an  $H^+$  ion, forming atomic hydrogen. The ions that are collisionally excited by the electron before its recombination eventually de-excite via forbidden transitions, emitting radiation that escapes the H II region.

In equilibrium, the total energy input into the gas via radiation from the star (that ionizes the hydrogen),  $E_{photoionization}$ , must be equal to the energy required to recombine the electron and the  $H^+$  ion,  $E_{recombination}$ , and the energy radiated away from the H II region by the forbidden transitions,  $E_{radiation}$ . So

$$E_{photoionization} = E_{recombination} + E_{radiation} \quad (A.1)$$

For radiative de-excitation to dominate over collisional de-excitation, the electron density  $n_e \ll 10^{8-10} \text{ cm}^{-3}$  (De Robertis et al., 1987). Typical H II regions have an electron density  $n_e \approx 100 \text{ cm}^{-3}$ , so radiative de-excitation dominates; this is why we observe forbidden transitions in outer space but not here on Earth.

### A.1 Forbidden transitions

Similar to a fingerprint, every element has a unique set of wavelengths seen in its spectrum that corresponds to the atom's different energy levels. Since electrons within an atom have discrete energy levels, transitions between levels result in either the emission (when decreasing in energy)

or absorption (an increase in energy) of a photon of the same energy as the difference between the two levels. The separation between energy levels is unique for each element, so the photon energies either emitted or absorbed by each element are unique.

### A.1.1 Time-dependent perturbation theory

A time-dependent potential is added to the Schrödinger equation in order to solve for the forbidden transition energies. We can assume that the time perturbation is a minor component of the potential and can thus treat it as a perturbation. For a simplified two-level system, the time-dependent wave function is

$$\Psi(t) = c_a(t)\psi_a e^{iE_a t/\hbar} + c_b(t)\psi_b e^{-iE_b t/\hbar} \quad (\text{A.2})$$

To solve for the time-dependent coefficients, we substitute  $\Psi(t)$  into the time-dependent Schrödinger equation

$$H\Psi = i\hbar \frac{\partial \Psi}{\partial t}, \quad \text{where } H = H^0 + H'(t) \quad (\text{A.3})$$

We find that

$$\dot{c}_a = -\frac{i}{\hbar} H'_{ab} e^{-i\omega_0 t} c_b \quad (\text{A.4})$$

$$\dot{c}_b = -\frac{i}{\hbar} H'_{ba} e^{-i\omega_0 t} c_a \quad (\text{A.5})$$

where

$$\omega_0 \equiv \frac{E_b - E_a}{\hbar} \quad (\text{A.6})$$

and

$$H_{ij} \equiv \langle \psi_i | H' | \psi_j \rangle \quad (\text{A.7})$$

The probability that transmission has occurred is  $P_{b \rightarrow a} = |c_a(t)|^2$ . The transmission rate  $R_{a \rightarrow b} = dP/dt$ .

For an electromagnetic wave, the atom experiences an oscillating electric field. For visible light, it is typically safe to assume that the spatial portion of the field does not change, since the wavelength

is much longer than the size of the atom. However, if we consider a spatial variation of the field, we find that the true electric field is

$$\vec{E}(\vec{r}, t) = \vec{E}_0 \cos(\vec{k} \cdot \vec{r} - \omega t) \quad (\text{A.8})$$

Expanding this to first order, we find that

$$\vec{E}(\vec{r}, t) = \vec{E}_0 [\cos(\omega t) + (\vec{k} \cdot \vec{r} \sin(\omega t))] \quad (\text{A.9})$$

The first term in this approximation represents the electric dipole transitions (the canonical electron transitions). The second term leads to forbidden transitions (magnetic dipole and electric quadrupole).

## A.2 Temperatures from the forbidden emission lines

Expanding the description in Sec. 2.2.2, we can define the ratio of the emission line strengths for [O III]  $\lambda 4363$  and [O III]  $\lambda \lambda 4959, 5007$  as

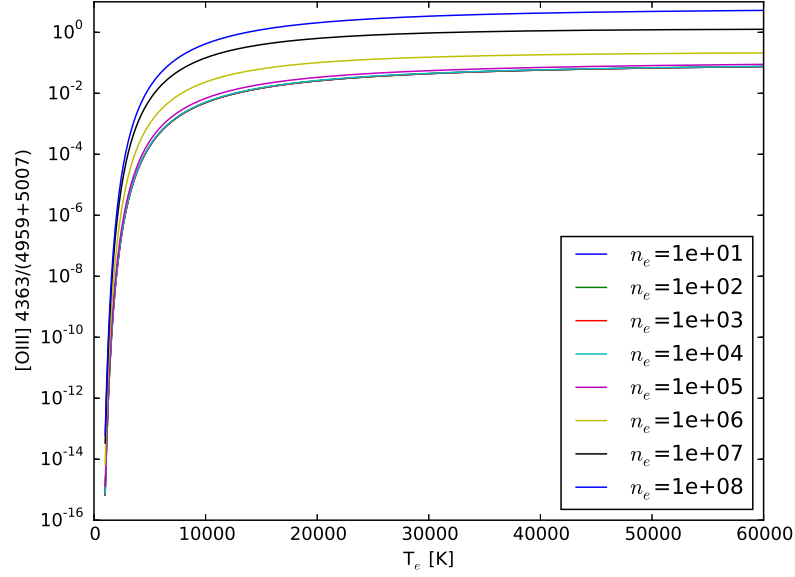
$$\frac{j_{4959} + j_{5007}}{j_{4363}} = \frac{\Omega(^3\text{P}, ^1\text{D})}{\Omega(^3\text{P}, ^1\text{S})} \left[ \frac{A_{1\text{S}, ^1\text{D}} + A_{1\text{S}, ^3\text{P}}}{A_{1\text{S}, ^1\text{D}}} \right] \frac{\bar{\nu}(^3\text{P}, ^1\text{D})}{\nu(^1\text{D}, ^1\text{S})} e^{\Delta E/kT} \quad (\text{A.10})$$

where

$$\bar{\nu}(^3\text{P}, ^1\text{D}) = \frac{A_{1\text{D}_2, ^3\text{P}_2} \nu(\lambda 5007) + A_{1\text{D}_2, ^3\text{P}_1} \nu(\lambda 4959)}{A_{1\text{D}_2, ^3\text{P}_2} + A_{1\text{D}_2, ^3\text{P}_1}} \quad (\text{A.11})$$

and  $j_\lambda$  is the emissivity of the emission line,  $\Omega(i, j)$  is the collision strength between levels  $i$  and  $j$ ,  $A_{i,j}$  is the radiative transition probability between an upper level  $i$  and a lower level  $j$ ,  $\nu$  is the frequency of the transition,  $\Delta E$  is the energy difference between the  $^1\text{D}_2$  and  $^1\text{S}_0$  levels, and  $T$  is the electron temperature (Osterbrock, 1989). The transition probabilities  $A_{i,j}$  do not depend on the temperature, but the collision strength  $\Omega(i, j)$  is temperature-dependent. We can define

$$\bar{\nu}(^3\text{P}, ^1\text{D}) = \frac{A_{1\text{D}_2, ^3\text{P}_2} \nu(5007) + A_{1\text{D}_2, ^3\text{P}_1} \nu(4959)}{A_{1\text{D}_2, ^3\text{P}_2} + A_{1\text{D}_2, ^3\text{P}_1}} \quad (\text{A.12})$$



**Figure A.1:** The [O III]  $\lambda 4363/(\lambda 4959 + \lambda 5007)$  ratio as a function of temperature for various densities (Luridiana et al., 2015). Typical electron densities in H II regions are  $\sim 100 \text{ cm}^{-3}$ .

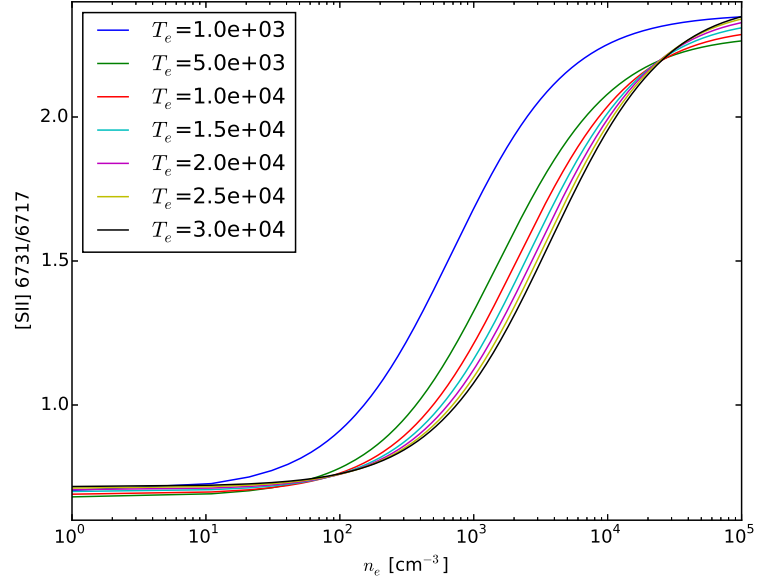
Inserting numerical values for the collision strengths and transition probabilities from Osterbrock (1989), the ratio becomes

$$\frac{j_{4959} + j_{5007}}{j_{4363}} = \frac{7.73 \exp[(3.29 \times 10^4)/T]}{1 + 4.5 \times 10^{-4}(N_e/T^{1/2})} \quad (\text{A.13})$$

Fig. A.1 shows how this ratio depends on the temperature. By measuring the flux of these emission lines, we can then solve for the temperature of the gas.

### A.3 [S II]

Similar to the sensitivity of the doubly-ionized oxygen ion transitions to the electron temperature of the surrounding gas, the transitions for singly-ionized sulfur are sensitive to electron number density. The relative excitation rates depend only on the ratio of the collision strengths when two emission lines (from the same ion) with nearly identical excitation energies are compared (Osterbrock, 1989). If the two levels have different transition probabilities and/or different collisional de-excitation rates, their ratio depends on the density.



**Figure A.2:** The [S II]  $\lambda 6731/\lambda 6717$  ratio as a function of density for various temperatures (Luridiana et al., 2015). The gas temperatures of H II regions are typically around  $10^4$  K.

The relative excitation rates of the two lines shown in Fig. 2.1 are proportional to their statistical weights; thus, the ratio of the line intensities is a constant in the low-density limit (Osterbrock, 1989). In the high-density regime, this ratio is best accurately described by a Boltzmann population ratio. There is a critical density for the energy levels which describes the turning point between these two extremes. A graphical representation of this ratio as a function of density can be seen in Fig. A.2.