03-28-22 - HW 4

Sunday, March 27, 2022 6:19 PM

5.4. a. Representing the equations in state space form

$$\begin{bmatrix} x_{p,k+1} \\ x_{g,k+1} \end{bmatrix} = \begin{bmatrix} 1-k_2 & k_1 \\ -k_3 & 1 \end{bmatrix} \begin{bmatrix} x_{p,k} \\ x_{g,k} \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_k + \begin{bmatrix} w_{p,k} \\ w_{g,k} \end{bmatrix}$$

$$= \begin{bmatrix} 1/2 & 1 \\ -1/2 & 1 \end{bmatrix} x_k + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_k + w_k$$

$$y_k = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_{p,k} \\ x_{g,k} \end{bmatrix} + v_k$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} x_k + v_k$$

From info in the question, we also know that

$$Q = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \text{ and } R = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

So $w_k \sim N(0, Q)$ and $v_k \sim N(0, R)$

b. Given that initial population count is perfect

$$\therefore P_0^+ = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Iteratively using the Kalman Filter equations

$$P_1^- = FP_0^+F^T + Q = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$K_1 = P_1^- H^T (HP_1^- H^T + R)^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1/2 \end{bmatrix}$$

$$P_1^+ = (I - K_1 H) P_1^- = \begin{bmatrix} 0 & 0 \\ 0 & 1/2 \end{bmatrix}$$

$$P_2^- = FP_1^+ F^T + Q = \begin{bmatrix} 3/2 & 1/2 \\ 1/2 & 3/2 \end{bmatrix}$$

$$K_2 = P_2^- H^T (HP_2^- H^T + R)^{-1} = \begin{bmatrix} 1 & 0 \\ 1/7 & 4/7 \end{bmatrix}$$

$$K_2 = P_2^- H^T (H P_2^- H^T + R)^{-1} = \begin{bmatrix} 1 & 0 \\ 1/7 & 4/7 \end{bmatrix}$$

$$P_2^+ = (I - K_2 H) P_2^- = \begin{bmatrix} 0 & 0 \\ 0 & 4/7 \end{bmatrix}$$

$\therefore$ After 1 week the variance of the guppy population is $1/2$ and $4/7$ after two weeks

c. As $k \to \infty$ $x_{k+1} = x_k$ (i.e. steady-state)

$$x_k = F x_k + G u_k$$

$$(I - F) x_k = G u_k$$

$$x_k = (I - F)^{-1} G u_k$$

$$= \begin{bmatrix} 2 \\ 1 \end{bmatrix} u_k$$

$\therefore$ The ratio of the piranha population to the guppy population is 2:1

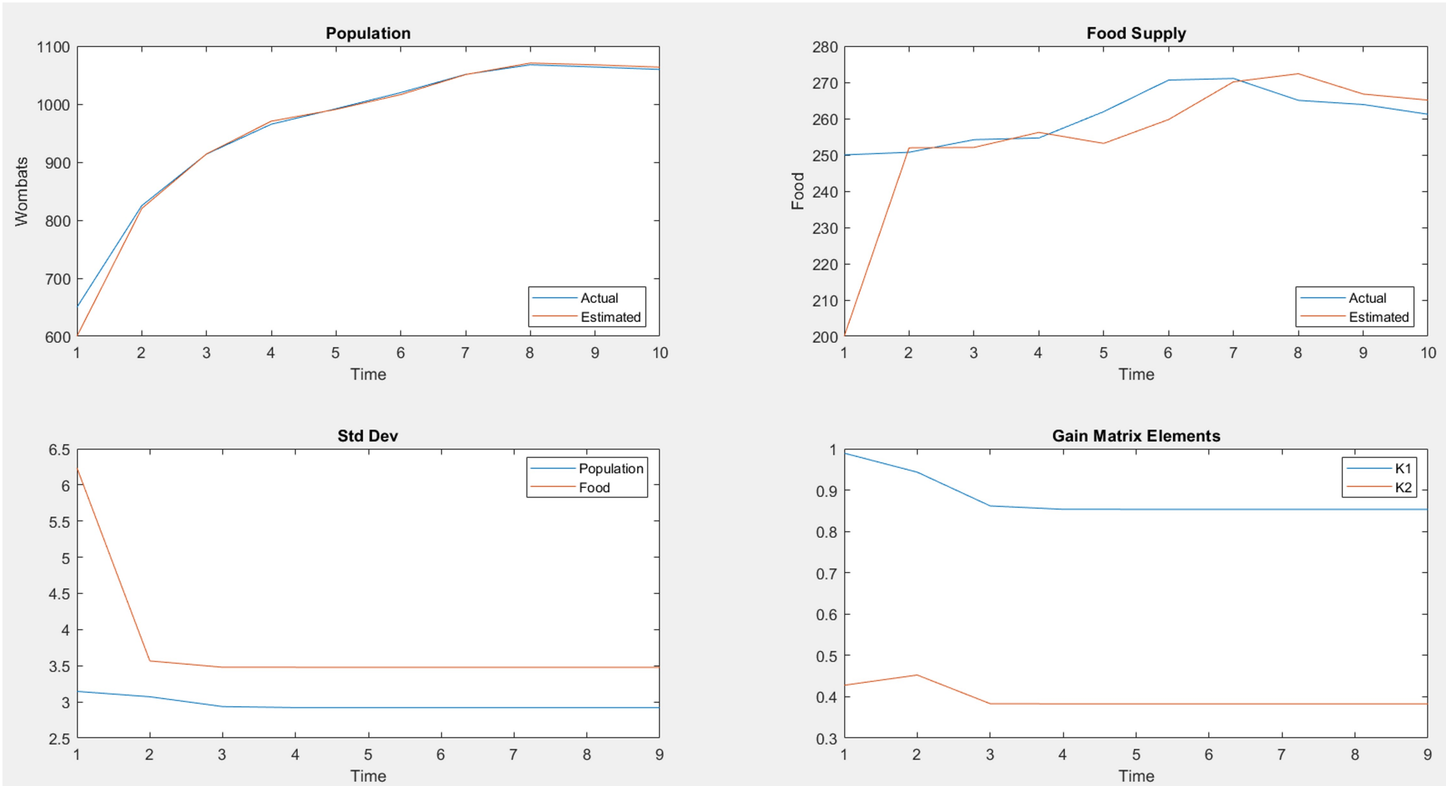
5.11. $$\begin{bmatrix} p_{k+1} \\ f_{k+1} \end{bmatrix} = \begin{bmatrix} 1/2 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} p_k \\ f_k \end{bmatrix} + \omega_k$$

$$\omega_{f,k} \sim N(0, Q) \quad \text{where } Q = \begin{bmatrix} 0 & 0 \\ 0 & 10 \end{bmatrix}$$

$$y_k = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} p_k \\ f_k \end{bmatrix} + v_k$$

$$v_k \sim N(0, R) \quad \text{where } R = 10$$

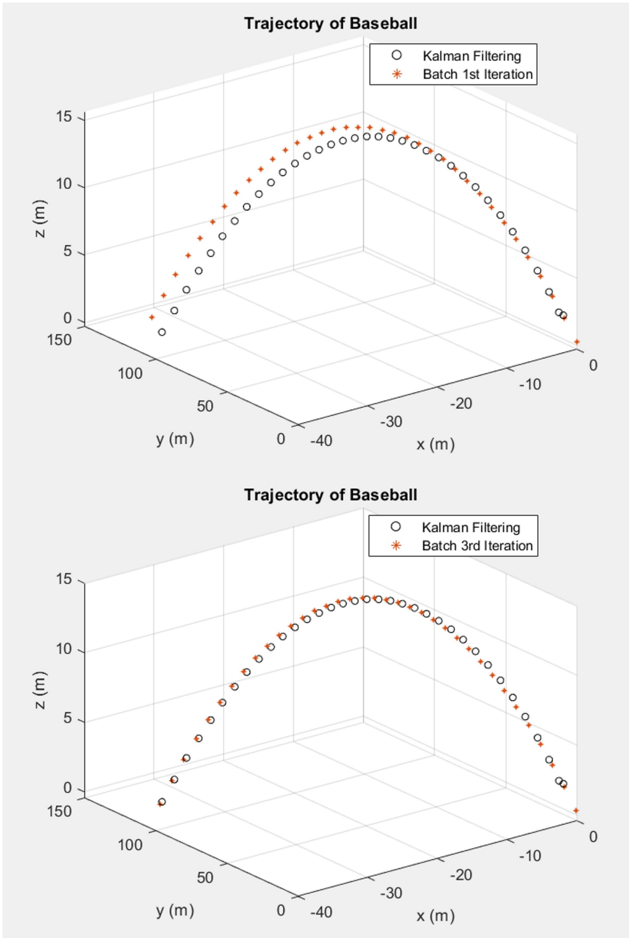
After assembling this in MATLAB and using Eq (5.17) through (5.19) from Simon, we get the following plots



b. The std. devs of the population and food supply estimation errors starts around 15. The steady state std. devs. settle at ~2.92 for the wombat population and ~3.47 for the food supply. This difference is due to the large error in the initial estimate and the small number of simulation iterations.

c. As the number of timesteps increases, the std dev. moves closer to the theoretical std. dev because the initial estimation errors are dealt with as more simulations are run.

2. • Made necessary changes to batch approach for a priori information
   • Implemented Kalman Filter code
   (Code in appendix)



Trajectory of Baseball
Kalman Filtering
Batch 1st Iteration
15
10
5
0
z (m)
150
100
50
0
y (m)
-40
-30
-20
-10
0
x (m)
Trajectory of Baseball
Kalman Filtering
Batch 3rd Iteration
15
10
5
0
z (m)
150
100
50
0
y (m)
-40
-30
-20
-10
0
x (m)