Homework #3
AE6505 Kalman Filtering, Spring 2022
Prof. Gunter
Assigned: 2-15-22
Due: 2-25-22 (2-28-22 for DL Students)

Homework is due by 11:59p on the indicated due date, and should be submitted electronically via Canvas. Late homework will not be accepted without prior permission from the instructor. In-class verbal due date announcements override projected dates in the lecture plan. Please submit your materials as two files. The first should be a writeup of your solutions, complete with any figures, explanations, etc., in .pdf form. This is the document that will be graded, i.e., do not embed solutions in your code, or require the grader to run your code to get any results. Homework should be professional, legible, indicate units, and sufficiently describe all important steps in a solution. Your final answer for each problem should be boxed or clearly indicated. You are welcome to scan any pages that are handwritten, but please make sure any such pages are clear and legible. Deductions will be made for incomplete solutions and improper formats. In addition to the .pdf file, upload any Matlab files that you have developed to generate the results described in your writeup as a single .zip file. The Matlab code you submit should be able to be run without modification, so do not include hardcoded file paths.

1. Exercises 4.5, 4.8, 4.11, and 4.13 from the text by Simon. Do not do part b of 4.13. Instead, show how

$$F = e^{AT} = \begin{bmatrix} (2e^{-T} - e^{-2T}) & (2e^{-T} - 2e^{-2T}) \\ (-e^{-T} + e^{-2T}) & (-e^{-T} + 2e^{-2T}) \end{bmatrix}$$

where T here represents the discretization step size. For this, you'll need to compute the matrix exponential as shown in the third expression in Eqn 1.71 (i.e., you'll need to analytically compute the Jordan form of A).

2. Write a Matlab program to proces range and range-data for the spring-mass problem covered in the lecture and generate an estimate of X_0 and V_0. Use the following data set, which has Gaussian noise with mean zero and $\sigma_\rho = 0.25$ m and $\sigma_{\dot\rho} = 0.10$ m/s,

```
ρ = [ 6.3118 6.0217 5.1295 6.3685 5.5422 5.5095 5.9740 5.4930 6.8653 6.6661 5.0692]';
ρ̇ = [ 0.3035 1.3854 -1.5512 0.6064 0.8642 -1.5736 1.1588 0.4580 -1.2328 1.4348 -0.4229 ]';
```

and the following initial conditions,

$$X_0^* = \begin{bmatrix} 3.9 \\ 0.5 \end{bmatrix}$$

$$\bar{P}_0 = \begin{bmatrix} 1000 & 0 \\ 0 & 100 \end{bmatrix}$$

,

and measurement noise covariance

$$R = \begin{bmatrix} \sigma_\rho^2 & 0 \\ 0 & \sigma_{\dot\rho}^2 \end{bmatrix} = \begin{bmatrix} 0.0625 & 0 \\ 0 & 0.01 \end{bmatrix}$$

,

Assume $k_1 = 2.6$, $k_2 = 3.5$, $m = 1.8$ and $h = 6.4$. Show your estimate after three iterations. Assume the measurements are taken in one second intervals starting at t = 0.

3. You've been hired by the Atlanta Braves baseball team to determine the distance homerun balls travel during home games (one of the many statistics tracked in baseball). Because a homerun ball can hit the stadium, or be caught by fans, before completing its full trajectory, you will be estimating the total horizontal distance the homerun ball would have travelled using a special lidar ranging system that has been set up. The lidar system gathers range (ρ), azimuth (α), and elevation (β) information on the baseball as it travels with respect to home plate, but only during part of the initial upward trajectory (it loses track of it as the baseball gets far away). When hit, assume the baseball follows the classical parabolic path, i.e., assume the only force acting on it after contact is gravity and ignore any atmospheric effects. Independent calibration of your tracking system shows that the range values are typically accurate to 5 ft., while the azimuth and elevation angles are typically accurate to 0.1 deg.

The state vector is $X^T = [x, y, z, v_x, v_y, v_z]$, and the system dynamics can be described by the following:

$$x = x_0 + v_{x0}t$$

$$y = y_0 + v_{y0}t$$

$$z = z_0 + v_{z0}t - \frac{1}{2}gt^2$$

$$v_x = v_{x0}$$

$$v_y = v_{y0}$$

$$v_z = v_{z0} - gt$$

(a) Determine the 6x6 state transition matrix, $\Phi(t, 0)$. Hint: use $\Phi(t, 0) = \frac{\partial x}{\partial x_0}$.

(b) Determine the measurement models, $G(X)$, noting that the measurement vector $Y^T = [\rho, \alpha, \beta]$.

(c) Now derive the elements of the 3x6 $\tilde{H}$ matrix

(d) To test the system, data was collected from the lidar system for a homerun ball hit during a training session. The data is in the file "homerun_data.txt," and is plotted in the figure below. File columns are time (s), range (ft), azimuth (deg), and elevation (deg). Using the data, and your knowledge of the dynamics, estimate the total horizontal distance travelled by the baseball. Use the following as initial conditions:

X_0^T `= [0.4921 0.4921 2.0013 -26.2467 114.3051 65.9941; % units in ft`

(e) What is the uncertainty on your estimate?