02-09-22 - HW 2

Wednesday, February 9, 2022 1:51 PM

3.5.a. $\hat{x} = (H^T W H)^{-1} H^T W y$

$H = [1; 1]$

$W = \begin{bmatrix} 1 & 0 \\ 0 & 1/4 \end{bmatrix}$

$y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$

$\therefore \hat{x}_0 = [4/5 \quad 1/5] \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \frac{4}{5} y_1 + \frac{1}{5} y_2$

b. $\hat{x}_0 = [0]$

$P_0 = [1]$

$H_1 = [1]$

$R_1 = [1]$

Using

$K_k = P_{k-1} H_k^T (H_k P_{k-1} H_k^T + R_k)^{-1}$

$P_k = (I - K_k H_k) P_{k-1} (I - K_k H_k)^T + K_k R_k K_k^T$

* Code included in Appendix

$P_2 = 0.5$

$P_3 = 0.444$

3.7.

	Est 1	Est 2	Est 3
RMS	2.9439	3.6968	6.0553
Average absolute error	2	1	4
Standard deviation	1.5275	2.6458	1

Estimate 1 has the smallest RMS value.
Estimate 2 has the smallest average absolute error.
Estimate 3 has the smallest standard deviation.

Estimate 3 is the best estimate because it involves a simple bias

offset to bring it closer to the true values.
Estimate 1 is the worst estimate because there is no obvious trend in the data.

3.9. It would be better to combine three present radar systems (P)

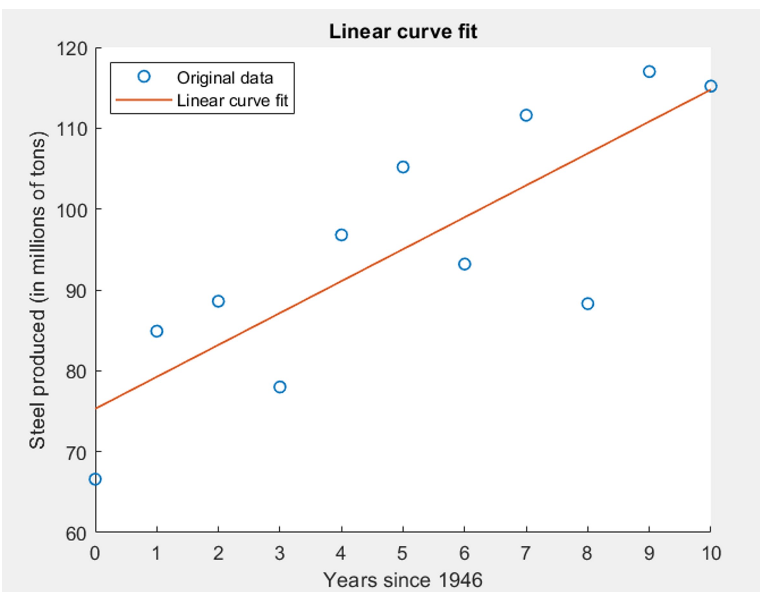
This is because

$$Var\left(\frac{P+N}{2}\right) = \frac{Var(P)}{2^2} + \frac{Var(N)}{2^2} = \frac{10+6}{4} = 4$$

$$Var\left(\frac{P+P+P}{3}\right) = \frac{3Var(P)}{3^2} = \frac{10}{3} = 3.333$$

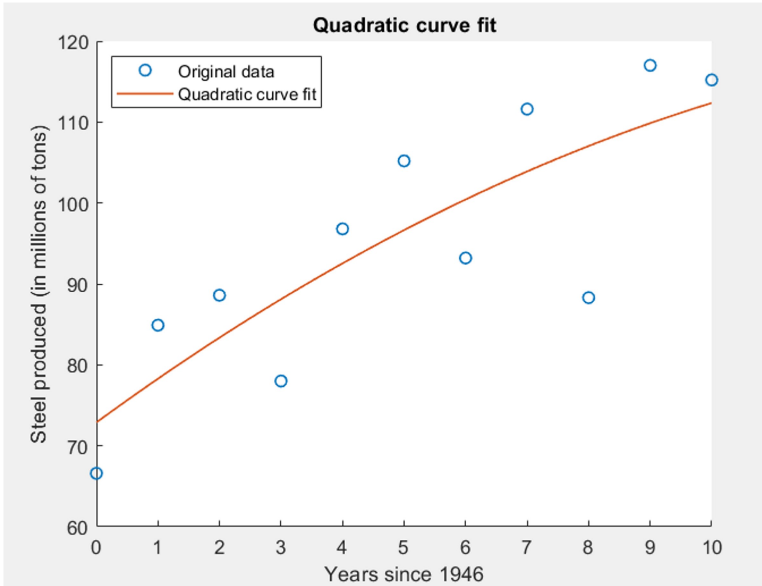
Since combining three present systems has the lowest variance, that is the best option.

3.13.a.



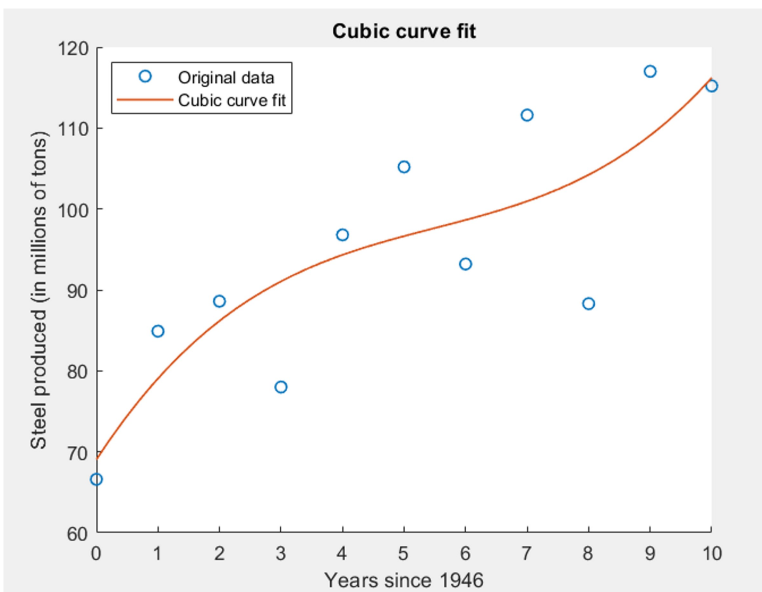
RMS = 8.7823

prediction = A(1) + 12A(2) = 122.66



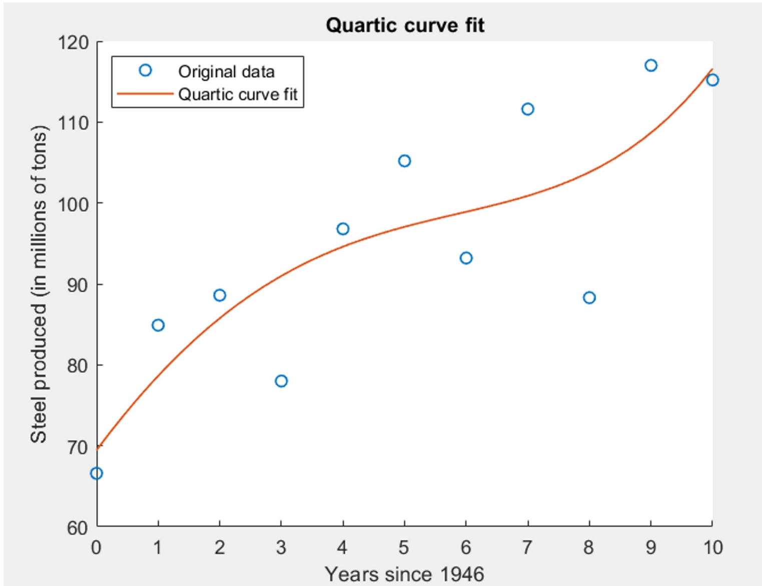
RMS = 8.665

prediction = $A(1) + 12A(2) + 12^2 A(3) = 116.38$



RMS = 8.2889

prediction = $A(1) + 12A(2) + 12^2 A(3) + 12^3 A(4) = 139.7$



RMS = 8.2816

prediction = $A(1) + 12A(2) + 12^2 A(3) + 12^3 A(3) + 12^4(5) = 146.83$

2.a. $A = \begin{bmatrix} b \\ m \\ A_1 \\ B_1 \\ A_2 \\ B_2 \end{bmatrix}$ where b is the bias and m is the drift (from fitting $y = mt + b$ to the data), A_1 and B_1 are the coefficients of the once-per-rev periodic (from fitting $A_1 \cos(t/5677) + B_1 \sin(t/5677)$) and A_2 and B_2 are the coefficients of the twice-per-rev periodic (from fitting $A_2 \cos(t/2838.5) + B_2 \sin(t/2838.5)$)

$$H = \begin{bmatrix} 1 & t_1 & \cos(t_1/5677) & \sin(t_1/5677) & \cos(t_1/2838.5) & \sin(t_1/2838.5) \\ 1 & t_2 & \cos(t_2/5677) & \sin(t_2/5677) & \cos(t_2/2838.5) & \sin(t_2/2838.5) \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & t_m & \cos(t_m/5677) & \sin(t_m/5677) & \cos(t_m/2838.5) & \sin(t_m/2838.5) \end{bmatrix}$$

$$X = \begin{bmatrix} t_1 \\ t_2 \\ \vdots \\ t_m \end{bmatrix}$$

$$G(X) = A(1) + A(2)X + A(3)\cos(X/5677) + A(4)\sin(X/5677) +$$

$A(5)\cos(x/2838.5) + A(6)\sin(x/2838.5)$

b. $b = 3.0741E{-7}$

$m = -2.2397E{-12}$

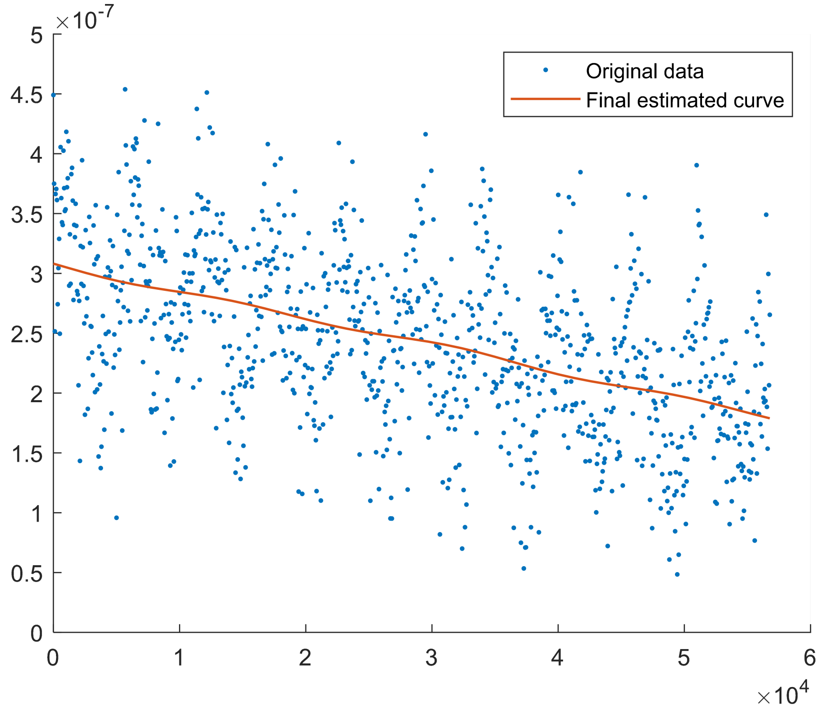
$A_1 = 2.4379E{-10}$

$B_1 = -6.8337E{-10}$

$A_2 = 5.0691E{-10}$

$B_2 = -1.8935E{-9}$

c.



d.

	Mean	Std dev	RMS	
Pre-fit	2.4367E-7	7.4825E-8	2.5489E-7 $= r^2$	
Post-fit	2.4367E-7	3.6541E-8	2.4639E-7	6.5263E-8 $= s^2$

Quality of the estimate $= \dfrac{r^2 - s^2}{r^2} = 0.9344$