Articulation Points

Biconnectedness

A graph is biconnected if every pair of vertices is connected by two vertex-disjoint paths

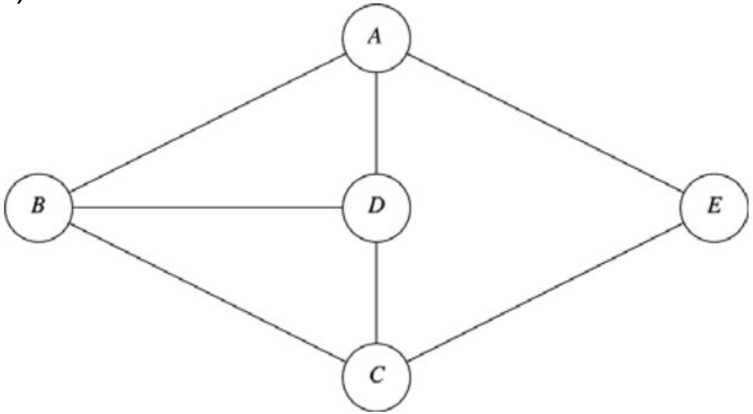
- A graph is biconnected if it has no articulation points

An articulation point (or cut vertex) in a connected graph is a vertex that would disconnect the graph if it (and its incident edges) were removed.

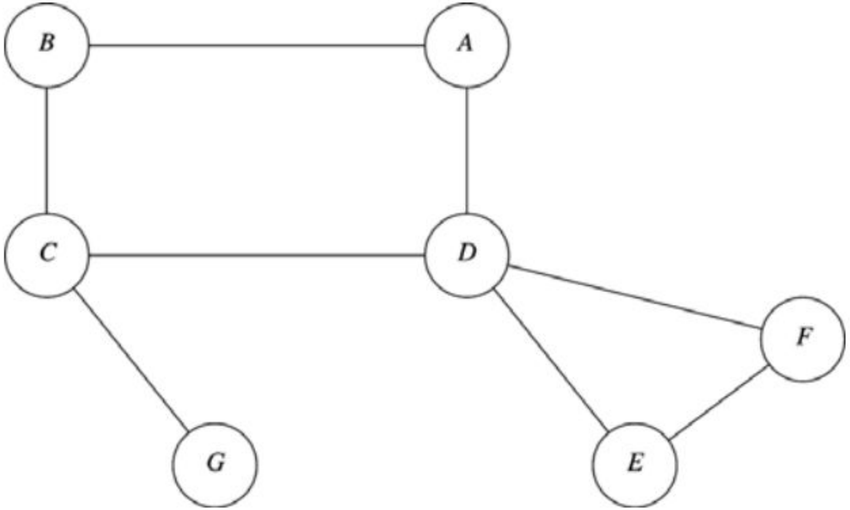
- Can be thought of as a single point of failure

Testing for Articulation Points

1)



2)



Brute Force

- Temporarily remove each vertex v from the graph
- DFS/BFS traversal to determine if graph is still connected

Finding Articulation Points

- Variation of DFS
- Build spanning tree
    - Directed with back edges

High Level Idea

1. Compute Num(v)
2. Compute Low(v)
3. Determine Articulation Points

Finding Articulation Points

- Num(V) is the visit number
- Low(v) = lowest-numbered vertex reachable from v using 0 or more spanning tree edges and then at most one back edge
  - Low(v) = minimum of
    - Num(v)
    - Lowest Num(w) among all back edges (v,w)
    - Lowest Low(w) among all tree edges (v,w)

Finding Articulation Points

- Root is articulation point iff it has more than one child
- Any other vertex v is an articulation point iff v has some child w such that Low(w) ≥ Num(v) □
  - Is there a child w of v that cannot reach a vertex visited before v? □
  - If yes, then removing v will disconnect w (and v is an articulation point)