Portfolio Optimization

Portfolio optimization is the process of selecting the optimal combination of assets to achieve a desired risk-return profile. We will explore how to use the SciPy library in Python to perform portfolio optimization.

Before we dive into the specifics of using SciPy, let's first define some important terms related to portfolio optimization:

- Return: The gain or loss on an investment, expressed as a percentage of the initial investment.
- Risk: The likelihood that an investment's actual return will differ from its expected return.
- Covariance: The measure of how two assets move in relation to each other.
- Correlation: A statistical measure that indicates the extent to which two or more variables are related. Now, let's move on to how to use SciPy for portfolio optimization.

First, we need to import the necessary libraries. We will use the following libraries:

In [72]:
```
import numpy as np
import scipy.optimize as sco
import pandas as pd
import warnings
warnings.simplefilter('ignore')
```

Next, we need to obtain our asset data. We will use a sample dataset of three stocks over seven periods.

In [73]:
```
df = pd.read_excel("Stock Price Data (1).xlsx",sheet_name = 'Sheet2')
print(df.columns)
headings = [df.columns[i] for i in range(1,len(df.columns),2)]
print(headings)
```

```
Index(['Unnamed: 0', 'JSE:NPN', 'Unnamed: 2', 'JSE:SBK', 'Unnamed: 4',
       'JSE:SHP', 'Unnamed: 6', 'JSE:APN', 'Unnamed: 8', 'JSE:SOL',
       'Unnamed: 10', 'JSE:MTN', 'Unnamed: 12', 'JSE:BVT', 'Unnamed: 14',
       'JSE:AGL', 'Unnamed: 16', 'NASDAQ:AAPL', 'Unnamed: 18', 'NYSE:PG'],
      dtype='object')
['JSE:NPN', 'JSE:SBK', 'JSE:SHP', 'JSE:APN', 'JSE:SOL', 'JSE:MTN', 'JSE:BVT', 'JS
E:AGL', 'NASDAQ:AAPL', 'NYSE:PG']
```

In [74]:
```
new_dataframes = []
num_columns = len(df.columns)

for i in range(0, num_columns, 2):
    # Select two columns using iloc (integer-location based indexing)
    # Ensure that the slice doesn't go out of bounds if there's an odd number of
    if i + 2 <= num_columns:
        new_df = df.iloc[1:, i:i+2].copy()
        new_dataframes.append(new_df)
    else:
```

```
        # Handle the case of an odd number of columns (last single column)
        new_df = df.iloc[:, i:i+1].copy()
        new_dataframes.append(new_df)
```

In [75]:
```
merged_df = new_dataframes[0].set_index('Unnamed: 0')
merged_df.index.name = None
for i in range(1, len(new_dataframes)):
    merged_df = pd.merge(merged_df, new_dataframes[i].set_index('Unnamed: '+ str
merged_df
```

Out[75]:

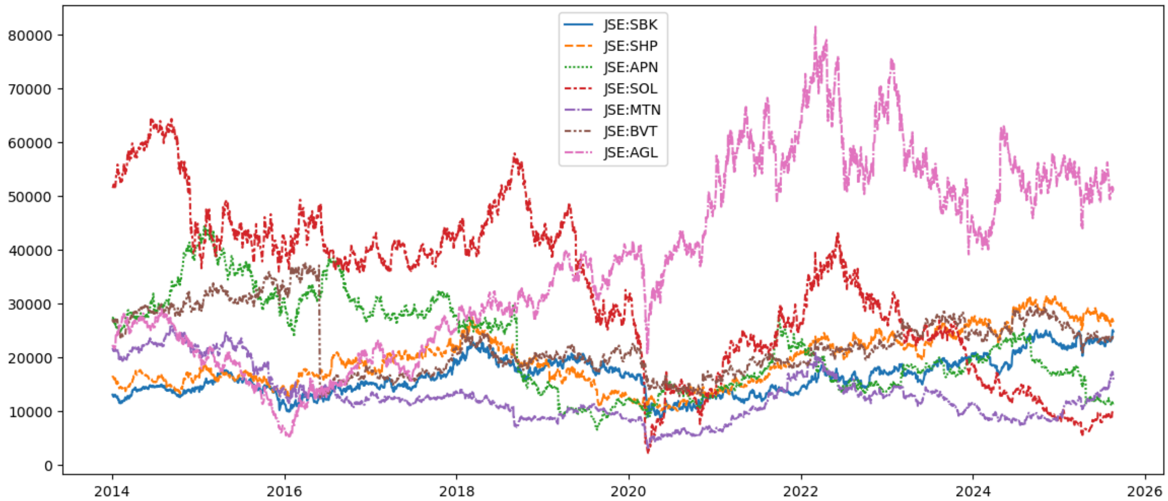
	JSE:NPN	JSE:SBK	JSE:SHP	JSE:APN	JSE:SOL	JSE:MTN	JSE:BVT	JSE:AGL
2014-01-02 17:00:00	107236.48	13031	16550	27305	51704	21714	27074	22275.81
2014-01-03 17:00:00	105178	12853	16351	27353	51611	21525	26701	22080.52
2014-01-06 17:00:00	104945.3	12630	16199	26801	51610	21145	26550	21988.27
2014-01-07 17:00:00	104717.44	12886	16173	27089	52105	21447	26811	21824.38
2014-01-08 17:00:00	105543.54	12792	15897	26961	51768	21224	26548	21902.89
...	...	...	...	...	...	...	...	...
2025-08-12 17:00:00	561633	23643	27050	11342	9418	17200	23903	51615
2025-08-13 17:00:00	585466	24202	27128	11635	9790	17212	24029	51329
2025-08-14 17:00:00	580976	24911	26959	11201	9820	17268	23573	50729
2025-08-15 17:00:00	583030	25040	26858	11245	9953	17178	23766	51565
2025-08-18 17:00:00	580716	24755	26514	11428	10083	15734	23626	50805

2803 rows × 10 columns

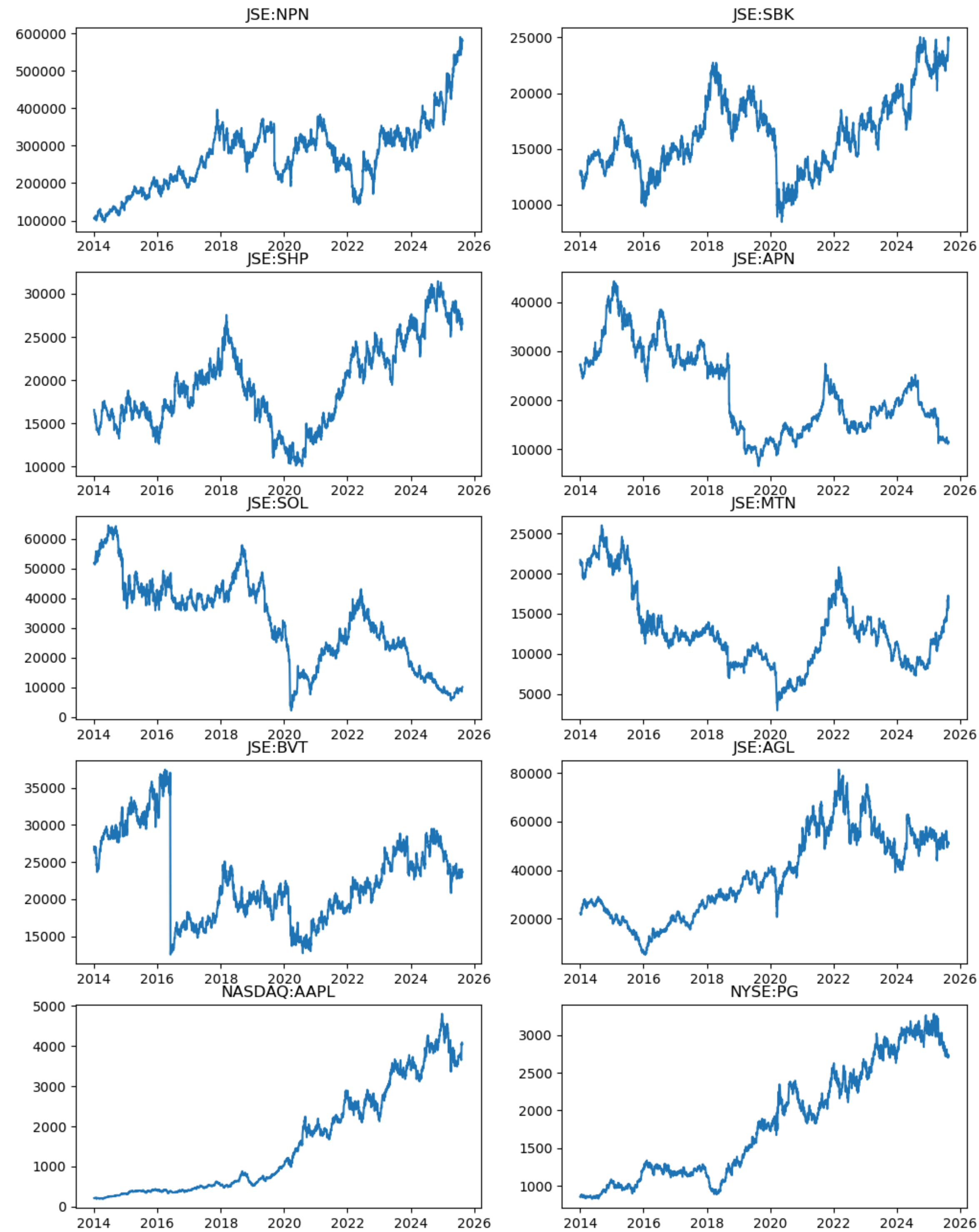
We can plot the price movement using the `lineplot` function from the `seaborn` library as follows:

In [76]:
```
import matplotlib.pyplot as plt
import seaborn as sns

plt.figure(figsize=(14,6))

sns.lineplot(data=merged_df.drop(["JSE:NPN",'NASDAQ:AAPL',"NYSE:PG"],axis=1))
```

Out[76]: <Axes: >

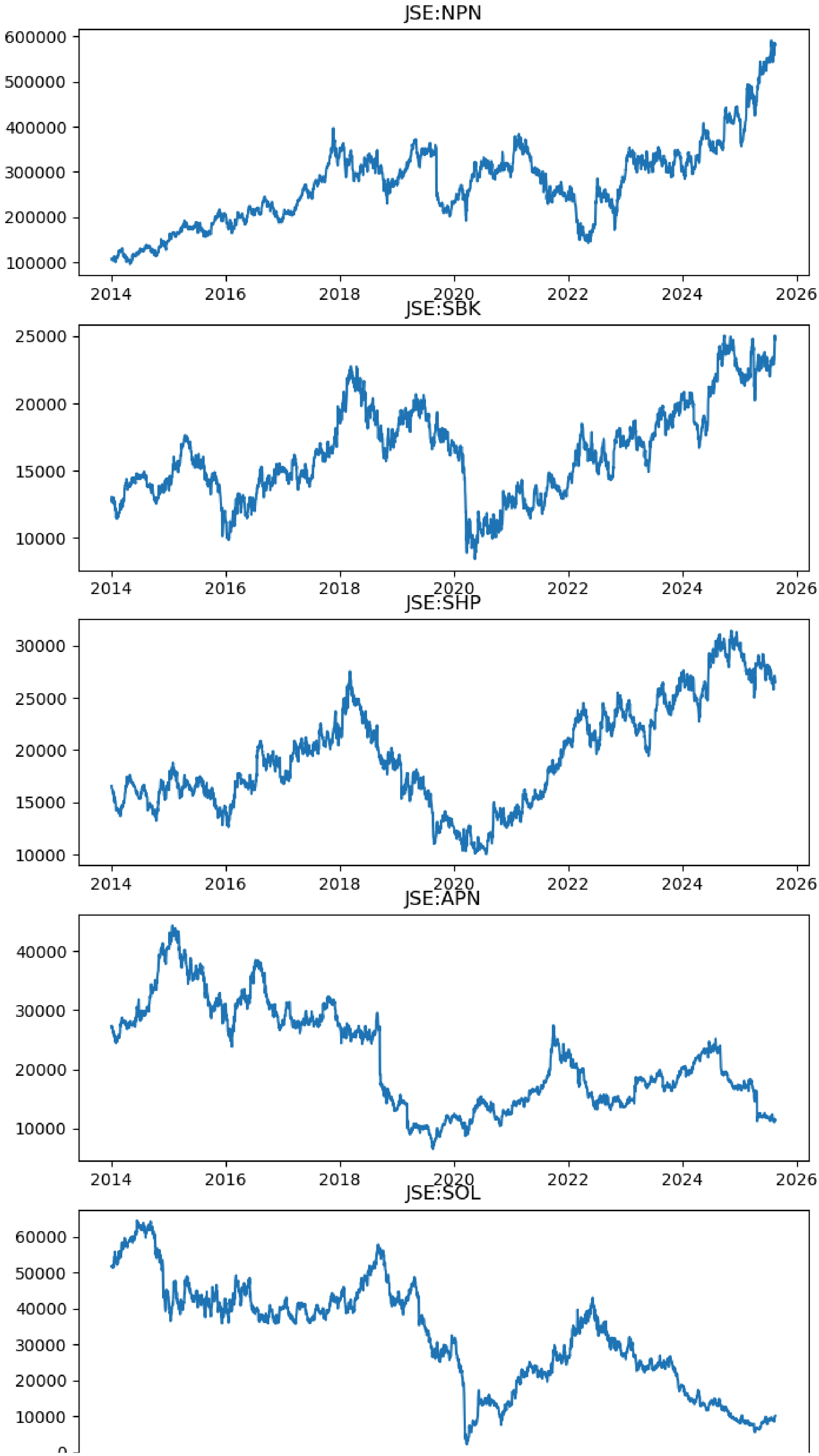


In [77]:
```
fig, axs = plt.subplots(5,2,figsize=(12,16))
for i in range(5):
    for j in range(2):
        axs[i,j].set_title(merged_df.columns[i*2+j])
        axs[i,j].plot(merged_df.reset_index()['index'], merged_df.iloc[:,i*2+j])
```

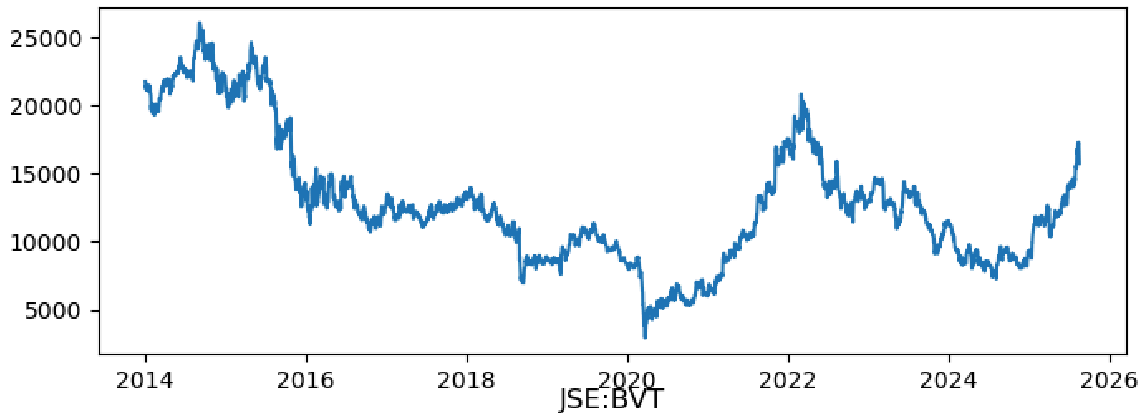


In [78]:

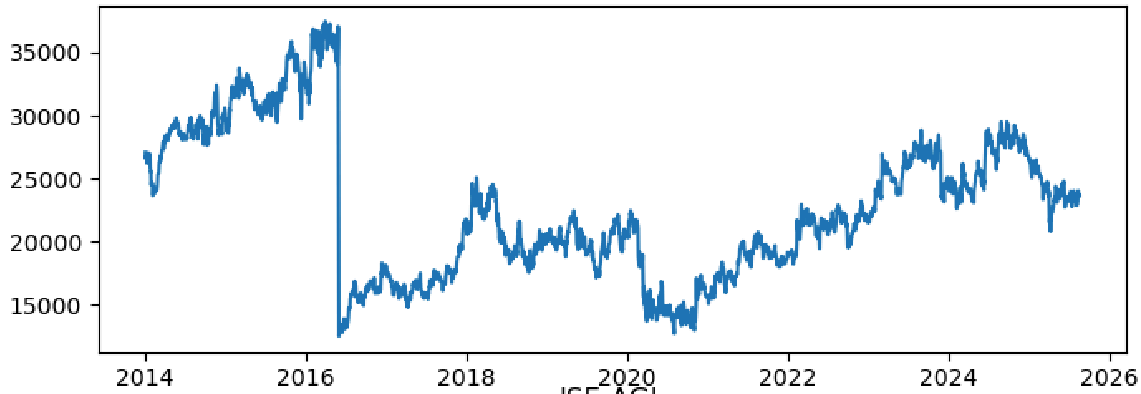
```
fig, axs = plt.subplots(10,1,figsize=(8,32))
for i in range(10):
    axs[i].set_title(merged_df.columns[i])
    axs[i].plot(merged_df.reset_index()['index'], merged_df.iloc[:,i])
```


JSE:NPN
600000
500000
400000
300000
200000
100000
2014 2016 2018 2020 2022 2024 2026
JSE:SBK
25000
20000
15000
10000
2014 2016 2018 2020 2022 2024 2026
JSE:SHP
30000
25000
20000
15000
10000
2014 2016 2018 2020 2022 2024 2026
JSE:APN
40000
30000
20000
10000
2014 2016 2018 2020 2022 2024 2026
JSE:SOL
60000
50000
40000
30000
20000
10000
0

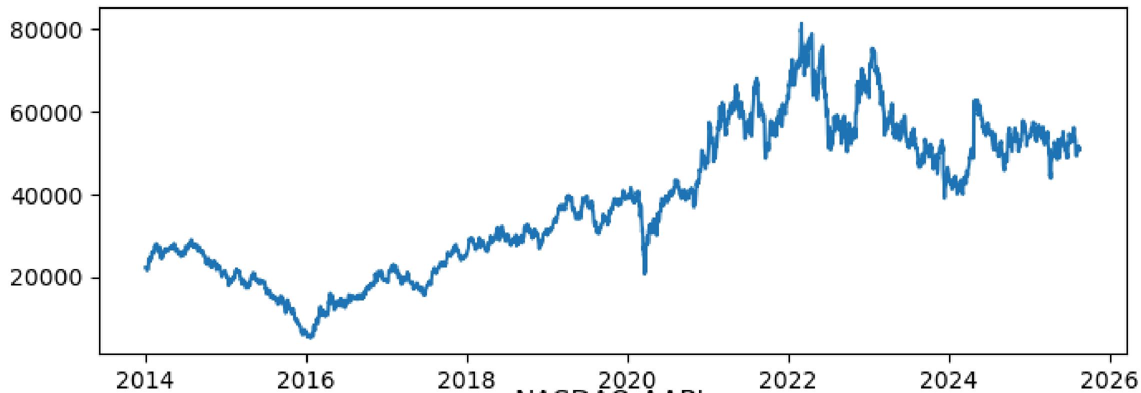
JSE:MTN



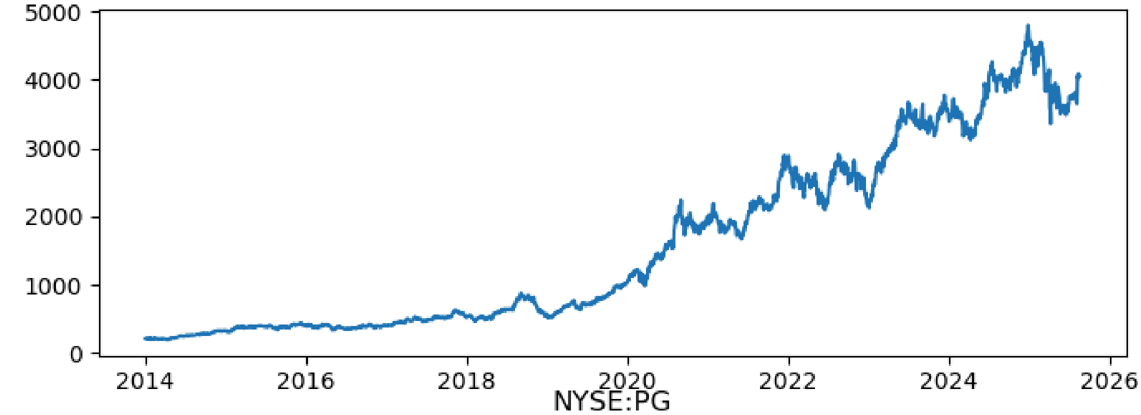
JSE:BVT



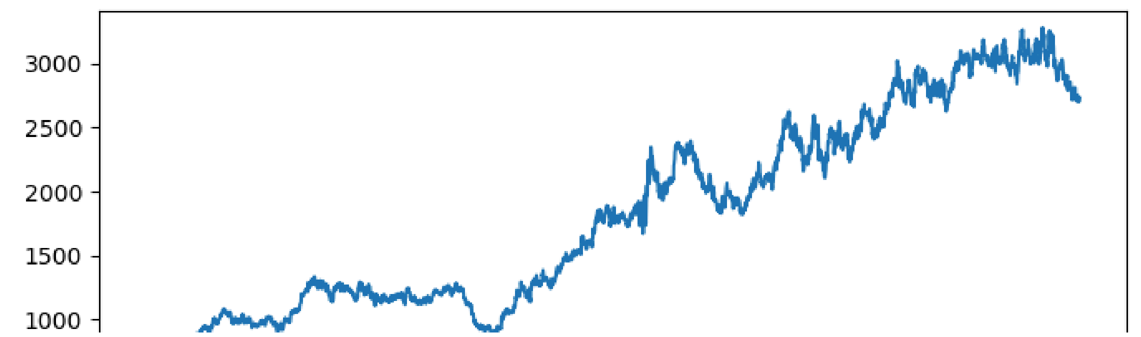
JSE:AGL

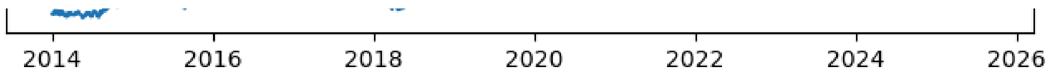


NASDAQ:AAPL



NYSE:PG





The `returns` can be calculated by subtracting the current price from the previous price using the `shift` function. Remember to remove `NaN` values which occur from trying to subtract the initial price from a previous price which does not exist. The leaves six return periods.

In [79]:
```
data =merged_df
returns = data/data.shift(1)-1
returns = returns.dropna()
returns#i think this is the daily returns
```

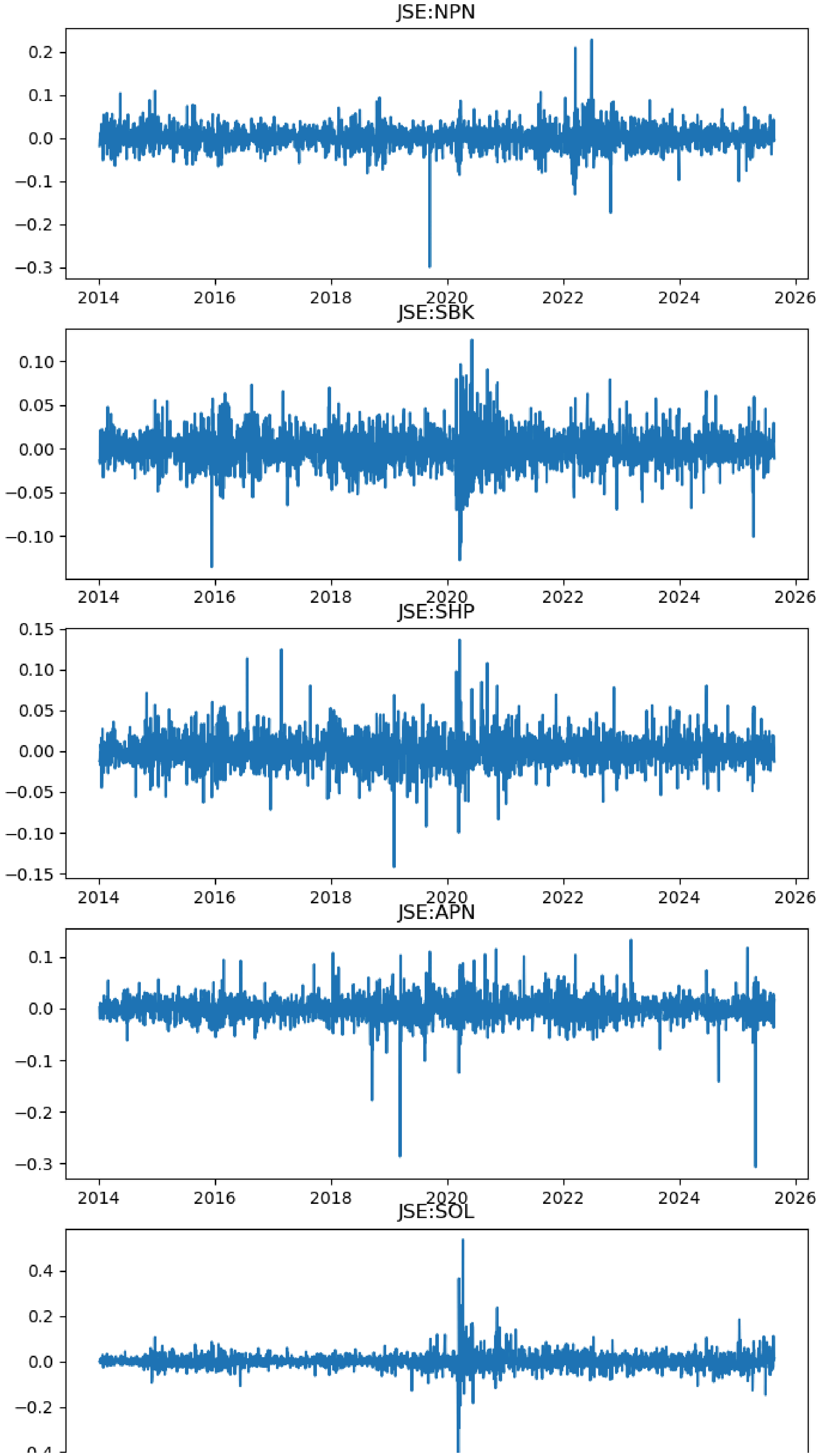
Out[79]:

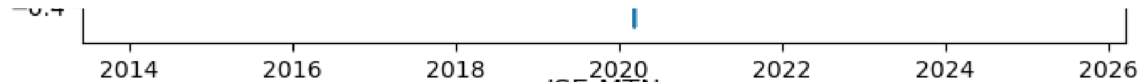
	JSE:NPN	JSE:SBK	JSE:SHP	JSE:APN	JSE:SOL	JSE:MTN	JSE:BVT	J
2014-01-03 17:00:00	-0.019196	-0.01366	-0.012024	0.001758	-0.001799	-0.008704	-0.013777	-0.
2014-01-06 17:00:00	-0.002212	-0.01735	-0.009296	-0.020181	-0.000019	-0.017654	-0.005655	-0.
2014-01-07 17:00:00	-0.002171	0.020269	-0.001605	0.010746	0.009591	0.014282	0.009831	-0.
2014-01-08 17:00:00	0.007889	-0.007295	-0.017065	-0.004725	-0.006468	-0.010398	-0.009809	0.
2014-01-09 17:00:00	-0.015516	-0.007583	0.000189	-0.017099	-0.008074	-0.004853	-0.006064	-0.
...	...	...	...	...	...	...	...	...
2025-08-12 17:00:00	0.005705	0.016903	0.018641	0.027727	0.110875	0.030928	0.031013	0.
2025-08-13 17:00:00	0.042435	0.023643	0.002884	0.025833	0.039499	0.000698	0.005271	-0.
2025-08-14 17:00:00	-0.007669	0.029295	-0.00623	-0.037301	0.003064	0.003254	-0.018977	-0.
2025-08-15 17:00:00	0.003535	0.005178	-0.003746	0.003928	0.013544	-0.005212	0.008187	(
2025-08-18 17:00:00	-0.003969	-0.011382	-0.012808	0.016274	0.013061	-0.084061	-0.005891	-0.

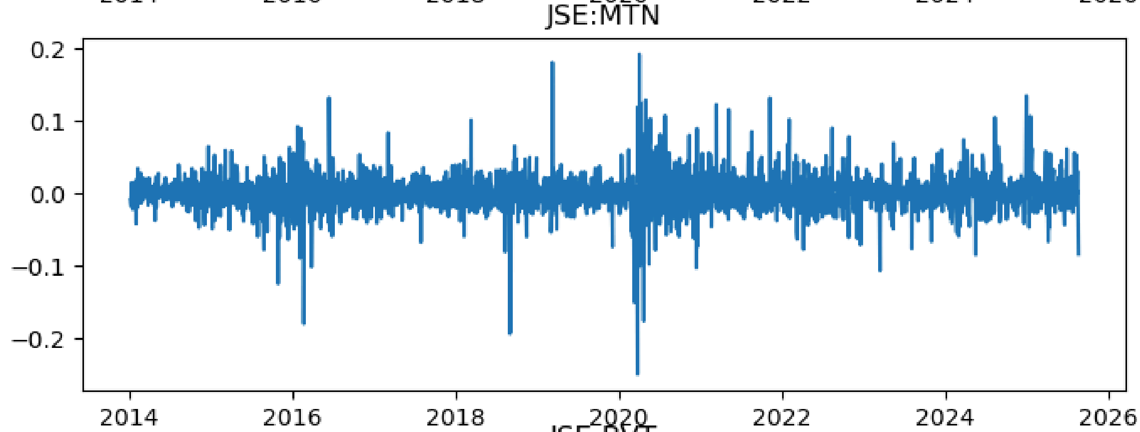
2802 rows × 10 columns

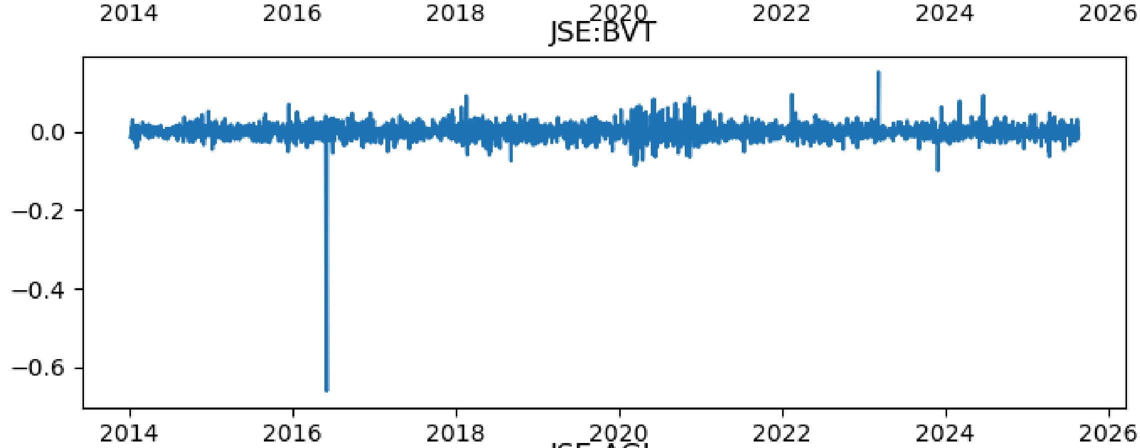
We can plot the returns using the `lineplot` function from the `seaborn` library as follows:

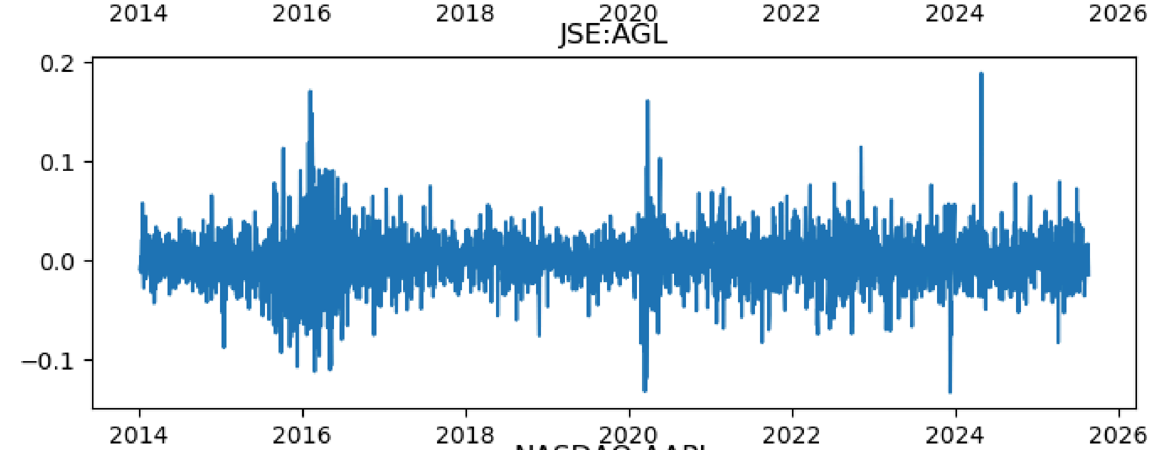
In [80]:
```
fig, axs = plt.subplots(10,1,figsize=(8,32))
for i in range(10):
    axs[i].set_title(returns.columns[i])
    axs[i].plot(returns.reset_index()['index'], returns.iloc[:,i])
```

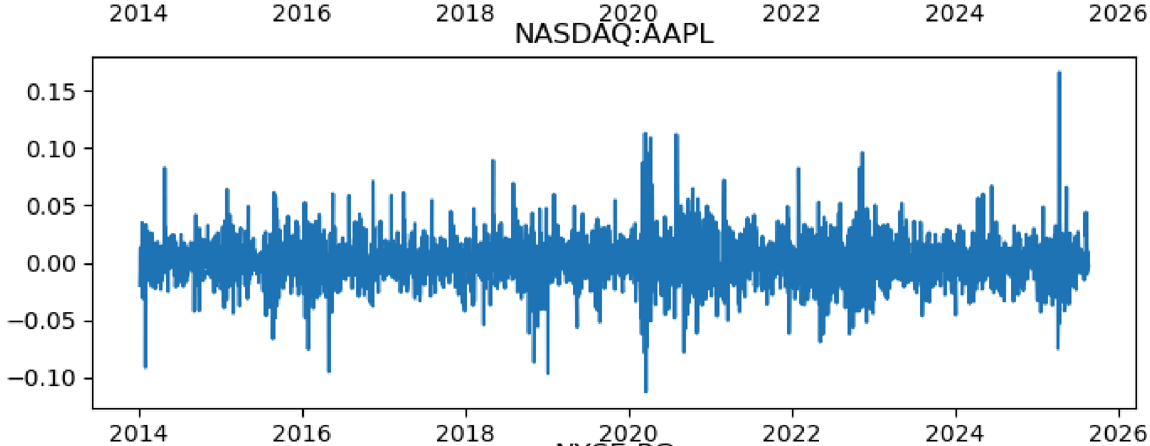

JSE:NPN
0.2
0.1
0.0
−0.1
−0.2
−0.3
2014 2016 2018 2020 2022 2024 2026
JSE:SBK
0.10
0.05
0.00
−0.05
−0.10
2014 2016 2018 2020 2022 2024 2026
JSE:SHP
0.15
0.10
0.05
0.00
−0.05
−0.10
−0.15
2014 2016 2018 2020 2022 2024 2026
JSE:APN
0.1
0.0
−0.1
−0.2
−0.3
2014 2016 2018 2020 2022 2024 2026
JSE:SOL
0.4
0.2
0.0
−0.2

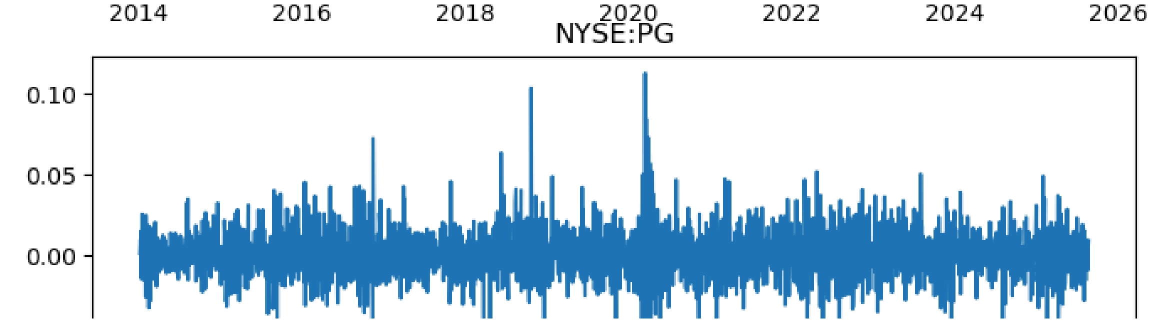

2014
2016
2018
2020
2022
2024
2026

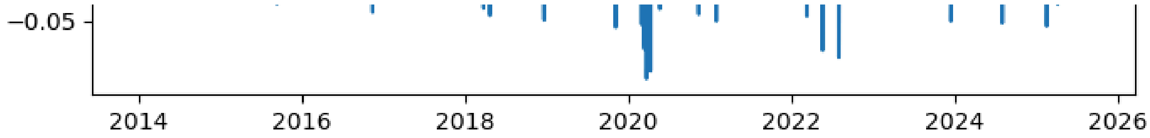

JSE:MTN
0.2
0.1
0.0
−0.1
−0.2
2014
2016
2018
2020
2022
2024
2026


JSE:BVT
0.0
−0.2
−0.4
−0.6
2014
2016
2018
2020
2022
2024
2026

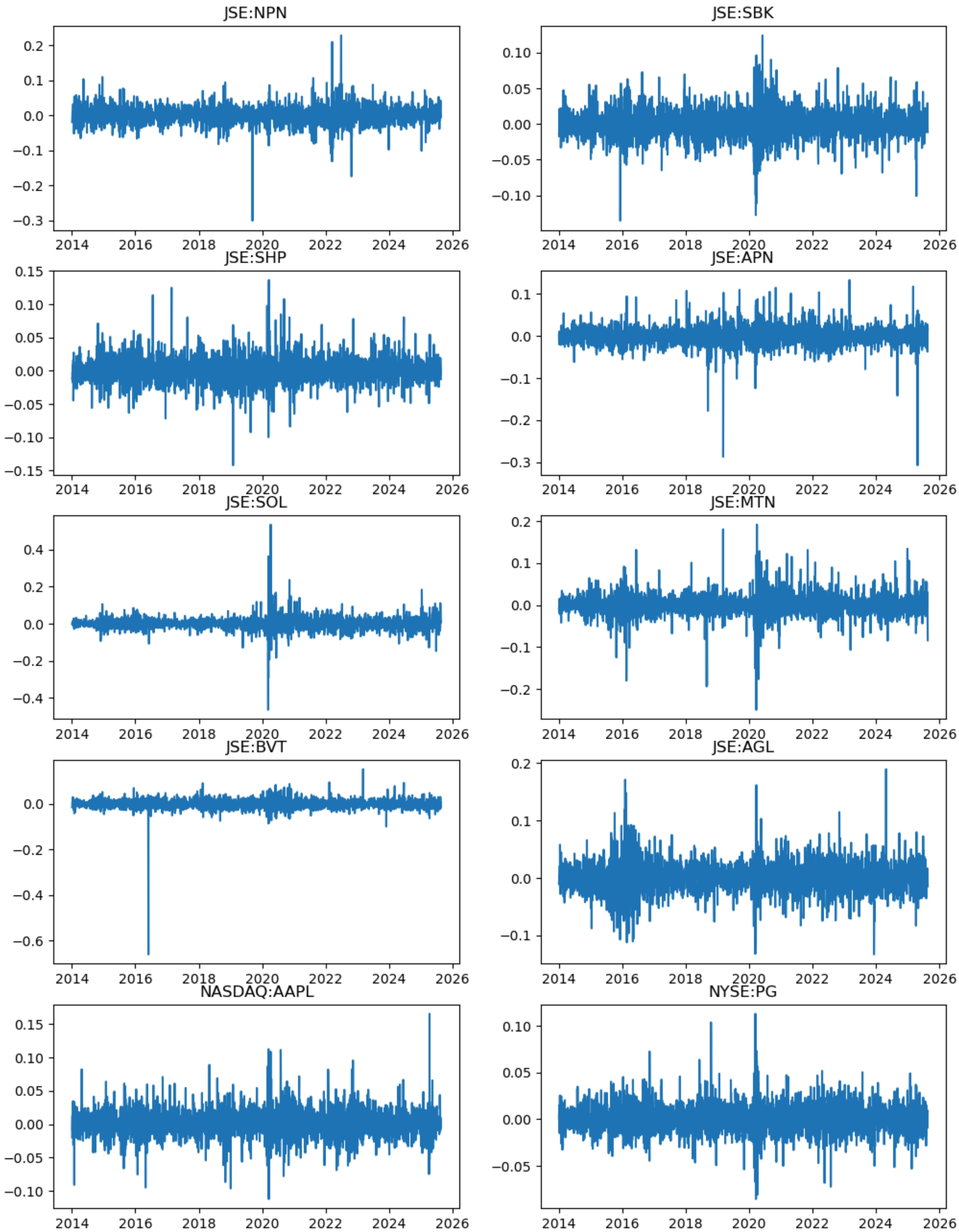

JSE:AGL
0.2
0.1
0.0
−0.1
2014
2016
2018
2020
2022
2024
2026


NASDAQ:AAPL
0.15
0.10
0.05
0.00
−0.05
−0.10
2014
2016
2018
2020
2022
2024
2026


NYSE:PG
0.10
0.05
0.00

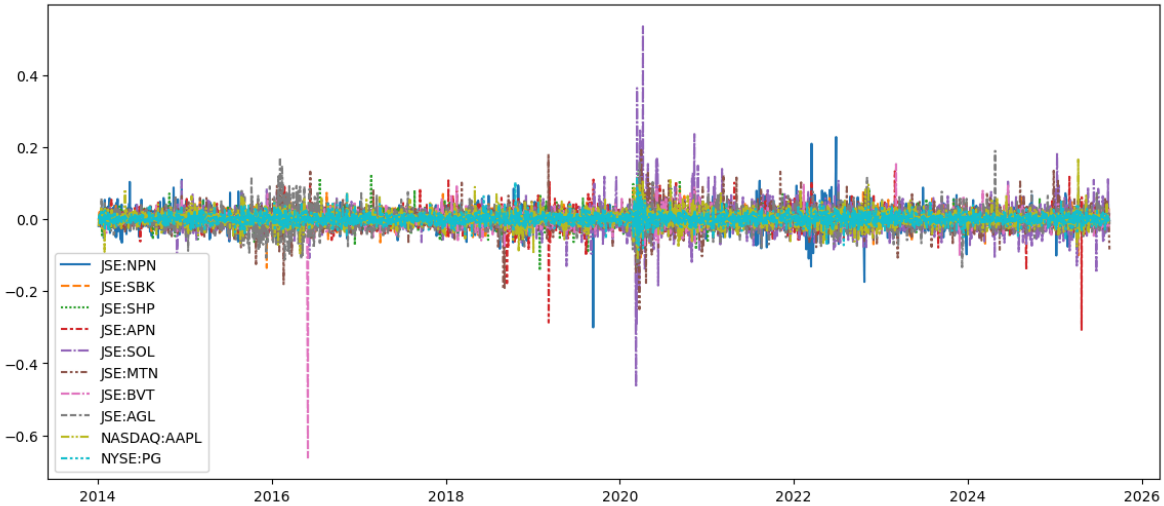


In [81]:
```
fig, axs = plt.subplots(5,2,figsize=(12,16))
for i in range(5):
    for j in range(2):
        axs[i,j].set_title(returns.columns[i*2+j])
        axs[i,j].plot(returns.reset_index()['index'], returns.iloc[:,i*2+j])
```



In [82]:
```
plt.figure(figsize=(14,6))

sns.lineplot(data=returns)
```

Out[82]: <Axes: >



`Stock A` has the highest return from period 3 to period 4 but also the highest drop in price from period 4 to period 5.

We can calculate the average returns using the `mean` function.

In [83]:
```
mean_returns = returns.mean()
mean_returns
```

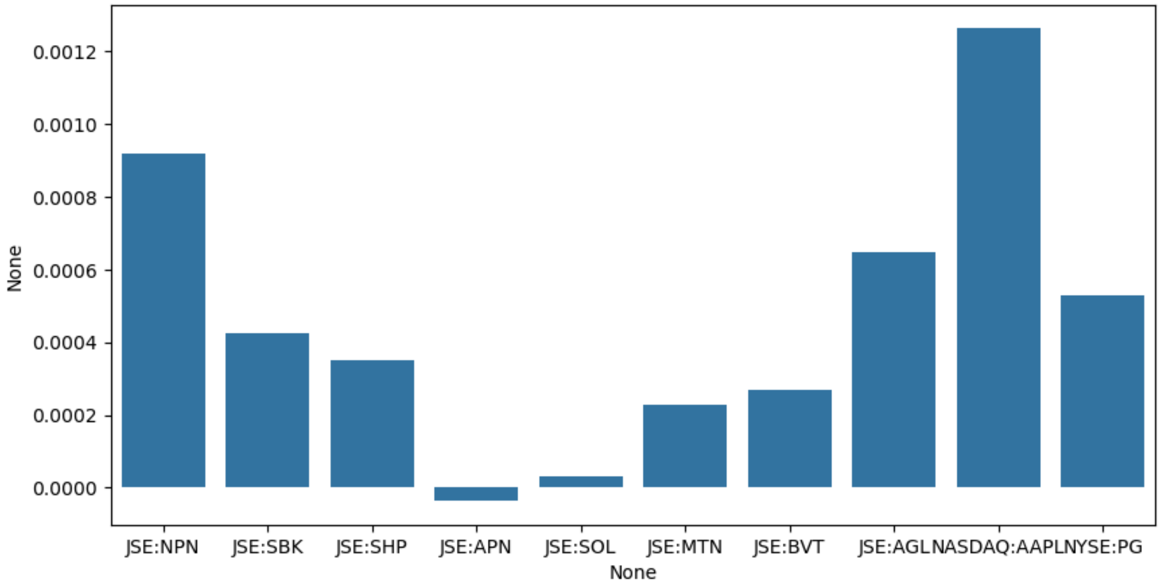
Out[83]:
```
JSE:NPN         0.00092
JSE:SBK        0.000425
JSE:SHP        0.000351
JSE:APN       -0.000037
JSE:SOL        0.000033
JSE:MTN        0.000228
JSE:BVT        0.000269
JSE:AGL         0.00065
NASDAQ:AAPL    0.001264
NYSE:PG        0.000528
dtype: object
```

In terms of the average returns, `Stock A` performs the best and `Stock B` performs the worst.

We can visualise this by deriving a bar chart using the `barplot` function from the `seaborn` library.

In [84]:
```
plt.figure(figsize=(10,5))

sns.barplot(x=mean_returns.index, y=mean_returns)
```

Out[84]: <Axes: xlabel='None', ylabel='None'>



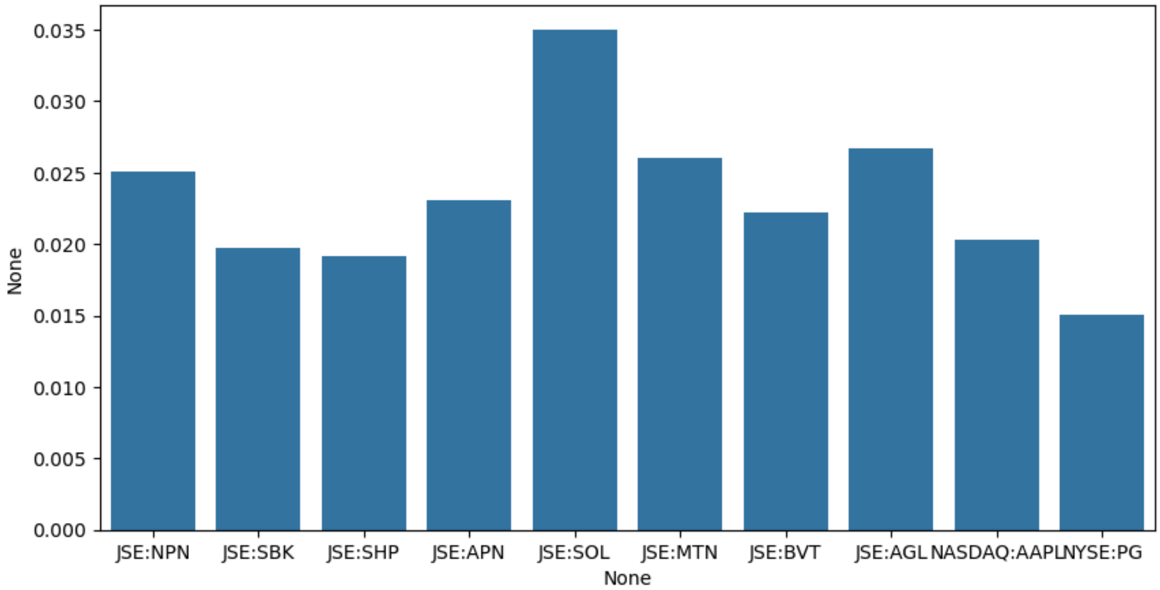
According to Markowitz, return is not the only metric that should be considered when selecting a portfolio of assets. These returns are not guaranteed due to volatility in price movement. Hence we can calculate the standard deviation using the `std` function and plot the volatility on a bar chart.

In [85]:
```
st_dev = returns.std()
st_dev
```

Out[85]:
```
JSE:NPN        0.025093
JSE:SBK        0.019776
JSE:SHP        0.019146
JSE:APN        0.023062
JSE:SOL        0.034996
JSE:MTN        0.026073
JSE:BVT        0.022195
JSE:AGL        0.026704
NASDAQ:AAPL    0.020315
NYSE:PG        0.015014
dtype: object
```

In [86]:
```
plt.figure(figsize=(10,5))

sns.barplot(x=st_dev.index, y=st_dev)
```

Out[86]:
```
<Axes: xlabel='None', ylabel='None'>
```



The graph above indicates that higher volatilites are rewarded with higher returns. To determine which stock has a higher reward for a given level of volatility, we can calculate the Sharpe Ratio which determines which the return per unit of risk

In [87]:

```
var = st_dev**2
var
```

Out[87]:

```
JSE:NPN         0.00063
JSE:SBK        0.000391
JSE:SHP        0.000367
JSE:APN        0.000532
JSE:SOL        0.001225
JSE:MTN         0.00068
JSE:BVT        0.000493
JSE:AGL        0.000713
NASDAQ:AAPL    0.000413
NYSE:PG        0.000225
dtype: object
```

In [88]:

```
sharpe_ratio = mean_returns/st_dev
sharpe_ratio
```

Out[88]:

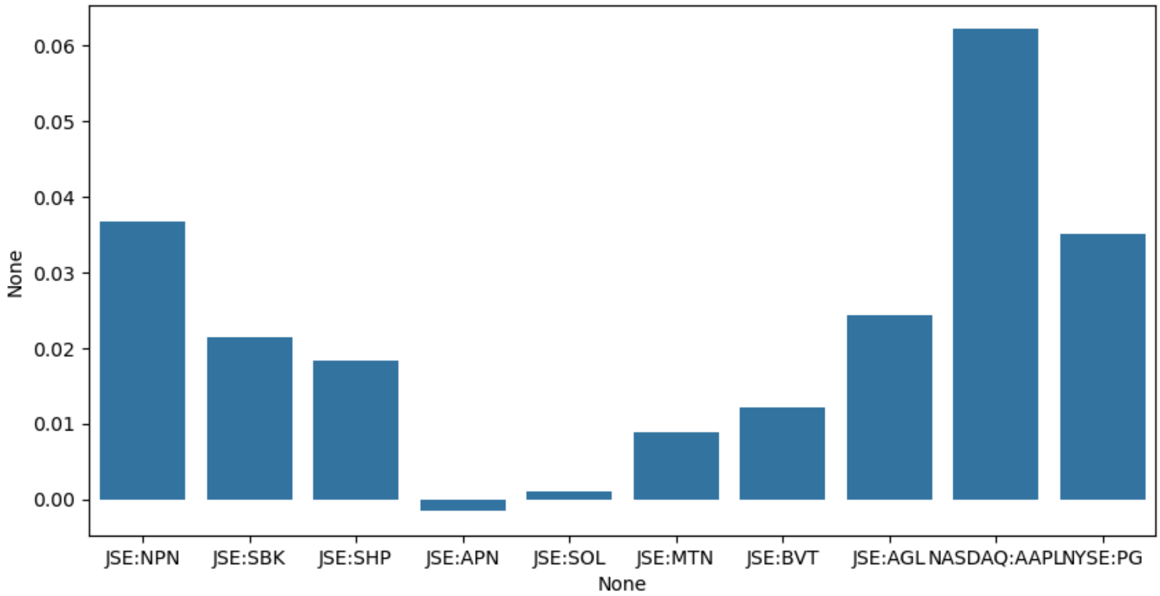
```
JSE:NPN        0.036679
JSE:SBK        0.021485
JSE:SHP        0.018318
JSE:APN       -0.001588
JSE:SOL        0.000932
JSE:MTN        0.008761
JSE:BVT        0.012129
JSE:AGL        0.024323
NASDAQ:AAPL    0.062204
NYSE:PG        0.035154
dtype: object
```

In [89]:

```
plt.figure(figsize=(10,5))

sns.barplot(x=sharpe_ratio.index, y=sharpe_ratio)
```

Out[89]:

```
<Axes: xlabel='None', ylabel='None'>
```



The above graphic shows that `Stock C` is the best performing stock according to the Sharpe Ratio, even though `Stock A` produces the highest return on average.

However, even though the graphics are a good indicator of how the portfolio should be constructed, they do not account for portfolio diversification. Hence we need to calculate the variance-covariance matrix using the `cov` function which will be used to calculate portfolio volatility. Portfolio volatility is used as the objective function for the minimum variance portfolio and accounts for the denominator of the objective function for the maximum Sharpe Ratio portfolio.

In [90]:
```
cov_matrix=((returns-mean_returns).T@(returns-mean_returns))/(len(returns)-1)
cov_matrix
```

Out[90]:

	JSE:NPN	JSE:SBK	JSE:SHP	JSE:APN	JSE:SOL	JSE:MTN	JSE:BV
JSE:NPN	0.00063	0.000106	0.00007	0.000098	0.000124	0.000104	0.00007
JSE:SBK	0.000106	0.000391	0.000175	0.000121	0.00022	0.000226	0.00020
JSE:SHP	0.00007	0.000175	0.000367	0.000109	0.000108	0.000144	0.00015
JSE:APN	0.000098	0.000121	0.000109	0.000532	0.000123	0.000134	0.0001
JSE:SOL	0.000124	0.00022	0.000108	0.000123	0.001225	0.000297	0.00015
JSE:MTN	0.000104	0.000226	0.000144	0.000134	0.000297	0.00068	0.00017
JSE:BVT	0.000077	0.000208	0.000159	0.00012	0.000157	0.000171	0.00049
JSE:AGL	0.000159	0.000144	0.000079	0.0001	0.000364	0.000179	0.00009
NASDAQ:AAPL	0.000059	-0.000011	0.000004	-0.000004	0.000067	0.000016	-0.00000
NYSE:PG	-0.000019	-0.00004	-0.000008	-0.00002	0.000004	-0.000016	-0.00003

Using `seaborn` to produce a heatmap using the `heatmap` function, we can see that the all the stocks have negative covariance with each other. Along the principal diagonal, we can see the variance of each stock (covariance with itself). `Stock A` is the most volatile.

In [91]:
```
portfolio_returns = [np.sum(np.array([0.1 for i in range(10)])*mean_returns)]
portfolio_weights = {'Equally Weighted':[0.1 for i in range(10)]}
portfolio_variance = [(np.array([0.1 for i in range(10)])@cov_matrix)@(np.array(
```

In [92]:
```
portfolio_returns
```

Out[92]: [0.00046306202305196483]

In [93]:
```
portfolio_weights
```

Out[93]: {'Equally Weighted': [0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1]}

In [94]:
```
portfolio_variance
```

Out[94]: [0.00014398129086071456]

Maximum Sharpe Ratio

The maximum Sharpe Ratio portfolio maximizes the portfolio Sharpe Ratio by adjusting the weights of each stock within the portfolio. Since `scipy.optimize` does not have a maxmization function, we need to multiply the objective function by negative 1 such that minimization of this new function is actually maximizing the original function. We can create the objective function by creating a python function

In [95]:
```
def neg_sharpe_ratio(weights, mean_returns, cov_matrix):
    portfolio_return = np.sum(mean_returns * weights)#is this the return
    portfolio_var = np.dot(weights.T, np.dot(cov_matrix, weights))
    portfolio_std_dev = np.sqrt(portfolio_var)
    sharpe_ratio = portfolio_return / portfolio_std_dev
    return -sharpe_ratio
```

Next, we need to create the constraints using Python functions. First constraint is that the weights need to sum up to 1 and the second constraint indicates that there is no shorting of assets i.e. each asset needs to have a non-negative weight.

In [96]:
```
def weight_constraint(weights):
    return np.sum(weights) - 1

def individual_weight_constraint(weights):
    return weights
```

Finally, we need to combine the constraints into a dictionary, set the intial weights (most likely equally weighted or an arbitrary non-zero weight), indicate the bounds as a tuple for each weight (lower limit of zero and upper limit of one) and then feed these parameters into the `minimize` function using the `Sequential Least Squares Programming - SLSQP`

In [97]:
```
constraints = ({'type': 'eq', 'fun': weight_constraint},
               {'type': 'ineq', 'fun': individual_weight_constraint})
```

```
initial_weights = np.array([1/len(data.columns) for i in range(len(data.columns)
bounds = tuple((0, 1) for i in range(len(data.columns)))

result = sco.minimize(neg_sharpe_ratio, initial_weights, args=(mean_returns, cov
result
```

Out[97]:

```
 message: Optimization terminated successfully
 success: True
  status: 0
     fun: -0.07245889050872087
       x: [ 1.743e-01  1.193e-01  5.685e-02  0.000e+00  0.000e+00
            1.277e-18  1.552e-17  6.370e-02  4.523e-01  1.335e-01]
     nit: 13
     jac: [ 2.527e-04  3.200e-05 -4.783e-04  2.093e-02  4.660e-02
            1.363e-02  6.566e-04  1.730e-04 -1.017e-04  1.070e-04]
    nfev: 143
    njev: 13
```

In [98]:

```
print('The Maximum Sharpe Ratio Portfolio produces a maximum return of ', -np.rc
print('With a weight vector as follows: ', np.round(result.x,2))

portfolio_returns.append(-result.fun)
portfolio_weights['Maximum Sharpe Ratio'] = result.x
portfolio_variance.append((result.x@cov_matrix)@(result.x).T)
```

```
The Maximum Sharpe Ratio Portfolio produces a maximum return of  0.07
With a weight vector as follows:  [0.17 0.12 0.06 0.   0.   0.   0.   0.06 0.45
0.13]
```

In [99]:

```
portfolio_variance
```

Out[99]:

```
[0.00014398129086071456, 0.000159289007605047]
```

Minimum Variance

The Minimum Variance portfolio minimizes portfolio variance by adjusting the weights of each stock within the portfolio. `scipy.optimize` can be used to minimize the portfolio variance. We can create the objective function by creating a python function which takes `weights` and `cov_matrix` as arguments.

In [100…

```
def portfolio_var(weights, cov_matrix):
    portfolio_var = np.sqrt(np.dot(weights.T, np.dot(cov_matrix, weights)))
    return portfolio_var
```

In [101…

```
def weight_constraint(weights):
    return np.sum(weights) - 1

def individual_weight_constraint(weights):
    return weights
```

In [102…

```
constraints = ({'type': 'eq', 'fun': weight_constraint},
               {'type': 'ineq', 'fun': individual_weight_constraint})

initial_weights = np.array([1/len(data.columns) for i in range(len(data.columns)
bounds = tuple((0, 1) for i in range(len(data.columns)))
```

```
result = sco.minimize(portfolio_var, initial_weights, args=(cov_matrix), method=
result
```

Out[102…

```
 message: Optimization terminated successfully
 success: True
  status: 0
     fun: 0.009492610752576782
       x: [ 8.542e-02  1.025e-01  1.141e-01  9.169e-02  8.674e-19
            1.908e-02  7.529e-02  5.179e-02  4.223e-02  4.179e-01]
     nit: 17
     jac: [ 9.696e-03  9.518e-03  9.662e-03  9.496e-03  1.029e-02
            9.464e-03  9.398e-03  9.685e-03  9.472e-03  9.394e-03]
    nfev: 187
    njev: 17
```

In [103…

```
print('The Minimum Variance Portfolio produces a minimum variance of ', np.round
print('With a weight vector as follows: ', np.round(result.x,2))

portfolio_returns.append(-result.fun)
portfolio_weights['Minimum Variance'] = result.x
portfolio_variance.append((result.x@cov_matrix)@(result.x).T)
```

```
The Minimum Variance Portfolio produces a minimum variance of  0.01
With a weight vector as follows:  [0.09 0.1  0.11 0.09 0.   0.02 0.08 0.05 0.04
0.42]
```

Maximum Return

The Maximum Return portfolio maximizes portfolio return by adjusting the weights of each stock within the portfolio. `scipy.optimize` can be used to minimize the negative of the portfolio return. We can create the objective function by creating a python function which takes `weights` and `mean_returns` as arguments.

In [104…

```
def neg_portfolio_return(weights, mean_returns):
    portfolio_return = np.sum(mean_returns * weights)
    return -portfolio_return#return on portfoliio
```

In [105…

```
def weight_constraint(weights):
    return np.sum(weights) - 1

def individual_weight_constraint(weights):
    return weights
```

In [106…

```
constraints = ({'type': 'eq', 'fun': weight_constraint},
               {'type': 'ineq', 'fun': individual_weight_constraint})

initial_weights = np.array([1/len(data.columns) for i in range(len(data.columns))
bounds = tuple((0, 1) for i in range(len(data.columns)))

result = sco.minimize(neg_portfolio_return, initial_weights, args=(mean_returns)
result
```

Out[106…

```
 message: Optimization terminated successfully
 success: True
  status: 0
     fun: -0.0010964341696626902
       x: [ 3.140e-01  0.000e+00  0.000e+00  8.825e-17  3.496e-16
            5.443e-17  2.906e-17  9.681e-02  5.892e-01  3.469e-18]
     nit: 23
     jac: [-9.204e-04 -4.249e-04 -3.507e-04  3.661e-05 -3.262e-05
           -2.284e-04 -2.692e-04 -6.495e-04 -1.264e-03 -5.278e-04]
    nfev: 253
    njev: 23
```

In [107…

```
print('The Maximum Return Portfolio produces a maximum return of ', -np.round(re
print('With a weight vector as follows: ', np.round(result.x,2))

portfolio_returns.append(-result.fun)
portfolio_weights['Maximum Return'] = result.x
portfolio_variance.append((result.x@cov_matrix)@(result.x).T)
```

```
The Maximum Return Portfolio produces a maximum return of  0.0
With a weight vector as follows:  [0.31 0.   0.   0.   0.   0.   0.   0.1  0.59
0.  ]
```

`sco.minimize` function takes several arguments. `

In [108…

```
portfolio_returns
```

Out[108…

```
[0.00046306202305196483,
 np.float64(0.07245889050872087),
 np.float64(-0.009492610752576782),
 0.0010964341696626902]
```

In [109…

```
portfolio_variance
```

Out[109…

```
[0.00014398129086071456,
 0.000159289007605047,
 9.010965889993635e-05,
 0.00024874873473091231]
```

In [110…

```
portfolio_weights
```

Out[110…

```
{'Equally Weighted': [0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1],
 'Maximum Sharpe Ratio': array([1.74341397e-01, 1.19276188e-01, 5.68456681e-02,
0.00000000e+00,
        0.00000000e+00, 1.27732568e-18, 1.55244199e-17, 6.37002053e-02,
        4.52313855e-01, 1.33522687e-01]),
 'Minimum Variance': array([8.54159995e-02, 1.02523949e-01, 1.14103320e-01, 9.1
6867676e-02,
        8.67361738e-19, 1.90818056e-02, 7.52915031e-02, 5.17888055e-02,
        4.22264993e-02, 4.17881351e-01]),
 'Maximum Return': array([3.13972093e-01, 0.00000000e+00, 0.00000000e+00, 8.825
40568e-17,
        3.49587438e-16, 5.44269491e-17, 2.90566182e-17, 9.68106627e-02,
        5.89217244e-01, 3.46944695e-18])}
```

In [111…

```
portfolio_weights = pd.DataFrame(portfolio_weights, index=merged_df.columns)
portfolio_weights
```

Out[111…

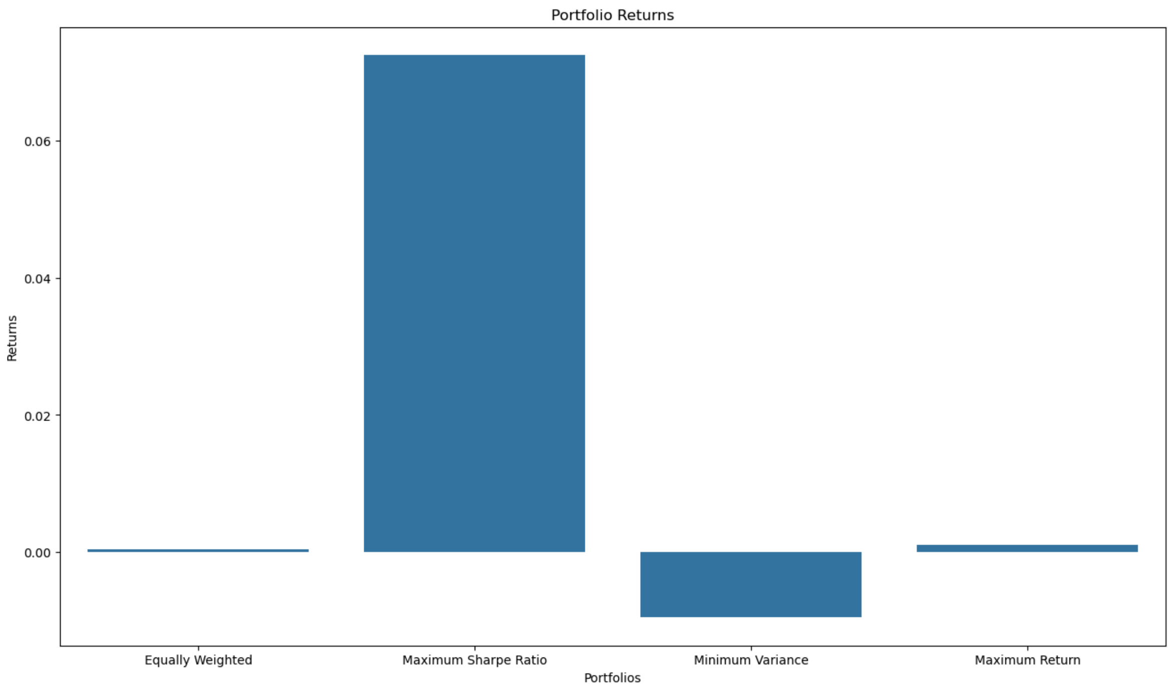
	Equally Weighted	Maximum Sharpe Ratio	Minimum Variance	Maximum Return
JSE:NPN	0.1	1.743414e-01	8.541600e-02	3.139721e-01
JSE:SBK	0.1	1.192762e-01	1.025239e-01	0.000000e+00
JSE:SHP	0.1	5.684567e-02	1.141033e-01	0.000000e+00
JSE:APN	0.1	0.000000e+00	9.168677e-02	8.825406e-17
JSE:SOL	0.1	0.000000e+00	8.673617e-19	3.495874e-16
JSE:MTN	0.1	1.277326e-18	1.908181e-02	5.442695e-17
JSE:BVT	0.1	1.552442e-17	7.529150e-02	2.905662e-17
JSE:AGL	0.1	6.370021e-02	5.178881e-02	9.681066e-02
NASDAQ:AAPL	0.1	4.523139e-01	4.222650e-02	5.892172e-01
NYSE:PG	0.1	1.335227e-01	4.178814e-01	3.469447e-18

In [112…

```
plt.figure(figsize=(16,9))
plt.title('Portfolio Returns')
ax = sns.barplot(x=portfolio_weights.columns, y=portfolio_returns)
ax.set_xlabel("Portfolios")
ax.set_ylabel("Returns")
```

Out[112…

```
Text(0, 0.5, 'Returns')
```



Sharpe Ratio

Most widely used performance metric which involves assessing the returns generated for each unti of risk taken.

$$S = \frac{R_p}{\sigma_p}$$

Where:

- R_p is the return of the portfolio
- σ_p is the portfolio variance

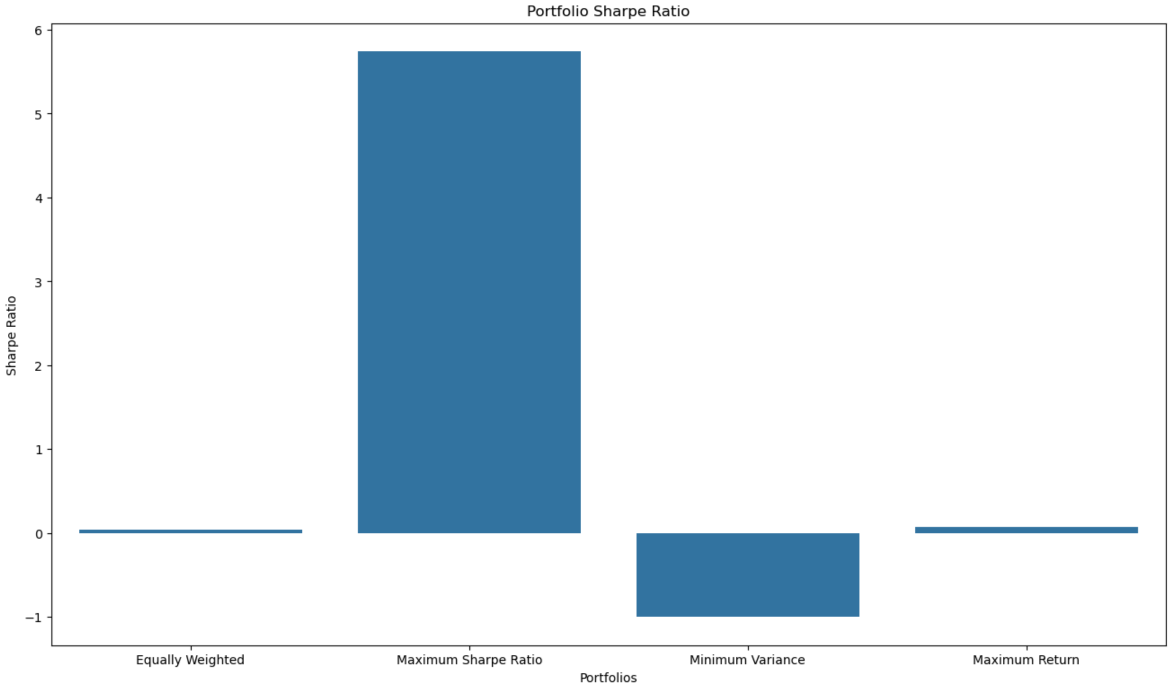
A higher Sharpe Ratio indicates better performance for the amount of total risk taken.

In [113…

```
plt.figure(figsize=(16,9))
portfolio_sharpe_ratio = np.array(portfolio_returns)/(np.sqrt(portfolio_variance
plt.title('Portfolio Sharpe Ratio')
ax = sns.barplot(x=portfolio_weights.columns, y=portfolio_sharpe_ratio)
ax.set_xlabel("Portfolios")
ax.set_ylabel("Sharpe Ratio")
```

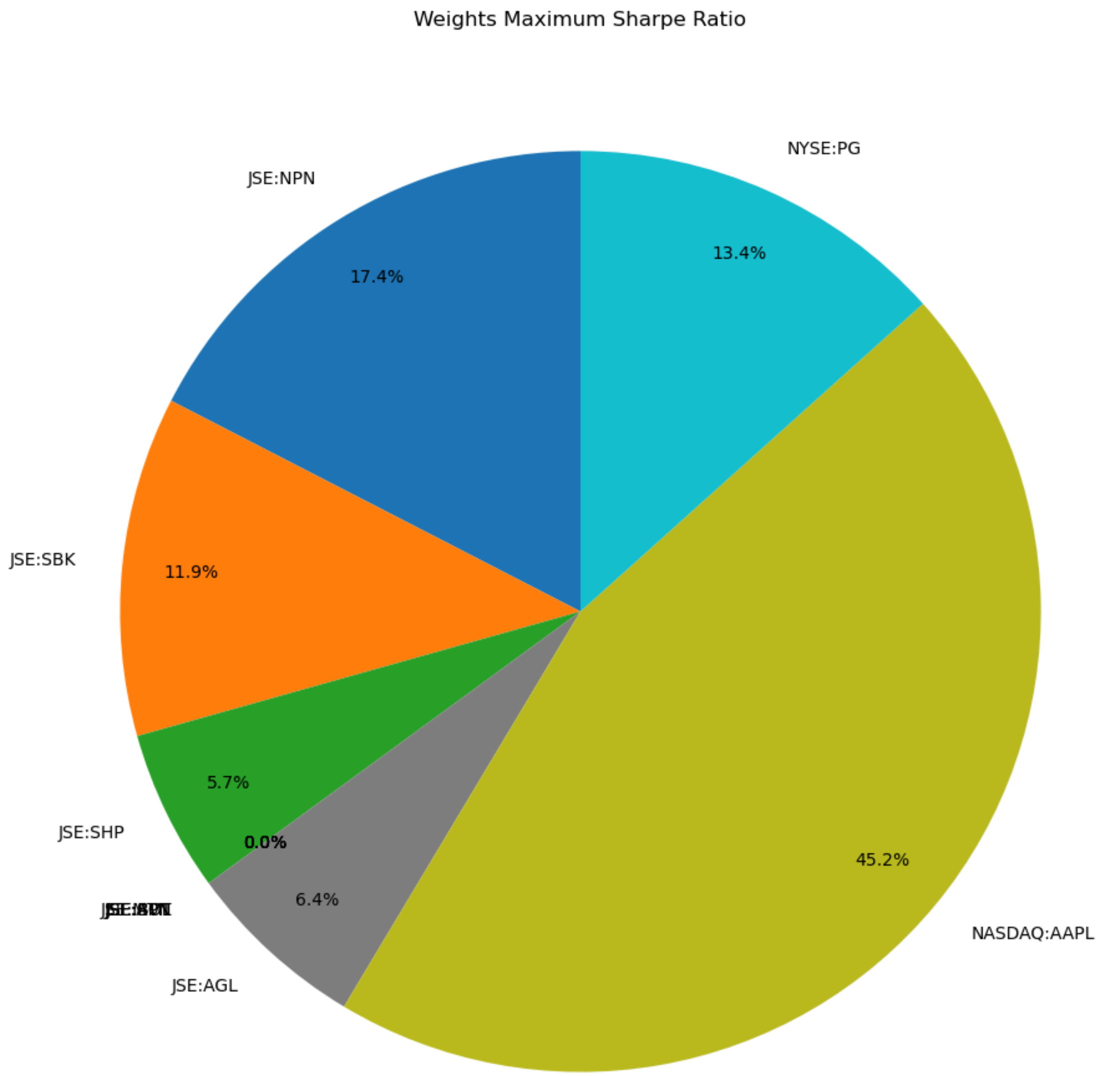
Out[113…

```
Text(0, 0.5, 'Sharpe Ratio')
```



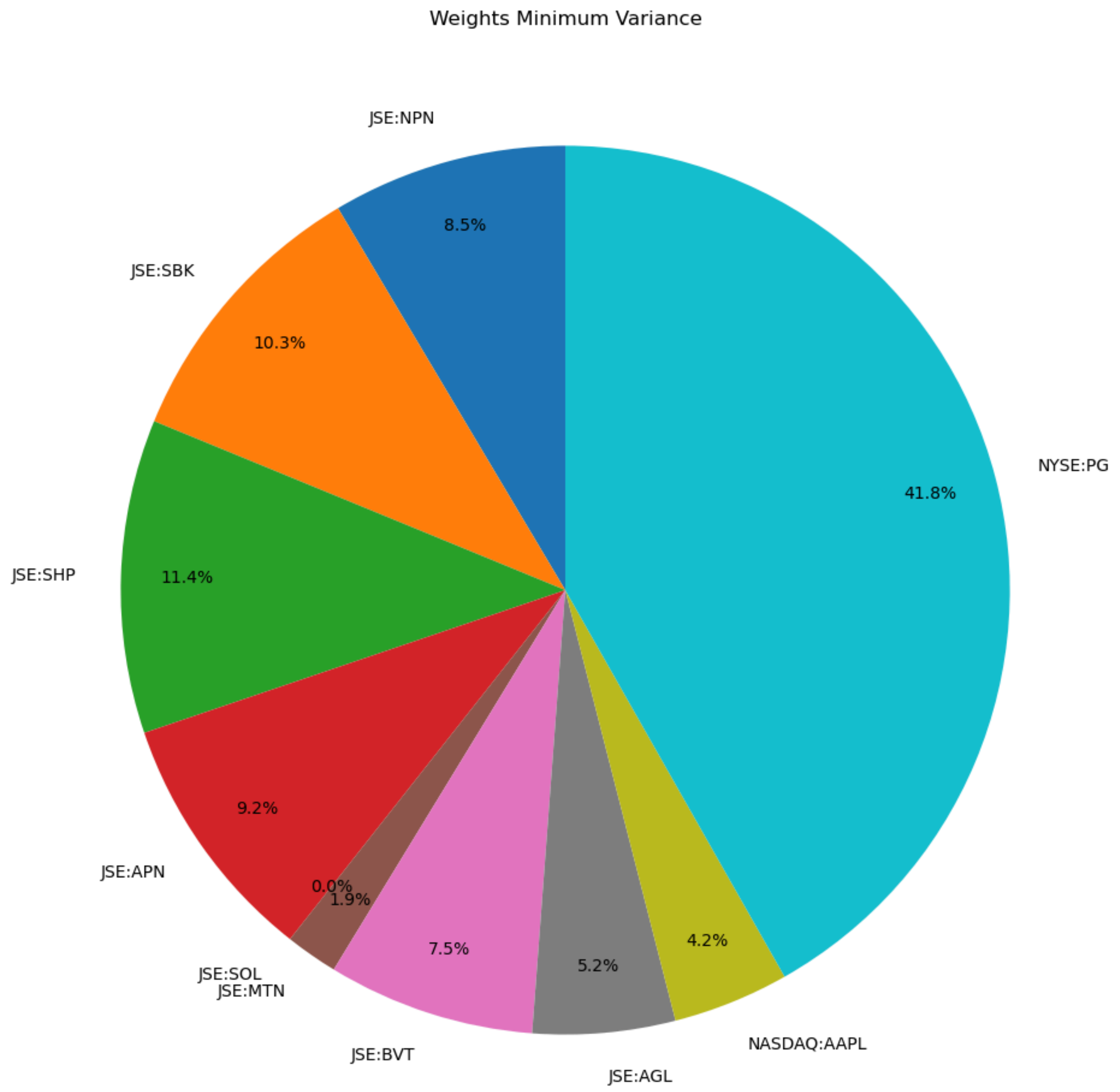
In [114…

```
plt.figure(figsize=(12,12))
plt.title('Weights Maximum Sharpe Ratio')
percentage_threshold = 1
plt.pie(portfolio_weights['Maximum Sharpe Ratio'],labels=portfolio_weights.reset
                                  startangle=90, pctdistance=0.85)
plt.show()
plt.savefig('Weights Maximum Sharpe Ratio.jpg')
```



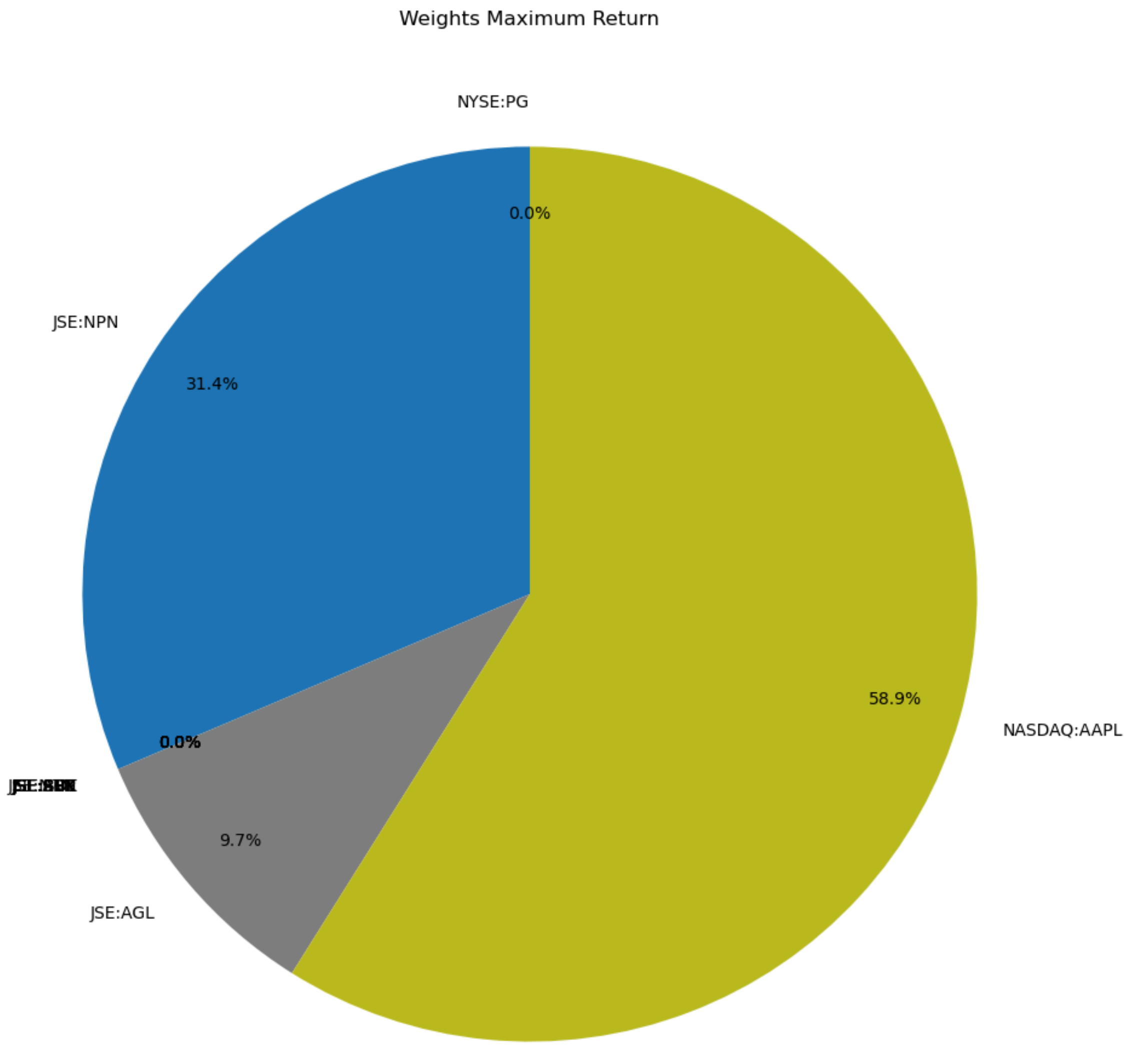
In [115…

```
plt.figure(figsize=(12,12))
plt.title('Weights Minimum Variance')
percentage_threshold = 1
plt.pie(portfolio_weights['Minimum Variance'],labels=portfolio_weights.reset_ind
                                  startangle=90, pctdistance=0.85)
plt.show()
plt.savefig('Weights Minimum Variance.jpg')
```



In [116…

```
plt.figure(figsize=(12,12))
plt.title('Weights Maximum Return')
percentage_threshold = 1
plt.pie(portfolio_weights['Maximum Return'],labels=portfolio_weights.reset_index
                                  startangle=90, pctdistance=0.85)
plt.show()
plt.savefig('Weights Maximum Return.jpg')
```




```
<Figure size 640x480 with 0 Axes>
```

In [46]:
```
portfolio_weights = {
    "Equally Weighted": np.array([0.1]*10),
    "Maximum Sharpe Ratio": np.array([0.17,0.12,0.06,0,0,0,0,0.06,0.45,0.14]),
    "Minimum Variance": np.array([0.09,0.1,0.11,0.09,0,0.02,0.08,0.05,0.04,0.42]
    "Maximum Return": np.array([0.31,0,0,0,0,0,0,0.1,0.59,0])
}
```

In [47]:
```
portfolio_daily_returns = {}

for name, weights in portfolio_weights.items():
    # Multiply each stock's daily return by its weight and sum across stocks
    portfolio_daily_returns[name] = returns.dot(weights)
```

In [48]:
```
max_drawdowns = {}

for name, daily_returns in portfolio_daily_returns.items():
    cumulative_returns = (1 + daily_returns).cumprod()
    rolling_max = cumulative_returns.cummax()
    drawdowns = (cumulative_returns - rolling_max) / rolling_max
    # max_drawdowns[name] = drawdowns.min()
    max_drawdowns[name] = drawdowns
# print("Maximum Drawdowns per Portfolio:")
```

```
# print(max_drawdowns)
max_drawdowns
```

Out[48]:
```
{'Equally Weighted': 2014-01-03 17:00:00         0.0
 2014-01-06 17:00:00   -0.005379
 2014-01-07 17:00:00   -0.000309
 2014-01-08 17:00:00   -0.005545
 2014-01-09 17:00:00   -0.011154
                          ...
 2025-08-12 17:00:00   -0.012355
 2025-08-13 17:00:00         0.0
 2025-08-14 17:00:00   -0.005636
 2025-08-15 17:00:00   -0.001759
 2025-08-18 17:00:00   -0.011514
 Length: 2802, dtype: object,
 'Maximum Sharpe Ratio': 2014-01-03 17:00:00         0.0
 2014-01-06 17:00:00         0.0
 2014-01-07 17:00:00   -0.001832
 2014-01-08 17:00:00   -0.001355
 2014-01-09 17:00:00   -0.003662
                          ...
 2025-08-12 17:00:00   -0.027312
 2025-08-13 17:00:00   -0.016025
 2025-08-14 17:00:00   -0.016561
 2025-08-15 17:00:00   -0.016842
 2025-08-18 17:00:00   -0.020597
 Length: 2802, dtype: object,
 'Minimum Variance': 2014-01-03 17:00:00         0.0
 2014-01-06 17:00:00   -0.001192
 2014-01-07 17:00:00         0.0
 2014-01-08 17:00:00   -0.008935
 2014-01-09 17:00:00   -0.007525
                          ...
 2025-08-12 17:00:00   -0.075689
 2025-08-13 17:00:00   -0.070124
 2025-08-14 17:00:00   -0.077342
 2025-08-15 17:00:00    -0.07437
 2025-08-18 17:00:00   -0.075064
 Length: 2802, dtype: object,
 'Maximum Return': 2014-01-03 17:00:00         0.0
 2014-01-06 17:00:00         0.0
 2014-01-07 17:00:00   -0.007127
 2014-01-08 17:00:00   -0.000648
 2014-01-09 17:00:00   -0.006854
                          ...
 2025-08-12 17:00:00   -0.013762
 2025-08-13 17:00:00         0.0
 2025-08-14 17:00:00   -0.004051
 2025-08-15 17:00:00   -0.004855
 2025-08-18 17:00:00    -0.00932
 Length: 2802, dtype: object}
```

max_drawdowns.keys()

In [49]:
```
max_drawdowns.values()
```

Out[49]:
```
dict_values([2014-01-03 17:00:00         0.0
2014-01-06 17:00:00   -0.005379
2014-01-07 17:00:00   -0.000309
2014-01-08 17:00:00   -0.005545
2014-01-09 17:00:00   -0.011154
                         ...
2025-08-12 17:00:00   -0.012355
2025-08-13 17:00:00         0.0
2025-08-14 17:00:00   -0.005636
2025-08-15 17:00:00   -0.001759
2025-08-18 17:00:00   -0.011514
Length: 2802, dtype: object, 2014-01-03 17:00:00         0.0
2014-01-06 17:00:00         0.0
2014-01-07 17:00:00   -0.001832
2014-01-08 17:00:00   -0.001355
2014-01-09 17:00:00   -0.003662
                         ...
2025-08-12 17:00:00   -0.027312
2025-08-13 17:00:00   -0.016025
2025-08-14 17:00:00   -0.016561
2025-08-15 17:00:00   -0.016842
2025-08-18 17:00:00   -0.020597
Length: 2802, dtype: object, 2014-01-03 17:00:00         0.0
2014-01-06 17:00:00   -0.001192
2014-01-07 17:00:00         0.0
2014-01-08 17:00:00   -0.008935
2014-01-09 17:00:00   -0.007525
                         ...
2025-08-12 17:00:00   -0.075689
2025-08-13 17:00:00   -0.070124
2025-08-14 17:00:00   -0.077342
2025-08-15 17:00:00    -0.07437
2025-08-18 17:00:00   -0.075064
Length: 2802, dtype: object, 2014-01-03 17:00:00         0.0
2014-01-06 17:00:00         0.0
2014-01-07 17:00:00   -0.007127
2014-01-08 17:00:00   -0.000648
2014-01-09 17:00:00   -0.006854
                         ...
2025-08-12 17:00:00   -0.013762
2025-08-13 17:00:00         0.0
2025-08-14 17:00:00   -0.004051
2025-08-15 17:00:00   -0.004855
2025-08-18 17:00:00    -0.00932
Length: 2802, dtype: object])
```

In [50]:
```
drawdowns=pd.DataFrame(max_drawdowns)
drawdowns
```

Out[50]:

	Equally Weighted	Maximum Sharpe Ratio	Minimum Variance	Maximum Return
2014-01-03 17:00:00	0.0	0.0	0.0	0.0
2014-01-06 17:00:00	-0.005379	0.0	-0.001192	0.0
2014-01-07 17:00:00	-0.000309	-0.001832	0.0	-0.007127
2014-01-08 17:00:00	-0.005545	-0.001355	-0.008935	-0.000648
2014-01-09 17:00:00	-0.011154	-0.003662	-0.007525	-0.006854
...	...	...	...	...
2025-08-12 17:00:00	-0.012355	-0.027312	-0.075689	-0.013762
2025-08-13 17:00:00	0.0	-0.016025	-0.070124	0.0
2025-08-14 17:00:00	-0.005636	-0.016561	-0.077342	-0.004051
2025-08-15 17:00:00	-0.001759	-0.016842	-0.07437	-0.004855
2025-08-18 17:00:00	-0.011514	-0.020597	-0.075064	-0.00932

2802 rows × 4 columns

Maximum Drawdown

Measurement of greatest peak-to-trough decline in a portfolio's value before a new peak is achieved. It represents the worst case loss an investor would have experienced had they invested at the absolute top and sold at the absolute bottom during a specific period.

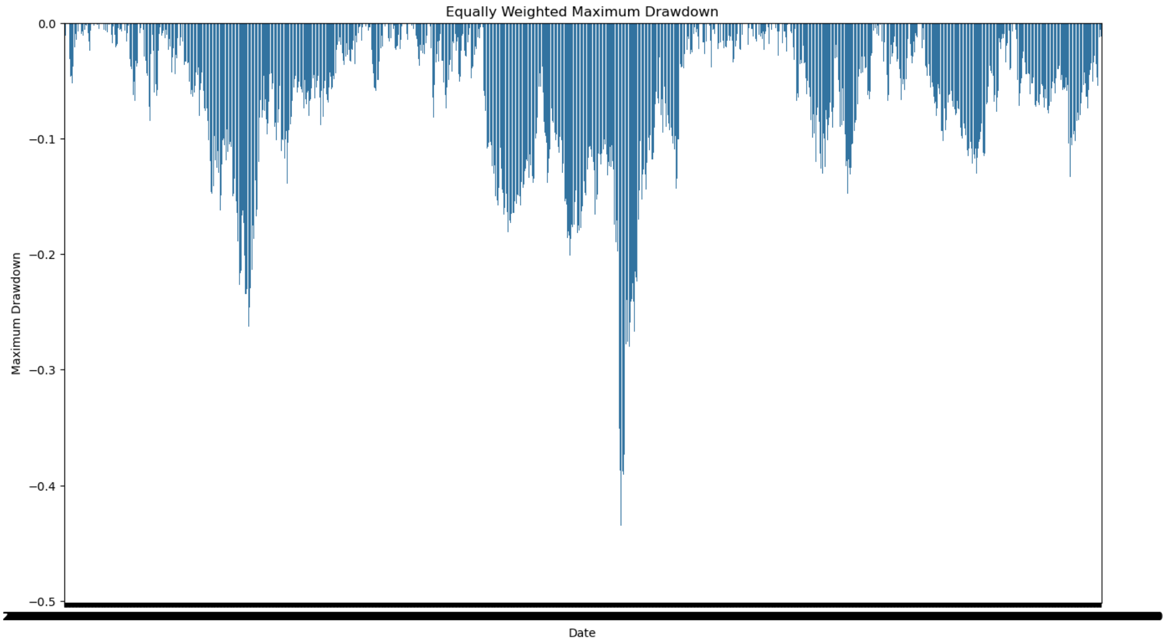
Unlike volatility, which is a statistical measure of dispersion, drawdown is a tangible representation of risk that investors feel acutely.

High drawdowns, for example 50%, could cause investors to panic and sell at the worst possible time.

In [51]:

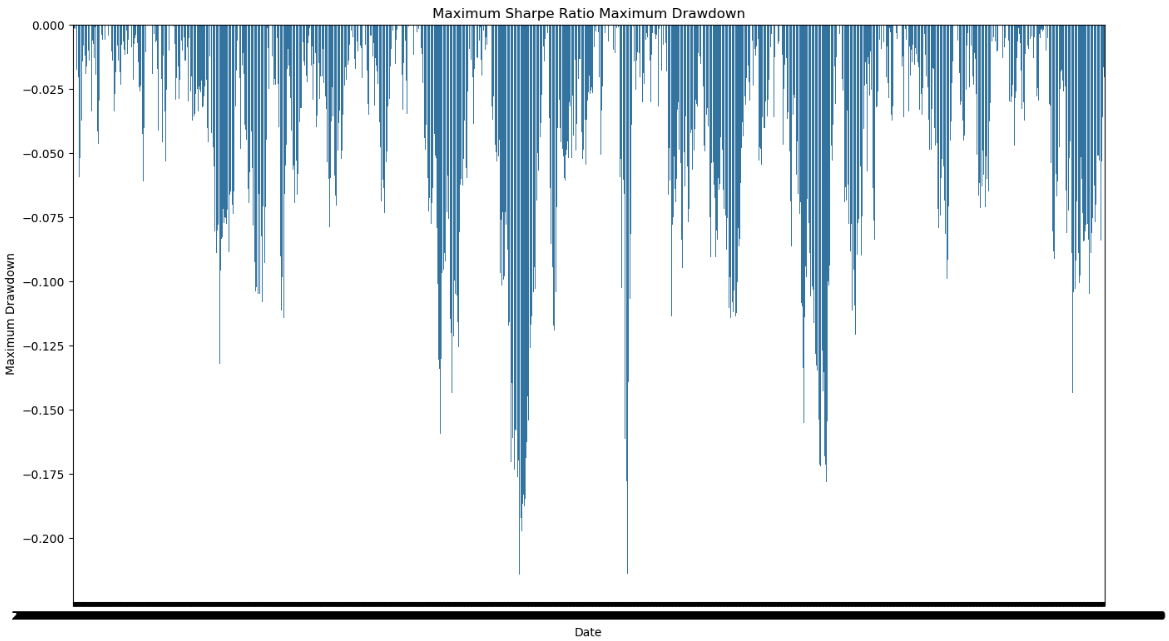
```
port = 'Equally Weighted'
plt.figure(figsize=(16,9))
plt.title(port + ' Maximum Drawdown')
ax = sns.barplot(x=drawdowns.index, y=drawdowns[port])
ax.set_xlabel("Date")
ax.set_ylabel("Maximum Drawdown")
```

Out[51]: Text(0, 0.5, 'Maximum Drawdown')



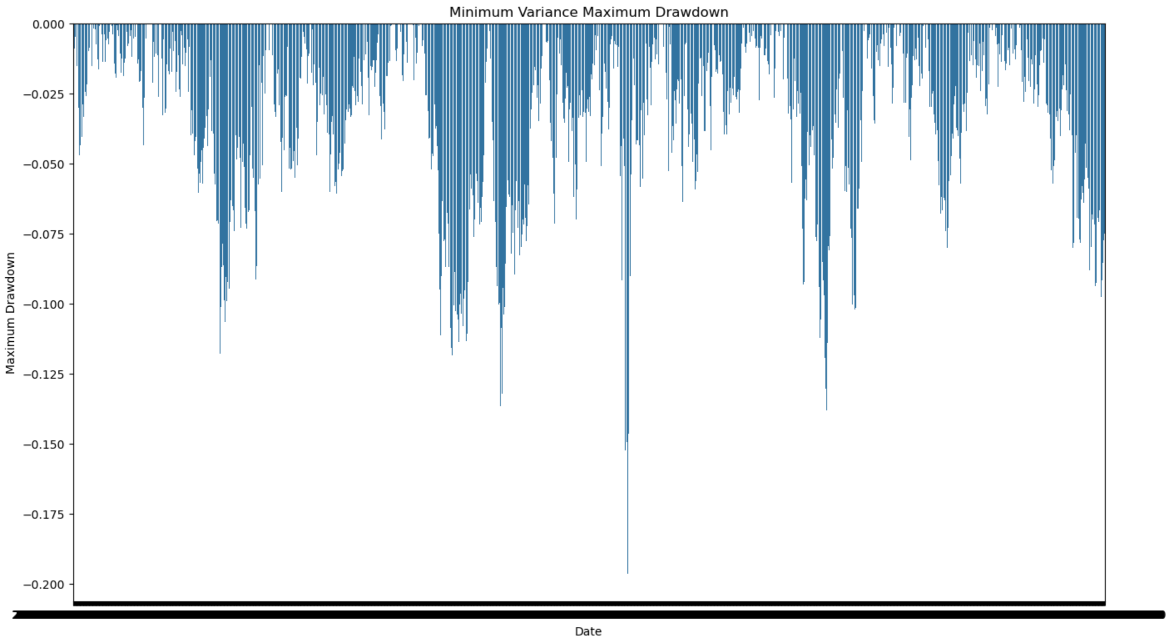
In [52]:
```
port = 'Maximum Sharpe Ratio'
plt.figure(figsize=(16,9))
plt.title(port + ' Maximum Drawdown')
ax = sns.barplot(x=drawdowns.index, y=drawdowns[port])
ax.set_xlabel("Date")
ax.set_ylabel("Maximum Drawdown")
```

Out[52]: Text(0, 0.5, 'Maximum Drawdown')



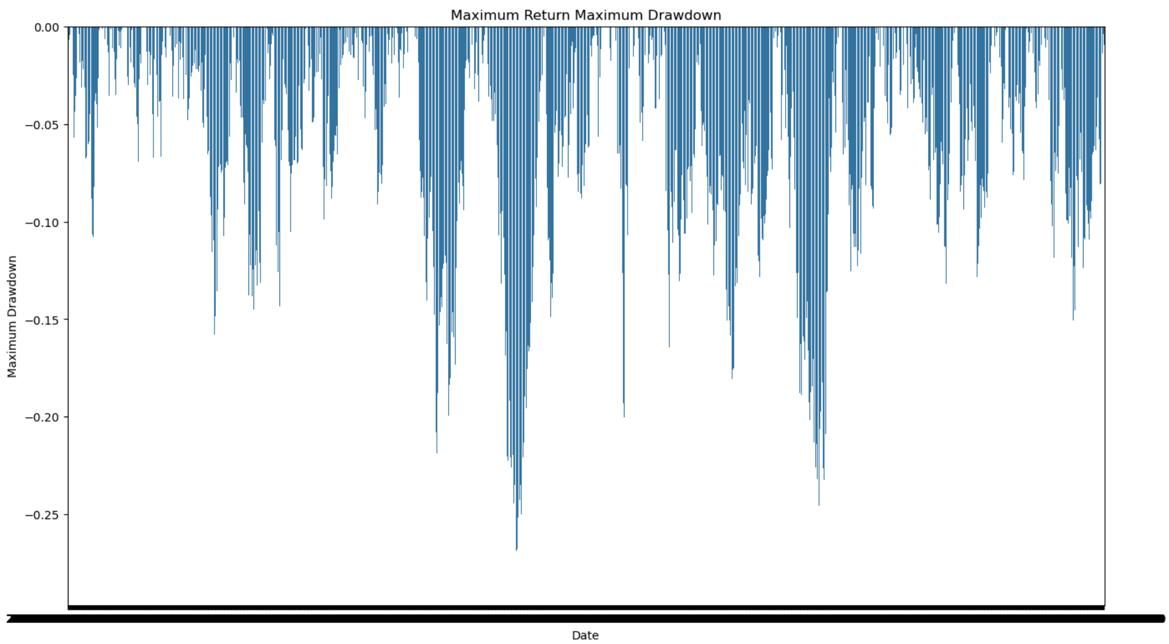
In [53]:
```
port = 'Minimum Variance'
plt.figure(figsize=(16,9))
plt.title(port + ' Maximum Drawdown')
ax = sns.barplot(x=drawdowns.index, y=drawdowns[port])
ax.set_xlabel("Date")
ax.set_ylabel("Maximum Drawdown")
```

Out[53]: Text(0, 0.5, 'Maximum Drawdown')



In [54]:
```
port = 'Maximum Return'
plt.figure(figsize=(16,9))
plt.title(port + ' Maximum Drawdown')
ax = sns.barplot(x=drawdowns.index, y=drawdowns[port])
ax.set_xlabel("Date")
ax.set_ylabel("Maximum Drawdown")
```

Out[54]:
```
Text(0, 0.5, 'Maximum Drawdown')
```



In [55]:
```
drawdowns.mean()
```

Out[55]:
```
Equally Weighted         -0.06423
Maximum Sharpe Ratio    -0.041482
Minimum Variance         -0.03262
Maximum Return          -0.059547
dtype: object
```

In [56]:
```
recovery_periods = {}

for name, daily_returns in portfolio_daily_returns.items():
```

```
    cumulative_returns = (1 + daily_returns).cumprod()
    rolling_max = cumulative_returns.cummax()
    drawdowns = (cumulative_returns - rolling_max) / rolling_max

    drawdown_end = drawdowns.idxmin()
    recovery_point = cumulative_returns[drawdown_end:].idxmax()
    recovery_days = (recovery_point - drawdown_end).days  # number of days to re

    recovery_periods[name] = recovery_days

print("Recovery Periods (days) per Portfolio:")
print(recovery_periods)
recovery = pd.DataFrame(recovery_periods,index =np.arange(len(recovery_periods)))
recovery = recovery.loc[recovery.index==0]
recovery.index = ['Portfolio Recovery Period']
recovery
```

```
Recovery Periods (days) per Portfolio:
{'Equally Weighted': 1969, 'Maximum Sharpe Ratio': 2185, 'Minimum Variance': 180
6, 'Maximum Return': 2414}
```

Out[56]:

	Equally Weighted	Maximum Sharpe Ratio	Minimum Variance	Maximum Return
Portfolio Recovery Period	1969	2185	1806	2414

Recovery Period

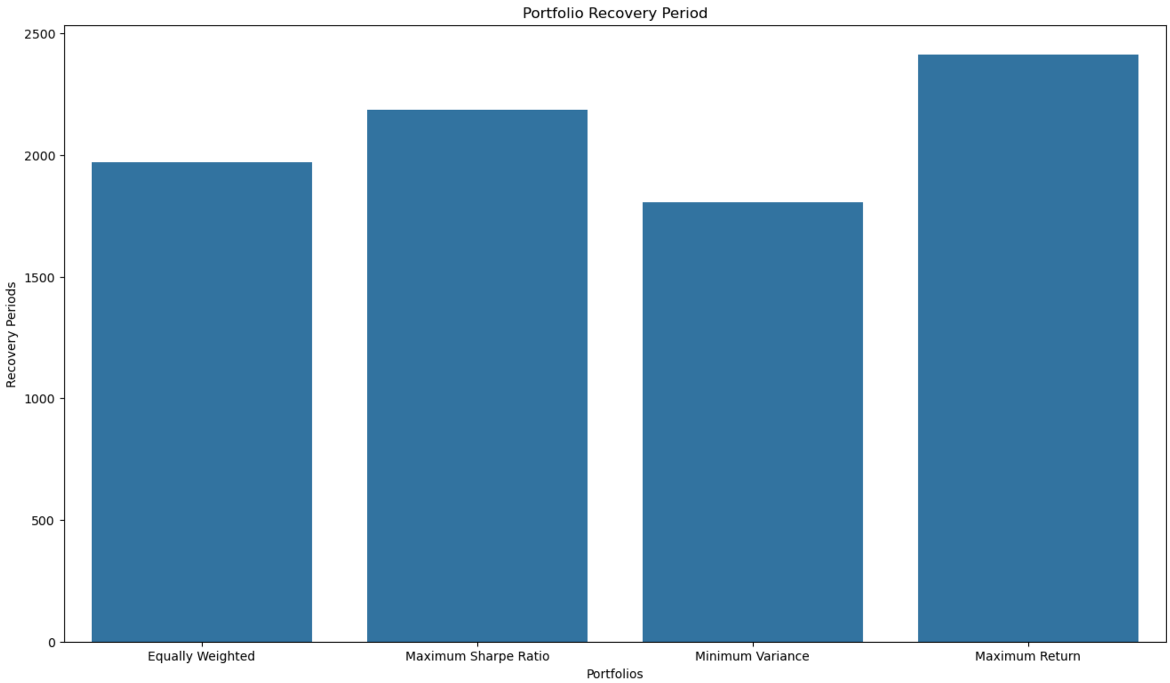
The time it takes for a portfolio to regain its value and surpass the previous peak after hitting the trough of a drawdown. It measures the resilience of the portfolio, a quick recovery is much more palatable than a loss that lingers for years.

The recovery time metric adds context to MDD. A steep drop of 25% that recovers in 6 months is different from a 25% drops that takes 5 years to brean even.

In [57]:

```
plt.figure(figsize=(16,9))
plt.title('Portfolio Recovery Period')
ax = sns.barplot(x=recovery.columns, y=recovery.iloc[0])
ax.set_xlabel("Portfolios")
ax.set_ylabel("Recovery Periods")
```

Out[57]: `Text(0, 0.5, 'Recovery Periods')`



Sortino Ratio

Modification of the Sharpe Ratio that only considers downside deviations (harmful volatility) rather than total volatility.

Sortino = $\frac{R_p}{\sigma_d}$

Where:

- σ_d is the downside deviations

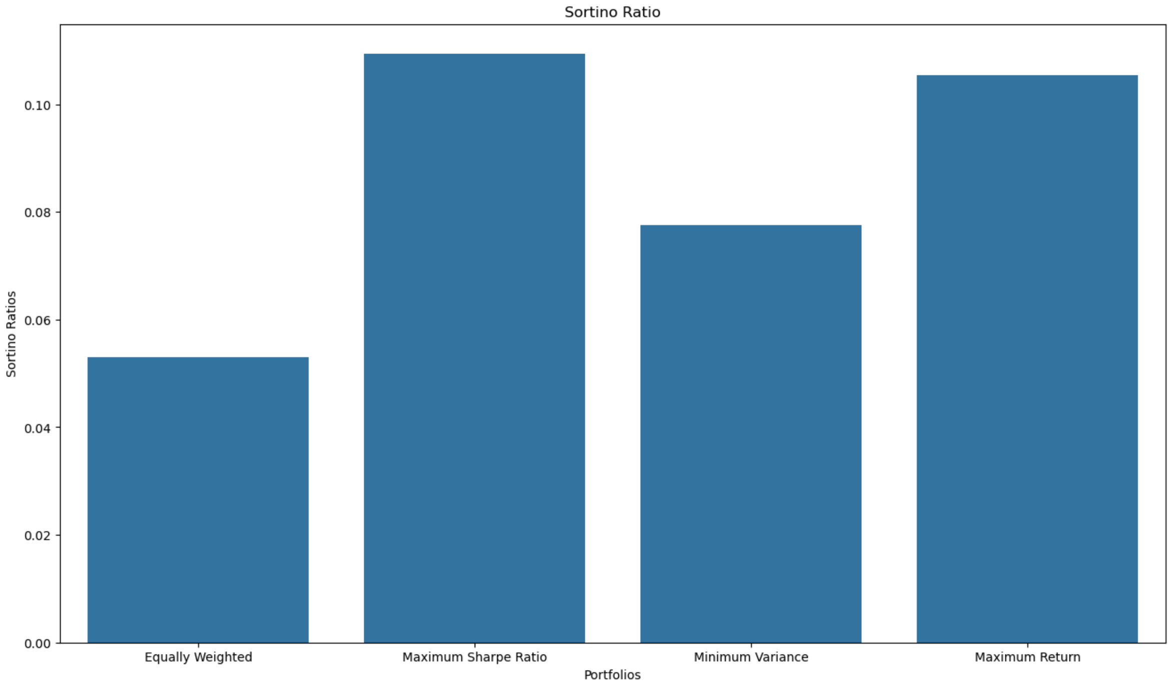
Useful for investors who are concerned with protecting against losses than with the overall volatility of their investment. A higher Sortino ratio indicates a better risk-adjusted return on the downside.

In [58]:
```
risk_free_rate = 0  # can adjust if you have a value but for now lets use zero b
sortino_ratios = {}

for name, daily_returns in portfolio_daily_returns.items():
    downside_returns = daily_returns[daily_returns < 0]

    if len(downside_returns) > 0:
        downside_std = downside_returns.std()
        sortino_ratio = (daily_returns.mean() - risk_free_rate) / downside_std
    else:
        sortino_ratio = np.nan  # no negative returns

    sortino_ratios[name] = sortino_ratio

print("Sortino Ratios per Portfolio:")
print(sortino_ratios)
```

```
Sortino Ratios per Portfolio:
{'Equally Weighted': np.float64(0.05300742497587133), 'Maximum Sharpe Ratio': np.
float64(0.10942318048837414), 'Minimum Variance': np.float64(0.0776145246265094
9), 'Maximum Return': np.float64(0.10538925949182061)}
```

In [59]:
```
plt.figure(figsize=(16,9))
plt.title('Sortino Ratio')
ax = sns.barplot(x=list(sortino_ratios.keys()), y=list(sortino_ratios.values()))
ax.set_xlabel("Portfolios")
ax.set_ylabel("Sortino Ratios")
```

Out[59]: `Text(0, 0.5, 'Sortino Ratios')`

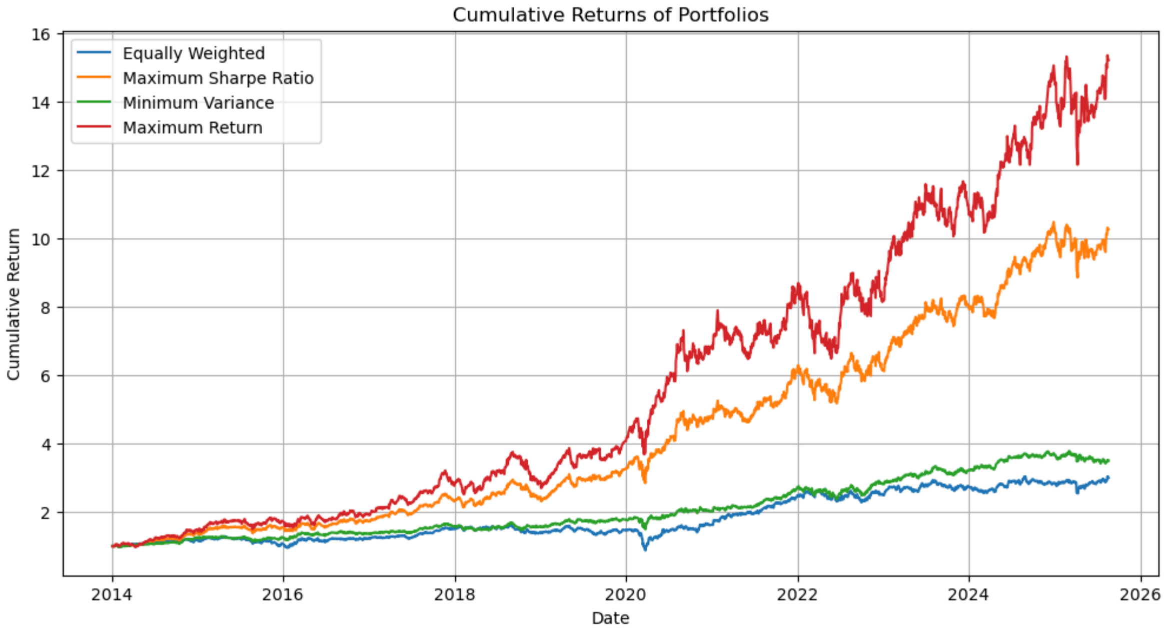


In [60]:
```
results_df = pd.DataFrame({
    "Max Drawdown": max_drawdowns,
    "Recovery Period (days)": recovery_periods,
    "Sortino Ratio": sortino_ratios
})

# Optional: round numbers for readability
results_df = results_df.round({
    "Max Drawdown": 4,
    "Recovery Period (days)": 0,
    "Sortino Ratio": 4
})

print("Portfolio Performance Metrics:")
print(results_df)
```

```
Portfolio Performance Metrics:
                                                         Max Drawdown  \
Equally Weighted      2014-01-03 17:00:00         0.0
2014-01-06 17:...
Maximum Sharpe Ratio  2014-01-03 17:00:00         0.0
2014-01-06 17:...
Minimum Variance      2014-01-03 17:00:00         0.0
2014-01-06 17:...
Maximum Return        2014-01-03 17:00:00         0.0
2014-01-06 17:...

                      Recovery Period (days)  Sortino Ratio
Equally Weighted                        1969         0.0530
Maximum Sharpe Ratio                    2185         0.1094
Minimum Variance                        1806         0.0776
Maximum Return                          2414         0.1054
```

In [61]:

```python
plt.figure(figsize=(12,6))

for name, daily_returns in portfolio_daily_returns.items():
    cumulative_returns = (1 + daily_returns).cumprod()
    plt.plot(cumulative_returns, label=name)

plt.title("Cumulative Returns of Portfolios")
plt.xlabel("Date")
plt.ylabel("Cumulative Return")
plt.legend()
plt.grid(True)
plt.show()
```



Portfolio Combinations

$$\frac{(10+20-1)!}{((10+20-1-10)!10!)} = \frac{(29)!}{(19!10!)}$$

$$\frac{29\times28\times27\times26\times25\times24\times23\times22\times21\times20}{10\times9\times8\times7\times6\times5\times4\times3\times2} = 20030010$$

$$\frac{20030010}{2} = 10015005$$

5% increments

In [62]:
```
from math import comb
from tqdm import tqdm

def generate_stock_weight_combinations(n_stocks=27, step=10, total=100):
    max_units = total // step
    valid_combinations = []

    total_paths = comb(max_units + n_stocks - 1, n_stocks - 1)
    pbar = tqdm(total=total_paths, desc="Generating Combinations", leave=False)

    def backtrack(current, remaining, depth):
        if remaining < 0:
            return  # prune invalid branches
        if depth == n_stocks:
            if remaining == 0:
                valid_combinations.append([w * step for w in current])
            return
        max_i = remaining  # max units we can assign at this depth
        for i in range(max_i + 1):
            pbar.update(1)
            backtrack(current + [i], remaining - i, depth + 1)

    backtrack([], max_units, 0)
    pbar.close()
    return valid_combinations
```

In [63]:
```
stocks = 10
weights = generate_stock_weight_combinations(n_stocks=stocks, step=5, total = 10
```

In [68]:
```
len(weights)
```

Out[68]: 10015005

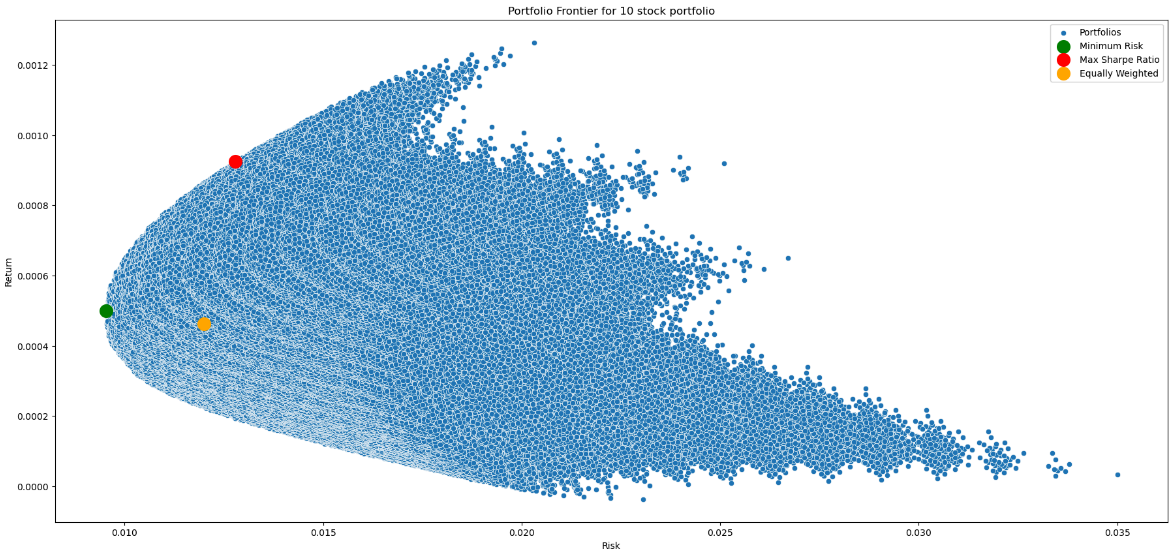
In [64]:
```
from tqdm import tqdm

total_iterations = len(weights) * len(weights[0])
with tqdm(total=total_iterations, desc="Normalizing Weights",leave=False) as pba
    for i in range(len(weights)):
        for j in range(len(weights[0])):
            weights[i][j] = weights[i][j] / 100
            pbar.update(1)
```

In [65]:
```
returns_plot = []
risk_plot = []
with tqdm(total=len(weights), desc="Plotting EF",leave=False) as pbar:
    for i in range(len(weights)):
        w = np.array(weights[i])
        returns_plot.append(w@mean_returns[:stocks])
        risk_plot.append(np.sqrt((w@cov_matrix.iloc[:stocks,:stocks])@w.T))
        pbar.update(1)
```

In [66]:
```
sharpe_ratios = []
for i in range(len(returns_plot)):
    sharpe_ratios.append(returns_plot[i]/risk_plot[i])

equal_weights = np.ones(stocks) / stocks
equal_return = np.dot(equal_weights, mean_returns[:stocks])
equal_risk = np.sqrt(np.dot(equal_weights.T, np.dot(cov_matrix.iloc[:stocks,:stc
```

In [67]:
```
plt.figure(figsize=(22,10))
plt.title('Portfolio Frontier for ' + str(stocks) + ' stock portfolio')
min_risk_idx = np.argmin(risk_plot)

max_sharpe_idx = np.argmax(sharpe_ratios)

ax = sns.scatterplot(x=risk_plot,y=returns_plot, label='Portfolios')
ax.set_xlabel('Risk')
ax.set_ylabel('Return')

plt.scatter(risk_plot[min_risk_idx], returns_plot[min_risk_idx], color='green',
plt.scatter(risk_plot[max_sharpe_idx], returns_plot[max_sharpe_idx], color='red'
plt.scatter(equal_risk, equal_return, color='orange', s=200, label='Equally Weig
plt.legend()
```

Out[67]: <matplotlib.legend.Legend at 0x226b7c20e10>



In [ ]: