

Rates of Change in Polynomial Functions

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$$\begin{aligned}\text{Average rate of change} &= \frac{\Delta y}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1} \\ \text{Instantaneous rate of change} &= m = \frac{f(a+h) - f(a)}{h}\end{aligned}$$

Ex 1

Find the average rate of change for the interval $x \in [1, 6]$ on the function $f(x) = (x - 3)^3 - 1$

$$\begin{aligned}f(6) &= (6 - 3)^3 - 1 \\ &= (3)^3 - 1 \\ &= 26 \\ f(1) &= (1 - 3)^3 - 1 \\ &= (-2)^3 - 1 \\ &= -9\end{aligned}$$

$$\begin{aligned}Aroc &= \frac{f(x_2) - f(x_1)}{x_2 - x_1} \\ &= \frac{f(6) - f(1)}{6 - 1} \\ &= \frac{26 - (-9)}{5} \\ &= \frac{35}{5} \\ &= 7\end{aligned}$$

Ex 2

Estimate the instantaneous rate of change in $f(x)$ at the point $(3,3)$

$$f(x) = a(x+0)(x-2)^2(x-4)$$

$$3 = a(1)(1-2)^2(1-4)$$

$$3 = a(1)(1)(-3)$$

$$3 = -3a$$

$$a = -1$$

$$f(x) = -1(x)(x-2)^2(x-4)$$

$$f(3) = 3$$

$$\begin{aligned} f(3.01) &= -(3.01)(3.01-2)^2(3.01-4) \\ &= 3.0398 \end{aligned}$$

$$\begin{aligned} I_{roc} &= \frac{f(3+0.01) - f(3)}{0.01} \\ &= \frac{3.0398 - 3}{0.01} \\ &= 3.98 \end{aligned}$$