Computational Neuroscience - Project 8

Modeling Spike Trains as a Poisson Process

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Introduction

In this project, we will be modelling spike trains of action potentials as a Poisson random process.

A spike train is a sequence of times at which a neuron produces an action potential, or spike. Modeling a spike train as a Poisson process involves assuming that the timing of the spikes is determined by a Poisson process with a certain rate parameter (lambda in our case).

What is a Poisson process? A Poisson process is a random process in which events occur independently with low probability over continuous time. While the Poisson process does meet most criteria for a neural spike train, the refractory period and other features prevent statistical independence, meaning that the firing rates in reality are not totally independent from each other. A Poisson distribution is parameterized by a single value, lambda. Lambda is often interpreted as the rate of success. The reason is that the expectation value (or mean) of a Poisson random variable is lambda. In other words, for a random variable describing the number of events per interval, the mean value is lambda. Equation shows the probability mass function for a Poisson distribution mathematically, which maps success rates to probabilities. An interesting property of a Poisson process is that it has a varience of lambda, where also a Poisson random variable has an expectation value of lambda.

The refractory period for firing neurons limits activity to one per refractory period. If we were to take a 1 ms refractory period, the time interval can be divided into a sequence of 1 ms periods. Each 1 ms period can be examined as a potential firing event for the neuron.

$$P(x = k) \equiv \frac{\lambda^k}{k!} e^{-\lambda} \qquad (1)$$

Exercises

Exercise 26.4

In this exercise, we are asked to estimate the distribution of spikes in two separate ways; by generating samples from a Poisson distribution representing the spike count over the 10 second interval, and by dividing the 10 second interval into 1 ms pieces and calculating an appropriate probability of the single event for each 1 ms interval.

For the lambda, the number of events per interval, we take an average rate of 25 spikes per second. Then, estimating for 10 seconds, we have $\lambda = 25 * 10$.

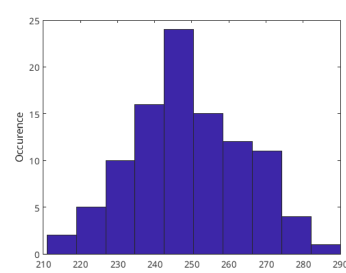

Figure 1: Histogram of the Poisson distribution estimate.

In figure 1, we see the histogram for the Poisson distribution estimation and figure 2 shows the histogram of our second way of estimating

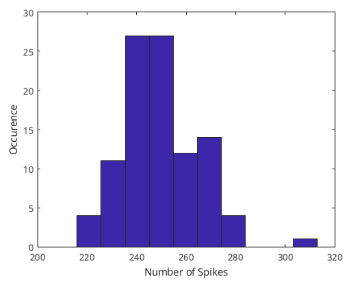

Figure 2: Histogram of the event probability estimate.

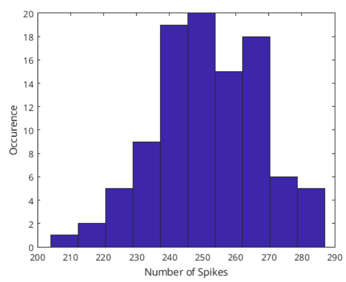

Figure 3: Histogram of the exponential distribution estimate.

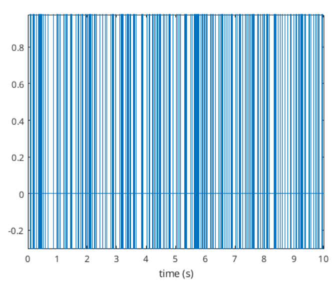

Figure 4: Spike trains over 10 seconds.

the spike trains. Note that both of these histograms change from estimation to estimation since they are randomly generated.

As expected, both have a peak in 250 total spikes over 10 seconds, since on average we have 25 spikes per second.

Exercise 26.5

In this one, we are asked to use the exponential distribution of the interspike intervals to generate a simulated spike train. We draw intervals from an exponential distribution with the desired lambda while the total of the intervals remains below the overall spike train duration. We can use the *exprnd* function to sample from the exponential distribution in MATLAB.

In figure 3, we can see the generated histogram plot of this estimation. Again, this is randomly generated, and also as expected, has a peak at 250 spikes over 10 seconds for the same reason.

We also plot the spike trains, as seen in figure 4, which shows when the events (spikes) happened over 10 seconds.

How does the spike train compare to that generated with an explicit Poisson variable?

The spike train generated by sampling from the exponential distribution of interspike intervals is equivalent to that generated by a Poisson process with the same rate parameter. This is because the interspike intervals in a Poisson process are exponentially distributed, so sampling from the exponential distribution with the same rate parameter as the Poisson process will produce a spike train with the same statistical properties as the one generated by the Poisson process.