IE 306 - Fall 2023
Assignment 1
Due date: Tuesday, October 24, 17:00

1. Consider an inlet valve mechanism that is used to fill a water reservoir. The valve works such that the water filling rate is a function of the gap between the actual water height (h) and the target water height (h*). The valve is produced to provide an inflow that will close the 10% of the gap between h* and h per hour. Additionally, there is a small hole in the reservoir through which some water leaks out. The leakage rate is proportional to the water height, and this hole alone leads to a 5% fall in the water height per hour. Initial height of water is 10 cm, and the target water height is 60 cm.
    a. Identify the state variables involved in this problem and give the model equation(s) that capture the dynamics of the state variable(s).
    b. Next give the simulation equations and simulate the model for 10 hours, using a step size dt=0.5, *tabulating* the water height (h). (You may do it by calculator or using a spreadsheet software, or by computer programming).
    c. Next solve the model equations analytically in order to find the exact water height value at t=5 and t=10. Compare the exact values with the simulated values you find in (b)

2. a) Consider the pdf of standard Normal random variable z, which is known, but impossible to integrate analytically. Using the simulation method discussed in class, approximately compute the integral of this function in the interval (0.56, 2.4). Use the random generator of Excel, R or any other programming language and do the simulation for 1000 random number pairs. At the end, how well do you think is the approximation?

    b) The famous central limit theorem in statistics states that if you take the sum of N identical independent random variables, the sum would be Normally distributed, if N is large enough. If we apply this to (say) 10 independent U(0,1) random variables, then the sum (Y) should be Normal(5, 0.83), since the mean of U(0,1) is 0.5 and its variance is 1/12. Write a computer program to generate 750 such Y values. To see if the theorem works, estimate the mean and variance of Y, and plot its histogram (using a suitable number of classes).

    In both parts (a) and (b), you should turn in not only your results, but also your formulas used in the cells (if you do it by spreadsheet), or you code.

3. Consider the following inventory system;
    a. Whenever the inventory falls to or below 10 units, an order is placed. Only one order can be outstanding at a time (i.e. no new orders until the placed order arrives).
    b. The size of each order is equal to target inventor level (20) minus the current inventory level (I)
    c. If a demand occurs during a period when the inventory level is zero, the sale is lost
    d. Daily demand is discrete between 2 and 7, where probabilities are given as;

Demand	2	3	4	5	6	7
Probability	0,10	0,18	0,20	0,22	0,20	0,10

    e. Lead time of orders is discrete uniform distributed between zero and 5 days. For simplicity, assume that orders are placed at the close of the business day and receive after the lead time has occurred. Thus, if lead time is one day, the order is available for distribution on the morning of the second day of business following the placement of the order.
    f. The unsatisfied portion of the daily demand is considered as lost sales

Estimate by simulation, the average number of lost sales per day, and the average inventory level fort his inventory system. The simulation will start with 18 unit in inventory and let the simulation run for 10 days.