BİL3014

Algoritma Analizi

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Recursive Algoritmalar ve Recurrence Bağıntısı

$$x^{\log_y n} \Longrightarrow n^{\log_y x}$$

$$x^0 + x^1 + x^2 + \cdots + x^n = \frac{x^{n+1} - 1}{x-1} \qquad for\ x \neq 1$$

$$x^0 + x^1 + x^2 + \cdots = \frac{1}{1-x} \qquad for\ |x| < 1$$

Recursion Tree Method: Example 2

$$T(n) = \begin{cases} 1 & n = 1 \\ 2\,T\left(\frac{n}{2}\right) + n^2 & n > 1 \end{cases}$$

Solve the following recurrence using the Recurrence Tree Method.

Assumption: We assume that n is exact power of 2.

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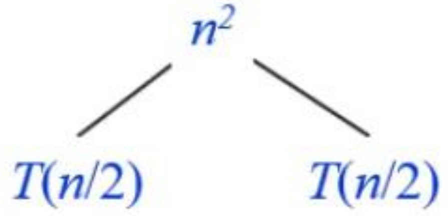

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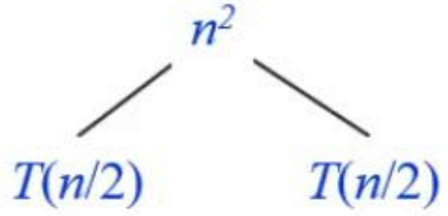

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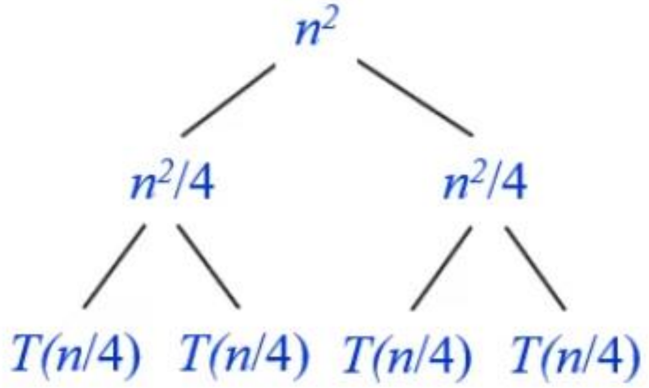

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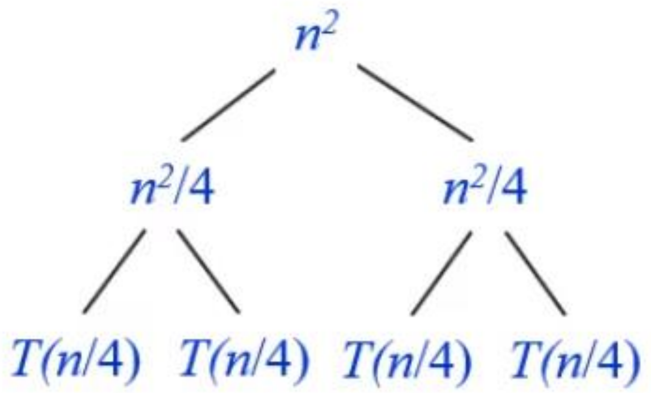

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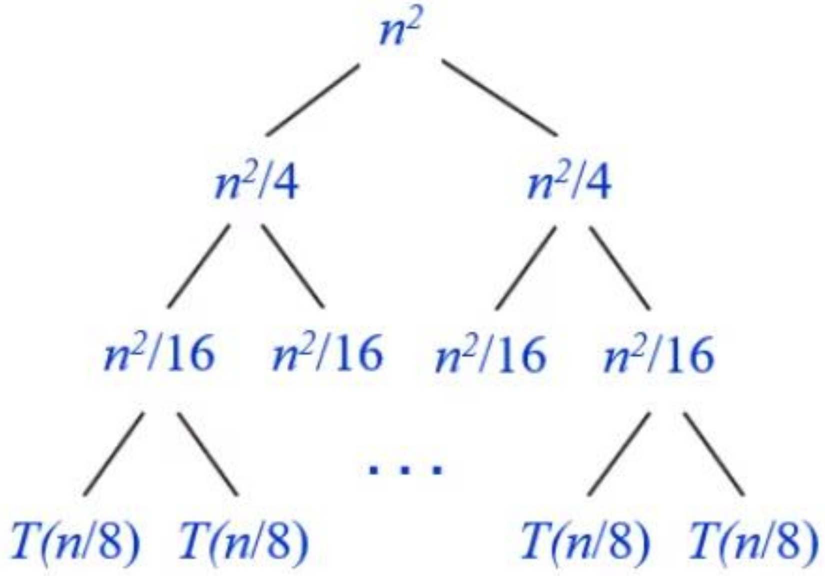

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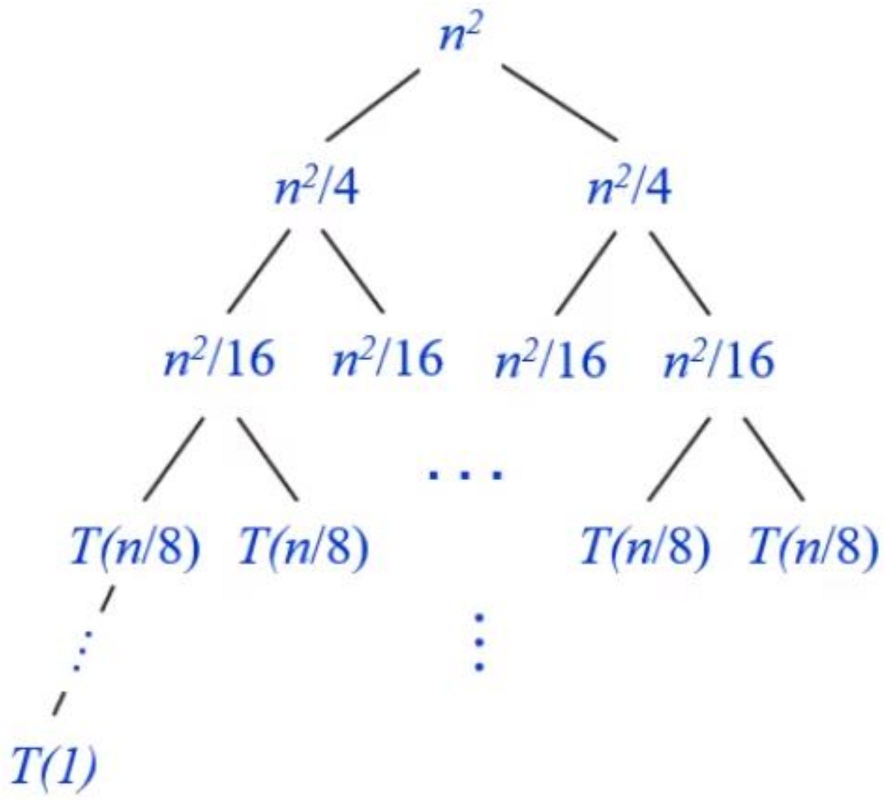

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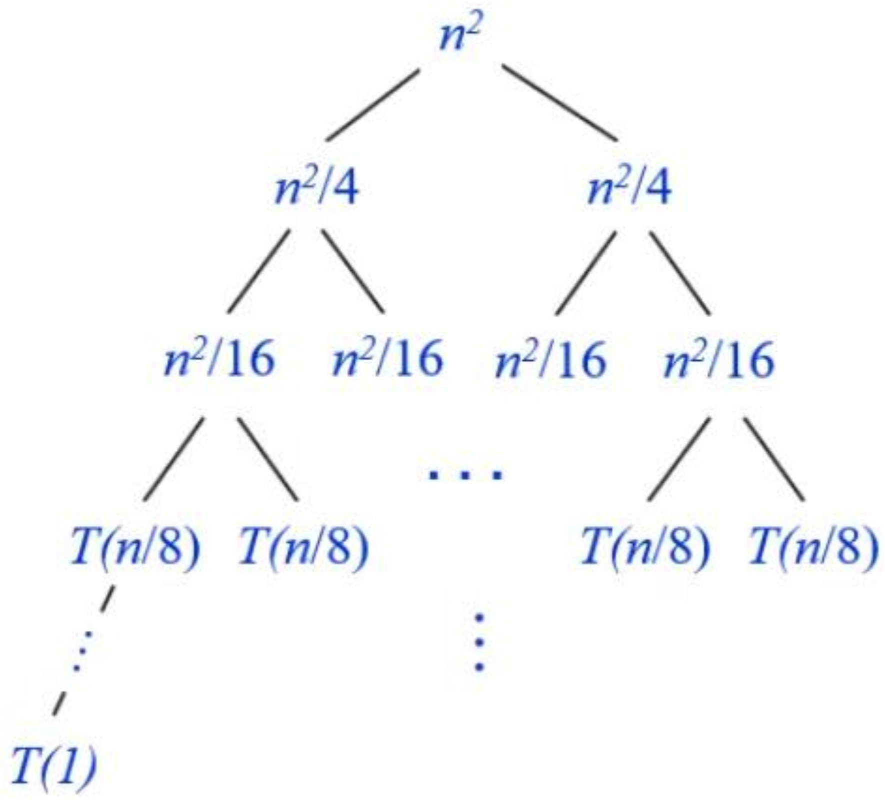

Total Cost $= L_c + I_c$

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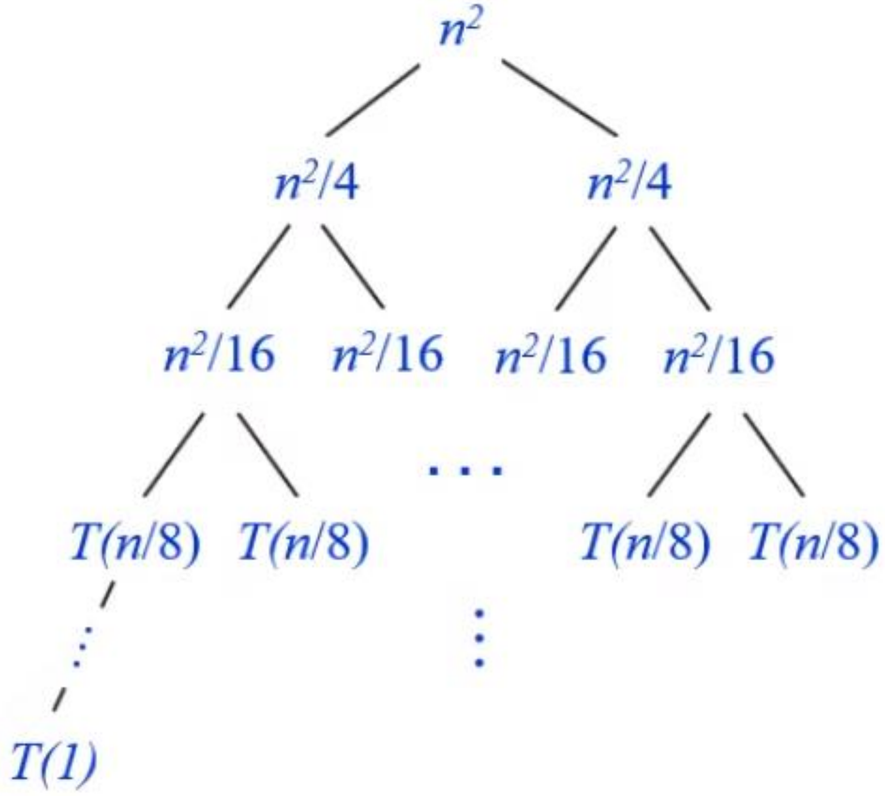

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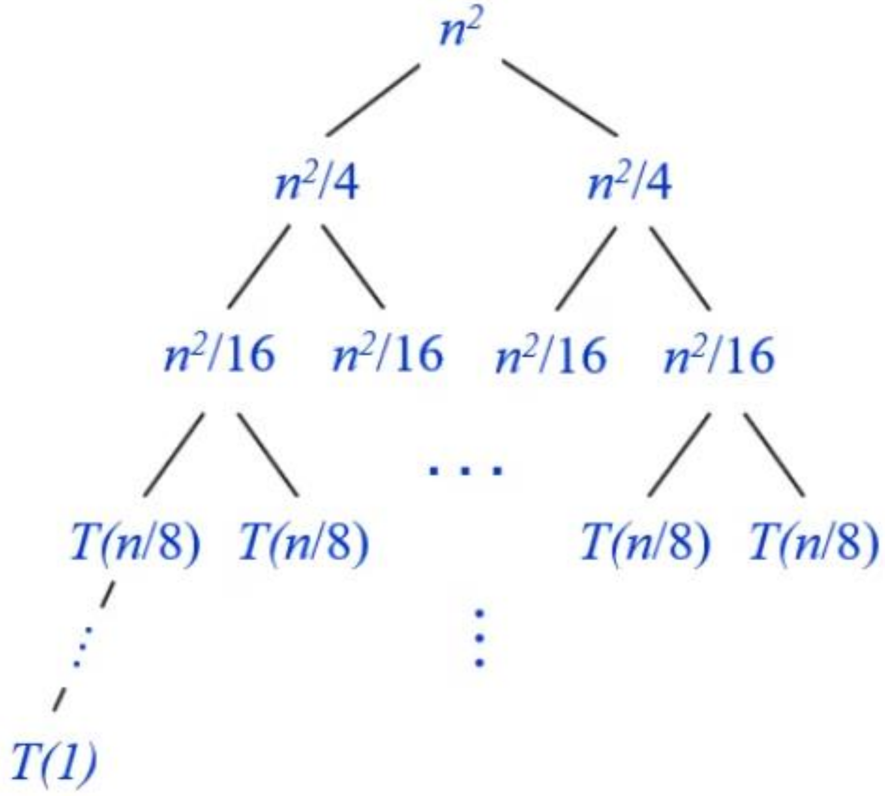

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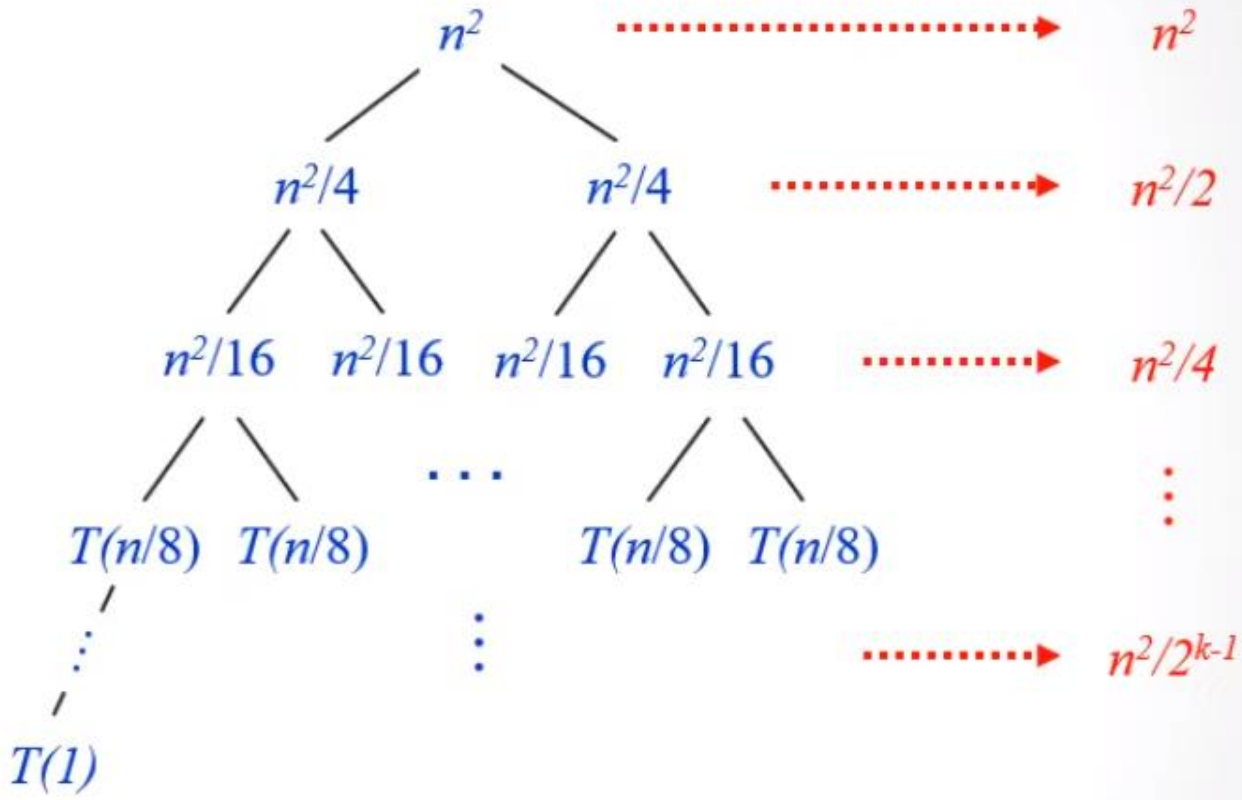

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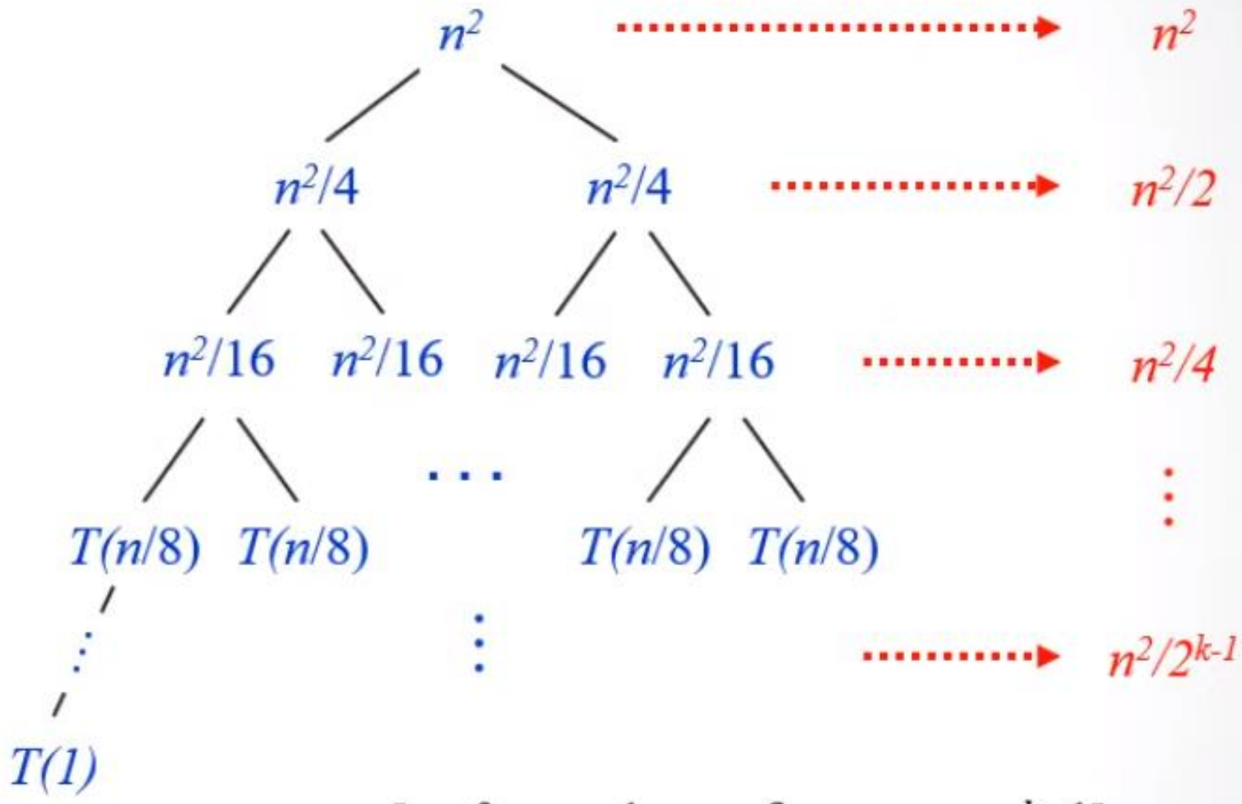

$$I_c = n^2 \cdot \left[\left(\frac{1}{2}\right)^0 + \left(\frac{1}{2}\right)^1 + \left(\frac{1}{2}\right)^2 + \cdots + \left(\frac{1}{2}\right)^{k-1}\right]$$

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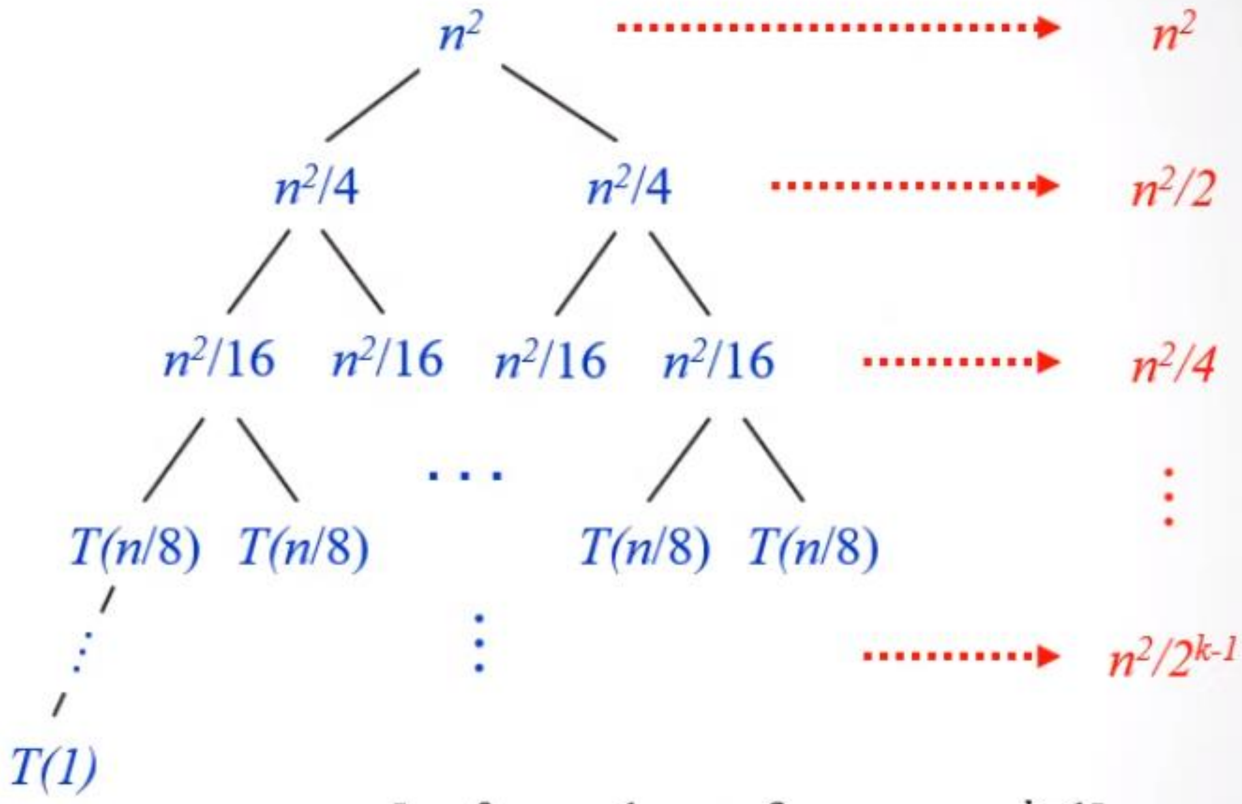

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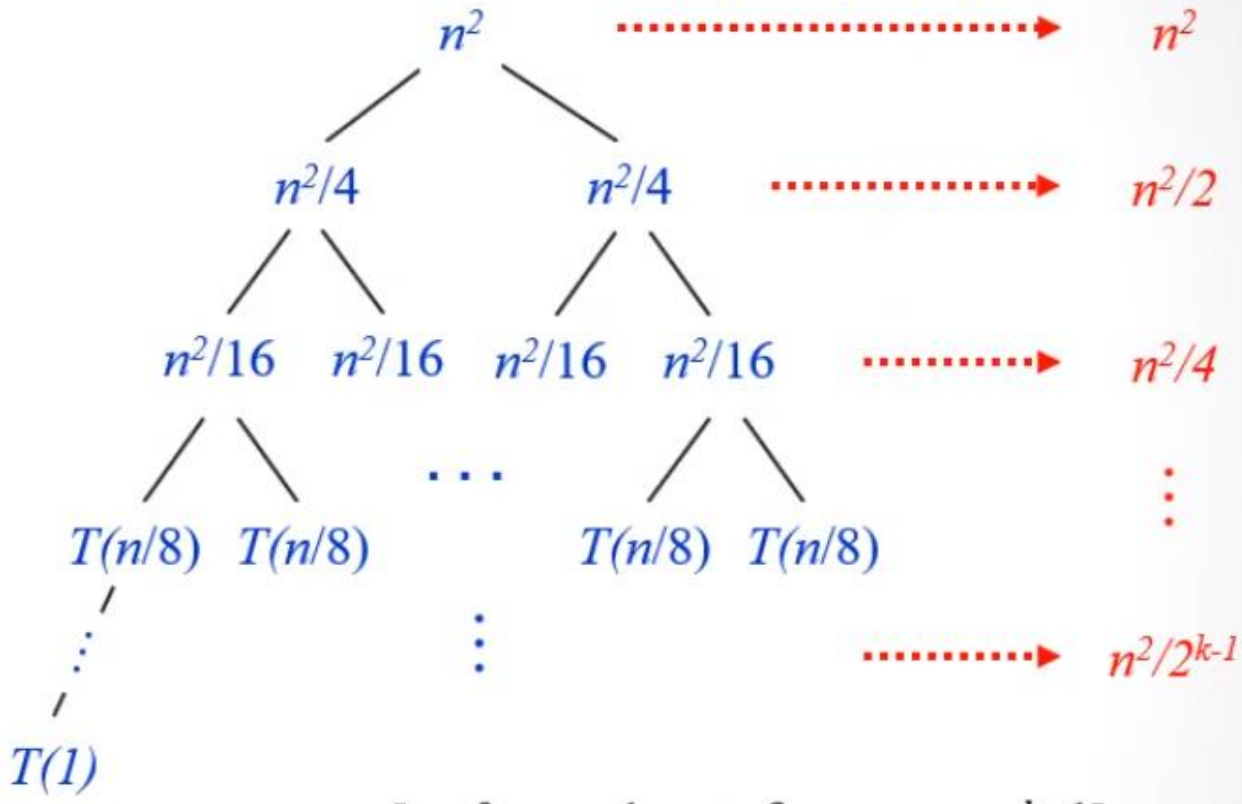

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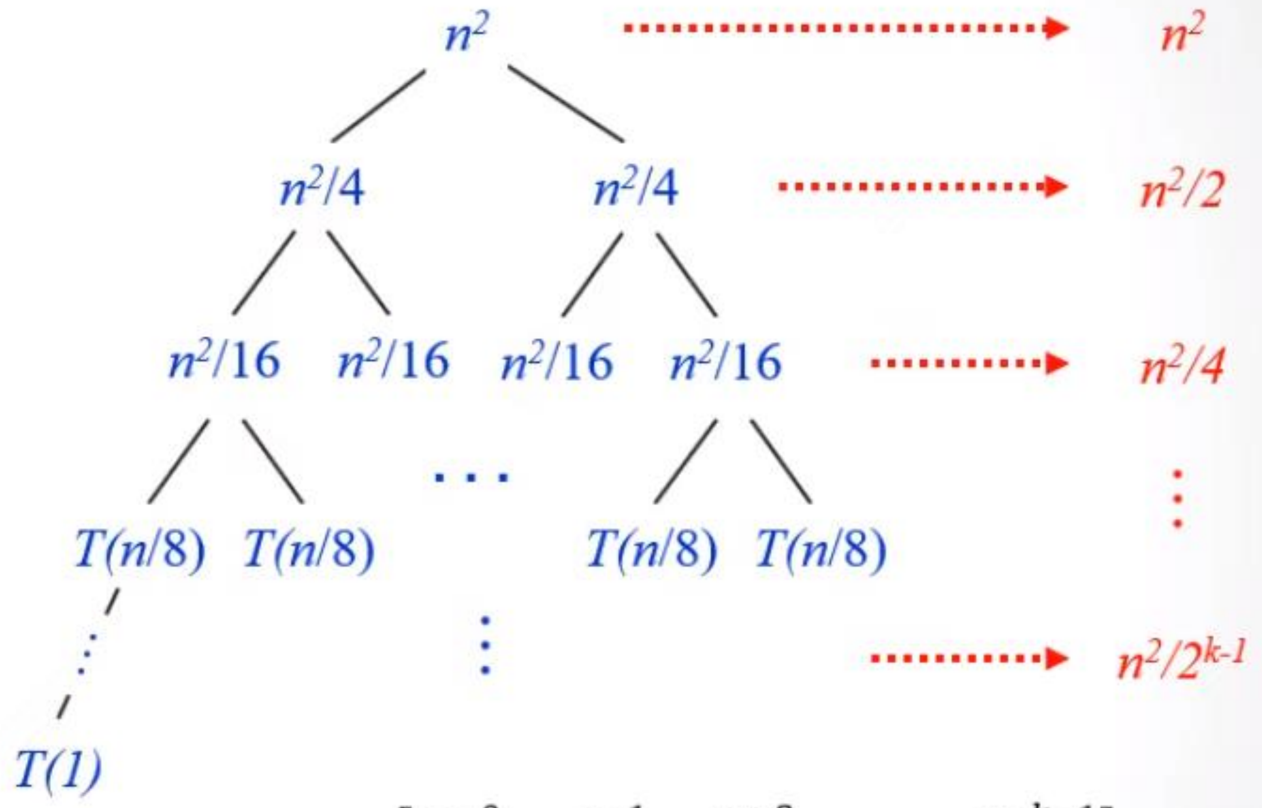

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$Hence: T(n) \in O(n^2)$

Recursion Tree Method: Example 3

$$T(n) = \begin{cases} 1 & n = 1 \\ 3\,T\left(\frac{n}{4}\right) + n^2 & n > 1 \end{cases}$$

Solve the following recurrence using the Recurrence Tree Method.

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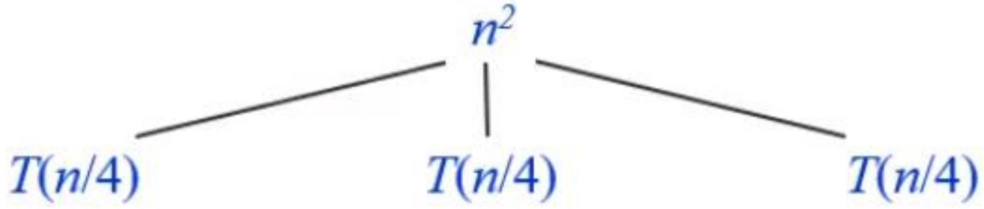

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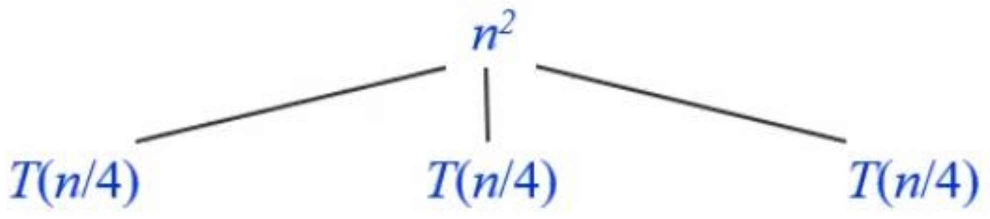

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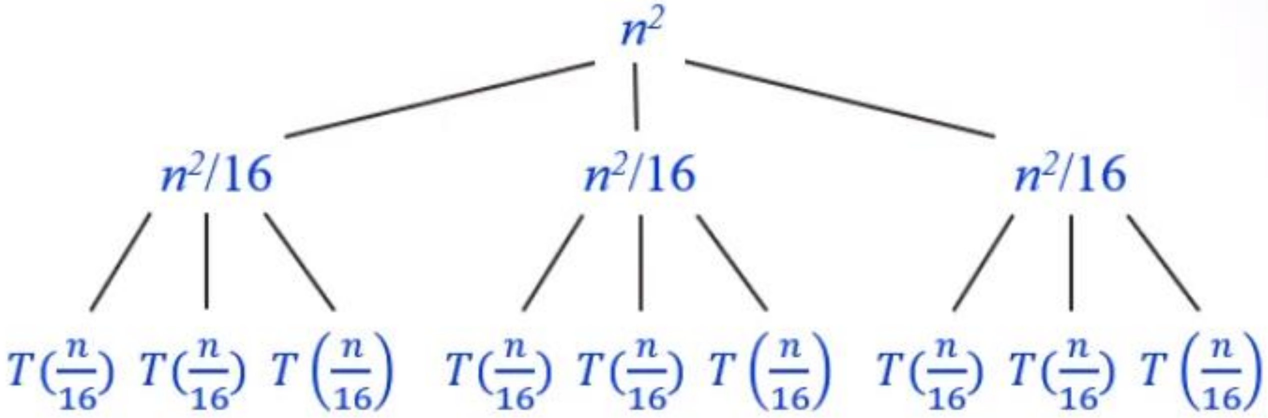

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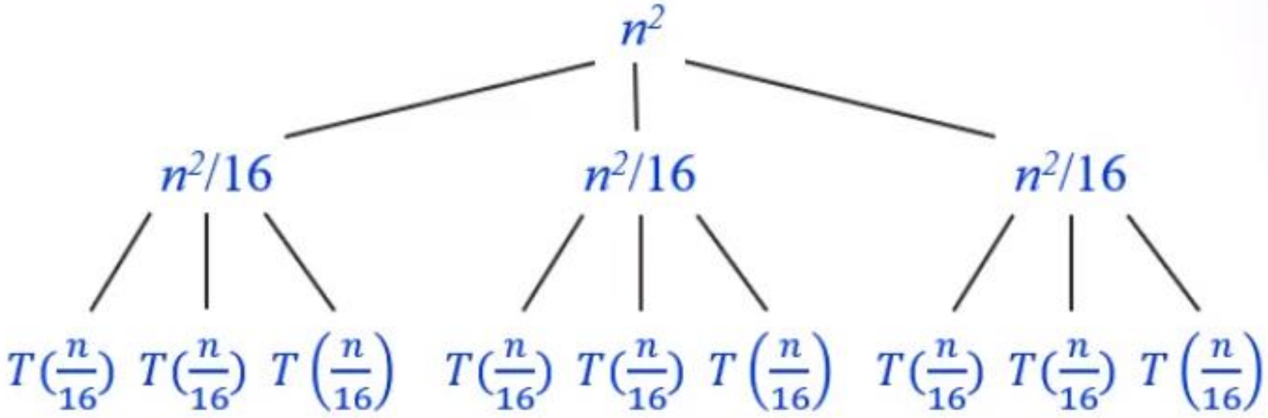

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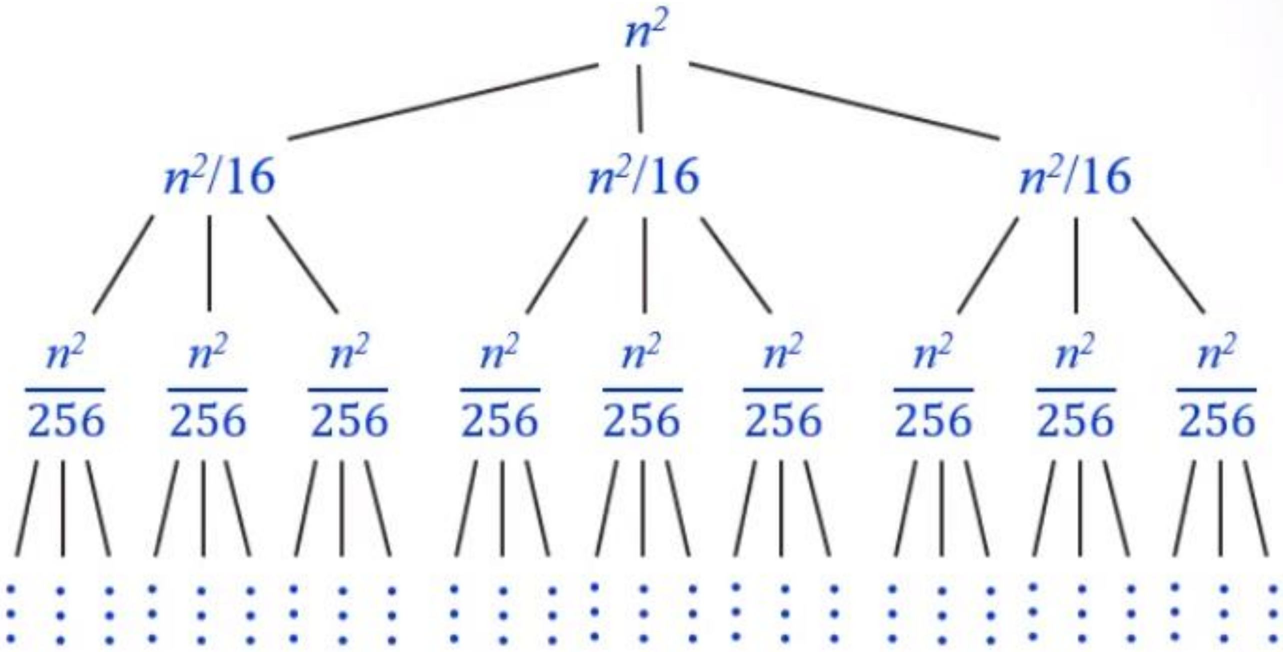

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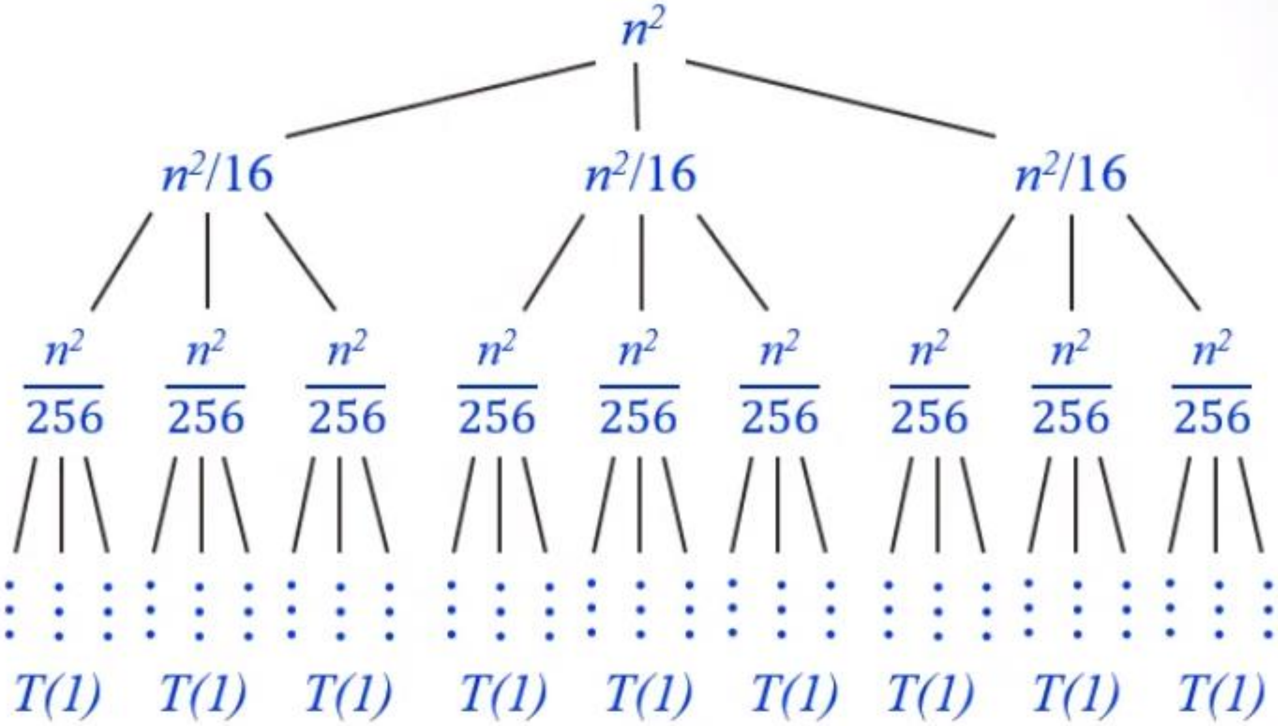

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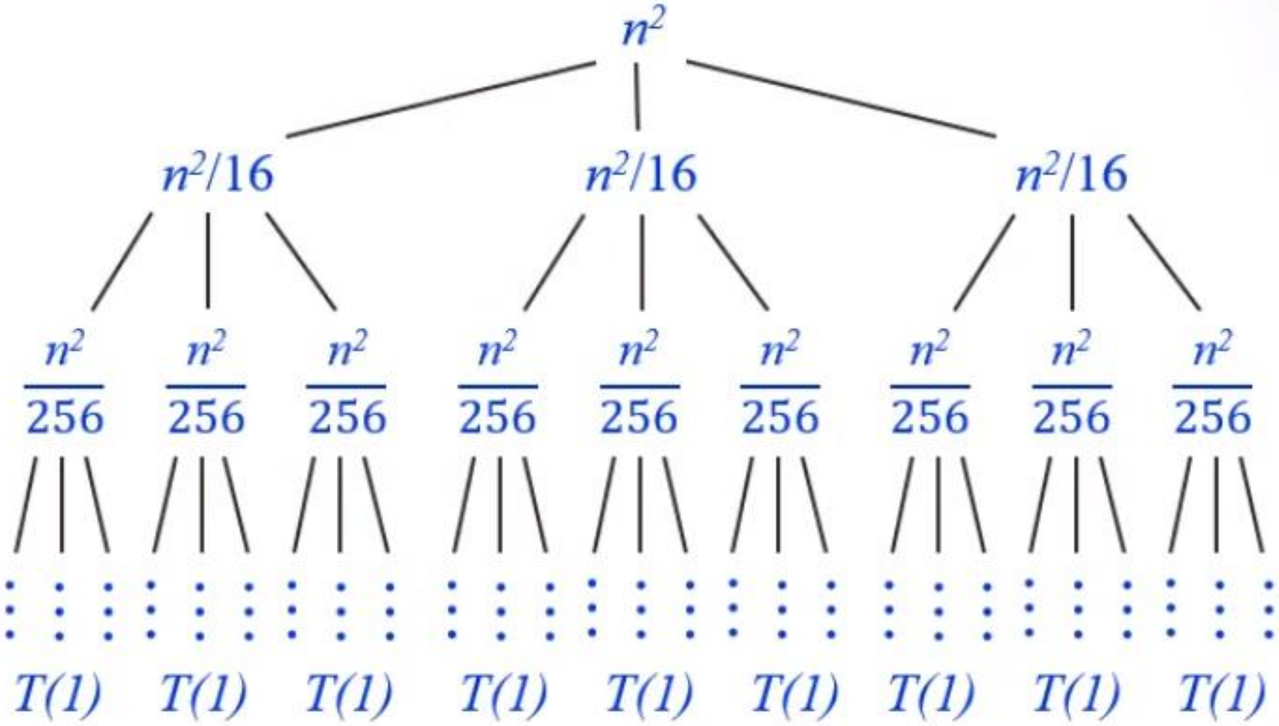

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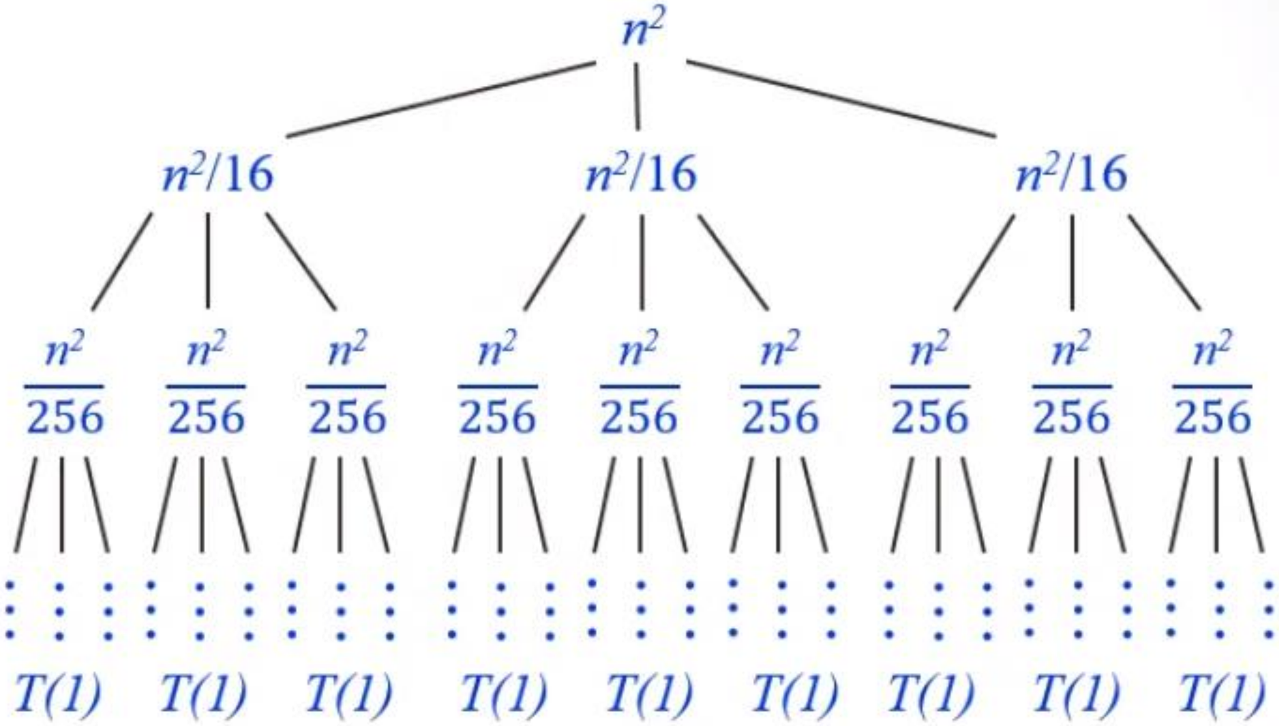

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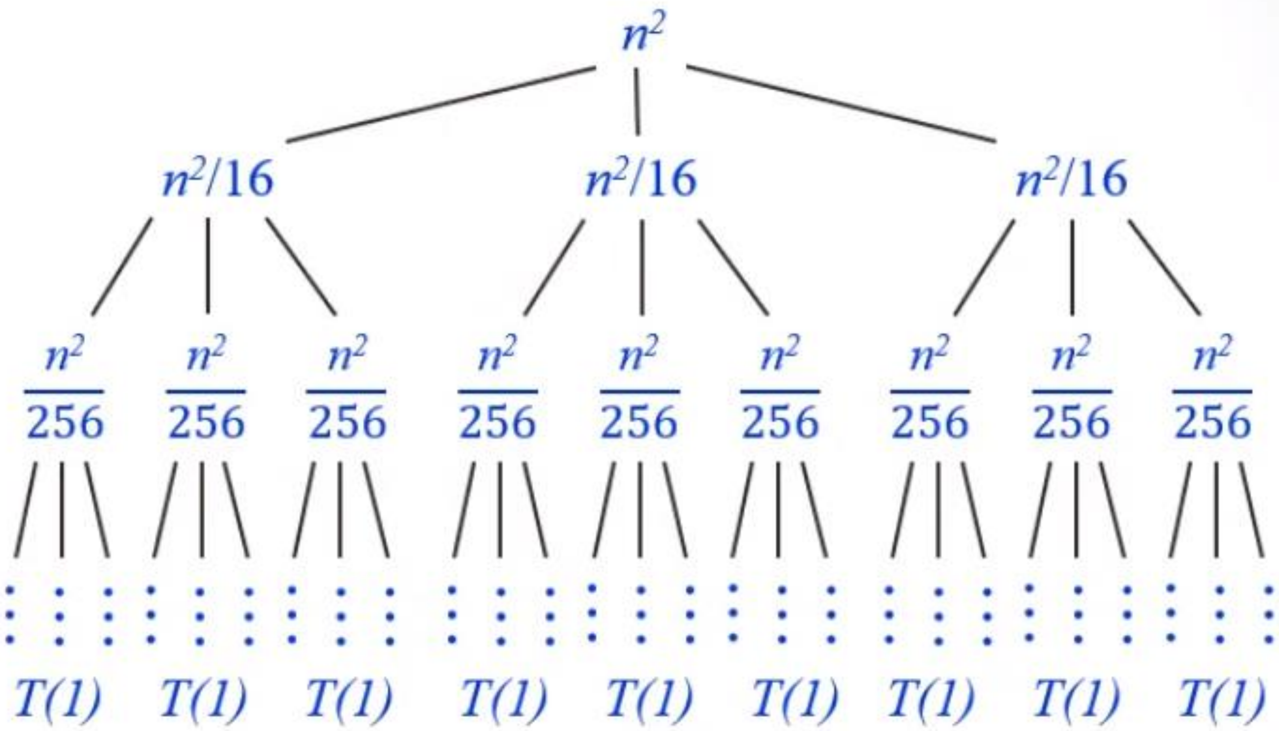

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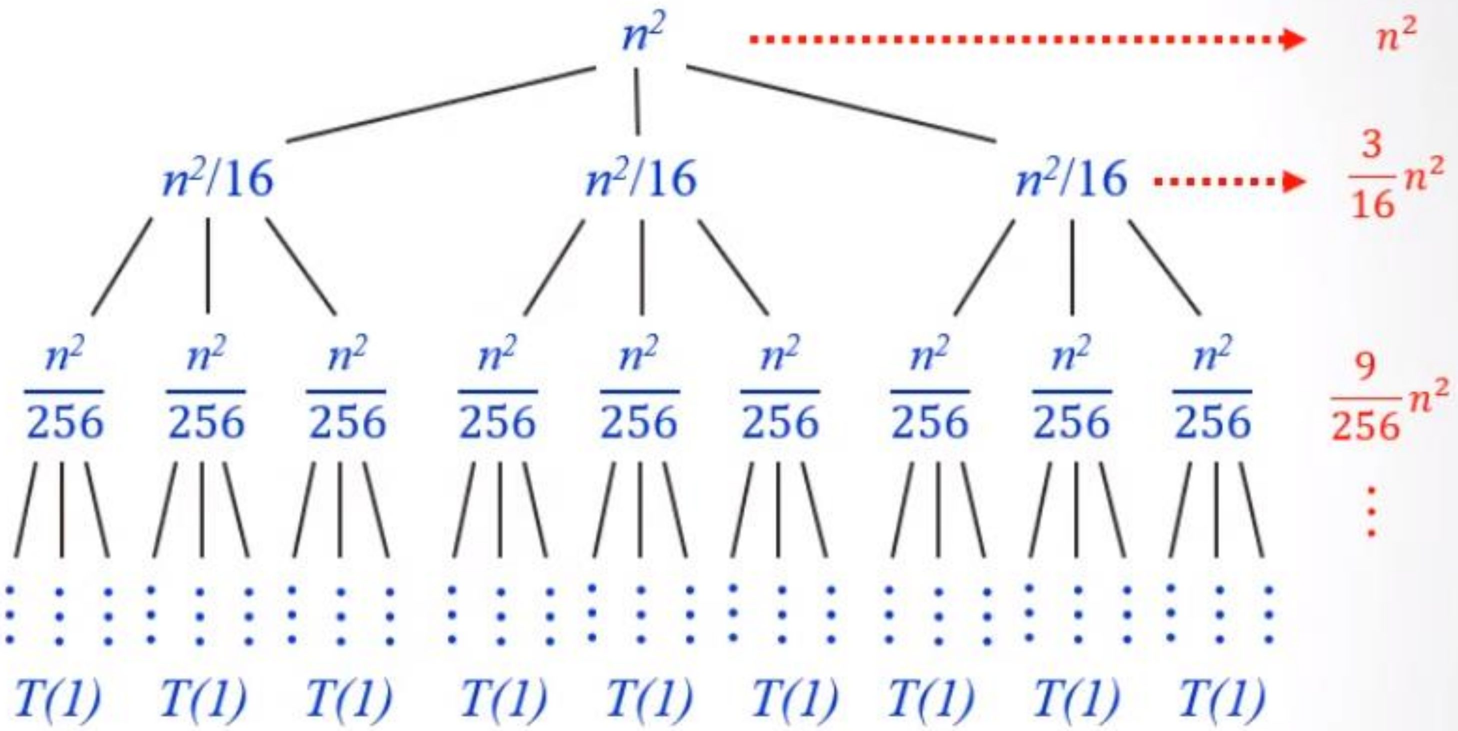

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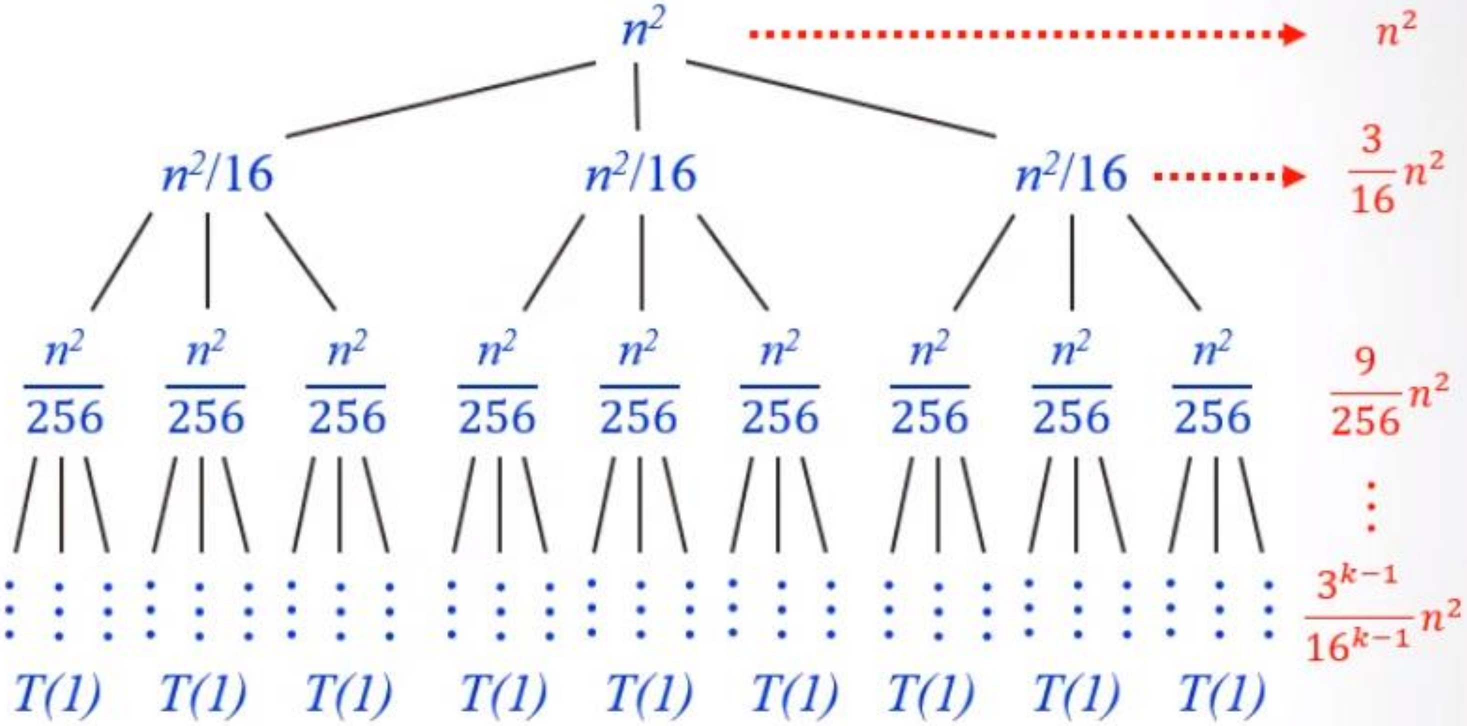

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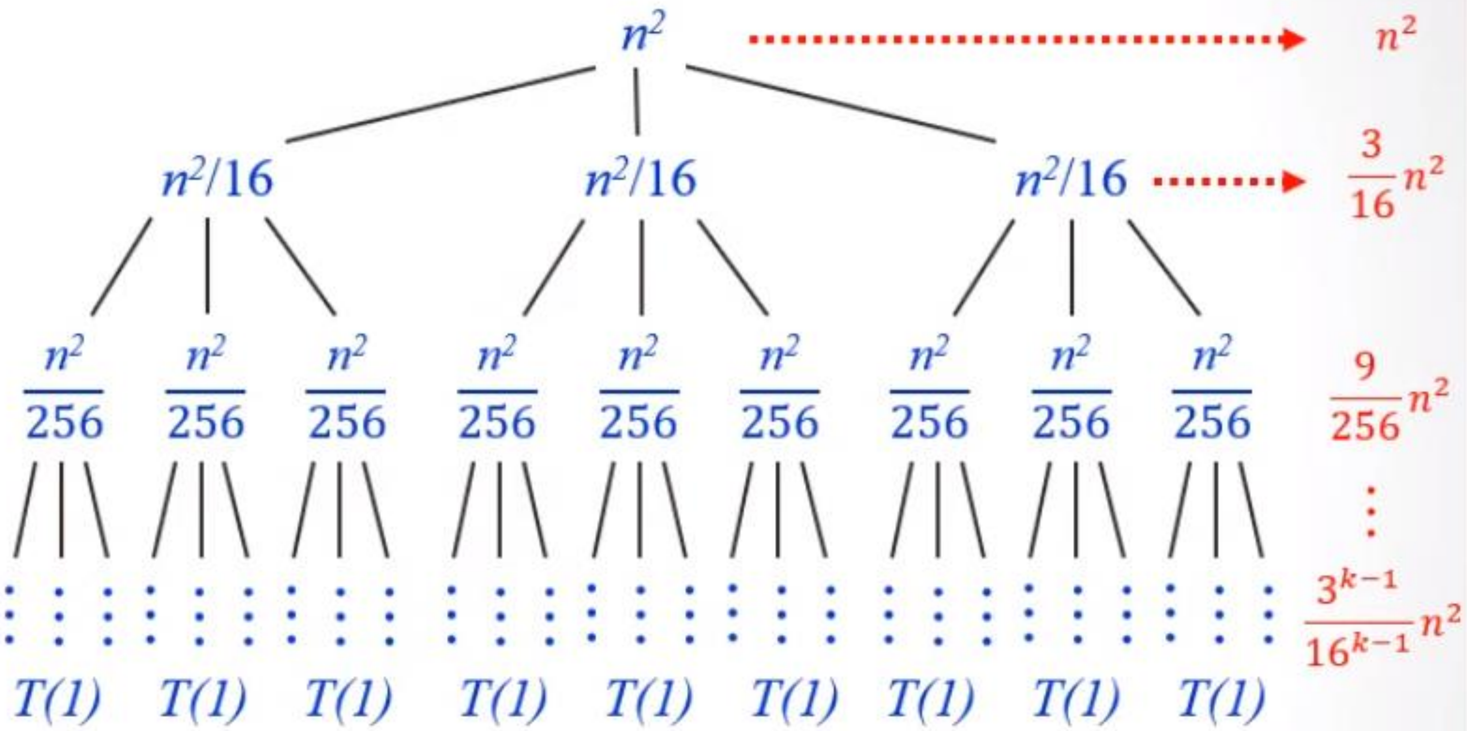

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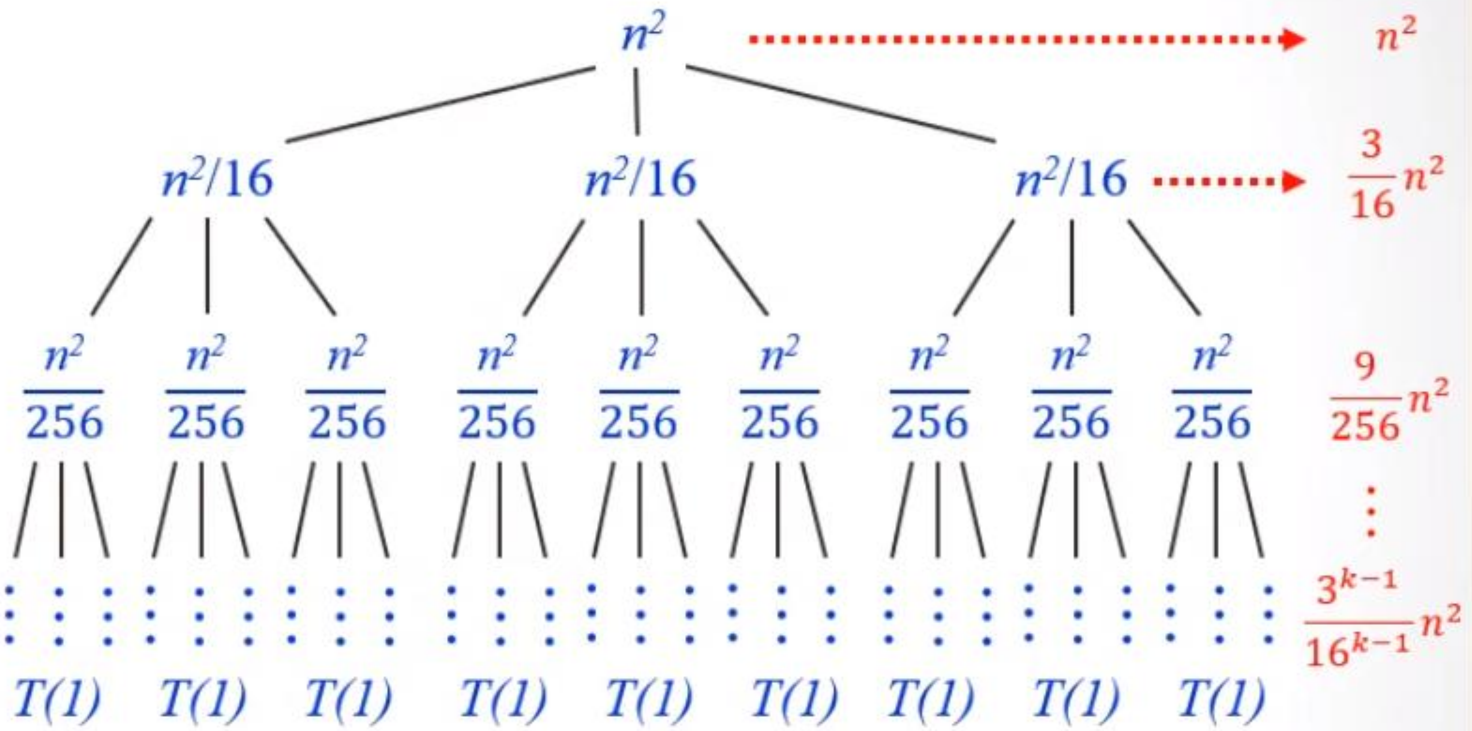

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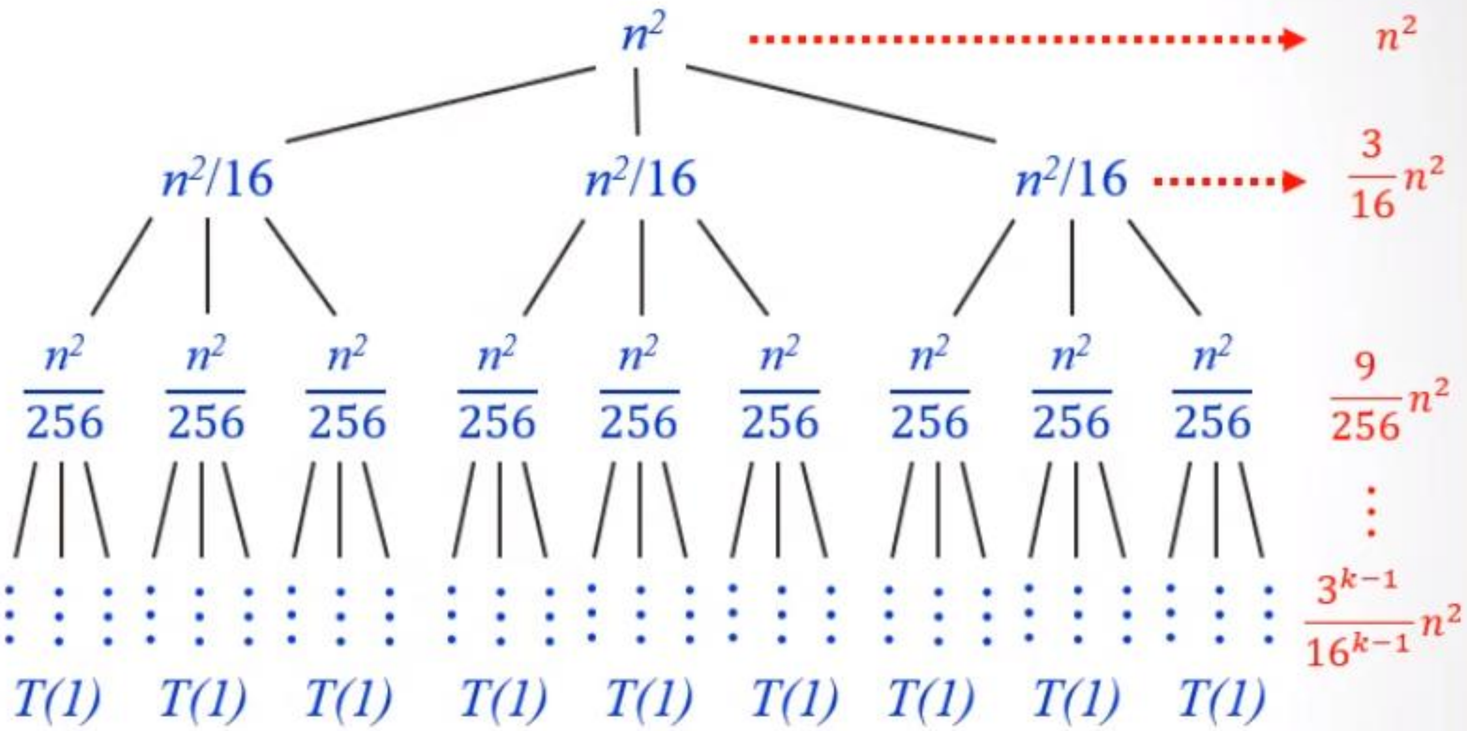

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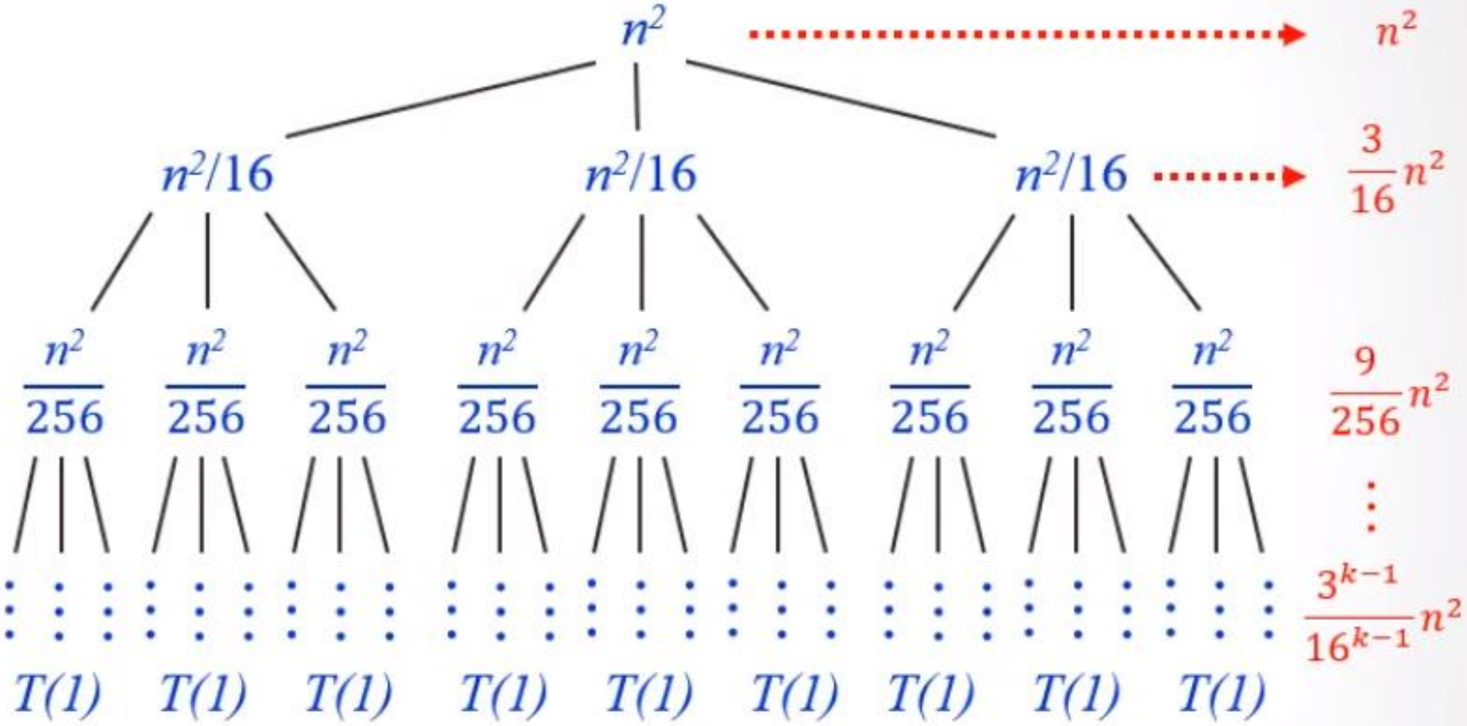
Recursion Tree Method: Example 3

$$T(n) = 3\,T\left(\frac{n}{4}\right) + n^2$$

$$T(\frac{n}{4}) = 3.T(\frac{n}{4^2}) + \frac{n^2}{4^2}$$

$$T(\frac{n}{4^2}) = 3.T(\frac{n}{4^3}) + \frac{n^2}{16^2}$$

$$T(\frac{n}{4^k}) = T(1)$$



$$n = 4^k$$

$$k = \log_4 n$$

$$L_c = 3^k \implies 3^{\log_4 n} \implies n^{\log_4 3}$$

$$I_c = n^2 \cdot \left[\left(\frac{3}{16}\right)^0 + \left(\frac{3}{16}\right)^1 + \left(\frac{3}{16}\right)^2 + \cdots + \left(\frac{3}{16}\right)^{k-1}\right]$$

$$I_c = n^2 \cdot \left[\frac{1}{1 - {}^{3}/_{16}}\right] \Rightarrow \frac{16}{13} n^2$$

Total Cost $= L_c + I_c \implies n^{\log_4 3} + \frac{16}{13} n^2$

$Hence: T(n) \in O(n^2)$

Recursion Tree Method: Example 4

$$T(n) = \begin{cases} 1 & n = 1 \\ T\left(\frac{n}{4}\right) + T\left(\frac{n}{2}\right) + n^2 & n > 1 \end{cases}$$

Solve the following recurrence using the Recurrence Tree Method.

Recursion Tree Method: Example 5

$$T(n) = \begin{cases} 1 & n = 1 \\ T\left(\frac{n}{3}\right) + T\left(\frac{2n}{3}\right) + n & n > 1 \end{cases}$$

Solve the following recurrence using the Recurrence Tree Method.