(REVISED COURSE) QP Code: NP-17762

(3 Hours)

[Total Marks: 80

N.B.: (1) Question No.1 is compulsory.

(2) Attempt any three question from remaining five questions.

- 1. (a) Evaluate $\int_{0}^{1} \sqrt{(\sqrt{x} x)} dx$ 3

 (b) Solve $[D^4 4D^3 + 8D^2 8D + 4]y = 0$ (c) Prove that $(1 + \Delta)(1 \nabla) = 1$ 3

 (d) Change to polar co-ordinate and evaluate $\int_{0}^{a} \int_{0}^{\sqrt{a^2 x^2}} (x^2 + y^2) dy dx$ 3

 (e) Solve $(x^2 4xy 2y^2) dx + (y^2 4xy 2x^2) dy = 0$ 4

 (f) Evaluate $\int_{0}^{a} \int_{0}^{\sqrt{1 + x^2}} \frac{1}{1 + x^2 + y^2} dy dx$ 4
- 2. (a) Solve $xy(1+xy^2)\frac{dy}{dx} = 1$
 - (b) Change the order of integration and evaluate $\int_{0}^{\infty} \int_{0}^{x} x e^{-x^2/y} dy dx$
 - (c) Evaluate $\int_{0}^{\pi/2} \frac{dx}{a^2 \sin^2 x + b^2 \cos^2 x}$ and show that

$$\int_{0}^{\pi/2} \frac{dx}{\left(a^{2} \sin^{2} x + b^{2} \cos^{2} x\right)^{2}} = \frac{\pi}{4ab} \left(\frac{1}{a^{2}} + \frac{1}{b^{2}}\right)$$

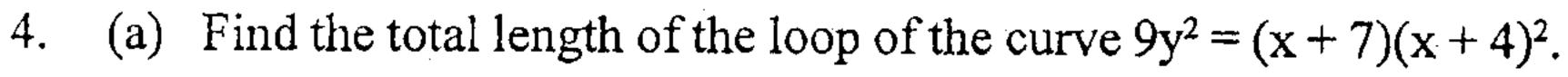
- 3. (a) Evaluate $\iiint x^2 yz \, dx \, dy \, dz$ throughout the volume bounded by x = 0, y = 0, z = 0, 6 x + y + z = 1.
 - (b) Find the area bounded by parabola $y^2 = 4x$ and the line y = 2x 4.
 - (c) Use the method of variation of parameter to solve

$$\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = e^{-2x} \sec^2 x (1 + 2\tan x)$$

[TURN OVER



2



6

(b) Solve
$$\frac{d^2y}{dx^2} + 2y = x^2e^{3x} + e^x - \cos 2x$$

6

- (c) Apply Runge-kutta method of fourth order to find an approximate value of y at x = 0.2 8

 if $\frac{dy}{dx} = x + y^2$ give that y = 1, when x = 0 in step of h = 0.1.
- 5. (a) Solve $y(x^2y + e^x) dx e^x dy = 0$.

6

- (b) Using Taylor's series method solve $\frac{dy}{dx} = 1 2xy$ given that y(0) = 0 and hence find y(0.2) and y(0.4).
- (c) Compute the value of the definite integral $\int_{0.2}^{1.4} (\sin x \log_e x + e^x) dx$, by

8

- (i) Trapezoidal Rule
- (ii) Simpson's one third Rule
- (iii) Simpson's three-eigth Rule.
- 6. (a) The motion of a particle is given by $\frac{d^2x}{dt^2} = -k^2x 2h\frac{dx}{dt}$, solve the equation when h = 5, k = 4 taking x = 0, $v = v_0$ at t = 0. Show that the time of maximum displacement is independent of the initial velocity.
 - (b) Evaluate $\iint (x^2 + y^2) dx$ dy over the area of triangle whose vertices are (0, 0), (1, 0), (1, 2).
 - (c) Find the volume bounded by $y^2 = x$, $x^2 = y$ and the planes z = 0 and x + y + z = 1.