22/5/14

SE/EXTC/III CBGS
A.M. III

QP Code : **NP-18646**

(3 Hours) [Total Marks : 80

N. B. : (1) Question No. **1** (one) is **compulsory**.
(2) Attempt any **3** (three) questions from the remaining questions.
(3) Assume suitable data, if necessary.

1. (a) Evaluate $\int_0^\infty \frac{(\cos 6t - \cos 4t)}{t}\,dt$ — 5

(b) Obtain complex form of fourier series for $f(x) = e^{ax}$ in $(-1,1)$ — 5

(c) Find the work done in moving a particle in a force field given by $\overline{F} = 3xy\,\hat{i} - 5z\hat{j} + 10x\hat{k}$ along the curve $x = t^2 + 1$, $y = 2t^2$, $z = t^3$ from $t = 1$ to $t = 2$. — 5

(d) Find the orthogonal trajectory of the curves $3x^2y + 2x^2 - y^3 - 2y^2 = \alpha$, where α is a constant. — 5

2. (a) Evaluate $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} - 3y = \sin t$, $y(o) = 0$, $y'(o) = 0$, by Laplace transform — 6

(b) Show that $J_{5/2} = \sqrt{\frac{2}{\pi x}}\left[\frac{3 - x^2}{x^2}\sin x - \frac{3}{x}\cos x\right]$ — 6

(c) (i) Find the constants a, b, c so that $\overline{F} = (x + 2y + az)\hat{i} + (bx - 3y - z)\hat{j} + (4x + (y + 2z)\hat{k}$ is irrotational. — 4

(ii) Prove that the angle between two surfaces $x^2 + y^2 + z^2 = 9$ and $x^2 + y^2 - z = 3$ at the point $(2,-1,2)$ is $\cos^{-1}\left(\frac{8}{3\sqrt{21}}\right)$ — 4

3. (a) Obtain the fourier series of f(x) given by — 6

$$f(x) = \begin{cases} 0 & , -\pi \le x \le 0 \\ x^2 & 0 \le x \le \pi \end{cases}$$

(b) Find the analylic function $f(z) = u + iv$ where $u = r^2\cos 2\theta - r\cos\theta + 2$ — 6

(c) Find Laplace transform of — 8

(i) $te^{-3t}\cos 2t.\cos 3t$

(ii) $\frac{d}{dt}\left[\frac{\sin 3t}{t}\right]$

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4. (a) Evaluate $\int J_3(x)dx$ and Express the result in terms of J_0 and J_1 6

(b) Find half range sine series for 6

$f(x) = \pi x - x^2$ in $(0, \pi)$

Hence deduce that $\frac{\pi^3}{32} = \frac{1}{1^2} - \frac{1}{3^2} + \frac{1}{5^2} - \frac{1}{7^2} + \ldots$

(c) Find inverse Laplace transform of 8

(i) $\frac{1}{s}\tanh^{-1}(s)$ (ii) $\frac{se^{-2s}}{(s^2+2s+2)}$

5. (a) Under the transformation $w + 2i = z + \frac{1}{z}$, show that the map of the circle $|z| = 2$ is an ellipse in w-plane. 6

(b) Find half range cosine series of $f(x) = \sin x$ in $0 \le x \le \pi$. 6

Hence deduce that

$\frac{1}{1.3} + \frac{1}{3.5} + \frac{1}{5.7} + \ldots = \frac{1}{2}$

(c) Verify Green's theorem, for 8

$\oint_c (3x^2 - 8y^2)\, dx + (4y - 6xy)\, dy$ where c is boundary of the region defined by $x=0$, $y=0$, and $x+y = 1$.

6. (a) Using convolution theorem; evaluate 6

$L^{-1}\left\{\frac{1}{(s-1)(s^2+4)}\right\}$

(b) Find the bilinear transformation which maps the points 6

$z = 1, i, -1$ onto $w = 0, 1, \infty$

(c) By using the appropriate theorem, Evaluate the following :– 8

(i) $\int \overline{F} \cdot d\overline{r}$ where $\overline{F} = (2x - y)\hat{i} - (yz^2)\hat{j} - (y^2 z)\hat{k}$

and c is the boundary of the upper half of the sphere $x^2 + y^2 + z^2 = 4$

(ii) $\iint_s (9x\hat{i} + 6y\hat{j} - 10z\hat{k}) \cdot d\overline{s}$ where s is the surface of sphere with radius 2 units.
