Sem IV / App. Maths-IV / MECH & PROD

19-05-14

Q P Code : **NP-19673**

(3 Hours) [Total Marks : 80

N.B. : (1) Question **No. 1** is **compulsory.**
(2) **Solve** any **three** questions out of **remaining.**
(3) **Each** question carries **equal** marks.
(4) **Use** of **statistical tables** is **allowed.**

1. (a) Find a, b, c if $\bar{F} = (axy + bz^3)\bar{i} + (3x^2 - cz)\bar{j} + (3xz^2 - y)\bar{k}$ is irrotational. 5

(b) Find $A^5 - 4A^4 - 7A^3 + 11A^2 - A - 10\,I$ interms of A using Cayley-Hamilton theorem for $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$ 5

(c) A continuous random variable X has the p.d.f. defined by $f(x) = A + Bx,\ 0 \le x \le 1$. if the mean of the distribution is $\frac{1}{3}$ find A and B. 5

(d) A sample of 50 pieces of certain type of string was tested. The mean breaking strength turned out to be 14.5 pounds. Test whether the sample is from a batch of a string having a mean breaking strength of 15.6 pound and S.D. of 2.2 pounds. 5

2. (a) Obtain the rank correlation coefficient from the follwoing data :— 6

X	10	12	18	18	15	40
Y	12	18	25	25	50	25

(b) The marks of 1000 students of university are found to be normally distrubuted with mean 70 & SD 5. Estimate the number of students whose marks will be (i) between 60 & 75 (ii) more than 75. 6

(c) Show that the matrix $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$ is diagonalisable. 8

Find the diagonal form and transforming matrix.

3. (a) A certain injection administered to 12 patients resulted in the following changes of blood pressure :
5, 2, 8, -1, 3, 0, 6, -2, 1, 5, 0, 4. Can it be concluded that the injection will be in general accompanied by an increase in blood pressure? 6

(b) Optimize $Z = x_1^2 + x_2^2 + x_3^2 - 6x_1 - 8x_2 - 10x_3$ 6

(c) Verify Green's theorem in the plane for 8
$\oint (x^2 - y)dx + (2y^2 + x)dy$
around the boundary of the region defined by $y = x^2$ and $y = 4$.

4. (a) A car hire firm has two cars which it hires out day by day. The number of demands for a car on each day is distributed as poisson variate with mean 1.5. Calculate the proportion of days on which (i) neither car is used (ii) some demand is refused. 6

(b) Evaluate $\iint_S (\nabla \times \bar{F}) \cdot d\bar{s}$ where 6

$\bar{F} = (2x - y + z))i + (x + y - z^2)j + (3x - 2y + 4z)k$ and S is the surface of the cylinder $x^2 + y^2 = 4$ bounded by the plane Z =9 and open at the other end.

(c) Table below shows the performances of students in Mathematics and Physics. Test the hypothesis that the performance in Mathematics is independent of performance in physics. 8

Grades in Physics	Grades in Maths		
	High	Medium	Low
High	56	71	12
Medium	47	163	38
Low	14	42	81

5. (a) The ratio of the probability of 3 successes in 5 independent trials to the probability of 2 successes in 5 independent trials is ¼. What is the probability of 4 successes in 6 independent trials? 6

(b) Evaluate $\iint_S \bar{F}.d\bar{s}$ where $\bar{F} = 4xi - 2y^2j + z^2k$ and S is the region bounded by 6

$y^2 = 4x$, $x = 1$, $z = 0$, $z = 3$.

(c) Find (i) the lines of regression (ii) coefficient of correlation for the following data. 8

X	65, 66, 67, 67, 68, 69, 70, 72
Y	67, 68, 65, 66, 72, 72, 69, 71

6. (a) A group of 10 rats fed on diet A and another group of 8 rats fed on different diet B, recorded the follwoing increase in weight 6

Diet A : 5, 6, 8, 1, 12, 4, 3, 9, 6, 10gms

Diet B : 2, 3, 6, 8, 1, 10, 2, 8 gms

Find if the variances are significantly different?

(b) If $A = \begin{bmatrix} -1 & 4 \\ 2 & 1 \end{bmatrix}$ then prove that 3 tan A = A tan 3. 6

(c) Using the kuhn-Tucker conditions solve the follwoing N.L.P.P. 8

Maximize $Z = -x_1^2 - x_2^2 - x_3^2 + 4x_1 + 6x_2$

Subject to $x_1 + x_2 \le 2$

$2x_1 + 3x_2 \le 12$

$x_1, x_2 \ge 0$
