

(3 hours)

Total Marks: 80

N.B.

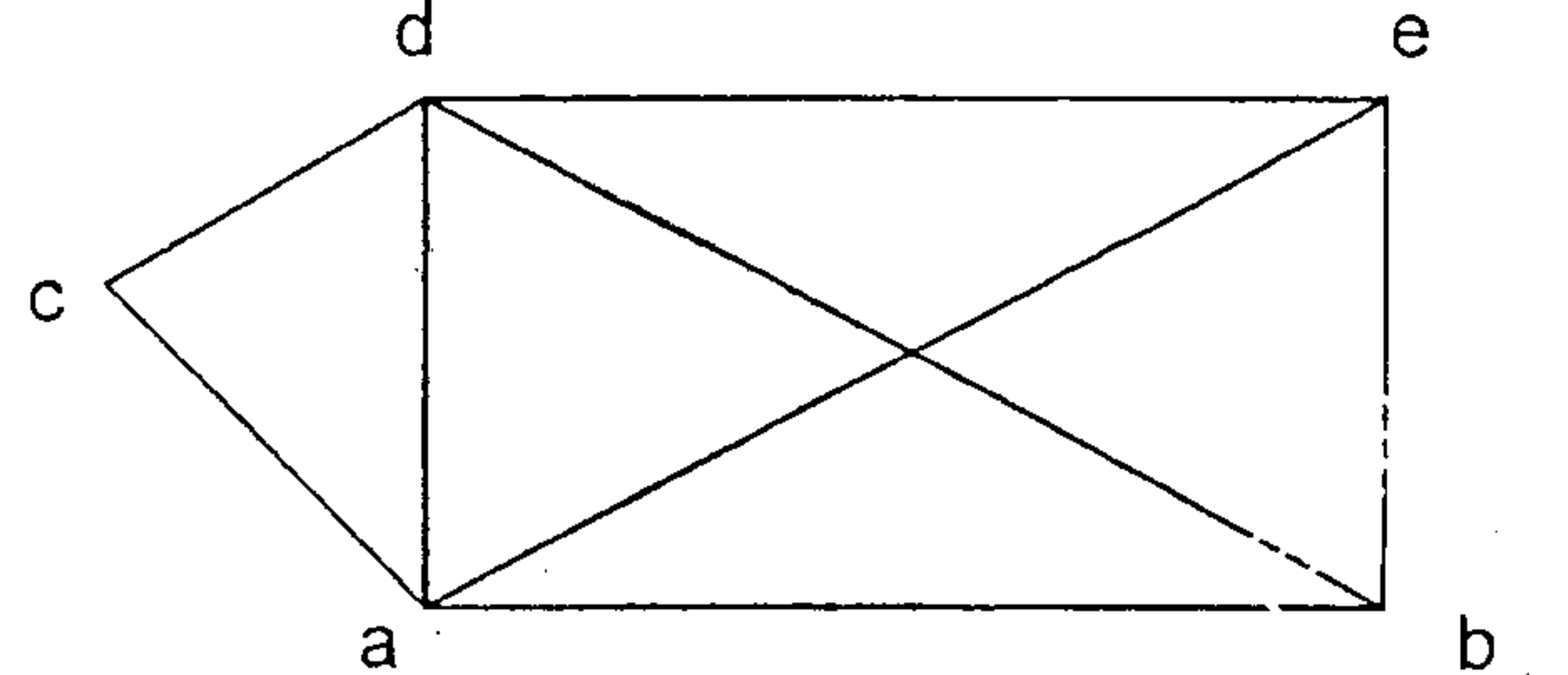
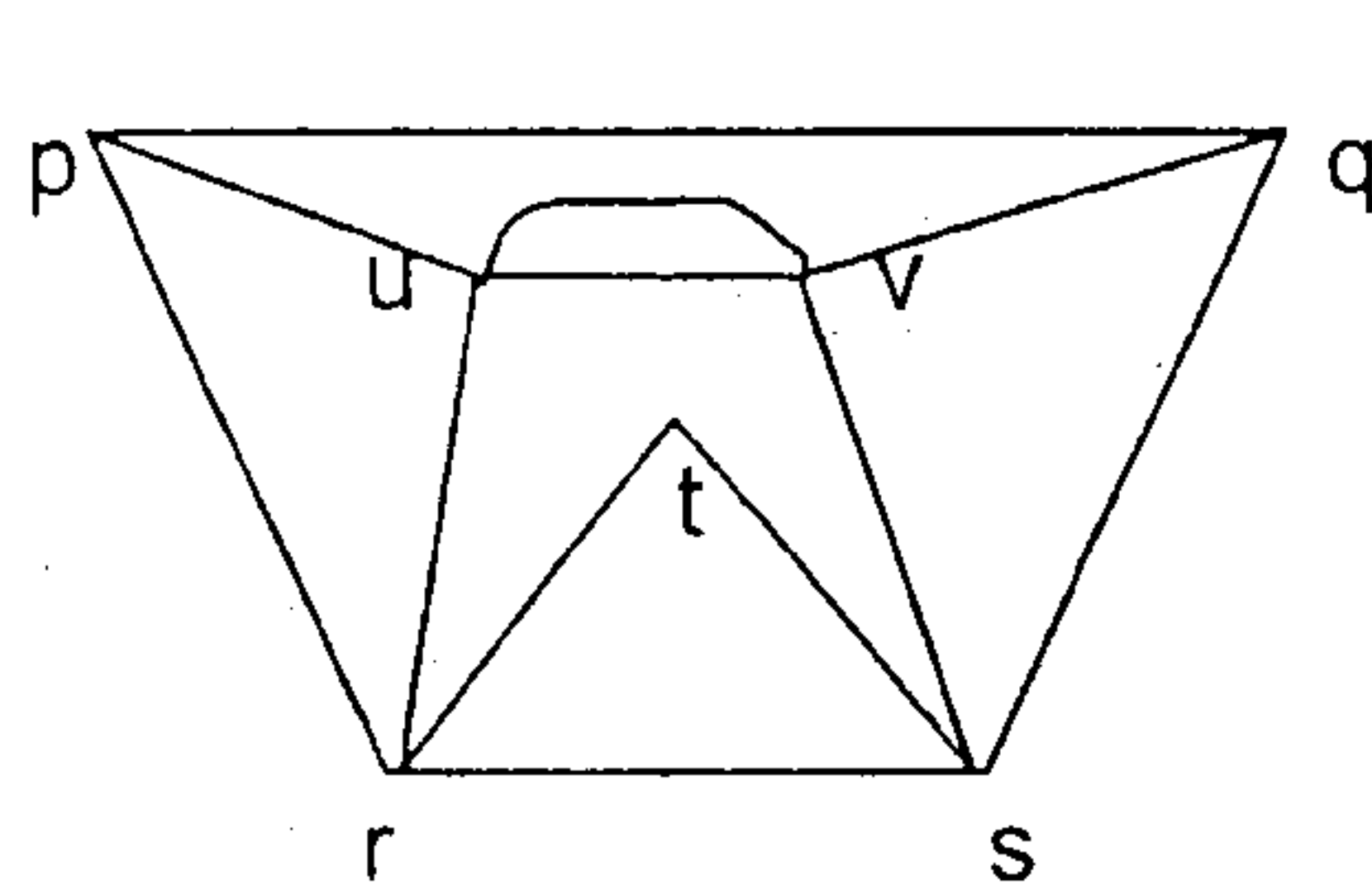
1. Question No 1 is compulsory
2. Solve any three question out of remaining five questions
3. Assumption made should be clearly stated
4. Figure to the right indicates full marks

- 1 (a) Prove that  $8^n - 3^n$  is a multiple of 5 by mathematical induction,  $n \geq 1$  5
  - (b) Show that if a relation on set A is transitive and irreflexive, then it is asymmetric. 5
  - (c) Function  $f(x) = (4x + 3)/(5x - 2)$ . Find  $f^{-1}$  5
  - (d) What is the total number of vertices in a full binary tree with 20 leaves? 5
  - 2 (a) Let  $f(x) = x + 2$ ,  $g(x) = x - 2$  and  $h(x) = 3x$  for all  $x \in R$ . ( $R$  is the set of real number). Find i)  $f \circ g \circ h$  ii)  $h \circ g \circ f$  iii)  $f \circ f \circ f$  8
  - (b) Let  $R$  be a relation on the set of integers  $Z$  defined by  $aRb$  if and only if  $a \equiv b \pmod{5}$ . Prove that  $R$  is an equivalence relation. Find  $Z/R$ . 8
  - (c) Show that  $A \times (B \cap C) = (A \times B) \cap (A \times C)$  4
  - 3 (a) Let  $A = \{1, 2, 3, 4\}$  and  $R = \{(1,2), (2,3), (3,4), (2,1)\}$ . Find the transitive closure using Warshall's algorithm. 6
  - (b) Consider the lattices  $L1 = \{1, 2, 4\}$ ,  $L2 = \{1, 3, 9\}$  under divisibility. Draw the lattice  $L1 \times L2$ . 7
  - (c) Solve the recurrence relation  $a_n = -3(a_{n-1} + a_{n-2}) - a_{n-3}$  with  $a_0 = 5$ ,  $a_1 = -9$  and  $a_2 = 15$  7
  - 4 (a) Show that a group  $G$  is abelian if and only if  $(ab)^2 = a^2b^2$  for all  $a, b \in G$  6
  - (b) Prove that the set  $G = \{1, 2, 3, 4, 5, 6\}$  is an abelian group under multiplication modulo 7. 6
  - (c) Find the generating function for the following series 8
    - i)  $\{0, 1, 2, 3, 4, \dots\}$
    - ii)  $\{1, 2, 3, 4, 5, \dots\}$
    - iii)  $\{2, 2, 2, 2, 2, \dots\}$
    - iv)  $\{0, 0, 0, 1, 1, 1, 1, \dots\}$
  - 5 (a) Let  $H = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$  be parity check matrix. 8
- Decode the following words relative to maximum likelihood decoding function.
- i) 011001
  - ii) 101011
  - iii) 111010
  - iv) 110110

[TURN OVER]

- (b) Determine the Eulerian and Hamiltonian path, if exists, in the following graphs:

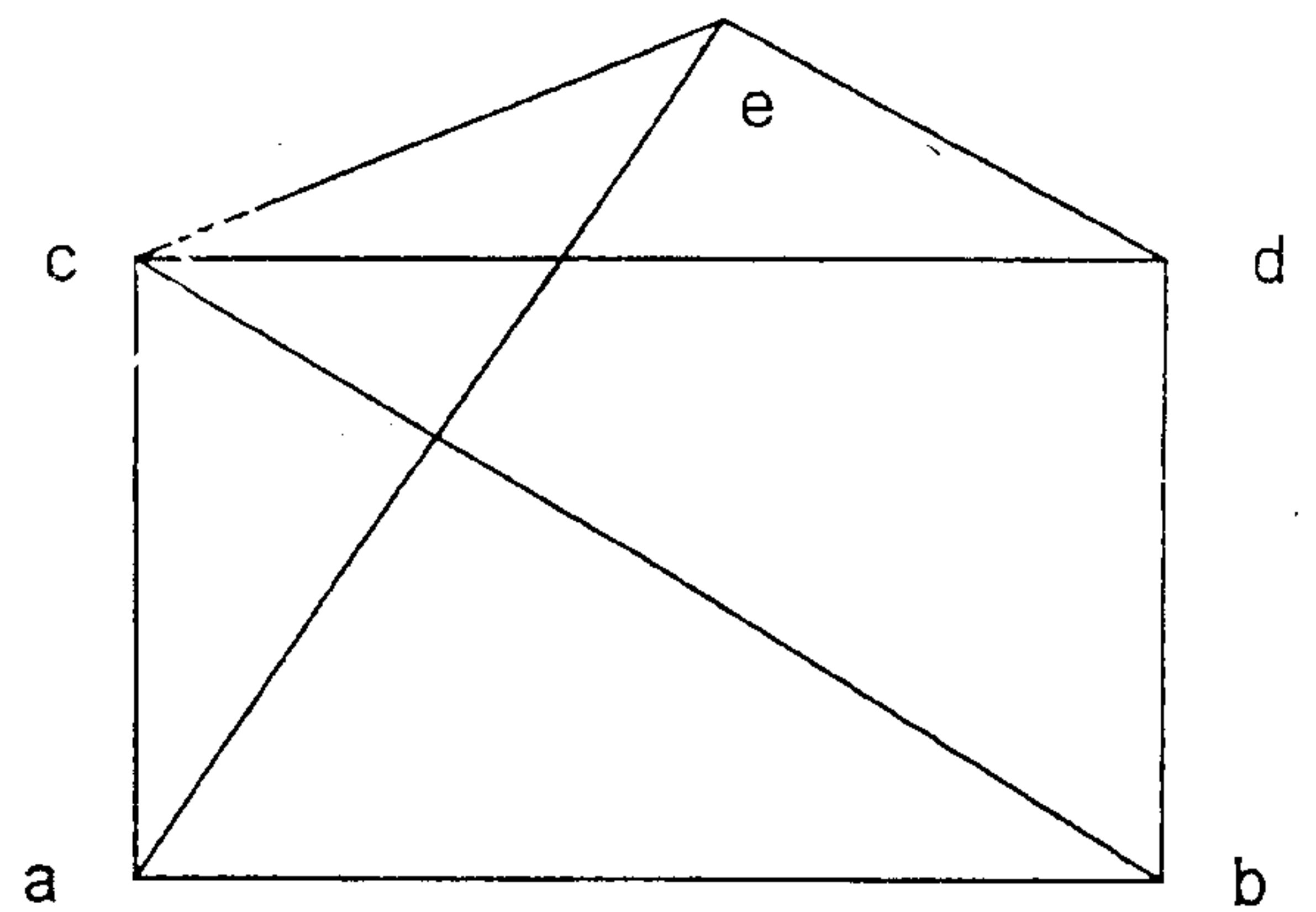
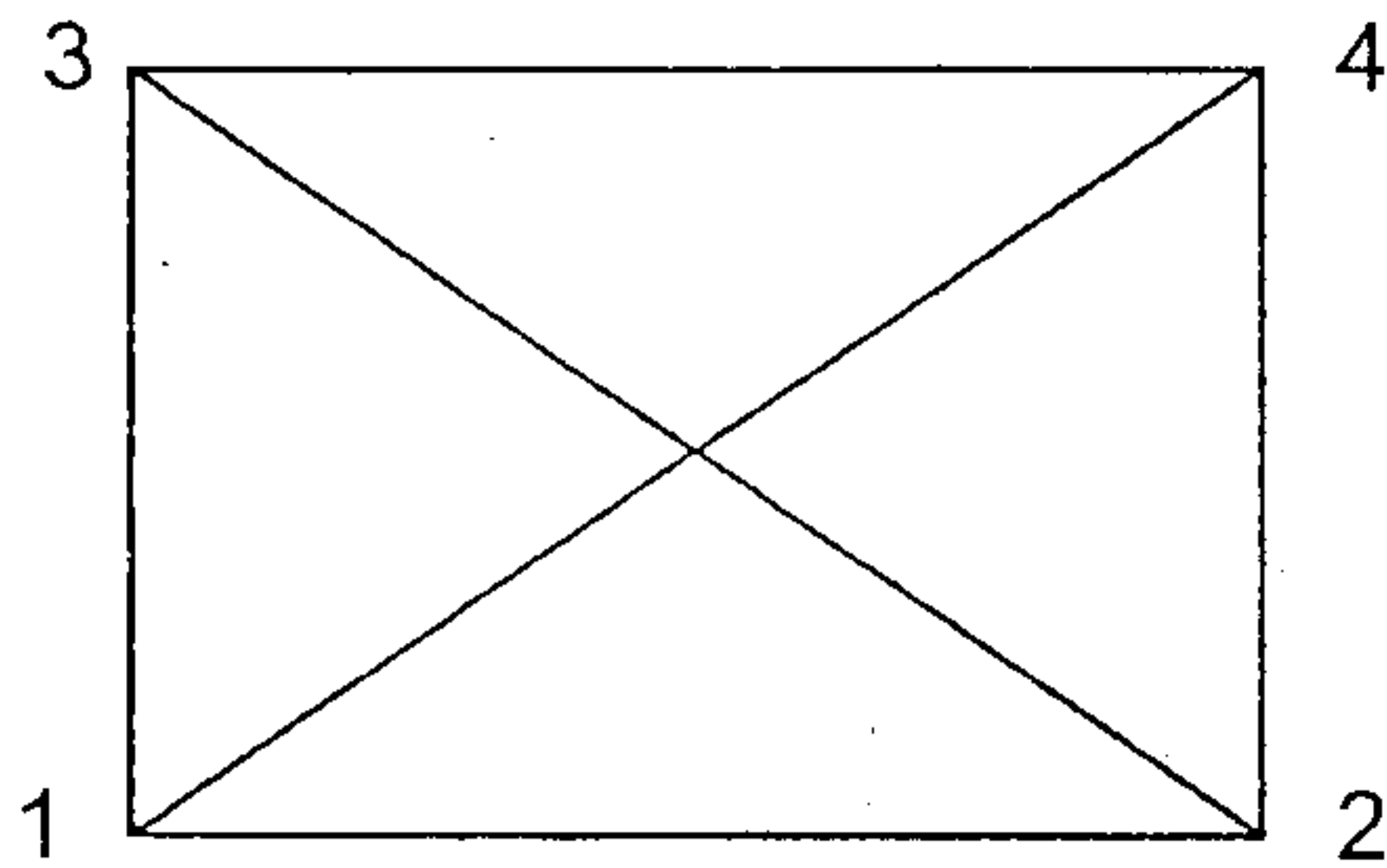
6



- (c) Let  $G$  be the set of real numbers and let  $a * b = ab/2$ . Show that  $(G, *)$  is an abelian group.

6

6 (a)



8

- (b) Use the laws of logic to determine the following expression as tautology or contradiction.

6

$$[p \wedge (p \Rightarrow q)] \Rightarrow q$$

- (c) Draw the Hasse Diagram of the following:

6

a)  $D_{105}$ b)  $D_{72}$