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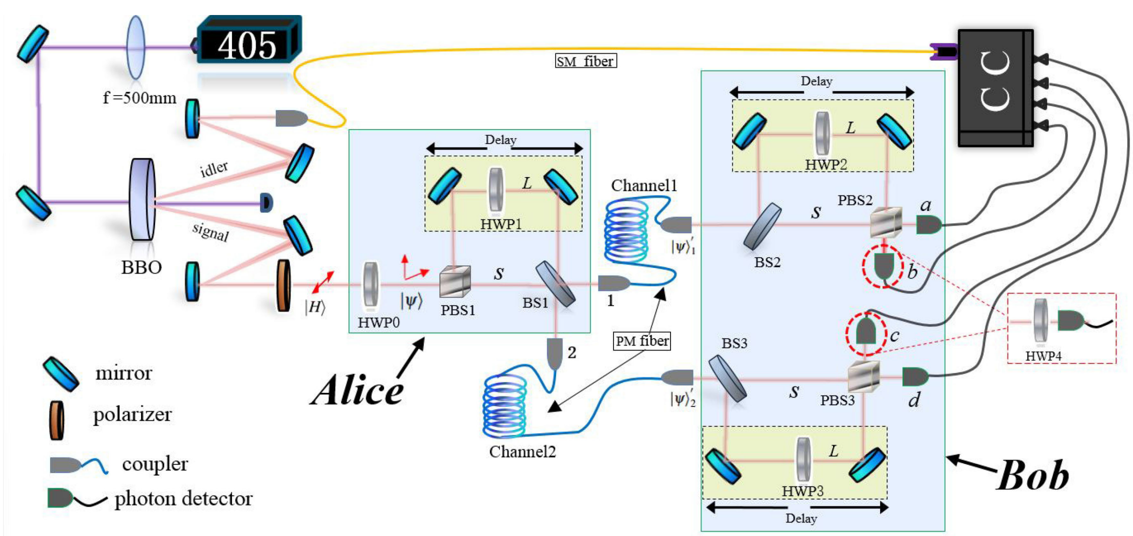

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Faithful Transmission of Single-Photon Qubits Using Error-Rejection Coding

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Abstract: Here, we experimentally demonstrate a faithful single-photon qubits transmission using error-rejecting coding, resorting to neither ancillary photons nor entangled photon resources. Using unbalanced polarization interferometer, the polarization qubits are transferred into the state on time-bin degree of freedom. Additionally, the deterministic transmission of single photon quantum states can be achieved with high fidelity. The immunity of time-bin encoded qubits against collective noise paves the way to practical quantum error-correction and error-rejection in quantum information science.

Index Terms: Faithful transmission, error-rejection coding, single-photon.

1. Introduction

Photons as flying qubits are widely studied in quantum information science, such as quantum key distribution(QKD) [1]–[9], quantum teleportation [10]–[13], and quantum secure direct communications [14]–[21], by the merit that photon possesses significant advantages like high information capacity [22]–[24], fast response rates [25], [26], and robustness against decoherence in free space [27]–[29]. In the context of optical fiber based schemes [30], [31], however, coherence of photon polarization will suffer from environmental noise due to the mode dispersion, which leads to uncertain transformation of polarization states of photons. Such noise, referred usually to as collective noise, oftentimes limits the reliability of quantum communication in practice. Much efforts insofar has been put on developing error-correction and error-rejection schemes to overcome such difficulties. For instance, decoherence-free subspace can be constructed in error-rejection as proposed by Wanlton *et al.* [32]. Resorting to ancillary qubits, errors could be corrected by using quantum redundancy code. More recently, Kalamidas [33] presented two single-photon schemes for quantum error-rejection and error-correction with linear optics, but the success probability is only 0.5 for

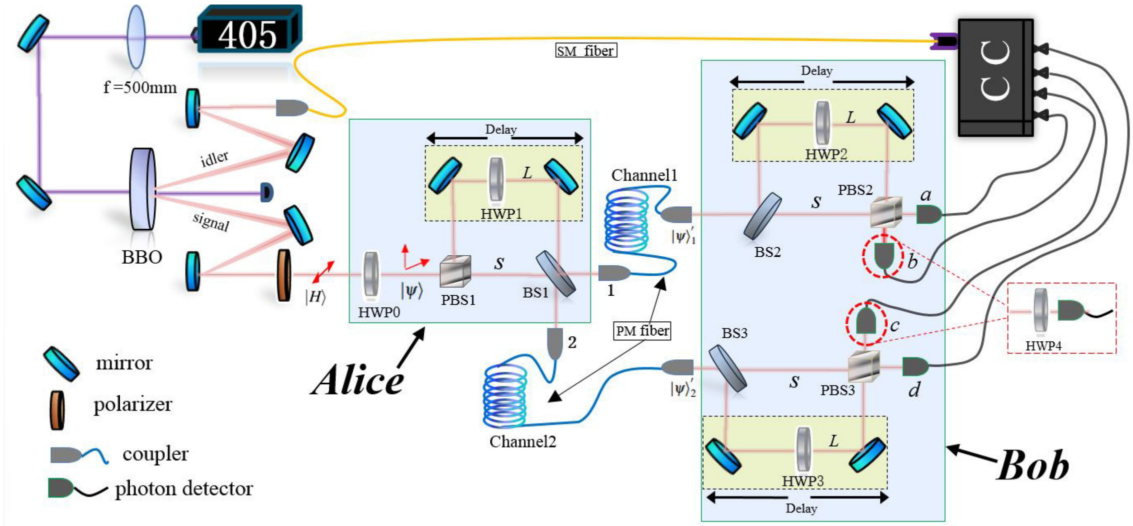

Fig. 1. **Experimental setup for single-photon qubit quantum error rejection**. The initial state $|\psi\rangle$ of the qubit is prepared by selecting one of the entangled photon pair generated via type-I parametric down conversion process in BBO crystal and with the help of the HWP0. Alice encodes the initial qubit using the unbalanced polarization interferometer into two time bins, and sends it to Bob's side through noisy channels, where Bob is equipped with the setup similar to Alice and selects the uncorrupted state at definite time slots with an additional HWP4. Two polarization-maintaining optical fibers with lossy fiber couplers are employed as quantum noise channels. Key to components: PBS, polarizing beam splitter; HWP, half-wave plate; BS, 50:50 beam splitter; BBO, β-BaB_2O_4 crystal; CC, coincidence counting module.

the first scheme, and it depends on these noise parameters. Although Han *et al.* [34] subsequently showed that these two schemes are suitable as well for mixed states, it was not easy to realize them because of the technologies required being not easily available.

On the other hand, the time-bin encoded qubits are much better immune to this decoherence mechanism, which facilitates their applications in qubits transmission, especially for long-distance quantum communication and fundamental proof-of-principle experiments. In 2007, Li *et al.* [35] proposed in theory a faithful qubit transmission scheme against collective channel noise using only linear optics and without resorting to ancillary qubits, which shows high success probability, fidelity and ingenious device. Here in this study, we experimentally demonstrate the scheme of deterministic distribution of single photon state. Requiring neither entangled resource nor ancillary qubits, the current protocol brings great convenience and feasibility to the delivery of quantum technologies. Our experiment shows that time-bin encoded qubits transmitted through different channels preserved high fidelity using conventional and commercially available optics, which is of indispensable importance for practical quantum communication.

2. Faithful Transmission of Single Qubit Over Noisy Channels

Sketched in Fig. 1 is the experiment setup employed in this work. Single photons are prepared by spontaneous parametric down conversion(SPDC) process in nonlinear crystal, and measured by coincidence detection process. A continuous-wave pumping laser operating at 405 nm, 55 mW is fed to a 2 mm thick type-I β-BaB_2O_4 crystal at room temperature and a pair of 810 nm photons are generated with polarization entanglement. The idler photon is coupled into a single mode fiber linked directly to the coincidence counting module, and the signal photon is filtered by horizontally aligned polarizer to make sure the initial photons are in the state denoted as $|H\rangle$. Half wave plate (HWP0) is used by the sender Alice to encode qubits, after which the initial state changes to $|\psi\rangle = \cos 2\theta|H\rangle + \sin 2\theta|V\rangle$, where θ is the rotation angle set from 0° to 45°. For simplicity, we rewrite the state $|\psi\rangle$ in the form $|\psi\rangle = \alpha|H\rangle + \beta|V\rangle$, with $|\alpha|^2 + |\beta|^2 = 1$. After preparation Alice sends the qubit to the unbalanced interferometer to transfer quantum information encoded in polarization to time-bin degree of freedom. The unbalanced interferometer consists of polarizing beam splitter, HWP and 50-50 beam splitter, with which the following operations is completed,

$$\begin{aligned}|\psi\rangle = \alpha|H\rangle + \beta|V\rangle &\xrightarrow{PBS1} \alpha|H\rangle_S + \beta|V\rangle_L \xrightarrow{HWP1} \alpha|H\rangle_S + \beta|H\rangle_L \\ &\xrightarrow{BS1} \frac{1}{\sqrt{2}}(\alpha|H\rangle_S + i\beta|H\rangle_L)_1 + \frac{1}{\sqrt{2}} i(\alpha|H\rangle_S - i\beta|H\rangle_L)_2 = |\psi\rangle_1 + |\psi\rangle_2 \end{aligned} \quad (1)$$

where $|\psi\rangle_{1(2)}$ describes the state of the photon at output port 1(2) of BS1. The output photons subsequently pass through channel 1 and channel 2, respectively. Many factors such as random birefringence, thermal fluctuations, mechanical vibrations, and the imperfections of the fiber itself could induce unclear transformation of the polarization state. Nevertheless such collective noise has the same effect for different time-bin qubits [36]. Thus the evolution of the states in channel 1 and channel 2 can be expressed with random unitary operations as $|H\rangle_{k,channel1} = (\delta|H\rangle_k + \eta|V\rangle_k)_{channel1}$ and $|H\rangle_{k,channel2} = (\mu|H\rangle_k + \nu|V\rangle_k)_{channel2}$ ($|\delta|^2 + |\eta|^2 = 1$, $|\mu|^2 + |\nu|^2 = 1$ represent the distortion parameters caused by above noises), with the subscript $k = S, L$ denoting the path information. Then, the evolution of the states $|\psi\rangle_1$ and $|\psi\rangle_2$ from Alice to Bob can be written as

$$\begin{aligned}|\psi\rangle_1 + |\psi\rangle_2 &\xrightarrow[channel2]{channel1} \frac{1}{\sqrt{2}}(\alpha\delta|H\rangle_S + \alpha\eta|V\rangle_S + i\beta\delta|H\rangle_L + i\beta\eta|V\rangle_L)_1 \\ &+ \frac{1}{\sqrt{2}}(i\alpha\mu|H\rangle_S + i\alpha\nu|V\rangle_S + \beta\mu|H\rangle_L + \beta\nu|V\rangle_L)_2 = |\psi\rangle_1^{'} + |\psi\rangle_2^{'}. \end{aligned} \quad (2)$$

Here $|\psi\rangle_{1(2)}^{'}$ denotes the quantum state of the photon received by Bob after passing through channel 1(2). As shown in Fig. 1, the qubits subsequently go through unbalanced polarization interferometer on Bob's side, and the evolution of the state could be described as

$$\begin{aligned}|\psi\rangle_{1,2}^{'} \rightarrow &\frac{\delta}{2}[i(\beta|H\rangle_{LS} + \alpha|V\rangle_{SL}) + (\alpha|H\rangle_{SS} - \beta|V\rangle_{LL})]_a \\ &+ \frac{\eta}{2}[i(\beta|V\rangle_{LS} + \alpha|H\rangle_{SL}) + (\alpha|V\rangle_{SS} - \beta|H\rangle_{LL})]_b \\ &+ \frac{\nu}{2}[(\beta|V\rangle_{LS} - \alpha|H\rangle_{SL}) + i(\alpha|V\rangle_{SS} + \beta|H\rangle_{LL})]_c \\ &+ \frac{\mu}{2}[(\beta|H\rangle_{LS} + \alpha|V\rangle_{SL}) + i(\alpha|H\rangle_{SS} + \beta|V\rangle_{LL})]_d. \end{aligned} \quad (3)$$

Here the subscripts a, b and c, d denote the two output ports of PBS2 and PBS3, respectively. As shown in Eq. (3), Bob eventually recovers quantum information encoded in either polarization $\frac{\eta}{2}i(\beta|V\rangle_{LS} + \alpha|H\rangle_{SL}$ at port b, $\frac{\nu}{2}(\beta|V\rangle_{LS} - \alpha|H\rangle_{SL})$ at port c, or time-bin degree of freedom $\frac{\delta}{2}(\alpha|H\rangle_{SS} - \beta|V\rangle_{LL}$ at port a, $\frac{\mu}{2}i(\alpha|H\rangle_{SS} + \beta|V\rangle_{LL})$ at port d. Note that in the later case time-bin and polarization degrees of freedom are entangled, which nevertheless donot affect the complex coefficients encoded on polarization. For the rest of the resulting states, local linear optics can be employed to transform them into the same states aforementioned, with no photons being dumped, making this scheme work in a deterministic way. For instance, by applying HWP with a definite angle (22.5°) after port a and port d, the following states can be obtained as

$$\begin{aligned}&\frac{\delta}{2}i(\beta|V\rangle_{LS} + \alpha|H\rangle_{SL})_a + \frac{\eta}{2}i(\beta|V\rangle_{LS} + \alpha|H\rangle_{SL})_b \\ &+ \frac{\nu}{2}(\beta|V\rangle_{LS} - \alpha|H\rangle_{SL})_c + \frac{\mu}{2}(\beta|V\rangle_{LS} + \alpha|H\rangle_{SL})_d, \end{aligned} \quad (4)$$

TABLE 1

EXPERIMENTAL RESULTS: FAITHFUL SINGLE-PHOTON QUBIT TRANSMISSION WITH HIGH SUCCESS PROBABILITY. TWO VALID PARAMETERS ARE INTRODUCED, $\lambda = \cos^2(2\theta)/\sin^2(2\theta) = \alpha^2/\beta^2$ AND $\varrho_a = \mathbf{n}_{|H\rangle_{SS}}/\mathbf{n}_{|V\rangle_{LL}}$, $\varrho_b = \mathbf{n}_{|H\rangle_{SS}}/\mathbf{n}_{|V\rangle_{LL}}$, $\varrho_c = \mathbf{n}_{|H\rangle_{SS}}/\mathbf{n}_{|V\rangle_{LL}}$, $\varrho_d = \mathbf{n}_{|H\rangle_{SS}}/\mathbf{n}_{|V\rangle_{LL}}$, DENOTE THE INITIAL STATE $|\psi\rangle = \cos 2\theta|H\rangle + \sin 2\theta|V\rangle$ AND THE FOUR TEMPORALLY SEPARATED UNCORRUPTED STATE $\frac{\delta}{2}(\alpha|H\rangle_{SS} - \beta|V\rangle_{LL})_a$, $\frac{\eta}{2}(\alpha|H\rangle_{SS} - \beta|V\rangle_{LL})_b$, $\frac{\nu}{2}i(\alpha|H\rangle_{SS} + \beta|V\rangle_{LL})_c$, $\frac{\mu}{2}i(\alpha|H\rangle_{SS} + \beta|V\rangle_{LL})_d$ AT THE BOB'S SIDE, RESPECTIVELY. A SERIES OF INITIAL STATES PREPARED WITH THE θ_{HWP0} SET AS $\pi/36$, $\pi/18$, $\pi/12$, $\pi/8$, $\pi/6$, $7\pi/36$, $2\pi/9$, WHICH OF THE CORRESPONDING MEASUREMENT RESULTS RECORDED IN THE ABOVE TABLE

		Alice(Theoretical value)	***Bob(Measurements)***			
			channel 1		channel 2	
		$\lambda = cot^2(2\theta) = \frac{\alpha^2}{\beta^2}$	port **a**: $\varrho_a = \mathbf{n}_{\|H\rangle_{SS}}/\mathbf{n}_{\|V\rangle_{LL}}$	port **b**: $\varrho_b = \mathbf{n}_{\|H\rangle_{SS}}/\mathbf{n}_{\|V\rangle_{LL}}$	port **c**: $\varrho_c = \mathbf{n}_{\|H\rangle_{SS}}/\mathbf{n}_{\|V\rangle_{LL}}$	port **d**: $\varrho_d = \mathbf{n}_{\|H\rangle_{SS}}/\mathbf{n}_{\|V\rangle_{LL}}$
θ_{HWP0}	$\pi/36$	32.258	30.900	29.995	34.062	33.841
	$\pi/18$	7.548	7.615	7.802	7.349	7.611
	$\pi/12$	3.000	2.967	3.090	3.102	3.099
	$\pi/8$	1.000	1.023	1.007	1.018	1.005
	$\pi/6$	0.333	0.329	0.341	0.326	0.315
	$7\pi/36$	0.132	0.129	0.131	0.135	0.137
	$2\pi/9$	0.031	0.029	0.033	0.032	0.033

while applying the same operation after port *b* and port *c* at a different time window as shown in Fig. 1, one has

$$\frac{\delta}{2}(\alpha|H\rangle_{SS} - \beta|V\rangle_{LL})_a + \frac{\eta}{2}(\alpha|H\rangle_{SS} - \beta|V\rangle_{LL})_b + \frac{\nu}{2}i(\alpha|H\rangle_{SS} + \beta|V\rangle_{LL})_c + \frac{\mu}{2}i(\alpha|H\rangle_{SS} + \beta|V\rangle_{LL})_d. \tag{5}$$

Here the subscripts SL and LS indicate different path information of the interferometer but arrived on the same time, while subscripts SS and LL indicate short path and long path information of the interferometer and exhibit different arriving time. At this stage the quantum state is not exactly the one prepared by Alice, which thus is different from the case of teleportation. However, the time-bin encoded state can be transformed to the original one with active switching and additional delay-line followed by 50:50 beam splitter, in which case Bob will be able to recover the same state sent by Alice. Neglecting the channel loss, for each qubit Alice sent, Bob has a probability of 0.5 in total to detect the photon time-bin encoded at one of the four ports (a, b, c, d), and another 0.5 probability to see it polarization encoded (Eq. (5)). In both cases quantum information can be recovered by Bob, thus this protocol works in a deterministic way.

Here the heralded single photon source is used in our experiment, and the measurement of single photons is carried out by coincidence counting of signal and idler photons. MAESTRO multi-channel analyzer software from ORTEC is used for data acquisition, and a time-amplitude-converter is employed for coincidence counting. Path difference between LL and SS is set to be equivalent to 10 ns delay, which is much wider than the time window chosen for optimizing the counting rate, meanwhile it could avoid crosstalk between the signals from different time-bins. SPCM-AQRH-14-FC single-photon detectors from EXCELITAS technologies are used, with dark count rate below 100 cps.

3. Results and Discussions

Analysis of measurement results between states prepared by Alice and those received by Bob is listed in Table 1, where θ is the rotation angle of HWP0 set at $\pi/36$, $\pi/18$, $\pi/12$, $\pi/8$, $\pi/6$, $7\pi/36$, $2\pi/9$, encoding the photon polarization state as $|\psi\rangle = \cos 2\theta|H\rangle + \sin 2\theta|V\rangle$, and $\varrho_i = (\mathbf{n}_{|H\rangle_{SS}}/\mathbf{n}_{|V\rangle_{LL}})_i$, ($i = a, b, c, d$), with $\mathbf{n}_{|H\rangle_{SS}}$ and $\mathbf{n}_{|V\rangle_{LL}}$ denoting the counting rate of the horizontally polarized single photons detected at earlier time and of those vertically polarized photons detected at later time, respectively. The density matrix of the time-bin encoded states $\cos\pi/18|H\rangle + \sin\pi/18|V\rangle$, $\cos\pi/6|H\rangle + \sin\pi/6|V\rangle$, $\cos\pi/3|H\rangle + \sin\pi/3|V\rangle$, $\cos 7\pi/18|H\rangle + \sin 7\pi/18|V\rangle$ are

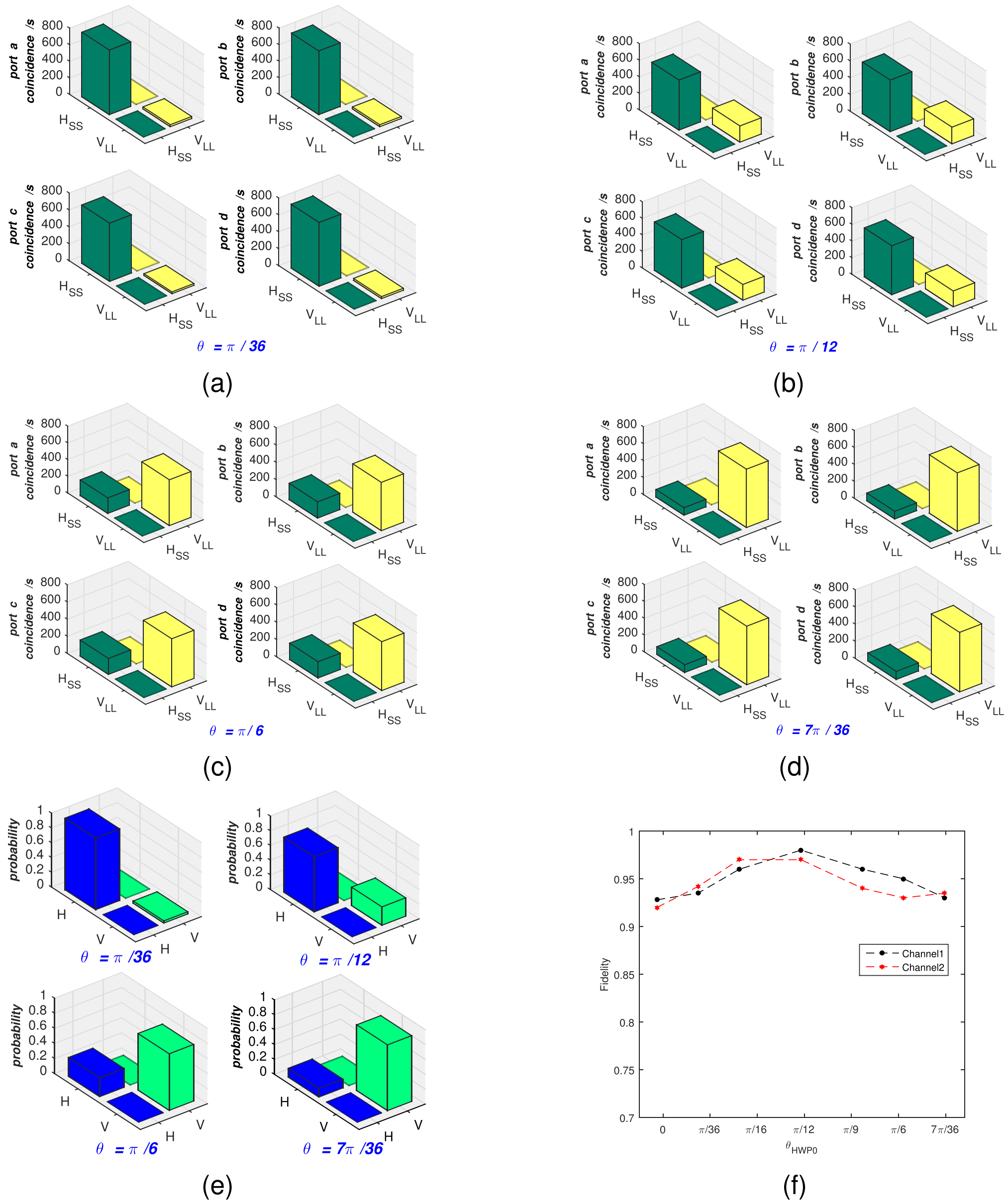

Fig. 2. **Experimental results: high fidelity transmission of qubit through noise channels**. (a)–(d) indicates that recombination density matrix of the uncorrupted time-bin states of the receiver is given by experimental measurement of the coincidence counting between the idler and the temporally separated qubits selected with definite time slots,where the initial four state are: $\cos\pi/18|H\rangle + \sin\pi/18|V\rangle$, $\cos\pi/6|H\rangle + \sin\pi/6|V\rangle$, $\cos\pi/3|H\rangle + \sin\pi/3|V\rangle$, $\cos 7\pi/18|H\rangle + \sin 7\pi/18|V\rangle$. (e) the theoretical results of the four output states as a comparison with (a)–(d). The fidelity of the series of qubits transmitted through the noisy channels is exhibited in (f).

presented in Fig. 2. Time-bin encoded photons from different output ports show nearly identical density matrix, which are presented respectively in Fig. 2(a), (b), (c), (d), and compared with the theoretically calculated value shown in Fig. 2(e). The fidelity of the received photon states exceeds 94.6%, which is averaged over the two channels as ultra-fast switching [37]. In addition, the lower bounds on the fidelity [38] for the experimental setup can be described as $\mathrm{F} = \mathrm{Tr}(\rho_{\mathrm{Alice}}\rho_{\mathrm{Bob}}) + \sqrt{2[\mathrm{Tr}(\rho_{\mathrm{Alice}}\rho_{\mathrm{Bob}})]^2 - 2\mathrm{Tr}[(\rho_{\mathrm{Alice}}\rho_{\mathrm{Bob}})^2]}$. Here the density matrices ρ_{Alice} and ρ_{Bob} are the initial input state at port Alice and the measured output state at port Bob, respectively. It should be emphasized that our experimental fidelity is slightly smaller than the theoretical scheme [35] due to difficulty in preparing highly entangled photon pair and not very high single photon detection efficiency.

4. Summary

In summary, we realized the faithful single-photon qubit transmission against the channel noise via one-way quantum communication. Based on the unbalanced polarization interferometer, the polarization degrees of freedom could be transferred to time-bin degrees of freedom which enables an undisturbed state through the transmission. The success probability of the scheme approaches to 100% for arbitrary initial qubit transmission through different channels. Here we precisely set the optical path difference to control the delay time ($\Delta t = 10$ ns) instead of using optical delay line. We show that the time-bin edcoded qubit is immune to collective channel noise which could be further applied for quantum error-correction and error-rejection of long-distance quantum communication.

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