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On-demand Microwave Controlled Single-photon Source

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Abstract: The microwave single-photon source plays a fundamental role in quantum information processing regime. In this paper, we demonstrate a theoretical model of the controlled single-photon source. The source is consisted of a transmon qubit, a control line and an output line. The source be drived by a short microwave pulse applied through the control line, single photon is emitted through the output line. Based on theoretical calculation and numerical simulation, we obtain the single-photon source, where the photon number and shape can be controlled. This is important to enhance the efficiency of quantum communication.

Key Words: Microwave Controlled, Single-photon Source, Shape of photons

1 Introduction

In recent years, quantum communication and quantum computation has a great development [1]. For quantum networks and quantum computation[2, 3, 4], the generation of single photons play an important part[5, 6]. Because the single photons can transfer information between different nodes or devices in quantum information processing as uesful information sequence, which can enhance the efficiency of information transfer by tuning the wave packet of single photons. But until now, an controllable single photon generation is one of the most important challenges in quantum communication and quantum computation. Therefore, a controllable model to demonstrate the single-photon source is consequential.

The superconducting quantum circuits (SQCs) is one of the most popular methods to realize quantum computation[7], has the advantage of easy extending. In superconducting quantum circuits, the artificial atoms and resonators replace the natural atoms and optical cavities. The microwave photonics transfer information between superconduction artificial atoms and resonators or transmission line. SQCs based on Josephson junctions can be classified as three kinds of circuits, including charge qubits, flux qubits and phase qubits. In 2007, the transmon qubit was proposed as one extending of the charge qubit[8, 9, 10]. It has an high potential to realize the generation of on-demand microwave single photons.

In this paper, we propose a method of generating demanded single photon by controlling the length of electric field of microwave pulse or control pulse, which is a π -pulse. The generator consists of a control line, a transmon qubit and an output line. The generator is drived by a short microwave pulse applied through the control line and emits single microwave photon through a output line[5, 6]. The length of the microwave pulse can be varied, but the shape of it is rectangle. When the angular frequency of the microwave pluse is the same with the transition frequency of transmon qubit, the qubit is at resonance with the microwave pulse. The qubit is excited to the first excited state from the ground state by absorbing the energy of the microwave and radiates spontaneously to environment. Most of the radiated photons enter into the output line, because the coupling coefficient between a control line and qubit is much smaller than that between qubit and output line. For one microwave pulse, the number of emitted photons from transmon qubit is tunable by the length of microwave pulse, therefore, the single photons can be obtained by optimal control, and the shape of photons also can be controlled by the same method. This work has an improment potential in quantum communication and quantum computation.

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2 The master equation of system

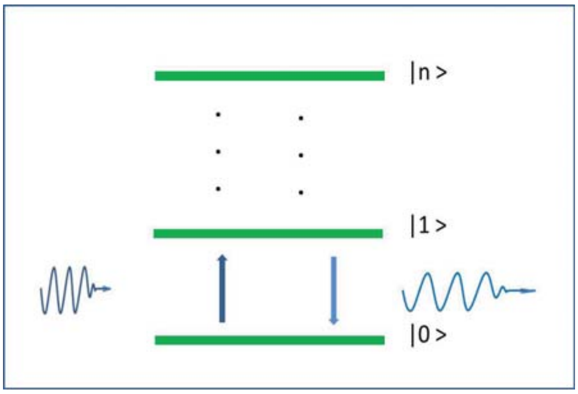

Fig. 1: The principle of the single-photon source. In the middle of the picture, it is a transmon qubit, which is a multiple levels system . The transition mainly happen between the first two energy levels $|0\rangle$, $|1\rangle$ under driven. A short microwave pulse (blue-black wavy line) from a control line to drive qubit. The qubit is excited from the first energy level to the second.At the same time, it radiates by releasing photons (light blue wave line) into output line.

In this section, we consider a transmon qubit driven by classically microwave pulse of single mode ω_d . The transmon qubit decays by emitting photons into an one-dimensional transmission line, which is named as output line. For the whole process, the Hamiltonian is:

$$H = H_S + H_R + H_1, \tag{1a}$$
$$H_S = \hbar\omega_A c^\dagger c - \hbar\beta(c^\dagger c)^2 + H_d, \tag{1b}$$
$$H_d = \hbar(g_1 \frac{\varepsilon^*(t)}{2} c e^{i\omega_d t} + g_1^* \frac{\varepsilon(t)}{2} c^\dagger e^{-i\omega_d t}), \tag{1c}$$
$$H_R = \hbar \sum_k \nu_k a_k^\dagger a_k, \tag{1d}$$
$$H_1 = \hbar \sum_k [g_{2k}(z_0) a_k^\dagger c + g_{2k}^*(z_0) c^\dagger a_k], \tag{1e}$$

Here, H_S is the Hamiltonian of system that is the qubit driven by microwave pulse. H_R is the Hamiltonian of the bath of the output line and H_1 is the interaction between the system and the bath. $\hbar$ is the Planck constant, ω_A is the transition frequence between the first energy level $|0\rangle$ and the second energy level $|1\rangle$. $c, c^\dagger$ are the annihilation operator and creation operator of the qubit respectively, β is the small contant than ω_A, H_d is the interation between the qubit and the microwave pulse. g_1 is the coupling constant between the control line and qubit, $\varepsilon(t)$ is the amplitude of electric field of microwave pulse, ω_d is the frequence of microwave pulse. $a, a^\dagger$ are the annihilation operator and creation operator of the bath respectively, ν is the frequence of bath and k is the mode of bath. The g_2 is the coupling constant between qubit and output line. $g_{2k}(z_0) = g_{2k} e^{-ikz_0}$, where we have used the spatial dependence explicity. Here, z_0 is the location of the qubit, and we can set $z_0 = 0$. Then we make the first Markov approximation by assuming g_{2k} is slowly varying function arounding ω, so

$$H_1 = \hbar \sum_k [g_2 a_k^\dagger c + g_2^*(z_0) c^\dagger a_k], \tag{2a}$$
$$H_0 = \hbar\omega_A c^\dagger c + H_R, \tag{2b}$$
$$U_0 = e^{-i\frac{H_0}{\hbar}t}, \tag{2c}$$
$$U_0^\dagger = e^{i\frac{H_0}{\hbar}t}, \tag{2d}$$

where $\omega = \omega_A = \omega_d$.

Let $\rho_{SR}^S(t)$ be the total density operator of the system with reservoir in the Schrödinger picture and $\rho_{SR}^I(t)$ be the total density operator of the system with reservoir in the interaction picture. So we can obtain the equation of motion of the total density operator of the system with reservoir in the Schrödinger pictrue is

$$\frac{d\rho_{SR}^S(t)}{dt} = -\frac{i}{\hbar}[H, \rho_{SR}^S(t)], \tag{3}$$

the equation of motion in the interaction picture is

$$\frac{d\rho_{SR}^I(t)}{dt} = -\frac{i}{\hbar}[V(t), \rho_{SR}^I(t)], \tag{4}$$

where

$$\begin{aligned} V(t) &= U_0^\dagger(H_1 + H_d - \hbar\beta(c^\dagger c)^2)U_0 \\ &= U_0^\dagger(H_1 + H_d)U_0 - \hbar\beta(c^\dagger c)^2 \\ &= \hbar\sum_k [g_{2k} a_k^\dagger c e^{-i(\omega-\nu_k)t} + g_{2k}^* c^\dagger a_k e^{i(\omega-\nu_k)t}] \\ &+ \hbar(g_1\frac{\varepsilon^*(t)}{2}c + g_1^*\frac{\varepsilon(t)}{2}c^\dagger) - \hbar\beta(c^\dagger c)^2, \end{aligned} \tag{5}$$

Integrating (4) we obtain the solution

$$\begin{aligned} \rho_{SR}^I(t) &= \rho_{SR}^I(0) - \frac{i}{\hbar}\int_0^t dt_1[V(t_1), \rho_{SR}^I(t_1)] \\ &= \rho_{SR}^I(0) - \frac{i}{\hbar}\int_0^t dt_1[H_d^I(t_1), \rho_{SR}^I(t_1)] \\ &- \frac{i}{\hbar}\int_0^t dt_1[H_1^I(t_1), \rho_{SR}^I(t_1)] \\ &+ \frac{i}{\hbar}\int_0^t dt_1[\hbar\beta(c^\dagger c)^2, \rho_{SR}^I(t_1)] \end{aligned} \tag{6}$$

where

$$H_d^I(t) = U_0^\dagger H_d(t) U_0, \tag{7}$$
$$H_1^I(t) = U_0^\dagger H_1(t) U_0, \tag{8}$$

simiarly, integrating continuly, we obtain the solution

$$\begin{aligned} \rho_{SR}^I(t_1) &= \rho_{SR}^I(0) - \frac{i}{\hbar}\int_0^{t_1} dt_2[V(t_2), \rho_{SR}^I(t_2)] \\ &= \rho_{SR}^I(0) - \frac{i}{\hbar}\int_0^{t_1} dt_2[H_d^I(t_2), \rho_{SR}^I(t_2)] \\ &- \frac{i}{\hbar}\int_0^{t_1} dt_2[H_1^I(t_2), \rho_{SR}^I(t_2)], \\ &+ \frac{i}{\hbar}\int_0^{t_1} dt_2[\hbar\beta(c^\dagger c)^2, \rho_{SR}^I(t_2)], \end{aligned} \tag{9}$$

substituting the third item in Eq.(6) by Eq.(9), we can obtain the more precise solution

$$\begin{aligned} &\rho_{SR}^I(t) - \rho_{SR}^I(0) \\ &= -\frac{i}{\hbar}\int_0^t dt_1[H_d^I(t_1), \rho_{SR}^I(t_1)] - \frac{i}{\hbar}\int_0^t dt_1[H_1^I(t_1), \rho_{SR}^I(0)] \\ &+ (-\frac{i}{\hbar})^2\int_0^t dt_1\int_0^{t_1} dt_2[H_1^I(t_1), [H_d^I(t_2), \rho_{SR}^I(t_2)]] \\ &+ (-\frac{i}{\hbar})^2\int_0^t dt_1\int_0^{t_1} dt_2[H_1^I(t_1), [H_1^I(t_2), \rho_{SR}^I(t_2)]] \\ &- (-\frac{i}{\hbar})^2\int_0^t dt_1\int_0^{t_1} dt_2[H_1^I(t_1), [\hbar\beta(c^\dagger c)^2, \rho_{SR}^I(t_2)]] \\ &+ \frac{i}{\hbar}\int_0^t dt_1[\hbar\beta(c^\dagger c)^2, \rho_{SR}^I(t_1)], \end{aligned} \tag{10}$$

The reduced density operator for the system is difined by

$$\rho_S(t) = Tr_R[\rho_{SR}(t)], \tag{11}$$

where Tr_R indicates a trace over bath variables. We assume that initially the system and bath are uncorrelated so that

$$\rho_{SR}(0) = \rho_S(0) \otimes \rho_R, \tag{12}$$

where ρ_R is the density operator of the bath, assuming the bath at equilibrium. Since H_1^I is small, we look for a solution of Eq.(4) in the form

$$\rho_{SR}(t) = \rho_S(t) \otimes \rho_R + \rho_c(t), \tag{13}$$

where ρ_c is of higher order in H_1^I. In order to satisfy Eq.(12), we require

$$Tr_R[\rho_c(t)] = 0, \tag{14}$$

so ρ_c can be ignored in the second-order perturbation process. After tracing over bath variables and making the time derivation, Eq.(10) becomes

$$\begin{aligned}\frac{d\rho_S^I(t)}{dt} =&\frac{i}{\hbar}[\hbar\beta(c^\dagger c)^2, \rho_S^I(t)] \\ &- i\frac{g_1^*\varepsilon(t)}{2}[c^\dagger, \rho_S^I(t)] - i\frac{g_1\varepsilon^*(t)}{2}[c, \rho_S^I(t)] \\ &+ \frac{\gamma}{2}[2c\rho_S^I(t)c^\dagger - \rho_S^I(t)c^\dagger c - c^\dagger c\rho_S^I(t)]\end{aligned} \quad (15)$$

where $\gamma = \gamma_1 + \gamma_2, \gamma_1 = |g_1|^2/2\pi, \gamma_2 = |g_2|^2/2\pi$. For the $\gamma_1 \ll \gamma_2$, and $\gamma \approx \gamma_2$, we may drop terms to the second order of H_1^I . If the perturbation is weak we can drop terms form H_1^I of order higher than two.In the interaction picture, the master equqation of the reduced density operator of the system is Eq.(15).

The transmon qubit is a system with multiple energy levels, which has small anharmonicity. For simplification, in this paper, it will be treated as an atom with two energy levels $|0\rangle, |1\rangle$. We consider an atom with two energy levels driven by classically microwave pulse of single mode ω_d . The atom can repeat with the front sentences by spontaneously emitting into an one-dimensional transmission line that is named as output line. The total Hamiltonian is

$$H = H_S + H_R + H_1, \quad (16a)$$

$$H_S = \frac{1}{2}\hbar\omega_A\sigma_z + H_d, \quad (16b)$$

$$H_d = \hbar(g_1\frac{\varepsilon^*(t)}{2}\sigma_- e^{i\omega_d t} + g_1^*\frac{\varepsilon(t)}{2}\sigma_+ e^{-i\omega_d t}), \quad (16c)$$

$$H_R = \hbar\sum_k \nu_k a_k^\dagger a_k, \quad (16d)$$

$$H_1 = \hbar\sum_k [g_{2k}(z_0)a_k^\dagger\sigma_- + g_{2k}^*(z_0)\sigma_+ a_k], \quad (16e)$$

here, σ_z is the Pauli operator, σ_- operator takes an atom in the upper state into the lower state whereas σ_+ operator takes an atom in the lower state into the upper state. By using the same method in the interaction picuture, the master equqation of the reduced density operator of the system be obtained as the Eq.(17),

$$\begin{aligned}\frac{d\rho_S^I(t)}{dt} =& - i\frac{\Delta\omega}{2}[\sigma_z, \rho_S^I(t)] - i\frac{g_1^*\varepsilon(t)}{2}[\sigma_+, \rho_S^I(t)] \\ &- i\frac{g_1\varepsilon^*(t)}{2}[\sigma_-, \rho_S^I(t)] \\ &+ \frac{\gamma}{2}[2\sigma_-\rho_S^I(t)\sigma_+ - \rho_S^I(t)\sigma_+\sigma_- - \sigma_+\sigma_-\rho_S^I(t)]\end{aligned} \quad (17)$$

Here, $\Delta\omega = \omega_A - \omega_d$.

3 the number and shape of output single photons

From the master equation we can derive the equations of motion for the density matrix elements

$$\dot\rho_{22} = -\gamma\rho_{22} + ig_1\varepsilon^*(t)\rho_{21}, \quad (18a)$$

$$\dot\rho_{11} = \gamma\rho_{22} - ig_1\varepsilon^*(t)\rho_{21}, \quad (18b)$$

$$\dot\rho_{21} = -(i\Delta\omega + \frac{\gamma}{2})\rho_{21} + i\frac{g_1^*\varepsilon(t)}{2}(\rho_{22} - \rho_{11}), \quad (18c)$$

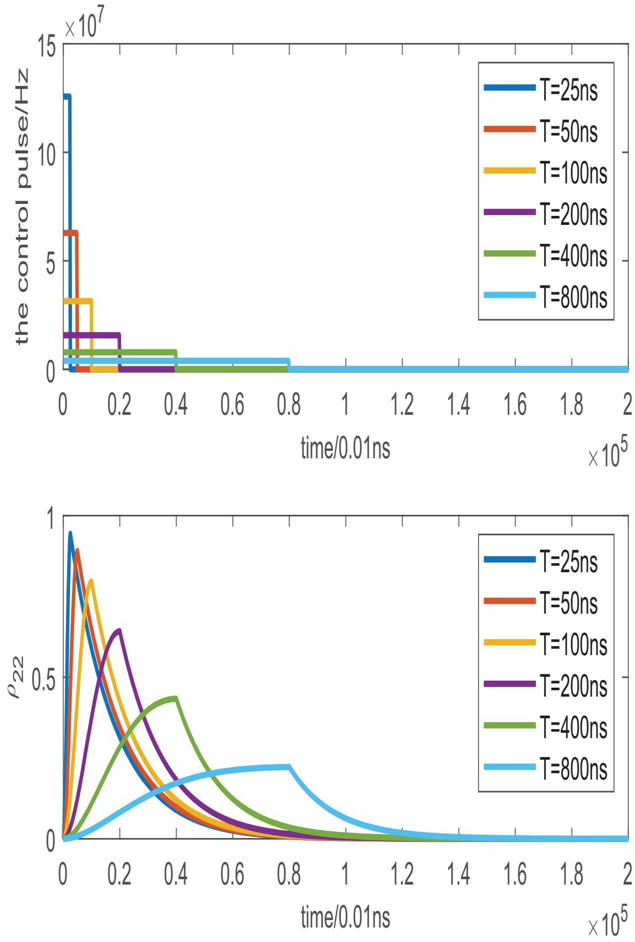

Fig. 2: The control pulse and the response of atom. The figure above is the control pulse and the figure below is the response of atom driven by control pulse with time.ρ_{22} is the probability of the atom at excited state. The T is stand for the length of pulse, it is 25ns(blue), 50ns(red), 100ns(orange), 200ns(purple), 400ns(cyan), 800ns(sky blue) respectively. The shape of pulse is rectangle.

Here, the ρ_{22} is the probability of the atom at excited state,ρ_{11} is the probability of the atom at ground state,ρ_{21} is the density matrix element. γ is the rate of radiation of atom, it is about $2\pi * 10^6 Hz$. On the basis of the equtions above. the response of the atom can obtain. The control pulse and the response are shown in Fig.2. The above of figure is the control pulse and the below is the response of atom driven by control pulse.ρ_{22} is the probability of the atom at excited state. The T is stand for the length of pulse, it is 25ns(blue), 50ns(red), 100ns(orange), 200ns(purple), 400ns(cyan), 800ns(sky blue) respectively. The size of pulse is constant. According to the figure, the relationship between the control pulse and ρ_{22} can obtain. The more small T, the more big maximum probability of the atom at excited state. The maximum probability can show the effect of producting single-photon. If the maximum probability is closed to 1, the single-photon could be obtained certainly after one control pulse. Here, the maximum ρ_{22} can get 0.95 at one control pulse with length of 20ns. The shape of the emitted single-photon can be reflected by the figure below of course.

4 Conclusion

In summary, we build the controllable model of the photon emission system by using master equation. Then we derive the equations of motion for the $\rho_{22}, \rho_{11}, \rho_{21}$. Through numerical simulation, the probability of the atom at excited state photon number and shape is obtained, it is a function of time. The more small T, the more big maximum probability of the atom at excited state. The maximum probability can show the effect of producting single-photon. The shape of the emitted single-photon can be reflected by the figure below of course. So, the effect and shape of single-photon can be controled by the control pulse. The situation under other shape control pulse will be reseach next. This is important to enhance the efficiency of quantum information. This work has a great potential in quantum communication and quantum computation.

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