# Equation Sheet Electronics 2 – EE 320

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## NMOS Characteristics

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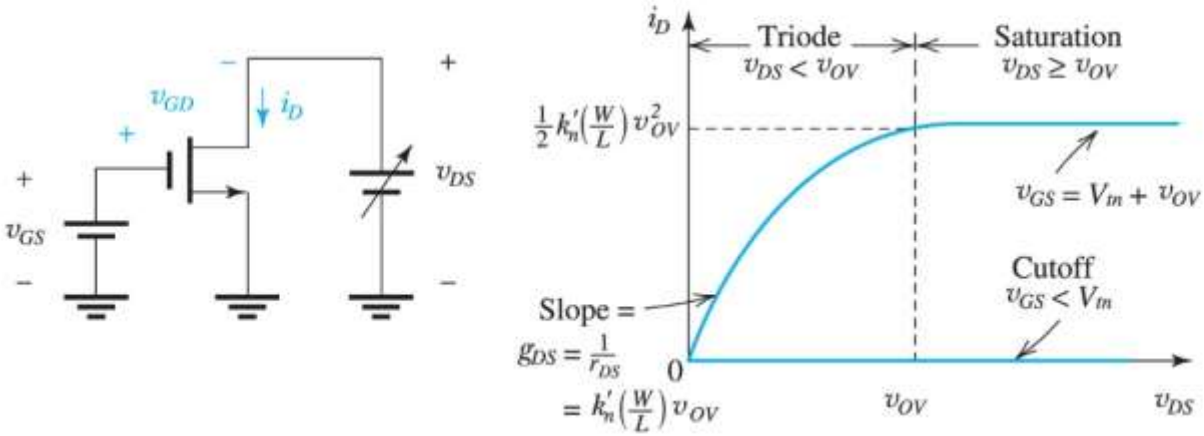
**Table 5.1** Regions of Operation of the Enhancement NMOS Transistor




- $v_{GS} < V_{tn}$ : no channel; transistor in cutoff; $i_D = 0$
- $v_{GS} = V_{tn} + v_{OV}$ : a channel is induced; transistor operates in the triode region or the saturation region depending on whether the channel is continuous or pinched off at the drain end;

*Triode Region*

Continuous channel, obtained by:

$$v_{GD} > V_{tn}$$

or equivalently:

$$v_{DS} < v_{OV}$$

Then,

$$i_D = k_n'\left(\frac{W}{L}\right)\left[(v_{GS} - V_{tn})v_{DS} - \frac{1}{2}v_{DS}^2\right]$$

or equivalently,

$$i_D = k_n'\left(\frac{W}{L}\right)\left(v_{OV} - \frac{1}{2}v_{DS}\right)v_{DS}$$

*Saturation Region*

Pinched-off channel, obtained by:

$$v_{GD} \leq V_{tn}$$

or equivalently:

$$v_{DS} \geq v_{OV}$$

Then

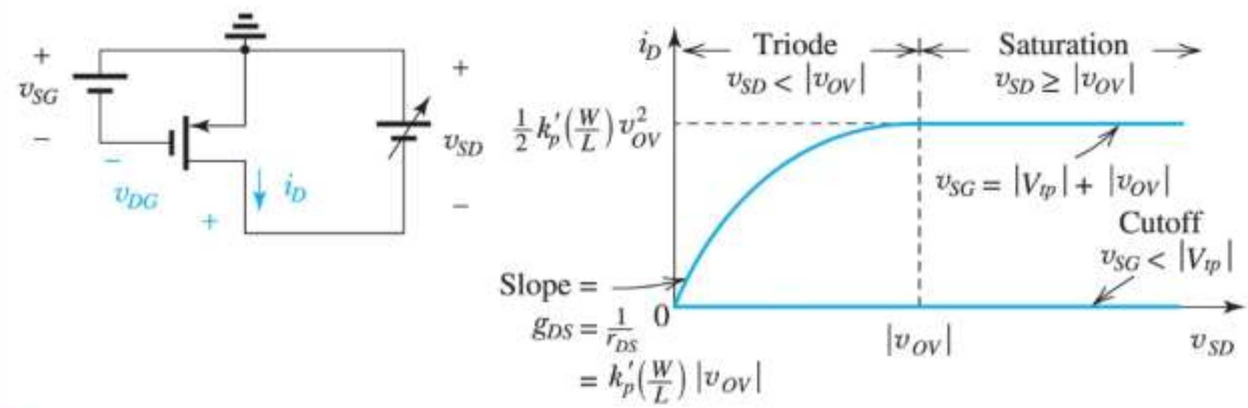
$$i_D = \frac{1}{2}k_n'\left(\frac{W}{L}\right)(v_{GS} - V_{tn})^2$$

or equivalently,

$$i_D = \frac{1}{2}k_n'\left(\frac{W}{L}\right)v_{OV}^2$$

# PMOS Characteristics

**Table 5.2** Regions of Operation of the Enhancement PMOS Transistor




- $v_{SG} < |V_{tp}|$: no channel; transistor in cutoff; $i_D = 0$
- $v_{SG} = |V_{tp}| + |v_{OV}|$ : a channel is induced; transistor operates in the triode region or in the saturation region depending on whether the channel is continuous or pinched off at the drain end;

*Triode Region*

Continuous channel, obtained by:

$$v_{DG} > |V_{tp}|$$

or equivalently

$$v_{SD} < |v_{OV}|$$

Then

$$i_D = k_p'\left(\frac{W}{L}\right)\left[(v_{SG} - |V_{tp}|)v_{SD} - \frac{1}{2}v_{SD}^2\right]$$

or equivalently

$$i_D = k_p'\left(\frac{W}{L}\right)\left(|v_{OV}| - \frac{1}{2}v_{SD}\right)v_{SD}$$

*Saturation Region*

Pinched-off channel, obtained by:

$$v_{DG} \leq |V_{tp}|$$

or equivalently

$$v_{SD} \geq |v_{OV}|$$

Then

$$i_D = \frac{1}{2}k_p'\left(\frac{W}{L}\right)(v_{SG} - |V_{tp}|)^2$$

or equivalently

$$i_D = \frac{1}{2}k_p'\left(\frac{W}{L}\right)v_{OV}^2$$

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# MOSFET Small Signal Characteristics

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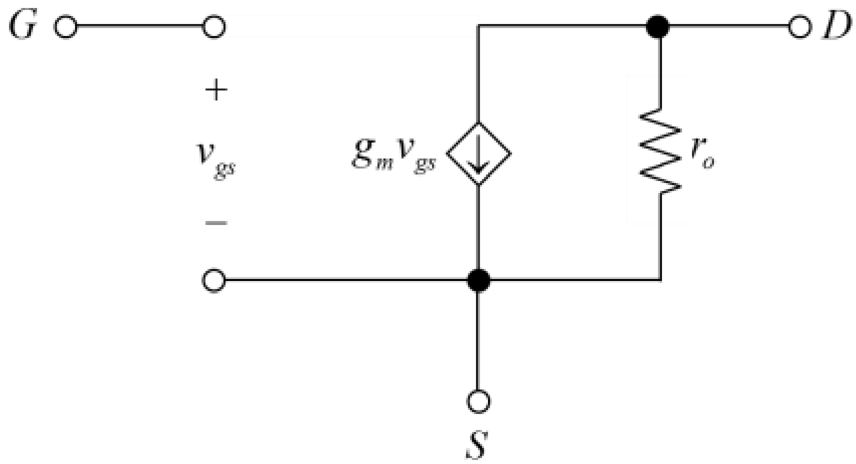


$\boxed{\text{NMOS}}$

$$g_m = \mu_n C_{ox}\left(\frac{W}{L}\right)V_{OV} = \sqrt{2\mu_n C_{ox}\left(\frac{W}{L}\right)I_D} = \frac{2I_D}{V_{OV}}$$

$$r_o = V_A / I_D = 1/\lambda I_D$$

$\boxed{\text{PMOS}}$

$$g_m = \mu_p C_{ox}\left(\frac{W}{L}\right)|V_{OV}| = \sqrt{2\mu_p C_{ox}\left(\frac{W}{L}\right)I_D} = \frac{2I_D}{|V_{OV}|}$$

$$r_o = |V_A| / I_D = 1/|\lambda| I_D$$

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# MOSFET Small Signal Equivalent Resistance

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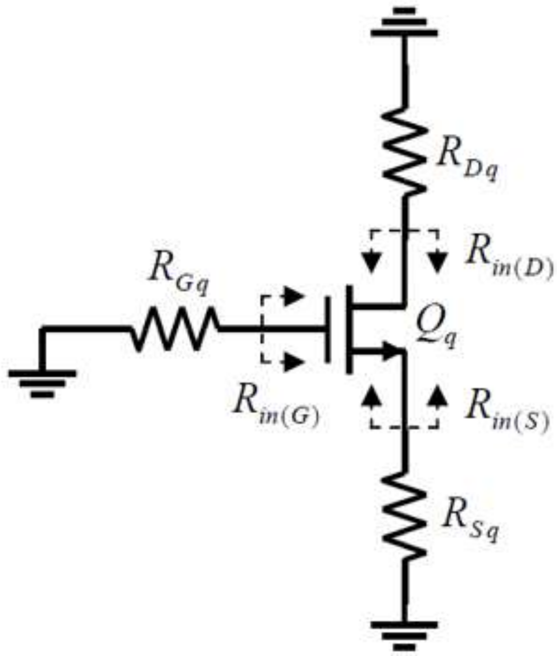


$$R_{in(\mathrm{G})} = \infty \qquad G_{in(\mathrm{G})} = 0$$

$$R_{in(D)} = \frac{g_{mq} + g_{oq} + G_{Sq}}{g_{oq} G_{Sq}} \qquad G_{in(D)} = \frac{g_{oq} G_{Sq}}{g_{mq} + g_{oq} + G_{Sq}}$$

$$R_{in(S)} = \frac{g_{oq} + G_{Dq}}{G_{Dq}\left(g_{mq} + g_{oq}\right)} \qquad G_{in(S)} = \frac{G_{Dq}\left(g_{mq} + g_{oq}\right)}{g_{oq} + G_{Dq}}$$

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# MOSFET Common Source Amplifier w/o Source Resistor

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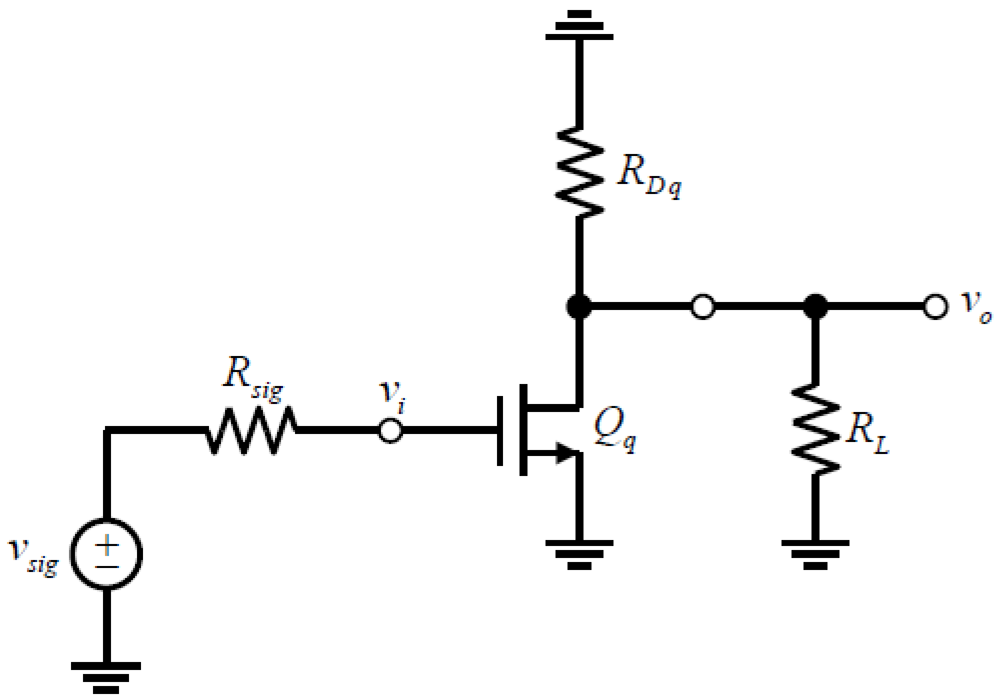


$$R_{in} = \infty \qquad G_{in} = 0$$

$$G_o = g_{oq} + G_{Dq} \qquad R_o = r_{oq} \parallel R_{Dq}$$

$$G_m = -g_{mq}$$

$$A_{vo} = G_m R_o \qquad A_v = \frac{G_m}{G_o + G_L}$$

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# MOSFET Common Source Amplifier w/ Source Resistor

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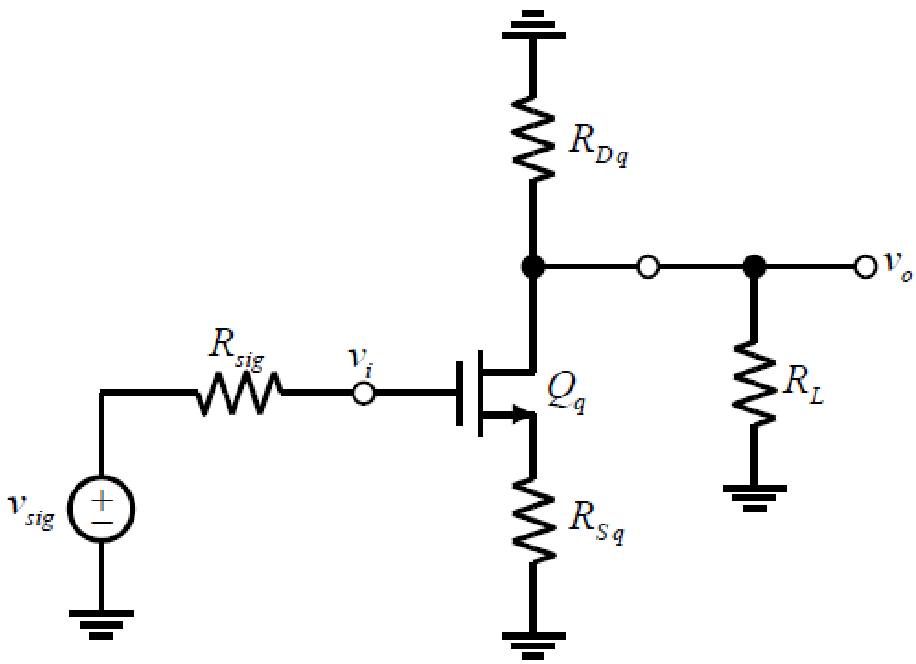


$$R_{in} = \infty \qquad G_{in} = 0$$

$$G_o = G_o^{'} + G_{Dq} \qquad R_o = R_o^{'} \parallel R_{Dq}$$

$$G_o^{'} = \frac{g_{oq} G_{Sq}}{g_{mq} + g_{oq} + G_{Sq}} \qquad R_o^{'} = R_{Sq} + r_{oq} + g_{mq} r_{oq} R_{Sq}$$

$$G_m = -\frac{g_{mq} G_{Sq}}{g_{mq} + g_{oq} + G_{Sq}}$$

$$A_{vo} = G_m R_o \qquad A_v = \frac{G_m}{G_o + G_L}$$

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# MOSFET Common Drain Amplifier w/o Drain Resistor

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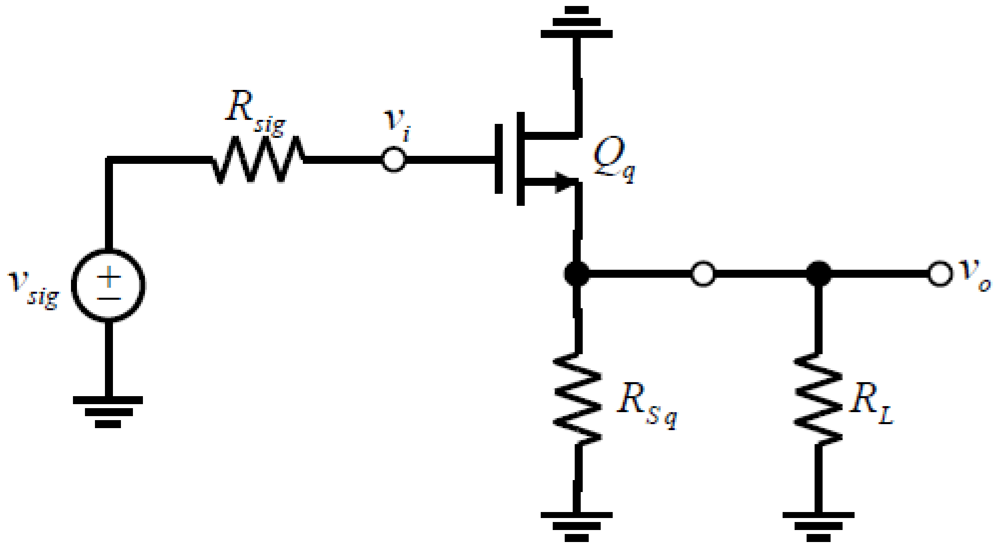


$$R_{in} = \infty \qquad G_{in} = 0$$

$$G_o = g_{mq} + g_{oq} + G_{Sq} \qquad R_o = \left(1/g_{mq}\right) \| r_{oq} \| R_{Sq}$$

$$G_m = g_{mq}$$

$$A_{vo} = G_m R_o \qquad A_v = \frac{G_m}{G_o + G_L}$$

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# MOSFET Common Drain Amplifier w/ Drain Resistor

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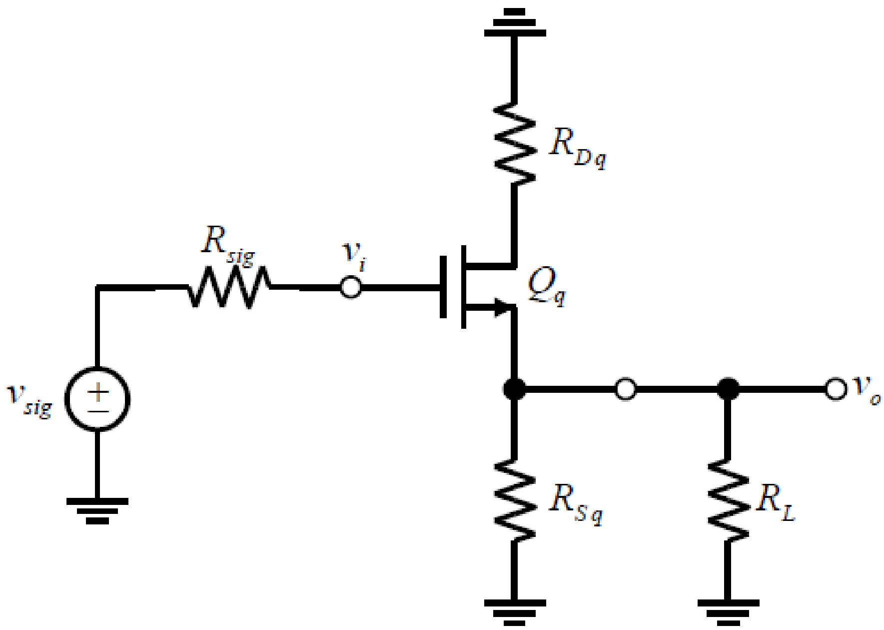



$$R_{in} = \infty \qquad G_{in} = 0$$

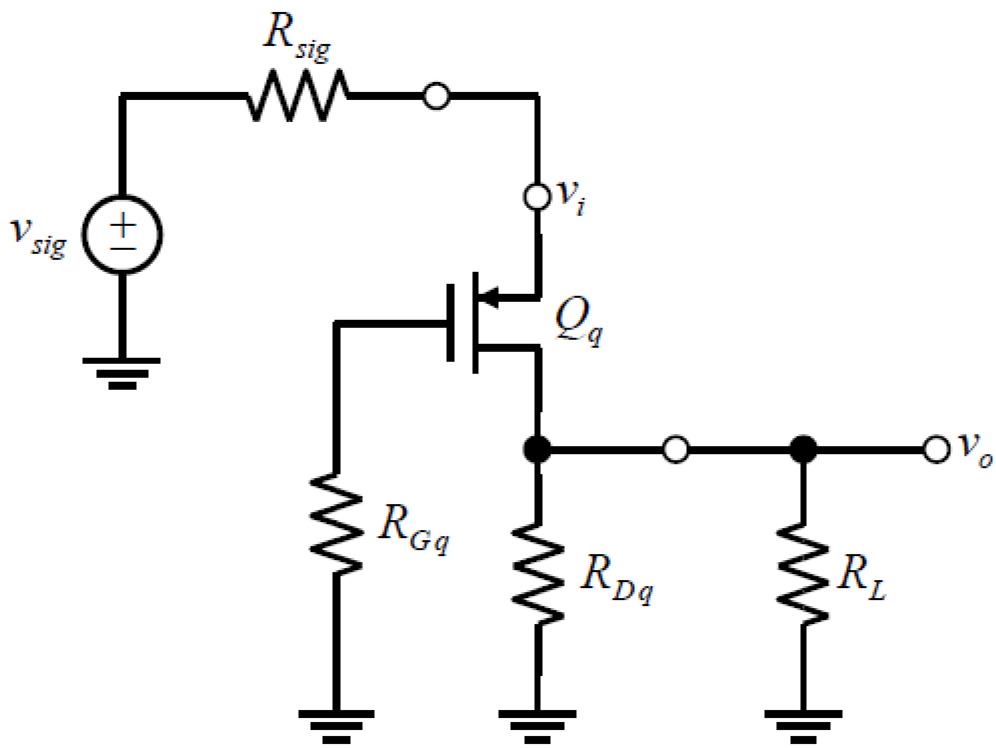
$$G_o = G_o' + G_{Sq} \qquad R_o = R_o' \parallel R_{Sq}$$

$$G_o' = \frac{G_{Dq}\left(g_{mq} + g_{oq}\right)}{g_{oq} + G_{Dq}} \qquad R_o' = \frac{g_{oq} + G_{Dq}}{G_{Dq}\left(g_{mq} + g_{oq}\right)}$$

$$G_m = \frac{g_{mq} G_{Dq}}{g_{oq} + G_{Dq}}$$

$$A_{vo} = G_m R_o \qquad A_v = \frac{G_m}{G_o + G_L}$$

# MOSFET Common Gate Amplifier




$$R_{in} = \frac{g_{oq} + G_{Dq} + G_L}{\left(g_{mq} + g_{oq}\right)\left(G_{Dq} + G_L\right)} \qquad G_{in} = \frac{\left(g_{mq} + g_{oq}\right)\left(G_{Dq} + G_L\right)}{g_{oq} + G_{Dq} + G_L}$$

$$G_o = g_{oq} + G_{Dq} \qquad R_o = r_{oq} \parallel R_{Dq}$$

$$G_m = g_{mq} + g_{oq}$$

$$A_{vo} = G_m R_o \qquad A_v = \frac{G_m}{G_o + G_L}$$

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# MOSFET Differential Amplifier

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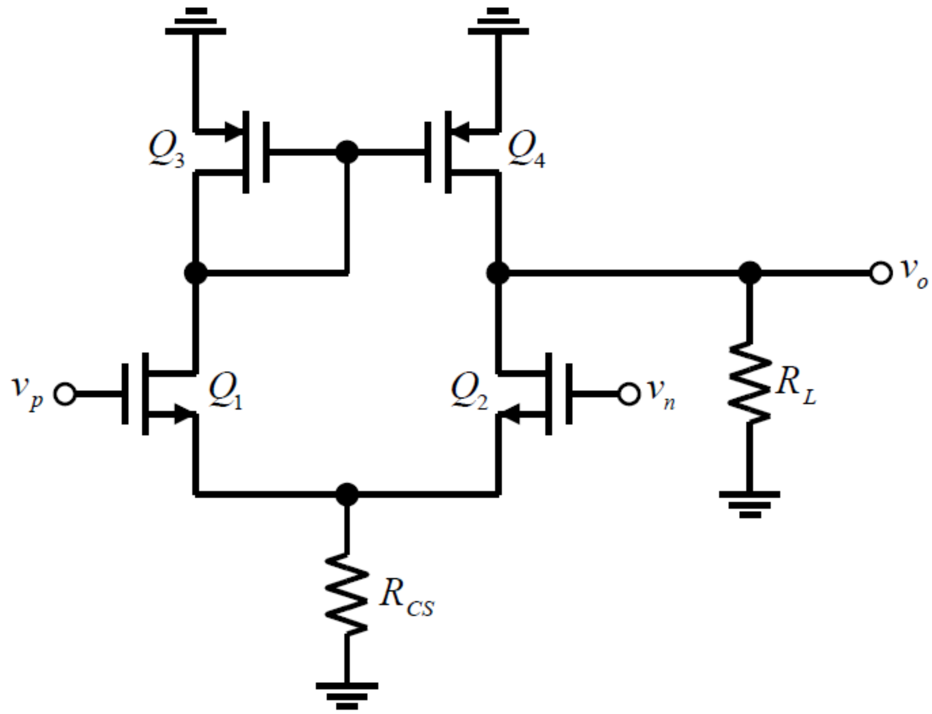


$$R_{in} = \infty \qquad G_{in} = 0$$

$$G_o = Xg_{o2} + g_{o4} \qquad R_o = \frac{1}{Xg_{o2} + g_{o4}}$$

$$G_m = Xg_{m2}$$

$$A_{vo} = G_m R_o \qquad A_v = \frac{G_m}{G_o + G_L}$$

$$X = \frac{1}{2}\left[1 + \frac{(W/L)_4}{(W/L)_3}\right]$$

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# BJT Equations in Active Mode

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$\boxed{npn}$ | $\boxed{pnp}$

$$v_{BE} = 0.7\ \text{V}$$

$$v_{EB} = 0.7\ \text{V}$$

$$v_{CE} \geq V_{CE(\text{SAT})} = 0.3\text{V}$$

$$v_{EC} \geq V_{EC(\text{SAT})} = 0.3\text{V}$$

$$i_C = I_S e^{v_{BE}/V_T}$$

$$i_C = I_S e^{v_{EB}/V_T}$$

$$\beta = \frac{\alpha}{1-\alpha} \qquad \alpha = \frac{\beta}{\beta+1}$$

$$i_C = \beta i_B \qquad i_C = \alpha i_E$$

$$i_E = (\beta+1) i_B \qquad i_E = i_C / \alpha$$

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# BJT Equations in Saturation Mode

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*npn* | *pnp*

$$v_{BE} = 0.7\ \text{V}$$

$$v_{EB} = 0.7\ \text{V}$$

$$v_{CE} \cong V_{CE(\text{SAT})} = 0.2\text{V}$$

$$v_{EC} \cong V_{EC(\text{SAT})} = 0.2\text{V}$$

$$I_{C(\text{SAT})} = \beta_{\text{Forced}} I_B$$

$$\beta_{\text{Forced}} \leq \beta$$

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# BJT Small Signal Characteristics

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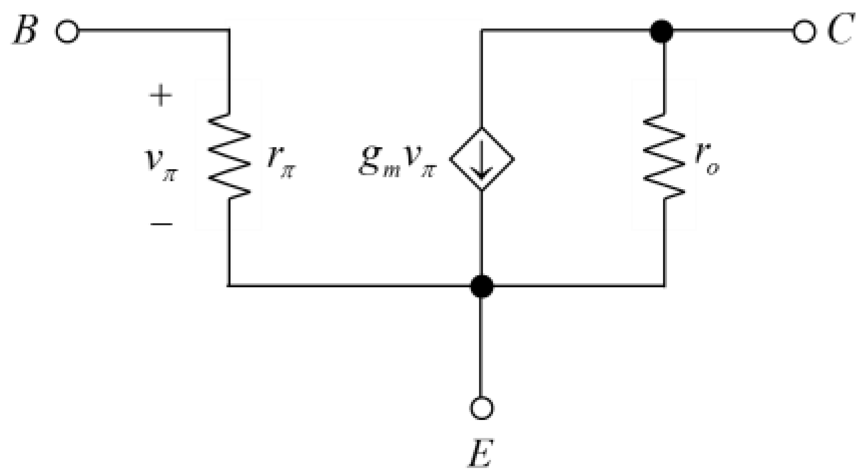


NPN & PNP

$$g_m = \frac{I_C}{V_T} \qquad r_o = \frac{|V_A|}{I_C} \qquad r_\pi = \frac{V_T}{I_B}$$

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# BJT Small Signal Equivalent Resistance

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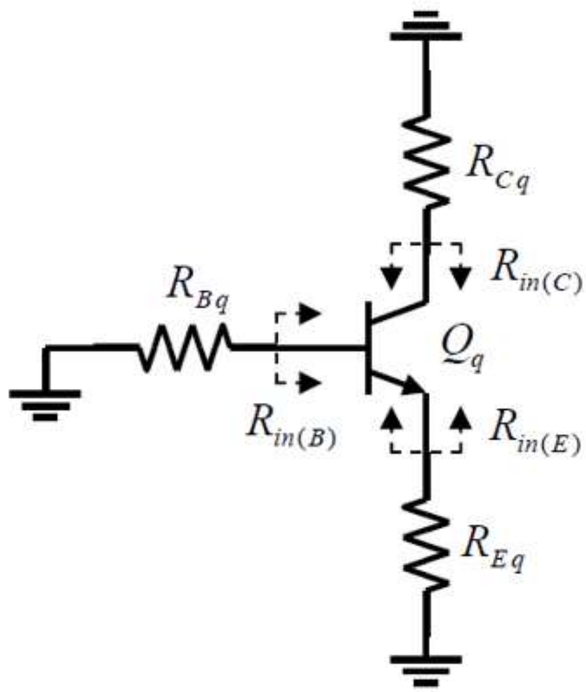


$$R_{in}^{'} = \frac{G_{Cq} + g_{oq} + G_{Cq} g_{mq} r_{\pi q}}{G_{Eq} G_{Cq} + G_{Eq} g_{oq} + G_{Cq} g_{oq}} \to R_{in(B)} = R_{in}^{'} + r_{\pi q}$$

$$R_{in(E)} = \frac{R_x g_{oq} + R_x G_{Cq}}{G_{Cq} + g_{oq} + G_{Cq} g_{oq} R_x + G_{Cq} g_{mq} r_{\pi q}} \qquad R_x = R_{Bq} + r_{\pi q}$$

$$R_{in(C)} = \frac{1 + G_{Eq} R_x + g_{oq} R_x + g_{mq} r_{\pi q}}{g_{oq} \left(1 + G_{Eq} R_x\right)} \qquad R_x = R_{Bq} + r_{\pi q}$$

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# BJT Common Emitter Amplifier w/o Emitter Resistor

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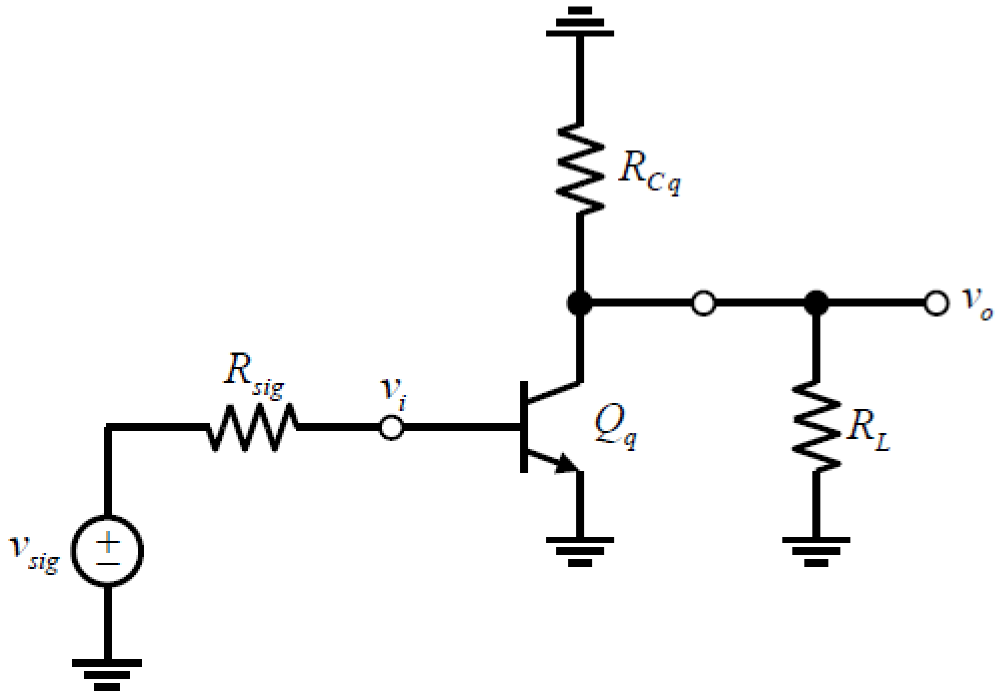


$$R_{in} = r_{\pi q} \qquad G_{in} = g_{\pi q}$$

$$G_o = g_{oq} + G_{Cq} \qquad R_o = r_{oq} \parallel R_{Cq}$$

$$G_m = -g_{mq}$$

$$A_{vo} = G_m R_o \qquad A_v = \frac{G_m}{G_o + G_L}$$

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# BJT Common Emitter Amplifier w/ Emitter Resistor

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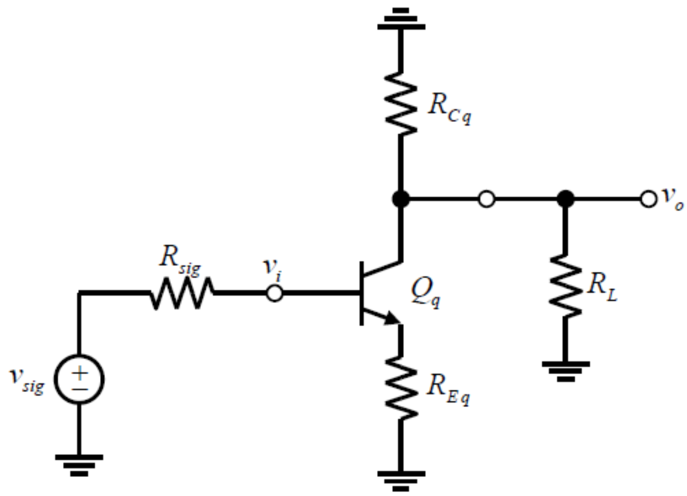


$$R_{in} = r_{\pi q} + R_{in}^{'} \qquad G_{in} = \frac{g_{\pi} G_{in}^{'}}{g_{\pi} + G_{in}^{'}}$$

$$G_{in}^{'} = \frac{\left(G_{Cq}^{'} + G_{Eq}\right) g_{oq} + G_{C}^{'} G_{Eq}}{g_{oq} + \left(\beta_q + 1\right) G_{C}^{'}} \qquad G_{C}^{'} = G_{Cq} + G_L$$

$$G_o = G_o^{'} + G_{Cq} \qquad R_o = R_o^{'} \parallel R_{Cq}$$

$$G_o^{'} = \frac{g_{oq} G_{xq}}{g_{mq} + g_{oq} + G_{xq}} \qquad R_o^{'} = R_{xq} + r_{oq} + g_{mq} r_{oq} R_{xq}$$

$$G_{xq} = g_{\pi q} + G_{Eq}$$

$$G_m = \frac{g_{\pi q} g_{oq} - g_{mq} G_{Eq}}{g_{mq} + g_{oq} + G_{Eq}}$$

$$A_{vo} = G_m R_o \qquad A_v = \frac{G_m}{G_o + G_L}$$

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# BJT Common Collector Amplifier w/o Collector Resistor

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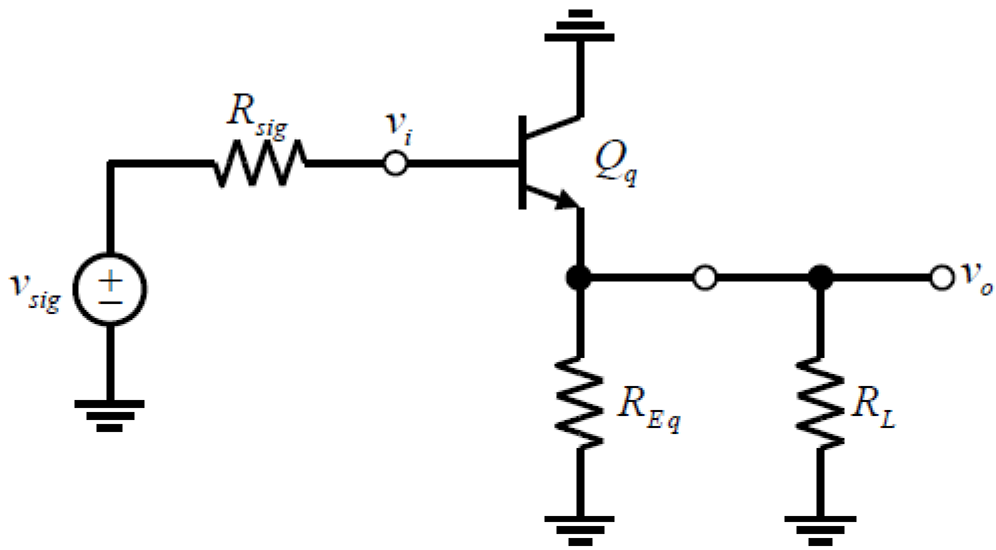



$$R_{in} = r_{\pi q} + R_{in}^{'} \qquad G_{in} = \frac{g_{\pi} G_{in}^{'}}{g_{\pi} + G_{in}^{'}}$$

$$G_{in}^{'} = \frac{g_{oq} + G_{Eq}^{'}}{\left(\beta_q + 1\right)} \qquad G_{Eq}^{'} = G_{Eq} + G_L$$

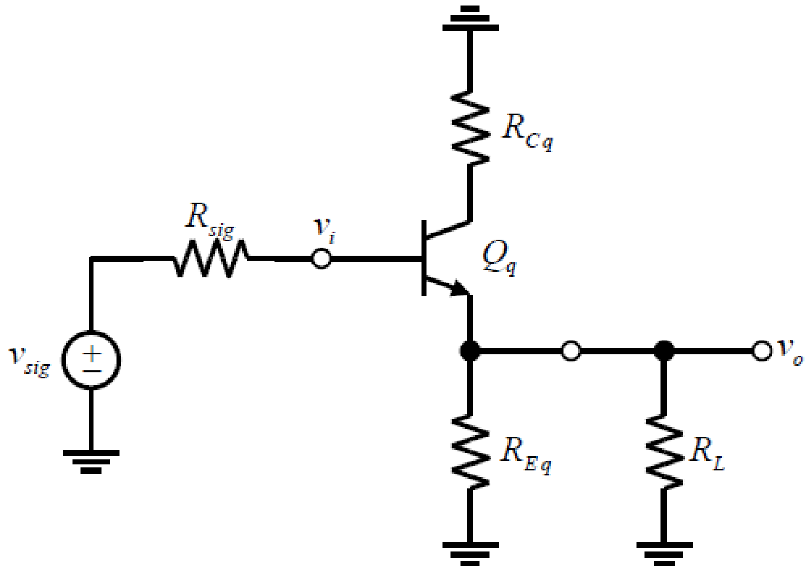
$$G_o = G_o^{'} + G_{Eq} \qquad R_o = R_o^{'} \parallel R_{Eq}$$

$$G_o^{'} = g_{mq} + g_{\pi q} + g_{oq}$$

$$G_m = g_{mq} + g_{oq}$$

$$A_{vo} = G_m R_o \qquad A_v = \frac{G_m}{G_o + G_L}$$

# BJT Common Collector Amplifier w/ Collector Resistor




$$R_{in} = r_{\pi q} + R_{in}^{'} \qquad G_{in} = \frac{g_{\pi} G_{in}^{'}}{g_{\pi} + G_{in}^{'}}$$

$$G_{in}^{'} = \frac{\left(G_{Eq}^{'} + G_{Cq}\right) g_{oq} + G_C G_{Eq}^{'}}{g_{oq} + \left(\beta_q + 1\right) G_C} \qquad G_{Eq}^{'} = G_{Eq} + G_L$$

$$G_o = G_o^{'} + G_{Eq} \qquad R_o = R_o^{'} \parallel R_{Eq}$$

$$G_o^{'} = g_{\pi} + \frac{\left(g_{mq} + g_{oq}\right) G_{Cq}}{g_{oq} + G_{Cq}}$$

$$G_m = \frac{g_{mq} G_{Cq} + g_{\pi q}\left(g_{oq} + G_{Cq}\right)}{g_{oq} + G_{Cq}}$$

$$A_{vo} = G_m R_o \qquad A_v = \frac{G_m}{G_o + G_L}$$

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# BJT Common Base Amplifier w/o Base Resistor

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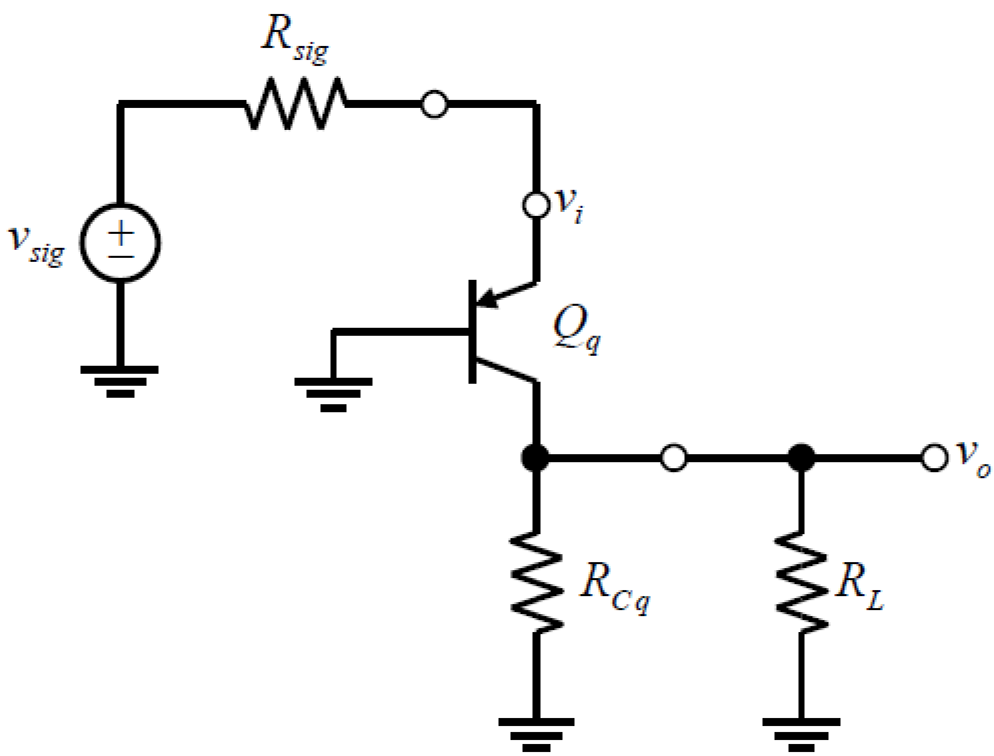


$$R_{in} = r_{\pi q} + R_{in}^{'} \qquad G_{in} = \frac{g_{\pi} G_{in}^{'}}{g_{\pi} + G_{in}^{'}}$$

$$G_{in}^{'} = \frac{\left(g_{mq} + g_{oq}\right)\left(G_{Cq} + G_L\right)}{g_{oq} + G_{Cq} + G_L}$$

$$G_o = g_{oq} + G_{Cq} \qquad R_o = r_{oq} \parallel R_{Cq}$$

$$G_m = g_{mq} + g_{oq}$$

$$A_{vo} = G_m R_o \qquad A_v = \frac{G_m}{G_o + G_L}$$

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# BJT Common Base Amplifier w/ Base Resistor

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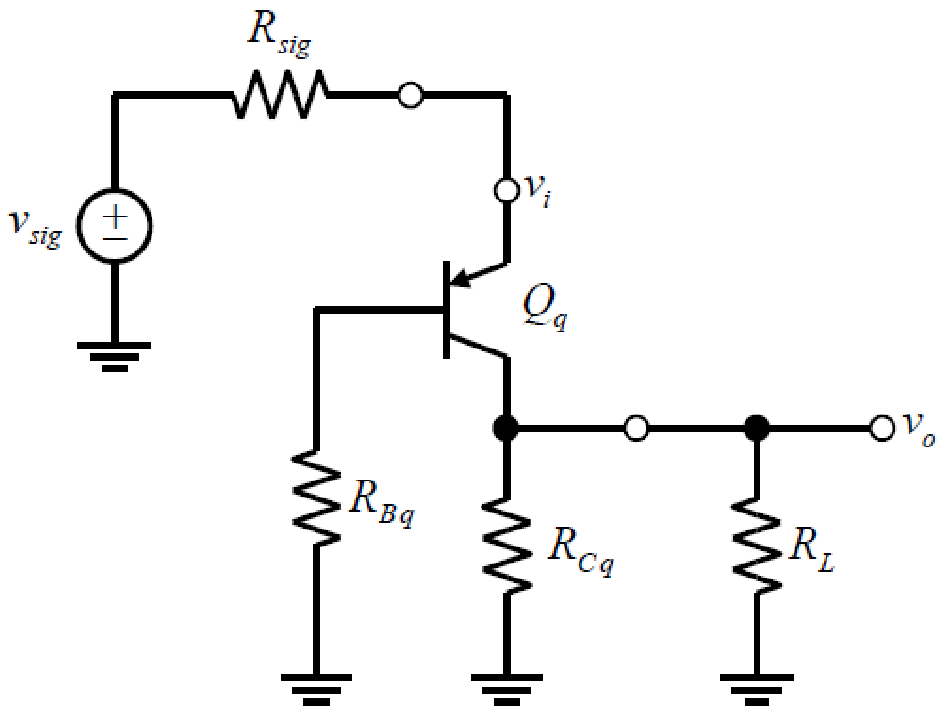



$$R_{in} = r'_{\pi q} \parallel R'_{in} \qquad G_{in} = g'_{\pi} + G'_{in}$$

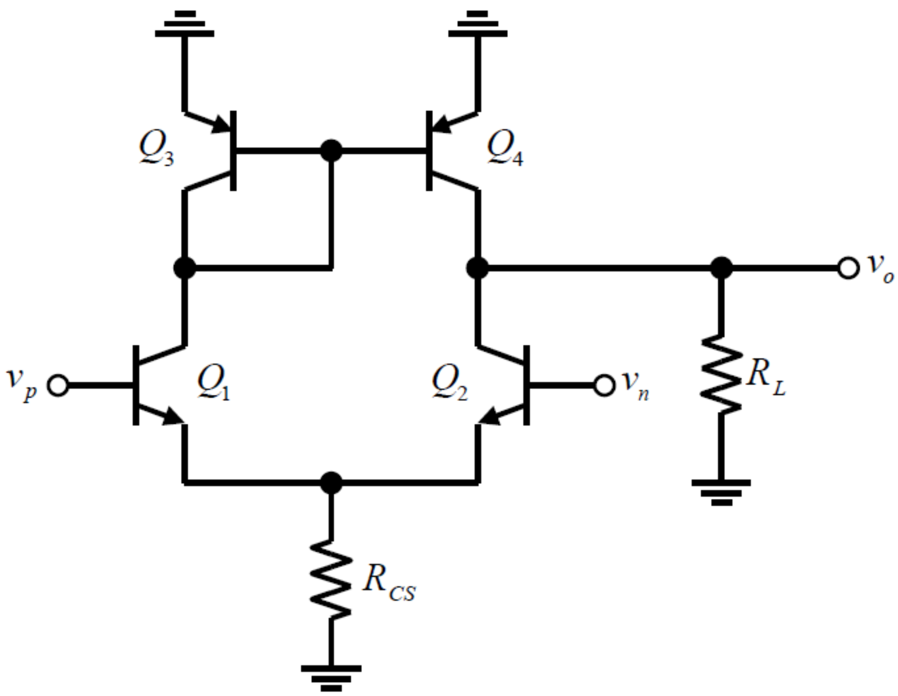
$$G'_{in} = \frac{\left(g'_{mq} + g_{oq}\right)\left(G_{Cq} + G_L\right)}{g_{oq} + G_{Cq} + G_L} \qquad g'_{\pi q} = \frac{g_{\pi} G_{Bq}}{g_{\pi q} + G_{Bq}} \qquad g'_{mq} = \frac{g'_{\pi q} g_{mq}}{g_{\pi q}}$$

$$G_o = g_{oq} + G_{Cq} \qquad R_o = r_{oq} \parallel R_{Cq}$$

$$G_m = g'_{mq} + g_{oq}$$

$$A_{vo} = G_m R_o \qquad A_v = \frac{G_m}{G_o + G_L}$$

# BJT Differential Amplifier




$$R_{in} = 2r_{\pi 2} \qquad G_{in} = g_{\pi}/2$$

$$G_o = Xg_{o2} + g_{o4} \qquad R_o = \frac{1}{Xg_{o2} + g_{o4}}$$

$$G_m = Xg_{m2}$$

$$A_{vo} = G_m R_o \qquad A_v = \frac{G_m}{G_o + G_L}$$

$$X = \frac{1}{2}\left[1 + \frac{A_{E4}}{A_{E3}}\right]$$