We will now be attempting to model a playthrough of what a game would look like.

As stated before, each player has a 6x6 grid and 3 ships. The positions of the ships are shown in figure 1. The red blocks signify the positions taken by the Battleships, the blue blocks represent the positions taken by the Cruisers and the green blocks represent the position taken by the Destroyers.

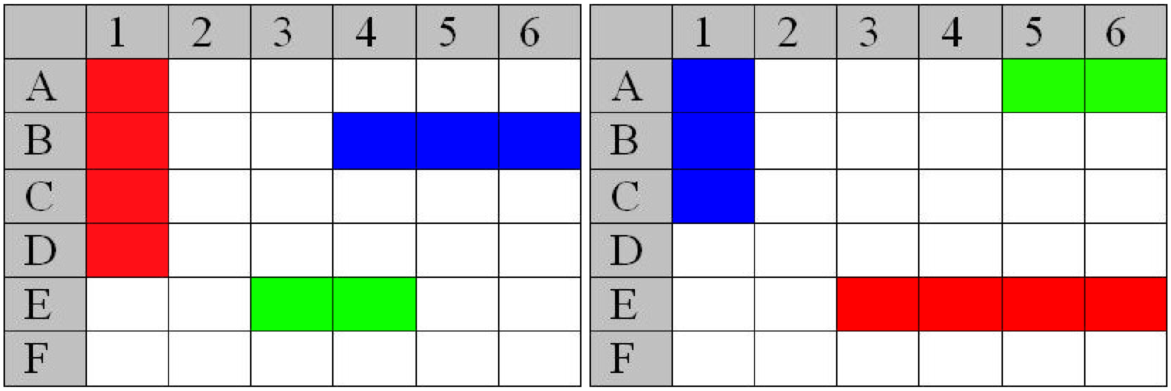

Figure 1: Board of agent 1 on the left. Board of agent 2 on the right.

Round 1 starts and agent 1 goes first. Agent 1 announces $\neg K_1 p_{Ship}(A, 6, 2)$. The world is unchanged, but agent 2 now replies with the announcement $\neg K_1 K_2 p_{Ship}(A, 6, 2)$. From the original $M \models \neg K_1 p_{Ship}(A, 6, 2)$, after the announcement we get $M', \neg K_1 K_2 p_{Ship}(A, 6, 2) \models C p_{Ship}(A, 6, 2) \wedge K_1(p_{Ship}(A, 5, 2) \vee p_{Ship}(B, 6, 2))$, because it is common knowledge that the ships are positioned either vertically or horizontally, and since the ship wasn't sunk, that means that there are more parts left. It is now the turn of agent 2. Agent 2 announces $\neg K_2 p_{Ship}(A, 1, 1)$. The world is unchanged, but agent 1 now replies with the announcement $\neg K_2 K_1 p_{Ship}(A, 1, 1)$. From the original $M \models \neg K_2 p_{Ship}(A, 1, 1)$, after the announcement we get $M', \neg K_2 K_1 p_{Ship}(A, 1, 1) \models C p_{Ship}(A, 1, 1) \wedge K_2(p_{Ship}(A, 2, 1) \vee p_{Ship}(B, 1, 1))$. Round 1 is over.

After round 1, the field, marked with the new knowledge looks as seen in figure 2. The yellow square indicates common knowledge about the slot.

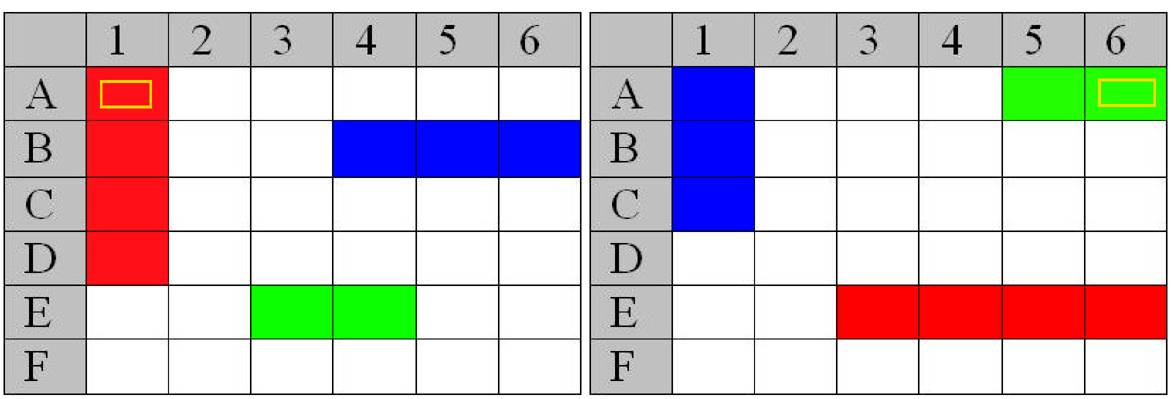

Figure 2: Board of agent 1 on the left. Board of agent 2 on the right.

Round 2 begins and agent 1 goes again. Agent 1 announces $\neg K_1 p_{Ship}(A,5,2)$ and agent 2 replies with $\neg K_1 K_2 p_{Ship}(A,5,2) \wedge \neg K_1 K_2 K_1 p_{Ship_{2,3}}$, which results in a change from $M' \models Cp_{Ship}(A,6,2) \wedge K_1(p_{Ship}(A,5,2) \vee p_{Ship}(B,6,2))$ to $M'', \neg K_1 K_2 p_{Ship}(A,5,2) \wedge \neg K_1 K_2 K_1 p_{Ship_{2,3}} \models Cp_{Ship}(A,6,2) \wedge Cp_{Ship}(A,5,2) \wedge Cp_{Ship_{2,3}}$. The turn of agent 1 is over and agent 2 begins with announcing $\neg K_2(p_{Ship}(A,2,1)$. Agent 1 replies with $\neg K_2 K_1 \neg(p_{Ship}(A,2,1)$. From $M' \models Cp_{Ship}(A,1,1) \wedge K_2(p_{Ship}(A,2,1) \vee p_{Ship}(B,1,1))$, we end up with $M'', \neg K_2 K_1 \neg(p_{Ship}(A,2,1) \models Cp_{Ship}(A,1,1) \wedge C\neg(p_{Ship}(A,2,1) \wedge K_2 p_{Ship}(B,1,1))$. Round 2 ends.

The results of round 2 can be seen in figure 3. The black circle indicates common knowledge of the absence of a ship on the given slot and the purple square indicates knowledge of both agents that a ship is there, but not common knowledge yet.

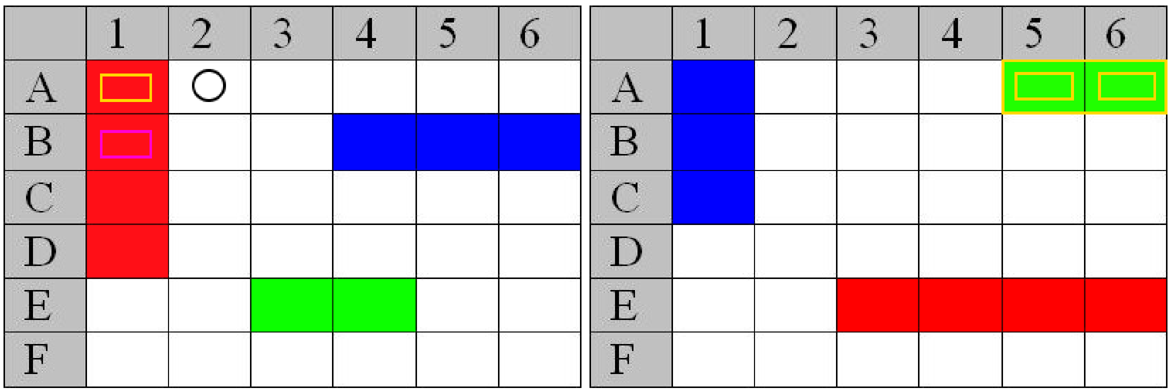

Figure 3: Board of agent 1 on the left. Board of agent 2 on the right.

Round 3 begins. Agent 1 announces $\neg K_1 p_{Ship}(C,6,2)$ and agent 2 replies with $\neg K_1 K_2 \neg p_{Ship}(C,6,2)$. The state of knowledge after the announcement becomes $M''', \neg K_1 K_2 \neg p_{Ship}(C,6,2) \models Cp_{Ship}(A,6,2) \wedge Cp_{Ship}(A,5,2) \wedge Cp_{Ship_{2,3}} \wedge C\neg p_{Ship}(C,6,2)$. Agent 2 announces $K_2 p_{Ship}(B,1,1))$ to which Agent 1 replies with $K_1 K_2 p_{Ship}(B,1,1))$, which acts more as an acknowledgement. The reason why this was necessary is because there could still be more ship parts(and there are). If that was the whole ship the reply announcement would've included that as well. The world still changes after that announcement and we now have $M''', K_1 K_2 p_{Ship}(B,1,1)) \models Cp_{Ship}(A,1,1) \wedge C\neg(p_{Ship}(A,2,1) \wedge Cp_{Ship}(B,1,1)) \wedge K_2 p_{Ship}(C,1,1)$..

As modelling every single round would take too long we will be doing a skip, while explaining what happens meanwhile and stop at relevant points that showcase different noteworthy behaviours.

Round 4 starts. During round 4, agent 1 finds out that D6 is empty and agent 2 confirms that it's common knowledge that there's a ship at C1. Agent 2 also now knows that there's a ship at D1 and it knows that that is the battleship ($Ship_{1,1}$), because the longest ship has length 4 and that is the length of the current ship's span. Round 4 ends and round 5 starts.

During round 5, agent 1 announces $negK_1p_{Ship}(E,6,2)$ and agent 2 replies $negK_1K_2p_{Ship}(E,6,2)$. This causes the update $M^{(5)}, negK_1K_2p_{Ship}(E,6,2) \models Cp_{Ship}(A,6,2) \wedge Cp_{Ship}(A,5,2) \wedge Cp_{Ship_{2,3}} \wedge C\neg p_{Ship}(C,6,2) \wedge C\neg p_{Ship}(D,6,2) \wedge Cp_{Ship}(E,6,2) \wedge K_1p_{Ship}(E,5,2)$, the last part is because in the case of the remaining ships, F6 would not fit since we already know that D6 is empty and that A5 and A6 hosts the length 2 ship, so that only leaves E5 as a block with a ship on it. Then Agent 2 announces $K_2p_{Ship}(D,1,1)$, agent 1 replies with $K_1K_2p_{Ship}(D,1,1) \wedge K_1K_2K_1p_{Ship_{1,1}}$, which is an acknowledgement of $Ship_{1,1}$ becoming common knowledge. The state for agent 2 now looks like this: $M^{(5)}, K_1K_2p_{Ship}(D,1,1) \wedge K_1K_2K_1p_{Ship_{1,1}} \models Cp_{Ship}(A,1,1) \wedge C\neg(p_{Ship}(A,2,1) \wedge Cp_{Ship}(B,1,1)) \wedge Cp_{Ship}(C,1,1) \wedge Cp_{Ship}(D,1,1) \wedge Cp_{Ship_{1,1}}$..

We can see how the board looks at the end of round 5 in figure 4.

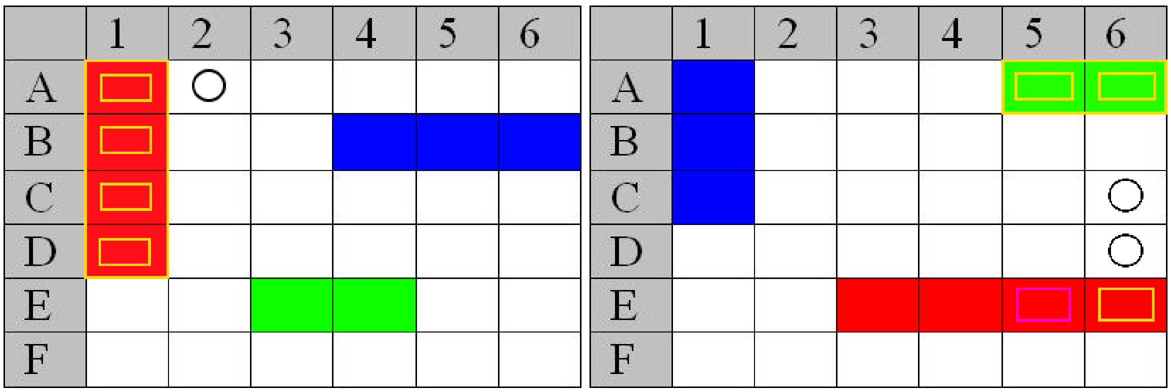

Figure 4: Board of agent 1 on the left. Board of agent 2 on the right.

Round 6 begins. During this round, agent 1 confirms that E5 has a ship part on it and learns that E4 has a ship part too and agent 2 finds out that D2 is empty. Round 6 ends and round 7 begins.

During round 7, agent 1 confirms that E4 has a ship on it and learns that E3 also has a ship on it and that the ship that spans E3-E6 is the Battleship ($Ship_{2,1}$). Agent 2, on the other hand, announces $\neg K_2p_{Ship}(E,3,1)$, to which Agent 1 replies $\neg K_2K_1p_{Ship}(E,3,1)$, which prompts the state to change into $M^{(7)}, \neg K_2K_1p_{Ship}(E,3,1) \models Cp_{Ship}(A,1,1) \wedge C\neg(p_{Ship}(A,2,1) \wedge Cp_{Ship}(B,1,1)) \wedge Cp_{Ship}(C,1,1) \wedge Cp_{Ship}(D,1,1) \wedge Cp_{Ship_{1,1}} \wedge C\neg p_{Ship}(D,2,1) \wedge Cp_{Ship}(E,3,1) \wedge K_2(p_{Ship}(E,2,1) \vee p_{Ship}(E,4,1) \vee p_{Ship}(D,3,1) \vee p_{Ship}(F,3,1))$, because none of the adjacent tiles have been verified and the ships can be vertical or horizontal.

Round 8 starts and agent 1 confirms that E3 is part of a ship. Agent 2 responds by confirming and confirming that agent 1 now knows the location of $Ship_{2,1}$. Afterwards, agent 2 announce that he doesn't know if there's a ship part on E4. Agent 1 confirms and also announces that agent 2 now knows the location of $Ship_{1,3}$. The state of the boards at the end of round 8 can be seen in figure 5.

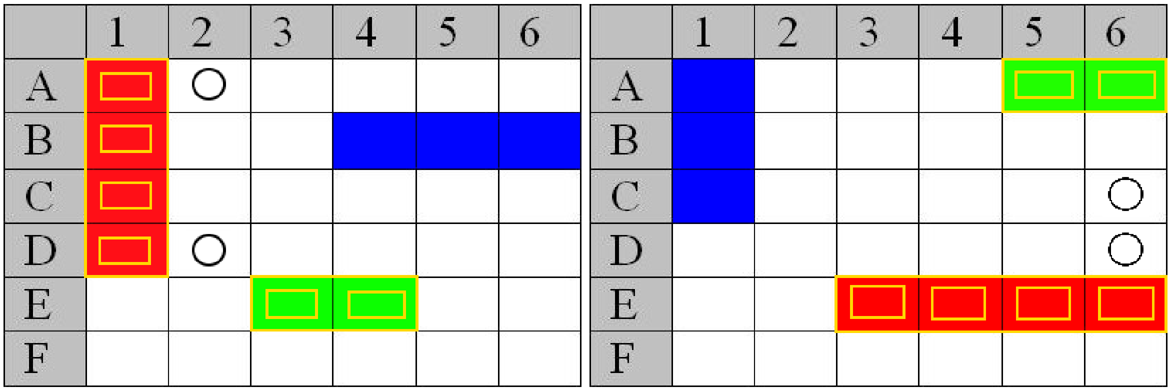

Figure 5: Board of agent 1 on the left. Board of agent 2 on the right.

Round 9 happens and agent 1 finds out that there's nothing on B2, while agent 2 finds out that there's nothing on C4.

Round 10 starts and agent 1 finds out that there's a ship piece on B1, while agent 2 finds out that there's a ship piece on B5.

Round 11 happens and agent 1 finds out that there's a ship piece on C1, while agent 2 finds out that there's a ship piece on B4. There is only 1 ship piece left to be found on each side and 2 possible spots for each. In the case of agent 1, the ship part could either be on A1 or on D1. For agent 2, the ship part could be on B3 or on B6. Both agents have equal chances of winning. We can see the current state of the board in figure 6, where "?" means that an agent knows that a ship part could be there.

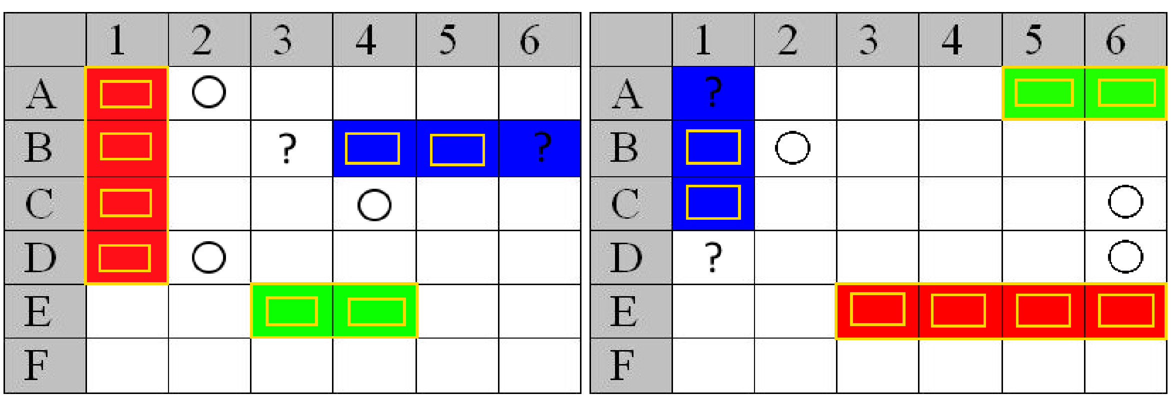

Figure 6: Board of agent 1 on the left. Board of agent 2 on the right.

Obviously, with the state being the way it is, either the next round will be the final one, with either or both potentially winning, or the round after it ending in a draw. Assuming we are still on round 12, the potential outcomes are: For agent 1 to win, agent 1 needs to guess A1 and agent 2 needs to guess B3; For agent 2 to win, agent 1 needs to guess D1 and agent 2 needs to guess B6; For it to be a draw, agent 1 needs to guess A1 and agent 2 needs to guess B6; To continue in round 13 and end up with a draw, for round 12 agent 1 needs to guess D1 and agent 2 needs to guess B3.

For the sake of having a proper finale modeled, we will go with the scenario of agent 1 winning. The round will play out as follows: agent 1 announces $\neg K_1 p_{Ship}(A,1,2)$, agent 2 replies with $\neg K_1 K_2 p_{Ship}(A,1,2) \wedge \neg K_1 K_2 K_1 p_{Ship_{2,2}}$, the model gets updated to $M^{(12)}, \neg K_1 K_2 p_{Ship}(A,1,1) \wedge \neg K_1 K_2 K_1 p_{Ship_{2,2}} \models Cp_{Ship_{2,1}} \wedge Cp_{Ship_{2,2}} \wedge Cp_{Ship_{2,3}}$; then agent 2 announces $\neg K_2 p_{Ship}(B,3,1)$, agent 1 replies $\neg K_2 K_1 \neg p_{Ship}(B,3,1)$ and on top of what was previously modeled we also get $M^{(12)}, \neg K_2 K_1 \neg p_{Ship}(B,3,1) \models C\neg p_{Ship}(B,3,1) \wedge K_2 p_{Ship}(B,6,1)$. Since all of the ships of agent 2 are common knowledge now, the game is over and agent 1 wins. The final state of the board can be seen in figure 7.

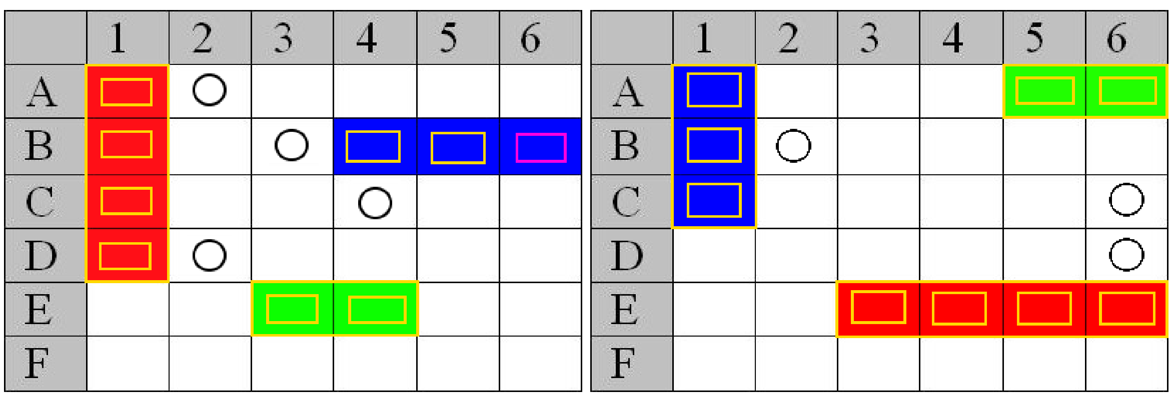

Figure 7: Board of agent 1 on the left. Board of agent 2 on the right.