Action Models

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1 Proposition

We define atoms P here:
$p_{Hit}(x,y) = t$ iff "There is a part of any ship on grid(x,y)"
$p_{Ship_n}(x,y) =$ "There is a part of ship n on grid (x,y)"

and $P = \{p_{Hit}(x,y)|x \in Xy \in Y\}$
Where $X = \{1,2,3,4,5,6\}$ and $Y = \{A,B,C,D,E,F,G\}$

There are 3 ships in 6x6 grid:
$Ship_1$ with size of 1x2
$Ship_2$ with size of 1x3
$Ship_3$ with size of 1x4

The proposition p_{Hit} is also $\vee_{n=1}^{3}(p_{ship_n})$. But a grid can only be used by one ship and does not allow overlap so $p_{ship_n} \rightarrow \neg \vee (p_{ship_m})$ where $m \neq n$, holds through the game

There are 2 agents: $A = \{1,2\}$
Then we have $L_{KCotimes}^{stat}(A,P)$ that is a union of *formula* and *epistemic actions*

$M = \{S, \pi, R_1, R_2\}$
where S is the all possible states s and $s = \{s_1, s_2\}$ where s_1 is the player1's broad and s_2 is the player2's broad and also $s_n = \{s_{(1,A)}, ...s_{(6,G)}\}$ where $s_{(x,y)}$ represent state of a particular position on the broad.
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$R_1 = (s_1(x,y), s_1(x,y))$ for all (x,y) and $(s_2(x,y), z_2(x,y))$ for all $z_2 \in S$
$R_2 = (s_2(x,y), s_2(x,y))$ for all (x,y) and $(s_1(x,y), z_1(x,y))$ for all $z_1 \in S$. In other words, relation represents reflection within its own broad state and all state with the same coordinate for other agent's broad.

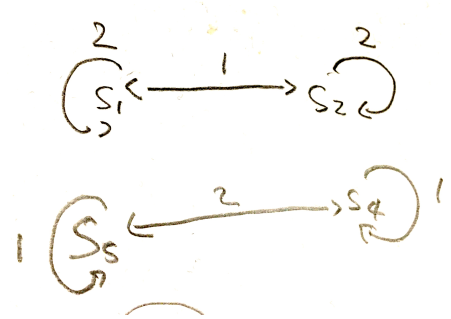
2 Actions

we will be creating a action model:
$[pub(\varphi)]$ which is equivalent to $[\varphi]$
Then agents *shoot* by requesting a grid coordinate to other agent. This actions should be always be able to perform so $pre[[shoot]] = \top$. The action *hit* is possible when the agent announcing that requested grid occupied so and *miss* is when requested grid is empty.

3 Procedure

For our convenience, we introduce Exclusive operator $\oplus$. It is in valid in $L^{stat}_{KC\otimes}(A, P)$, because: $p \oplus q = \neg(p \wedge q) \wedge (p \vee q)$
The rules of the game suggest following formula:
$p_{hit}(x, y) \leftrightarrow \vee(p_{ship_n}(x, y)) \wedge \neg \wedge (p_{ship_n}(x, y))$
$p_{ship_n}(x, y) \to \neg(p_{ship_n}(x, y) \vee p_{ship_n}(x, y))$ with $m, z \neq n$: No overlap
$p_{ship_1}(x, y) \to p_{ship_1}(x+1, y) \oplus p_{ship_1}(x, y+1) \oplus p_{ship_1}(x-1, y) \oplus p_{ship_1}(x, y-1)$
Ship 1 only covers 2 grids
$p_{ship_2} \to$ Ship 2 only covers 3 grids
$p_{ship_3} \to$ Ship 3 only covers 4 grids

The game starts with both agents not knowing anything about the other agents' broad. so for an instance for state $s^0_1(x, y)$, it always holds that
$(M, s_1(x, y)) \models \neg K_2 P_{Hit}(x, y)$
From semantic definition of the knowledge operator
$(M, s_1(x, y)) \not\models K_2 P_{Hit}(x, y)$
$(M, t) \not\models P_{Hit}(x, y)$ for at least one of the states in t s.t. $(s_1(x, y), t) \in R_2$
and since there is are relation to all possible state $s_1(x, y)$ for agent2, there is a state that this formula holds.



The figure representing simplified states where for $s_1|p_{hit}, s_2|\neg p_{hit}$ and are the state of agent2's broad on one grid and $s_3|p_{hit}, s_4|\neg p_{hit}$ are state of agent1's broad. And this state is expanded for all grid. This suggests that for every action the agent takes, hit or miss, the global state is a least 1/4 for given grid and also in term of goal. The agents want to know only the other agents broad, then the interested state is halved. By knowing that one grid is not occupied, the agent can eliminate $2 * (2 + 3 + 4)$

global state.
By knowing that one gird is occupied, the agent knows that there must be a ship. $M, s \models K_i p_{hit}(x, y)$. .