Differentiation

Partial Differentiation in Linear System
(선형시스템에서의 편미분, aka 행렬미분)

소프트웨어 꼰대 강의

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Course Overview

Topic	Contents
01. Orientation 오리엔테이션	Course introduction, motivations, final objectives 과정 소개, 동기부여, 최종 목표
02. Learning in deeplearning 딥러닝 학습	How does the deeplearing learns knowledge from data 어떻게 딥러닝은 데이터로부터 지식을 배우는가?
03. Principle of differentiation 미분의 원리	Basics of differentiation (concepts, notation, operations) 미분 기본지식 (개념, 표기, 연산)
04. Partial differentiation 편미분	Concept & operation of partial differenciation 편미분 개념, 연산
05. Gradient descent 경사 하강법	Concept, interpretation and learning in gradient descent 경사하강 알고리즘 개념, 해석 및 학습
06. Chain rule 연쇄법칙	Concept & operation of chain rule 연쇄법칙 개념 및 연산
07. Matrix differentiation 행렬미분	Partial differentiation in linear system 선형시스템에서의 편미분
08. Back propagation 역전파 학습	The mechanism of back propagation 역전파 학습의 작동 방법
09. Gradient vanishing 기울기 소실	Quick overview on activation function, cause root of gradient vanishing and its counter-measure 활성함수 간단 소개, 기울기 소실 근본원인과 대책

Beautiful References - 강추!

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  - https://www.youtube.com/watch?v=tIeHLnjs5U8
- **Brent Scarff, Understanding Backpropagation, Web blog, Jan. 13, 2021.**
  - A visual derivation of the equations that allow neural networks to learn
  - https://towardsdatascience.com/understanding-backpropagation-abcc509ca9d0
- **Miclael Nielsen, How the backpropagation algorithm works, Sept. 2019.**
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- **42bsk, Is there an easier way to compute derivatives of vector-valued functions with respect to parameter matrices in Pytorch?, StackOverflow, May 18, 2023.**
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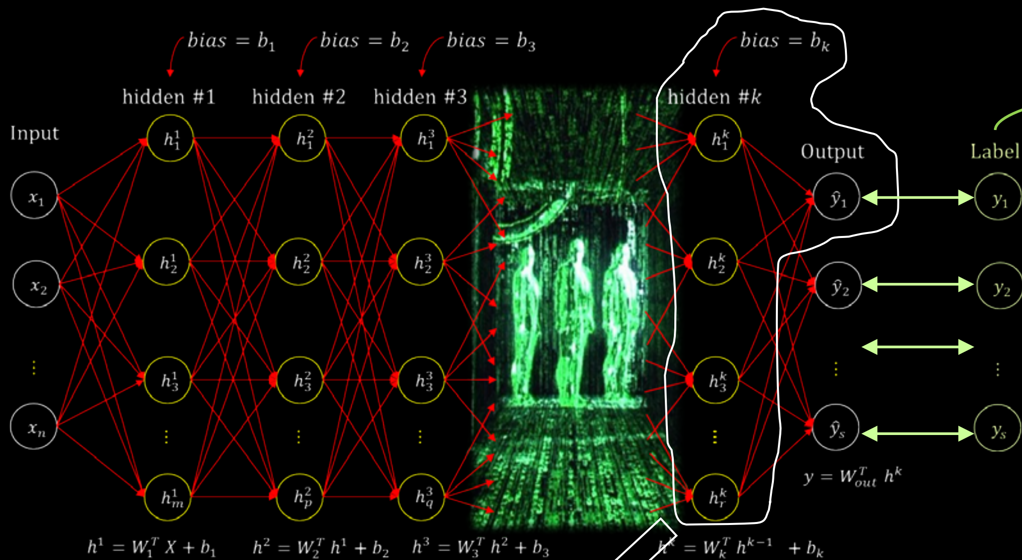
  - https://tutorials.pytorch.kr/beginner/blitz/autograd_tutorial.html

변수(파라미터)가 겁나게 많은 경우



Loss:
How much
DL is wrong

$$L(\theta) = \sum_{i=1}^{n} \boxed{(\hat{y}_i - y_i)^2}$$

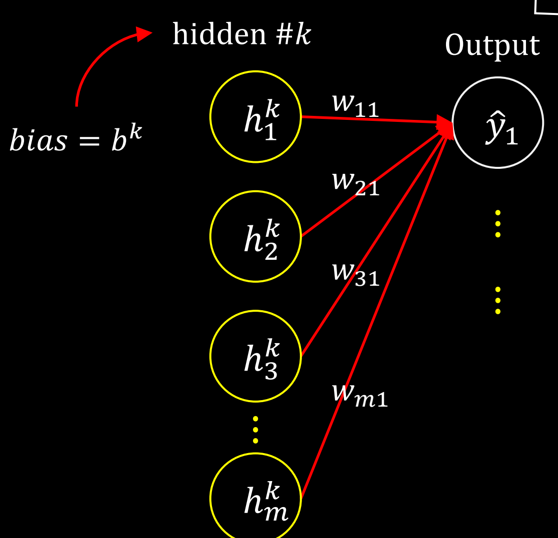
w_{11}에 대하여 미분

w_{21}에 대하여 미분

⋮

w_{m1}에 대하여 미분

$\hat{y}_1$에 대입



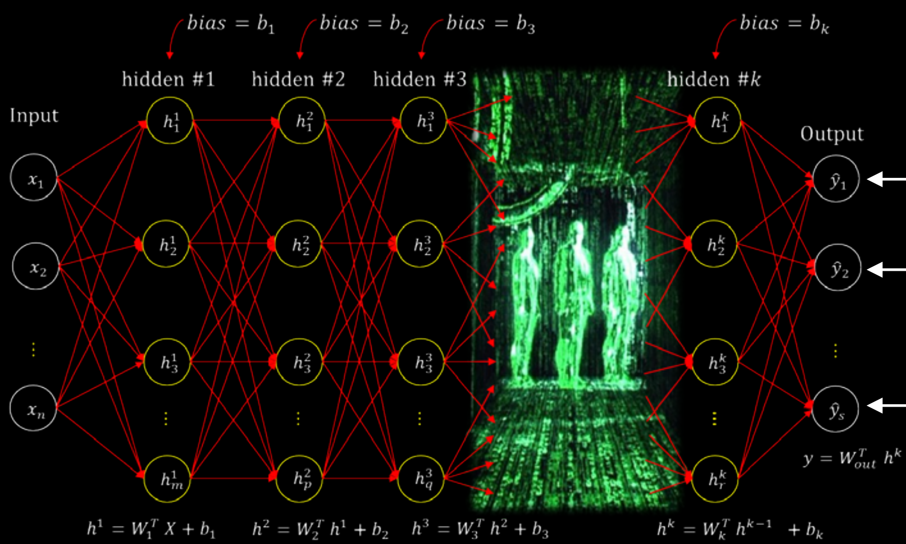
$$\hat{y}_1 = \boxed{w_{11}h_1^k + w_{21}h_2^k + \cdots + w_{m1}h_m^k + b_1^k}$$

$$= (w_{11} \quad w_{12} \quad \cdots \quad w_{m1}) \begin{pmatrix} h_1^k \\ h_2^k \\ \vdots \\ h_m^k \end{pmatrix} + b_1^k$$

어차피 선형 시스템
즉, matrix 이다.

뭔가 한번에 처리할 방법이 필요하다.

딥러닝에서의 미분구조



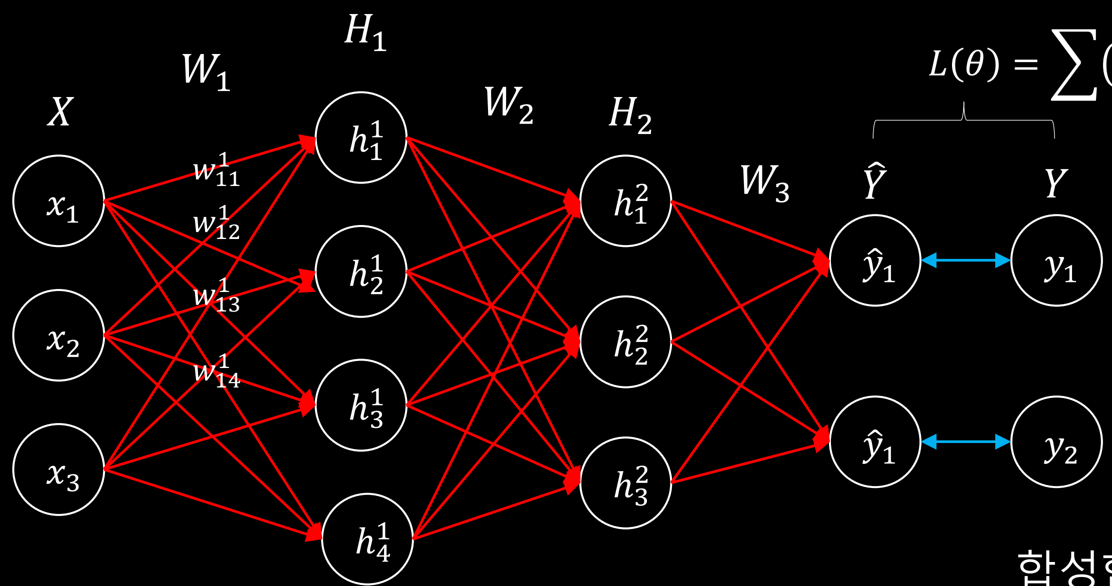
Loss:
How much
DL is wrong

$$L(\theta) = \sum_{i=1}^{n} (\hat{y}_i - y_i)^2$$

그림이 복잡하네요...

좀 더 간단히 표현해 보겠습니다 ^^

선형 시스템을 합성함수로 바라보기



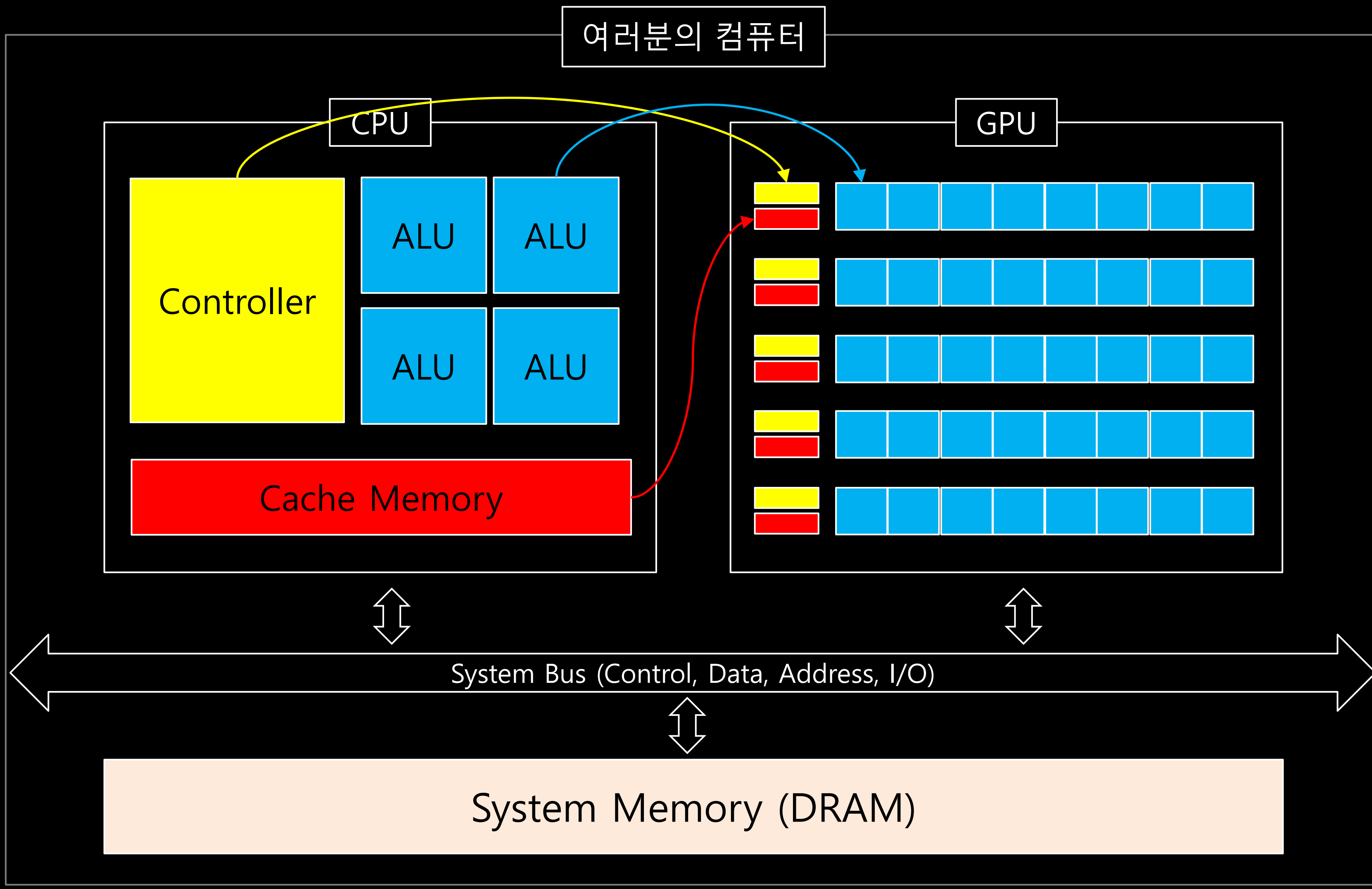
$$L(\theta) = \sum \left(\hat{Y} - Y\right)^2 = \sum_{i=1}^{n} (\ \hat{y}_i\ -\ y_i\ )^2$$

이제 준비 끝~

합성함수의 미분은 이미 배웠다!

$H_1 = W_1^T X$	$H_2 = W_2^T H_1$ $= W_1^T (W_1^T X_1)$	$\hat{Y} = W_3^T H_2$ $= W_3^T \left(W_1^T (W_1^T X_1) \right)$	$L(\theta) = \sum \left(\hat{Y} - Y\right)^2$ $= \sum \left(W_3^T \left(W_1^T (W_1^T X_1)\right) - Y \right)^2$
$H_1 = f(X)$	$H_2 = g(f(X))$ $= g \circ f$	$H_2 = p\left(g(X)\right)$ $= p \circ g \circ f$	$L = l\left(p(X)\right) = l(p(g(f(X)))$ $= l \circ p \circ g \circ f$

한꺼번에 처리? - Computer Architecture (a.k.a 컴구)



선형 시스템의 미분 (partial derivative scalor w.r.t vector)

Scalor를 vector로 미분

[Note]

∇ : 'nabla (나블라)' 라고 읽음

편미분을 간단하게 표현하는 기호^^

$$X = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} \xrightarrow{f : \mathbb{R}^n \rightarrow \mathbb{R}} Y = y$$

$$Y = f(X) = f\left(\begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}\right)$$

$$\nabla_X Y = \frac{\partial Y}{\partial X} = \begin{pmatrix} \dfrac{\partial y}{\partial x_1} \\ \dfrac{\partial y}{\partial x_2} \\ \vdots \\ \dfrac{\partial y}{\partial x_n} \end{pmatrix}$$

$\mathbb{R}^n$

선형 시스템의 미분 (partial derivative scalor w.r.t matrix)

Scalor를 matrix로 미분

[Note]

∇ : 'nabla (나블라)' 라고 읽음

편미분을 간단하게 표현하는 기호^^

$$X = \begin{pmatrix} x_{11} & \cdots & x_{1m} \\ \vdots & \ddots & \vdots \\ x_{n1} & \cdots & x_{nm} \end{pmatrix} \xrightarrow{f : \mathbb{R}^{n\times m} \to \mathbb{R}} Y = y$$

$$Y = f(X) = f\left(\begin{pmatrix} x_{11} & \cdots & x_{1m} \\ \vdots & \ddots & \vdots \\ x_{n1} & \cdots & x_{nm} \end{pmatrix}\right)$$

$$\nabla_X Y = \frac{\partial Y}{\partial X} = \begin{pmatrix} \frac{\partial y}{\partial x_{11}} & \cdots & \frac{\partial y}{\partial x_{1m}} \\ \vdots & \ddots & \vdots \\ \frac{\partial y}{\partial x_{n1}} & \cdots & \frac{\partial y}{\partial x_{nm}} \end{pmatrix}$$

$\mathbb{R}^{n\times m}$

선형 시스템의 미분 (partial derivative vector w.r.t vector)

Vector를 vector로 미분

$$X = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} \qquad Y = \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_m \end{pmatrix}$$

$$\begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_m \end{pmatrix} = Y = f(X) = f\left(\begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}\right)$$

$$f : \mathbb{R}^n \rightarrow \mathbb{R}^m$$

Recommendation: a beautiful video:
https://youtu.be/FCWrduAxf-Q?si=uPc3QjIMwUCIud_-

[Note]

∇ : 'nabla (나블라)' 또는 'del(델)' 라고 읽음

편미분을 간단하게 표현하는 기호^^

Scalor를 vector로 미분하는 것과 똑같다.

$$\nabla_X Y = \frac{\partial Y}{\partial X} = \left(\boxed{\frac{\partial y_1}{\partial X}}, \boxed{\frac{\partial y_2}{\partial X}}, \cdots, \boxed{\frac{\partial y_m}{\partial X}} \right)$$

$\mathbb{R}^{n \times m}$

$$= \begin{pmatrix} \boxed{\begin{matrix} \frac{\partial y_1}{\partial x_1} \\ \vdots \\ \frac{\partial y_1}{\partial x_n} \end{matrix}} & \boxed{\begin{matrix} \frac{\partial y_2}{\partial x_{11}} \\ \vdots \\ \frac{\partial y_2}{\partial x_n} \end{matrix}} & \begin{matrix} \cdots \\ \ddots \\ \cdots \end{matrix} & \boxed{\begin{matrix} \frac{\partial y_m}{\partial x_1} \\ \vdots \\ \frac{\partial y_m}{\partial x_n} \end{matrix}} \end{pmatrix}$$

선형 시스템의 미분 (partial derivative vector w.r.t matrix)

Vector를 matrix로 미분

[Note]

∇ : 'nabla (나블라)' 또는 'del(델)' 라고 읽음

편미분을 간단하게 표현하는 기호^^

$$X = \begin{pmatrix} x_{11} & \cdots & x_{1m} \\ \vdots & \ddots & \vdots \\ x_{n1} & \cdots & x_{nm} \end{pmatrix} \xrightarrow{f: \mathbb{R}^{n\times m} \to \mathbb{R}^m} Y = \begin{pmatrix} y_1 \\ \vdots \\ y_m \end{pmatrix}$$

$$\begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_m \end{pmatrix} = Y = f(X) = f\left(\begin{pmatrix} x_{11} & \cdots & x_{1m} \\ \vdots & \ddots & \vdots \\ x_{n1} & \cdots & x_{nm} \end{pmatrix}\right)$$

Scalor를 matrix로 미분하는 것과 똑같다.

$$\nabla_X Y = \frac{\partial Y}{\partial X} = \left(\overbrace{\frac{\partial y_1}{\partial X}, \cdots, \frac{\partial y_m}{\partial X}} \right)$$

다음 슬라이드로....

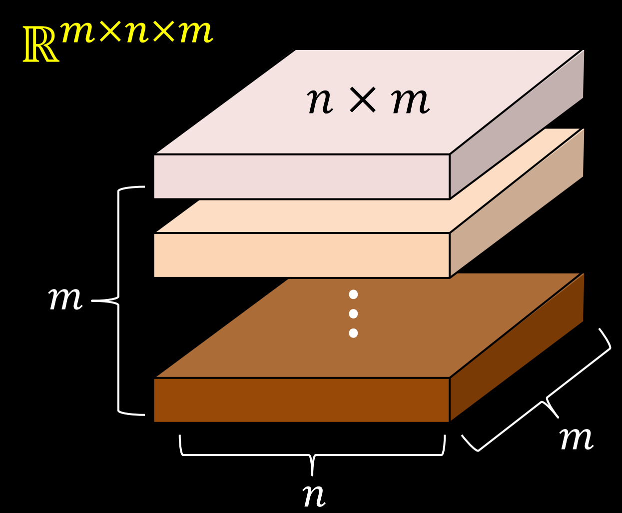
Recommendation: a beautiful video: https://youtu.be/7oDHThXECt8?si=8U43T_77nvXX32vv

선형 시스템의 미분 (partial derivative vector w.r.t matrix) - cont.

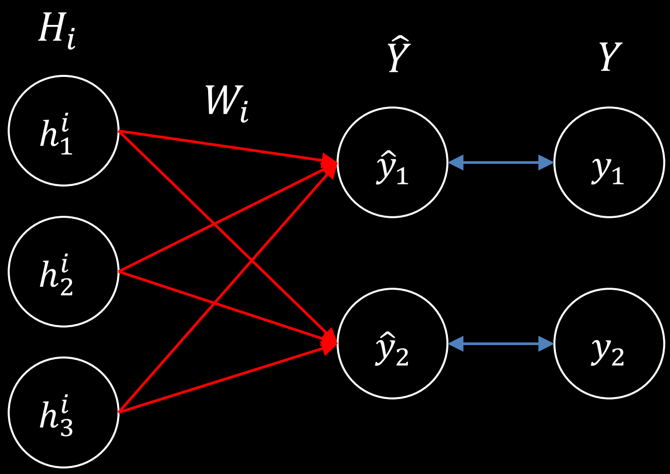
$$X = \begin{pmatrix} x_{11} & \cdots & x_{1m} \\ \vdots & \ddots & \vdots \\ x_{n1} & \cdots & x_{nm} \end{pmatrix} \qquad Y = \begin{pmatrix} y_1 \\ \vdots \\ y_m \end{pmatrix}$$

$$\nabla_X Y = \frac{\partial Y}{\partial X} = \left(\frac{\partial y_1}{\partial X}, \cdots, \frac{\partial y_m}{\partial X} \right)$$

$$= \left(\begin{pmatrix} \frac{\partial y_1}{\partial x_{11}} & \cdots & \frac{\partial y_1}{\partial x_{1m}} \\ \vdots & \ddots & \vdots \\ \frac{\partial y_1}{\partial x_{n1}} & \cdots & \frac{\partial y_1}{\partial x_{nm}} \end{pmatrix} \cdots \begin{pmatrix} \frac{\partial y_m}{\partial x_{11}} & \cdots & \frac{\partial y_m}{\partial x_{1m}} \\ \vdots & \ddots & \vdots \\ \frac{\partial y_m}{\partial x_{n1}} & \cdots & \frac{\partial m}{\partial x_{nm}} \end{pmatrix} \right)$$



Play with Toy Example (1/8)



$$L(\theta) = \frac{1}{2}\left(\hat{Y} - Y\right)^2 = \frac{1}{2}\sum_{i=1}^{n}(\ \hat{y}_i\ -\ y_i\ )^2$$

Loss $L(\theta)$를 W_i에 대하여 미분하려면?

Scalar를 vector로 미분

Vector를 Matrix로 미분

$$\nabla_{W_i} L = \frac{\partial L(\theta)}{\partial W_i} = \boxed{\frac{\partial L(\theta)}{\partial \hat{Y}}} \cdot \boxed{\frac{\partial \hat{Y}}{\partial W_i}}$$

합성함수의 미분

다음 슬라이드에서
구체적으로....

Play with Toy Example (2/8)

Loss $L(\theta)$를 W_i에 대하여 미분하려면?

$$\nabla_{W_i} L = \frac{\partial L(\theta)}{\partial W_i} = \boxed{\frac{\partial L(\theta)}{\partial \hat{Y}}} \cdot \frac{\partial \hat{Y}}{\partial W_i}$$

Scalar를 vector로 미분

이미 공부한 내용^^

$$\nabla_{\hat{Y}} L(\theta) = \frac{\partial L(\theta)}{\partial \hat{Y}} = \begin{pmatrix} \frac{L(\theta)}{\partial \hat{y}_1} \\ \frac{L(\theta)}{\partial \hat{y}_2} \end{pmatrix}$$

Play with Toy Example (3/8)

Loss $L(\theta)$를 W_i에 대하여 미분하려면?

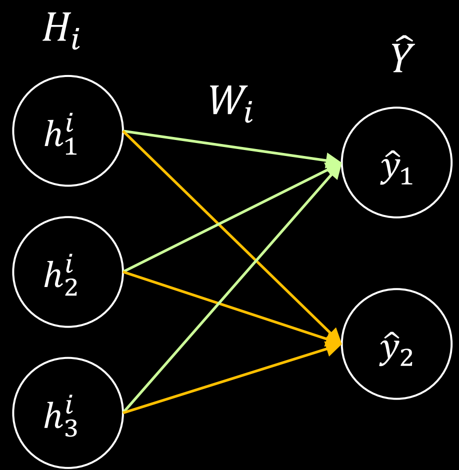
$$\nabla_{W_i} L = \frac{\partial L(\theta)}{\partial W_i} = \frac{\partial L(\theta)}{\partial \hat{Y}} \cdot \boxed{\frac{\partial \hat{Y}}{\partial W_i}}$$

Vector를 Matrix로 미분

이미 공부한 내용^^

다음 슬라이드에서 살펴보겠습니다 ^^

Play with Toy Example (4/8)



$$W_i = \begin{pmatrix} w_{11} & w_{12} \\ w_{21} & w_{22} \\ w_{31} & w_{32} \end{pmatrix} \qquad W_i^T = \begin{pmatrix} w_{11} & w_{21} & w_{31} \\ w_{12} & w_{22} & w_{32} \end{pmatrix} \qquad \hat{Y} = \begin{pmatrix} \hat{y}_1 \\ \hat{y}_2 \end{pmatrix}$$

$$\hat{Y} = W_i^T \cdot H_i = \begin{pmatrix} w_{11}h_1^i + w_{21}h_2^i + w_{31}h_3^i \\ w_{12}h_1^i + w_{22}h_2^i + w_{32}h_3^i \end{pmatrix} = \begin{pmatrix} \hat{y}_1 \\ \hat{y}_2 \end{pmatrix}$$

$$\frac{\partial \hat{Y}}{\partial W_i} = \left(\boxed{\frac{\partial \hat{y}_1}{\partial W_i}}, \boxed{\frac{\partial \hat{y}_2}{\partial W_i}} \right) = \left(\begin{pmatrix} \frac{\partial \hat{y}_1}{\partial w_{11}} & \frac{\partial \hat{y}_1}{\partial w_{12}} \\ \frac{\partial \hat{y}_1}{\partial w_{21}} & \frac{\partial \hat{y}_1}{\partial w_{22}} \\ \frac{\partial \hat{y}_1}{\partial w_{31}} & \frac{\partial \hat{y}_1}{\partial w_{32}} \end{pmatrix}, \begin{pmatrix} \frac{\partial \hat{y}_2}{\partial w_{11}} & \frac{\partial \hat{y}_2}{\partial w_{12}} \\ \frac{\partial \hat{y}_2}{\partial w_{21}} & \frac{\partial \hat{y}_2}{\partial w_{22}} \\ \frac{\partial \hat{y}_2}{\partial w_{31}} & \frac{\partial \hat{y}_2}{\partial w_{32}} \end{pmatrix} \right)$$

사실… 각각의 값에 대하여
행렬로 미분한 것을
모아놓은 것에 불과함

$$= \left(\begin{pmatrix} h_1^i & 0 \\ h_2^i & 0 \\ h_3^i & 0 \end{pmatrix}, \begin{pmatrix} 0 & h_1^i \\ 0 & h_2^i \\ 0 & h_3^i \end{pmatrix} \right)$$

실제 손으로
미분해 보기

Play with Toy Example (5/8)

최종 값 구하기

Scalar를 vector로 미분

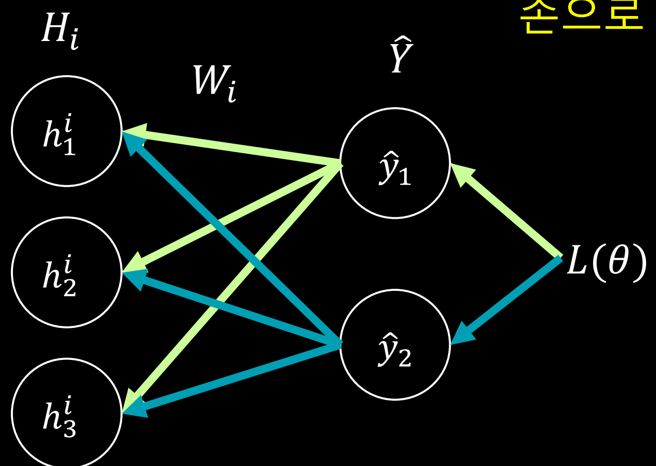
Vector를 Matrix로 미분

$$\nabla_{W_i} L = \frac{\partial L(\theta)}{\partial W_i} = \boxed{\frac{\partial L(\theta)}{\partial \hat{Y}}} \cdot \boxed{\frac{\partial \hat{Y}}{\partial W_i}}$$

$$\nabla_{W_i} \hat{Y} = \frac{\partial \hat{Y}}{\partial W_i} = \left(\frac{\partial \hat{y}_1}{\partial W_i}, \frac{\partial \hat{y}_2}{\partial W_i} \right)$$

$$= \left(\begin{pmatrix} h_1^i & 0 \\ h_2^i & 0 \\ h_3^i & 0 \end{pmatrix}, \begin{pmatrix} 0 & h_1^i \\ 0 & h_2^i \\ 0 & h_3^i \end{pmatrix} \right)$$

$$\nabla_{\hat{Y}} L(\theta) = \frac{\partial L(\theta)}{\partial \hat{Y}} = \begin{pmatrix} \frac{L(\theta)}{\partial \hat{y}_1} \\ \frac{L(\theta)}{\partial \hat{y}_2} \end{pmatrix}$$



손으로 직접 최종 값 구해보기 ^^

$$\frac{\partial L(\theta)}{\partial w_{11}} = \frac{L(\theta)}{\partial \hat{y}_1} \times h_1^i \qquad \frac{\partial L(\theta)}{\partial w_{12}} = \frac{L(\theta)}{\partial \hat{y}_2} \times h_1^i$$

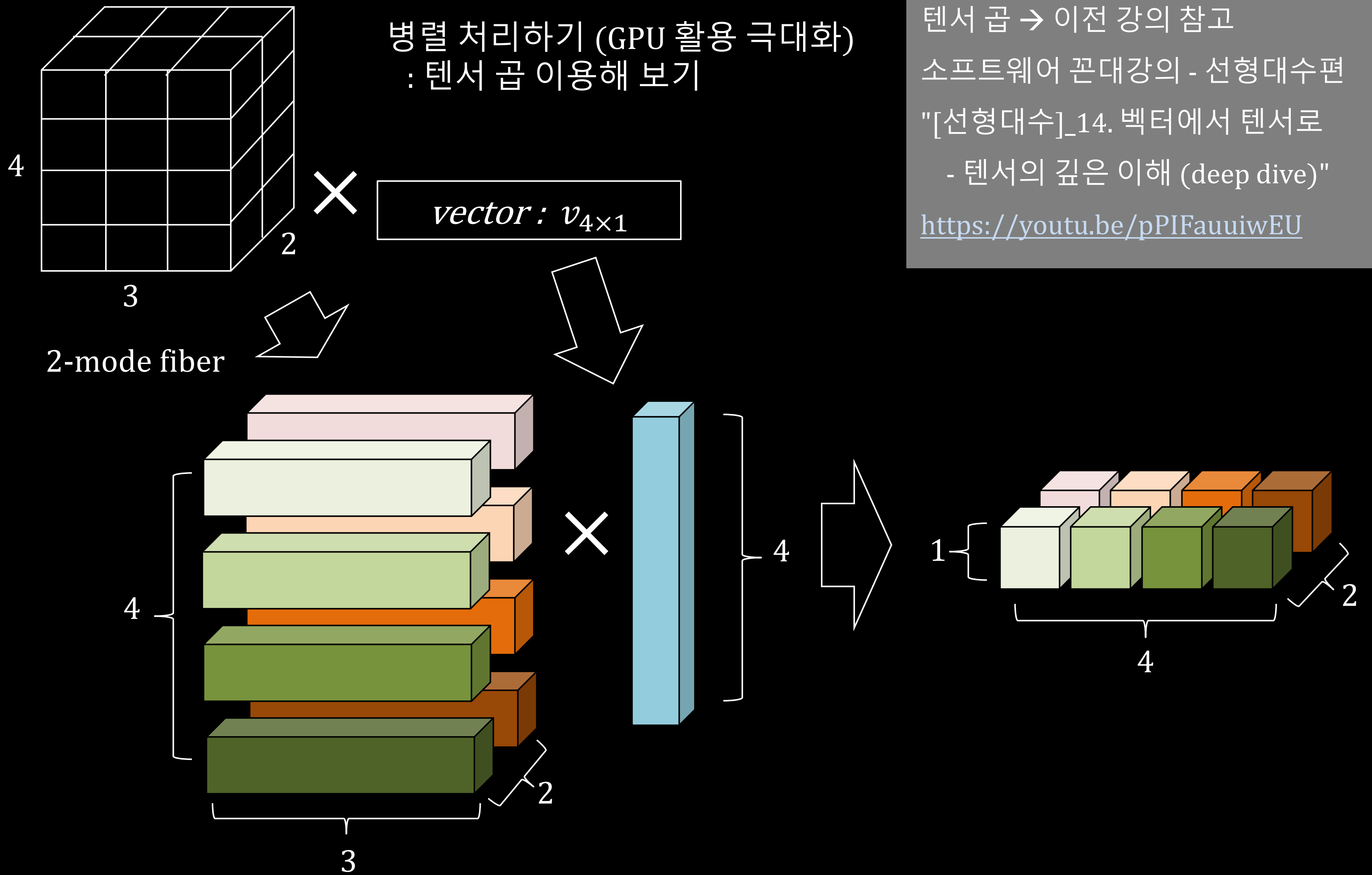
$$\frac{\partial L(\theta)}{\partial w_{21}} = \frac{L(\theta)}{\partial \hat{y}_1} \times h_2^i \qquad \frac{\partial L(\theta)}{\partial w_{22}} = \frac{L(\theta)}{\partial \hat{y}_2} \times h_2^i$$

$$\frac{\partial L(\theta)}{\partial w_{31}} = \frac{L(\theta)}{\partial \hat{y}_1} \times h_3^i \qquad \frac{\partial L(\theta)}{\partial w_{32}} = \frac{L(\theta)}{\partial \hat{y}_2} \times h_3^i$$

Play with Toy Example (6/8) → Tensor Product 잠깐 복습하기

병렬 처리하기 (GPU 활용 극대화)
: 텐서 곱 이용해 보기

텐서 곱 → 이전 강의 참고
소프트웨어 꼰대강의 - 선형대수편
"[선형대수]_14. 벡터에서 텐서로
- 텐서의 깊은 이해 (deep dive)"
https://youtu.be/pPIFauuiwEU



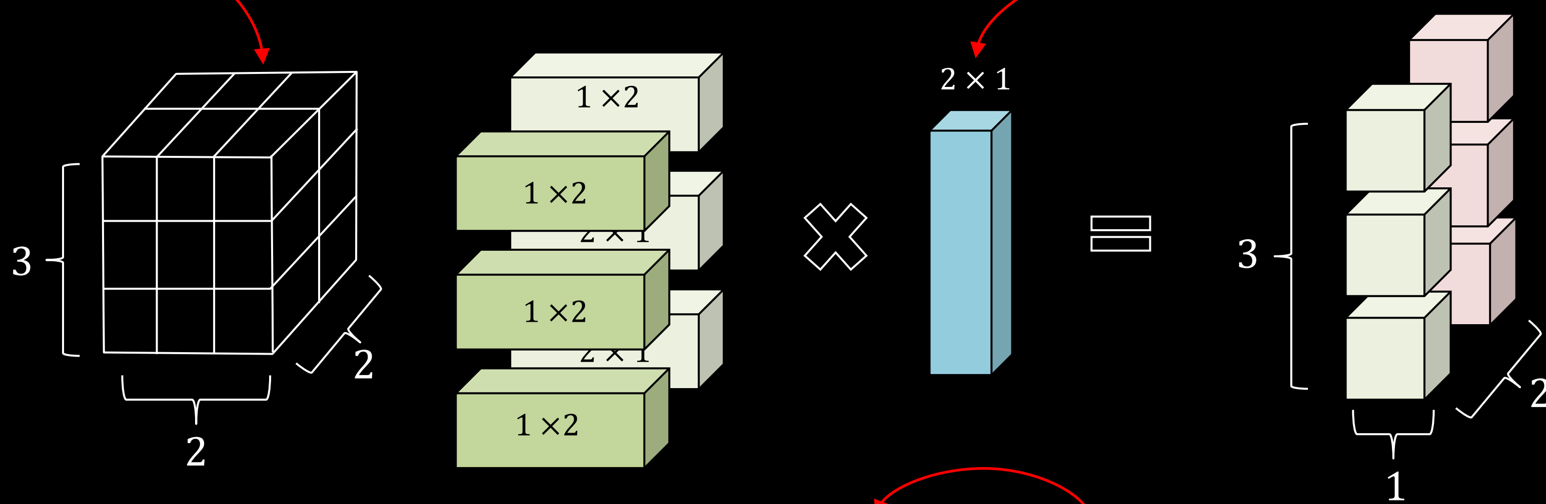
Play with Toy Example (7/8)

$$\frac{\partial \hat{Y}}{\partial W_i} = \left(\frac{\partial \hat{y}_1}{\partial W_i}, \frac{\partial \hat{y}_2}{\partial W_i}\right) = \left(\begin{pmatrix} \frac{\partial \hat{y}_1}{\partial w_{11}} & \frac{\partial \hat{y}_1}{\partial w_{12}} \\ \frac{\partial \hat{y}_1}{\partial w_{21}} & \frac{\partial \hat{y}_1}{\partial w_{22}} \\ \frac{\partial \hat{y}_1}{\partial w_{31}} & \frac{\partial \hat{y}_1}{\partial w_{32}} \end{pmatrix}, \begin{pmatrix} \frac{\partial \hat{y}_2}{\partial w_{11}} & \frac{\partial \hat{y}_2}{\partial w_{12}} \\ \frac{\partial \hat{y}_2}{\partial w_{21}} & \frac{\partial \hat{y}_2}{\partial w_{22}} \\ \frac{\partial \hat{y}_2}{\partial w_{31}} & \frac{\partial \hat{y}_2}{\partial w_{32}} \end{pmatrix} \right)$$

Shape: $(3 \times 2 \times 2)$

$$\nabla_{\hat{Y}} L(\theta) = \frac{\partial L(\theta)}{\partial \hat{Y}} = \begin{pmatrix} \frac{L(\theta)}{\partial \hat{y}_1} \\ \frac{L(\theta)}{\partial \hat{y}_2} \end{pmatrix}$$

Shape: (2×1)



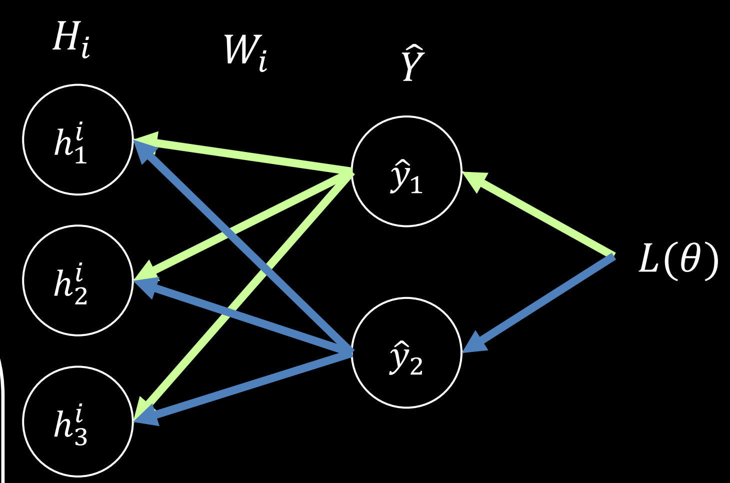
Shape: $(3 \times 1 \times 2)$

$$\frac{\partial \hat{Y}}{\partial W_i} \rightarrow (3 \times 2)$$

Play with Toy Example (8/8): 수식으로 이해하기

$$
\left(\begin{pmatrix} \frac{\partial \hat{y}_1}{\partial w_{11}} & \frac{\partial \hat{y}_1}{\partial w_{12}} \\ \frac{\partial \hat{y}_1}{\partial w_{21}} & \frac{\partial \hat{y}_1}{\partial w_{22}} \\ \frac{\partial \hat{y}_1}{\partial w_{31}} & \frac{\partial \hat{y}_1}{\partial w_{32}} \end{pmatrix}, \begin{pmatrix} \frac{\partial \hat{y}_2}{\partial w_{11}} & \frac{\partial \hat{y}_2}{\partial w_{12}} \\ \frac{\partial \hat{y}_2}{\partial w_{21}} & \frac{\partial \hat{y}_2}{\partial w_{22}} \\ \frac{\partial \hat{y}_2}{\partial w_{31}} & \frac{\partial \hat{y}_2}{\partial w_{32}} \end{pmatrix}\right) \begin{pmatrix} \frac{L(\theta)}{\partial \hat{y}_1} \\ \frac{L(\theta)}{\partial \hat{y}_2} \end{pmatrix}
$$

$$
= \left(\begin{pmatrix} \frac{\partial L(\theta)}{\partial \hat{y}_1}\frac{\partial \hat{y}_1}{\partial w_{11}} + \frac{\partial L(\theta)}{\partial \hat{y}_2}\frac{\partial \hat{y}_1}{\partial w_{12}} \\ \frac{\partial L(\theta)}{\partial \hat{y}_1}\frac{\partial \hat{y}_1}{\partial w_{21}} + \frac{\partial L(\theta)}{\partial \hat{y}_2}\frac{\partial \hat{y}_1}{\partial w_{22}} \\ \frac{\partial L(\theta)}{\partial 1}\frac{\partial \hat{y}_1}{\partial w_{31}} + \frac{\partial L(\theta)}{\partial \hat{y}_2}\frac{\partial \hat{y}_1}{\partial w_{32}} \end{pmatrix}, \begin{pmatrix} \frac{\partial L(\theta)}{\partial \hat{y}_1}\frac{\partial \hat{y}_2}{\partial w_{11}} + \frac{\partial L(\theta)}{\partial \hat{y}_2}\frac{\partial \hat{y}_2}{\partial w_{12}} \\ \frac{\partial L(\theta)}{\partial \hat{y}_1}\frac{\partial \hat{y}_2}{\partial w_{21}} + \frac{\partial L(\theta)}{\partial \hat{y}_2}\frac{\partial \hat{y}_2}{\partial w_{22}} \\ \frac{\partial L(\theta)}{\partial 1}\frac{\partial \hat{y}_2}{\partial w_{31}} + \frac{\partial L(\theta)}{\partial \hat{y}_2}\frac{\partial \hat{y}_2}{\partial w_{32}} \end{pmatrix}\right)
$$



$$
= \left(\begin{pmatrix} \frac{\partial L(\theta)}{\partial \hat{y}_1}\frac{\partial \hat{y}_1}{\partial w_{11}} + \cancel{\frac{\partial L(\theta)}{\partial \hat{y}_2}\frac{\partial \hat{y}_1}{\partial w_{12}}} \\ \frac{\partial L(\theta)}{\partial \hat{y}_1}\frac{\partial \hat{y}_1}{\partial w_{21}} + \cancel{\frac{\partial L(\theta)}{\partial \hat{y}_2}\frac{\partial \hat{y}_1}{\partial w_{22}}} \\ \frac{\partial L(\theta)}{\partial 1}\frac{\partial \hat{y}_1}{\partial w_{31}} + \cancel{\frac{\partial L(\theta)}{\partial \hat{y}_2}\frac{\partial \hat{y}_1}{\partial w_{32}}} \end{pmatrix}, \begin{pmatrix} \cancel{\frac{\partial L(\theta)}{\partial \hat{y}_1}\frac{\partial \hat{y}_2}{\partial w_{11}}} + \frac{\partial L(\theta)}{\partial \hat{y}_2}\frac{\partial \hat{y}_2}{\partial w_{12}} \\ \cancel{\frac{\partial L(\theta)}{\partial \hat{y}_1}\frac{\partial \hat{y}_2}{\partial w_{21}}} + \frac{\partial L(\theta)}{\partial \hat{y}_2}\frac{\partial \hat{y}_2}{\partial w_{22}} \\ \cancel{\frac{\partial L(\theta)}{\partial 1}\frac{\partial \hat{y}_2}{\partial w_{31}}} + \frac{\partial L(\theta)}{\partial \hat{y}_2}\frac{\partial \hat{y}_2}{\partial w_{32}} \end{pmatrix}\right) = \begin{pmatrix} \overbrace{\boxed{\begin{matrix} \frac{\partial L(\theta)}{\partial \hat{y}_1}\frac{\partial \hat{y}_1}{\partial w_{11}} \\ \frac{\partial L(\theta)}{\partial \hat{y}_1}\frac{\partial \hat{y}_1}{\partial w_{21}} \\ \frac{\partial L(\theta)}{\partial 1}\frac{\partial \hat{y}_1}{\partial w_{31}} \end{matrix}}}^{\text{의미?}} & \overbrace{\boxed{\begin{matrix} \frac{\partial L(\theta)}{\partial \hat{y}_2}\frac{\partial \hat{y}_2}{\partial w_{12}} \\ \frac{\partial L(\theta)}{\partial \hat{y}_2}\frac{\partial \hat{y}_2}{\partial w_{22}} \\ \frac{\partial L(\theta)}{\partial \hat{y}_2}\frac{\partial \hat{y}_2}{\partial w_{32}} \end{matrix}}}^{\text{의미?}} \end{pmatrix} = \begin{pmatrix} \frac{\partial L(\theta)}{\partial \hat{y}_1} h_1^i & \frac{\partial L(\theta)}{\partial \hat{y}_2} h_1^i \\ \frac{\partial L(\theta)}{\partial \hat{y}_1} h_2^i & \frac{\partial L(\theta)}{\partial \hat{y}_2} h_2^i \\ \frac{\partial L(\theta)}{\partial 1} h_3^i & \frac{\partial L(\theta)}{\partial \hat{y}_2} h_3^i \end{pmatrix}
$$

교수님...ㅠ 간단한 연산 방법은 없나요?

교수님.... 너무 복잡해요 ㅠㅠ
간단한 방법이 있을 것 같아요 ㅠㅠ

너무 깊게 들어가는 것
같아서 약간 미안......

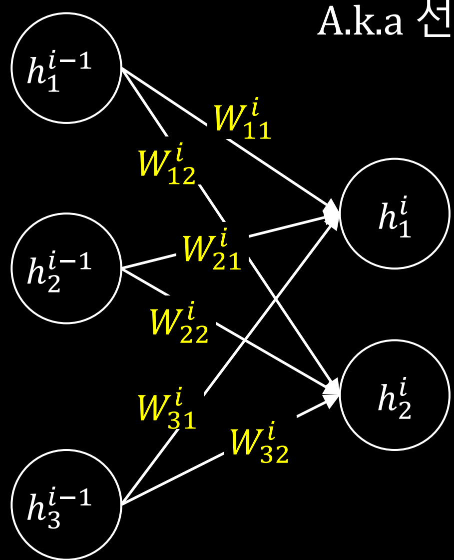
Jacobian matrix (야코비안 행렬) 이라는 것이 있습니다. ^^

$$f: \mathbb{R}^n \rightarrow \mathbb{R}^m$$

$$\mathcal{J} = \left(\frac{\partial f}{\partial x_1} \cdots \frac{\partial f}{\partial x_n} \right) = \begin{pmatrix} \nabla^T f_1 \\ \vdots \\ \nabla^T f_m \end{pmatrix} = \begin{pmatrix} \frac{\partial f_1}{\partial x_1} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial f_m}{\partial x_1} & \cdots & \frac{\partial f_m}{\partial x_n} \end{pmatrix}$$

, where $\nabla^T f_1$ is the transpose of the gradient of $i - th$ component

Jacobian Example



A.k.a 선형변환 함수

$$W^i = \begin{pmatrix} w_{11} & w_{12} \\ w_{21} & w_{22} \\ w_{31} & w_{32} \end{pmatrix} \qquad f : \mathbb{R}^3 \xrightarrow{W^i} \mathbb{R}^2$$

$$Input\ \vec{x} = \begin{pmatrix} h_1^{i-1} \\ h_2^{i-1} \\ h_3^{i-1} \end{pmatrix} \qquad Output\ \vec{y} = \begin{pmatrix} h_1^i \\ h_2^i \end{pmatrix}$$

$$\mathcal{J} = \begin{pmatrix} \dfrac{\partial f}{\partial x_1} & \cdots & \dfrac{\partial f}{\partial x_n} \end{pmatrix} = \begin{pmatrix} \nabla^T f_1 \\ \vdots \\ \nabla^T f_m \end{pmatrix} = \begin{pmatrix} \dfrac{\partial h_1^i}{\partial h_1^{i-1}} & \dfrac{\partial h_1^i}{\partial h_2^{i-1}} & \dfrac{\partial h_1^i}{\partial h_3^{i-1}} \\ \dfrac{\partial h_2^i}{\partial h_1^{i-1}} & \dfrac{\partial h_2^i}{\partial h_2^{i-1}} & \dfrac{\partial h_2^i}{\partial h_3^{i-1}} \end{pmatrix}$$

, where $\nabla^T f_1$ is the transpose of the gradient of $i-th$ component

딥러닝 프레임워크 구현 - Pytorch Case

Tutorials > PyTorch로 딥러닝하기: 60분만에 끝장내기 > `torch.autograd` 에 대한 간단한 소개

선택적으로 읽기(Optional Reading) - `autograd` 를 사용한 벡터 미적분(calculus)

수학적으로, 벡터 함수 $\vec{y} = f(\vec{x})$ 에서 $\vec{x}$ 에 대한 $\vec{y}$ 의 변화도는 야코비안 행렬(Jacobian Matrix) J: 입니다:

$$J = \begin{pmatrix} \frac{\partial \mathbf{y}}{\partial x_1} & \cdots & \frac{\partial \mathbf{y}}{\partial x_n} \end{pmatrix} = \begin{pmatrix} \frac{\partial y_1}{\partial x_1} & \cdots & \frac{\partial y_1}{\partial x_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial y_m}{\partial x_1} & \cdots & \frac{\partial y_m}{\partial x_n} \end{pmatrix}$$

일반적으로, `torch.autograd` 는 벡터-야코비안 곱을 계산하는 엔진입니다. 이는, 주어진 어떤 벡터 $\vec{v}$ 에 대해 $J^T \cdot \vec{v}$ 을 연산합니다.

만약 $\vec{v}$ 가 스칼라 함수 $l = g(\vec{y})$ 의 변화도(gradient)인 경우:

$$\vec{v} = \begin{pmatrix} \frac{\partial l}{\partial y_1} & \cdots & \frac{\partial l}{\partial y_m} \end{pmatrix}^T$$

이며, 연쇄 법칙(chain rule)에 따라, 벡터-야코비안 곱은 $\vec{x}$ 에 대한 l 의 변화도(gradient)가 됩니다:

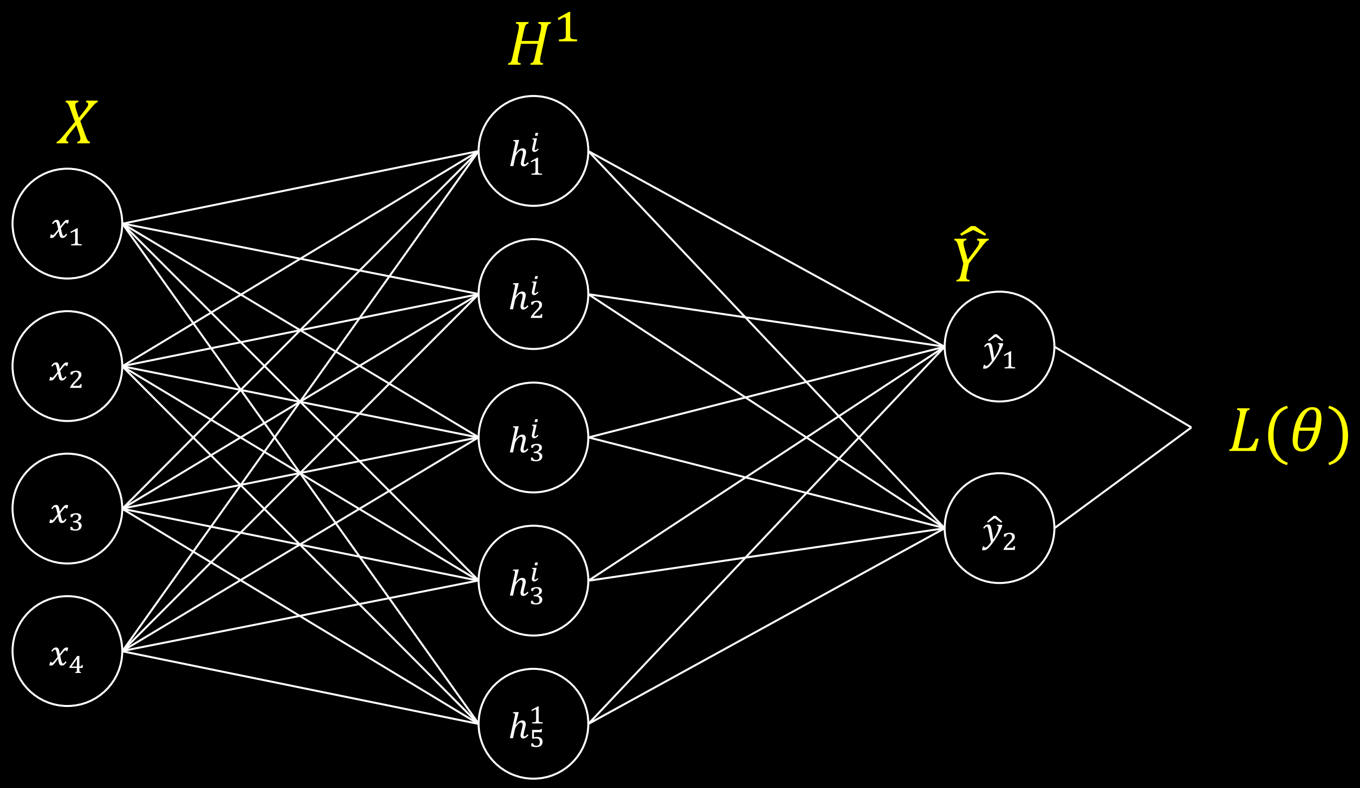
$$J^T \cdot \vec{v} = \begin{pmatrix} \frac{\partial y_1}{\partial x_1} & \cdots & \frac{\partial y_m}{\partial x_1} \\ \vdots & \ddots & \vdots \\ \frac{\partial y_1}{\partial x_n} & \cdots & \frac{\partial y_m}{\partial x_n} \end{pmatrix} \begin{pmatrix} \frac{\partial l}{\partial y_1} \\ \vdots \\ \frac{\partial l}{\partial y_m} \end{pmatrix} = \begin{pmatrix} \frac{\partial l}{\partial x_1} \\ \vdots \\ \frac{\partial l}{\partial x_n} \end{pmatrix}$$

위 예제에서 벡터-야코비안 곱의 이러한 특성을 사용했습니다; `external_grad` 가 $\vec{v}$ 를 뜻합니다.

자료출처: https://tutorials.pytorch.kr/beginner/blitz/autograd_tutorial.html

실습 with Toy Example

Target Network



수고하셨습니다 ..^^..