

**Expected Value? 그게 뭔~ 가요?
(기댓값??)**

소프트웨어 공대 강의

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이게 도대체 뭐래요? 헐!!!

$$\hat{\theta} = \arg \min_{\theta \in \Theta} \mathbb{E}_{x \sim P(x)} \left[\mathbb{E}_{y \sim P(y|x)} [\ln P(y|x; \theta)] \right]$$

머신러닝(특히 딥러닝)에서 많이 보게 됩니다.

웬... 무슨... 이런 만행을!!
외계어나?

\mathbb{E} 는 기댓값을 의미합니다.

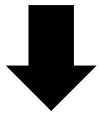
오늘 강의를 들으면 무슨 뜻인지 감잡을 수 있어요^^



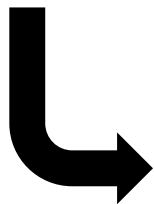
우리가 사는 세상.... 어떤 현상을 표현하려면?



내가 아는 어떤 인물



이 사진을
표현할 수 있는 방법?



이름
주민번호
군번
사원번호
⋮



자료출처: <https://hangyo.com/mobile/article.html?no=85073>

초등학교 교실의 학생들



이 사진을
표현할 수 있는 방법?



"학년 - 반" 활용?

5학년 3반

5-3

0503

⋮

데이터를 표현하는 방법은?

	A	B	C	D	E	F	G	H	I	J	K	
	File Name	EX_COEF_355	BA_COEF_355	EX_COEF_332	BA_COEF_332	BA_COEF_1064	mw	nm	real	imaginary	reff	
1	File: r12001000-0in140410 ID=d0.01	m=1.200 - .000E+001	2.02E-06	5.38E-09	1.56E-06	3.00E-09	2.70E-09	14	410	1.2	0.00E+00	5.44E-01
2	File: r12001100-0in140410 ID=d0.01	m=1.200 - .100E+01	1.61E-06	9.46E-09	1.67E-06	9.56E-09	1.02E-08	14	410	1.2	1.00E+00	5.44E-01
3	File: r12001200-1in140410 ID=d0.01	m=1.200 - .100E+00	1.57E-08	5.74E-10	1.41E-08	6.89E-10	9.98E-10	14	410	1.2	1.00E-01	5.44E-01
4	File: r12001300-2in140410 ID=d0.01	m=1.200 - .100E-01	1.94E-06	2.56E-09	1.53E-06	2.06E-09	2.25E-09	14	410	1.2	1.00E-02	5.44E-01
5	File: r12001400-3in140410 ID=d0.01	m=1.200 - .100E-02	2.01E-06	4.98E-09	1.56E-06	2.88E-09	2.65E-09	14	410	1.2	1.00E-03	5.44E-01
6	File: r12001500-4in140410 ID=d0.01	m=1.200 - .100E-03	2.01E-06	4.98E-09	1.56E-06	2.88E-09	2.65E-09	14	410	1.2	1.00E-03	5.44E-01
7	File: r12001600-5in140410 ID=d0.01	m=1.200 - .100E-04	2.01E-06	4.98E-09	1.56E-06	2.88E-09	2.65E-09	14	410	1.2	1.00E-03	5.44E-01
8	File: r12001700-6in140410 ID=d0.01	m=1.200 - .100E-05	2.01E-06	4.98E-09	1.56E-06	2.88E-09	2.65E-09	14	410	1.2	1.00E-03	5.44E-01
9	File: r12001800-7in140410 ID=d0.01	m=1.200 - .100E-06	2.01E-06	4.98E-09	1.56E-06	2.88E-09	2.65E-09	14	410	1.2	1.00E-03	5.44E-01
10	File: r12001900-8in140410 ID=d0.01	m=1.200 - .100E-07	2.01E-06	4.98E-09	1.56E-06	2.88E-09	2.65E-09	14	410	1.2	1.00E-03	5.44E-01
11	File: r12002000-9in140410 ID=d0.01	m=1.200 - .100E-08	2.01E-06	4.98E-09	1.56E-06	2.88E-09	2.65E-09	14	410	1.2	1.00E-03	5.44E-01
12	File: r12002100-10in140410 ID=d0.01	m=1.200 - .100E-09	2.01E-06	4.98E-09	1.56E-06	2.88E-09	2.65E-09	14	410	1.2	1.00E-03	5.44E-01
13	File: r12002200-11in140410 ID=d0.01	m=1.200 - .100E-10	2.01E-06	4.98E-09	1.56E-06	2.88E-09	2.65E-09	14	410	1.2	1.00E-03	5.44E-01
14	File: r12002300-12in140410 ID=d0.01	m=1.200 - .100E-11	2.01E-06	4.98E-09	1.56E-06	2.88E-09	2.65E-09	14	410	1.2	1.00E-03	5.44E-01
15	File: r12002400-13in140410 ID=d0.01	m=1.200 - .100E-12	2.01E-06	4.98E-09	1.56E-06	2.88E-09	2.65E-09	14	410	1.2	1.00E-03	5.44E-01
16	File: r12002500-14in140410 ID=d0.01	m=1.200 - .100E-13	2.01E-06	4.98E-09	1.56E-06	2.88E-09	2.65E-09	14	410	1.2	1.00E-03	5.44E-01
17	File: r12002600-15in140410 ID=d0.01	m=1.200 - .100E-14	2.01E-06	4.98E-09	1.56E-06	2.88E-09	2.65E-09	14	410	1.2	1.00E-03	5.44E-01
18	File: r12002700-16in140410 ID=d0.01	m=1.200 - .100E-15	2.01E-06	4.98E-09	1.56E-06	2.88E-09	2.65E-09	14	410	1.2	1.00E-03	5.44E-01
19	File: r12002800-17in140410 ID=d0.01	m=1.200 - .100E-16	2.01E-06	4.98E-09	1.56E-06	2.88E-09	2.65E-09	14	410	1.2	1.00E-03	5.44E-01
20	File: r12002900-18in140410 ID=d0.01	m=1.200 - .100E-17	2.01E-06	4.98E-09	1.56E-06	2.88E-09	2.65E-09	14	410	1.2	1.00E-03	5.44E-01
21	File: r12003000-19in140410 ID=d0.01	m=1.200 - .100E-18	2.01E-06	4.98E-09	1.56E-06	2.88E-09	2.65E-09	14	410	1.2	1.00E-03	5.44E-01
22	File: r12003100-20in140410 ID=d0.01	m=1.200 - .100E-19	2.01E-06	4.98E-09	1.56E-06	2.88E-09	2.65E-09	14	410	1.2	1.00E-03	5.44E-01
23	File: r12003200-21in140410 ID=d0.01	m=1.200 - .100E-20	2.01E-06	4.98E-09	1.56E-06	2.88E-09	2.65E-09	14	410	1.2	1.00E-03	5.44E-01
24	File: r12003300-22in140410 ID=d0.01	m=1.200 - .100E-21	2.01E-06	4.98E-09	1.56E-06	2.88E-09	2.65E-09	14	410	1.2	1.00E-03	5.44E-01

ID	Case Number	Date	Block	IUCR	Primary Type	Description	Location Description	Arrest	Domestic	Beat	District	Ward	Community Area	FBI Case No.	Coordinate	Updated On	Latitude	Longitude	Location
1	4570006	HM158744	02/02/2006	07:35:00	PM	0M00XX	E PLEASANT ST,0610,BURLGARY, FORCIBLE ENTRY,OTHER,false,false,1833,018,42,8,05,1176455,1906042,2006,04/15/2016	08:55:02	AM	41,89753369,-87.627343939	7	1	1	1	1	1	1	1	1
2	4570009	HM158673	02/01/2006	07:00:00	PM	1100X	S LEX AVE,0890,THEFT, FROM BUILDING, RESIDENCE, false, false, 2233,022,34,49,06,1167944,1831616,2006,04/15/2016	08:55:02	AM	41,69335737,-87.638775051	7	1	1	1	1	1	1	1	1
3	4570011	HM157834	02/01/2006	06:30:00	PM	013XX	W 18TH ST,0486,BATTERY,DOMESTIC BATTERY SIMPLE, RESIDENCE, false, false, 1222,012,31,08,1167625,1891525,2006,04/15/2016	08:55:02	AM	41,857892731,-87.650111111	7	1	1	1	1	1	1	1	1
4	4570012	HM182478	12/27/2005	07:58:38	PM	083X	S YATES BLVD,0811,NARCOTICS, POSS: CANNABIS 30GMS OR LESS, STREET, true, false, 0412,004,8,18,1193623,1850241,2005,04/15/2016	08:55:02	AM	41,74400813	7	1	1	1	1	1	1	1	1
5	4570013	HM157884	01/31/2006	05:34:00	PM	011XX	S HAMILTON AVE,4387,OTHER OFFENSE,VIOULATE ORDER OF PROTECTION,GOVERNMENT BUILDING/PROPERTY, true, true, 1224,012,2,28,26,1162121,1895023,2006,04/15/2016	08:55:02	AM	41,89753369,-87.627343939	7	1	1	1	1	1	1	1	1
6	4570015	HM158624	02/02/2006	09:00:00	AM	045XX	S WESTERN BLVD,0860,THEFT,RETAIL THEFT,DEPARTMENT STORE,true,false,0914,009,12,61,06,1161289,1874276,2006,04/15/2016	08:55:02	AM	41,810693516,-87.627343939	7	1	1	1	1	1	1	1	1
7	4570016	HM159122	02/02/2006	11:00:00	AM	018XX	W 10TH PL,0890,THEFT, FROM BUILDING, RESIDENCE, false, false, 2212,022,19,75,06,1166059,1832730,2006,04/15/2016	08:55:02	AM	41,69658521,-87.667612276	7	1	1	1	1	1	1	1	1
8	4570017	HM158803	02/02/2006	11:31:31	AM	076XX	S CONSTANCE AVE,0484,BATTERY, PRO EMP HANDS NO/MIN INJURY, 'SCHOOL, PUBLIC, BUILDING, false, false, 0414,004,8,43,088,1189862,1854688,2006,04/15/2016	08:55:02	AM	41,89753369,-87.627343939	7	1	1	1	1	1	1	1	1
9	4570019	HM159615	02/02/2006	06:05:00	PM	060XX	N RIDGE AVE,0810,THEFT,OVER \$500,APARTMENT, false, false, 2433,024,40,47,06,1163480,1940257,2006,04/15/2016	08:55:02	AM	41,891704561,-87.67403274	7	1	1	1	1	1	1	1	1
10	4570021	HL78034	11/27/2005	11:28:00	AM	07XX	W W FINGTON ST,2017,NARCOTICS, MANU/DFI NVR:CRACK VFNHIC F NON-COMMERCIAL, true, false, 1135,011,2,27,18,1158251,1896590,2005,04/15/2016	08:55:02	AM	41,89753369,-87.627343939	7	1	1	1	1	1	1	1	1
11	4570022	HL770136	12/03/2005	06:02:00	AM	07XX	W W FINGTON ST,2017,NARCOTICS, MANU/DFI NVR:CRACK VFNHIC F NON-COMMERCIAL, true, false, 1135,011,2,27,18,1158251,1896590,2005,04/15/2016	08:55:02	AM	41,89753369,-87.627343939	7	1	1	1	1	1	1	1	1
12	4570023	HM158587	02/02/2006	08:08:00	AM	07XX	W W FINGTON ST,2017,NARCOTICS, MANU/DFI NVR:CRACK VFNHIC F NON-COMMERCIAL, true, false, 1135,011,2,27,18,1158251,1896590,2005,04/15/2016	08:55:02	AM	41,89753369,-87.627343939	7	1	1	1	1	1	1	1	1
13	4570025	HM159372	01/26/2006	03:08:00	AM	07XX	W W FINGTON ST,2017,NARCOTICS, MANU/DFI NVR:CRACK VFNHIC F NON-COMMERCIAL, true, false, 1135,011,2,27,18,1158251,1896590,2005,04/15/2016	08:55:02	AM	41,89753369,-87.627343939	7	1	1	1	1	1	1	1	1
14	4570026	HM151987	01/29/2006	04:04:00	AM	07XX	W W FINGTON ST,2017,NARCOTICS, MANU/DFI NVR:CRACK VFNHIC F NON-COMMERCIAL, true, false, 1135,011,2,27,18,1158251,1896590,2005,04/15/2016	08:55:02	AM	41,89753369,-87.627343939	7	1	1	1	1	1	1	1	1
15	4570027	HM159713	02/02/2006	06:05:00	PM	060XX	N RIDGE AVE,0810,THEFT,OVER \$500,APARTMENT, false, false, 2433,024,40,47,06,1163480,1940257,2006,04/15/2016	08:55:02	AM	41,891704561,-87.67403274	7	1	1	1	1	1	1	1	1
16	4570028	HM159975	02/02/2006	06:05:00	PM	060XX	N RIDGE AVE,0810,THEFT,OVER \$500,APARTMENT, false, false, 2433,024,40,47,06,1163480,1940257,2006,04/15/2016	08:55:02	AM	41,891704561,-87.67403274	7	1	1	1	1	1	1	1	1
17	4570032	HM154606	01/31/2006	01:01:00	AM	07XX	W W FINGTON ST,2017,NARCOTICS, MANU/DFI NVR:CRACK VFNHIC F NON-COMMERCIAL, true, false, 1135,011,2,27,18,1158251,1896590,2005,04/15/2016	08:55:02	AM	41,89753369,-87.627343939	7	1	1	1	1	1	1	1	1
18	4570035	HL770140	12/03/2005	06:02:00	AM	07XX	W W FINGTON ST,2017,NARCOTICS, MANU/DFI NVR:CRACK VFNHIC F NON-COMMERCIAL, true, false, 1135,011,2,27,18,1158251,1896590,2005,04/15/2016	08:55:02	AM	41,89753369,-87.627343939	7	1	1	1	1	1	1	1	1
19	4570036	HL802826	12/21/2005	10:10:00	AM	07XX	W W FINGTON ST,2017,NARCOTICS, MANU/DFI NVR:CRACK VFNHIC F NON-COMMERCIAL, true, false, 1135,011,2,27,18,1158251,1896590,2005,04/15/2016	08:55:02	AM	41,89753369,-87.627343939	7	1	1	1	1	1	1	1	1
20	4570037	HM156609	01/31/2006	01:01:00	AM	07XX	W W FINGTON ST,2017,NARCOTICS, MANU/DFI NVR:CRACK VFNHIC F NON-COMMERCIAL, true, false, 1135,011,2,27,18,1158251,1896590,2005,04/15/2016	08:55:02	AM	41,89753369,-87.627343939	7	1	1	1	1	1	1	1	1
21	4570038	HM159975	02/02/2006	06:05:00	PM	060XX	N RIDGE AVE,0810,THEFT,OVER \$500,APARTMENT, false, false, 2433,024,40,47,06,1163480,1940257,2006,04/15/2016	08:55:02	AM	41,891704561,-87.67403274	7	1	1	1	1	1	1	1	1
22	4570039	HM159975	02/02/2006	06:05:00	PM	060XX	N RIDGE AVE,0810,THEFT,OVER \$500,APARTMENT, false, false, 2433,024,40,47,06,1163480,1940257,2006,04/15/2016	08:55:02	AM	41,891704561,-87.67403274	7	1	1	1	1	1	1	1	1
23	4570040	HM159975	02/02/2006	06:05:00	PM	060XX	N RIDGE AVE,0810,THEFT,OVER \$500,APARTMENT, false, false, 2433,024,40,47,06,1163480,1940257,2006,04/15/2016	08:55:02	AM	41,891704561,-87.67403274	7	1	1	1	1	1	1	1	1
24	4570041	HM159975	02/02/2006	06:05:00	PM	060XX	N RIDGE AVE,0810,THEFT,OVER \$500,APARTMENT, false, false, 2433,024,40,47,06,1163480,1940257,2006,04/15/2016	08:55:02	AM	41,891704561,-87.67403274	7	1	1	1	1	1	1	1	1

Order #	First Name	Last Name	Email	Country	IP address	Total	Item #	Payment	Shipping	Status
1	Dalton	Kramer	dalton@email.com	France	211.91.226.108	99	868	Card	Regular	In progress
2	Gita	Tetterton	gita@email.com	USA	222.153.179.100	99	537	Card	Regular	Delivered
3	Weston	Jurgens	weston@email.com	Spain	203.123.236.1	99	616	Paypal	Regular	Delivered
4	Brad	Chupp	brad@email.com	France	202.183.111.122	49	673	Card	Fast	Delivered
5	Marybeth	Baumann	marybeth@email.com	Italy	214.132.168.129	199	829	Bank	Regular	In progress
6	Allyson	Federn	allyson@email.com	Italy	182.108.190.85	29	40	Card	Regular	In progress
7	Lucile	Folks	lucile@email.com	Greece	18.64.161.62	199	548	Paypal	Fast	In progress
8	Mickey	Rusk	mickey@email.com	Canada	40.18.115.207	49	53	Paypal	Fast	Delivered
9	Clarine	Esslinger	clarine@email.com	Greece	185.134.23.86	49	817	Bank	Regular	Delivered
10	Kimberly	Penny	kimberly@email.com	France	34.72.165.11	99	998	Bank	Regular	In progress
11	Colleen	Kellough	colleen@email.com	USA	73.51.152.185	49	14	Paypal	Regular	In progress
12	Nettie	Edmonds	nettie@email.com	Spain	94.133.138.234	99	670	Card	Fast	Delivered
13	Duncan	Rickenbacker	duncan@email.com	France	211.91.226.108	199	869	Card	Regular	Delivered
14	Marchelle	Diedrich	marchelle@email.com	Italy	222.153.179.100	29	536	Paypal	Fast	Delivered
15	Mariano	Murrell	mariano@email.com	Italy	203.123.236.1	99	477	Card	Regular	Delivered

Dataset...

데이터를 표현할 수 있는 방법?

갯수
최대/최솟값
평균
표준편차

...

이런 질문을 자주 받습니다. ^^

■ 사실은... 저도 늘 헷갈렸던 부분입니다...

- 음... 뭐냐면....



이미지 출처:
https://pngtree.com/freepng/education--thinking--question-mark_5923515.html

교수님....
평균과 기댓값은
같은 건가요?
ㅠㅠ

기댓값을 이해하려면
확률변수 개념을 먼저 이해하셔야 해요^^

평균과 기댓값은 다릅니다.
하지만 같은 값을 가질수도 있습니다.

결론부터 이야기 하면,



이미지 출처:
<https://www.cleanpng.com/png-professor-teacher-clip-art-mad-professor-5552425/>

평균 vs. 기댓값?

■ 평균 (Mean)

- 어떤 집합의 모든 원소 값을 더한 후 원소의 갯수로 나눈 값

$$\text{Mean } (\mu) = \frac{x_1 + x_2 + \dots + x_n}{n} = \frac{1}{n} \sum_{i=1}^n x_i$$

Note:

평균의 종류는 많습니다^^
여기서 '평균'은 산술평균을
의미합니다.

■ 기댓값 (Expected Value)

expected

형용사 예상되는 (→unexpected)

미국·영국 [ɪkˈspektɪd] 미국식 [ɪkˈspektɪd]



Expected value: 기대하는 값

정확히 값을 안다면 '기대하는' 표현 X

'기대하는' → 확률적으로 계산해야 할 것임

value

1. 명사 (경제적인) 가치 (→market value, street value)
2. 명사 (가격·비용 대비) 가치
3. 동사 소중하게[가치 있게] 생각하다[여기다]
4. 동사 (가치·가격을) 평가하다

미국·영국 [ˈvæljuː]



기댓값 표기

■ Definition of expected value (https://en.wikipedia.org/wiki/Expected_value)

- In probability theory, the expected value (also called expectation, expectancy, expectation operator, mathematical expectation, mean, average, or first moment) is a **generalization of the weighted average**.
- Informally, the expected value is the **arithmetic mean of a large number of independently selected outcomes of a random variable**.

■ 요거를 수학 기호로 표시하면, 이렇게 됩니다.

$$E[X] = x_1 p_1 + x_2 p_2 + \cdots + x_n p_n = \sum_{i=1}^n x_i p_i$$

- X : a random variable
- x_i : possible outcome of X
- p_i : probability of x_i

기댓값은 확률 변수로부터!
(확률변수는 확률과 값을 갖는다)

굳이 기댓값을 사용하는 이유

■ 뭐하러 복잡하게 기댓값을 만들어서 쓰나요?

- 그냥 **평균**, **중앙값** 이런 거 쓰면 되잖아요?

■ 물론 그래도 되죠 ㅎㅎ

■ 어떤 일을 결정할 때 (확률적으로) 예상되는 값을 알게 된다면...

- 더 좋지 않을까요?

(예) 도박을 하는 경우

- 이길 확률: 60% → 이기면 1만원 받기
- 질 확률 : 100% - 60% = 40% → 지면 2만원 주기
- 현재까지 데이터

게임 데이터	결과	수익
Game1	Win	+1만원
Game2	Win	+1만원
Game3	Lose	-2만원
Game4	Win	+1만원

평균수익

$$\begin{aligned} \text{Mean}(\mu) &= \frac{1}{n} \sum_{i=1}^n x_i \\ &= \frac{1+1-2+1}{4} = \frac{1}{4} = +0.25 \text{ 만원} \end{aligned}$$

평균 수익이 플러스네... = 계속 해서 부자 되야지^^



기댓값

$$E[X] = \sum_{i=1}^n x_i \cdot p(x_i) = 0.6 \times 1 + 0.4 \times (-2)$$

$$= 0.6 - 0.8 = -0.2 \text{ 만원}$$

이런 게임 해야 할까요?



평균을 기댓값으로 해석한다면?

■ 수학 성적

$$E[X] = \sum_{i=1}^n x_i p(x_i) = 100 \cdot \frac{1}{6} + 90 \cdot \frac{1}{6} + 80 \cdot \frac{1}{6} + 70 \cdot \frac{1}{6} + 60 \cdot \frac{1}{6} + 50 \cdot \frac{1}{6}$$

$$= \frac{100 + 90 + 80 + 70 + 60 + 50}{6} = \frac{450}{6} = 75$$

가능한 점수

Ω	X	E	확률
100	→	100	1/6
90	→	90	1/6
80	→	80	1/6
70	→	70	1/6
60	→	60	1/6

가능한 점수가 나올 확률을

동일하게 지정하면,

기댓값과 평균값은 같다.

평균은 기댓값의 Special Case

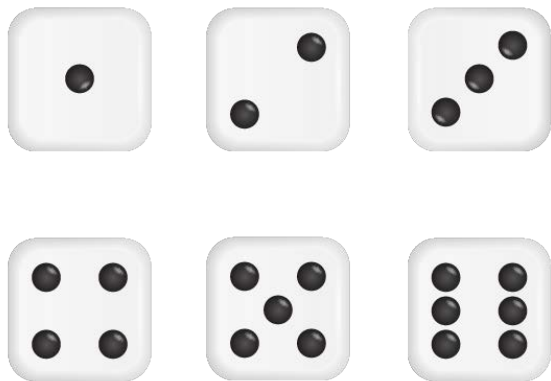
(동일한 가중치!)

$$\text{Mean}(\mu) = \frac{1}{n} \sum_{i=1}^n x_i = \frac{100 + 90 + 80 + 70 + 60 + 50}{6} = \frac{450}{6} = 75$$

기댓값 구하는 간단한 예제

■ 실험: 주사위 1개를 던지는 실험

- 먼저, Random variable X (맵핑 함수) 정하기
- X : 주사위 던졌을 때 나온 값이라고 하자...



Ω	X	E
1	→	1
2	→	2
3	→	3
4	→	4
5	→	5
6	→	6


Random variable X 의 기댓값

$$E[X] = \sum_{i=1}^n x_i p_i = 1 \cdot \frac{1}{6} + 2 \cdot \frac{1}{6} + 3 \cdot \frac{1}{6} + 4 \cdot \frac{1}{6} + 5 \cdot \frac{1}{6} + 6 \cdot \frac{1}{6} = \frac{21}{6} = 3.5$$

기댓값과 평균 예시 - 조금만 더 복잡한 걸루...(1/2)

■ 주사위를 던지는 실험

- 먼저, Random variable X (맵핑 함수) 정하기
- X : 주사위 2개의 합이라고 하자...



	Ω	X	E
	1, 1	→	2
	1, 2	→	3
	⋮	→	⋮
	2, 1	→	3
	⋮	→	⋮
	6, 6	→	12

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

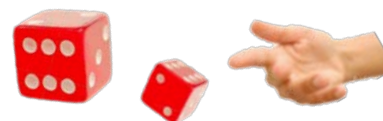
- 주사위 2개를 던졌을 때, 합이 7이 될 확률은? $P(X = 7) = \frac{6}{36} = \frac{1}{6}$
- 주사위 2개를 던졌을 때, 합이 9가 될 확률은? $P(X = 9) = \frac{4}{36} = \frac{1}{9}$
- 기댓값은?

$$E[X] = \sum_{i=1}^n x_i p_i = 2 \cdot \frac{1}{36} + 3 \cdot \frac{2}{36} + 4 \cdot \frac{3}{36} + \cdots + 12 \cdot \frac{1}{36} = 7$$

평균과 비교하면?
(다음 슬라이드)

기댓값과 평균 예시 - 조금만 더 복잡한걸루... (2/2)

- 주사위 2개를 던진 실험을 10회 dataset 확보했다.



- Dataset의 평균은?

- Dataset #1

평균과 기댓값이 같은 경우

실험	1	2	3	4	5	6	7	8	9	10
결과값	3, 4	1, 5	5, 1	3, 3	1, 2	6, 3	6, 6	1, 6	2, 4	4, 4
RV X	7	6	6	6	3	9	12	7	6	8

$$\text{Mean}(\mu) = \frac{1}{n} \sum_{i=1}^n x_i = \frac{7 + 6 + 6 + 6 + 3 + 9 + 12 + 7 + 6 + 8}{10} = \frac{70}{10} = 7$$

- Dataset #2

평균과 기댓값이 다른 경우

실험	1	2	3	4	5	6	7	8	9	10
결과값	3, 4	3, 6	6, 6	5, 6	1, 2	2, 6	5, 1	1, 6	2, 4	4, 4
RV X	7	9	12	11	7	8	6	7	6	8

$$\text{Mean}(\mu) = \frac{1}{n} \sum_{i=1}^n x_i = \frac{7 + 9 + 12 + 11 + 7 + 8 + 6 + 7 + 6 + 8}{10} = \frac{81}{10} = 8.1$$

기댓값의 재밌는 성질

X and Y are random variables.

- $E[X + Y] = E[X] + E[Y]$

- $E[XY] = E[X] \cdot E[Y]$

- $E[aX + b] = aE[X] + b$

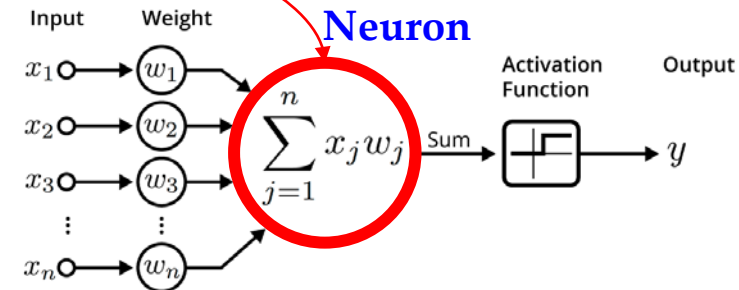
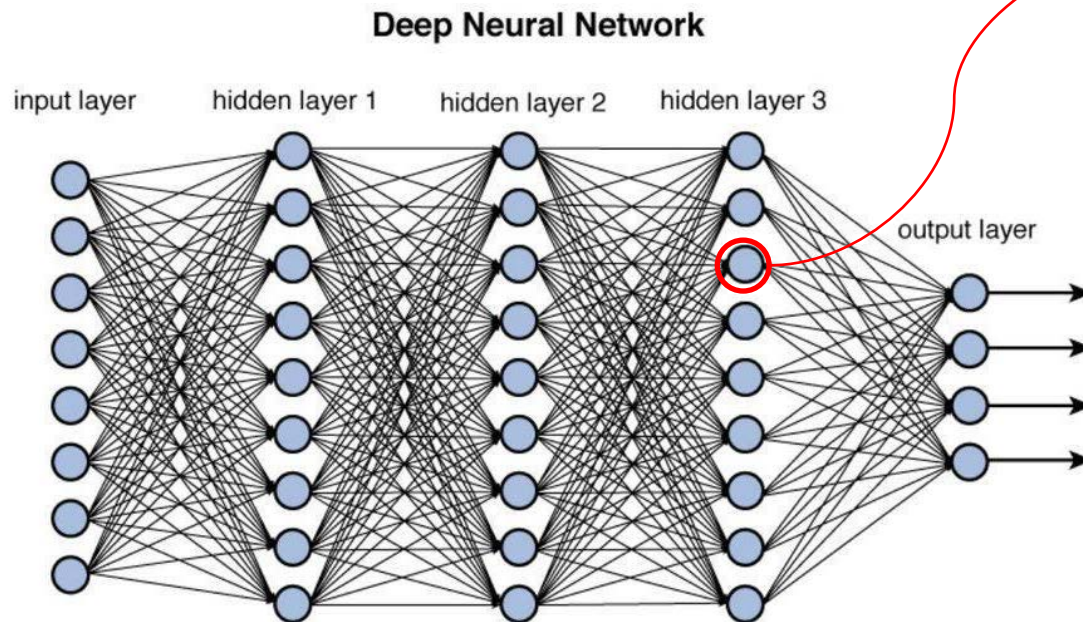
- $E[aX^2 + bX + c] = aE[X^2] + bE[X] + c$, where $a, b, c \in \mathbb{R}$

- $E[aX + bY] = aE[X] + E[Y]$, where $a, b, c \in \mathbb{R}$

딥러닝 & 기댓값

■ 딥러닝에도 기댓값 개념이 있나요?

- 아직 먼 이야기인 것은 맞습니다. 파라미터, 신경망 개념을 알아야 합니다. π
- 하지만 기댓값이 어떻게 쓰이는지 맛보기는 할 수 있습니다.



An illustration of an artificial neuron. Source: Becoming Human.

이미지 출처: <https://www.freecodecamp.org/news/deep-learning-neural-networks-explained-in-plain-english/>

이미지 출처:
<https://towardsdatascience.com/training-deep-neural-networks-9fdb1964b964>

$$\hat{\theta} = \arg \max_{\theta \in \Theta} \mathbb{E}_{x \sim P(x)} \left[\mathbb{E}_{y \sim P(y|x)} [\ln P(y|x; \theta)] \right]$$



수고하셨습니다 ..^^..