

The following are a summary of relevant sparse linear solvers available from PETSc (I included what I thought we will use based on what I learned from the thesis).

In short, the types of matrices are:

- sparse matrices
- block sparse matrices
- sequential sparse matrices, based on compressed sparse row format
- sequential block sparse matrices, based on block sparse compressed row format

## 1 Krylov Methods: Conjugate Gradients (KSPCG)

<http://www.mcs.anl.gov/petsc/petsc-current/docs/manualpages/KSP/KSPCG.html>

The Preconditioned Conjugate Gradient (PCG) iterative method.

- Requires both the matrix and preconditioner to be symmetric positive (or negative) (semi) definite.
- Only left preconditioning is supported.
- Parallel and complex.
- For complex numbers there are two different CG methods, one for Hermitian symmetric matrices and one for non-Hermitian symmetric matrices.

## 2 Krylov Methods: GMRES (KSPGMRES)

<http://www.mcs.anl.gov/petsc/petsc-current/docs/manualpages/KSP/KSPGMRES.html>

Left and right preconditioning are supported, but not symmetric preconditioning.

## 3 CG for Least Squares (KSPCGLS)

<http://www.mcs.anl.gov/petsc/petsc-current/docs/manualpages/KSP/KSPCGLS.html> Conjugate Gradient method for Least-Squares problems

- Supports non-square (rectangular) matrices.
- Parallel and complex.

## 4 Preconditioner: Jacobi (PCJACOBI)

<http://www.mcs.anl.gov/petsc/petsc-current/docs/manualpages/PC/PCJACOBI.html> Diagonal scaling preconditioning.

Parallel and complex.

Matrix Types:

1. MATAIJ = "aij" - A matrix type to be used for sparse matrices.
2. MATBAIJ = "baij" - A matrix type to be used for block sparse matrices.
3. MATSBAIJ = "sbaij" - A matrix type to be used for symmetric block sparse matrices.
4. MATDENSE = "dense" - A matrix type to be used for dense matrices.

## 5 Preconditioner: Point Block Jacobi (PCPBJACOBI)

<http://www.mcs.anl.gov/petsc/petsc-current/docs/manualpages/PC/PCPBJACOBI.html>

Uses dense LU factorization with partial pivoting to invert the blocks; if a zero pivot is detected a PETSc error is generated.

Parallel and complex.

Matrix Types:

1. AIJ
2. BAIJ

## 6 Preconditioner: Block Jacobi (PCBJACOBI)

<http://www.mcs.anl.gov/petsc/petsc-current/docs/manualpages/PC/PCBJACOBI.html>

Use block Jacobi preconditioning, each block is (approximately) solved with its own KSP object.

Each processor can have one or more blocks, or a single block can be shared by several processes. Defaults to one block per processor.

Parallel and complex.

Matrix Types:

1. AIJ
2. BAIJ
3. SBAIJ

## 7 Preconditioner: ILU (PCILU)

<http://www.mcs.anl.gov/petsc/petsc-current/docs/manualpages/PC/PCILU.html>

For BAIJ matrices this implements a point block ILU.

Parallel and complex.

Matrix Types:

1. MATSEQAIJ = "seqaij" - A matrix type to be used for sequential sparse matrices, based on compressed sparse row format.
2. MATSEQBAIJ = "seqbaij" - A matrix type to be used for sequential block sparse matrices, based on block sparse compressed row format.

## 8 Direct Solvers: Cholesky (PCCHOLESKY)

<http://www.mcs.anl.gov/petsc/petsc-current/docs/manualpages/PC/PCCHOLESKY.html>

Uses a direct solver, based on Cholesky factorization, as a preconditioner.

Usually this will compute an "exact" solution in one iteration.

Matrix Types:

1. seqaij
2. seqbaij

## 9 Matrix Collection

Set of widely used set of sparse matrix benchmarks collected from a wide range of applications. Link: <https://sparse.tamu.edu/>

## 10 Free Linear Algebra Software

<http://www.netlib.org/utk/people/JackDongarra/la-sw.html>

[http://www.netlib.org/lapack/#\\_presentation](http://www.netlib.org/lapack/#_presentation)

CG using Jacobi preconditioner:

<https://www.npmjs.com/package/conjugate-gradient>

Iterative methods library:

<https://math.nist.gov/iml++/>

## 11 CG

Github repository: High Performance Computing Conjugate Gradients: The original Mantevo miniapp

<https://github.com/Mantevo/HPCCG>

- Generates a 27-point finite difference matrix with a user-prescribed sub-block size on each processor
- Code compiles with MPI support and can be run on one or more processors
- Input:  $n_x$ ,  $n_y$ ,  $n_z$  are the number of nodes in the x, y and z dimension respectively on a each processor. The global grid dimensions will be  $n_x$ ,  $n_y$  and  $\text{numproc} * n_z$ . In other words, the domains are stacked in the z direction.

## 12 B-CG

Paper: An Implementation of Block Conjugate Gradient Algorithm on CPU-GPU Processors

Link to paper:

<https://ieeexplore.ieee.org/stamp/stamp.jsp?tp=&arnumber=7017966>