MA 162 Quiz 4
June 27, 2019

You have **15 minutes** to complete this quiz. Each correct answer will award you **five points**. Show your work **neatly** and you will receive **two** to **three points** depending on your level of correctness.

Problem 4.1. Evaluate the integral

$$\int_0^1 xe^{2x}\,dx.$$

(A) $1+2e^2$ (B) $\dfrac{1+3e^2}{2}$ (C) $\dfrac{1+e^2}{4}$ (D) $1+3e^2$ (E) $\dfrac{1+2e^2}{4}$

Solution. Using integration by parts with $u = x$ and $dv = e^{2x}$,

$$\begin{aligned}\int_0^1 xe^{2x}\,dx &= \frac{1}{2}xe^{2x}\Big|_0^1 - \frac{1}{2}\int_0^1 e^{2x}\,dx \\ &= \frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x}\Big|_0^1 \\ &= \underline{\frac{1+e^2}{4}}.\end{aligned}$$

◇

Problem 4.2. Evaluate the integral

$$\int_0^{\pi/4} 5\sec^4 x\tan^2 x\,dx.$$

(A) $\dfrac{3}{4}$ (B) $\dfrac{3}{8}$ (C) $\dfrac{8}{3}$ (D) $\dfrac{8}{5}$ (E) $\dfrac{5}{8}$

Solution. We evaluate this by using methods we learned for dealing with trigonometric functions, as follows: first, use the identity $1+\tan^2 x = \sec^2 x$

$$\int_0^{\pi/4} 5\sec^4 x\tan^2 x\,dx. = \int_0^{\pi/4} 5\sec^4 x(1+\tan^2 x)\tan^2 x\,dx$$

make the substitution $u = \tan x$, $du = \sec^2 x\,dx$ to get (taking note to change the bounds to $u_0 = \tan(0) = 0$ and $u_1 = \tan(\pi/4) = 1$)

$$= \int_0^1 (1+u^2)u^2\,du$$
$$= 5\left(\frac{u^3}{3} + \frac{u^5}{5}\right)\Big|_0^1$$
$$= \underline{\frac{8}{3}}.$$

◇

Problem 4.3. Evaluate the integral

$$\int_0^{\pi/2} \cos^4 x\,dx.$$

(*Hint:* Use the identity $2\cos^2(x) = 1 + \cos(2x)$.)

(A) $\frac{\pi}{6}$ (B) $\frac{2\pi}{9}$ (C) $\frac{3\pi}{16}$ (D) $\frac{\pi}{5}$ (E) $\frac{5\pi}{8}$

Solution. By applying the identity, we get

$$\int_0^{\pi/2} \cos^4 x\,dx = \int_0^{\pi/2} \left(\frac{1+\cos(2x)}{2}\right)^2 dx$$
$$= \frac{1}{4}\int_0^{\pi/2} 1 + 2\cos(2x) + \cos^2(2x)\,dx$$

applying the identity again on $\cos^2(2x)$, the above becomes

$$= \frac{1}{4}\int_0^{\pi/2} \left(1 + 2\cos(2x) + \frac{1+\cos(4x)}{2}\right) dx$$
$$= \frac{1}{4}\left(\frac{3}{2}x + \sin(2x) + \frac{1}{8}\sin(4x)\right)\Big|_0^1$$
$$= \underline{\frac{3\pi}{16}}.$$

◇