MA161 Quiz 21

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Problem 21.1. What is the maximum vertical distance between the line $y = x + 12$ and the parabola $y = x^2$ for $-3 \leq x \leq 4$?

Solution. The distance from a point on $y = x + 12$ and $y = x^2$ is given by the distance formula

$$\begin{aligned} d &= \sqrt{(x_1 - x_0)^2 + (y_1 - y_0)^2} \\ &= \sqrt{(x - x)^2 + (x^2 - x - 12)^2} \\ &= |x^2 - x - 12|. \end{aligned}$$

Since x is between -3 and 4 and $x^2 - x - 12$ is positive there, we are free to drop the absolute value. Then, to find the maximum distance, we have to take the derivative

$$d' = 2x - 1.$$

Therefore, $x = 1/2$ is a possible candidate. Now we use the Extreme Value Theorem, which tells us that the extrema (minima and maxima) happen at the endpoints and at the critical points. That is,

$$d(-3) = 6, \quad d(4) = 0, \quad d(1/2) = 12.25.$$

Therefore, the maximum happens at $d(1/2) = 12.25$. ☺

Problem 21.2. A box with a square base and open top must have a volume of 62500 cm^3. Find the dimensions of the box that minimize the amount of material used.

Solution. The volume of the box is

$$V = 62500 = x^2 h$$

and therefore $h = 62500/x^2$. Now, we want to minimize the material used, which is the same as minimizing the surface area which is $S = x^2 + 4xh$. By the equation above,

$$S = x^2 + 4\frac{62500x}{x^2} = x^2 + \frac{4 \cdot 62500}{x}.$$

To minimize this value, we must take a derivative, as we now do

$$S' = 2x - \frac{4 \cdot 62500}{x^2} = 0.$$

Thus,

$$2x^3 - 4 \cdot 62500 = 0.$$

Therefore,

$$x = \sqrt[3]{2 \cdot 62500} = 50.$$

And so,

$$h = \frac{62500}{50} = 1250.$$

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