

MA 261 QUIZ 7

MARCH 5, 2018

If you do not know how to do any one of these problems, circle “(E) I don’t know” as your answer choice. You will receive **two points** for doing that. **Each problem** is worth **five points**. You get **two points** for writing your **full name** and **three points** for writing your **section number**.

Problem 7.1. Let D be the region bounded by $y = x^2$, $y = 0$, and $x = 1$. If the density is $\rho(x, y) = x$, find \bar{y}

(A) $1/3$

(B) $1/4$

(C) $2/5$

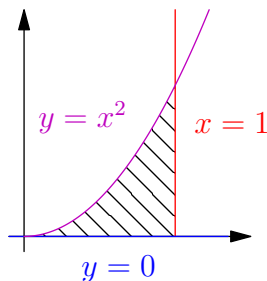
(D) $3/4$

(E) I don’t know how to do this

Solution. Recall that $\bar{y} = M_x/M$, where

$$M = \iint_D \rho(x, y) dA, \text{ and } M_x = \iint_D y\rho(x, y) dA.$$

No, if we sketch the region, as we do below



it is not too difficult to see that the bounds for the double integrals are $0 \leq x \leq 1$ and $0 \leq y \leq x^2$. Thus,

$$\begin{aligned} M &= \int_0^1 \int_0^{x^2} x dy dx, \\ &= \int_0^1 x^3 dx \\ &= \frac{1}{4} \end{aligned}$$

$$\begin{aligned} M_x &= \int_0^1 \int_0^{x^2} yx dy dx \\ &= \frac{1}{2} \int_0^1 x^5 dx \\ &= \frac{1}{12} \end{aligned}$$

so

$$\bar{y} = M_x/M = \frac{1/12}{1/4} = \frac{4}{12} = \frac{1}{3}.$$

Answer: (A).

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Problem 7.2. Compute $\iint_D e^{x^2+y^2} dA$, $D = \{x^2 + y^2 \leq 1\}$ by changing to polar coordinates.

To clarify, D is the unit disk centered at the origin.

- (A) 1
- (B) $\pi(e - 1)$
- (C) e^π
- (D) πe
- (E) I don't know how to do this

Solution. By changing to polar coordinates, the computation follows easily:

$$\begin{aligned}\iint_D e^{x^2+y^2} dA &= \int_0^{2\pi} \int_0^1 r e^{r^2} dr d\theta \\ &= 2\pi \int_0^1 r e^{r^2} dr \\ &= 2\pi \left[e^{r^2}/2 \right]_0^1 \\ &= \pi(e - 1).\end{aligned}$$

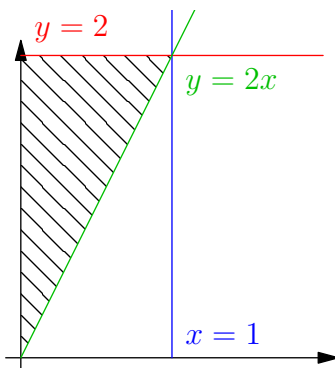
Answer: (B).

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Problem 7.3. Compute $\int_0^1 \int_{2x}^2 e^{y^2} dy dx$ by reversing the order of integration.

- (A) $e^4/4$
- (B) e^4
- (C) $e^4 - 1$
- (D) $e^4/4 - 1/4$
- (E) I don't know how to do this

Solution. First we sketch the region of integration, as below



Having done this, it is quite easy to see that

$$\begin{aligned}\int_0^1 \int_{2x}^2 e^{y^2} dy dx &= \int_0^2 \int_0^{y/2} e^{y^2} dx dy \\ &= \frac{1}{2} \int_0^2 y e^{y^2} dy\end{aligned}$$

(by u -substitution, with $u = y^2$)

$$\begin{aligned}&= \frac{1}{4} [e^{y^2}]_0^2 \\ &= e^4/4 - 1/4.\end{aligned}$$

Answer: (D)

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