MA161 Quiz 17 Solutions

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Problem 17.1. Find the absolute maximum and absolute minimum of $f(x) = x - \ln(9x)$ on the interval $[1/2, 2]$.

Solution. Most of us got this problem right. By the Extreme Value Theorem, we have to check at the critical points, i.e. where $f'(x) = 0$, and at the endpoints $x = 1/2$ and $x = 2$. So let us do just that. First,

$$f(1/2) = 1/2 - \ln(9/2), \tag{17.1}$$
$$f(2) = 2 - \ln(18). \tag{17.2}$$

What about the critical points? First we need to take the derivative as we now do:

$$f'(x) = 1 - \frac{9}{9x} = 1 - \frac{1}{x}.$$

The only place where the derivative is zero happens when $x = 1$. Now we must check what happens at $x = 1$. At $x = 1$

$$f(1) = 1 - \ln(9). \tag{17.3}$$

The max was $2 - \ln(18)$ and the min was $1 - \ln(9)$. ♢

Remark 1. It is difficult to determine which of Equations (17.1), (17.2), and (17.3) are the min and max, so I did not take off any points for mislabeling the extrema.

Here's a general method for dealing with logarithms. Let's look at Equations (17.1), (17.2), and (17.3). The first thing you should try to do when you come across a logarithm is try to see if you can write what is on the inside as a power or a product of numbers. In this case,

$$\begin{aligned} f(1/2) &= 1/2 - \ln(9/2) \\ &= 1/2 - (\ln(9) - \ln(2)) \\ &= 1/2 - \ln(3^2) + \ln(2), \\ &= 1/2 - 2\ln(3) + \ln(2), \\ f(2) &= 2 - \ln(18) \\ &= 2 - 2\ln(3) - \ln(2), \\ f(1) &= 1 - \ln(9) \\ &= 1 - 2\ln(3), \end{aligned}$$

Now here's another tip. To see which number between a and b is bigger, try to subtract them. For example

$$f(1/2) - f(2) = 1/2 - 2 + 2\ln(2) = -3/2 + 2\ln(2)$$

Now $\ln(2) \approx 0.7$ so this number is negative, i.e. $f(2)$ is bigger than $f(1/2)$.

Problem 17.2. Find the absolute maximum and absolute minimum of $f(x) = (x^2 - 1)^3$ on the interval $[-1, 5]$.

Solution. By the Extreme Value Theorem, we must check at the end points and where the derivative of f equals 0. First, let us take the derivative

$$f'(x) = 6x(x^2 - 1)^2. \tag{17.4}$$

The zeros of Equation (17.4) happen at $x = 0, \pm 1$. Luckily this means that we only have to check $x = -1$ once. Now, let us find all the extrema.

$$\begin{aligned} f(-1) &= 0, \\ f(1) &= 0, \\ f(0) &= -1, \\ f(5) &= 24^3. \end{aligned}$$

Clearly $f(5) = 24^3$ is our maximum and $f(0) = -1$ our minimum. $\Diamond$

Problem 17.3. Let $f(x) = x + 4/x$. If we denote by M the absolute maximum of f on $[1, 4]$ and by m the absolute minimum, what is their product Mm?

Solution. First let us find the derivative. The derivative of f is

$$f'(x) = 1 - \frac{4}{x^2},$$

which has zero at $x = 2$ and $x = -2$. Since $x = -2$ is outside our interval, which was $[1, 4]$, we disregard it. So the extrema are

$$\begin{aligned} f(1) &= 5, \\ f(2) &= 4, \\ f(4) &= 5. \end{aligned}$$

Then $M = 5$ and $m = 4$ so $Mm = 20$. $\Diamond$