

# MA 261 QUIZ 1

## JANUARY 15, 2018

If you do not know how to do any one of these problems, circle “**(E) I don’t know**” as your answer choice. You will receive **two points** for doing that. **Each problem** is worth **five points**. You get **two points** for writing your **full name** and **three points** for writing your **section number**.

**Problem 1.1.** Find a pair of unit vectors  $\mathbf{u}_1$  and  $\mathbf{u}_2$  which make a  $60^\circ$  angle with  $\mathbf{v} = \langle \sqrt{3}, 1 \rangle$ .

- (A)  $\mathbf{u}_1 = \langle 1, 0 \rangle, \mathbf{u}_2 = \langle -\sqrt{3}, -1 \rangle$ .
- (B)  $\mathbf{u}_1 = \langle \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \rangle, \mathbf{u}_2 = \langle \frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2} \rangle$ .
- (C)  $\mathbf{u}_1 = \langle 1, 0 \rangle, \mathbf{u}_2 = \langle \sqrt{3}, -1 \rangle$ .
- (D)  $\mathbf{u}_1 = \langle 0, 1 \rangle, \mathbf{u}_2 = \langle \frac{\sqrt{3}}{2}, -\frac{1}{2} \rangle$ .
- (E) I don’t know.

*Solution.* To solve this problem, remember that we can find the angle the vector  $\mathbf{v} = \langle \sqrt{3}, 1 \rangle$  makes with respect to the origin, using the following equation

$$\tan \theta = \frac{1}{\sqrt{3}}.$$

Since both the  $x$  and  $y$  component of  $\mathbf{v}$  are positive, this happens in the first quadrant, and the angle must be  $30^\circ$ . Now that we know this angle, we need only add and subtract  $60^\circ$  to figure out the angle the vectors  $\mathbf{u}_1$  and  $\mathbf{u}_2$  should make with respect to the origin. These are,  $90^\circ$  and  $-30^\circ$ , respectively. After we convert these angles to unit vectors by putting them into the formula  $\langle \cos \theta, \sin \theta \rangle$ , we get

$$\mathbf{u}_1 = \langle 0, 1 \rangle, \quad \mathbf{u}_2 = \left\langle \frac{\sqrt{3}}{2}, -\frac{1}{2} \right\rangle.$$

Therefore, the answer is (D). ◇

**Problem 1.2.** Find the area of the triangle with vertices at  $(2, 1, 1)$ ,  $(1, 2, 1)$ , and  $(1, 1, 2)$ .

- (A)  $7/2$ .
- (B)  $3/2$ .
- (C)  $\sqrt{2}$ .

- (D)  $\sqrt{3}/2$ .  
 (E) I don't know.

*Solution.* By fixing a point, say,  $(2, 1, 1)$ , and finding the direction from  $(2, 1, 1)$  to  $(1, 2, 1)$  and  $(1, 1, 2)$ , we may find the area of the parallelogram made by these vectors by taking the magnitude of their cross-product; the area of the triangle will be half of this. In symbols,

$$\begin{aligned}\mathbf{u} &= (1, 2, 1) - (2, 1, 1) = \langle -1, 1, 0 \rangle \\ \mathbf{v} &= (1, 1, 2) - (2, 1, 1) = \langle -1, 0, -1 \rangle \\ \mathbf{v} \times \mathbf{u} &= \langle -1, -1, 1 \rangle.\end{aligned}$$

Therefore, the area of the parallelogram is  $\sqrt{3}$ , so the area of the triangle is  $\sqrt{3}/2$ .

The correct answer choice was (D).  $\diamond$

**Problem 1.3.** Find parameterization for the line passing through the points  $(1, -2, 1)$  and  $(2, 3, -1)$ .

- (A)  $\mathbf{r}(t) = \langle t + 2, 5t + 3, 2t - 1 \rangle$   
 (B)  $\mathbf{r}(t) = \langle t + 1, 5t - 2, -2t + 1 \rangle$   
 (C)  $\mathbf{r}(t) = \langle t, 5t, -2t \rangle$   
 (D)  $\mathbf{r}(t) = \langle t + 1, -5t - 2, -2t \rangle$   
 (E) I don't know.

*Hint:* The parameterization is not unique, but there is only one correct answer choice.

*Solution.* To begin solving this problem, we need to know the direction from one point to the other; in this case, let us go from  $(1, -2, 1)$  to  $(2, 3, -1)$ . The vector we get upon subtracting one point from the other is

$$(2, 3, -1) - (1, -2, 1) = \langle 1, 5, -2 \rangle.$$

Therefore, if we parameterize the line, it should go in the direction  $\langle 1, 5, -2 \rangle$  and it must pass through the point  $(1, -2, 1)$  (or  $(2, 3, -1)$ , we only need to check one of them since we are already headed in the direction of the other). The line

$$\mathbf{r}(t) = \langle t, 5t, -2t \rangle + \langle 1, -2, 1 \rangle = \langle t + 1, 5t - 2, -2t + 1 \rangle$$

satisfies these conditions.

A very quick way to do this problem would have been to check which one of the parameterization passes through both points.

The correct answer was (B).

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