MA 261 Quiz 1

January 15, 2018

If you do not know how to do any one of these problems, circle **"(E) I don't know"** as your answer choice. You will receive **two points** for doing that. **Each problem** is worth **five points**. You get **two points** for writing your **full name** and **three points** for writing your **section number**.

Problem 1.1. Find a pair of unit vectors $\mathbf{u}_1$ and $\mathbf{u}_2$ which make a $60°$ angle with $\mathbf{v} = \langle\sqrt{3}, 1\rangle$.

(A) $\mathbf{u}_1 = \langle 1, 0\rangle, \mathbf{u}_2 = \langle -\sqrt{3}, -1\rangle$.
(B) $\mathbf{u}_1 = \left\langle \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right\rangle, \mathbf{u}_2 = \left\langle \frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right\rangle$.
(C) $\mathbf{u}_1 = \langle 1, 0\rangle, \mathbf{u}_2 = \langle \sqrt{3}, -1\rangle$.
(D) $\mathbf{u}_1 = \langle 0, 1\rangle, \mathbf{u}_2 = \left\langle \frac{\sqrt{3}}{2}, -\frac{1}{2}\right\rangle$.
(E) I don't know.

Solution. To solve this problem, remember that we can find the angle the vector $\mathbf{v} = \langle\sqrt{3}, 1\rangle$ makes with respect to the origin, using the following equation

$$\tan\theta = \frac{1}{\sqrt{3}}.$$

Since both the x and y component of $\mathbf{v}$ are positive, this happens in the first quadrant, and the angle must be $30°$. Now that we know this angle, we need only add and subtract $60°$ to figure out the angle the vectors $\mathbf{u}_1$ and $\mathbf{u}_2$ should make with respect to the origin. These are, $90°$ and $-30°$, respectively. After we convert these angles to unit vectors by putting them into the formula $\langle\cos\theta, \sin\theta\rangle$, we get

$$\mathbf{u}_1 = \langle 0, 1\rangle, \quad \mathbf{u}_2 = \left\langle \frac{\sqrt{3}}{2}, -\frac{1}{2}\right\rangle.$$

Therefore, the answer is (D). ◇

Problem 1.2. Find the area of the triangle with vertices at $(2, 1, 1)$, $(1, 2, 1)$, and $(1, 1, 2)$.

(A) $7/2$.
(B) $3/2$.
(C) $\sqrt{2}$.
(D) $\sqrt{3}/2$.
(E) I don't know.

Solution. By fixing a point, say, $(2,1,1)$, and finding the direction from $(2,1,1)$ to $(1,2,1)$ and $(1,1,2)$, we may find the area of the parallelogram made by these vectors by taking the magnitude of their cross-product; the area of the triangle will be half of this. In symbols,

$$\begin{aligned}\mathbf{u} &= (1,2,1) - (2,1,1) = \langle -1,1,0\rangle \\ \mathbf{v} &= (1,1,2) - (2,1,1) = \langle -1,0,-1\rangle \\ \mathbf{v}\times\mathbf{u} &= \langle -1,-1,1\rangle.\end{aligned}$$

Therefore, the area of the parallelogram is $\sqrt{3}$, so the area of the triangle is $\sqrt{3}/2$.

The correct answer choice was (D). ◇

Problem 1.3. Find parameterization for the line passing through the points $(1,-2,1)$ and $(2,3,-1)$.

(A) $\mathbf{r}(t) = \langle t+2, 5t+3, 2t-1\rangle$
(B) $\mathbf{r}(t) = \langle t+1, 5t-2, -2t+1\rangle$
(C) $\mathbf{r}(t) = \langle t, 5t, -2t\rangle$
(D) $\mathbf{r}(t) = \langle t+1, -5t-2, -2t\rangle$
(E) I don't know.

Hint: The parameterization is not unique, but there is only one correct answer choice.

Solution. To begin solving this problem, we need to know the direction from one point to the other; in this case, let us go from $(1,-2,1)$ to $(2,3,-1)$. The vector we get upon subtracting one point from the other is

$$(2,3,-1) - (1,-2,1) = \langle 1,5,-2\rangle.$$

Therefore, if we parameterize the line, it should go in the direction $\langle 1,5,-2\rangle$ and it must pass through the point $(1,-2,1)$ (or $(2,3,-1)$, we only need to check one of them since we are already headed in the direction of the other). The line

$$r(t) = \langle t, 5t, -2t\rangle + \langle 1,-2,1\rangle = \langle t+1, 5t-2, -2t+1\rangle$$

satisfies these conditions.

A very quick way to do this problem would have been to check which one of the parameterization passes through both points.

The correct answer was (B). ◇