MA 261 Quiz 4
February 5, 2018

If you do not know how to do any one of these problems, circle **"(E) I don't know"** as your answer choice. You will receive **two points** for doing that. **Each problem** is worth **five points**. You get **two points** for writing your **full name** and **three points** for writing your **section number**.

Problem 4.1. What is the domain of the function $f(x, y) = \ln(x/(y+2))$?
(A) $x > 0$, $y > -2$
(B) $x > 0$, $y > 2$
(C) $x > 0$, $y > -2$ or $x < 0$, $y < -2$
(D) $x > 0$, $y > 2$ or $x < 0$, $y < 2$
(E) I don't know how to do this

Solution. This problem is a bit tricky if you don't notice that $x/(y+2)$ is positive when $x < 0$ and $y < -2$.

What's even more troubling is that if you try to separate $\ln(x/(y+2))$ into $\ln x + \ln(y+2)$, as you were taught to do, the domain of the function changes since $\ln x$ is undefined for negative values of x. The thing to note here is that $\ln(x - y) = \ln x - \ln y$ only makes sense for positive x and y. This identity no longer holds when x or y is less that 0.

Answer: (C). ◇

Problem 4.2. Find dy/dx for $3y^4 + x^7 = 5x$.
(A) $(7x^6 - 5)/(12y^3)$
(B) $(5 - 7x^6)/(12y^3)$
(C) $12y^3/(5 - 7x^6)$
(D) $2x^3/(1 - x^6)$
(E) I don't know how to do this

Solution. A very simple way to compute this is as we showed in class, that is,

$$\begin{aligned} \frac{d}{dx}\left(3y^4 + x^7\right) &= \frac{d}{dx}(5x) \\ 12y^3\frac{dy}{dx} + 7x^6 &= 5 \\ \frac{dy}{dx} &= \frac{5 - 7x^6}{12y^3}. \end{aligned}$$

Answer: (B). $\diamond$

Problem 4.3. What are the level curves of $f(x, y) = \sqrt{x^2 + 4y^2 + 4} - x$?
(A) hyperbolas
(B) ellipses
(C) parabolas
(D) circles
(E) I don't know how to do this

Solution. We provided a method for tackling these types of problems in class. The method goes as follows. Fix a number k and let $f(x, y) = k$. Then

$$k = \sqrt{x^2 + 4y^2 + 4} - x$$

and we play around with this equation until we arrive at some conic section we recognize. That is,

$$\begin{aligned} k &= \sqrt{x^2 + 4y^2 + 4} - x \\ (k + x)^2 &= x^2 + 4y^2 + 4 \\ k^2 + 2kx + x^2 &= x^2 + 4y^2 + 4 \\ x &= \frac{4y^2 + 4 - k^2}{2k}. \end{aligned}$$

The last equation is that of a *parabola* increasing along the x axis.

Answer: (C). $\diamond$