MA161 Quiz 20 Solutions

TA: Carlos Salinas

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Problem 20.1. Use **L'Hôpital's Rule** to determine the following limits

(a) $\displaystyle\lim_{x\to 0} \frac{1-\cos(4x)}{x^2}$;

(b) $\displaystyle\lim_{x\to 0^+} (1+\sin(x))^{1/x}$;

(c) $\displaystyle\lim_{x\to 0} \frac{\ln(e^3+3x)-3}{x}$;

(d) $\displaystyle\lim_{x\to 0} (1+2x)^{\cot(x)}$.

Solution. For part (a),

$$\begin{aligned}\lim_{x\to 0} \frac{1-\cos(4x)}{x^2} &= \lim_{x\to 0} \frac{2\sin(4x)}{x} \\ &= \lim_{x\to 0} 8\cos(4x) \\ &= \boxed{8}.\end{aligned}$$

For part (b),

$$\begin{aligned}\ln(L) &= \ln\left(\lim_{x\to 0^+} (1+\sin(x))^{1/x}\right) \\ &= \lim_{x\to 0^+} \frac{\ln(1+\sin(x))}{x} \\ &= \lim_{x\to 0^+} \frac{\cos(x)}{1+\sin(x)} \\ &= 1.\end{aligned}$$

Therefore, $L = e^{\ln(L)} = \boxed{e^1}$.

For part (b),

$$\begin{aligned}\lim_{x\to 0}\frac{\ln(e^3+3x)-3}{x} &= \lim_{x\to 0}\frac{3}{e^3+3x}\\ &= \boxed{\frac{3}{e^3}}.\end{aligned}$$

For part (d),

$$\begin{aligned}\ln(L) &= \lim_{x\to 0}\cot(x)\ln(1+2x)\\ &= \lim_{x\to 0}\frac{\ln(1+2x)}{\tan(x)}\\ &= \lim_{x\to 0}\frac{2}{(1+2x)\sec^2(x)}\\ &= 2.\end{aligned}$$

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Therefore, $L = e^{\ln(L)} = \boxed{e^2}$.