MA 261 Practice Midterm 1

October 2, 2018

Problem 1. Find the angle between the planes given by the equations $x + y = 2$ and $x + y + \sqrt{2}z = \sqrt{6}$.

(A) $\pi/2$
(B) $\pi/4$
(C) $\pi/6$
(D) π
(E) $\pi/3$

Problem 2. Find the length of the curve

$$\mathbf{r}(t) = \langle t - \sin(t), 1 - \cos(t) \rangle$$

on $0 \leq t \leq \pi$. (*Hint*: Use the double angle formula $\cos(2\theta) = 1 - 2\sin^2(\theta)$.)

(A) 4
(B) 8
(C) -4
(D) -5
(E) 2π

Problem 3. Find the area of the triangle with vertices $P(2, 2, 1)$, $Q(1, -1, 2)$, and $R(0, 1, -1)$.

(A) $\sqrt{5}$
(B) $3\sqrt{10}/2$
(C) $\sqrt{31}/2$
(D) $2\sqrt{5}$
(E) $\sqrt{69}/2$

Problem 4. The absolute minimum value of

$$f(x, y) = 2 + x^2y^2$$

in the region $x^2/2 + y^2 \leq 1$ equals 2. The absolute maximum value of f in this region is?

(A) 4.5
(B) 4
(C) 3.5
(D) 3
(E) 2.5

Problem 5. Find $f'(1)$, where $f(t) = \mathbf{u}(t) \cdot \mathbf{v}(t)$, $\mathbf{u}(1) = \langle 1, 1, 1 \rangle$, $\mathbf{u}'(1) = \langle 1, 2, 3 \rangle$, and $\mathbf{v}(t) = \langle t, t^2, t^3 \rangle$.

(A) 6
(B) 14
(C) 28
(D) 12
(E) 24

Problem 6. Find the tangent plane to the level surface $xy^2z^3 = 12$ at $(3, 2, 1)$.

(A) $x + 2y + 3z = 10$
(B) $x + y + z = 6$
(C) $3x + 2y + z = 14$
(D) $x + 3y + 6z = 15$
(E) $x + 3y + 9z = 18$

Problem 7. Find the directional derivative for

$$T(x, y) = \frac{y - 1}{x - 2}$$

at $(3, -2)$ in the direction toward the origin.

(A) 7
(B) -7
(C) $7/\sqrt{13}$
(D) $-7/\sqrt{13}$
(E) 5

Problem 8. Suppose the graph of $z = g(x, y)$ intersects the plane $x = 0$ along the curve $z = y^3 + 2y^2 + 1$. What is $g_y(0, 2)$?

(A) 1
(B) 4
(C) 8
(D) 17
(E) 20

Solutions: 1–(B), 2–(A), 3–(B), 4–(E), 5. (D), 6–(E), 7–(C), 8–(E).