MA161 Quiz 12 Solutions

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Problem 12.1. Find the derivative of

$$y = x^{\tan^{-1}(x)}.$$

Solution. As we saw in the previous quiz, we can write y as

$$y = x^{\tan^{-1}(x)} = e^{\ln(x)\tan^{-1}(x)}.$$

Then

$$y' = \left(\frac{\tan^{-1}(x)}{x} + \frac{\ln(x)}{x^2+1}\right)x^{\tan^{-1}(x)}. \qquad \smiley$$

Problem 12.2. Find dy/dx by implicit differentiation

(a) $9\sqrt{x} + 9\sqrt{y} = 8$;

(b) $4x^2 + 3xy - y^2 = 3$.

Solution. For part (a),

$$8 = 9\sqrt{x} + 9\sqrt{y},$$
$$0 = \frac{9}{2\sqrt{x}} + \frac{9y'}{2\sqrt{y}}$$
$$y' = -\frac{\sqrt{y}}{\sqrt{x}}$$
$$= -\sqrt{\frac{y}{x}}.$$

For part (b),

$$3 = 4x^2 + 3xy - y^2,$$
$$0 = 8x + 3xy' + 3y - 2yy'$$
$$-8x - 3y(3x - 2y)y'$$
$$y' = \frac{8x + 3y}{2y - 3x}. \qquad \text{☺}$$

Problem 12.3. Find the derivative of the function

$$5^{\ln(x)+a^2}.$$

Solution. Here is a simple solution. Write

$$y = 5^{\ln(x)+a^2}$$
$$= 5^{a^2}e^{\ln(5)\ln(x)}$$
$$= 5^{a^2}\left(e^{\ln(x)}\right)^{\ln(5)}$$
$$= 5^{a^2}x^{\ln(5)}.$$

Therefore,

$$y' = 5^{a^2}\ln(5)x^{\ln(5)-1}. \qquad \text{☺}$$