

# MA161 Quiz 21

TA: Carlos Salinas

April 10, 2018

**Problem 21.1.** What is the maximum vertical distance between the line  $y = x + 12$  and the parabola  $y = x^2$  for  $-3 \leq x \leq 4$ ?

*Solution.* The distance from a point on  $y = x + 12$  and  $y = x^2$  is given by the distance formula

$$\begin{aligned} d &= \sqrt{(x_1 - x_0)^2 + (y_1 - y_0)^2} \\ &= \sqrt{(x - x)^2 + (x^2 - x - 12)^2} \\ &= |x^2 - x - 12|. \end{aligned}$$

Since  $x$  is between  $-3$  and  $4$  and  $x^2 - x - 12$  is positive there, we are free to drop the absolute value. Then, to find the maximum distance, we have to take the derivative

$$d' = 2x - 1.$$

Therefore,  $x = 1/2$  is a possible candidate. Now we use the Extreme Value Theorem, which tells us that the extrema (minima and maxima) happen at the endpoints and at the critical points. That is,

$$d(-3) = 6, \quad d(4) = 0, \quad d(1/2) = 11.25.$$

Therefore, the maximum happens at  $d(1/2) = 11.25$ .

☺

**Problem 21.2.** A box with a square base and open top must have a volume of 62500 cm<sup>3</sup>. Find the dimensions of the box that minimize the amount of material used.

*Solution.* The volume of the box is

$$V = 62500 = x^2 h$$

and therefore  $h = 62500/x^2$ . Now, we want to minimize the material used, which is the same as minimizing the surface area which is  $S = x^2 + 4xh$ . By the equation above,

$$S = x^2 + \frac{250000x}{x^2}.$$

To minimize this value, we must take a derivative, as we now do

$$S' = 2x + \frac{250000}{x^2} = 0.$$

Thus,

$$2x^3 - 250000 = 0.$$

Therefore,

$$x = \frac{\sqrt[3]{250000}}{2}.$$

And so,

$$h = \frac{125000}{\sqrt[3]{250000}}.$$

☺