

MA161 Quiz 20 Solutions

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Problem 20.1. Use **L'Hôpital's Rule** to determine the following limits

(a) $\lim_{x \rightarrow 0} \frac{1 - \cos(4x)}{x^2};$

(c) $\lim_{x \rightarrow 0} \frac{\ln(e^3 + 3x) - 3}{x};$

(b) $\lim_{x \rightarrow 0^+} (1 + \sin(x))^{1/x};$

(d) $\lim_{x \rightarrow 0} (1 + 2x)^{\cot(x)}.$

Solution. For part (a),

$$\begin{aligned} \lim_{x \rightarrow 0} \frac{1 - \cos(4x)}{x^2} &= \lim_{x \rightarrow 0} \frac{2 \sin(4x)}{x} \\ &= \lim_{x \rightarrow 0} 8 \cos(4x) \\ &= \boxed{8}. \end{aligned}$$

For part (b),

$$\begin{aligned} \ln(L) &= \ln\left(\lim_{x \rightarrow 0^+} (1 + \sin(x))^{1/x}\right) \\ &= \lim_{x \rightarrow 0^+} \frac{\ln(1 + \sin(x))}{x} \\ &= \lim_{x \rightarrow 0^+} \frac{\cos(x)}{1 + \sin(x)} \\ &= 1. \end{aligned}$$

Therefore, $L = e^{\ln(L)} = \boxed{e^1}.$

For part (b),

$$\begin{aligned}\lim_{x \rightarrow 0} \frac{\ln(e^3 + 3x) - 3}{x} &= \lim_{x \rightarrow 0} \frac{3}{e^3 + 3x} \\ &= \boxed{\frac{3}{e^3}}.\end{aligned}$$

For part (d),

$$\begin{aligned}\ln(L) &= \lim_{x \rightarrow 0} \cot(x) \ln(1 + 2x) \\ &= \lim_{x \rightarrow 0} \frac{\ln(1 + 2x)}{\tan(x)} \\ &= \lim_{x \rightarrow 0} \frac{2}{(1 + 2x) \sec^2(x)} \\ &= 2.\end{aligned}$$

Therefore, $L = e^{\ln(L)} = \boxed{e^2}$.

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