

MA 261 QUIZ 5

FEBRUARY 12, 2018

If you do not know how to do any one of these problems, circle “**(E) I don’t know**” as your answer choice. You will receive **two points** for doing that. **Each problem** is worth **five points**. You get **two points** for writing your **full name** and **three points** for writing your **section number**.

Problem 5.1. Find an equation for the plane tangent to $xy^2z^3 = 12$ at $(3, 2, 1)$.

- (A) $x + 2y + 3z = 10$
- (B) $x + y + z = 6$
- (C) $3x + 2y + z = 14$
- (D) $x + 3y + 9z = 18$
- (E) I don’t know how to do this

Solution. As we mentioned in class, to find the plane tangent to $xy^2z^3 = 12$ at the point $(3, 2, 1)$, we need to find the gradient of the function $F(x, y, z) = xy^2z^3 - 12$ at the point $(3, 2, 1)$ and then put it into the tangent plane formula

$$\text{grad } f \cdot \langle x - x_0, y - y_0, z - z_0 \rangle = 0.$$

First, it is easy to compute that $\text{grad } f(x, y, z) = \langle y^2z^3, 2xyz^3, 3xy^2z^2 \rangle$ so $\text{grad } f(3, 2, 1) = \langle 4, 12, 36 \rangle$. Therefore, the equation for the tangent plane is

$$\langle 4, 12, 36 \rangle \cdot \langle x - 3, y - 2, z - 1 \rangle = 0,$$

or, if we simplify this,

$$x + 3y + 9z = 18.$$

Answer: (D). ◇

Problem 5.2. Use linear approximation on $f(x, y) = \sqrt{x^2 + 3y}$ at $(4, 3)$ to approximate the value of $\sqrt{(4.02)^2 + 3(2.97)}$.

- (A) 5.004
- (B) 5.05
- (C) 4.093
- (D) 5.007
- (E) I don’t know how to do this

Solution. Recall from class that to find the linear approximation of a function f we must find its tangent line. That is, using f as given above, first we compute its gradient

$$\text{grad } f(x, y) = \frac{\langle x, 3/2 \rangle}{\sqrt{x^2 + 3y}}.$$

Therefore, the tangent line at the point $(4, 3)$ is given by

$$L(x, y) = \frac{4}{5}(x - 4) + \frac{3}{10}(y - 3) + 5,$$

and thus,

$$\begin{aligned} \sqrt{(4.02)^2 + 3(2.97)} &\approx L(4.02, 2.97) \\ &= 5 + (0.8)(0.02) - (0.3)(0.03) \\ &= 5.007. \end{aligned}$$

Answer: (D)

◇

Problem 5.3. Find the directional derivative of $f(x, y) = \sin(4x + 2y)$ at the point $(-4, 8)$ in the direction $\mathbf{v} = \sqrt{3}\mathbf{i} - \mathbf{j}$.

- (A) 0
- (B) $4\sqrt{3} - 2$
- (C) $2\sqrt{3} - 1$
- (D) $2 - 4\sqrt{3}$
- (E) I don't know how to do this

Solution. By the directional derivative formula, first we need to find the unit direction, \mathbf{u} , of \mathbf{v} , which is

$$\mathbf{u} = \frac{\mathbf{v}}{|\mathbf{v}|} = \frac{\sqrt{3}}{2}\mathbf{i} - \frac{1}{2}\mathbf{j}.$$

We also need the gradient at $(-4, 8)$, which is

$$\text{grad } f(x, y) = \langle 4 \cos(4x + 2y), 2 \cos(4x + 2y) \rangle,$$

so

$$\text{grad } f(-4, 8) = 4\mathbf{i} + 2\mathbf{j}.$$

Therefore,

$$D_{\mathbf{u}}f(-4, 8) = \left(\frac{\sqrt{3}}{2}\mathbf{i} - \frac{1}{2}\mathbf{j} \right) \cdot (4\mathbf{i} + 2\mathbf{j}) = 2\sqrt{3} - 1.$$

Answer: (C).

