$\boxed{\text{2. CVIKO}}$ ODR I. řádu - Separovatelné ($y(x)$ neznámá)

a) $f(y', y, x) = 0$ nejobecnější (Př.: $\ln(y') + \sqrt{y'} - y + x = 0$)

b) $y' = g(y, x)$ rozřešená vzhledem k y' (Př.: $y' = \frac{y+x}{y-x}$)

c) $y' = u(y) \cdot v(x)$ Separovatelné - Teď

Př. Já $y' = \frac{y}{x}$ a $y(1) = 2$ (Cauchyho úloha)

$$\frac{dy}{dx} = \frac{y}{x} \quad / : y / \cdot dx \quad \to y = 0 \text{ je řeš.!!}$$

$$\frac{dy}{y} = \frac{dx}{x}$$

$$\int \frac{dy}{y} = \int \frac{dx}{x}$$

$$\ln|y| = \ln|x| + c_1; \quad c_1 \in R$$

$$\begin{pmatrix} \ln|y| = \ln|x| + \ln c_2 \\ \ln|y| = \ln[c_2 |x|] \\ |y| = c_2 |x| \\ y = cx \end{pmatrix}$$

$$|y| = e^{[\ln|x| + c_1]}$$

$$|y| = e^{c_1} \cdot |x|$$

Platí: $\ln|y| = \ln|x| + c_1 \Leftrightarrow y = cx$

$$y = \pm e^{c_1} \cdot x$$

$$\boxed{y = cx}$$

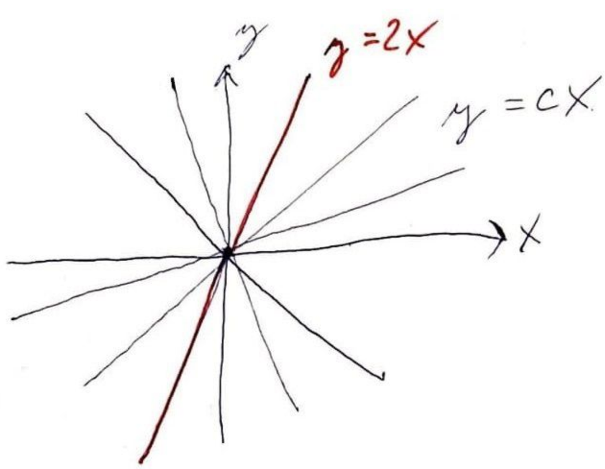
$c = \pm e^{c_1} \cup \{0\}$ ($c = 0$ odpovídá $y = 0$ a to víme, že je řešení)

$\underline{c \in R}$

$$y(\overset{x_0}{1}) = \overset{y_0}{2}$$

$$2 = c \cdot 1$$

$$\boxed{c = 2} \Rightarrow \boxed{y = 2x}$$



1. $y' = \dfrac{y-1}{x^2 y^2}$ $\longrightarrow$ $\underline{y=1 \text{ je řeš!!}}$

$\dfrac{dy}{dx} = \dfrac{y-1}{x^2 y^2} \quad / : y-1 \;/ \cdot y^2 dx$

$\dfrac{y^2}{y-1}\, dy = \dfrac{dx}{x^2}$

$\displaystyle\int \frac{(y^2-1+1)\,dy}{y-1} = \int x^{-2}\, dx$

$\displaystyle\int \left(y+1+\frac{1}{y-1}\right) dy = \int x^{-2}\, dx$

$\dfrac{y^2}{2} + y + \ln|y-1| = -\dfrac{1}{x} + c$

$\boxed{\dfrac{y^2}{2} + y + \ln|y-1| + \dfrac{1}{x} = c}$; $\boxed{y=1}$ $\longrightarrow$ je singulární řešení

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2. $y' \operatorname{cotg} x + y = 2$ a $\boxed{y(0) = -1}$

$\dfrac{dy}{dx}\,\dfrac{\cos x}{\sin x} = 2 - y \quad / : 2-y \;/ : \cos x \;/ \cdot \sin x\, dx$

$\longrightarrow$ $\underline{y = 2 \text{ je řeš!!}}$

$-\displaystyle\int \frac{-dy}{2-y} = -\int \frac{-\sin x}{\cos x}\, dx$

$-\ln|2-y| = -\ln|\cos x| - c_1 \qquad c_1 \in \mathbb{R}$

$\ln|2-y| = \ln|\cos x| + c_1$

$\vdots$

$2 - y = c \cdot \cos x \qquad c \in \mathbb{R}$

$y = 2 - c\cdot\cos x \qquad \underline{y(0) = -1} : \; -1 = 2 - c$

$\underline{\underline{c = 3}}$

$\boxed{y = 2 - 3\cos x}$

(3) $y' \sin^2 x \cos^2 x = e^{-y}$

$$\frac{dy}{dx} \sin^2 x \cos^2 x = e^{-y} \quad \Big/ \frac{e^{y}\, dx}{\sin^2 x \cos^2 x}$$

$$\int e^{y} dy = \int \frac{1}{\sin^2 x \cos^2 x}\, dx$$

$$\int e^{y} dy = \int \frac{\sin^2 x + \cos^2 x}{\sin^2 x \cos^2 x}\, dx$$

$$\int e^{y} dy = \int \left(\frac{1}{\cos^2 x} + \frac{1}{\sin^2 x} \right) dx$$

$$\boxed{e^{y} = \operatorname{tg} x - \operatorname{cotg} x + c; \qquad c \in \mathbb{R} \text{ vhodná}}$$

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(4) $1 + y^2 - xy(1+x^2)\, y' = 0$

$$xy(1+x^2) \frac{dy}{dx} = 1 + y^2 \quad \Big/ \frac{dx}{x(x^2+1)(y^2+1)}$$

$$\frac{1}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+1}$$

$$1 = Ax^2 + A + Bx^2 + Cx$$

$\underline{\underline{x^0}}: \; 1 = A \Rightarrow \boxed{A = 1}$

$\underline{\underline{x^1}}: \; 0 = C \Rightarrow \boxed{C = 0}$

$\underline{\underline{x^2}}: \; 0 = A + B \quad \boxed{B = -1}$

$$\frac{1}{2} \int \frac{2y}{y^2+1}\, dy = \int \frac{1}{x(x^2+1)}\, dx$$

$$\frac{1}{2} \ln(y^2+1) = \int \frac{1}{x} - \frac{1}{2} \int \frac{2x}{x^2+1}$$

$$\frac{1}{2} \ln(y^2+1) = \ln|x| - \frac{1}{2} \ln(x^2+1) + c_1$$

$$\ln(y^2+1)^{\frac{1}{2}} = \ln|x| - \ln(x^2+1)^{\frac{1}{2}} + c_1$$

$$\ln\sqrt{y^2+1} = \ln \frac{|x|}{\sqrt{x^2+1}} + c_1 \qquad (c_1 \in \mathbb{R})$$

$$\vdots$$

$$\sqrt{y^2+1} = c_2 \frac{|x|}{\sqrt{x^2+1}} \qquad (c_2 \in \mathbb{R})$$

$$y^2 + 1 = \frac{c_2^2 x^2}{x^2+1}$$

$$\boxed{y = \pm \sqrt{\frac{c x^2}{x^2+1} - 1}\,; \qquad c = c_2^2 > 0, \quad c \in \mathbb{R}^+}$$

5) $y' = \dfrac{y \ln y}{\sin x}$ [Asi DÚ]

$y=0$ Není řeš.!!

$y=1$ Je řeš!!

$$\frac{dy}{dx} = \frac{y \ln y}{\sin x} \quad / \cdot \frac{dx}{y \ln y}$$

$$\int \frac{dy}{y \ln y} = \int \frac{dx}{\sin x}$$

$$\int \frac{dy}{y \ln y} \left| \begin{array}{l} \ln y = s \\ \frac{1}{y} dy = ds \end{array} \right| = \int \frac{1}{s}\, ds = \ln|s| = \ln|\ln y|$$

$$\ln|\ln y| = \ln \sqrt{\left|\frac{\cos x - 1}{\cos x + 1}\right|} + c_1 \quad (c_1 \in \mathbb{R})$$

$$\vdots$$

$$\ln y = c_2 \sqrt{\left|\frac{\cos x - 1}{\cos x + 1}\right|} \quad c_2 \in \mathbb{R} \quad (c_2 = 0 \;\; y = 0)$$

$$y = e^{c_2 \sqrt{\left|\frac{\cos x - 1}{\cos x + 1}\right|}}$$

$$\boxed{y = c^{\sqrt{\left|\frac{\cos x - 1}{\cos x + 1}\right|}} \quad c > 0} \quad (c = e^{c_2})$$

$$\int \frac{1}{\sin x}\, dx = \int \frac{\sin x\, dx}{1 - \cos^2 x} \left| \begin{array}{l} \cos x = u \\ -\sin x\, dx = du \end{array} \right| = \int \frac{-du}{1-u^2} = \int \frac{1}{u^2 - 1}\, du =$$

$$= \int \left(\frac{\frac{1}{2}}{u-1} - \frac{\frac{1}{2}}{u+1} \right) du = \frac{1}{2} \left[\ln|u-1| - \ln|u+1| \right] = \frac{1}{2} \ln \left| \frac{u-1}{u+1} \right| = \ln \sqrt{\left| \frac{\cos x - 1}{\cos x + 1} \right|}$$

DÚ

- $y' = e^{x+y}$ $\quad [\, e^x + e^{-y} = c \,]$
- $y' + \sqrt{\dfrac{1-y^2}{1-x^2}} = 0$ $\quad [\, \arcsin x + \arcsin y = c \,]$
- $y' = (y-1)(y-2)$ $\quad \left[\, \ln \left| \dfrac{y-2}{y-1} \right| = x + c \,\right]$