$\boxed{\text{3.CVIKO}}$ ODR I. řádu - Lineární

$$\boxed{y' + f(x)\,y = g(x)} \quad (*)$$

a) homogenní: Nejdříve řešíme rovnici $\boxed{y' + f(x)\,y = 0}$. Její řešení je ve tvaru $\boxed{y_h = c \cdot F(x)}$.

b) nehomogenní: Řešení nehomogenní rovnice $(*)$ hledáme ve tvaru $\boxed{y = c(x) \cdot F(x)}$ tzv. metodou variace konstanty. Dosadíme tento tvar do rovnice $(*)$ a dopočítáme $c(x)$.

(Př) Já $y' \cos x + y \sin x = 1 \quad /: \cos x$

$$y' + \frac{\sin x}{\cos x}\, y = \frac{1}{\cos x}$$

a) homogenní

$$y' = -\frac{\sin x}{\cos x}\, y$$

$$\frac{dy}{dx} = -\frac{\sin x}{\cos x}\, y$$

$$\int \frac{dy}{y} = \int \frac{-\sin x}{\cos x}\, dx$$

$$\ln|y| = \ln|\cos x| + c_1 \quad (c_1 \in \mathbb{R})$$

$\vdots$

$$\boxed{y_h = c \cdot \cos x} \quad (c \in \mathbb{R})$$

b) nehomogenní

$$\boxed{y = c(x) \cos x}$$

$$\boxed{y' = c'(x) \cos x - c(x) \sin x}$$

$$c'(x)\cos x - \cancel{c(x)\sin x} + \cancel{\frac{\sin x}{\cos x} c(x) \cos x} = \frac{1}{\cos x}$$

$$c'(x) = \frac{1}{\cos^2 x}$$

$$c(x) = \int \frac{1}{\cos^2 x}\, dx$$

$$\boxed{c(x) = \operatorname{tg} x + k}$$

$$\boxed{y = (\operatorname{tg} x + k) \cos x}$$

(1) $xy' + y = x^2$ a $y(1) = -2$

$$y' + \frac{1}{x}y = x$$

a) homogenní

$$y' + \frac{1}{x}y = 0$$

$$\frac{dy}{dx} = -\frac{y}{x} \quad / \cdot \frac{dx}{y}$$

$$\int \frac{dy}{y} = -\int \frac{dx}{x}$$

$$\ln|y| = -\ln|x| + c_1$$

$$\ln|y| = \ln\left|\frac{1}{x}\right| + c_1$$

$$\vdots$$

$$\boxed{y_h = \frac{c}{x}}$$

b) nehomogenní

$$\boxed{y = \frac{c(x)}{x}} \Rightarrow y' = \frac{c'(x)x - c(x)}{x^2} = \frac{c'(x)}{x} - \frac{c(x)}{x^2}$$

$$\frac{c'(x)}{x} - \cancel{\frac{c(x)}{x^2}} + \cancel{\frac{1}{x} \cdot \frac{c(x)}{x}} = x$$

$$c'(x) = x^2$$

$$\boxed{c(x) = \frac{x^3}{3} + k}$$

$$\boxed{y = \frac{\frac{x^3}{3} + k}{x}} \qquad -2 = \frac{1}{3} + k \qquad \boxed{k = -\frac{7}{3}}$$

$$y = \frac{\frac{x^3}{3} - \frac{7}{3}}{x} \Rightarrow \boxed{y = \frac{x^3 - 7}{3x}}$$

(2) $y' = 2xy + x - x^3$

homogenní

$$y' = 2xy$$

$$\frac{dy}{dx} = 2xy \quad / \cdot \frac{dx}{y}$$

$$\int \frac{dy}{y} = \int 2x\,dx$$

$$\ln|y| = x^2 + c_1$$

$$|y| = e^{x^2 + c_1}$$

$$y = \pm e^{c_1} \cdot e^{x^2}$$

$$\boxed{y_h = c \cdot e^{x^2}}$$

nehomogenní

$$\boxed{y = c(x) \cdot e^{x^2}} \Rightarrow \boxed{y' = c'(x) \cdot e^{x^2} + c(x)e^{x^2} \cdot 2x}$$

$$c'(x)e^{x^2} + \cancel{c(x)e^{x^2} \cdot 2x} = \cancel{2x\,c(x) \cdot e^{x^2}} + x - x^3$$

$$c'(x) = (x - x^3)e^{-x^2}$$

$$c(x) = \int x(1 - x^2)e^{-x^2}dx \left| \begin{array}{l} \boxed{s = -x^2} \\ ds = -2x\,dx \\ x\,dx = -\frac{1}{2}ds \end{array} \right| =$$

$$= -\frac{1}{2}\int (1+s)e^s ds \left| \begin{array}{ll} u = 1+s & u' = 1 \\ v' = e^s & v = e^s \end{array} \right| =$$

$$= -\frac{1}{2}\left[e^s(1+s) - e^s\right] = -\frac{1}{2}\left[s e^s\right] =$$

$$= -\frac{1}{2}(-x^2 e^{-x^2}) = \boxed{\frac{1}{2}x^2 e^{-x^2} + k}$$

$$y = \left(\frac{1}{2}x^2 e^{-x^2} + k\right)e^{x^2} \Rightarrow \boxed{y = \frac{1}{2}x^2 + ke^{x^2}}$$

③ $y' - \frac{y}{x+1} = x-1$ a $y(0)=0$ [Asi DÚ]

a) homogenní

$$y' - \frac{y}{x+1} = 0$$

$$\frac{dy}{dx} = \frac{y}{x+1} \quad \Big| \cdot \frac{dx}{y}$$

$$\frac{dy}{y} = \frac{dx}{x+1} \qquad (c_1 \in \mathbb{R})$$

$$\ln|y| = \ln|x+1| + c_1$$

$$\vdots$$

$$\boxed{y_h = c\,(x+1)} \quad (c \in \mathbb{R})$$

b) nehomogenní

$$\boxed{y = c(x)\cdot(x+1)}$$

$$\boxed{y' = c'(x)\cdot(x+1) + c(x)}$$

$$c'(x)\cdot(x+1) + \cancel{c(x)} - \cancel{\frac{c(x)(x+1)}{x+1}} = x-1$$

$$c'(x) = \frac{x-1}{x+1}$$

$$c(x) = \int \frac{x-1}{x+1}\,dx = \int \frac{x+1-2}{x+1}\,dx = \int \left(1 - \frac{2}{x+1}\right)dx$$

$$= \boxed{x - 2\ln|x+1| + k}$$

$$y = (x - 2\ln|x+1| + k)\cdot(x+1)$$

$$0 = (0 - 2\ln 1 + k)\cdot 1$$

$$\boxed{k=0} \Rightarrow \boxed{y = (x - 2\ln|x+1|)\cdot(x+1)}$$

DÚ

- $y' - y\,\mathrm{tg}\,x = \frac{1}{\cos^3 x}$ $\left[y = \frac{\mathrm{tg}\,x + k}{\cos x}\right]$
- $y' x \ln x - y = 3x^3 \ln^2 x$ $\left[y = (x^3 + k)\ln x\right]$
- $y' = x^2 - x^2 y$ $\left[y = 1 + k e^{-\frac{x^3}{3}}\right]$

JINÝ ZPŮSOB řešení rovnice $\boxed{y' + f(x)\,y = g(x)}$ přes

Integrační Faktor :

$$\boxed{y(x,c) = c\cdot e^{-\int f(x)dx} + e^{-\int f(x)dx}\cdot \int \left(g(x)\cdot e^{\int f(x)dx}\right)dx}$$

(všechny integrály bereme bez integračních konstant)

Toto ne! (čas...) $\boxed{\text{ODR vyšších řádu - úvod}}$

$$\boxed{y^{(n)} = f(x)}$$

(Př) (Já)

$$y''' = 18e^{3x} + \sin x$$

$$y'' = 6e^{3x} - \cos x + c_1$$

$$y' = 2e^{3x} - \sin x + c_1 x + c_2$$

$$\boxed{y = \frac{2}{3}e^{3x} + \cos x + \frac{c_1 x^2}{2} + c_2 x + c_3}$$

(1) $y'' = 12x^3 + 8$ a $y(0) = 0, \; y'(0) = 1$

$$y' = 3x^4 + 8x + c_1$$

$$y = \frac{3}{5}x^5 + 4x^2 + c_1 x + c_2$$

$0 = c_2 \Rightarrow \boxed{c_2 = 0}$

$1 = c_1 \Rightarrow \boxed{c_1 = 1}$

$$\boxed{y = \frac{3}{5}x^5 + 4x^2 + x}$$

(Př) Není čas...!! Řešte substitucí $\boxed{u = \frac{y}{x}}$

rovnici $\boxed{y' = \frac{y^2}{x^2} + \frac{y}{x} + 1}$

$u = \frac{y}{x}$

$y = u \cdot x$

$\boxed{y' = u'x + u}$

$$u'x + u = u^2 + u + 1$$

$$u' \cdot x = u^2 + 1$$

$$\frac{du}{dx} = \frac{u^2+1}{x}$$

$$\int \frac{du}{u^2+1} = \int \frac{dx}{x}$$

$$\operatorname{arctg} u = \ln|x| + c$$

$$\operatorname{arctg} \frac{y}{x} = \ln|x| + c$$

$$\frac{y}{x} = \operatorname{tg}(\ln|x| + c) \Rightarrow \boxed{y = x \operatorname{tg}(\ln|x| + c)}$$