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Adaptive proportional fairness resource allocation for OFDM-based cognitive radio networks

Shaowei Wang · Fangjiang Huang · Chonggang Wang

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Abstract In this paper, we study the resource allocation problem in multiuser Orthogonal Frequency Division Multiplexing (OFDM)-based cognitive radio networks. The interference introduced to Primary Users (PUs) is fully considered, as well as a set of proportional rate constraints to ensure fairness among Secondary Users (SUs). Since it is extremely computationally complex to obtain the optimal solution because of integer constraints, we adopt a two-step method to address the formulated problem. Firstly, a heuristic subchannel assignment is developed based on the normalized capacity of each OFDM subchannel by jointly considering channel gain and the interference to PUs, which approaches a rough proportional fairness and removes the intractable integer constraints. Secondly, for a given subchannel assignment, we derive a fast optimal power distribution algorithm that has a complexity of $\boldsymbol{O}(L^2N)$ by exploiting the problem's structure, which is much lower than standard convex optimization techniques that generally have a complexity of $\boldsymbol{O}((N+K)^3)$, where N, L and K are the number of subchannels, PUs and SUs, respectively. We also develop a simple power distribution algorithm with complexity of only $\boldsymbol{O}(L+N)$, while achieving above 90 % sum capacity of the upper bound. Experiments show that our proposed algorithms work quite well in practical wireless scenarios. A significant capacity gain is obtained and the proportional fairness is satisfied perfectly.

Keywords Cognitive radio · OFDM · Proportional fairness · Resource allocation

S. Wang (✉) · F. Huang
School of Electronic Science and Engineering, Nanjing University, Nanjing 210093, China
e-mail: wangsw@nju.edu.cn

F. Huang
e-mail: huangfangjiang@163.com

C. Wang
InterDigital Communications, King of Prussia, PA 19406, USA
e-mail: cgwang@ieee.org

1 Introduction

Spectrum scarcity crisis exists for many wireless applications, especially in the band below 6 GHz. But, on the other hand, investigations show that a large portion of licensed spectrum is far underutilized [1]. Cognitive Radio (CR) [2, 3] is deemed as a highly promising technology to improve the utilization efficiency of radio spectrum and gained more and more attentions in recent years. In a CR system, although a certain part of spectrum has been assigned to a licensed Primary User (PU), a CR user, also called as a Secondary User (SU), is allowed to adjust its operation parameters and reuse the licensed spectrum, as long as the interference introduced to the PU is below a predefined threshold, such as interference temperature [4].

Orthogonal Frequency Division Multiplexing (OFDM), which offers a high flexibility in radio resource allocation, is widely recognized as a practicable air interface of CR systems [5]. As one of the most important issues in conventional OFDM systems, adaptive Resource Allocation (RA) has been studied intensively in the past decade [6–10] and a comprehensive survey can be found in [11]. In [6], multiuser subcarrier, bit and power allocation algorithms for OFDM systems are proposed, and greedy bit loading strategy is proven to be the optimal for a given subcarrier assignment. Multicast services, heterogenous traffic demand and minimal user rate requirements are considered in [7–9], respectively. In [10], proportional rate constraint is introduced and an optimal power allocation algorithm is developed. For an OFDM-based CR network, dynamic RA is much more important than that in conventional OFDM systems because the CR system has to adjust the usage mode of radio resource frequently to adapt to the changes of spectrum environment. However, when PUs do not adopt OFDM modulation, mutual interference between PUs and SUs, which stems from the non-orthogonality of the transmitted signals, should be concerned carefully. As a result, RA in the CR networks is more complex and the RA algorithms for conventional OFDM systems may be no longer suitable.

Adaptive RA in OFDM-based CR networks is an attractive research issue in the CR field and many methods have been proposed with different degree of success. In [12], both optimal and suboptimal algorithms are developed to maximize the sum capacity of a CR network. However, the transmission power limitation is not considered. In [13], RA in OFDM-based CR systems is formulated as a multidimensional knapsack problem with consideration of transmission power and interference constraints. A greedy algorithm, named Max–Min, is proposed to allocate bits on each OFDM subchannel. Simulation results show that the performance of the Max–Min is close to the optimal. However, its complexity is much higher than the method in [14], where an approximately constant complexity algorithm is developed. Unfortunately, there is a significant capacity gap between the algorithm in [14] and the optimal. In [15], an efficient power allocation algorithm is proposed, which can approach the optimal solution. The algorithms proposed in [12–15] are for the single SU case, limiting their applications.

RA for multiuser OFDM-based CR networks is investigated in [16–22]. To maximize the sum capacity of a CR network, subchannel allocation and power allocation are carried out sequentially in [16], and the performance of the proposed algorithm is close to the optimal. However, fairness among SUs is not considered in this work. In [17], an algorithm for non-real-time application is proposed to ensure that the rate of a CR user is proportional to a target value. Service delay is allowed and average data rates are required to maintained proportionally among SUs. In [18], a cooperative game theory based RA algorithm is proposed, where the primary system is assumed to adopt OFDM modulation. This assumption is not practical for many wireless systems. In [19], RA for a CR network with imperfect spectrum sensing is studied, but no rate requirements of the SUs are mentioned. In [20], RA in a finite queue backlogs CR system is investigated. Both real-time and non-real-time services are considered in [21], and fast RA algorithms are developed. However, fairness among SUs is ignored in both [20] and [21]. A heuristic method is developed in [22]. Though fairness is considered in this work, there is a significant capacity gap between the proposed algorithm and the optimal.

In this paper, we consider two co-existing cellular systems which share the same base station site, one of which is for PUs, the other is for SUs. Sharing base station is practical for services providers because it is difficult to obtain a new site to install base stations in the future [23]. The system that serves for SUs adopts OFDM modulation and PUs in the licensed system are not necessary to employ OFDM. We focus on maximizing the sum capacity of the CR system, while keeping proportional rates of the SUs satisfied. The formulated optimization is extremely hard to solve because of integer constraints. Jointly considering the state-of-the-art optimization techniques [24–26], we separate it into two subproblems: subchannel allocation and power distribution. We propose an efficient subchannel allocation algorithm to obtain a rough proportional fairness, as well as removing the integer constraints to make the problem intractable. Then we derive a fast *barrier* method to distribute power in an optimal manner by exploiting the structure of the problem. Besides, a heuristic power allocation algorithm is also developed, which has much lower complexity while producing a good approximation of the optimal solution.

The remainder of this paper is organized as follows. In Sect. 2, we illustrate system model and formulate the optimization problem. In Sect. 3, subchannel allocation scheme is presented. Optimal and suboptimal power distribution algorithms are developed in Sects. 4 and 5 with complexity analysis, respectively. Simulation results and discussions are given in Sect. 6 Conclusion is drawn in Sect. 7.

2 System model and problem formulation

To make this paper easy to follow, we list terminologies and symbol notations in Table 1.

We consider the downlink of a multiuser OFDM-based CR network. Assume that perfect instantaneous channel state information is available. There are also L PUs in the licensed system and the licensed spectrum is equally divided into N OFDM subchannels shared by K SUs in the CR system. Recall that PUs may not adopt OFDM modulation. $\mathcal{K} = \{1, 2, \ldots, K\}$, $\mathcal{L} = \{1, 2, \ldots, L\}$ and $\mathcal{N} = \{1, 2, \ldots, N\}$ denote the sets of SUs, PUs and OFDM subchannels, respectively.

Suppose the bandwidth of each subchannel is B. The nominal spectrum of subchannel n spans from $f_s + (n-1)B$ to $f_s + nB$, where f_s is the starting frequency. The nominal band of PU l ranges from f_l to $f_l + B_l$, where f_l and B_l are the starting frequency and the bandwidth occupied by PU l, respectively.

Table 1 List of terminologies and symbol notations

B	The bandwidth of each subchannel
B_l	The bandwidth occupied by PU l
f_s	The starting frequency of the total bandwidth
f_l	The starting frequency of PU l
$g_{l,n}^{BP}$	The power gain over subchannel n from the BS to the receiver of PU l
$g_{k,n}^{BS}$	The power gain of subchannel n from the BS to the receiver of SU k
$h_{k,n}$	The SNR of the nth subchannel used by the kth SU with unit power
$I_{k,l,n}^{PS}$	The interference that casts into subchannel n because of the transmission of the BS in the primary system with unit power
I_l	The interference threshold of PU l
$I_{l,n}^{SP}$	The interference imposed on subchannel n generated by an SU with unit transmission power
$\mathcal{K}$	The set of SUs
K	The number of SUs
$\mathcal{L}$	The set of PUs
L	The number of PUs
$\mathcal{N}$	The set of OFDM subchannels
N	The number of OFDM subchannels
N_k	The number of OFDM subchannels used by SU k
$p_{k,n}$	The power allocated to subchannel n by SU k
$p_{k,n}^{M}$	The maximum possible power allocated to subchannel n by SU K
P	The maximum available transmission power in the CR system
$r_{k,n}$	The rate that SU k can transmit over subchannel n
R_k	The sum rate of SU k
$r_{k,n}^{M}$	The highest achievable rate of the subchannel n for SU k
T_s	The OFDM symbol duration
$\phi_l^{PU}(f)$	The power spectrum density (PSD) of the lth PU's signal
Ω_k	The set of subchannels allocated to SU k
$\rho_{k,n}$	The subchannel allocation index
Γ	The signal-to-noise gap

The interference imposed on subchannel n generated by an SU with unit transmission power, which is also fallen into the bandwidth of PU l, can be expressed as

$$I_{l,n}^{SP} = \int_{f_l-(n-\frac{1}{2})B}^{f_l+B_l-(n-\frac{1}{2})B} g_{l,n}^{BP}\phi(f)df, \quad (1)$$

where $g_{l,n}^{BP}$ is the power gain over subchannel n from the base station to the receiver of PU l, $\phi(f)$ is the power spectrum density of OFDM signal and $\phi(f) = T_s\left(\frac{\sin(\pi f T_s)}{\pi f T_s}\right)^2$, where T_s is OFDM symbol duration.

Similarly, the interference that casts into subchannel n because of the transmission of the base station in the primary system with unit power is

$$I_{k,l,n}^{PS} = \int_{(n-1)B-(f_l+\frac{1}{2}B_l)}^{nB-(f_l+\frac{1}{2}B_l)} g_{k,n}^{BS}\phi_l^{PU}(f)df, \quad (2)$$

where $g_{k,n}^{BS}$ is the power gain of subchannel n from the base station to the receiver of SU k. $\phi_l^{PU}(f)$ is the power spectrum density of PU l's signal and assumed to be processed by elliptically filter [27], i.e. $\phi_l^{PU}(f) = [1+\epsilon_l^2 R_n^2(\xi_l, f/f_{0,l})]^{-1}$, where n, ϵ_l, $f_{0,l}$ are the filter parameters and $R_n(.)$ is an n-order elliptically rational function.

The maximum rate that SU k can transmit over subchannel n is given by

$$r_{k,n} = B\log\left(1+p_{k,n}h_{k,n}\right). \quad (3)$$

In turn, given a transmission rate $r_{k,n}$, the required transmission power can be expressed as follows,

$$p_{k,n} = \frac{e^{r_{k,n}}-1}{h_{k,n}}, \quad (4)$$

where $h_{k,n} = \frac{g_{k,n}^{BS}}{\Gamma\left(\sum_{l=1}^{L} I_{k,l,n}^{PS}+N_0B\right)}$, N_0 is the power spectrum density of additive white Gaussian noise and Γ is the signal-to-noise ratio (SNR) gap, which can be represented as $\Gamma = -\ln(5BER)/1.5$ for an uncoded multilevel quadrature amplitude modulation (MQAM) with a specified bit-error-rate (BER) [28].

The optimization objective is to maximize the sum capacity of the CR system under the constraints of transmission power budget, the interference threshold of the PUs and the proportional rates of the SUs. Mathematically, the optimization problem can be formulated as follows,

$$\max_{\rho_{k,n}, r_{k,n}} \sum_{k=1}^{K}\sum_{n=1}^{N}\rho_{k,n}r_{k,n}$$

subject to:

$$\begin{aligned} &\text{C1:} r_{k,n} \geq 0, \forall k \in \mathcal{K}, n \in \mathcal{N} \\ &\text{C2:} \sum_{k=1}^{K}\sum_{n=1}^{N} p_{k,n} \leq P \\ &\text{C3:} \sum_{k=1}^{K}\sum_{n=1}^{N} p_{k,n}I_{l,n}^{SP} \leq I_l, \forall l \in \mathcal{L} \\ &\text{C4:} \rho_{k,n} = \{0,1\}, \forall k \in \mathcal{K}, n \in \mathcal{N} \\ &\text{C5:} \sum_{k=1}^{K}\rho_{k,n} = 1, \forall n \in \mathcal{N} \\ &\text{C6:} R_1 : R_2 : \ldots : R_K = \beta_1 : \beta_2 : \ldots : \beta_K, \end{aligned} \quad (5)$$

where P is transmission power budget, I_l is the interference threshold of PU l, $\rho_{k,n}$ is either 1 or 0, indicating whether subchannel n is used by SU k or not. C5 shows that each subchannel can be used by only one SU. $\{\beta_i\}_{i=1}^{K}$'s are predefined values to ensure the proportional fairness among the SUs. R_k denotes the sum rate of user k,

$$R_k = \sum_{n=1}^{N} \rho_{k,n} r_{k,n}. \tag{6}$$

The optimization problem defined by (5) involves both continuous variables $r_{k,n}$ and binary variables $\rho_{k,n}$. It is a mixed binary integer programming problem that is generally very hard to solve. Furthermore, the proportional fairness constraint C6 makes it more complex. In our considered CR system, N subchannels are possibly allocated to any one of K SUs, which leads to K^N possible combinations even if a subchannel can be used by just one SU.

In [5], a similar problem is studied except for the interference constraint C3 in (5), where a subchannel is always allocated to the user with the largest SNR in order to obtain the highest transmission rate with a given power budget. But in a cognitive OFDM network, a subchannel with high SNR may also generate more interference to the PUs, which makes it impossible to transmit with the maximum available power over this subchannel. The SNR of a subchannel and the interference generated to the PUs should be jointly considered in OFDM-based CR networks. Besides, power distribution among subchannels should also consider the interference constraints because the greedy bit loading strategy for conventional OFDM systems [6] is no longer the optimal for the CR networks.

3 Subchannel allocation

Obviously, optimal subchannel assignment can be found by exhaustive search. The complexity increases exponentially with the number of subchannels, which makes it impractical even for a moderate scale number of subchannels. In this section, we present a subchannel allocation algorithm which is based on the quality of OFDM subchannels by jointly considering the SNR of each subchannels and the interference to the PUs.

Consider the transmission power budget of the CR system and the interference threshold of the PUs, the maximum power allocated to subchannel n for SU k is given by

$$p_{k,n}^{M} = \min\left(P, \min_{l \in \mathcal{L}}\left(\frac{I_l}{I_{l,n}^{PU}}\right)\right). \tag{7}$$

(7) means that the power over subchannel n is bounded by the total power P of the CR system and the interference constraints laid by the PUs, which can be seen from the constraints C2 and C3 in (5). Consequently, the highest achievable rate of subchannel n for SU k is

$$r_{k,n}^{M} = \log\left(1 + p_{n,k}^{M} h_{k,n}\right). \tag{8}$$

We propose a two-round procedure to allocate OFDM subchannels among the SUs. In the first round, each SU is allocated to a subchannel, over which the SU can achieve the highest possible rate among all available subchannels. Furthermore, to minimize the potential consumed power, the SU with higher achievable maximum rate has a priority to obtain a subchannel at this stage. In the second round, the user who suffers the severest unjustness, is given a priority to choose a subchannel among the remaining ones. Again, the subchanel with the high achievable rate for this user is picked out with priority. This allocation process is repeated until all subchannels are consumed. Our proposed subchannel allocation algorithm can reach a coarse proportional fairness. The exact satisfaction of the proportional fairness among the SUs, as well as the maximization of sum capacity, will be ultimately accomplished after power disribution among subchannels.

The subchannel allocation algorthm is described in Table 2. Recall that $\mathcal{K}, \mathcal{L}$ and $\mathcal{N}$ are the sets of SUs, PUs and subchannels, respectively. The set of subchannels assigned to user k is denoted as Ω_k. N_k is the number of subchannels in Ω_k.

4 Optimal power allocation: a fast *barrier* method

Given a subchannel assignment, the constraints C4 and C5 in (5) vanish, and the optimization problem is transformed into the following form,

Table 2 Subchannel allocation algorithm

Initialization
  Set $R_k = 0, \Omega_k = \emptyset, \forall k \in \mathcal{K}, \mathcal{K}_t = \mathcal{K}, \mathcal{N}_t = \mathcal{N}$;
  Calculate $r_{k,n}^M, \forall k \in \mathcal{K}, \forall n \in \mathcal{N}$.

Subchannel Allocation
  for i=1 to K
    Find k^*, n^* that $r_{k^*,n^*}^M \geq r_{k,n}^M, \forall k \in \mathcal{K}_t, \forall n \in \mathcal{N}_t$;
    $\Omega_{k^*} := \Omega_{k^*} \cup n^*, \mathcal{K}_t := \mathcal{K}_t \setminus k^*, \mathcal{N}_t := \mathcal{N}_t \setminus n^*$;
    $R_{k^*} = r_{k^*,n^*}^M$.
  end for
  while*(true)*
    if $\mathcal{N}_t = \emptyset$
      break;
    else
      Find $k^* = \min_{k \in \mathcal{K}}(\frac{R_k}{\beta_k})$;
      Find n^* satisfying $r_{k*,n^*}^M \geq r_{k*,n}^M, \forall n \in \mathcal{N}_t$;
      $\Omega_{k^*} := \Omega_{k^*} \cup \{n^*\}, \mathcal{N}_t := \mathcal{N}_t \setminus \{n^*\}$;
      $R_k = R_k + r_{k^*,n^*}^M$.
    end if
  end while

$$\max_{r_{k,n}} \sum_{k=1}^{K} \sum_{n \in \Omega_k} r_{k,n}$$

subject to:

$$\begin{aligned} &\text{C1:} r_{k,n} \geq 0, \forall k \in \mathcal{K}, n \in \mathcal{N} \\ &\text{C2:} \sum_{k=1}^{K} \sum_{n=1}^{N} p_{k,n} \leq P \\ &\text{C3:} \sum_{k=1}^{K} \sum_{n=1}^{N} p_{k,n} I_{l,n}^{SP} \leq I_l, \forall l \in \mathcal{L} \\ &\text{C4:} R_1 : R_2 : \ldots : R_K = \beta_1 : \beta_2 : \ldots : \beta_K. \end{aligned} \tag{9}$$

Based on (3), we know that the solution of (9) is essentially the optimal power allocation because $r_{k,n}$ is a monotropic function of $p_{k,n}$. Obviously, (9) defines a convex problem [24] and can be solved by standard convex optimization techniques, such as *barrier* method, subgradient method. Generally, the complexity of these methods is about $\boldsymbol{O}\ (N+K)^3$ for (9). Since the number of subchannels in practical wireless systems is usually very large, these methods are not suitable in practice. We should develop more efficient algorithms to address this optimization problem. In this section, we propose a fast *barrier* method to obtain the optimal power distribution by exploiting the structure of (9).

4.1 The *barrier* method

First, we convert (9) into the form that can be tackled by standard convex optimization techniques. Rearrange the set of allocated subchannels as $\mathcal{N} := \{\Omega_1, \Omega_2, \ldots, \Omega_K\}$, recall that Ω_k is the set of subchannels allocated to SU k, (9) can be also rewritten as follows,

$$\max_{r_n} \sum_{n=1}^{N} r_n$$

subject to:

$$\begin{aligned} &\text{C1:} r_n \geq 0, n = 1, 2, \ldots, N \\ &\text{C2:} \sum_{n=1}^{N} p_n \leq P \\ &\text{C3:} \sum_{n=1}^{N} p_n I_{l,n}^{SP} \leq I_l, l = 1, 2, \ldots, L \\ &\text{C4:} \beta_k R_1 - \beta_1 R_k = 0, k = 2, \ldots, K, \end{aligned} \tag{10}$$

where $R_k = \sum_{n \in \Omega_k} r_n$, and $p_n = \frac{e^{r_n} - 1}{h_n}$. Note that constraint C4 in (10) is also the corresponding constraint C4 in (9), except for their forms. Collect the variables of (10) into a vector $r = (r_1, r_2, \ldots, r_n)^T$, (10) can be rewritten as

$$\begin{aligned} &\max_{r} f(r) \\ &\text{subject to:} \\ &\quad r \geq 0 \\ &\quad F_0(r) \leq P \\ &\quad F_l(r) \leq I_l, l = 1, 2, \ldots, L \\ &\quad Ar = 0, \end{aligned} \tag{11}$$

where $f(r) = \sum_{n=1}^{N} r_n$, $F_0(r) = \sum_{n=1}^{N} p_n$, $F_l(r) = \sum_{n=1}^{N} p_n I_{l,n}^{PU}$ and $l = 1, 2, \ldots, L$. A is a $(K-1) \times N$ matrix whose jth element of the ith row is given by $A_{ij} = \begin{cases} \beta_{i+1} & for \quad j = 1, \ldots, N_1 \\ -\beta_1 & for \quad j = \pi_i + 1, \ldots, \pi_i + N_{i+1} \end{cases}$, where $\pi_i = \sum_{k=1}^{i} N_k$. It is easily proved that the rank of the matrix A is $(K-1)$.

Note that $f, F_0, F_1, \ldots, F_L : \mathbf{R}^n \to \mathbf{R}$ are convex and twice continuously differentiable, so (11) defines a convex optimization problem with N variables, $N + L + 1$ inequality constraints and $K - 1$ equality constraints. Theoretically, the optimal solution of (11), can be obtained by standard convex optimization methods [24]. However, as mentioned above, these methods typically have a complexity of $\boldsymbol{O}\ (N+K)^3$, which is too high to apply in practical wireless systems. We exploit the structure of (11) and derive a fast *barrier* algorithm to update Newton step, the main computation load of the *barrier* method. Analytically, the proposed fast *barrier* method has a complexity of $\boldsymbol{O}\ (L^2 N)$. Since L is the number of PUs and much smaller than the number of subchannels N for practical CR systems, the complexity is reduced dramatically.

Firstly, convert (11) into an equality constrained problem,

$$\begin{aligned} &\min_{r} \psi_t(r) = -tf(r) + \phi(r) \\ &\text{subject to:}\ A\, r = 0, \end{aligned} \tag{12}$$

where $\phi(r)$ is called as the *barrier* function [24] and

$$\phi(r) = -\sum_{n=1}^{N} \log(r_n) - \log(P - F_0(r)) - \sum_{l=1}^{L} \log(I_l - F_l(r)).$$

The parameter t decides the accuracy of the approximation to the original problem. As t increases, the central point $r^*(t)$ obtained by solving (12) becomes a more and more accurate approximation to the solution of (11) as discussed in [24]. The procedure of the *barrier* method is illustrated in Table 3.

The computational load of the *barrier* method mainly lies in the computation of the Newton step Δr at r, which needs to solve the following equations,

Table 3 Procedure of the *barrier* method

Give feasible r, t, $\mu > 1$ and tolerance ϵ_b;
while $(N+L+K)/t > \epsilon_b$ /* Outer loop */
  Give tolerance ϵ_n, $\alpha \in (0, 1/2)$, $\gamma \in (0, 1)$;
  while *(true)* /* Inner loop */
    if $\lambda^2/2 \leq \epsilon_n$
      break;
    else
      Calculate Δr and $\lambda^2 = -\psi_t(r)\Delta r$;
      Set $\sigma := 1$;
      while $\psi_t(r+\sigma\Delta r) > \psi_t(r) - \alpha\sigma\lambda^2$
        $\sigma := \gamma\,\sigma$;
      end while
    end if
    $r := r + \sigma\Delta r.$
  end while
  $t := \mu\, t.$
end while

$$\begin{bmatrix} \nabla^2\psi_t(r) & A^T \\ A & \mathbf{0}_m \end{bmatrix}\begin{bmatrix} \Delta r \\ \omega \end{bmatrix} = \begin{bmatrix} -\nabla\psi_t(r) \\ \mathbf{0}_v \end{bmatrix}, \tag{13}$$

where $\mathbf{0}_m \in \mathbf{R}^{(K-1)\times(K-1)}$ is a zero matrix and $\mathbf{0}_v \in \mathbf{R}^{(K-1)\times 1}$ is a zero vector. Typically, it needs the inversion of the Karush-Kuhn-Tucker (KKT) matrix on the left side of (13).

4.2 Fast computation of newton step

In general, it has a cost of $\boldsymbol{O}((N+K)^3)$ to solve (13) because of the computation of matrix inversion. We will show how it can be solved with $\boldsymbol{O}(L^2N)$ complexity by exploiting the structure.

The gradient of $\psi_t(r)$ is given by

$$\begin{aligned}\nabla\psi_t(r) &= -t\nabla f(r) + \nabla\phi(r) \\ &= -t - \frac{1}{r_n} + \frac{e^{r_n}/h_n}{P - F_0(r)} + \sum_{l=1}^{L}\frac{I^{PU}_{l,n}e^{r_n}/h_n}{I_l - F_l(r)}.\end{aligned}$$

And the Hessian of $\psi_t(r)$ is

$$\begin{aligned}\nabla^2\psi_t(r) &= -t\nabla^2 f(r) + \nabla^2\phi(r) \\ &= \frac{\nabla F_0(r)\nabla F_0(r)^T}{(P-F_0(r))^2} + \frac{\nabla^2 F_0(r)}{P-F_0(r)} \\ &\quad + \sum_{l=1}^{L}\left(\frac{\nabla F_l(r)\nabla F_l(r)^T}{(I_l-F_l(r))^2} + \frac{\nabla^2 F_l(r)}{I_l-F_l(r)}\right) \\ &\quad + \begin{bmatrix}\lambda_1 & & & \\ & \lambda_2 & & \\ & & \ddots & \\ & & & \lambda_N\end{bmatrix},\end{aligned}$$

where $\lambda_n = \frac{1}{r_n^2} + \left\{\frac{1}{P-F_0(r)} + \sum_{l=1}^{L}\frac{I^{PU}_{l,n}}{I_l-F_l(r)}\right\}\frac{e^{r_n}}{h_n}$.

Denote $D = diag(\lambda_1, \lambda_2, \ldots, \lambda_N)$, the Hessian can be expressed as

$$\nabla^2\psi_t(r) = D + \sum_{l=1}^{L+1} g_l g_l^T, \tag{14}$$

where $g_l = \begin{cases}\frac{1}{I_l-F_l(r)}\left[\frac{I^{PU}_{l,1}e^{r_1}}{h_1}, \ldots, \frac{I^{PU}_{l,N}e^{r_N}}{h_N}\right]^T, l = 1, \ldots, L. \\ \frac{1}{P-F_0(r)}\left[\frac{e^{r_1}}{h_1}, \ldots, \frac{e^{r_N}}{h_N}\right]^T, l = L+1.\end{cases}$

Using (14), the first matrix on the left of (13) can be denote as

$$H_l = H + \sum_{i=l}^{L+1} G_i G_i^T, l = 1, 2, \ldots, L+1, \tag{15}$$

where $H = \begin{bmatrix} D & A^T \\ A & \mathbf{0}_m\end{bmatrix}$ and $G_l = \begin{bmatrix} g_l \\ \mathbf{0}_v\end{bmatrix}, l = 1, \ldots, L+1.$

In diagonal matrix D, diagonal entries $\lambda_n > 0$, $n = 1, 2, \ldots, N$. Meanwhile, the rank of matrix A is $K-1$, it follows that H is nonsingular. Since $G_l G_l^T \geq 0$, H_l's are also nonsingular and invertible. Rewrite (13) as

$$H_1 x = G_0, \tag{16}$$

where $x = \begin{bmatrix}\Delta r \\ \omega\end{bmatrix}$ and $G_0 = \begin{bmatrix} -\nabla\psi_t(r) \\ \mathbf{0}_v\end{bmatrix}$, from (15) we have $H_l = H_{l+1} + G_l G_l^T$ and H_l is nonsingular. By exploiting its special structure, we have the following theorem:

Theorem 1 *(16) can be solved with complexity of* $\boldsymbol{O}(L^2N)$.

The proof is placed in Appendix. Generally, $L \ll N$ in practical wireless systems. So the complexity of the algorithm is almost linearly related to N if the number of PUs is small.

5 Low complexity power distribution

In this section, we develop an approximation scheme which offers a performance close to the optimal with lower complexity. In [14], a discrete normalized cost function is introduced to measure the cost of a possible bit allocated to a subchannel. Motivated by the concept of cost function, we define a continuous normalized cost function as

$$f_c(r_{k,n}) = \frac{e^{r_{k,n}} - 1}{e^{r^M_{k,n}} - 1}, \tag{17}$$

where $r_{k,n} = \log(1 + p_{k,n}h_{k,n})$. The cost function can measure the quality of a subchannel, just as the incremental power required to add a bit to a subchannel in conventional OFDM systems [6]. Now we consider the following optimization problem,

$$\max_{r_{k,n}} \sum_{k=1}^{K} \sum_{n\in\Omega_k} r_{k,n}$$

subject to:

$$\begin{aligned} &\text{C1:} r_{k,n} \geq 0, \forall k \in \mathcal{K}, n \in \mathcal{N} \\ &\text{C2:} \sum_{k=1}^{K} \sum_{n\in\Omega_k} f_c(r_{k,n}) \leq C \\ &\text{C3:} R_1 : R_2 : \ldots : R_K = \beta_1 : \beta_2 : \ldots : \beta_K \end{aligned} \tag{18}$$

where C is a constant. Compared to (9), we use the constraint C2 in (18) to substitute the transmission power budget C2 and interference constraints C3 in (9). Note that both transmission power and interference constraints can be satisfied if we meet the constraint C2 in (18). As a result, (18) is an approximation to (9). We can find the approximation is very close to the optimal solution of (9) in the next section. Note that we do not know the value of C in advance. We will show that it is not necessary to know C when working out $r_{k,n}$'s.

Put constraints C1 in (18) aside, which we will discuss in the next step, the Lagrangian of (18) can be written as

$$\begin{aligned} L \ &= -\sum_{k=1}^{K} \sum_{n\in\Omega_k} r_{k,n} + \varphi_1 \left(\sum_{k=1}^{K} \sum_{n\in\Omega_k} f_c(r_{k,n}) - C \right) \\ &\quad + \sum_{k=2}^{K} \varphi_k \left(\sum_{n\in\Omega_1} r_{1,n} - \frac{\beta_1}{\beta_k} \sum_{n\in\Omega_k} r_{k,n} \right), \end{aligned}$$

where $\{\beta_k\}_{i=1}^{K}$'s are Lagrangian multipliers. Using the KKT conditions, we can obtain the following equations,

$$\begin{aligned} \frac{\partial L}{\partial r_{1,n}} &= \frac{\varphi_1 e^{r_{1,n}}}{e^{r^M_{1,n}} - 1} + \sum_{k=2}^{K} \varphi_k - 1 = 0 \\ \frac{\partial L}{\partial r_{k,n}} &= \frac{\varphi_k e^{r_{k,n}}}{e^{r^M_{k,n}} - 1} - \frac{\beta_1}{\beta_k} \varphi_k - 1 = 0 \end{aligned} \tag{19}$$

for $k = 2, 3, \ldots, K$, and $n \in \Omega_k$. We set $r_{k,n} = 0$ if $r_{k,n} < 0$.

5.1 Power allocation among subchannels of an SU

Firstly, we derive the power distribution for SU k. For subchannel $m, n \in \Omega_k$, $\forall k \in \mathcal{K}$, from (8) and (19), we obtain

$$r_{k,m} - r_{k,n} = \log\left(\frac{p^M_{k,m} h_{k,m}}{p^M_{k,n} h_{k,n}} \right). \tag{20}$$

Without loss of generality, assume that $r^M_{k,1} \leq r^M_{k,2} \leq \cdots \leq r^M_{k,N_k}$, and $\omega_{k,n} = \frac{p^M_{k,n} h_{k,n}}{p^M_{k,1} h_{k,1}}$, (20) can be written as

$$r_{k,n} = r_{k,1} + \log(\omega_{k,n}). \tag{21}$$

Consequently, R_k can be expressed as

$$R_k = \sum_{n=1}^{N_k} r_{k,n} = N_k r_{k,1} + \sum_{n=1}^{N_k} \log(\omega_{k,n}). \tag{22}$$

Denote P_k as the total power allocated to user k and I^l_k as the total interference caused to PU l, we have

$$\begin{aligned} P_k &= \sum_{n=1}^{N_k} p_{k,n} = \sum_{n=1}^{N_k} \frac{e^{r_{k,n}} - 1}{h_{k,n}} \\ &= e^{r_{k,1}} \sum_{n=1}^{N_k} \frac{\omega_{k,n}}{h_{k,n}} - \sum_{n=1}^{N_k} \frac{1}{h_{k,n}} \\ &= e^{r_{k,1}} W^0_k - H^0_k \\ I^l_k &= \sum_{n=1}^{N_k} p_{k,n} I^{SP}_{l,n} \\ &= e^{r_{k,1}} \sum_{n=1}^{N_k} \frac{I^{SP}_{l,n} \omega_{k,n}}{h_{k,n}} - \sum_{n=1}^{N_k} \frac{I^{SP}_{l,n}}{h_{k,n}} \\ &= e^{r_{k,1}} W^l_k - H^l_k, l = 1, 2, \ldots, L, \end{aligned} \tag{23}$$

where

$$\begin{aligned} W^0_k &= \sum_{n=1}^{N_k} \frac{\omega_{k,n}}{h_{k,n}},\ H^0_k = \sum_{n=1}^{N_k} \frac{1}{h_{k,n}}, \\ W^l_k &= \sum_{n=1}^{N_k} \frac{I^{SP}_{l,n} \omega_{k,n}}{h_{k,n}},\ H^l_k = \sum_{n=1}^{N_k} \frac{I^{SP}_{l,n}}{h_{k,n}}. \end{aligned} \tag{24}$$

5.2 Power allocation among different SUs

With (21) and the rate constraint, $\frac{R_k}{R_1} = \frac{\beta_k}{\beta_1}$, we can get

$$r_{k,1} = \frac{N_1 \alpha_k}{N_k \alpha_1} r_{1,1} + \frac{\alpha_k \omega_1 - \alpha_1 \omega_k}{N_1 \alpha_1} = V_k r_{1,1} + U_k, \tag{25}$$

where V_k and U_k are defined as

$$\begin{aligned} V_k &= \frac{N_1 \alpha_k}{N_k \alpha_1} \\ U_k &= \frac{\alpha_k \omega_1 - \alpha_1 \omega_k}{N_1 \alpha_1}. \end{aligned} \tag{26}$$

Consequently, with (21) and (25), we have

$$\begin{aligned} R_k &= 2^{V_k r_{1,1} + U_k} \sum_{n=1}^{N_k} \frac{\omega_{k,n}}{h_{k,n}} - \sum_{n=1}^{N_k} \frac{1}{h_{k,n}} \\ I^l_k &= 2^{V_k r_{1,1} + U_k} W^l_k - H^l_k. \end{aligned} \tag{27}$$

According to the power limitation and interference constraints, there is a set of inequality functions on $r_{1,1}$,

$$\begin{aligned} \sum_{k=1}^{K} P_k \ &= \sum_{k=1}^{K} e^{V_k r_{1,1} + U_k} W^0_k - H^0_k \leq P \\ \sum_{k=1}^{K} I^l_k \ &= \sum_{k=1}^{K} e^{V_k r_{1,1} + U_k} W^l_k - H^l_k \leq I_l, \end{aligned} \tag{28}$$

for $l = 1, 2, \ldots, L$.

Note that there is only a variable, $r_{1,1}$ in the set of inequality functions and the left of each inequality in (28) increases monotonically with $r_{1,1}$. We can use bisection method to solve (28). When $r_{1,1}$ is obtained, the solution to (18) can be worked out with (21) and (25).

Now we take into account the first set of constraints in (18). For SU k, $\forall k \in \mathcal{K}$, there is no power allocation if $r_{k,n} \leq 0$, which means that a subchannel is allocated to an SU who contributes no capacity to the CR system. When this case happens, the sets of Ω_k, U_k, V_k, W_k^i and H_k^i, need to be updated, as well as power distribution, as shown in Fig. 1.

To solve the $L+1$ inequalities in (28), the number of iterations of bisection method required for convergence is bounded by $(L+1)\log_2(1/\epsilon)$, where ϵ is a small value indicating the error tolerance. The power allocation of each subchannel is obtained with $\boldsymbol{O}\,(1)$ complexity. So the complexity of our proposed power allocation algorithm is approximately $\boldsymbol{O}\,(N+L)$.

6 Numerical results

Experiments are performed to evaluate the performance of our RA algorithms. Consider a CR system, where SUs and PUs are uniformly distributed within a 1 km circle area around a base station. The channel fading is frequency-selective, the path loss exponent is 4, the variance of logarithmic normal shadow fading is 10dB and the amplitude of multipath fading is Rayleigh. The total bandwidth of the CR system is the product of the number of OFDM subchannels and the bandwidth of each subchannel is 62.5 kHz. The bandwidth of each PU is generated randomly with uniform distribution on the interval [0, 2N*62.5/3L]kHz. The transmission power of each PU is set to the number of subchannels crossed over by the PU. The noise power is 10^{-13}W. The interference thresholds of all PUs are set to 5×10^{-13}W.

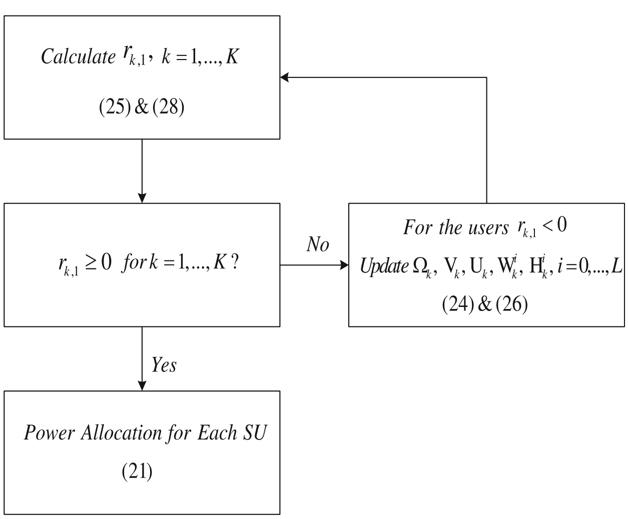


Fig. 1 Flowchart of suboptimal power allocation

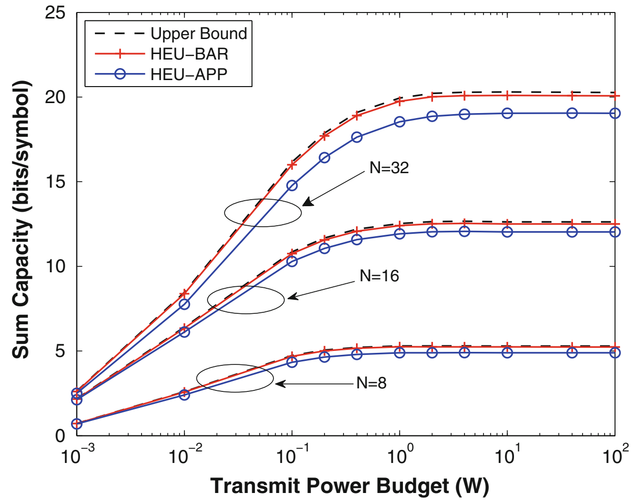


Fig. 2 Sum capacity as a function of transmission power budget. $L=2$, $K=4$ and β_1: β_2: β_3: β_4 = 1:1:1:1

First, we compare the sum capacity between our proposed RA schemes and the upper bound. The HEU-BAR denotes the heuristic subchannel allocation algorithm proposed in Sect. 3, followed by the fast *barrier* method proposed in Sect. 4. HEU-APP denotes the approximate power distribution method with the same subchannel allocation scheme as the HEU-BAR. The upper bound is produced by a commercial software that can tackle problem (5). Consider a CR system with 2 PUs and 4 SUs,[1] the number of subchannels is 8, 16 and 32. β_1: β_2: β_4: β_4 = 1:1:1:1. The results in Fig. 2 are averaged over 1,000 channel realizations and the sum capacity is normalized to bits/symbol. It can be seen that our proposed HEU-BAR and HEU-APP can achieve over 98 and 90 % of the upper bound for the considered CR system, respectively. Note that RA in OFDM-based CR networks is a time sensitive problem which should be solved in an online manner, low complexity algorithm with negligible capacity loss is more promising than the time-consuming method, even the latter can produce the optimal solution. Especially for the HEU-BAR scheme, it can achieve almost the upper bound capacity with a reasonable complexity.

Next, we compare the performance of our proposed subchannel allocation algorithm with other representative ones: equal power allocation (EPA) and maximum SNR

[1] It consumes too much time to work out the solutions for the commercial software to get the upper bound, so we only consider a small scale of users.

priority (MSP). The EPA assumes that equal power distribution is across all subchannels with consideration of fairness constraints, as proposed in [10]. The MSP always allocates a subchannel to the user which has the highest SNR on this subchannel. The EPA and the MSP adopt the optimal power allocation. Figure 3 shows the sum capacity of the CR network as a function of transmission power budget. There are 128 subchannels in the CR system. The number of PUs is 4. The SUs have the same rate requirements. All results are also obtained by averaging over 1,000 realizations. From Fig. 3 we can see that our proposed Heuristic subchannel allocation (HEU) outperforms the others. The reason is that the HEU jointly considers the SNR and the interference level of each subchannel, while the EPA or the MSP only take one of them into consideration. It can be also seen that the sum capacity increases as the number of SUs becomes larger. It indicates the the system benefits from multiuser diversity. A subchannel is more likely to allocate to an SU which has good channel gain over this channel.

Third, we evaluate the performance of the fast *barrier* method proposed in Sect. 4 (BARRIER) which can work out the optimal power distribution and the approximate scheme proposed in Sect. 5 (APPROX), both of which adopt the subchannel assignment method proposed in Sect. 3. The transmission power limit $P = 1W$. All SUs have the same rate requirements. Figure 4 shows the sum capacity as a function of the number of subchannels N. From Fig. 4 we observe the capacity increases as N increases. Again, we can see that multiuser diversity effect results that the capacity of the CR system is higher in more SUs cases for a given number of subchannels. It can be also seen that the capacity gap between the APPROX and the BARRIER is not more than 10 % in all cases. Consider the low complexity of the APPROX, it is promising for strictly time-limit scenarios.

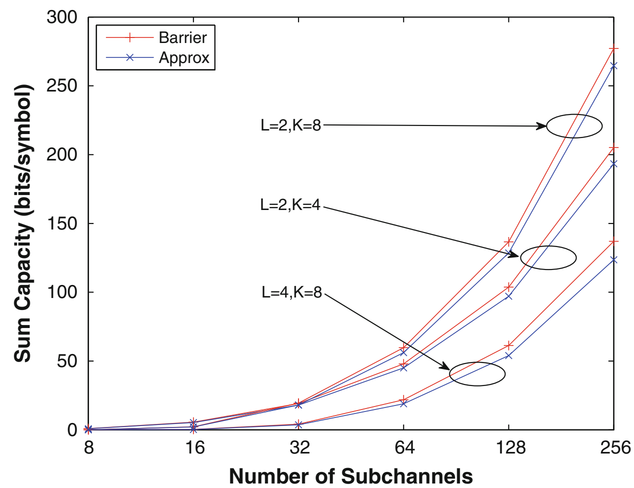


Fig. 4 Performance of the proposed power allocation algorithms. $P = 1W$

Then we compare the proportional fairness of our proposed algorithms with the sum capacity maximization [16] and static TDMA [10]. The statistic TDMA assigns a fixed time slot to each user. $\beta_1{:}\beta_2{:}\beta_3{:}\beta_4 = 4{:}1{:}1{:}1$. The number of PUs, SUs and subchannels are 2,4 and 32, respectively. Figure 5 shows the normalized ergodic rate of each SU. It can be seen the proportional fairness of our proposed HEU-BAR and HEU-APP achieves an ideal one. While, the sum capacity maximization and the static TDMA can not maintain fairness among SUs since there is no efficient fairness control in both of the schemes.

Let D_c be the dissatisfaction rate defined as

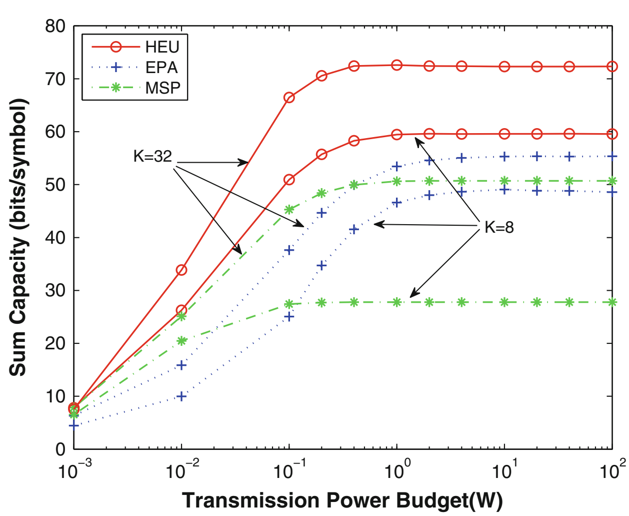


Fig. 3 Performance of different subchannel allocation algorithms. $L = 4$, $N = 128$

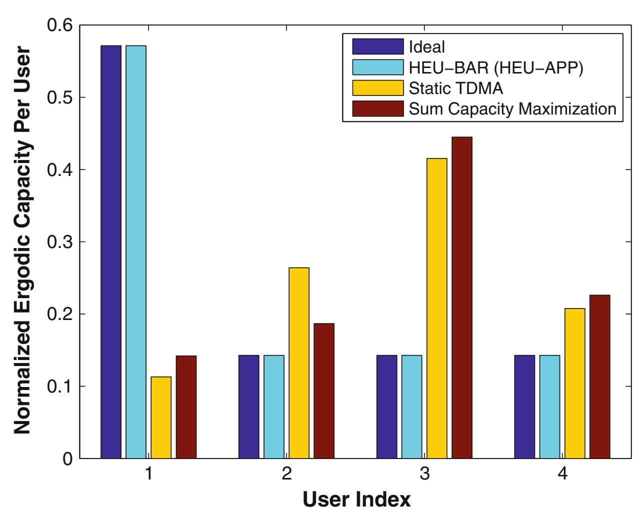


Fig. 5 Normalized ergodic sum capacity distribution among SUs. $L = 2,\ K = 4,\ N = 32,\ P = 1W$ and $\beta_1{:}\beta_2{:}\beta_3{:}\beta_4 = 4{:}1{:}1{:}1$

$$D_c = \frac{1}{I}\sum_{i=1}^{I}\sum_{k=1}^{K}\left|\frac{R_k^i}{\sum_{k=1}^{K} R_k^i} - \frac{\beta_k}{\sum_{k=1}^{K}\beta_k}\right|, \tag{29}$$

where R_k^i is the sum rate of SU k of the ith instance and I is the number of instances. Table 4 shows the average dissatisfaction rate of each RA method over 1,000 instances with $L = 2$, $K = 4$ and $N = 32$. The proportional constraint dissatisfaction coefficients of the HEU-BAR and the HEU-APP are nearly zeros, which means the rate constraint is satisfied perfectly.

Finally, we investigate the convergence of our proposed *barrier* method. As mentioned above, the computational load is mainly caused by Newton iterations. Figure 6 shows the number of Newton iterations required for a guaranteed duality gap of less than 10^{-3} for 100 channel realizations. We can see that the Newton iterations are less than 40 for most instances. Figure 7 shows the Cumulative Distribution Function (CDF) curve as a function of Newton iterations. The number of Newton iterations is less then 60 for 95 % instances for different wireless scenarios, which means the fast *barrier* method is effective and efficient.

Table 4 Dissatisfaction coefficients in different scenarios

β_1:β_2:β_3:β_4	1:1:1:1	2:1:1:1	4:1:1:1	8:1:1:1
HEU-BAR	0.0000	0.0000	0.0000	0.0000
HEU-APP	0.0000	0.0000	0.0000	0.0000
Static TDMA	0.2944	0.3978	0.6616	0.9500
Sum Cap. Max.	0.5759	0.6340	0.8093	1.0308

7 Conclusions

In this paper, we studied the adaptive resource allocation problem in multiuser OFDM-based cognitive radio networks. We try to maximize the sum capacity of the CR system while keeping proportional rate constraints satisfied to guarantee the fairness among SUs. Due to find the optimal solution is computationally expensive, we proposed two low complexity algorithms, in which subchannel and power allocation are carried out separately. For subchannel allocation, we convert the channel gain and interference introduced to the PUs into a normalized

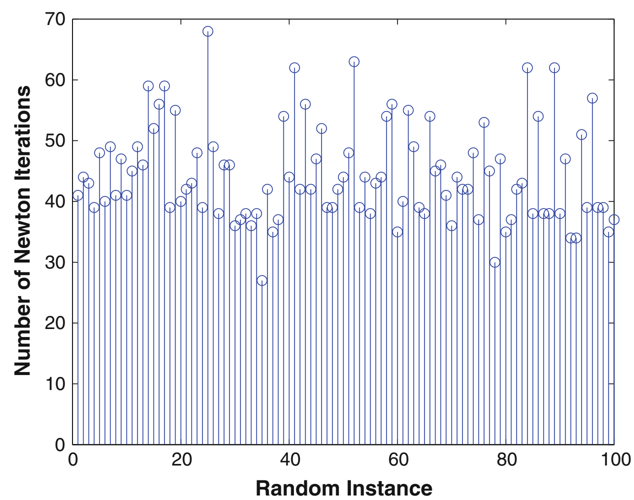
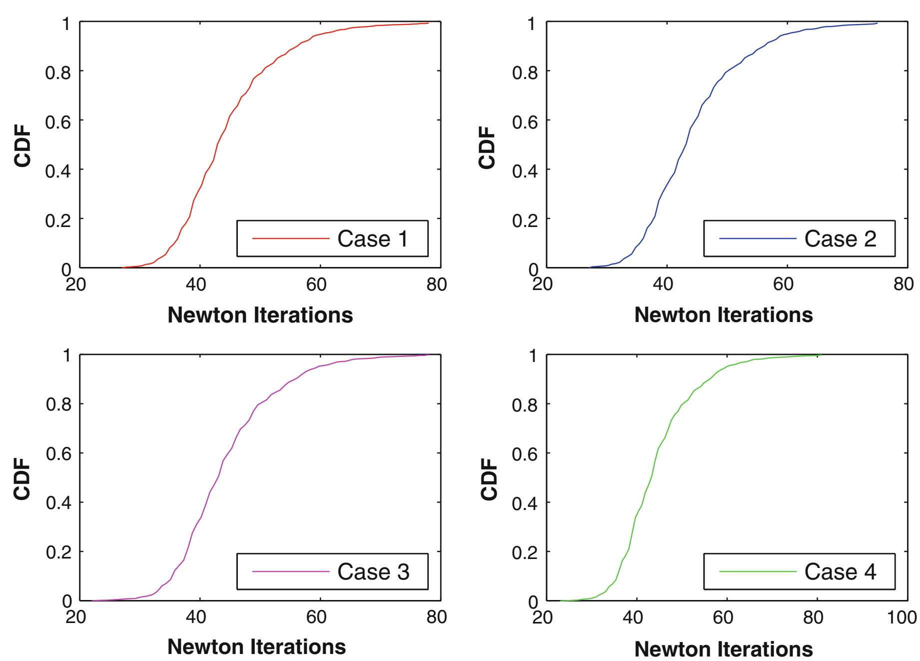


Fig. 6 Number of Newton iterations required for convergence during 100 channel realizations. $L = 2$, $K = 4$, β_1: β_2:β_3: β_4 = 1:1:1:1

Fig. 7 Cumulative distribution function of the number of Newton iterations for convergence over 1,000 instances. $L = 2$, $K = 4$ and $N = 32$ and $P = 1W$. Case 1: β_1:β_2:β_3:β_4 = 1:1:1:1; Case 2: β_1:β_2:β_3:β_4 = 2:1:1:1; Case 3: β_1:β_2:β_3:β_4 = 4:1:1:1; Case 4: β_1:β_2:β_3:β_4 = 8:1:1:1



capacity, based on which each subchannel is allocated to the SU with the highest achievable rate over it. Given a subchannel assignment, we proposed a fast *barrier* method with complexity of $\boldsymbol{O}\,(L^2N)$ to find the optimal power distribution by exploiting the structure of the optimization problem. We also presented an approximate power allocation algorithm with complexity of only $\boldsymbol{O}\,(N+L)$, which can approach the optimal power distribution. Simulation results show both of the proposed algorithms work well. The fast *barrier* method can can achieve the upper bound, while the approximate one can obtain above 90 % capacity of the optimal. Furthermore, the proportional rate requirements are strictly satisfied by using our proposed methods, which makes them promising for applications.

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Appendix: Proof of Theorem 1

We propose an $L+1$ iterations procedure to solve (16). Define $s+1$ intermediate variables, $u_1^s, u_2^s, \ldots, u_{s+1}^s \in \mathbf{R}^{N+K-1}$ at step s, $s = 1, 2, \ldots, L+1$, the procedure is illustrated as follows. Step 1: since $H_1 = H_2 + G_1G_1^T$, we have

$$x = u_1^1 - \frac{G_1u_1^1}{1 + G_1^Tu_2^1}u_2^1, \tag{30}$$

and u_1^1, u_2^1 can be calculated as

$$\begin{aligned} H_2u_1^1 &= G_0 \\ H_2u_2^1 &= G_1. \end{aligned} \tag{31}$$

Step $s:\ s = 2, \ldots, L$. As $H_s = H_{s+1} + G_sG_s^T$, we have

$$u_i^{s-1} = u_i^s - \frac{G_s^Tu_i^s}{1 + G_s^Tu_{s+1}^s}u_{s+1}^s, i = 1, \ldots, s, \tag{32}$$

and we need to solve the following $s+1$ equations

$$H_{s+1}u_j^s = G_{j-1}, j = 1, 2, \ldots, s+1. \tag{33}$$

Step $L+1$: we need to solve the following $L+2$ equations at this step

$$Hu_j^{L+1} = G_{j-1}, j = 1, 2, \ldots, L+2. \tag{34}$$

Without loss of generality, each equation in (34) can be written as

$$\begin{bmatrix} D & A^T \\ A & \mathbf{0}_m \end{bmatrix}\begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} G \\ \mathbf{0}_v \end{bmatrix}, \tag{35}$$

where $x, G \in \mathbf{R}^{N\times 1}$ and $v \in \mathbf{R}^{(K-1)\times 1}$. Recall that D is a diagonal matrix, denote $\theta_k = N - \sum_{i=k}^{N} N_i$, we have

$$\begin{aligned} &\lambda_{\theta_k+i}u_{\theta_k+i} - \beta_1 v_k = h_{\theta_k+i} \\ &\beta_k\sum_{i=1}^{N_1}u_i - \beta_1\sum_{i=1}^{N_k}u_{\theta_k+i} = 0, \end{aligned} \tag{36}$$

$i = 1, 2, \ldots, N_k$ for $k = 1, 2, \ldots, K$. And from (36), we have

$$\begin{aligned} X_k &= a_k + b_k(\beta_1 v_k) \\ X_k &= \tfrac{\beta_k}{\beta_1}X_1, \end{aligned} \tag{37}$$

where $X_k = \sum_{i=1}^{N_k} u_{\theta_k+i}$, $a_k = \sum_{i=1}^{N_k}\frac{h_{\theta_k+i}}{\lambda_{\theta_k+i}}$, $b_k = \sum_{i=1}^{N_k}\frac{1}{\lambda_{\theta_k+i}}$. Using the set of equations in (37), we can obtain

$$\begin{aligned} X_1 &= \left(\sum_{k=1}^{K}\frac{a_k\beta_k}{b_k}\right)\Big/\left(\sum_{k=1}^{K}\frac{\beta_k^2}{b_k\beta_1}\right) \\ v_k &= \frac{\beta_k}{\beta_1^2 b_k}X_1 - \frac{a_k}{b_k\beta_1}, k = 2, \ldots, K. \end{aligned} \tag{38}$$

As v_k is worked out by (38), we can insert it to (36) to obtain u. The computation cost is $\boldsymbol{O}\,(K+N)$. Since $K \ll N$ in practice, the complexity can be denoted as $\boldsymbol{O}\,(N)$.

Note that there are $L+2$ equations in (34), each equation can be solved with complexity $\boldsymbol{O}\,(N)$. We carry out an inverse process by using the intermediate variables until Newton step x is worked out. In total, we should calculate $L^2 + 3L + 3$ variables and the complexity of computing each variable is $\boldsymbol{O}\,(N)$. So the complexity of our proposed power allocation algorithm is $\boldsymbol{O}\,(L^2N)$.

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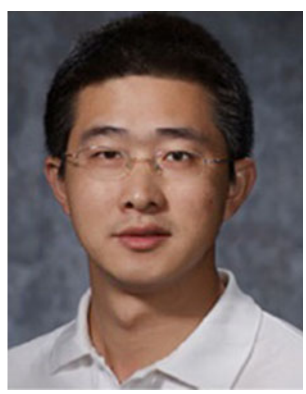
Author Biographies

Shaowei Wang received the B.S., M.S. and Ph.D. degrees in electronic engineering from Wuhan University, China, in 1997, 2003, and 2006, respectively. From 1997 to 2001, he was a research scientist of China Telecom. He joined the School of Electronic Science and Engineering of Nanjing University, China, in 2006, where he has been an Associate Professor since 2008. From 2009 to 2011, He was also with LAMDA group, Nanjing University. Currently, he is also with Stanford University, CA. His research interests include wireless communications and networks, optimization and machine learning. Currently, his research focuses on resource allocation in wireless networks, cellular networks planning, and traffic engineering in telecommunications.

Fangjiang Huang received the B.S. degree in electronic engineering from Nanjing University, China, in 2009. He is currently working toward the M.S. degree at the School of Electronic Science and Engineering, Nanjing University, China. His research interests include wireless networks and convex optimization. Currently, his research focuses on the resource allocation in cognitive radio networks.



Chonggang Wang received his Ph.D. degree in computer science department from Beijing University of Posts and Telecommunications, China in 2002. He is currently with InterDigital Communications with focuses on Machine-to-Machine (M2M) communications and Internet of Things (IoT) R&D activities including technology development, standardization, and external collaboration. Before joining InterDigital, he had conducted various researches on optical networks, cognitive radio, RFID and sensor networks, P2P networking, etc, respectively with NEC Laboratories America, AT&T Labs Research, University of Arkansas, and Hong Kong University of Science and Technology.