Online Channel Selection and User Association in High-density WiFi Networks

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Abstract—**In this paper, we consider the emerging deployment of WiFi networks in sports and entertainment venues characterized by high-density, large capacity, and real-time service delivery. Due to extremely high user density, channel allocation and user association should be carefully managed so that co-channel inference can be mitigated. To this end, we propose a channel selection and user association (CSUA) solution based on the Adversarial Multi-armed Bandit (AMAB) framework, which captures not only the uncertainty of channel states, but also the selfishness of individual stations (STAs) and access points (APs). An exponentially weighted average strategy is adopted to design an online algorithm for this problem, which is guaranteed to converge to a set of correlated equilibria with vanishing regrets. Simulation results show the convergence of the proposed algorithm and its performance under different settings.**

I. Introduction

To meet the growing demands for data service due to the rapid penetration of mobile devices (smart phones, iPad, laptop, etc.), WiFi networks have been selected by many service providers as a cost-effective 3G/4G offloading solution in public places such as shopping malls, bars, hotels, and public squares. In sports and entertainment venues, the availability of WiFi networks is also seen as an opportunity to improve the experience of customers and pursue revenue growth opportunities. Unlike common enterprise networks, most stadiums have tens of thousands of seats, user density will be hundreds of times greater than in common office networks. For example, for a 50,000-seat stadium, around 400 APs would be needed to provide sufficient capacity and coverage throughout the venue area[1]. Therefore, WiFi networks in these venues are characterized by high-density, large capacity, and real-time service delivery, making its deployment and management complicated. Recognizing the challenges arising in high-density WiFi networks, the IEEE 802.11 working group has formed a new study group with the objective to define a highly efficient WLAN standard, which was approved as 802.11ax project by the IEEE standard board in March 2014[2].

The main limiting factor in designing high-density networks is co-channel interference since the number of available orthogonal channels is far less than the number of neighboring APs in this case. Therefore, channels should be carefully assigned to APs so that co-channel interference from neighboring APs can be mitigated, and has been studied extensively in literature. For example, in [3], the authors formulate AP channel allocation as a min-max optimization problem with the objective of minimizing the channel utilization of the AP with the maximum traffic load and propose dynamic radio channel allocation strategy. Based on the same framework, a channel allocation strategy is proposed for WLANs with multiple data rates[4]. The channel assignment algorithm proposed in [5] aims to maximize the signal-to-interference ratio at user level. In [6], the channel allocation problem is reduced to a bandwidth allocation problem, which is then formulated as a non-cooperative game whereby the desired allocation result corresponds to a Nash equilibrium. A comprehensive survey on the channel allocation strategy for WLANs can be found in [7].

On the other hand, due to high user density, the number of stations (STAs) associated with an AP can be dozens or even hundreds and may distribute unevenly geographically. The performance of STAs served by the overly-loaded APs degrades dramatically due to the CSMA/CA based channel access mechanism. To address this problem, many user association schemes have been proposed in literature that attempt to balance traffic load across APs. In [8], the authors analyze the convergence and stability of a user association strategy based on the congestion game theoretical model. The authors in [9] propose a solution to determine the user associations so that the max-min fair bandwidth allocation is ensured. Recently, user association and channel allocation have been jointly considered to achieve better performance. For example, in [10], an algorithm is designed for this joint optimization problem based on the Gibbs sampler technique. In our previous work [11], we model the channel allocation and user association problem as a non-cooperative game, and the proposed algorithm is proven to converge to a Nash equilibrium.

Due to the dynamics of wireless channels (e.g., fast-fading, shadowing, and co-channel interference, etc.), the channel state information is unknown a priori. The problem of joint channel selection and user association is non-trivial since the co-channel interference and contention level can only be estimated (or observed) after the decisions have been made, which leads to the challenging trade-off between exploration (*learning the statistics of individual channels and APs*) and exploitation (*utilizing the best channel and AP*). Moreover, the interference and contention also depend on the decision of

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other APs and STAs, which are unknown since their decisions are made online. In this paper, we adopt the Adversarial Multi-armed Bandit (AMAB) framework for this problem, which handles not only the uncertainty of channel states, but also the selfishness of individual STAs and APs. An online channel algorithm is designed based on the exponentially weighted average strategy, which is guaranteed to converge to a set of correlated equilibria with vanishing regrets as validated by theoretical analysis and simulation results. The main novelty in the proposed solution lies in its decentralized nature where individual APs and STAs in multiple contention domains make decisions entirely based on local measurements.

The rest of paper is organized as follows. In Section II, we introduce the system model and formally define the CSUA problem. In Section III, the AMAB framework is introduced and the proposed decentralized CSUA algorithm is presented. Simulation results are provided in Section IV to evaluate the performance of the proposed scheme and finally we conclude this paper in SectionV.

II. System Models and Problem Statement

A. Network model

We consider an IEEE 802.11 wireless network consisting of a set of APs $\mathcal{A} = \{1, \cdots, A\}$ and a set of stations (STAs) $\mathcal{S} = \{1, \cdots, S\}$. Let A_s denote the set of APs who are within the communication range of STA s. Similarly, S_a denotes the set of STAs associated with AP a.

Each AP can operate in a set of M orthogonal channels, and all STAs associated with an AP should operate in the same channel as the AP. Let $\mathcal{N}_a$ denote the set of neighboring APs who are in the contention range of AP a, which will lead to co-channel interference to AP a if they are operating in the same channel.

B. Airtime Cost

We adopt the *airtime cost* as a measure of the delay between a pair of AP and STA, which was firstly proposed as a routing metric for 802.11s wireless mesh networks [12]. Specifically, for a STA s and its associated AP a, the airtime cost is defined as:

$$T_s^a(t) = T_{ca} + T_p + \frac{B_t}{C_s^a(t)}, \tag{1}$$

where T_{ca} is the channel access overhead, T_p is the protocol overhead, B_t is the number of bits in the data frame at t, and $C_s^a(t)$ is the transmission rate between AP a and STA s at time t. Under saturated condition, T_{ca}, T_p and T_t are constants, $T_{ca} + T_p = 1.25ms$ and $B_t = 8224$ bits [8].

The airtime cost of an AP a is the aggregated airtime cost of its associated STAs, that is,

$$T(a,t) = \sum_{s \in S_a} T_s^a(t) = \sum_{s \in \mathcal{S}_i} \left[T_{ca} + T_p + \frac{B_t}{C_s^a(t)} \right], \tag{2}$$

which is a good approximation of the uplink transmission delay due to the contention among STAs associated with the same AP[8].

In high-density WiFi networks, the number of orthogonal channels can be far less than the number of neighboring APs. As a result, multiple neighboring APs may have to share the same channel, so the contention is not only from the STAs of the same AP, but also from other co-channel neighboring APs and their associated STAs. Therefore, the airtime cost should also take into account the co-channel contention delay from neighboring APs. Specifically, for an AP a operating in channel i, its airtime cost is given by:

$$\begin{aligned} T(a,i,t) &= T(a,t) + \sum_{a' \in \mathcal{N}_a^i} T_(a',t) \\ &= \sum_{a' \in \mathcal{N}_a^i \bigcup \{a\}} \sum_{s \in \mathcal{S}_j} [T_{ca} + T_p + \frac{B_t}{C_s^{a'}(t)}], \end{aligned} \tag{3}$$

where $\mathcal{N}_a^i$ denotes the set of neighboring APs operating in the same channel as AP a.

C. Channel Selection and User Association (CSUA) Problem

As seen from (3), the airtime cost is dominated by two factors, one is the intra-AP contention between STAs associated with the same AP; the other is the inter-AP contention between co-channel neighboring APs. Therefore, from an AP's perspective, an effective way to mitigate co-channel contention is to select a less congested channel so that its airtime cost is minimized, while from a STA's perspective, it prefers an AP with smaller airtime cost so that the overall contention is minimized. Mathematically, we have the following channel selection problem for an AP a:

$$i_a^* = \arg \min_{i=1,\cdot,M} T(a,i,t), \tag{4}$$

and the user association problem for a STA s:

$$a_s^* = \arg \min_{a \in \mathcal{A}_s} T(a,i,t). \tag{5}$$

Unfortunately, the airtime cost is unknown a priori. The reason is two-fold. Firstly, the transmission rate between a STA s and its associated AP a depends on the its perceived signal-to-noise-ratio (SNR), i.e., $C_s^a(t) = f(SNR_s^a)$. For example, for AWGN channels, the transmit rate can be upper bounded by the Shannon's equation, that is, $C_s^a(t) \leq C \log\left(1 + \frac{P_s G_s^a(t)}{N_0}\right)$, where C is the channel bandwidth in hertz, P_s is the transmit power, $G_s^a(t)$ is the instantaneous channel gain at time t, and N_0 is the noise power. Due to fast-fading and shadowing effects, the channel gain changes over time following an unknown distribution, and thus $C_s^a(1), \cdots, C_s^a(t)$ can be modeled as i.i.d. random variables. Secondly, the co-channel contention depends on the number of neighboring APs and STAs operating in the same channel, which is unknown since the decisions of other APs and STAs are made online.

Therefore, the CSUA problem is nontrivial since the airtime cost *can only be estimated after all APs and STAs has made their decision*. In next section, we adopt the AMAB framework to address this problem.

III. Adversarial Multi-armed Bandit Framework for CSUA Problem

In this section, we first introduce the AMAB framework for the CSUA problem, and then propose an algorithm based on the exponentially weighted average strategy.

A. Adversarial Multi-armed Bandit

The CSUA problem can be modeled as an AMAB problem consisting of two kinds of players (APs and STAs), who can choose an action from a set of actions (a set of M channels for an AP a, and a set of APs $\mathcal{A}_s$ for a STA s). This problem can be seen as a game with two kinds of agents: for an AP a (or a STA s), the first agent is itself, and the second agent is the set of all other players whose channel selection (user association) affect its airtime cost.

In the following, we focus on the channel selection problem from an AP's perspective. The same strategy is applicable to the user association problem as well. Specifically, each AP a chooses a channel i in successive rounds. Upon operating on channel i in round t, AP a observes its airtime cost $T(a,i,t)$, while no other information can be accessed. In order to measure the utility of channel selection sequences, the concept of *regret* is introduced to denote the difference between the airtime cost which could have been achieved if the AP selects the optimal channel, and the airtime cost which is achieved in the selected channel. The objective of AP a is to minimize its cumulative regret $R_a(n)$ up to time n, which is defined as follows:

$$R_a(n) = \max_{i=1,\cdots,M} \sum_{t=1}^{n} W(a,i,t) - \sum_{t=1}^{n} W(a,I_t,t), \quad (6)$$

where I_t denotes the channel selected by AP a at time t and $W(a,i,t) = \frac{1}{T(a,i,t)}$ denotes the reward of channel i selected by AP a at time t.

This definition can be generalized to the case where AP a selects channel using mixed strategies, which is characterized by a probability distribution $\mathbf{P_t} = (p_{1,t}, p_{2,t}, ..., p_{M,t})$ over the set of M possible channels, where each channel i is selected with probability $p_{i,t}$. In this case, the concept of *external regret* [13] can be adopted to measure the difference between the expected cost of the actual mixed strategy and the optimal channel selection:

$$\begin{aligned} R_a^{Ext}(n) &= \max_{i=1,\cdots,M} \sum_{t=1}^{n} W(a,i,t) - \sum_{t=1}^{n} \bar{W}(a,\mathbf{P_t},t) \\ &= \max_{i=1,\cdots,M} \sum_{t=1}^{n} \sum_{j=1}^{M} p_{j,t}(W(a,i,t) - W_a(a,j,t)) \end{aligned} \quad (7)$$

where $\bar{W}(a,\mathbf{P_t},t) = E_t[W(a,\mathbf{P_t},t)] = \sum_{j=1}^{M}[p_{j,t}W(a,j,t)]$ denotes the expected airtime cost for AP a to operate on a channel according to the probability distribution $\mathbf{P_t}$.

In order to compare the channel selection in pair, we define the *internal regret* to measure the expected regret by choosing channel i instead of channel j, which is given by:

$$\begin{aligned} R_a^{Int}(n) &= \max_{i,j=1,\cdots,M} R_{(i\to j),n} \\ &= \max_{i,j=1,\cdots,M} \sum_{t=1}^{n} p_{i,t}[W(a,j,t) - W(a,i,t)] \end{aligned} \quad (8)$$

From (7) and (8), it can be seen that the external regret is bounded from above by the internal regret, that is:

$$\begin{aligned} R_a^{Ext}(n) &= \max_{i=1,\cdots,M} \sum_{j=1}^{M} R_{(i\to j),n} \\ &\le M \max_{i,j=1,\cdots,M} R_{(i\to j),n} \\ &= M R_a^{Int}(n), \end{aligned} \quad (9)$$

which suggests that if a channel selection strategy has a vanishing internal regret, it also has vanishing external regret.

In order to achieve vanishing external and internal regret, mixed strategies are necessary. In this game-theoretic formulation, the channel selection strategy approaches a Nash equilibrium (correlated equilibrium) if an AP cannot improve its airtime cost by unilaterally changing its choice. The relationship between internal regret and Nash equilibrium is shown as follows:

Theorem 1 ([13]): Consider an bandit game with a set of players $\mathcal{A}$, where each player $a \in \mathcal{A}$ select a channel from a set of M channels. Let ζ denote the set of correlated equilibria. We can define the empirical joint distribution of the game at time n as:

$$\begin{aligned} \hat{\pi}_n(i) &= \frac{1}{n} \sum_{t=1}^{m} \mathbb{I}_{\{\mathbf{I_t}=\mathbf{i}\}}, \\ \mathbf{i} &= (i_1, i_2, \ldots, i_A) \in \bigotimes_{a\in\mathcal{A}} \{1, \cdots, M\} \end{aligned} \quad (10)$$

Then, if all players play according to any strategy such that

$$\lim_{n\to\infty} \frac{1}{n} R_a^{Int}(n) = 0, \forall a \in \mathcal{A}, \quad (11)$$

then the distance $\inf_{\pi\in\zeta} \sum_i |\hat{\pi}_n(\mathbf{i}) - \pi_n(\mathbf{i})|$ between the empirical joint distribution of plays and the set of Nash equilibria converges to 0 almost surely. That is, if each AP selects a channel with vanishing internal regret, the joint distribution of channel selection converges to the set of correlated equilibria.

B. Algorithm

In this subsection, we propose algorithms for the CSUA problem based on the AMAB framework, which is the bandit version of the exponentially weighted average (EWA) strategy proposed in [14]. The core idea of the algorithm is that each player chooses an action using a mixed strategy where the probability is adjusted according to the internal regret. In order to guarantee that the correlated equilibria can be obtained, the algorithm needs to achieve vanishing internal regret as shown in (11). To this end, the algorithm needs to maintain two parameters, γ_t and η_t, which guarantees the Hannan-consistency and leads to vanishing internal regrets[13].

The algorithm consists of two stages, that is, channel selection stage and user association stage. In each round t, the channel selection stage is executed first, whereby each AP a runs an EWA based channel selection algorithm (EWA_CS) to select a channel and observes the corresponding airtime cost. The details of the EWA_CS algorithm is shown in Algorithm 1. Firstly, the parameters are set as $\gamma_t = t^{-\frac{1}{3}}$ and $\eta_t = \frac{\gamma_t^3}{M^2}$ (step 1), and a modified strategy is constructed based on the probability distribution $\mathbf{P}(t-1) = (p(1,t-1), p(2,t-1), \ldots, p(M,t-1))$, which yields a probability $P_a^{i\to j}(t-1)$ for each pair of i and j by transporting the probability mass from i to j (step 2). The internal regret $\tilde{R}_a(i \to j, t)$ and a weighted parameter $\delta_a^{i\to j}(t)$ can then be obtained (steps 3). Note that since each AP can only observe the airtime cost of the selected channel but not those of others, so the airtime cost can only be estimated using an unbiased estimator (14), that is $E_t(\widetilde{W}(a,k,t)) = W(a,k,t)$, where E denotes the expectation operator. The probability distribution $P_a(t)$ is obtained by solving a set of fixed equations (step 4) and adjusted using the EWA strategy (step 5). Finally, a channel is selected according to the updated probability and the corresponding airtime cost is observed. Note that since the purpose of our channel selection strategy is to minimize the airtime cost, when $\tilde{W}(a,i,t)$ is larger than $\tilde{W}(a,j,t)$, the AP can achieve a smaller airtime cost after changing channel selection from j to i, which will lead to the decrease in $p_{t,j}$ and the increase in $p_{t,i}$.

After all APs have selected their channels, the algorithm enters the user association stage, where each STA s runs an EWA based user association algorithm (EWA_UA) to select an AP and observes the corresponding airtime cost. The procedure is similar to Algorithm 1. The action set for each STA s is its neighboring AP set $\mathcal{A}_s$, and the parameters are set as $\gamma_t = t^{-\frac{1}{3}}$ and $\eta_t = \frac{\gamma_t^3}{|A_s|^2}$. Once all STAs have decided their association, the next round of the algorithm is executed and the same procedure is repeated.

Using a similar proof technique for Theorem 6.9 of [13], it can be proven that this algorithm have *vanishing internal regrets*, and the joint distribution of plays converges to a set of correlated equilibria. The proof is omitted here due to space limit.

IV. Performance Evaluation

A. Simulation Setup

In this section, we study the convergence of the CSUA algorithm and its performance under different settings. The network considered in the simulation consists of a set of APs and STAs, which are distributed randomly in 1000×1000m rectangular area. The large scale path loss model is adopted with the path loss exponent set to 4. The transmit power of each AP is set to 15dBm, and the noise level is assumed to be constant and set to -95dBm. The transmit rate between an STA and an AP is determined by the SNR of the link according to the IEEE 802.11a/g protocol as shown in Table I. The maximum transmit range is 300m, and the contention range is 500m.

Algorithm 1 EWA_CS Algorithm

1: Set $\gamma_t = t^{-\frac{1}{3}}$ and $\eta_t = \frac{\gamma^3}{M^2}$;

2: Construct a modified strategy for each pair of i and j as follows: $P_a^{i\to j}(t-1) = (p_{1,t-1}, \ldots, 0, \ldots, p_{j,t-1} + p_{i,t-1}, \cdots, p_{M,t-1})$ where $p_{i,t-1}$ is replaced by 0, and $p_{j,t-1}$ is replaced by $p_{j,t-1} + p_s i, t-1$;

3: Define

$$\delta_a^{i\to j}(t) = \frac{\exp(\eta_t \tilde{R}_a^{i\to j}(t-1))}{\sum_{k\to l, k\neq l} \exp(\eta_t \tilde{R}_a(k\to l, t-1))} \qquad (12)$$

where

$$\tilde{R}_a^{i\to j}(t-1) = \sum_{l=1}^{t-1} p_{i,l}[\tilde{W}(a,j,l) - \tilde{W}(a,i,l)] \qquad (13)$$

and for $k = 1, \cdots, M$:

$$\widetilde{W}(a,k,t) = \begin{cases} \frac{W(a,\boldsymbol{I}_t,t)}{p_{k,t}}, & \text{if } k = \boldsymbol{I}_t. \\ 0, & \text{otherwise.} \end{cases} \qquad (14)$$

4: Solve the following fixed point equation to find $P_a(t)$

$$P_a(t) = \sum_{i\to j, i\neq j} P_a^{i\to j}(t)\delta_a^{i\to j}(t) \qquad (15)$$

5: Finally yields:

$$P_a(t) = (1-\gamma_t)P_a(t) + \frac{\gamma_t}{M}. \qquad (16)$$

6: Select a channel i according to $P_a(t)$ and observe the corresponding airtime cost $T(a,i,t)$ in this channel.

TABLE I: SINR vs. link capacity in IEEE 802.11 a/g

Rate Index	0	1	2	3	4	5	6	7
SINR(dB)	6	7.8	9	10.8	17	18.8	24	24.6
Rate(Mbps)	6	9	12	18	24	36	48	54

B. Simulation Results

Firstly, we study the convergence of the CSUA algorithm. In Fig.1, we show the evolution of the channel selection probability vector for a specific AP. It can be seen that the channel selection probability vector converges to (1,0,0) in about 600 simulation rounds, in other words, the first channel is determined by the channel selection algorithm, which is in fact the optimal solution in equilibrium. Fig.2 shows the convergence of the user association probability vector for a specific STA, which has 4 APs in its transmit range. It can be seen that the probability vector converges to $(0,1,0,0)$ eventually, which suggests that the proposed user association algorithm is effective in finding the optimal AP in equilibrium. In Fig.3, we shows the convergence of the airtime cost of three APs. It can be seen that the airtime cost varies dramatically at beginning. However, as the channel selection of each AP converges, the airtime cost also becomes stable.

Secondly, we study the performance of the CSUA algorithm under different settings. In Fig.4-6, we show the airtime cost of APs under different number of APs, STAs and channels

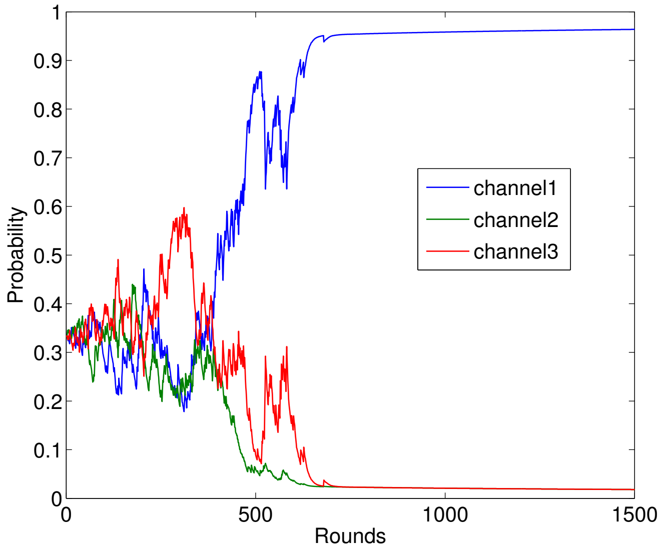

Fig. 1: Convergence of the channel selection probability vector

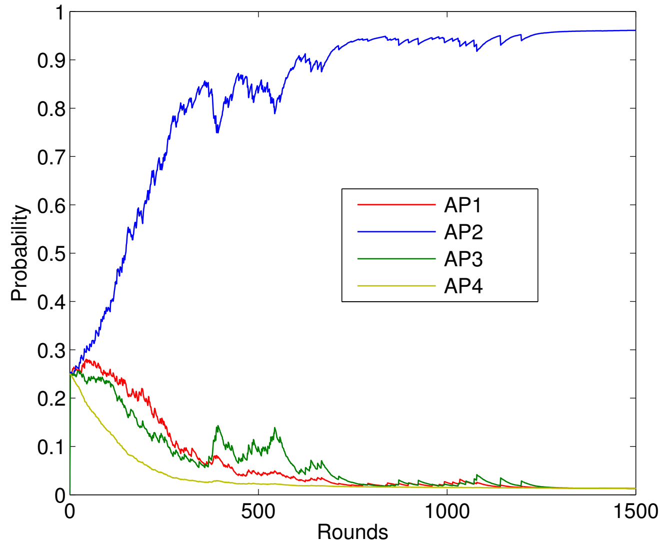

Fig. 2: Convergence of the user association probability vector

respectively, where the airtime cost is obtained after the simulation is ran for 1500 rounds (which is converged as shown in Fig. 1-3). In these figures, the horizontal lines are the average airtime of all APs, and the error bar indicates the maximum and minimum airtime costs of all APs.

In Fig.4, we show the airtime cost with 125 STAs and 3 channels, while the number of APs varies from 5 to 25. It can be seen that the airtime cost is gradually decreasing with the increase of APs. The reason is two-fold. Firstly, as more APs are deployed in the network area, the traffic load of existing APs is alleviated. Secondly, the transmit rate increases as the STAs and APs are closer, which in turn reduces the airtime cost. However, when the number of APs is beyond 15, the performance gain is compromised due to the increase of contention between APs at a fixed transmission power level. Therefore, the airtime cost cannot be further improved.

In Fig.5, we show the airtime cost with 10 APs and 3 channels, and the number of STAs varies from 50 to 200. It can be seen that the airtime cost increases gradually since the average traffic load of APs is increased as the user density increases.

Finally, Fig.6 shows the airtime cost with 10 APs and 125 STAs, and the number of channel varies from 2 to 8. It can be seen that the increase of channels can help mitigate co-channel interference, which leads to the decrease of the airtime cost. On the other hand, as the number of channels is beyond 5, the performance gain is no longer significant, which suggests the contention between APs can be well controlled by the channel allocation strategy given sufficient number of channels.

V. Conclusion

In this paper, the channel selection and user association problem is considered for high-density WiFi networks. The AMAB framework is adopted to capture both the uncertainty of channel states and the selfishness of individual STAs and APs. An exponentially weighted average strategy is adopted to design the online algorithms for this problem, which is guaranteed to converge to a set of correlated equilibria with vanishing regrets. Simulation results also show the performance of the proposed scheme under different settings.

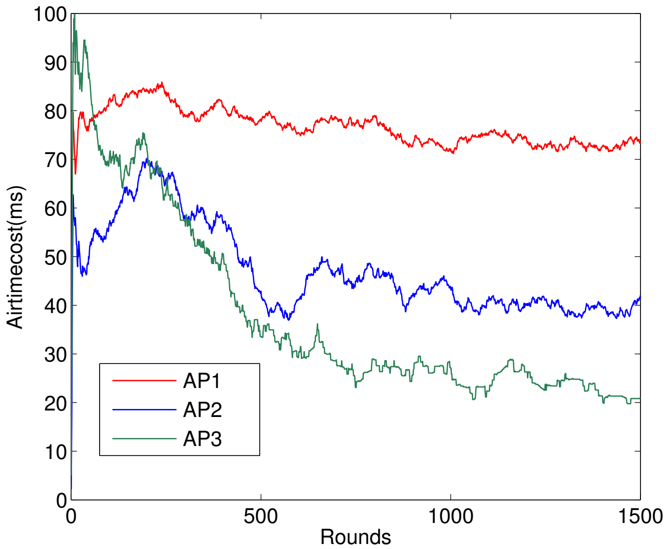

Fig. 3: Convergence of the airtime cost.

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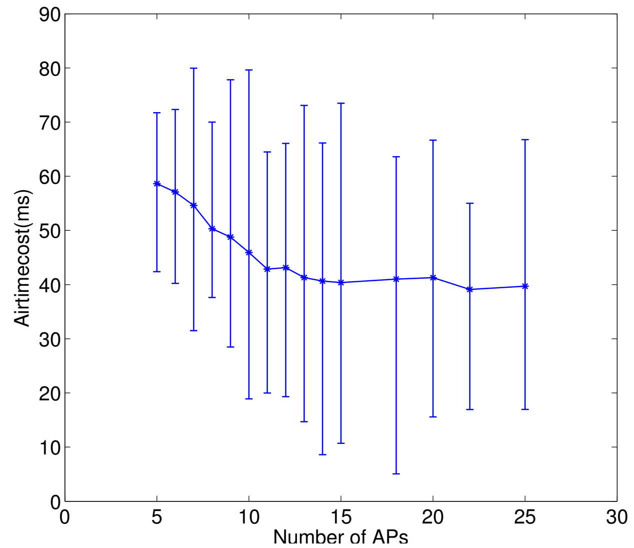

Fig. 4: Airtime cost vs. the number of APs (125 STAs, 3 channels)

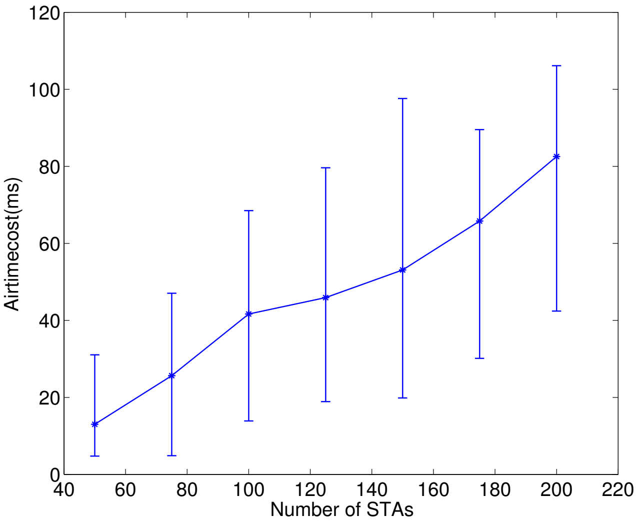

Fig. 5: Airtime cost vs. the number of STAs (10 APs,3 channels)

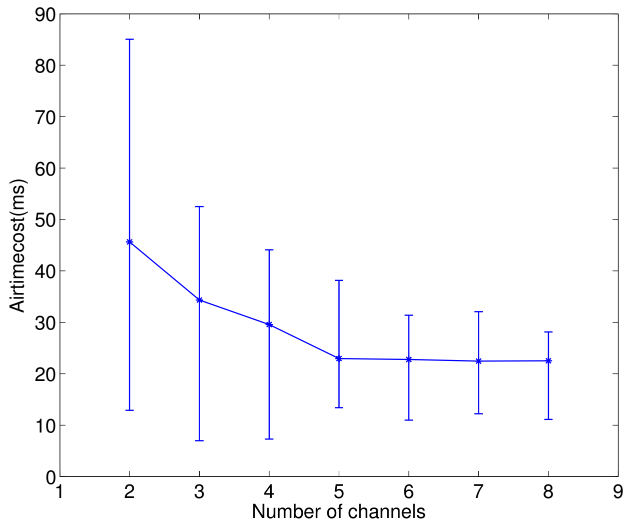

Fig. 6: Airtime cost vs. the number of channels (10 APs, 125 STAs)

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