Association Games in IEEE 802.11 Wireless Local Area Networks

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***Abstract*—In IEEE 802.11 wireless networks, users associate with access points that can provide the best service quality. In this paper, we analyze the convergence and steady state performance of a practically well performing load-based user association scheme. The analysis is based on a novel game theoretical model, which extends the results on atomic congestion games. We prove the existence of and convergence to a Nash equilibrium for this game. The bounds on the efficiency of equilibrium compared to centralized optimum solutions are established under different system costs.**

***Index Terms*—Game theory, price of anarchy, atomic non-cooperative games, distributed optimization.**

I. INTRODUCTION

IEEE 802.11 wireless local area networks (WLANs) have widespread deployment in enterprises, public areas and homes. In current implementations of WLANs, each station (STA) scans the wireless channel to detect the access points (APs) nearby, and associate itself with the AP that has the strongest received signal strength indicator (RSSI). Thus, it is expected that a STA associates itself with the closest/strongest AP. It is known that this association policy can lead to inefficient use of the network resources [1], [2], [3]. The most important disadvantage of RSSI-based user association is that RSSI does not provide any information about the current load of the AP in terms of number of users associated with it. It has been well established that when there are multiple STAs connected to the same AP with different physical transmission rates, then the saturation throughput of all users is bounded by the slowest transmission rate [4]. Therefore, even though there are other less loaded APs in the region, most STAs may associate with the same AP, and experience congestion.

In [5], a new association scheme is proposed to resolve this problem. In this scheme, each STA collects usage statistics from the APs, calculates *airtime cost* for each AP, and associates with the AP that has the minimum cost. The airtime cost is first proposed as a default metric for routing in wireless mesh networks according to the draft 802.11s standard [6], and it reflects the average latency a packet experiences during a transmission. If all APs operate at separate orthogonal channels, the airtime cost metric depends only on the individual physical transmission rates and the number of users associated with the AP. In [5], it has been shown by extensive simulation that the airtime cost based user association scheme can significantly improve average user throughput and end-to-end delay compared to RSSI based user association.

The airtime cost based association scheme proposed in [5], is a greedy scheme, where each STA chooses to associate with the AP from which it expects to receive the best performance. In this setting, the users are non-cooperative and behave selfishly to optimize their own performance. The non-cooperative and selfish behavior may have negative consequences for the whole system. Foremost, when a new STA associates with an arbitrary AP S, those users already connected to the same AP may experience performance degradation. Thus, this new association may trigger re-associations in the network, since those STAs associated with AP S may now find some other AP with lower airtime cost than AP S. It is important to understand whether such a system ever achieves an equilibrium within a finite time, and if such an equilibrium is achieved, then the efficiency of the equilibrium should be compared to the centralized optimum solution.

We have two main contributions in this paper. First, in order to understand the interactions between the users, we provide a novel game theoretical model for the user association problem in 802.11 WLANs. In so called "user association game," the user's utility obtained from an AP depends on the number and transmission rates of other associated users. We prove that this game has a Nash equilibrium solution, where no user attempts to change its association given the decisions of other users, and the equilibrium is reached within finite number of steps. Second, in order to understand the efficiency of the Nash equilibrium solution, we consider different system (social) costs, and determine the prices of anarchy for these costs. We also show that the lexicographical optimum solution of the airtime costs of the users is a Nash equilibrium solution.

The rest of the paper is organized as follows: In Section II, we summarize the previous work on user association schemes in 802.11 wireless networks, and game theoretical analysis of wireless resource sharing. In Section III, we discuss the model of the system considered in the paper. In the following section, we show that the airtime metric is an approximation of uplink average packet delay, and argue that it is a measure of congestion of an AP. In Section V, we discuss the airtime metric based user association algorithm, and model the association algorithm as a form of *atomic congestion game*. Unlike the original atomic congestion games previously discussed in the literature, the resource is not shared equally among the users. The share of resource consumed by a user varies according to the identity of the resource. We extend the results on the

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worst case efficiency of atomic congestion games for this more general model under different social objectives in Section VI. Finally, we summarize our contributions and discuss our conclusions in Section VII.

II. Related Work

With the proliferation of 802.11 wireless networks, the number of APs has increased exponentially, and it is often the case that a user has the option to associate with any of the several APs in the region. Due to the observed inefficiency of RSSI based association scheme, there has been recent interest to enhance the default association mechanism and improve the user experience by balancing the loads of the APs [3], [5], [7], [8]. In [3], the authors study a new STA association policy that ensures network-wide max-min fair bandwidth allocation to the users. The work assumes that all users are cooperative; however, in reality users usually have no particular incentive to cooperate with each other and would be interested in maximizing their individual payoffs. In [7], the users are again assumed to be cooperative and the problem of optimal user association to the available APs is formulated as a utility maximization problem. The work in [8] presents self-configuring algorithms that provide improved client association and fair resource sharing in the wireless network. Athanasiou et al., present a new association mechanism that estimates the load of APs in terms of average packet delay and associate each user with the AP that has the minimum load [5]. The authors demonstrate by simulations that the proposed association mechanism can significantly improve the performance of the network in terms of throughput and average packet delay.

In real networks, users are usually non-cooperative and selfish entities that make decisions to maximize their individual utilities. When interactions between the users are taken into account, game theory emerges as a natural modeling framework. In computer networks, game theory is mainly considered to determine if the multi-user non-cooperative system achieves an equilibrium. There has been recent interest in understanding the behavior of wireless networks using game theoretical models. For example, game theory is used to determine efficient power allocation ([9], [10], [11]), to determine pricing and incentive mechanisms for enabling cooperation and relaying packets ([12], [13], [14]) and for access control on common shared channel ([15], [16]).

In the context of user association problem in wireless networks, [17] is the first work to model the users as non-cooperative players. In [17], the user association problem is modeled as a population game by assuming that a user may associate with more than one AP, and the traffic can be split among these APs. The APs are assumed to operate at orthogonal channels, and users are charged according to the duration they occupy the channel. The asymptotic throughput of the network is maximized under the assumption that there are infinitely many users with each having negligible effect on the throughput. However, in reality, there may be only a few nodes associated with each AP, and thus, the individual effects of the users may not be negligible.

Another important limitation of the work in [17] is that flow bifurcation is permitted, i.e., each user may split its traffic among several APs. However, according to 802.11 standard, each station may associate with only one AP at any given time, and thus, it is often impractical to implement flow bifurcation. In such cases, we say that resource sharing is *atomic*, in the sense that a demand cannot be split among two or more resources. Noncooperative networks with atomic resource sharing were the subject of several recent works, e.g., [18], [19], [20]. The main objective of these works is to calculate the worst case efficiency of having noncooperative selfish behavior in the system. A special case is when the cost of using a machine or an edge depends on its load. Such games are called congestion games and were introduced by Rosenthal [21] as a broad class of games possessing pure equilibria. The main results on congestion games depend on the assumption that the utility/payoff of player i from each of the resource depends only on the number of players sharing the resource. However, in 802.11 wireless networks this assumption is not satisfied, since the physical transmission rate and the contention observed by a user depend on the relative locations of other users and APs. Those users with better channel quality and closer relative location to the AP can transmit with higher physical transmission rates, and thus, occupy the channel for a shorter period of time. Therefore, in the user association game, the utility/payoff of a user obtained by associating with an AP is defined as a function of the average throughput of that AP. As shown in Section IV, the utility depends not only on the number of users associated with the same AP, but also on the physical transmission rates of those users. In our work, we extend the results on atomic congestion games for this more general payoff function where the load of the resource depends on the number and types of users sharing the resource. We assume that there are finite number of types of users, with each type of user inflicting a particular given load on the AP.

III. System Model

Assume that a geographical area is covered by S APs, and there are N STAs distributed randomly in this area. The APs periodically broadcast beacon frames containing relevant information necessary for association. The APs operate at non-overlapping frequencies, so that there is no interference from STAs in the adjacent cells. Every STA scans wireless channels and collects beacon frames from the APs. As shown in Figure 1, there may be more than one AP from which a STA receives beacon frames, and the STA may associate with any of those APs. The STAs transmit at some predefined rates $\underline{R} = \{R_1, R_2, \dots, R_I\}$ (e.g., according to 802.11g standard with data rates of 6, 9, 12, 18, 24, 36, 48, and 54 Mbps). The physical transmission rate of STA n when associated with AP s is determined by one of the rate adaptation algorithms available in the literature, e.g., [22]. Let R_n^s denote the rate of connection between STA n and AP s. Also, if STA n has no connection to AP s, i.e., if it does not receive a beacon frame, then $R_n^s = 0$. According to 802.11 standard, a STA can associate with only one AP at any particular instant. Let x_n^s be a binary variable which takes value 1, when STA n is associated with AP s, and 0 otherwise.

In this paper, we consider *user association games*, $\Gamma(P, S, C)$, where the players of the game, P, are the STAs

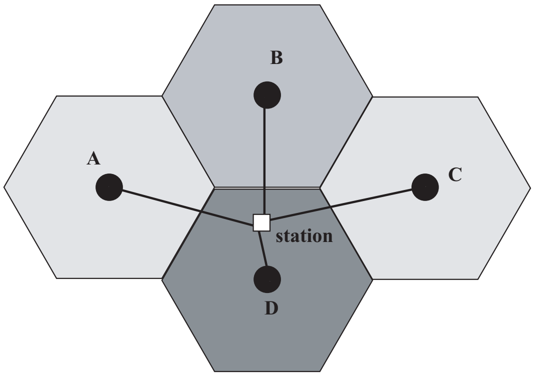

Fig. 1. Division of a geographical area into non-interfering cells using three independent non-overlapping channels. STAs can be in a position to associate with any of several access points.

in the network, the strategy set S_n for STA n is the set of APs that STA n can associate with, and the payoff function C_s for STA n is the airtime cost of associating with AP s, which is defined formally in the next section.

IV. Airtime Metric as a Measure of Congestion

In this section, we argue that airtime cost is an approximation of per packet latency on the uplink from STA to AP in 802.11 wireless networks. Thus, airtime cost reflects the level of congestion experienced at an AP. Let A_s be the set of STAs associated with AP s. The airtime metric of STA $n \in A_s$ is defined as [5]:

$$c_s^n = \left[O_{ca} + O_p + \frac{B_t}{R_n^s}\right] \frac{1}{1 - e_{pt}}, \quad (1)$$

where O_{ca} is the channel access overhead, O_p is the protocol overhead and B_t is the number of bits in the test frame. Some representative values for these constants (for 802.11b networks) are: $O_{ca} + O_p = 1.25$ ms and $B_t = 8224$ bits. The parameter e_{pt} represents the frame error rate for the test frame size B_t, but it is omitted from discussion in the remainder of this work to simplify the analysis. Let $C_s(A_s)$ denote the total airtime cost of AP s, and is defined as the sum of individual airtime costs of the STAs associated with s:

$$\begin{aligned} C_s(A_s) &= \sum_{n \in A_s} c_s^n, \\ &= \sum_{n \in A_s} \left[O_{ca} + O_p + \frac{B_t}{R_n^s}\right]. \end{aligned} \quad (2)$$

The fundamental steps of operation of the proposed association scheme in [5] are as follows: The STAs collect information about the candidate APs, e.g., number of STAs associated with the AP and their transmission rates, by listening to their beacon frames. The airtime metric for each AP is then calculated according to (2). Then, a STA associates with the AP that has the minimum total airtime cost. We refer the reader to [5] for more detailed description of the association protocol.

Next, we demonstrate that the total airtime cost of AP s, i.e., $C_s(A_s)$ is equal to the average uplink per packet delay for that AP. The average uplink throughput in a single cell environment is the topic of many recent works, e.g., [4], [24]. In order to calculate the average uplink throughput, two important approximations, i.e., saturation and decoupling approximations are employed. The saturation approximation states that the STAs always have packets backlogged in their buffers. Note that the total throughput of the AP is maximized under this assumption. Meanwhile, the decoupling approximation states that when there are N STAs associated with an AP, the aggregate attempt process of $(N-1)$ STAs is independent of the backoff process of any given STA. Let $\theta(n,s)$ be the average uplink saturation throughput of STA n when each STA is the transmitter of a single flow. $\theta(n,s)$ is determined as, [4]:

$$\begin{aligned} \frac{1}{\theta(n,s)} &= \frac{1}{\mu(1-\mu)^{N_s-1} L_n^s x_n^s} \times \\ &\left[1 + N_s \mu (1-\mu)^{N_s-1} \left(T_0 - T_c + 1/N_s \sum_{q=1}^{N} L_q^s / R_q^s x_q^s\right)\right. \\ &\left. +(1-(1-\mu)^{N_s-1}) T_c\right], \end{aligned} \quad (3)$$

where L_n^s is the size of the packets of STA n when transmitting to AP s, $N_s = \sum_{n=1}^{N} x_n^s$ is the number of STAs associated with AP s, T_c is the fixed overhead for an RTS collision, T_0 is the fixed overhead associated with packet transmission, and μ is the transmission attempt probability at the equilibrium. The value of μ at equilibrium is determined by observing the exponential backoff behavior of the STAs, and solving a fixed point equation governing the attempt probability. It can be shown that the transmission attempt probability, $G(\cdot)$, is a function of the transmission collision probability γ and is given as

$$G(\gamma) = \frac{\sum_{k=0}^{K} \gamma^k}{\sum_{k=0}^{K} \gamma^k b_k}, \quad (4)$$

where K is the maximum number of attempts allowed under the protocol, and b_k is the mean back-off at the k^{th} attempt. Meanwhile, under decoupling approximation, the probability of collision of an attempt by a node is given by $\Gamma(\mu) = 1-(1-\mu)^{n-1}$. The equilibrium behavior of the system is governed by the solution of the fixed point equation $\gamma = \Gamma(G(\gamma))$. The solution of this equation yields the collision probability from which the attempt rate in the equilibrium, i.e., μ can be determined from (4).

Note that the product of the mean packet length with the reciprocal of the average saturation throughput, i.e., (3), gives the average delay per packet at equilibrium. Comparing this product when $L_n^s = B_t$ with (2), one can easily see that the airtime metric is an approximation of average latency per packet transmission in a saturated 802.11 wireless network. In (2), $O_{ca} + O_p$ represents the delay due to channel contention and protocol overheads, and the last term in brackets represents the average transmission time of a B_t length packet from each user.

Therefore, in the user association scheme of [5], the users' objective is to determine the best AP that has the minimum average uplink latency. The operation of this system can be analyzed as a congestion game, which is explained in Section V.

V. User Association Game

The average latency in the network depends on the loads of the APs, and it is minimized if the loads are balanced. However, unlike traditional load balancing, in the user association game users are not interested in optimizing the social welfare (i.e., total system-wide latency). Instead, each STA selfishly tries to minimize its own latency. This setting gives rise to a noncooperative game, and the stable outcomes of this setting are called Nash Equilibria. Let a specific realization of STA associations, say $\mathbf{A} = \{A_s : s = 1, \ldots, S\}$, be defined as the set of STAs associated with each AP s in the network. Also note that $A_s = \{n : x_n^s = 1\}$. It is said that a realization is a *Nash Equilibrium Solution*, if each STA considers its chosen AP to be the best under the given choices of other STAs. Formally, STA n that is currently associated with AP s may decide to switch to AP t if and only if

$$C_s(A_s) > C_t(A_t \cup \{n\}), \tag{5}$$

under current realization $\mathbf{A}$.

The aforementioned user association game belongs to the general class of *congestion games* [21], and more specifically to the class of *atomic congestion games* [20]. In these games, a set of players compete for a set of resources, and the cost of each resource depends only on the number of players using it. In atomic games, one user is completely assigned to a single server. In computer science, perhaps the best known example on congestion games is network routing, studied first by Koutsoupias and Papadimitriou [19].

Unfortunately, there are no algorithms for computing an equilibrium, and thus, computing Nash equilibria remains a topic of current research. It is shown that many types of the congestion games can be defined as potential games, for which there is always an equilibrium solution. However, the *user association game* considered in this paper differs from those congestion games discussed in the literature. In user association game, the congestion observed by a user not only depends on the number of users but also on the types of users sharing the resource. Therefore, we first present a result that shows that user association game played among STAs in a 802.11 wireless network has a Nash equilibrium, and this equilibrium point is achieved in a finite time.

Theorem 1: The airtime metric based user association scheme converges to a Nash equilibrium solution after a finite number of steps.

Proof The proof of the theorem follows from a similar result given for unsplittable routing problem in [18]. Consider a move that a STA makes from AP l to m, and denote by C_l the airtime cost of AP l before the move. For the STA to have made the move, the airtime cost of m after the move should be less than C_l. Since the airtime cost of AP is an increasing function of the total number of STAs associated, the airtime cost of AP l after the move and the airtime cost of AP m before the move are also less than C_l. Let us observe the STA association realization after step t. Denote by $M(t)$ the set of APs with the maximum airtime cost, $M(t) = \arg\min_l C_l$, and $U(t)$ the set of STAs associated with those APs in $M(t)$. Suppose that a certain number of moves later, a STA from one of the APs in $M(t)$ makes a move. Due to our previous discussion, the move is made to an AP not in $M(t)$, and after that move one less AP remains in $M(t)$ with the original cost. Obviously, the airtime costs of other APs that are not involved in the move do not change. Hence, after $M(t)$ such moves the highest airtime cost in the network strictly decreases. The number of possible STA association realizations in the network is finite; therefore, if the network does not converge, STA eventually begin repeating previous realizations. However, we have just established that the highest airtime cost attained by a AP continues to decrease for as long as STAs make moves. Therefore, to avoid a contradiction, the network must come to a stable point after a finite time. ■

VI. Price of Anarchy of User Association Game

Given that user association game has an equilibrium, we are now interested in the efficiency of the equilibrium. A vast majority of literature on computer networks considers centralized optimal solutions; however, these solutions are not usually stable, since they do not take into account users' preferences. On the other hand, the cost of Nash equilibrium solutions achieved under selfish optimization of individual user's utility, can be much worse than that of centralized outcomes. For this case, a relevant measure to understand the performance of the outcome of the game is to consider the worst-case ratio between the Nash outcome and the social optimum, which is termed *price of anarchy* in [19]. In this section, we analyze the performance of the Nash equilibrium solutions by determining the *price of anarchy* of the user association game. The price of anarchy depends not only on the game itself, but also on the definition of the social (system) cost. Different system objectives can be selected depending on what is expected from the operation of the system. For example, the system administrator may either aim to balance the load of the APs in the network, to maximize the user performance or to provide fair service among users. In this paper, we consider four different system cost functions:

1) *Minimum Network Airtime Cost* minimizes the total average per packet delay of the APs in the network, which is given by
$$\sum_{s=1}^{S} C_s(A_s).$$
2) *Minimum User Airtime Cost* minimizes the total average per packet delay of the STAs in the network, which is given by
$$\sum_{s=1}^{S} n_s C_s(A_s).$$
3) *MinMax Airtime Cost* minimizes the maximum average delay observed by a user in the network, which is given by
$$\max_s C_s(A_s).$$
4) *Lexicographical Minimum of Airtime Costs* aims to provide a fair service to all users in the network by minimizing the maximum airtime cost of a user, then subject to this minimization, the second highest airtime cost is minimized, and so on.

For the first three types of system objectives, we determine the *price of anarchy*, by deriving a worst case bound for the Nash equilibrium solution. For lexicographical minimization of airtime costs as the system objective, we prove that the optimal solution is a Nash equilibrium solution.

A. Minimum Network Airtime Cost

We first consider an objective, where we minimize the total airtime cost of the APs in the network. We aim to determine the price of anarchy for this objective.

When the airtime cost of AP j as observed by STA i is considered as the congestion cost, it can be re-written in the following more general form:

$$f_i(A_j) = \alpha_j n_j + \beta_j l_j, \tag{6}$$

where $n_j = |A_j|$ represents the number of users associated with AP j, $l_j = \sum_{i \in A_j} \rho_{ij}$ is the total load of AP j and $\rho_{ij} = 1/R_i^j$ is the load of user i inflicted on AP j due to STA i's physical transmission rate R_i^j. Also note that $\alpha_j = O_{ca} + O_p$, and $\beta_j = B_t$. Therefore, for a given realization of user associations, $(A_1, A_2, \ldots, A_S)$, the total airtime cost of the APs in the network, τ, is

$$\tau = \sum_{j=1}^{S} \alpha_j n_j + \beta_j l_j.$$

Theorem 2: The price of anarchy of user association game when the social cost is the minimum network airtime cost is $O(N)$. Formally, the ratio of the worst cost of Nash solution and the optimal solution is

$$\frac{\sum_{i=1}^{N} \max_j \{1/R_i^j\}}{\min_{i,j}\{1/R_i^j\}} + 1.$$

Proof Under a Nash equilibrium solution, $\mathbf{A}$, assume that $i \in A_j$, and thus,

$$f_i(A_j) < f_i(A_k \cup \{i\}),\ k \neq j \tag{7}$$

$$\alpha_j n_j + \beta_j l_j < \alpha_k(n_k + 1) + \beta_k(l_k + \rho_{ik}),\ k \neq j. \tag{8}$$

Let $\mathcal{J}_i$ and $\mathcal{J}_i^*$ be the APs that user i associates with under Nash and optimal solutions, respectively. Then,

$$\alpha_{\mathcal{J}_i} n_{\mathcal{J}_i} + \beta_{\mathcal{J}_i} l_{\mathcal{J}_i} < \alpha_{\mathcal{J}_i^*}(n_{\mathcal{J}_i^*} + 1) + \beta_{\mathcal{J}_i^*}(l_{\mathcal{J}_i^*} + \rho_{i\mathcal{J}_i^*}). \tag{9}$$

Now, observe that $\alpha_j = \alpha$ and $\beta_j = \beta$ for all $j = 1, \ldots, S$, which is true if e_{pt} is the same for all APs. Multiply both sides of (9) by $\rho_{i\mathcal{J}_i^*}$ and sum it over i. In the following, let i correspond to the user index, whereas j represents the index for the APs. For clarity, we only give the index of the summations omitting their lower and upper bounds.

$$\sum_i \alpha n_{\mathcal{J}_i} \rho_{i\mathcal{J}_i^*} + \beta l_{\mathcal{J}_i} \rho_{i\mathcal{J}_i^*} < \sum_i \alpha((n_{\mathcal{J}_i^*} + 1)\rho_{i\mathcal{J}_i^*}) + \sum_i \beta(l_{\mathcal{J}_i^*} \rho_{i\mathcal{J}_i^*} + \rho_{i\mathcal{J}_i^*}^2). \tag{10}$$

We can re-write the summation terms in (10) with respect to index j, i.e., with respect to APs, by observing the following equalities.

$$\sum_i n_{\mathcal{J}_i} \rho_{i\mathcal{J}_i^*} = \sum_j \sum_{i \in A_j^*} n_j \rho_{ij} = \sum_j n_j l_j^*,$$
$$\sum_i l_{\mathcal{J}_i} \rho_{i\mathcal{J}_i^*} = \sum_j l_j l_j^*,$$
$$\sum_i n_{\mathcal{J}_i^*} \rho_{i\mathcal{J}_i^*} = \sum_j n_j^* l_j^*,$$
$$\sum_i l_{\mathcal{J}_i^*} \rho_{i\mathcal{J}_i^*} = \sum_j (l_j^*)^2,$$

where n_j^* and l_j^* are the number of users associated with AP j, and the load of AP j at the optimal solution. Therefore, it can be shown that the following holds,

$$\sum_j (\alpha n_j + \beta l_j) l_j^* < \sum_j (\alpha n_j^* + \beta l_j^*) l_j^* + \left(\alpha + \beta \sum_j l_j^*\right) \sum_j l_j^*, \tag{11}$$

by also observing that $\sum_i \rho_{i\mathcal{J}_i^*}^2 \leq \left(\sum_i \rho_{i\mathcal{J}_i^*}\right)^2$. Let us move the first term on the right hand side (RHS) to the left hand side (LHS) of (11),

$$\sum_j \left[(\alpha n_j + \beta l_j) - (\alpha n_j^* + \beta l_j^*)\right] l_j^* < \left(\alpha + \beta \sum_j l_j^*\right) \sum_j l_j^*.$$

Note that the LHS can be lower bounded by $\min_j\{l_j^*\} \sum_j (\alpha n_j + \beta l_j) - (\alpha n_j^* + \beta l_j^*)$, but $\sum_j (\alpha n_j + \beta l_j) - (\alpha n_j^* + \beta l_j^*) = \tau_{nash} - \tau_{opt}$, i.e., it is the difference between the total airtime cost with Nash and optimal solutions. Also, $\min_j\{l_j^*\} > \min_{i,j}\{\rho_{ij}\}$. Meanwhile, on the RHS, $\sum_j l_j^* < \sum_i \max_j\{\rho_{ij}\}$, and $\alpha + \beta \sum_j l_j^* < \alpha \sum_j n_j^* + \beta \sum_j l_j^* = \tau_{opt}$. Combining all these, we have

$$[\tau_{nash} - \tau_{opt}] \min_{i,j}\{\rho_{ij}\} < \tau_{opt} \sum_i \max_j \{\rho_{ij}\},$$
$$\frac{\tau_{nash}}{\tau_{opt}} < \frac{\sum_i \max_j\{\rho_{ij}\}}{\min_{i,j}\{\rho_{ij}\}} + 1. \tag{12}$$

■

For 802.11g networks, the price of anarchy is bounded as $9N + 1$, since the minimum and maximum transmission rates are 6Mbps and 54Mbps, respectively.

B. Minimum User Airtime Cost

Now, we consider the minimization of user airtime cost as the relevant social objective. For a given user association realization, $\mathbf{A} = (A_1, A_2, \ldots, A_S)$, the total airtime cost of the users in the network, τ', is

$$\tau' = \sum_{j=1}^{S} n_j(\alpha_j n_j + \beta_j l_j).$$

The following is a simple fact that will be used in the proof of next theorem.

Lemma 1: For every pair of nonnegative integers a, b, it holds

$$a(b+1) \leq \frac{2}{3}a^2 + b^2 + \frac{1}{3}.$$

Theorem 3: With minimum user airtime cost defined as the social cost, the price of anarchy is $O(S^2/N^2+S)$. Formally, the ratio of the worst cost of Nash solution and the optimal solution is

$$3+\frac{\alpha}{\alpha+\beta\check{\rho}}\frac{S^2}{N^2}+\frac{3\beta\hat{\rho}}{\alpha+\beta\check{\rho}}S,$$

where $\check{\rho}=\min_{i,k}\{\rho_{ik}\}$, and $\hat{\rho}=\max_{i,k}\{\rho_{ik}\}$.

Proof Let $\mathcal{J}_i$ be the strategy of STA i under Nash policy, whereas $\mathcal{J}_i^*$ be the strategy of the same STA under the optimal policy. Recall that the airtime cost of user i associating with AP j is $f_i(A_j)=\alpha n_j+\beta l_j$. Since $\mathcal{J}_i$ is Nash, the following holds

$$\alpha n_{\mathcal{J}_i}+\beta l_{\mathcal{J}_i}<\alpha(n_{\mathcal{J}_i^*}+1)+\beta(l_{\mathcal{J}_i^*}+\rho_{i\mathcal{J}_i^*}). \tag{13}$$

Observe that the following statements also hold,

$$\begin{aligned}\sum_i n_{\mathcal{J}_i}&=\sum_j n_j^2,\\ \sum_i n_{\mathcal{J}_i^*}&=\sum_j\sum_{i\in A_j^*}n_j=\sum_j n_jn_j^*,\\ \sum_i l_{\mathcal{J}_i}&=\sum_j\sum_{i\in A_j}l_j=\sum_j n_jl_j,\\ \sum_i l_{\mathcal{J}_i^*}&=\sum_j\sum_{i\in A_j^*}l_j=\sum_j n_j^*l_j,\\ \sum_i \rho_{i\mathcal{J}_i^*}&=\sum_j\sum_{i\in A_j^*}\rho_{ij}=\sum_j l_j^*.\end{aligned}$$

Taking the summation of both sides of (13) with respect to i, and using the statements given above we obtain

$$\alpha\sum_j n_j^2+\beta\sum_j n_jl_j<\alpha\sum_j n_jn_j^*+\alpha N+\beta\sum_j n_j^*l_j+\beta\sum_j l_j^*. \tag{14}$$

Also observe that since a STA can associate with only a single AP, $\sum_j n_j=\sum_j n_j^*=N$. Thus, we can rewrite (14) as

$$\alpha\sum_j n_j^2+\beta\sum_j n_jl_j<\alpha\sum_j(n_j^*+1)n_j+\beta\sum_j(n_j^*l_j+l_j^*). \tag{15}$$

By using Lemma 1, and since $\sum_j l_j^*\le\sum_j n_j^*l_j^*$, we can rewrite (15) as

$$\begin{aligned}\alpha\sum_j n_j^2+\beta\sum_j n_jl_j<&\sum_j n_j^*(\alpha n_j^*+\beta l_j^*)+\frac{2\alpha}{3}\sum_j n_j^2\\ &+\frac{\alpha}{3}S+\beta\sum_j n_j^*l_j.\end{aligned} \tag{16}$$

The total cost of Nash policy is $\tau_{nash}=\sum_j n_j(\alpha n_j+\beta l_j)$, whereas the total cost of the optimal policy is $\tau_{opt}=\sum_j n_j^*(\alpha n_j^*+\beta l_j^*)$. Therefore, the price of anarchy is

$$\begin{aligned}\frac{\tau_{nash}}{\tau_{opt}}&<1+\frac{2}{3}\frac{\tau_{nash}}{\tau_{opt}}+\frac{\alpha S/3+\beta\sum_j n_j^*l_j-2\beta/3\sum_j n_jl_j}{\tau_{opt}},\\ &<3+3\frac{\alpha S/3+\beta\sum_j n_j^*l_j-2\beta/3\sum_j n_jl_j}{\tau_{opt}}.\end{aligned} \tag{17}$$

Note that l_j satisfies $n_j\check{\rho}\le l_j\le n_j\hat{\rho}$, where $\check{\rho}=\min_{i,k}\{\rho_{ik}\}$, and $\hat{\rho}=\max_{i,k}\{\rho_{ik}\}$. Thus,

$$\beta\sum_j n_j^*l_j-2\beta/3\sum_j n_jl_j\le\beta\sum_j n_j(n_j^*\hat{\rho}-2/3n_j\check{\rho}). \tag{18}$$

Also note that $n_j^*\hat{\rho}-2/3n_j\check{\rho}\le N\hat{\rho}$, and thus, LHS of (18) can be bounded as

$$\beta\sum_j n_j^*l_j-2\beta/3\sum_j n_jl_j\le\beta\sum_j n_jN\hat{\rho}=\beta N^2\hat{\rho}. \tag{19}$$

Now, we derive a lower bound for the optimal cost by observing that

$$\tau_{opt}\ge\sum_j n_j^*(\alpha n_j^*+\beta n_j^*\check{\rho})=\sum_j\left(n_j^*\right)^2(\alpha+\beta\check{\rho}).$$

Also, $\sum_j\left(n_j^*\right)^2\ge\frac{\left(\sum_j n_j^*\right)^2}{S}$ holds, since $\sum_k\sum_j\left(n_j^*\right)^2=S\sum_j\left(n_j^*\right)^2$, $\left(\sum_j n_j^*\right)^2=\sum_k\sum_j n_k^*n_j^*$, and

$$\begin{aligned}\sum_k\sum_j\left(n_j^*\right)^2-\sum_k\sum_j n_k^*n_j^*&=\sum_j n_j^*\sum_k(n_j^*-n_k^*)\\ &=\sum_{j=1}^{S-1}\sum_{k=j+1}^{S}(n_j-n_k)^2\ge 0.\end{aligned}$$

Therefore,

$$\tau_{opt}\ge\frac{N^2}{S}(\alpha+\beta\check{\rho}), \tag{20}$$

Combining (17)-(20), we obtain the desired result

$$\frac{\tau_{nash}}{\tau_{opt}}<3+\frac{\alpha}{\alpha+\beta\check{\rho}}\frac{S^2}{N^2}+\frac{3\beta\hat{\rho}}{\alpha+\beta\check{\rho}}S.$$

■

C. Minmax Airtime Cost

We now turn our attention to the case where the social cost is the maximum airtime cost of the users in the network.

Theorem 4: The price of anarchy when the social objective is to minimize the maximum airtime cost is $O(\sqrt{S^2/N+NS})$.

Proof In the proof, we will use Theorem 3, which bounds the average user cost. Let $\mathbf{A}$ and $\mathbf{P}$ be the Nash equilibrium and optimal policies, respectively. Also let $\mathcal{J}_i$ be the AP user i associates with under policy $\mathbf{A}$, and $\mathcal{J}_i^*$ be the AP user i associates with under policy $\mathbf{P}$. Without loss of generality, the first STA has the maximum cost, i.e., $\arg\max_s C_s(A)=\mathcal{J}_1$. It suffices to bound $C_{\mathcal{J}_1}(A)$ with respect to $\widehat{P}=\max_s C_s(P)$.

Since $\mathbf{A}$ is Nash equilibrium, the cost of associating with $\mathcal{J}_1$ should be lower than the cost of associating with $\mathcal{J}_1^*$. Thus,

$$\begin{aligned}C_{\mathcal{J}_1}(A)&<\alpha(n_{\mathcal{J}_1^*}+1)+\beta(l_{\mathcal{J}_1^*}+\rho_{1\mathcal{J}_1^*})\\ &\le\alpha(n_{\mathcal{J}_1^*}+1)+\beta(n_{\mathcal{J}_1^*}\hat{\rho}+\hat{\rho}),\\ &=(\alpha+\beta\hat{\rho})(n_{\mathcal{J}_1^*}+1).\end{aligned} \tag{21}$$

Let $I\subset N$ be the subset of STAs under policy $\mathbf{A}$ using AP $\mathcal{J}_1^*$. The sum of their costs is

$$\begin{aligned}\sum_{i\in I}C_{\mathcal{J}_1^*}(A)&\ge n_{\mathcal{J}_1^*}(\alpha n_{\mathcal{J}_1^*}+\beta l_{\mathcal{J}_1^*})\ge n_{\mathcal{J}_1^*}(\alpha n_{\mathcal{J}_1^*}+\beta n_{\mathcal{J}_1^*}\check{\rho}),\\ &=n_{\mathcal{J}_1^*}^2(\alpha+\beta\check{\rho}).\end{aligned} \tag{22}$$

On the other hand, by Theorem 3,

$$\sum_{i=1}^{N} C_{\mathcal{J}_i}(A) \le \left(3 + \frac{\alpha}{\alpha+\beta\check{\rho}}\frac{S^2}{N^2} + \frac{3\beta\hat{\rho}}{\alpha+\beta\check{\rho}}S\right)\sum_{i=1}^{N} C_{\mathcal{J}_i^*}(P),$$
$$= B\sum_{i=1}^{N} C_{\mathcal{J}_i^*}(P), \quad (23)$$

where $B = 3 + \frac{\alpha}{\alpha+\beta\check{\rho}}\frac{S^2}{N^2} + \frac{3\beta\hat{\rho}}{\alpha+\beta\check{\rho}}S$. Combining (22) and (23), we have

$$(\alpha+\beta\check{\rho})n^2_{\mathcal{J}_1^*} \le \sum_{i\in I} C_{\mathcal{J}_1^*}(A) \le \sum_{i=1}^{N} C_{\mathcal{J}_i}(A) \le B\sum_{i=1}^{N} C_{\mathcal{J}_i^*}(P)$$

Together with (21), we get

$$C_{\mathcal{J}_1}(A) < (\alpha+\beta\hat{\rho})\left[\sqrt{\frac{B}{\alpha+\beta\check{\rho}}}N\sqrt{\widehat{P}}+1\right].$$

Inserting the value of B, we get the desired result. ∎

D. Lexicographical Optimum is a Nash Equilibrium

Consider the following minmax system optimization problem.

$$\min_{\{A_s\}_{s=1}^{S}} \max_{s} C_s \quad (24)$$
$$\exists s, \text{s.t. } n \in A_s, \text{while } n \ni A_{s'}, \text{for } s' \neq s, \forall n. \quad (25)$$

A lexicographically optimal association configuration is the one which minimizes the airtime cost of the AP with the maximum airtime cost, subject to this minimization, it minimizes the second maximum, etc. Thus, we repeatedly solve the minmax optimization problem in (24) over the remaining APs and STAs by removing the AP and its associated STAs determined by the optimum solution at each iteration.

Theorem 5: The solution of the lexicographical optimization problem is a Nash Equilibrium solution for the user association game.

Proof Let us assume without loss of generality that at the optimal solution the airtime costs of the APs are ordered as

$$C_1^* > C_2^* > \ldots > C_S^*, \quad (26)$$

for user association configuration $\mathbf{A}^*$.

If STA $i \in A_s^*$ is not at Nash equilibrium, then there are certain such configurations where i may move from s to $s' > s$ such that

$$C_{s'}(A_{s'}^* \cup \{i\}) < C_s(A_s^*).$$

Now, if $C_s(A_s^* - \{i\}) < C_{s'}(A_{s'}^* \cup \{i\})$, then the ordering in (26) has changed. Note that $C_s(A_s^* - \{i\}) < C_s(A_s^*)$ since the airtime cost increases with the number of users associated. Also note that $C_s(A_s^*) > C_{s'}(A_{s'}^*)$ at the optimum solution. However, this contradicts with the fact that $\mathbf{A}^*$ is lexicographically optimum. On the other hand, if $C_s(A_s^* - \{i\}) \ge C_{s'}(A_{s'}^* \cup \{i\})$, then the ordering in (26) has not changed but this still contradicts with the lexicographical optimality of $\mathbf{A}^*$, since $C_s(A_s^*)$ was not at its minimum in the first place. ∎

VII. Conclusions

In this paper, we started from a practical problem observed in wireless networks, and analyzed it using novel game theoretical models. Efficient association of users with the available access points is required to balance the loads of the access points in the network, and thus, to prevent congestion. A new promising greedy user association scheme is considered for this purpose, where each user is associated with the AP that can provide the lowest per packet latency. The algorithm is analyzed theoretically by modeling it as a form of an atomic congestion game. The results available in the literature on congestion games are not sufficient for analyzing this specific form of user association game, since in our case, the congestion of a resource depends on the number and types of users associated. Therefore, we first showed that the user association game achieves equilibrium in finite time, and then, we derived new results on price of anarchy for this more general version of atomic congestion games. Our results indicate that the price of anarchy of load-based user association is high, and in many cases depend on the number of users or the number of resources in the network. However, an encouraging result is that a fair solution for all users (lexicographic optimum) is also an equilibrium solution. Further studies are needed both to improve the given bounds, and to design mechanisms that can drive the system to more favorable equilibriums. The design of such mechanisms is a difficult problem to solve in congestion games, but a first initial attempt is made for unsplittable selfish routing problem in [25]. This work can be taken as a starting point for further research on this topic.

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