Access Point Selection Game for Mobile Wireless Users

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Abstract—Selecting a Access Point (AP) is an important task for a mobile wireless user to achieve the best possible quality of service. We consider AP selection as a game where players make choices selfishly and try to select the closest AP based on Minimum Path-Loss (MPL) criteria.

We formulate the AP selection problem as a game where players are mobile wireless users and they choose radio APs to connect to the network. We define a new parameter called Access Point Selection Parameter (APSP) based on Signal to Interference plus Noise Ratio (SINR). Each selfish user chooses an AP which maximizes its APSP value. This APSP value depends on both the distance to AP and the total number of connected users in AP. Furthermore, we extend two players game to n-players game by adopting the KP (Koutsoupias-Papadimitriou) model of parallel links. The performance of the proposed game is illustrated by using simulations.

I. INTRODUCTION

A wireless network consists of a large number of Access Points (APs) and an AP covers a certain geographical area to provide the wireless connectivity for users. Usually, these APs have overlapped coverage areas. The overlapping is required to handover the connection to other APs to provide seamless mobility. On the other hand, some wireless networks use multiple APs to provide the radio access service for a certain area in order to support large number of mobile users.

In these scenarios, a user may detect signals from multiple APs and a AP selection method is needed to prioritize the best one amongst them. In general, mobile users make choices selfishly and try to maximize their own Signal-to-Noise Ratio (SNR). Selecting the closest AP based on Minimum Path-Loss (MPL) criteria is not always the best possible strategy, as other players may already consumed the network resources of that AP. Recent research efforts on wireless communication try to address this AP selection problem. A proper AP selection decision helps to achieve various goals: to increase the throughput of users [1] [2] [3] [4] [5], to minimize the interference among users [6] [7] [8] and to minimizes the AP selection cost [9].

Furthermore, the most popular approach to enhance the system performance is proposing novel matrices to steer the selection decision and analyze the performance based on a game theory model [1] [2] [7] [10]. The game theory framework is well suited in this context since we have to allocate a limited amount of resources among a large number of users.

On the other hand, game theory framework improves the resilience of the access point selection process. For instance, there are open source smart phones where hackers could modify software to gain more resources and not follow the common protocol. In such cases, game-theoretic model is needed.

However, most of the AP selection congestion games were assumed that all users have an equal impact on the congestion. Therefore, all that matters is the total number of users associated with a particular AP. This however is not always true in wireless communication. The distance profiles of each connected user is also steering the performance of congestion game. For instance, the received signal strength and the interference from other users depend on the distance to AP. Thus, both the path loss and interference have to be considered in a AP selection procedure.

- Our Contribution

In this paper, we formulate the AP selection problem as a game where players are mobile wireless users and they choose radio APs to connect to the network. Strategies in the game are AP numbers or probabilities, which players use to choose APs. We propose a novel AP selection parameter called Access Point Selection Parameter (APSP). Each selfish user chooses an AP which maximizes the player's APSP value. The APSP value of a user depends on: 1) the distance between the player and the AP, 2) the total number of connected players to the selected AP. In contrast to the SINR, the novel APSP treats the user's own signal as interference to itself and it ultimately increases the performance of the game. Thus, we consider the total interference is a sum of all signals at the AP. Some constant noise (white noise) is also considered. Furthermore, the received signal strength is inversely propisitional to the square of the distance to the chosen AP. Thus, the proposed APSP includes both path loss and interference in the AP selection decision. We prove the Nash Equilibrium (NE) in this model with and without users knowing each other's location. Furthermore, we determine the NE in both pure and mixed strategies for each case. Then, we present a procedure to extend two players game to n-players game by adopting the KP model [11] of parallel links. Finally, we prove the performance advantage of the proposed game in various scenarios by using extended simulations.

The rest of the paper is organized as follows. In Section II, we formulate the problem for two players. Section III presents the adaptation of KP model to the AP selection game. Section IV contains the simulation results and Section V concludes the paper.

II. GAME-THEORETIC MODEL

In the proposed AP selection game, users are responsible to select a AP based on the information received from each AP. Hence, it is a distributed AP selection mechanism.

A. One Player Game

Consider one-dimensional model with one player moving on interval $[0,1]$. Two identical APs ("0" and "1") are situated at the corresponding ends of the interval. The white noise level is represented by c. At a particular moment player's coordinate is $x \in [0,1]$. Here, we assume the path loss exponent as 2 by considering free space communications.

APSP of the player can be calculated as follows.

$$
APSP = \begin{cases} \frac{1/x^2}{1/x^2+c} = \frac{1}{cx^2+1} & \text{if user connects to the AP 0,} \\ \frac{1}{1+c(1-x)^2} & \text{if user connects to the AP 1,} \end{cases} \tag{1}
$$

$$
APSP = \begin{cases} \frac{1/x^2}{1/x^2+c} = \frac{1}{cx^2+1} & \text{if user connects to the AP 0,} \\ \frac{1}{1+c(1-x)^2} & \text{if user connects to the AP 1,} \end{cases} \tag{2}
$$

An obvious solution is choosing AP 0 if $x \leq \frac{1}{2}$ and 1 otherwise, since $\frac{1}{cx^2+1} \geq \frac{1}{1+c(1-x)^2}$ if $x \leq \frac{1}{2}$.

B. Two Players Game

Consider a one-dimensional game with two players (0 and 1) moving on interval $[0,1]$. Two identical APs ("0" and "1") are situated at the corresponding ends of the interval. The white noise level is represented by c. At each time moment player 0's coordinate is some $x \in [0,1]$ and player 1's is some $y \in [0,1]$. Each player i in each situation (x,y) must determine the best AP to connect.
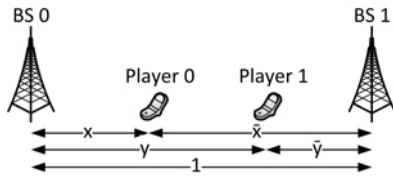

Fig. 1: The System Model

Let's consider that pure strategy for each player i is the l_i radio AP. Then (l_0, l_1) is the pure strategy profile. Furthermore, following mixed strategies are defined: p is a probability that player 0 chooses the AP 0, q is a probability that player 1 chooses the AP 0. For AP 1 probabilities are correspondingly $1-p$ and $1-q$. Thus, (p,q) defines the mixed strategy profile. Each pure strategy profile corresponds to a mixed profile, since

$p = P(l_0 = 0)$, $q = P(l_1 = 0)$. The payoff function for each player is an expectation of its APSP at the AP.

Pure APSP for the i-th player connecting to j-th AP is

$$
APSP_i^j = \begin{cases} \frac{\frac{1}{(\rho_i^j)^2}}{\sum_{k=0}^1 \frac{1}{(\rho_k^j)^2}+c} & \text{if } l_0 = l_1 = j, \\ \frac{\frac{1}{(\rho_i^j)^2}}{\frac{1}{(\rho_i^j)^2}+c} & \text{otherwise,} \end{cases} \tag{3}
$$

where ρ_i^j is the distance between player i and AP j. We simplify it and define in areas of ambiguity as follow:

$$
APSP_i^j = \begin{cases} \frac{1}{2} & \text{if } l_0 = l_1 = j \text{ and} \\ & \rho_i^j = \rho_{1-i}^j = 0, \\ \frac{1}{\frac{(\rho_i^j)^2}{(\rho_{1-i}^j)^2}+1+c(\rho_i^j)^2} & \text{if } l_0 = l_1 = j \text{ and} \\ & \rho_i^j + \rho_{1-i}^j > 0, \\ \frac{1}{1+c(\rho_i^j)^2} & \text{otherwise.} \end{cases} \tag{4}
$$

Here, we assume that values $u, v \in [0,1]$ $\frac{u}{v}|_{u=v=0} \overset{def}{=} 1$ and $\frac{1}{\frac{u^2}{v^2}+1+cu^2}|_{u=v=0} \overset{def}{=} \frac{1}{2}$. Also, we denote $\bar{u} \overset{def}{=} 1-u$. Furthermore, ρ_{1-i} represents the distance between other player (not player i) and AP j.

The two player model can be categorized into two games based on the information availability at the user.

1) Bimatrix Game with Full Information: In the model with full information, each player knows the location of its opponent. Each AP sends this information to each player. Here, we consider the game as a bimatrix model. The payoff matrix is following: A and B represent the payoffs of the player 0 and 1 respectively.

$$
(A,B) = \begin{pmatrix} \frac{1}{1+\frac{x^2}{y^2}+cx^2}, \frac{1}{1+\frac{y^2}{x^2}+cy^2} & \frac{1}{1+cx^2}, \frac{1}{1+c\bar{y}^2} \\ \frac{1}{1+c\bar{x}^2}, \frac{1}{1+cy^2} & \frac{1}{1+\frac{\bar{x}^2}{\bar{y}^2}+c\bar{x}^2}, \frac{1}{1+\frac{\bar{y}^2}{\bar{x}^2}+c\bar{y}^2} \end{pmatrix} \tag{5}
$$

and payoff functions for each player are:

$$
H^0_{x,y}(p,q) = \frac{pq}{1+\frac{x^2}{y^2}+cx^2} + \frac{p(1-q)}{1+cx^2} + \frac{(1-p)q}{1+c\bar{x}^2} + \dots
$$
$$
\dots + \frac{(1-p)(1-q)}{1+\frac{\bar{x}^2}{\bar{y}^2}+c\bar{x}^2}
$$

$$
H^1_{x,y}(p,q) = \frac{pq}{1+\frac{y^2}{x^2}+cy^2} + \frac{p(1-q)}{1+c\bar{y}^2} + \frac{(1-p)q}{1+cy^2} + \dots
$$
$$
\dots + \frac{(1-p)(1-q)}{1+\frac{\bar{y}^2}{\bar{x}^2}+c\bar{y}^2}
$$

• Pure Strategies

For each player's position (x,y) pure equilibrium is such choice of APs' numbers (l_0^*, l_1^*) that $a_{il_1^*} \leq a_{l_0^* l_1^*}$ and $b_{l_0^* j} \leq b_{l_0^* l_1^*}$. Here, a_{ij} and b_{ij} represent the corresponding payoffs in the payoff matrix of player 0 and 1 respectively.

(l_0^*, l_1^*)	is equilibrium at (x, y) such that
$(0,0)$	$c(1-2x) \geq \frac{x^2}{y^2}$ and $c(1-2y) \geq \frac{y^2}{x^2}$
$(0,1)$	$c(2x-1) \leq \frac{\bar{x}^2}{\bar{y}^2}$ and $c(1-2y) \leq \frac{y^2}{x^2}$
$(1,0)$	$c(1-2x) \leq \frac{x^2}{y^2}$ and $c(2y-1) \leq \frac{\bar{y}^2}{\bar{x}^2}$
$(1,1)$	$c(2x-1) \geq \frac{\bar{x}^2}{\bar{y}^2}$ and $c(2y-1) \geq \frac{\bar{y}^2}{\bar{x}^2}$

Each position on a square $[0,1] \times [0,1]$ has some pure equilibrium which is illustrated in Figure 2a and 2b. When $c \leq 1$, the couple of conditions for equilibrium $(0,0)$ provide only one feasible position $x = y = 0$ where this pure equilibrium exists. Indeed, curves $c(1 - 2x) = \frac{x^2}{y^2}$ and $c(1 - 2y) = \frac{y^2}{x^2}$ have only one intersection point on $[0,1] \times [0,1]$ when $c \leq 1$. However, they intersect at two points on $[0,1] \times [0,1]$ when $c > 1$. Therefore, this equilibrium can be achieved in some area for this case. Similarly, there are curves bounding for $(1,1)$-equilibrium existing area as well. When $c \leq 1$, this equilibrium exists only at position $x = y = 1$. Figure 2a and 2b illustrate this fact.
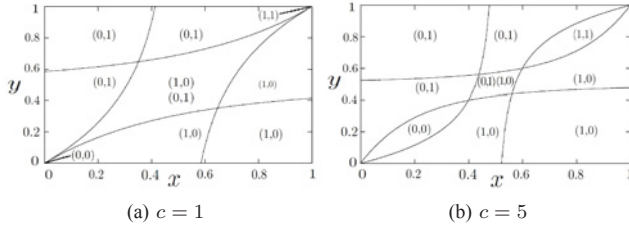

(a) $c = 1$ (b) $c = 5$

Fig. 2: Pure equilibria in bimatrix game with full info

- Mixed Strategies

Mixed strategies can be calculated by using the payoff matrix (Equation 5) of users and they are stated as follows,

$$p = \frac{b_{11} - b_{10}}{b_{00} + b_{11} - b_{01} - b_{10}} \tag{6}$$

$$q = \frac{a_{11} - a_{01}}{a_{00} + a_{11} - a_{01} - a_{10}} \tag{7}$$

Figures 3a,3b and 3c demonstrate players' mixed strategies depending on player 0's position x when the position of player 1 is fixed, i.e. $y = y_0$.

The arrow ends in each figure show bounds of interval $[x_l, x_r]$ for x where the mixed equilibria exists. An interesting observation is that each player in mixed equilibrium uses the most distant AP and lets its opponent to use the nearest AP.

2) Bimatrix Model with Incomplete Information: In the model with incomplete information, each player does not know the location of its opponent. Thus, each player assumes that the location of its opponent is uniformly distributed on the range $[0,1]$. Based on this assumption, each player generates his own expected payoff matrix specified by the player's choices. The player 0's expected payoffs matrix is calculated as follows.

$$(A', B') =$$

$$\begin{pmatrix} \int_0^1 \frac{1}{1+\frac{x^2}{y^2}+cx^2}dy, \int_0^1 \frac{1}{1+\frac{y^2}{x^2}+cy^2}dy & \int_0^1 \frac{1}{1+cx^2}dy, \int_0^1 \frac{1}{1+c\bar{y}^2}dy \\ \int_0^1 \frac{1}{1+c\bar{x}^2}dy, \int_0^1 \frac{1}{1+cy^2}dy & \int_0^1 \frac{1}{1+\frac{\bar{x}^2}{\bar{y}^2}+c\bar{x}^2}dy, \int_0^1 \frac{1}{1+\frac{\bar{y}^2}{\bar{x}^2}+c\bar{y}^2}dy \end{pmatrix}$$

or in simplified form:

$$A' = \begin{pmatrix} \frac{\sqrt{cx^2+1}-x\arctan\frac{\sqrt{cx^2+1}}{x}}{\sqrt{cx^2+1}(1+cx^2)} & \frac{1}{1+cx^2} \\ \frac{1}{1+c\bar{x}^2} & \frac{\sqrt{c\bar{x}^2+1}-\bar{x}\arctan\frac{\sqrt{c\bar{x}^2+1}}{\bar{x}}}{\sqrt{c\bar{x}^2+1}(c\bar{x}^2+1)} \end{pmatrix},$$

$$B' = \begin{pmatrix} \frac{x\arctan\frac{\sqrt{cx^2+1}}{x}}{\sqrt{cx^2+1}} & \frac{\arctan\sqrt{c}}{\sqrt{c}} \\ \frac{\arctan\sqrt{c}}{\sqrt{c}} & \frac{\bar{x}\arctan\frac{\sqrt{c\bar{x}^2+1}}{\bar{x}}}{\sqrt{c\bar{x}^2+1}} \end{pmatrix}.$$

Similar matrix (A'', B'') can be obtained for the player 1 by integrating (A, B) (Equation 5) with respect to x.

$$A'' = \begin{pmatrix} \frac{y\arctan\frac{\sqrt{cy^2+1}}{x}}{\sqrt{cy^2+1}} & \frac{\arctan\sqrt{c}}{\sqrt{c}} \\ \frac{\arctan\sqrt{c}}{\sqrt{c}} & \frac{\bar{y}\arctan\frac{\sqrt{c\bar{y}^2+1}}{\bar{y}}}{\sqrt{c\bar{y}^2+1}} \end{pmatrix},$$

$$B'' = \begin{pmatrix} \frac{\sqrt{cy^2+1}-y\arctan\frac{\sqrt{cy^2+1}}{y}}{\sqrt{cy^2+1}(1+cy^2)} & \frac{1}{1+c\bar{y}^2} \\ \frac{1}{1+cy^2} & \frac{\sqrt{c\bar{y}^2+1}-\bar{y}\arctan\frac{\sqrt{c\bar{y}^2+1}}{\bar{y}}}{\sqrt{c\bar{y}^2+1}(c\bar{y}^2+1)} \end{pmatrix}.$$

- Pure Strategies

Player 0 obtains expected equilibrium profile (l_0', l_1') such that $a'_{il_1'} \leq a'_{l_0'l_1'}$ and $b'_{l_0'j} \leq b'_{l_0'l_1'}$.

The function $\frac{\arctan x}{x}$ is monotonically decreasing for $x \geq 0$. Furthermore, $\frac{\sqrt{cx^2+1}}{x} = \sqrt{c + \frac{1}{x^2}} > \sqrt{c}$. Thus, $b'_{01} = b'_{10} = a''_{01} = a''_{10}$ are the maximal elements in B' and A''. Therefore, each player thinks that choosing the different AP will be the opponent's best answer on any of its choices. Each expects two possible types of equilibria: player 0 expects $(0,1)$ for $a'_{01} \geq a'_{11}$ and $(1,0)$ for $a'_{10} \geq a'_{00}$. Similarly, player 1 expects $(0,1)$ for $b''_{01} \geq b''_{00}$ and $(1,0)$ for $b''_{10} \geq b''_{11}$.

Let x_l be such that $a'_{01}(x_l) = a'_{11}(x_l)$, x_r such that $a'_{10}(x_r) = a'_{00}(x_r)$, y_l such that $b''_{10}(y_l) = b''_{11}(y_l)$ and y_h such that $b''_{01}(y_h) = b''_{00}(y_h)$. Numeric results show that for player 0 the first equilibria exist at $0 \leq x \leq x_r$ and second at $x_l \leq x \leq 1$ where $x_l < \frac{1}{2} < x_r$ are symmetric with respect to $\frac{1}{2}$ and tend to it with increasing of c. For player 1 first equilibria exist at $y_l \leq y \leq 1$ and second at $0 \leq y \leq y_h$ where $y_l < \frac{1}{2} < y_h$ are symmetric with respect to $\frac{1}{2}$ and tend to it with the increment of c.

Each player acts according to its mind about expected equilibrium and they achieve real equilibrium situation. The equilibria for different squares are presented in Figure 4a.

Let's consider $[0, x_l] \times [0, y_l]$ square. Here, $a'_{01} \geq a'_{11}$, $a'_{10} \leq a'_{00}$, $b''_{10} \geq b''_{11}$, $b''_{01} \leq b''_{00}$. Player 0 expects $(0,1)$, so it chooses AP 0. Player 1 expects $(1,0)$ and chooses AP 0. Thus, $(0,0)$ is the equilibria for $[0, x_l] \times [0, y_l]$ square. Furthermore, player 0 expects a'_{01}. However, it obtains $a'_{00} > a'_{10}$ and there is no reason to switch to AP 1. Similarly, player 1 expects b''_{10}. However, it obtains $b''_{00} \geq b''_{01}$ and so it has no reason to switch to AP 1.

(a) $c = 1$ and $y_0 = 0.3$ (b) $c = 1$ and $y_0 = 0.5$ (c) $c = 1$ and $y_0 = 0.7$
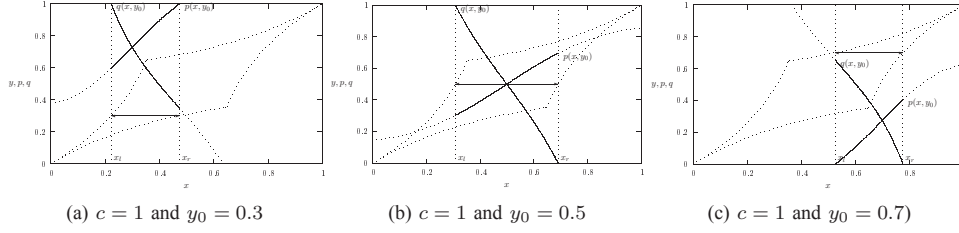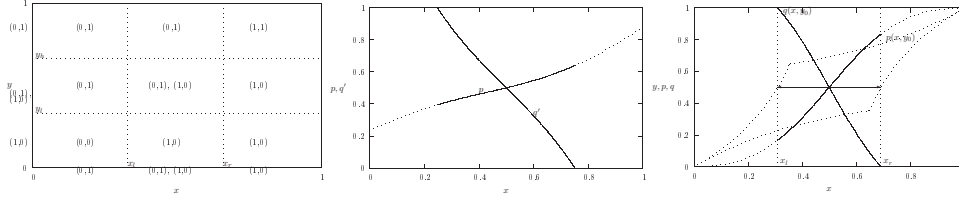
Fig. 3: Mixed equilibria in bimatrix game with full info

(a) Pure equilibria in bimatrix game with incomplete info   (b) Mixed equilibria in bimatrix game with incomplete info for $c = 1$ (c) Mixed equilibria in KP-like model for $c = 1$ and $y_0 = 0.5$)

Fig. 4: Bimatrix game with incomplete info and KP-like model

Let's consider $[x_l, x_r] \times [0, y_l]$ square. Here, $a'_{01} \geq a'_{11}$, $a'_{10} \geq a'_{00}$, $b''_{10} \geq b''_{11}$, $b''_{01} \leq b''_{00}$. Player 0 expects $(0, 1)$ or $(1, 0)$. Player 1 expects $(1, 0)$, so it chooses AP 0. If player 0 chooses AP 1 according to $(1, 0)$ players achieve equilibrium. If player 0 chooses AP 0 according to $(0, 1)$, it obtains $a'_{00} \leq a'_{10}$ instead of a'_{01}. Therefore, player 0 switches to AP 1.

Similarly, we can define equilibria for all remaining squares.

- Mixed Strategies

Mixed strategies are feasible exactly in a square $[x_l, x_r] \times [y_l, y_h]$ (Figure 4a) where two types of pure equilibria exist for both players.

Mixed strategies for players are also defined according to their expectations. Player 0's mixed strategy is

$$p = \frac{b'_{11} - b'_{10}}{b'_{00} + b'_{11} - b'_{01} - b'_{10}} \tag{8}$$

It depends only on player 0's position and not on opponent's position. Furthermore, $0 < p < 1$ for all x on $[0, 1]$ since $b'_{11} < b'_{10}$ and $b'_{00} < b'_{01}$. However, q' is connected with expected player 1's strategy.

$$q' = \frac{a'_{11} - a'_{01}}{a'_{00} + a'_{11} - a'_{01} - a'_{10}} \tag{9}$$

It exists when $0 \leq q' \leq 1$, and it is the feasibility condition for mixed strategy of player 0.

Since $a'_{00} = a'_{01}(1 - b'_{00})$ and $a'_{11} = a'_{10}(1 - b'_{11})$,

$$q' = \frac{a'_{10}(1 - b'_{11}) - a'_{01}}{-a'_{10}b'_{11} - a'_{01}b'_{00}} \tag{10}$$

Thus, it is non-negative when
$a'_{10}(1 - b'_{11}) - a'_{01} = a'_{11} - a'_{01} \leq 0$
and not greater than 1 when

$a'_{10}(1 - b'_{11}) - a'_{01} + a'_{10}b'_{11} + a'_{01}b'_{00} = a'_{10} - a'_{01} + a'_{01}b'_{00} = a'_{10} - a'_{00} \geq 0$.

Figure 4b demonstrates player's 0 perception about expected opponent's strategy q' and its own mixed strategy p.

Similarly, player 1's mixed strategy is

$$q = \frac{a''_{11} - a''_{01}}{a''_{00} + a''_{11} - a''_{01} - a''_{10}} \tag{11}$$

It depends only on player 1's position and not on opponent's position. It is feasible when $b''_{01} \geq b''_{00}$ and $b''_{10} \geq b''_{11}$.

III. KP-LIKE MODEL

We investigate the possibility to extend two players game to n-players game by adopting the KP (Koutsoupias and Papadimitriou) model [11] of parallel links. Our AP selection problem can be presented as KP-like model [11] with n players and two parallel links only when full information is available for the players. Each player is situated at a position x_i. Players' pure strategy profile is $(l_1, \ldots, l_n)$ where l_i is number of AP that is chosen by player i and mixed profile is $(p_1, \ldots, p_n)$ where p_i is a probability that i chooses AP 0. APSP for the i-th player connecting to j-th AP is calculated as follows,

$$APSP_i^j = \frac{1}{\sum\limits_{k \neq i: l_k = j} \frac{(\rho_k^j)^2}{(\rho_i^j)^2} + 1 + c(\rho_i^j)^2} \tag{12}$$

We also define here that $APSP_i^j = \frac{1}{n}$ if $l_k = j$ and $\rho_k^j = 0$ for $k = 1, \ldots, n$.

In case of pure strategies, i-th player's payoff is $\Lambda_i = APSP_i^{l_i}$. Pure strategy profile $(l_1, \ldots, l_n)$ is a NE if for each player $i = 1, \ldots, n$ its $\Lambda_i = \max\limits_{j=0,1} APSP_i^j$.

In case of mixed strategies, i-th player's payoff on AP is

$$\Lambda_i^j = \frac{1}{\sum\limits_{k \neq i} \frac{p_k^j (\rho_i^j)^2}{(\rho_k^j)^2} + 1 + c(\rho_i^j)^2}, \qquad (13)$$

Here $p_k^0 = p_k$ and $p_k^1 = 1 - p_k$. $\Lambda_i = \max\limits_{j=0,1} \Lambda_i^j$. Mixed strategy profile $(p_1, \ldots, p_n)$ is a NE for each player i and each AP j: $\Lambda_i^j = \Lambda_i$ if $p_i^j > 0$ and $\Lambda_i^j < \Lambda_i$ if $p_i^j = 0$.

The pure equilibria in two-players KP-like model with players' coordinates x and y fully coincide with pure equilibria in the bimatrix model with full information. Here, mixed equilibrium probabilities are:

$$p = \frac{\bar{y}^2 x^2 - c\bar{x}^2 x^2 (2y - 1)}{\bar{x}^2 y^2 + x^2 \bar{y}^2} \qquad (14)$$

$$q = \frac{\bar{x}^2 y^2 - c\bar{y}^2 y^2 (2x - 1)}{\bar{x}^2 y^2 + x^2 \bar{y}^2} \qquad (15)$$

They are feasible in the same area as in the bimatrix model with full information and have the same property, that each player in mixed equilibrium uses the most distant AP (see Figure 4c).

IV. NUMERICAL RESULTS

We conduct extensive simulations on Matlab to study the performance of the proposed AP selection game model. The used simulation setting is as follow. We consider two APs (AP 0 and AP 1) which have identical transmitting power. The distance between two APs is one unit. One-dimensional model with two players (player 0 and player 1) who are moving between two APs independently is considered. We compare the performance of the proposed two players game models and KP-like model against the widely used Minimum Path-Loss (MPL) strategy where each player selects the AP with the strongest received signal. The simulations are conducted for both low white noise (c=1) case and high white noise (c=5) situations.

Here, we measure APSP of each player. Based on APSP definition (Equation 3), we can derive that

$$APSP = \frac{SINR}{1 + SINR} \qquad (16)$$

Thus, APSP is a deceleration power function of SINR for $SINR \geq 0$. Therefore, a player with higher APSP gets the higher SINR.

A. Low White Noise Situation

Figure 5 shows APSP values of player 0 under different strategies for low noise situation (c=1). The game is symmetric. Therefore, we discuss the performance of only one player, namely player 0. We change the position of each player independently between APs and measure the APSP value for each setting under different AP selection strategies.

We observed that MPL (Figure 5a) strategy has the worst performance. The two player model without information is the most realistic scenario for a real world mobile network. Notably, this case also has better performance (Fig. 5b) than
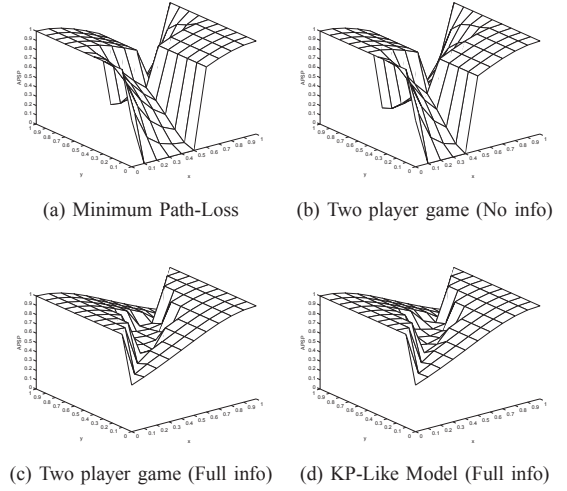

(a) Minimum Path-Loss   (b) Two player game (No info)

(c) Two player game (Full info)   (d) KP-Like Model (Full info)

Fig. 5: The APSP values of player 0 for low white noise situation (c=1).

the closest attachment strategy. Although the expected APSP is dropping to zero, the region with such performance is comparably smaller than MPL strategy.

When we assumed that players' positions are uniformly distributed between the two APs, average APSP for each strategy is presented in Table I. It indicates that the proposed two player game can achieve 11% increment over MPL strategy, even without having any information about the other player.

The advantage of using two player game is further increased with the availability of the full information about players. Statistically, the increment is over 18%. When the players have full information about other players, they can take intelligent decisions accordingly. Both two player model with info (Fig. 5c) and KP-like model (Fig. 5d) have similar performance. For both models, APSP did not lower beyond 0.5 for any setting.

There are two explanations for lower performance under MPL strategy. First, one player can be dominated by suppressing the performance of the other player. Generally, if the first player is very close to the AP than the second player and both connect to the same AP, second player's APSP will be very low. Meanwhile, second player may get better APSP by connecting to other AP although it is farer than the first AP. Furthermore, it helps to a proper utilization of network resources.

Second, the performance degrades for both players due to selfishness. For instance, if both players are close to the first AP rather than second AP, both players try to connect the first AP. However, both players can achieve better APSP, if one player chooses the second AP without being selfish.

We can see that all four strategies have identical performance regions. When players are closed to different AP, the attachment to the closest AP is the most effective choice. Thus, proposed game models are also converging to the MPL

strategy.

TABLE I: Average APSP values for player 0

	c=1	c=5
Minimum Path-Loss	0.7097	0.6140
Two Players model with full info.	0.8380	0.6634
Two Players model without info.	0.7874	0.6140
KP-Like Model with full info.	0.8461	0.6775

B. High White Noise Situation

Fig. 6 shows the APSP values of player 0 under different strategies for high noise situation (c=5).

(a) Minimum Path-Loss   (b) Two player game (No info)

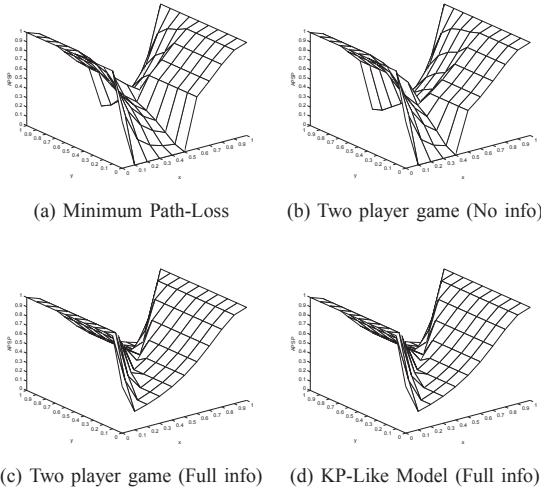(c) Two player game (Full info)   (d) KP-Like Model (Full info)

Fig. 6: The APSP values of player 0 for High White Noise Situation (c=5).

Here, we observed that MPL (Fig. 6a) strategy and two players model without info also have almost similar performance (Fig. 6b). The higher noise is dominating here. Therefore, the advantage that can be achieved by connecting to different AP is suppressed. As both strategies have similar APSP graphs, it is concluded that in the worst case our two player game model achieves at least the performance of the MPL strategy with the absent of the players information.

However, two players model and KP-like model are still able to enhance the performance with full information about players. Both two player models with info (Fig. 6c) and KP-Like model with info (Fig. 6d) have almost the same performance and in this case APSP performance increment is 9%. Dominant white noise has decreased the advantage here.

We can see that all four graphs have more identical performance regions than low noise situation. When the channel conditions are getting worse, game models are converging towards MPL strategy.

V. Conclusions

We have analyzed a two-player game where the users selfishly select one AP among the multiple APs to connect. Our main contributions are follows:

- We have modeled a congestion game by considering not only the number of users connected to the AP but also the distance to each user and the noise level
- We obtained Nash equilibrium for two-players game with and without knowledge of other player's location
- The player can improve its SINR up to 18% by using intelligent strategy (compared with common Minimum Path-Loss (MPL) strategy)
- In high noisy environments, the difference between strategies is smaller
- Unsurprisingly, knowing other player's location allows obtaining better SINR
- KP-model is used to extend the two player game with full info for n-players game.

In future work, we are planning to extend the model to cover n players and m access points located on a plane or three-dimensional coordinates. Furthermore, we will analyze the game by using different path loss exponents.

References

[1] M. Cesana, N. Gatti, and I. Malanchini, "Game Theoretic Analysis of Wireless Access Network Selection: Models, Inefficiency Bounds, and Algorithms," in *Proceedings of the 3rd International Conference on Performance Evaluation Methodologies and Tools*. ICST, 2008.
[2] L. Yen, J. Li, and C. Lin, "Stability and Fairness of AP Selection Games in IEEE 802.11 Access Networks," *IEEE Transactions on Vehicular Technology*, vol. 60, no. 3, pp. 1150–1160, 2011.
[3] F. Xu, C. Tan, Q. Li, G. Yan, and J. Wu, "Designing a Practical Access Point Association Protocol," in *Proc. of IEEE International Conference on Computer Communications : INFOCOM*. IEEE, 2010, pp. 1–9.
[4] M. Hong, A. Garcia, and J. Alviar, "Joint Distributed Access Point Selection and Power Allocation in Cognitive Radio Networks," in *Proceedings of IEEE International Conference on Computer Communications: INFOCOM, 2011*. IEEE, 2011, pp. 2516–2524.
[5] M. Haddad, S. Elayoubi, E. Altman, and Z. Altman, "A Hybrid Approach for Radio Resource Management in Heterogeneous Cognitive Networks," *Journal on Selected Areas in Communications*, vol. 29, no. 4, pp. 831–842, 2011.
[6] R. Southwell and J. Huang, "Convergence Dynamics of Resource-Homogeneous Congestion Games," in *Game Theory for Networks*. Springer, 2012, pp. 281–293.
[7] M. Cesana, I. Malanchini, and A. Capone, "Modelling Network Selection and Resource Allocation in Wireless Access Networks with Non-Cooperative Games," in *Proc. of 5th IEEE International Conference on Mobile Ad Hoc and Sensor Systems*. IEEE, 2008.
[8] M. Abusubaih and A. Wolisz, "Interference-Aware Decentralized Access Point Selection Policy for Multi-rate IEEE 802.11 Wireless LANs," in *Proc. of IEEE 19th International Symposium on Personal, Indoor and Mobile Radio Communications : PIMRC*. IEEE, 2008.
[9] I. Malanchini, M. Cesana, and N. Gatti, "Network Selection and Resource Allocation Games for Wireless Access Networks," *IEEE Transactions on Mobile Computing*, vol. 12, no. 12, pp. 2427–2440, 2013.
[10] R. Trestian, O. Ormond, and G. Muntean, "Game Theory-Based Network Selection: Solutions and Challenges," *IEEE Communications Surveys Tutorials,*, vol. 14, no. 4, pp. 1212–1231, 2012.
[11] E. Koutsoupias and C. Papadimitriou, "Worst-Case Equilibria," in *Proceedings of the 16th annual conference on Theoretical aspects of computer science*. Springer-Verlag, 1999, pp. 404–413.