Channel-Aware Weighted Proportional Fair Medium Access Control in Wireless LANs with MIMO links

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Abstract

MIMO techniques using multiple antennas for both transmitting and receiving have recently manifested themselves to be very promising for future broadband wireless networks. Aiming to leverage the impact of these MIMO techniques on network protocol design in wireless LANs (WLANs), we take a utility approach to study channel-aware weighted proportional fair medium access control (WPF MAC) for QoS provisioning. Simply put, in this utility based approach, every user in a WLAN attempts to maximize its own utility, and the optimization procedure takes place in two stages: a channel contention phase *that arbitrates fair channel access across the users via combining adaptive persistence mechanism with random backoff, and a* data transmission phase *that determines the transmission duration based on the channel conditions in each transmission round. Furthermore, adaptive beamforming is carried out by using the training signals embedded in the RTS/CTS handshake to enhance the spectral efficiency of the MIMO links. Using a stochastic approximation method, together with the Lyapunov stability theorems, we establish the stability of the adaptive persistence mechanism in the proposed WPF MAC and analyze the fairness across the users therein.*

1. Introduction

With the explosive growth in demand for heterogeneous wireless services, next generation wireless networks are expected to handle quality of service (QoS) requirements of a variety of multimedia applications. Indeed, the wireless technology evolution is taking place in many fronts: multiple antenna techniques, modulation/coding schemes, medium access control (MAC) design, routing protocols and security mechanisms. In particular, MIMO techniques using multiple antennas for both transmitting and receiving have manifested themselves as very promising candidates for future broadband wireless networks [11].

While the stand-alone performance of the MIMO technique is relatively well understood, it remains largely open to leverage the impact of these promising techniques on network protocol design and QoS provisioning in wireless networks. In particular, it is natural to ask the following questions. How should we take advantage of the new "network resources" such as spatial degrees of freedom in MIMO channels and integrate these new features of the PHY layer into the MAC design? How can we devise MAC protocols towards QoS provisioning? Thus motivated, we will study fair MAC design for WLANs with MIMO links, and a main objective is to achieve high throughput while meeting the QoS requirements.

The optimal design of fair MAC for wireless networks is challenging, partially due to the time-varying nature of the PHY-layer communication channels and the distributed nature of the access control. Indeed, in many practical wireless systems there exists a significant amount of multipath fading, and fading is regarded as an intrinsic feature of wireless channels. Unfortunately, there has been little work on the MAC design to take into account channel fading directly; and the time-varying fading channel is often "abstract" to be a constant rate link and this is likely to be oversimplified. A major challenge is how to design efficient and robust wireless MAC protocols and handle the QoS provisioning issues in WLANs with MIMO links, and this is the subject of this paper.

In this paper, we take a utility approach to study channel-aware weighted proportional fair MAC (WPF MAC) design for QoS provisioning. We focus on WPF MAC because the weighted proportional fairness is a very popular fairness model, and furthermore, we will show in Section 4 that the corresponding protocol design is amenable to easy implementation. Roughly speaking, in our utility based approach, every user attempts to maximize its own utility that

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is a function of its throughput determined by the MAC protocol. To this end, the optimization procedure takes place in two stages: a *channel contention phase* that arbitrates fair channel access across the users, and a *data transmission phase* that determines the transmission duration in each transmission round based on channel conditions . The main features of the WPF MIMO MAC can be summarized as follows:

1. In the channel contention phase, the contention resolution is performed via combining the adaptive persistence mechanism with constant backoff counters (cf. [9]). Specifically, based on the stochastic approximation algorithm [7], the distributed adaptation of the persistence probabilities is carried out to maximize the corresponding user utility functions. Using the mean ordinary differential equation (ODE) method, together with the Lyapunov theorems, we establish the stability of the proposed fair MAC schemes and analyze the fairness across the users [7]. In particular, we first use *Lyapunov's indirect method* to show the system stability for the proportional fair case. Then, we use *Lyapunov's direct method* to examine the stability and fairness of the proposed weighted proportional fair MAC.

2. In the data transmission phase, the transmission rate is first chosen based on adaptive beamforming over the MIMO link. The next key step is to determine the transmission duration that is obtained by solving the "local" optimization of user utility functions. The underlying rationale is that the channel utilization can be enhanced by adapting the transmission duration based on the transmission rate while taking into account the throughput fairness requirements. We use pilot signals in the RTS/CTS handshake to help implement adaptive beamforming to utilize the MIMO links more efficiently. Simply put, the RTS/CTS handshake is used not only to reserve the transmission floor, but also to provide a mechanism for the estimation the channel conditions.

Fair access control design has received much attention in the past few years. Particularly, contention-based MAC protocols in [10, 4] attempt to assign equal share of bandwidth to different users with an inherent assumption of equal weights. Recently, backoff-based schemes has been presented in [2, 16] to address weighted fair bandwidth allocation. Assuming constant link capacities, [9] has presented a novel framework of translating any pre-specified fairness model into a corresponding persistence/backoff-based contention resolution scheme. Along a different line, MIMO MAC protocols have recently appeared in [14]. Specifically, [14] has proposed a MIMO MAC scheme with stream control, assuming perfect closed-loop MIMO and ideal interference cancellation.

The rest of the paper is organized as follows. Section 2 contains a brief review of the MIMO concept and fair medium access design. In Section 3 we give the system models and present the basic ideas of our problem formulation. The detailed WPF MIMO MAC algorithm is explained in Section 4, followed by the analysis of the stability of the adaptive persistence mechanism in Section 5. We provide simulation results in Section 6. Section 7 concludes this paper and provides discussions of future work.

2. Background

2.1. The MIMO Concept

The past few years have witnessed a surge of interest in the multiple-antenna technology in the field of wireless communications. It has been shown that using multiple antennas at the wireless transmitter and the receiver, namely, the multiple-input multiple-output (MIMO) technique, can boost up the channel capacity significantly [15].

Specifically, consider the following MIMO channel model with flat fading [12]:

$$\mathbf{y} = \sqrt{\frac{E_s}{M}}\mathbf{GHFs} + \mathbf{Gn}$$

where $\mathbf{s}, \mathbf{y}$ are the $M \times 1$ transmitted and received signal vectors, E_s is the average power at the transmitter, $\mathbf{F}, \mathbf{G}$ are the precoder and decoder matrix, $\mathbf{H}$ is an $M \times M$ channel matrix, and $\mathbf{n}$ is the noise with covariance matrix $E[\mathbf{nn^H}] = N_0\mathbf{I}_M$. We assume that the channel has a coherence time T_C, i.e., the channel ($\mathbf{H}$) remains (more or less) constant for a duration of T_C. Note that the covariance matrix of $\mathbf{s}$ must satisfy the average power constraint, i.e., trace$(\mathbf{R_{ss}}) = M$. It has been shown that the channel capacity of the MIMO channel above can be given as [15]:

$$C = \log_2 \det\left(\mathbf{I}_M + \frac{E_s}{MN_0}\mathbf{GHFR_{ss}F}^H\mathbf{H}^H\mathbf{G}^H\right) bps/Hz.$$

Based on the channel estimation via the pilot symbols in the RTS/CTS handshake, a key objective is to find $\{\mathbf{F}^*, \mathbf{G}^*\}$ to maximize the instant transmission rate. This is called adaptive beamforming or pre-coding/pre-filtering [12, 11].

We note that the channel state information (CSI) is required at the transmitter to carry out the adaptive beamforming. To obtain the CSI at the transmitter, we can utilize the channel reciprocity principle in the time division duplexing (TDD) systems [11].

2.2. Fairness

Definition 2.1 Let $\vec{w} = [w_1, w_2, \ldots, w_m]^T$ denote the priorities. A vector of rates $\vec{x}^*$ is weighted proportionally fair if it is feasible and for any other feasible rates $\vec{x}$,

$$\sum_i w_i \frac{x_i - x_i^*}{x_i^*} \leq 0. \tag{1}$$

The utility function associated with the weighted proportional fairness is given as [6]:

$$U(x_i) = w_i \log x_i. \tag{2}$$

Another fairness metric, namely, the fairness index FI, has been introduced to measure the degree of fairness [2]:

$$FI = \max_{\{i,j\}} \left\{ \max\left(\frac{\theta_i}{w_i}, \frac{\theta_j}{w_j}\right) / \min\left(\frac{\theta_i}{w_i}, \frac{\theta_j}{w_j}\right) \right\}, \tag{3}$$

where θ_i is the throughput of node i, and w_i is the specified share node i needs to receive.

3. Problem Formulation

Consider a wireless LAN where all nodes can hear each other. We are interested in designing WPF MIMO MAC protocols so that nodes can share the wireless channel fairly and efficiently. To this end, we only consider per-node fairness.

Recall that a key goal of this study is to design a distributed fair MAC while taking into account fading. It has been shown that in wireline networks achieving some system-wide notion of fairness is equivalent to optimizing some utility functions of users [13, 6]. Building on [13, 6], we take a utility approach to the design of fair MAC that arbitrates the channel allocation to the users so as to maximize their utility functions. We should note that a major challenge here is to identify the key considerations that need to be incorporated to generalize the framework in [6] for wireline networks to wireless networks.

Since the channel condition of each flow is unknown before contending, it is plausible to decompose the optimization of the utility functions into two phases: the first phase is the channel contention phase; the main objective of this phase is to allocate the channel among the contending nodes in a "fair" manner so as to optimize the overall utility functions globally; the second phase is the data transmission phase, where the transmission duration is adapted using the information such as the instant transmission rate, the throughput and the fairness requirements, and the objective here is to maximize the utility functions locally.

3.1. The Channel Contention Phase

We now consider the channel contention phase for the WPF MAC design.

For convenience, we first give a brief review of the framework in [6] for fair rate allocation over the Internet. Then we show how to generalize it to wireless MAC design by establishing a mapping between the rate and the persistence probability.

It has been shown in [6] that the fair rate allocation in wireline networks can be carried out by using convex programming. Specifically, suppose that there are m nodes in a network, and node i has a utility function $U_i(r_i)$ where r_i is the transmission rate. The fair rate allocation can be obtained by solving the following optimization problem:

$$\begin{array}{lll} P_0: & \max\limits_{\{r_i\}} & \sum_i U_i(r_i) \\ & \text{subject to}: & \sum_i r_i \le 1, \\ & & r_i \ge 0, i \in \{1, 2, \ldots, m\}, \end{array}$$

where the overall bandwidth is normalized to 1. This is a convex programming problem that can be readily solved via the standard Lagrangian multiplier method, and alternatively by the penalty method. For instance, letting $U_i(r_i) = w_i \log(r_i)$, the optimal solution can be obtained by the following distributed rate adaptation algorithm [13]:

$$\dot{r}_i = w_i - (\sum_i r_i) r_i. \tag{4}$$

For TCP flow control in the Internet, the bandwidths of the links are well defined, and it has been shown that there exists an one-one mapping between the congestion window ($cwnd$) size adjustment and transmission rate control [8]. Therefore, the rate adaptation can be achieved by adjusting the $cwnd$ size based on the congestion feedback.

In contrast, the radio channel in wireless ad hoc networks is spatially shared by many users, and different users would experience different channel conditions. Therefore, it is not clear whether the rate control techniques developed for wireline networks would work well in wireless ad hoc networks. Furthermore, the rate allocation in ad hoc networks is controlled by the channel access algorithm, and it remains open to establish a mapping between the channel access and the rate allocation. Accordingly, the rate adaptation algorithm for wireline networks cannot be directly applied to the shared-channel wireless access control. It is also suggested in [5] that the fairness issue should be considered in the context of specific channel access protocols.

The proposed fair MAC design in this paper is built upon CSMA/CA. Specifically, the collision avoidance is done by carrier sensing and the RTS/CTS handshake. A key step in meeting the fairness requirements is to develop a mapping between the rate adaptation and the corresponding contention resolution. To this end, we use the persistence mechanism with a constant backoff counter for contention resolution (cf. [9]). More specifically, in the persistence mechanism, every node maintains a persistence probability and contends for the channel with this probability. In this backoff scheme with a constant counter (B), every contending node defers a random amount of time uniformly distributed in $[0, B]$ before accessing the channel. The persistence probability should be adjusted to match the contention level and fairness requirements based on the outcomes of the transmissions. Hence, the main task boils down to developing a mapping between the rate control and the persistence probability adaptation.

To this end, let $\theta_{n,i}$ denote the throughput of node i after n rounds of transmissions (T_n), which is given by

$$\begin{aligned} \theta_{n,i} &= \frac{\sum_{l=1}^{n} r_{l,i} t_{l,i} \mathbf{I}_{l,i}}{T_n}, \\ T_n &= \sum_{l=1}^{n} (\sum_{i=1}^{m} t_{l,i} \mathbf{I}_{l,i} + B \mathbf{I}_{\sum_i \mathbf{I}_{l,i}=0}), \end{aligned}$$

where $r_{l,i}$ is the transmission rate during the transmission period $t_{l,i}$; and $\mathbf{I}_{l,i}$ is the indicator function of the event that node i has occupied the channel during round l.

Assume for now that the transmission rates $\{r_{l,i}\}$ have the same statistics with mean C. If the transmission duration $t_{l,i}$ has the same mean value (denoted as D) for all links and is independent from the persistence probabilities, x_i's, then by the Law of Large Numbers, we have that as $n \to \infty$, $\theta_{n,i} \xrightarrow{P} C \cdot x_i \prod_{j \neq i}(1 - x_j)$, or $\theta_{n,i} \xrightarrow{P} C \cdot \frac{x_i}{1-x_i} \prod_j (1 - x_j) \approx R \cdot x_i$, where $R = C \prod_j (1 - x_j) \approx C/e$ is constant for all i and the approximation is based on the assumption that in a large network the x_i's are small around the equilibrium point.

In light of this linear relationship between the persistence probability x_i and the achieved throughput $\theta_{n,i}$, we observe that the persistence probability x_i can follow the same adaptation procedure as in the fair rate control.

In summary, in the channel contention phase, the persistence probabilities, x_i's, should be adapted so as to solve the following global optimization problem:

$$\begin{array}{lll} P_G: & \max\limits_{\{x_i\}} & \sum_i w_i \log(x_i) \\ & \text{subject to}: & \sum_i x_i \leq 1, \\ & & x_i \geq 0, i \in \{1, 2, \ldots, m\}. \end{array} \tag{5}$$

3.2. The Data Transmission Phase

Once a node successfully claims the channel, the instant channel condition ($\mathbf{H}$) can be estimated using the training symbols embedded in the RTS/CTS packets. Adaptive beamforming can be carried out to find the corresponding $\{\mathbf{G}^*, \mathbf{F}^*\}$ to maximize the transmission rate.

Another key objective of the fair MAC design is to utilize the channel conditions to improve spectral efficiency while taking into account the fairness requirements. To this end, we model the data transmission in each round as a local optimization problem, and the transmission duration for the $(n+1)$th transmission round can be obtained by solving the following problem:

$$\begin{array}{lll} P_L: & \max\limits_{t_{n+1,i}} & \sum_j w_j \log(\theta_{n+1,j}) \\ & \text{subject to}: & 0 \leq t_{n+1,i} \leq t_c, \end{array} \tag{6}$$

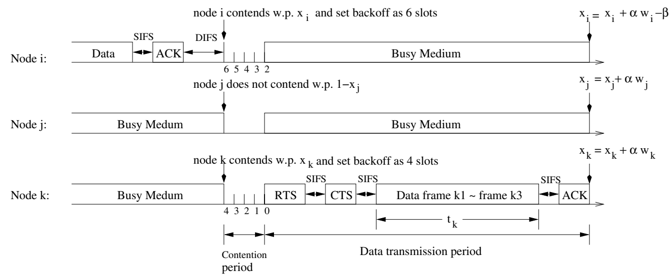

Figure 1. A sketch of one transmission round of the WPF MIMO MAC ($B = 10$)

where t_c denotes the channel coherence time, within which the channel condition remains more or less unchanged. It should be noted that the local problem P_L is relatively simple because the objective function of P_L is a continuous function on a compact set, so it must have a maximum point.

4. Channel-Aware Weighted Proportional Fair MIMO MAC

Solving (5) and (6), we are now ready to give the detailed description of the proposed WPF MIMO MAC protocol (see Fig. 1).

4.1. Adaptation of Persistence Probability

Using the stochastic approximation method together with the penalty method, we can show that the following algorithm solves (5):

$$x_{n+1,i} = x_{n,i} + \alpha w_i - \beta \cdot \mathbf{I}(\mathcal{C} \cup \mathcal{B}) \cdot \mathbf{I}(\mathcal{E}) \tag{7}$$

Based on (7), we describe the corresponding adaptation procedures of the persistence probability in the following and relegate the proofs of the fairness and the stability properties to Section 5.

The adaptive persistence mechanism with random backoff is performed as follows: If a node has packets to transmit, it competes for the channel with probability $x_{n,i}$ and waits a random back-off uniformly distributed in $[0, B]$, where B is the back-off interval. After the backoff, if the channel is free and there is no collision, the node starts transmission. If the channel is sensed busy or there is a collision (by the lack of the CTS packet), the node decreases its transmission probability by β. At the end of the transmission round, each user increases their transmission probability by αw_i.

4.2. Adaptive Beamforming for the MIMO Link

Once node i has acquired the channel, the RTS/CTS handshake will be performed and the channel matrix ($\mathbf{H}$) is estimated by using the training signals. Recall that one key step here is to find the precoder and decoder matrix $\{\mathbf{F}^*, \mathbf{G}^*\}$ that maximize the instant information rate. It can be shown that $\mathbf{G}^* = \mathbf{U}^H$ and $\mathbf{F}^* = \mathbf{V}$ are the solutions where $\mathbf{U}, \mathbf{V}$ are the singular vector matrices of $\mathbf{H}$, i.e., $\mathbf{H} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^H$ and $\mathbf{\Sigma} = diag\{\sigma_1, \sigma_2, \ldots, \sigma_M\}$ [11]. This decomposes the matrix channel into M parallel single-input single-output (SISO) channels:

$$y_i = \sqrt{\frac{E_s}{M}}\sigma_i s_i + n_i, \text{for } i = 1, 2, \ldots, M. \tag{8}$$

Therefore, the MIMO channel capacity is given as

$$C = \sum_{i=1}^{M} log_2 \left(1 + \frac{E_s \gamma_i}{M N_0}, \sigma^2\right) \tag{9}$$

where $\gamma_i = E[|s_i|^2]$ denotes the transmit power in the ith eigen-channel and $\sum_i \gamma_i = M$. The optimal power allocation can be obtained by the so-called water-filling scheme.

Therefore, we can use the decomposed eigen-channels to transmit the data packets by selecting the modulation (and hence the transmission rate) according to their corresponding channel conditions. The overall transmission rate is the summation of the transmission rates along the separate eigen channels, and is denoted as $r_{n,i}$.

4.3. Adaptation of Transmission Durations

After adaptive beamforming is done, the next important issue is to determine the transmission duration based on the transmission rate while taking into account the fairness requirements. Specifically, the transmission duration for the $(n+1)$th transmission round can be obtained by solving the following local problem P_L:

$$\begin{aligned} P_L: \quad \max_{t_{n+1,i}} \quad & w_i \log\left(\tfrac{\theta_{n,i}T_n + t_{n+1,i}r_{n+1,i}}{T_n + t_{n+1,i}}\right) \\ & + \textstyle\sum_{j\neq i} w_j \log\left(\tfrac{\theta_{n,j}T_n}{T_n + t_{n+1,i}}\right) \\ \text{subject to}: \quad & 0 \le t_{n+1,i} \le t_c. \end{aligned}$$

After some algebra, the corresponding optimum transmission duration is given by

$$t_{n+1,i} = \min\left\{\left[\left(\frac{w_i}{\sum_{j\neq i} w_j} - \frac{\sum_i w_i}{\sum_{j\neq i} w_j}\frac{\theta_{n,i}}{r_{n+1,i}}\right)T_n\right]^+, t_c\right\}. \tag{10}$$

A few important observations are in order:

1. To determine the transmission duration using the algorithm above, it suffices to know the weights pre-specified for different flows and local information such as the instant transmission rate $r_{n+1,i}$, the throughput $\theta_{n,i}$ after the round n and the system time T_n.

2. From (10), it can be seen that the optimum value of $t_{n+1,i}$ is an increasing function of the transmission rate $r_{n+1,i}$, indicating that the transmission duration is larger when the channel condition is good and is smaller when it is poor. Note that $t_{n+1,i}$ is also a decreasing function of $\theta_{n,i}$. Accordingly, the nodes who have not obtained its fair share would have a relatively longer transmission duration.

3. We use a sliding-window algorithm with a window size $T^* >> t_c$ to ensure the short-term fairness of the proposed fair MAC. Then, the transmission duration and the throughput are updated as follows:

$$t_{n+1,i} = \min\left\{\left[\left(\frac{w_i}{\sum_{j\neq i} w_j} - \frac{\sum_i w_i}{\sum_{j\neq i} w_j}\frac{\theta_{n,i}}{r_{n+1,i}}\right)T^*\right]^+, t_c\right\}$$

$$\theta_{n+1,j} = \begin{cases} \left(1 - \frac{t_{n+1,i}}{T^* + t_{n+1,i}}\right)\theta_{n+1,i} + \frac{\theta_{n+1,i}}{T^* + t_{n+1,i}} r_{n+1,i} & j = i \\ \left(1 - \frac{t_{n+1,i}}{T^* + t_{n+1,i}}\right)\theta_{n+1,j} & j \neq i. \end{cases}$$

Clearly, the value of T^* affects the fairness and the channel utilization which will be examined in the simulation part.

5. Fairness and Stability Analysis of the Adaptive Persistence Mechanism for the Channel-Aware WPF MIMO MAC

Recall that to achieve the weighted proportional fairness, the persistence probability for node i is updated by (7).

5.1. A Stochastic Approximation Formulation of the Adaptive Persistence Mechanism

It is not difficult to see that (7) can be modeled as a stochastic recursive algorithm, where the randomness is due to events $\mathcal{E}, \mathcal{B}, \mathcal{C}$ [7]. The convergence of this algorithm can be examined via the tool of the mean Ordinary Differential Equation (ODE) [7, 3].

The standard form of stochastic recursive algorithm is given by [3]

$$\vec{x}(t+1) = \vec{x}(t) + \gamma F(\vec{\epsilon}(t), \vec{x}(t)), \tag{11}$$

where $\vec{\epsilon}$ is the random inputs and γ is a small constant. The ODE associated with the algorithm is introduced as follows:

$$\frac{d\,\vec{x}(t)}{dt} = f(\vec{x}(t)), \tag{12}$$

$$f(\vec{x}(t)) = E[F(\vec{\epsilon}, \vec{x}(t))|\vec{x}(t)]. \tag{13}$$

The mean ODE method states that under the condition that γ is small, the stochastic algorithm in (11) converges in probability to the attractor of the ODE in (12) if the ODE

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is globally stable with a unique stable equilibrium (attractor) [3]. For ease of reference, we restate the result as follows [3].

Lemma 5.1 *If ODE (12) is globally stable, with a unique stable equilibrium $\vec{x^*}$, then for any sufficiently small $\gamma > 0$, for all $\epsilon > 0$, there exists a constant $C(\gamma)$ such that, under mild conditions,*

$$\limsup_{t\to\infty} \mathbb{P}\{\|\vec{x}(t) - \vec{x^*}\| > \epsilon\} \leq C(\gamma). \tag{14}$$

where $C(\gamma)$ tends to zero as γ tends to zero,

Appealing to Lemma 5.1, from (7), we have that

$$H_i(\vec{\epsilon}, \vec{x}) = \frac{1}{\gamma}\left(\alpha w_i - \beta \cdot \mathbf{I}(\mathcal{C} \cup \mathcal{B}) \cdot \mathbf{I}(\mathcal{E})\right). \tag{15}$$

Based on Lemma 5.1, γ affects only the convergence rate of the ODE provided that γ is sufficiently small. For simplicity, it can be "skipped" in the calculation of $h(\vec{x})$. As a result, from (15), the mean ODE for the algorithm is given by

$$\dot{x_i} = \alpha w_i - \beta E[\mathbf{I}(\mathcal{C} \cup \mathcal{B})\mathbf{I}(\mathcal{E})|\vec{x}], \tag{16}$$

where $E[\mathbf{I}(\mathcal{C} \cup \mathcal{B})\mathbf{I}(\mathcal{E})|\vec{x}]$ can be evaluated as follows

$$\begin{aligned} E[\mathbf{I}(\mathcal{C} \cup \mathcal{B})\mathbf{I}(\mathcal{E})|\vec{x}] &= E[\mathbf{I}(\mathcal{C} \cup \mathcal{B})|\vec{x}]E[\mathbf{I}(\mathcal{E})|\vec{x}] \\ &\overset{(a)}{=} x_i(P(\mathcal{C}) + P(\mathcal{B})) \\ &\overset{(b)}{=} x_i \frac{B+1}{2B}\sum_{j\neq i} x_j, \end{aligned} \tag{17}$$

where (a) comes from the fact that events $\mathcal{C}$ and $\mathcal{B}$ are mutually exclusive, and (b) from the following calculation:

$$\begin{aligned} & P(\mathcal{C}) + P(\mathcal{B}) \\ = \;& 1 - E[P(\mathcal{S}_i|\text{node i select slot l})] \\ = \;& 1 - \sum_{l=0}^{B-1}\frac{1}{B}\sum_{s_a\subseteq\{s\backslash i\}}\prod_{\substack{k\in s_a^c\\ j\in s_a}}\left(\frac{B-1-l}{B}\right)x_j(1-x_k), \end{aligned} \tag{18}$$

where $\mathcal{S}_i$ is the event that node i has occupied the channel after the contention, and $s_a \subseteq \{s\backslash i\} = \{1, \ldots, i-1, i+1, \ldots, m\}$ is the set of the nodes which contend for the channel. The exact characterization of right hand side of (18) involves tedious combinatorics, and in general does not yield much insight into determining the system stability. Instead, we consider an interesting yet practical scenario where there are many nodes inside the WLAN, and the transmission probabilities, $x_i, i \in \{1, \ldots, m\}$ are small compared to $\sum_i x_i$. Intuitively speaking, no single node would dominate the channel usage. We note that this approximation is applicable to many practical systems, and we have used numerical methods to corroborate that it works well when the number of users is larger than 5. It follows that

$$\begin{aligned} & 1 - \sum_{l=0}^{B-1}\frac{1}{B}\sum_{s_a\subseteq\{s\backslash i\}}\prod_{\substack{k\in s_a^c\\ j\in s_a}}\left(\frac{B-1-l}{B}\right)x_j(1-x_k) \\ \overset{(c)}{=} \;& 1 - \prod_{j\neq i}(1-x_j) + \sum_{j\neq i}\prod_{k\neq i,j}\frac{B-1-l}{B}x_j(1-x_k) \\ = \;& \frac{B+1}{2B}\sum_{j\neq i} x_j, \end{aligned} \tag{19}$$

where (c) follows from the assumption that x_i are small so that high orders (≥ 2) terms can be safely ignored.

Substitute (17) into (16), we have that

$$\dot{x_i} = \alpha w_i - \beta\frac{B+1}{2B}x_i\sum_{j\neq i} x_j. \tag{20}$$

5.2. Stability via Lyapunov's indirect Method: the Proportional Fair Case

We first examine the stability of an interesting case where $w_i = w_j = w, i \neq j$, i.e., the proportional fairness case, for which, we can exactly characterize its equilibrium point and study its stability property. However, it is usually difficult to find the equilibrium point for the weighted proportional fairness case, and the general proof of the weighted proportional fairness will be given in the following section using Lyapunov's direct method instead.

Let $\mathbf{1}$ denote all one column vector and $a = \sum_{i=1}^m x_i$. Therefore, (20) can be rewritten in the following matrix form:

$$\dot{X} = \alpha^* I - \beta^* \operatorname{diag}\{x_1, \ldots, x_m\}(a\mathbf{1} - X), \tag{21}$$

where $X = [x_1, \ldots, x_m]^T$, $\alpha^* = \alpha w$ and $\beta^* = \frac{B+1}{2B}\beta$. The equilibrium point X_0 can be found by the first order condition and is given as

$$X_0 = \sqrt{\frac{\alpha^*}{\beta^*(m-1)}}\mathbf{1}. \tag{22}$$

Since $X_0 \leq 1$, we have to choose $\alpha^* < \beta^*/2$.

We have the following proposition:

Proposition 5.1 *If parameters α^* and β^* are sufficiently small and $\alpha^* < \beta^*/2$, the system in (21) is locally stable around the equilibrium point (22).*

Due to space limitation, the proof is left in [17].

5.3. Stability via Lyapunov's Direct Method: the Weighted Proportional Fair Case

A more general approach to examining the fairness and stability is to use Lyapunov's Direct Method, which does not require explicit characterization of the equilibrium point. It should be noted that some relaxation is often required for applying this method.

To this end, we approximate (20) as

$$\dot{x_i} = \alpha^* w_i - \beta^* x_i \sum_j x_j. \tag{23}$$

The underlying rationale is that in a clique with many nodes, since the value of x_i is small, $\sum_{j \neq i} x_j$ can be well approximated by $\sum_j x_j$.

We state the following proposition:

Proposition 5.2 *The proposed WPF MAC scheme in (7) converges in probability to the unique globally stable equilibrium point of (23), and this equilibrium point also maximizes the Lyapunov function defined as*

$$U(X) = \sum_i \alpha^* w_i log(x_i) - \beta^* \int_0^{\sum_j x_j} x dx,$$

i.e., (7) achieves the weighted proportional fairness.

The proof of this proposition follows the same line in [13].

6. Simulation Study

In this section, we evaluate the performance of the proposed channel-aware WPF MIMO MAC by simulations using GloMoSim [1]. Specifically, we study a wireless LAN where there are 5 flows with different transmission weights ($\vec{w} = \{0.32, 0.32, 0.2, 0.12, 0.04\}$). The fairness performance is examined using the fairness index defined in (3).

The simulation parameters are outlined as follows. First, we assume that the transmission rates of data streams for a MIMO channel are 2 Mbps, 5.5Mbps, and 11 Mbps selected depending on channel conditions. These rates are used to transmit data frames, and all the control packets are using the basic rate 2 Mbps. We assume that the channel fading follows the Rayleigh distribution with a coherence time of $20ms$. Constant Bit Rate (CBR) traffic is generated between different nodes. Unless otherwise specified, the packet size is set to be 512 Bytes. Parameters α and β are 0.001 and 0.005 respectively. B is set to be 5 slots.

First, we plot in Fig. 2 the fairness index (FI) as a function of the packet arrival rate, and in Fig. 3 the individual throughput of each flow. From Fig. 2, we observe that in the WPF MAC scheme, the FI is close to 1 when the packet arrival rate is high. We should also note that when the packet arrival rate is small, all the flows achieve almost the same throughput due to possibly empty queues for higher priority flows (see Fig. 3); as a result, the weighted proportional fairness is not well demonstrated. We also observe that the WPF MAC achieves better fairness performance than the PFCR scheme proposed in [9].

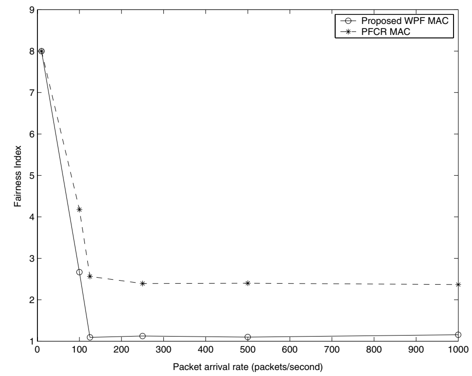

Figure 2. Fairness performance of the channel-aware WPF MIMO MAC

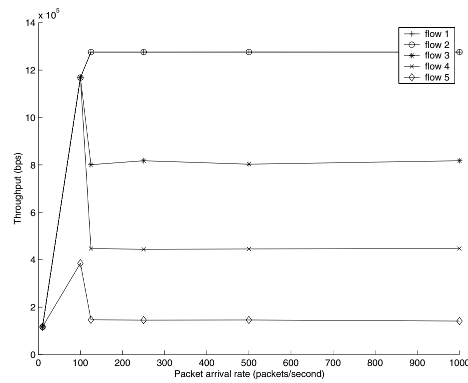

Figure 3. Throughput performance of the channel-aware WPF MIMO MAC

Table 1 summarizes the throughput gains by using adaptive beamforming over a 4×4 MIMO link, compared with the equal power transmissions. It is easy to see that the performance improvement is almost linear with the packet size. This is because the larger the packet size, the more efficient the utilization of adaptive beamforming.

Table 2 illustrates the performance gain of the channel-aware WPF MIMO MAC by using adaptive transmission duration, compared to the no adaptation case. From Fig. 4, it is clear that the sliding window size T^* is of critical importance to achieve both long-term and short-term fairness. Recall that the transmission duration $t_{n+1,i}$ is determined by the accumulated throughput $\theta_{n,i}$ and the weights $\{w_i\}$. If T^* is too small, then $t_{n+1,i}$ is given by $w_i/(\sum_j w_j - w_i)T_n$; as a result, the higher priority flows may occupy the channel longer than needed, resulting in possible unfairness (as illustrated in Fig. 4). In contrast, if T^* is too large (e.g., $T^* = 200ms$), the short-term fairness may become poorer,

Table 1. Throughput performance gain by the adaptive beamforming

Packet size (Bytes):	256	512	1024	1460
Throughput gain	7.80%	12.6%	29.9%	34.8%

Table 2. Throughput performance gain by the adaptation of transmission duration

Flows:	1	2	3	4	5
$T^* = 20MS$	78.9%	92.7%	57.7%	25.2%	8.10%
$T^* = 50MS$	81.7%	80.5%	63.6%	56.0%	47.3%
$T^* = 100MS$	80.8%	83.0%	79.0%	82.4%	85.0%
$T^* = 200MS$	74.3%	76.6%	68.3%	82.0%	83.3%

because a small positive difference of the first two terms in (10) would yield large $t_{n+1,i}$ in this case. Consequently, this may ruin the short-term fairness (see Fig. 4).

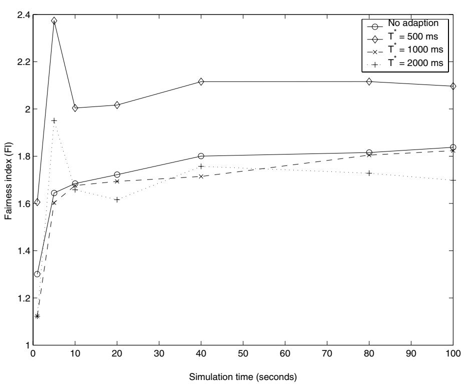

Figure 4. The WPF MIMO MAC fairness performance w.r.t. the sliding window size

7. Conclusion

We have taken a utility approach to study channel-aware weighted proportional fair medium access control for MIMO ad hoc networks. Summarizing, the optimization of user utility functions is decomposed into two phases: in the channel contention phase, the fair channel access is achieved by the adaptation of the persistence probability combined with backoff; in the data transmission phase, the transmission duration is adapted to maximize the utility functions based on the channel condition and throughput fairness requirements. Furthermore, to enhance the spectral efficiency of MIMO links, adaptive beamforming is carried out using the training signals embedded in the RTS/CTS handshake. We establish the stability of the adaptive persistence mechanism in the proposed WPF MAC scheme using the stochastic approximation method. We are currently pursuing to extend the above study to multi-hop networks with general utility functions.

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