

On two approaches in investigation of nonequilibrium effects of filtration in a porous medium.

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1. Introduction

Nonequilibrium effects in the displacement in a porous medium which manifest themselves in the dependence of relative phase permeabilities and capillary pressure on the rate of the displacement are well known (see, for example, [1-2]). There are at least two different approaches to account for this phenomenon in mathematical models. The first one is based on the use of dynamic relations $f(S, S_t)$ instead of equilibrium functions of saturation. The model of this type was developed by Barenblatt et al. [1]. Its great advantage is a small number of additional empirical parameters. Under some suggestions the characteristic time of capillary relaxation τ is the only additional parameter of the model and dynamic relative permeability has the following form $f_\alpha = f_\alpha^e(S + \tau S_t)$, where superscript 'e' denotes the equilibrium functions of the saturation. Another approach (see, [2]) is based on principles of nonequilibrium thermodynamics. In the framework of this approach a new thermodynamic parameter being an argument of thermodynamic potential is introduced to parameterize nonequilibrium states. For this parameter (hereafter denoted as ξ) the kinetic equation is formulated which provides non-negativity of dissipation of capillary forces. Relative phase permeabilities and capillary pressure are suggested to be functions of this parameter and saturation

$$f_\alpha = f_\alpha(S, \xi), p_c = p_c(S, \xi). \quad (1.1)$$

This way of modeling starts from relation between the thermodynamical potential and its arguments and even the simplest models include a relatively large number of empirical parameters. Development of the permeability function $f_\alpha(S, \xi)$ requires additional argumentation. In the present paper the dependence of the phase permeabilities on ξ is introduced such that the resulting model is compatible with Barenblatt's model at small relaxation time i.e.

$$f_\alpha \rightarrow f_\alpha^e(S + \tau S_t) \text{ as } \tau \rightarrow 0. \quad (1.2)$$

On the other hand the functions (1.1) can not be represented by the dynamic relations of the form $f(S, S_t)$ at finite τ . The present paper focuses on the model to model comparison and examination of properties of the solutions to some one-dimensional problems of immiscible displacement in a porous medium.

2. Description of models.

One of the most common models of the displacement is Muskat and Leverett's model (see [3], [4]). This model is based on the conservation of mass and Darcy's equation:

$$u_1 = -\frac{kk_1}{\mu_1} \nabla p_1, \quad u_2 = -\frac{kk_2}{\mu_2} \nabla p_2, \quad (2.1)$$

$$p_2 - p_1 = p_{cap}, \quad (2.2)$$

$$\phi \frac{\partial S}{\partial t} + \nabla \cdot u_1 = 0, \quad \phi \frac{\partial (1-S)}{\partial t} + \nabla \cdot u_2 = 0, \quad (2.3)$$

$$\nabla \cdot (u_1 + u_2) = 0, \quad (2.4)$$

where p_1 and p_2 are pressures of water and oil, p_{cap} is a capillary pressure, k , f_1 , f_2 are the permeability of the skeleton and relative phase permeabilities of water and oil respectively, μ_1 , μ_2 are the viscosities of water and oil, S is a water saturation, u_1 and u_2 are the velocities of filtration of water and oil respectively, ϕ is a porosity. In one-dimensional case (2.4) results in $u_1 + u_2 = u = \text{const}$. That leads to the following well-known equation

$$\frac{\partial S}{\partial t} + \frac{1}{\phi} \frac{\partial}{\partial x} ub(S) = \frac{\partial}{\partial x} \left[a(S) \frac{\partial S}{\partial x} \right], \quad (2.5)$$

$$\text{Where } b(S) = \frac{f_1}{f_1 + \mu f_2}, \quad \mu = \frac{\mu_1}{\mu_2} \quad \text{and} \quad a(S) = -\frac{kM}{\phi \mu_2} \frac{f_1(S)f_2(S)}{f_1(S) + \mu f_2(S)}.$$

It the model concerned relative phase permeabilities are supposed to be functions of the saturation. This is the case of local equilibrium between the fluids, i.e. the time which is required for flow paths to change their distribution is much smaller than the time of the significant change of the saturation. This assumption is not valid near the water-oil front and in the processes with time-dependent rate of the injection of the fluids.

To take nonequilibrium effects into account, Barenblatt et al., introduced the effective saturation (2.6) and supposed relative permeabilities to be functions of it.

$$\omega = S(t + \tau) \approx S + \tau \frac{\partial S}{\partial t}, \quad (2.6)$$

where τ is the relaxation time. The assumption above is based on the fact that in the water-oil displacement the permeability of water should be higher and the permeability of oil should be less than the equilibrium one. This happens because the non-wetting fluid easier displaces the wetting one in large scale pores then in pores with a small size. So, one may assume that relative phase permeabilities depend on a certain effective saturation, which is higher than the real one. The choice of the effective saturation in the form (2.6) pays attention to the effects listed above and allows one to take into account the dependence of the filtration on the velocity of the process.

Equations (2.5) and (2.6) lead to the equation on the effective saturation

$$\frac{\partial \omega}{\partial t} + \frac{1}{\phi} \frac{\partial}{\partial x} [ub(\omega)] + \tau \frac{\partial}{\partial t} \left[\frac{1}{\phi} \frac{\partial}{\partial x} [ub(\omega)] \right] = \frac{\partial}{\partial x} \left[a(\omega) \frac{\partial \omega}{\partial x} \right] + \tau \frac{\partial}{\partial t} \left[\frac{\partial}{\partial x} \left[a(\omega) \frac{\partial \omega}{\partial x} \right] \right], \quad (2.7)$$

with the initial condition

$$\omega + \tau \left[\frac{1}{\phi} \frac{\partial}{\partial x} [ub(\omega)] \right] - \tau \left[\frac{\partial}{\partial x} \left[a(\omega) \frac{\partial \omega}{\partial x} \right] \right] = S. \quad (2.8)$$

In the calculations below it is assumed that $\Delta p \gg \Delta p_{cap}$, which leads to the equations on the effective saturation in the form:

$$\frac{\partial \omega}{\partial t} + \frac{1}{\phi} \frac{\partial}{\partial x} [ub(\omega)] + \tau \frac{\partial}{\partial t} \left[\frac{1}{\phi} \frac{\partial}{\partial x} [ub(\omega)] \right] = 0. \quad (2.9)$$

and the initial conditions can be rewritten as:

$$\omega + \tau \left[\frac{1}{\phi} \frac{\partial}{\partial x} [ub(\omega)] \right] = S. \quad (2.10)$$

For the sake of convenience, let's introduce the dimensionless space and time coordinates. If x_0 is the scale of the domain of calculations, then the time scale is defined as $t_0 = \frac{\phi x_0}{u}$. So, after a substitution

$$\begin{aligned} t' &= t / t_0, \\ x' &= x / x_0, \end{aligned} \quad (2.11)$$

one obtains:

$$\partial_{t'} \omega + \partial_{x'} b(\omega) + \tau \partial_{t'}^2 b(\omega) = 0, \quad (2.12)$$

The equation (2.12) can be written in the form of the system of equations

$$\begin{aligned} \partial_{t'} v &= -\frac{v - g(z)}{\tau}, \\ \partial_{x'} z &= \frac{v - g(z)}{\tau}, \end{aligned} \quad (2.13)$$

where $v = \omega + \tau \partial_{x'} b(\omega)$, $z = b(\omega)$, $g(z) = b^{-1}(z)$ - the inverse to Buckley's functions.

The system (2.13) is solved numerically in the present paper.

Another non-equilibrium model has been developed by V.I. Kondaurov in [2]. Its key-point is an introduction of an auxiliary parameter (here and after nonequilibrium parameter) which is proportional to the area of the surface of the contact of the non-wetting fluid and the skeleton. The evolution of the nonequilibrium parameter is governed by the kinetic equation

$$\frac{\partial \xi}{\partial t} = -C \frac{\partial \Psi}{\partial \xi}, \quad (2.14)$$

where Ψ is a Helmholtz free energy of the fluids and the skeleton and C is a constant. Such choice of the kinetic equation guarantees nonnegative dissipation of capillary forces.

An explicit expression for Ψ is unknown, so one may consider its polynomial approximation. Taking into account terms of the first and the second order in the oil saturation and nonequilibrium parameter one may obtain:

$$\Psi = \frac{1}{2} \beta \xi^2 + \frac{1}{2} M s^2 - \alpha \xi s + \gamma s. \quad (2.15)$$

In accordance with (2.15), equation (2.14) takes form:

$$\frac{\partial \xi}{\partial t} = \frac{1}{\tau} \Lambda(s, \xi) = \frac{1}{\tau} \left(\frac{\alpha}{\beta} (1 - S) - \xi \right), \quad (2.16)$$

In the present paper the dependence of relative phase permeabilities on the saturation and nonequilibrium parameter is modified to achieve the coincidence of two models, when $\tau \rightarrow 0$, namely, (2.16) gives

$$\tau \frac{\beta}{\alpha} \frac{\partial \xi}{\partial t} = 1 - S - \frac{\beta}{\alpha} \xi. \quad (2.18)$$

The equation (2.16) gives rise to the following expression:

$$\frac{\beta}{\alpha} \tau \frac{\partial}{\partial t} \xi = -\tau^2 \frac{\beta}{\alpha} \frac{\partial^2 \xi}{\partial t^2} - \tau \frac{\partial}{\partial t} S.$$

Substituting this into (2.20), one derives

$$\frac{\beta}{\alpha} \xi = 1 - S + \tau \frac{\partial S}{\partial t} + \frac{\beta}{\alpha} \tau^2 \frac{\partial^2 \xi}{\partial t^2}.$$

Finally, the nonequilibrium parameter can be expressed as the power series in τ :

$$\frac{\beta}{\alpha} \xi = 1 - S + \tau \frac{\partial S}{\partial t} - \tau^2 \frac{\partial^2 S}{\partial t^2} + \tau^3 \frac{\partial^3 S}{\partial t^3} + \dots$$

Taking into account terms of the first and zero order in τ , we have

$$S + \tau \frac{\partial S}{\partial t} = 2S + \frac{\beta}{\alpha} \xi - 1. \quad (2.19)$$

It seems to be reasonable to hold calculations with Buckley's function of the following form:

$$b(S, \xi) = \frac{f_1(2S + \frac{\beta}{\alpha}\xi - 1)}{f_1(2S + \frac{\beta}{\alpha}\xi - 1) + \mu f_2(2 - 2S - \frac{\beta}{\alpha}\xi)} = b_e(S + \tau \frac{\partial S}{\partial t}). \quad (2.20)$$

The final form of the governing equations is

$$\begin{aligned} \frac{\partial S}{\partial t} + \frac{1}{\phi} \frac{\partial}{\partial x} ub(S, \xi) &= 0, \\ \frac{\partial \xi}{\partial t} &= \frac{1}{\tau} \Lambda(S, \xi). \end{aligned} \quad (2.21)$$

The initial and boundary conditions are

$$S(x, 0) = 0, \xi(x, 0) = \xi_0 = \frac{\alpha}{\beta}, \quad S(0, t) = 1, \xi(0, t) = 0, \quad S(x_0, t) = 0, \xi(x_0, t) = \xi_0. \quad (2.22)$$

The case of strong nonequilibrium effects is of special interest. If $\tau \rightarrow \infty$ then the system (2.21) can be written in the form

$$\begin{aligned} \frac{\partial S}{\partial t} + \frac{1}{\phi} \frac{\partial}{\partial x} ub(S, \xi) &= 0, \\ \frac{\partial \xi}{\partial t} &= 0. \end{aligned} \quad (2.23)$$

(2.22) gives:

$$\frac{\partial S}{\partial t} + \frac{1}{\phi} \frac{\partial}{\partial x} ub(S, \xi_0) = 0. \quad (2.24)$$

Finally, we obtain a classical model with modified Buckley's function.

3. Comparison of numerical results.

In the present section results of numerical investigations are discussed. It was assumed that water saturation is zero at the very beginning of the displacement and that water is injected in the point $x = 0$. The following values of parameters are used in calculations below: $x_0 = 100$ m, $u = 10^{-6}$ m/s, $\mu_1 = 10^{-2}$ Pa/s, porosity $\phi = 10^{-2}$, permeability $k = 10^{-14}$, capillary pressure constant

$$M = 10^4 \text{ Pa}, \alpha = \beta = 0.1M, \text{ then, } t_0 = \frac{x_0 \phi}{u} = 10^6 \text{ s.}$$

A numerical scheme of the second order of accuracy is used to calculate the spatial distribution of the saturation in a fixed moment of time. This scheme has been introduced in [5] and has the form;

$$\begin{aligned} z_{j+1}^n &= z_j^n + \frac{\Delta x}{2\tau} (v_j^n + v_{j+1}^n - g(z_{j+1}^n) - g(z_j^n)) , \\ v_j^{n+1} &= \theta v_j^n + \frac{\Delta t}{2\tau} (g(z_j^{n+1}) + \theta g(z_j^n)) . \end{aligned} \quad (3.1)$$

Where n, j - spatial and time indexes respectively, Δx and Δt are spatial and time steps and $\exp -\Delta t/\tau$ is denoted by θ .

WENO-5 scheme is used for the numerical solution of the system (2.21).

The results of the calculations are listed below. Here and after the dimensionless space and time coordinates are used, i.e. $t' = t/t_0$ and $x' = x/x_0$ is used instead of t and x .

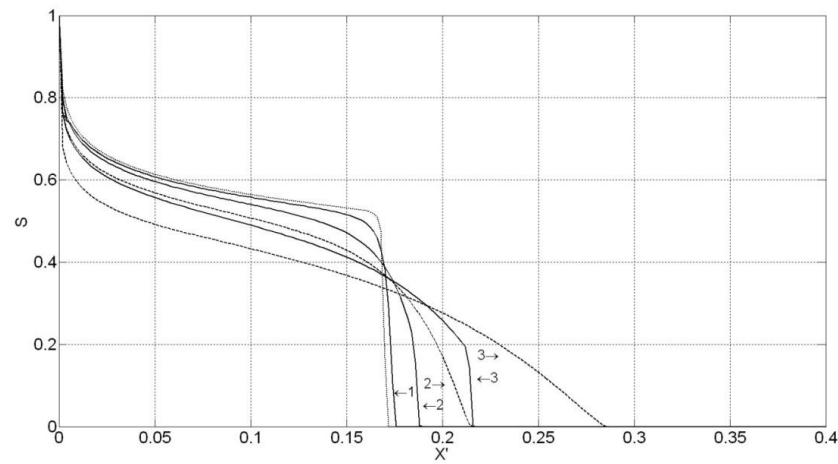


Figure1. The dependence of water saturation on spatial coordinate is depicted for the cases of the different models and different τ . Dashed lines correspond to the first model and solid lines represent the second model.

Lines 1, 2, 3 represent the cases of $\tau/t_0 = 0.001, 0.01, 0.1$. If $\tau/t_0 = 0.001$ then the distributions of water saturation are very similar.

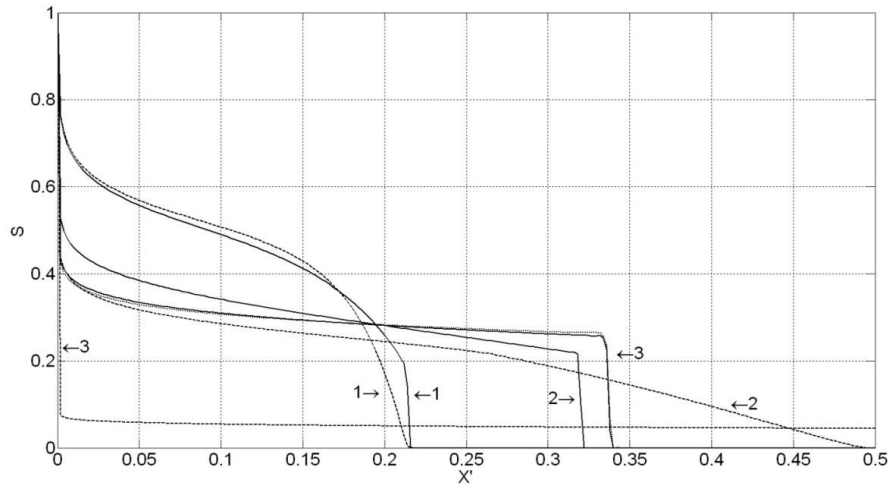


Figure 2. The dependence of water saturation on spatial coordinate is depicted for the cases of $\tau/t_0=0.01, 0.1, 1$ (lines 1,2,3 respectively). Barenblatt's and Kondraurov's models are represented by dashed and solid lines respectively. The case of $\tau/t_0=1000000$ is depicted by dotted line (it is slightly different from the case $\tau/t_0=1,0$)

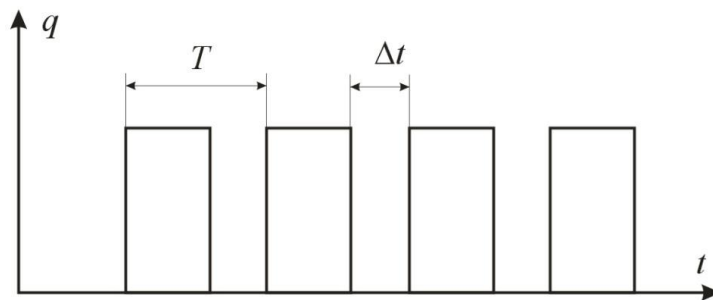


Figure3. Sketch of the pulse injection. q is a volume of water injected per unit time

The case of pulse injection of water is of great interest. According to (2.9) there is no change of the saturation according to Barenblatt's model, when $u = 0$. If latter is the case, the evolution of the saturation is described by the equation $\partial S / \partial t = 0$. So, the distribution of water saturation is completely determined by the total volume of the injected fluid and by the rate of the injection and doesn't depend on Δt (see Fig.3). This statement isn't valid for another non-equilibrium model and it is illustrated in the Fig.4.

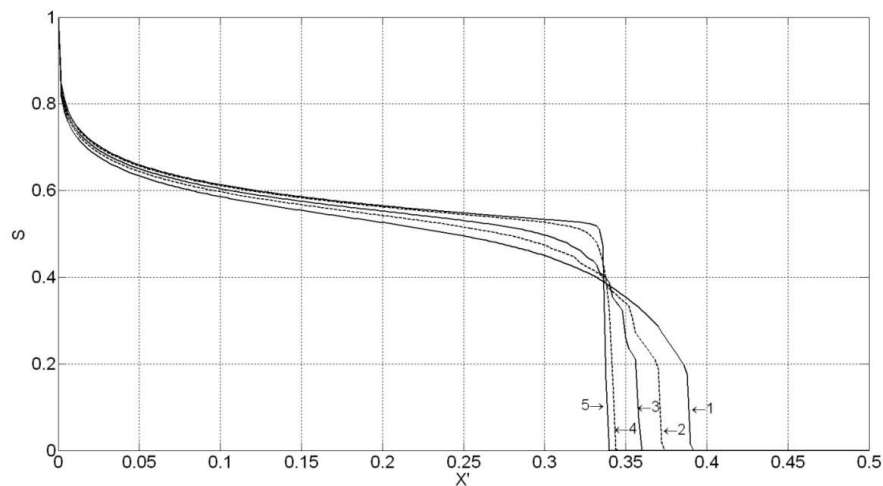


Figure 4. The distribution of the saturation according to the second model in the case $T = 2\tau$, $f = 0.05, 0.50, 0.75$, and 0.95 is represented by lines 1, 2, 3, 4 respectively. Line 5 is a classical solution

4. Conclusion.

The behavior of the solutions at infinite τ and the comparison of the solutions to the problem with piecewise constant rate of displacement gives ground for some physical arguments. At $\tau \rightarrow 0$ the both models behave in a similar way in accordance with derivation. As follows from the solutions as $\tau \rightarrow \infty$ the velocity of displacement front in the case of Barenblatt's model tends to infinity. On the other hand the similarity solution to the model [2] with modified permeabilities tends to some limiting solution with finite speed of the front propagation. From physical point of view the displacement at $\tau \rightarrow \infty$ is the displacement in the largest pores network (the liquids can not redistribute in the pore space under capillary forces). If the net volume of the network is finite the velocity of the displacement front must be finite too.

Another difference between the models is illustrated by solutions to the problem of displacement with the piecewise constant rate $W(t)$ shown in fig.1. The models of Barenblatt's type are unable to account for the effects of capillary relaxation which take place at the time intervals where $U(t)=0$ and accordingly $dS/dt=0$. Thus the solution depends only on U_0 and the total volume of the injected fluid and does not depend on the time interval between pulses Δt . Even the saturation is constant the redistribution of the fluids in the pore space under capillary forces takes place unless the equilibrium state is achieved. In the model (2) this is taken into account by the kinetic equation for the capillary nonequilibrium parameter ξ . The initial state at which the next pulse occurs depends on Δt , the effect is the most notable when $\Delta t \sim \tau$.

One may conclude that even with simple approximation of thermodynamic potential the model [2] can eliminate some shortcomings of models based on dynamical relations.

5. References.

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