# Fluid Mechanics

Kafi Shabbir

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# Chapter 1

# Pressure

## 1.1 Surface direction and pressure

In the the prism.

The forces are given by:

$$F_x = p_x \Delta y d \tag{1.1}$$

$$F_y = p_y \Delta x d \tag{1.2}$$

$$F_n = p_n \Delta s d \tag{1.3}$$

Relation between the surface areas:

$$\Delta x = \Delta s \cos(\theta) \tag{1.4}$$

$$\Delta y = \Delta s \sin(\theta) \tag{1.5}$$

Forces in terms of $\Delta s$:

$$F_x = p_x \Delta s \sin(\theta) d \tag{1.6}$$

$$F_y = p_y \Delta s \cos(\theta) d \tag{1.7}$$

$$F_n = p_n \Delta s d \tag{1.8}$$

Balance of forces:

$$F_x = F_n \sin(\theta) \tag{1.9}$$

$$F_y = F_n \cos(\theta) + \rho \frac{1}{2} \Delta x \Delta y d g \tag{1.10}$$

Balance of pressures:

$$p_x \Delta s \sin(\theta) d = (p_n \Delta s d) \sin(\theta) \tag{1.11}$$

$$p_y \Delta s \cos(\theta) d = (p_n \Delta s d) \cos(\theta) + \rho \frac{1}{2} \Delta x \Delta y d g \tag{1.12}$$

Simplifying:

$$p_x = p_n \tag{1.13}$$

$$p_y = p P_n + \rho \frac{1}{2} \frac{\Delta x \Delta y}{\Delta s \cos(\theta)} g \tag{1.14}$$

$$p_y = p_n + \rho \frac{1}{2} \Delta y g \tag{1.15}$$

For small prism:

$$\Delta y \to 0 \tag{1.16}$$

$$p_y = p_n \tag{1.17}$$

Pressure does not depend on the orientation of the surface.

## 1.2 Difference of pressures between two points

Consider a cube.

Along the $x - axis$, the forces:

$$F_x(x) = p(x) dy dz \tag{1.18}$$

$$F_x(x + dx) = -p(x + dx) dy dz \tag{1.19}$$

The net force:

$$dF_x = F_x(x) + F_x(x + dx) \tag{1.20}$$

$$dF_x = p(x) dy dz - p(x + dx) dy dz \tag{1.21}$$

$$dF_x = -(p(x + dx) - p(x)) dy dz \tag{1.22}$$

$$dF_x = -\frac{\partial p}{\partial x} dx dy dz \tag{1.23}$$

The infinitesimal volume is given by:

$$dV = dxdydz \tag{1.24}$$

Let $f$ be force per unit volume:

$$f = \frac{dF}{dV} \tag{1.25}$$

Then:

$$f_x = -\frac{\partial p}{\partial x} \tag{1.26}$$

$$\mathbf{f} = -\left(\frac{\partial p}{\partial x}\mathbf{i} + \frac{\partial p}{\partial y}\mathbf{j} + \frac{\partial p}{\partial z}\mathbf{k}\right) \tag{1.27}$$

$$\mathbf{f_p} = -\nabla p \tag{1.28}$$

$$p = p(x, y, z) \tag{1.29}$$

$$dp = \frac{\partial p}{\partial x}dx + \frac{\partial p}{\partial y}dy + \frac{\partial p}{\partial z}dz \tag{1.30}$$

$$d\mathbf{l} = dx\mathbf{i} + dy\mathbf{j}dz\mathbf{k} \tag{1.31}$$

$$dp = (\nabla p) \cdot d\mathbf{l} \tag{1.32}$$

Taking dot product of $d\mathbf{l}$ on both sides of equation 1.28

$$\mathbf{f_p} \cdot d\mathbf{l} = -(\nabla p) \cdot d\mathbf{l} \tag{1.33}$$

$$\mathbf{f_p} \cdot d\mathbf{l} = -dp \tag{1.34}$$

Force due to gravity per unit volume:

$$\mathbf{dF_g} = \rho dV\mathbf{g} \tag{1.35}$$

$$\mathbf{f_g} = \rho\mathbf{g} \tag{1.36}$$

The force due to the pressure difference is balanced by the weight:

$$\mathbf{f_p} = -f_g \tag{1.37}$$

Taking integral on both sides:

$$\int_{\mathbf{a}}^{\mathbf{b}} \mathbf{f_g} \cdot d\mathbf{l} = \int_{\mathbf{a}}^{\mathbf{b}} dp \tag{1.38}$$

$$p(\mathbf{b}) - p(\mathbf{a}) = \int_{\mathbf{a}}^{\mathbf{b}} \mathbf{f_g} \cdot d\mathbf{l} \tag{1.39}$$

Because of this, the fluid at the bottom of arbitary shaped tank is the same.

$$p(\mathbf{b}) - p(\mathbf{a}) = \int_{\mathbf{a}}^{\mathbf{b}} \rho\mathbf{g} \cdot d\mathbf{l} \tag{1.40}$$

For constant density and gravity:

# Chapter 2

# ch02

## 2.1 Hello

The world is a nice place.

# Chapter 3

# ch03

## 3.1 Hello

The world is a nice place.

# Chapter 4

# ch04

## 4.1 Hello

The world is a nice place.