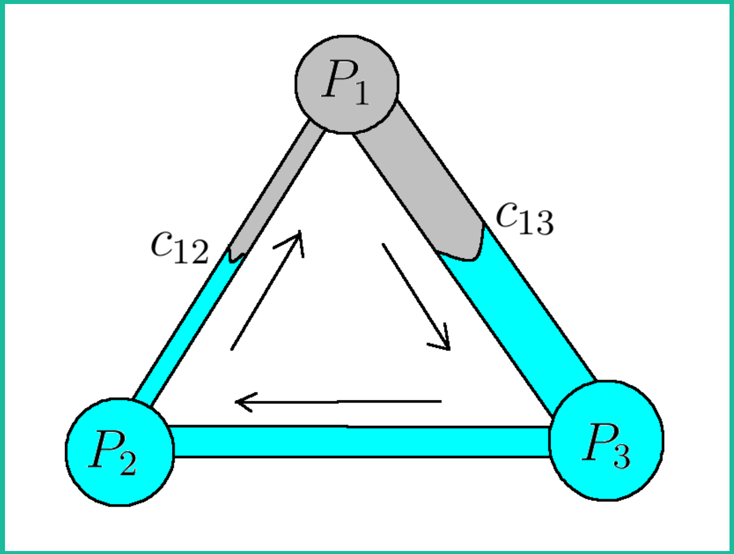

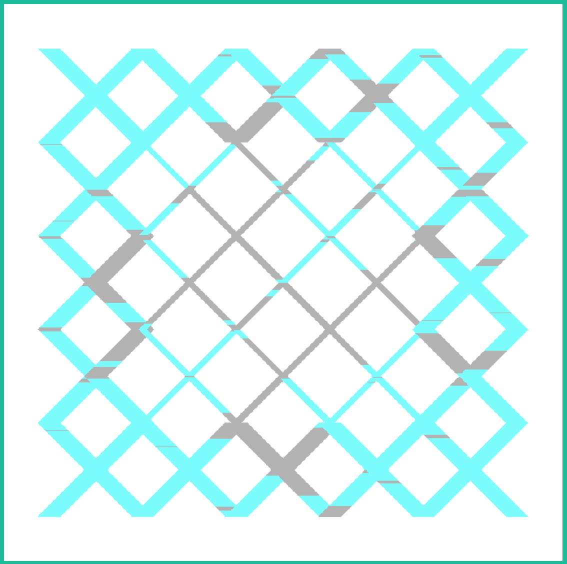

Models of nonequilibrium flow in porous medium

Bachelor Thesis Presentation

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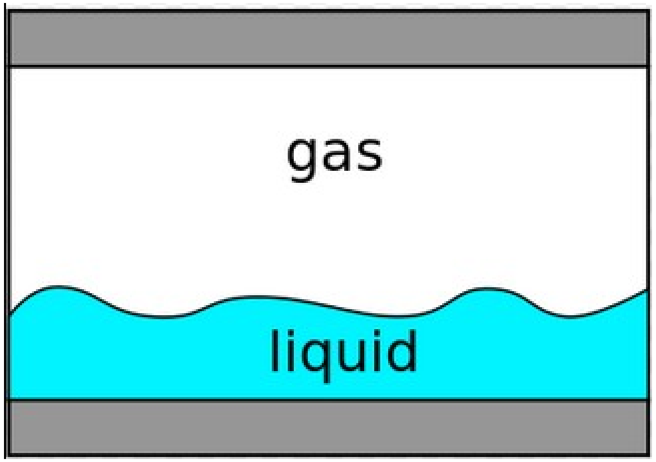
Group: Б03-910

Date: 21.06.2023

Department of Applied Mechanics, MIPT

Motivation – Two phase flow

- Oil recovery
- Hydrology
- Electricity production



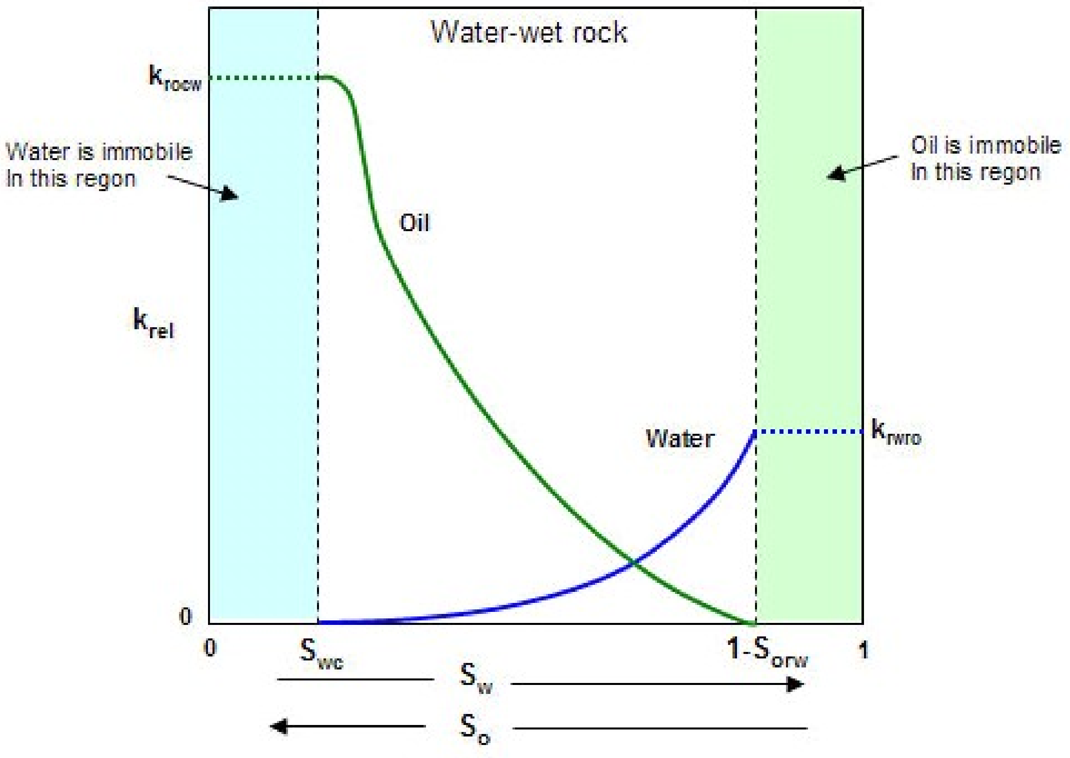
Classical Continuum models

Darcy's Law

- $$q = -\frac{k}{\mu}\nabla p$$

$$k = k(S)$$

$$S = \frac{dV_w}{dV_{void}}$$



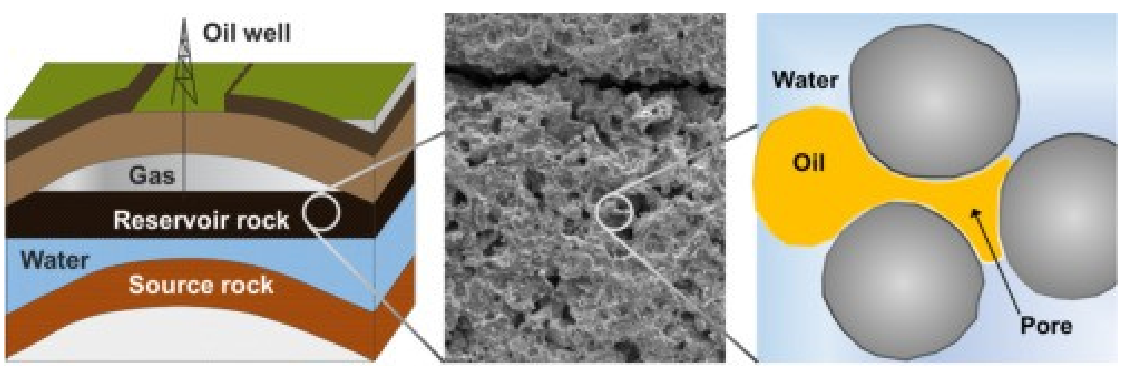
Limitation of Classical Models

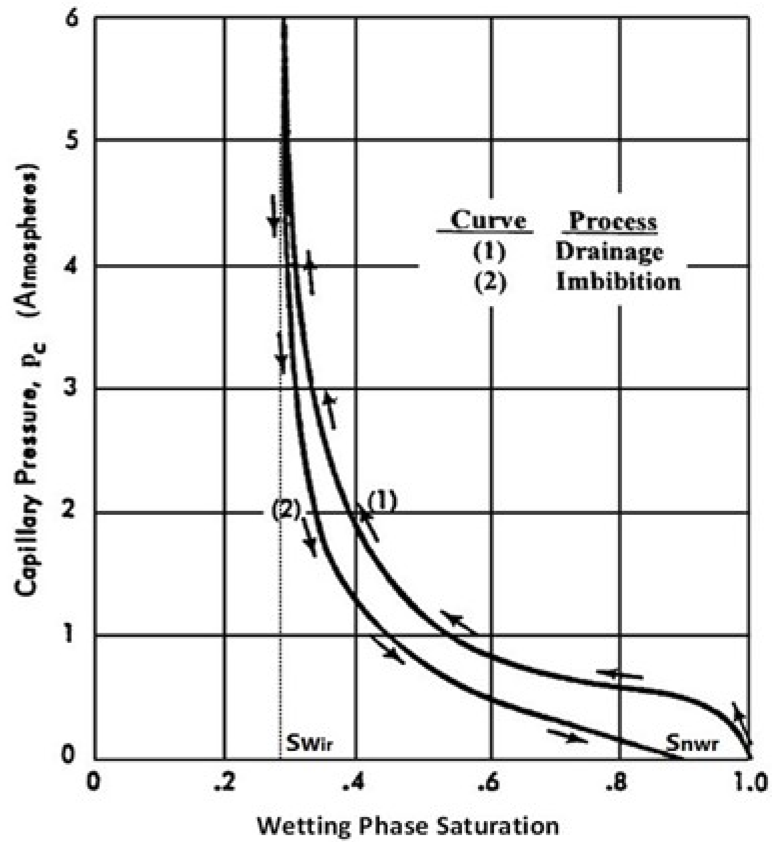
$$k = k(S)$$

Only true when

Characteristic time of the processes is much greater than characteristic time of fluid redistribution in the capillary space.

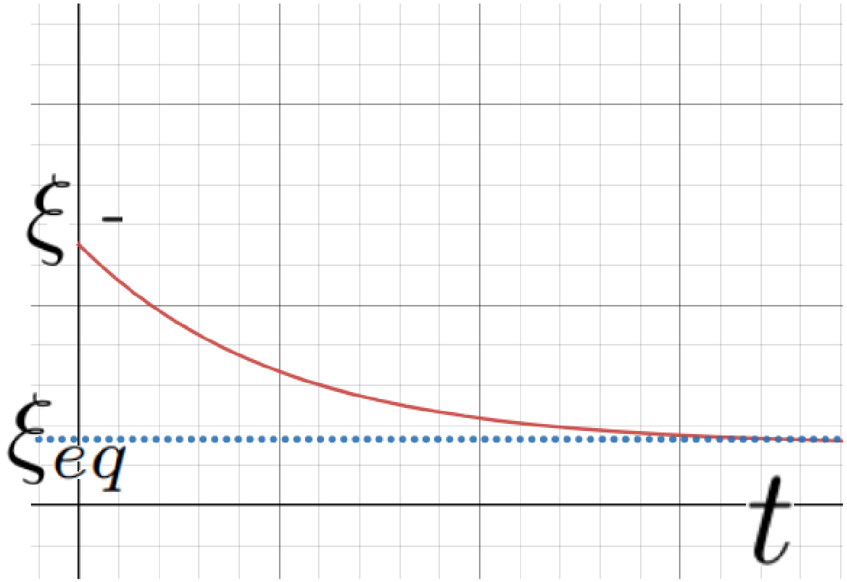
Fractured-porous medium with blocks and cracks





Advanced Continuum models

- **Hasanizadeh[6], Barenblatt[7]**
- **Kondaurov model[1]**



$$q = -\frac{k}{\mu}\nabla p$$

$$k = k(S, \frac{\partial S}{\partial t})$$

$$k = k(S, \xi)$$

$$\frac{\partial \xi}{\partial t} = \Omega(S(t), \xi)$$

Pore Scale

- **Direct Navier-Stokes Simulation**
- **Network Models**

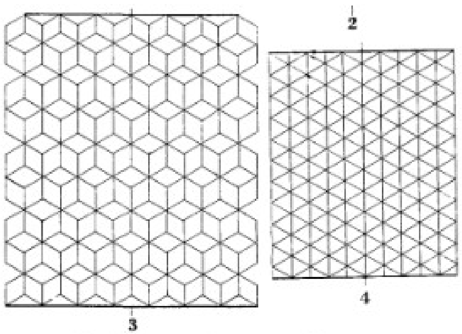


FIG. 1—SINGLE HEXAGONAL NETWORK.
FIG. 2—SQUARE NETWORK.
FIG. 3—DOUBLE HEXAGONAL NETWORK.
FIG. 4—TRIPLE HEXAGONAL NETWORK.

the same channel. Multiphase fluid flow in separate channels in porous media has been termed channel flow, while concentric flow in the same channel has been termed filament flow. The best observational support for the assumption of channel flow comes from the cinemicrophotographic studies of Chatenevar, and from the Stanolind group. Leverett has shown on theoretical grounds that the interfaces between two immiscible phases in porous media will distribute themselves so that the radii of curvature of the interfaces, r_1 and r_2, in the pore spaces will obey the LaPlace equation

$$P = \delta\left(\frac{1}{r_1} + \frac{1}{r_2}\right) \quad . \ . \ . \ . \ . \ . \ . \ . \ . \ (1)$$

[1] Fatt I. The network model of porous media: Model using Resistors 1956

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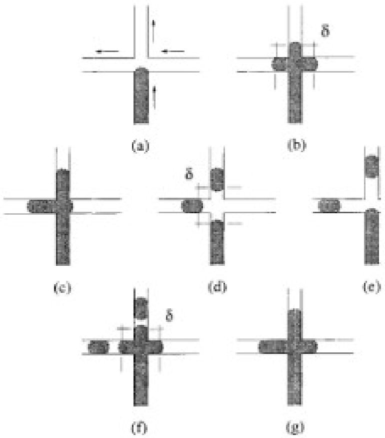


Figure 7. A 'mixture' of non-wetting (shaded) and wetting (white) which flow into the neighbor tubes. The different arrangements (a)–(g) are a result of applying the rules which are described earlier in this section. For all figures the fluids flow towards the node from the bottom and right tube while the fluids in the top and the left tube flow away from it (denoted by the arrows in (a)).

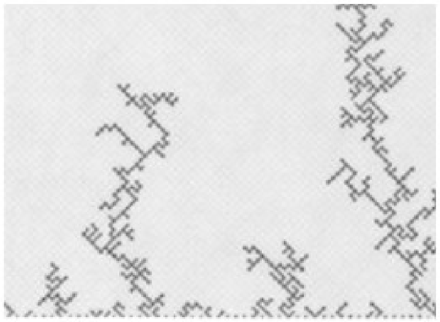
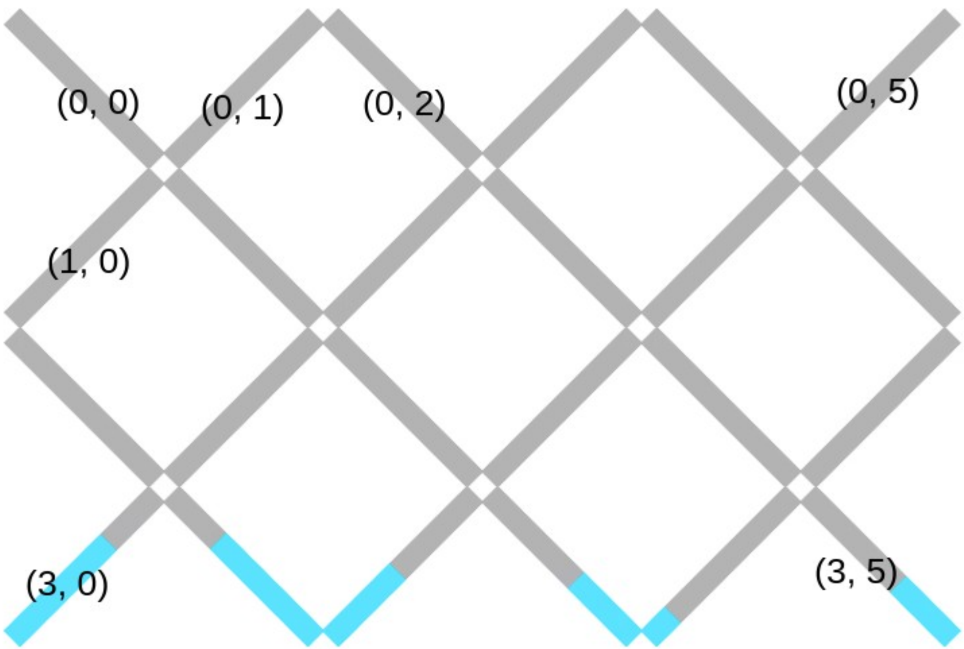


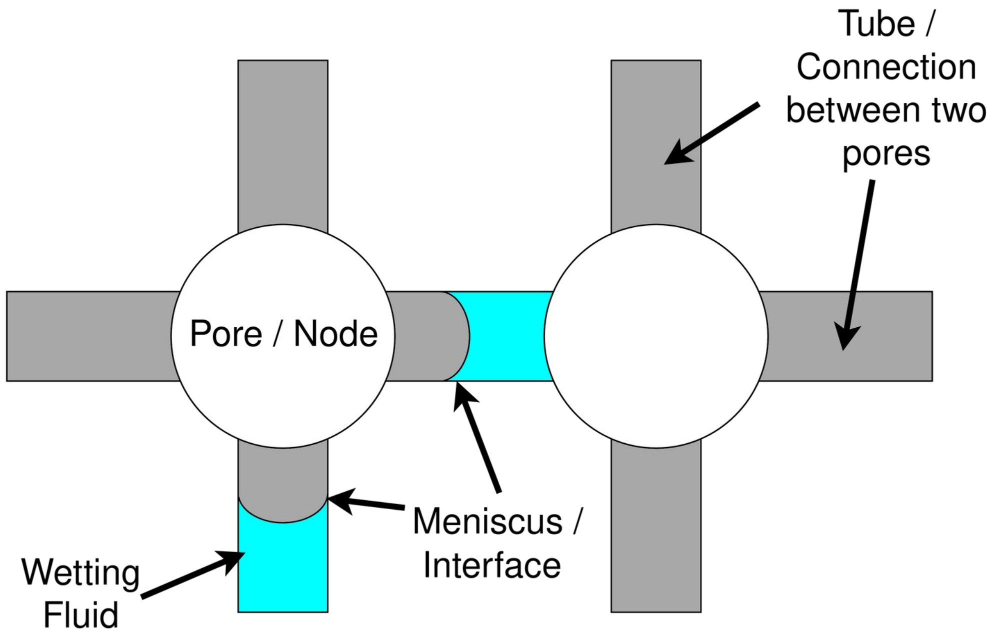
Figure 8. The pattern obtained of a simulation in the regin of 60 × 80 nodes. $C_a = 4.6 \times 10^{-3}$ and $M = 1.0 \times 10^{-}$ (black) displaces the defending wetting fluid (gray) from $7\frac{1}{2}$ h on a Cray T90 vector machine.

Aker, E., Måløy, K.J., Hansen, A., Batrouni, G.G. A two-dimensional network simulator for two-phase flow in porous media // Transp. Porous Med. 1998 V. 32 P. 163

Our Model

- **Zero node volume**
- **Cylinder tubes**
- **Flow due to capillary pressure**
- **Maximum two meniscus**





Algorithm implemented in C++

1) Read initial and radius
2) Linear Equations for Pressure
3) Solution using Gaussian-Elimination
4) Velocity
5) Time Step
6) Volume Distribution (New Method)
7) Recombination

$$\sum Q_i = 0 \quad (1)$$

$$Q = \frac{\pi R^4}{8 M l}\left(\Delta P + \frac{2 s \sigma}{R}\right) \quad (2)$$

$$M = \sum \mu_i \frac{l_i}{l} \quad (3)$$

$$Q = \frac{\pi R^4}{8 \mu} \frac{\Delta P}{l} \quad (4)$$

Flow Rate of a Node

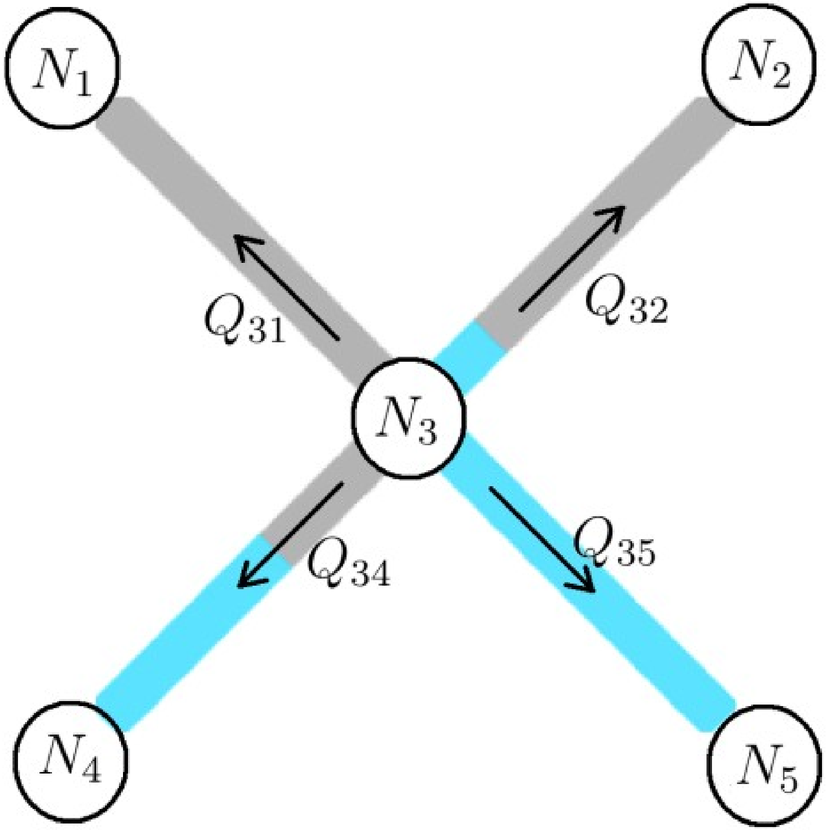
$$Q_{31} = \frac{\pi R_{31}^3}{8 l M_{31}} (R_{31} \Delta P_{31} + 2 s_{31} \sigma)$$

$$Q_{32} = \frac{\pi R_{32}^3}{8 l M_{32}} (R_{32} \Delta P_{32} + 2 s_{32} \sigma)$$

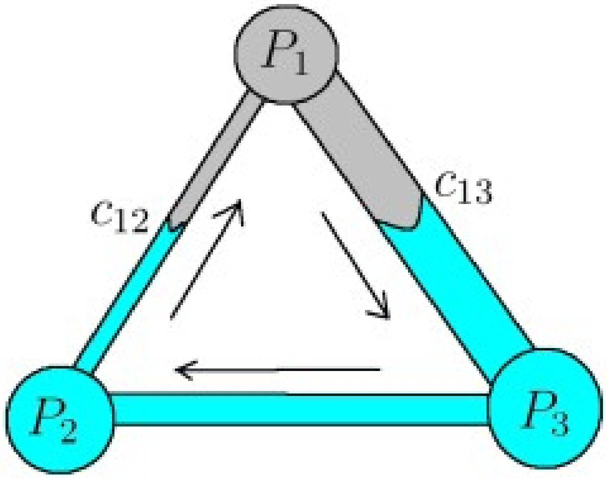
$$Q_{34} = \frac{\pi R_{34}^3}{8 l M_{34}} (R_{34} \Delta P_{34} + 2 s_{34} \sigma)$$

$$Q_{35} = \frac{\pi R_{35}^3}{8 l M_{35}} (R_{35} \Delta P_{35} + 2 s_{35} \sigma)$$

$$\sum_k Q_{3k} = 0$$



Case of Infinitely many solutions



$$q_{ij} = k_{ij}\Delta P + c_{ij}$$

$$q_{12} = k_{12}(P_1 - P_2) + c_{12}$$

$$q_{13} = k_{13}(P_1 - P_3) + c_{13}$$

$$q_{12} + q_{13} = 0$$

$$(k_{12} + k_{13})P_1 - k_{12}P_2 - k_{13}P_3 = -c_{12}$$

$$\begin{pmatrix} (k_{12}+k_{13}) & -k_{12} & -k_{13} & -c_{12}-c_{13} \\ -k_{21} & (k_{21}+k_{23}) & -k_{23} & -c_{21}-c_{23} \\ -k_{31} & -k_{32} & (k_{31}+k_{32}) & -c_{31}-c_{32} \end{pmatrix}$$

$$k_{ij} = k_{ji}$$

$$c_{ij} = -c_{ji}$$

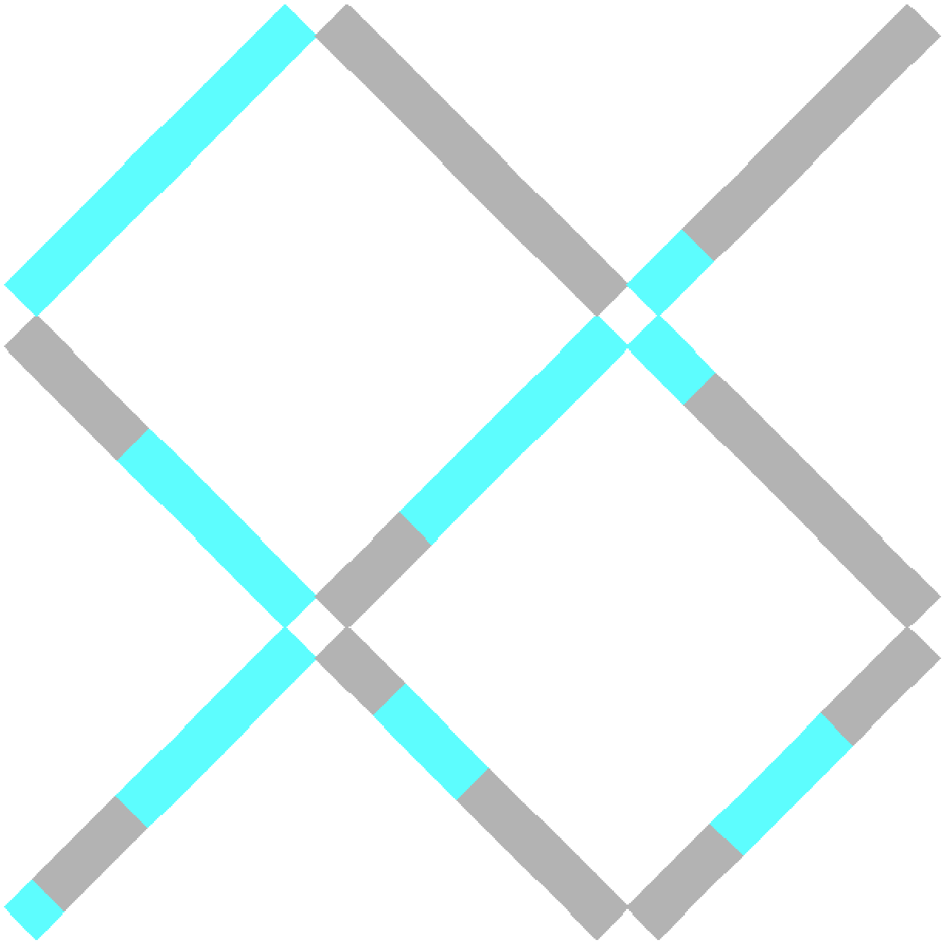
$$\begin{pmatrix} (k_{12}+k_{13}) & -k_{12} & -k_{13} & -c_{12}-c_{13} \\ -k_{21} & (k_{21}+k_{23}) & -k_{23} & -c_{21}-c_{23} \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} (k_{12}+k_{13}) & -k_{12} & -k_{13}+a & -c_{12}-c_{13} \\ -k_{21} & (k_{21}+k_{23}) & -k_{23}+a & -c_{21}-c_{23} \\ 0 & 0 & 3a & 0 \end{pmatrix}$$

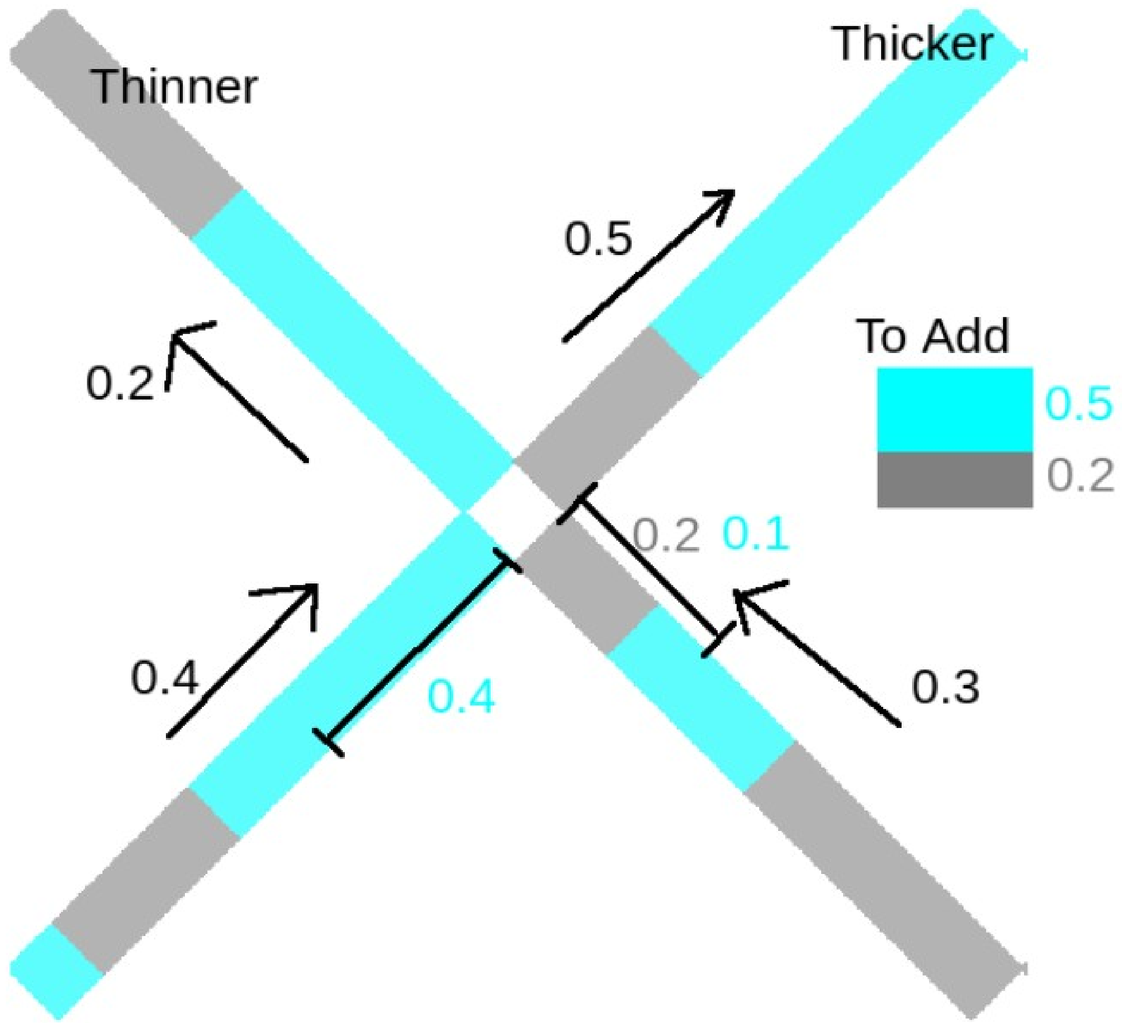
$$3aP_3 = 0$$

Meniscus Configuration Data

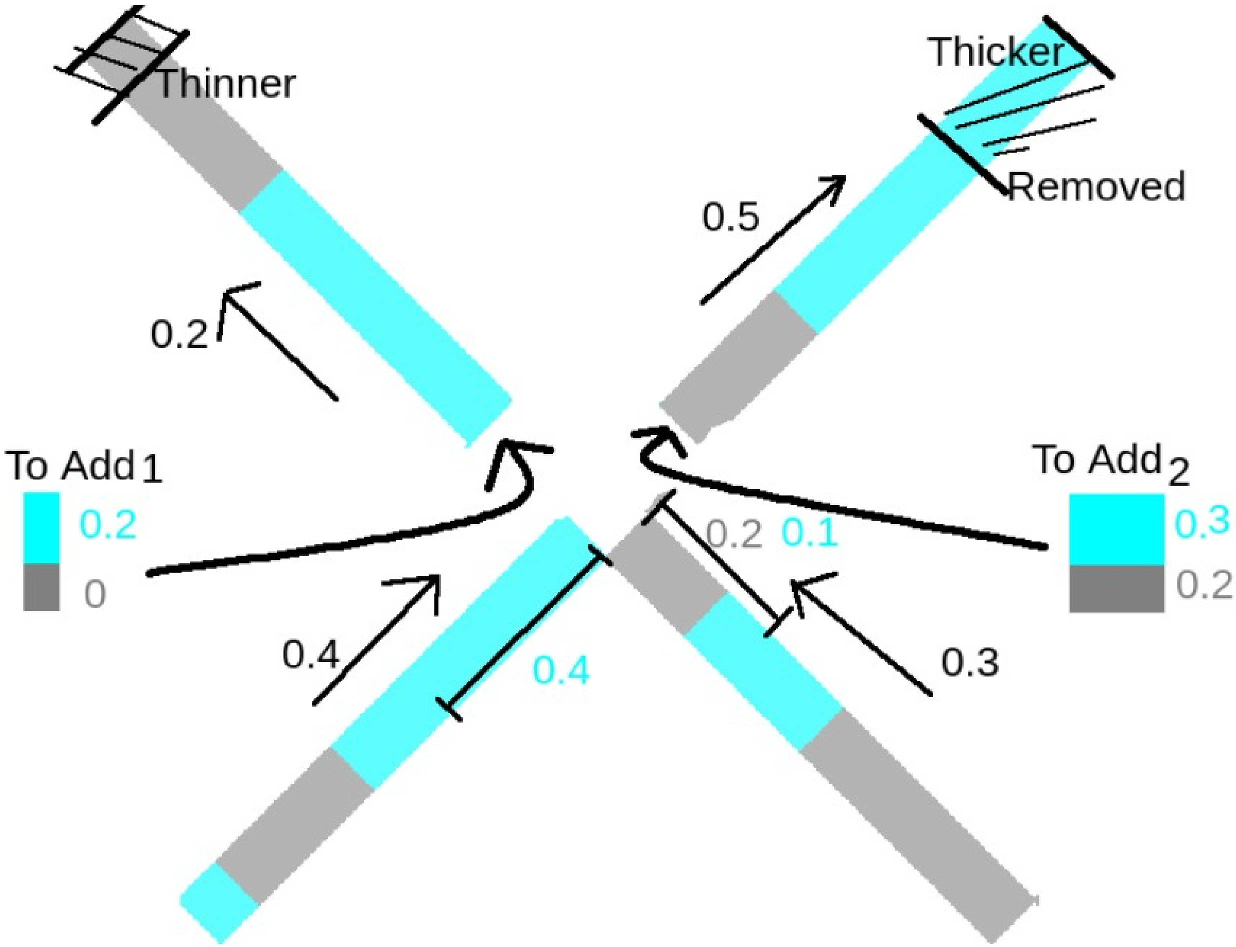
- [n, type, pos1, pos2]
- 0 0 0 0
- 0 1 0 0
- 1 0 0.2 0
-
- 1 0 0.6 0
- 1 1 0.3 0
- 1 1 0.8 0
-
- 2 0 0.1 0.4
- 2 1 0.5 0.8
- 2 1 0.3 0.7



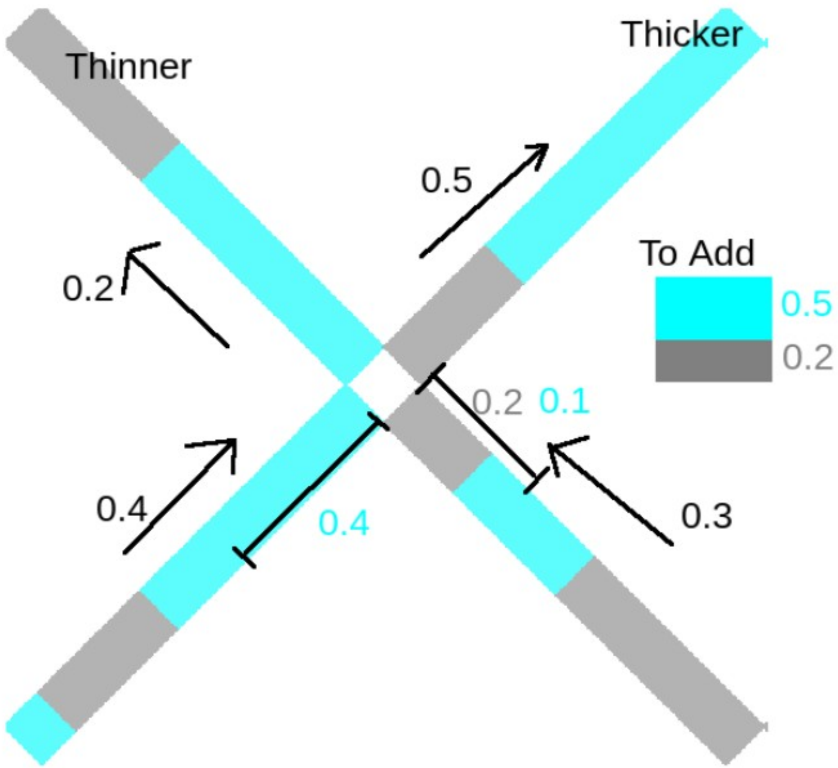
New Feature: Distribution of Phases at the nodes

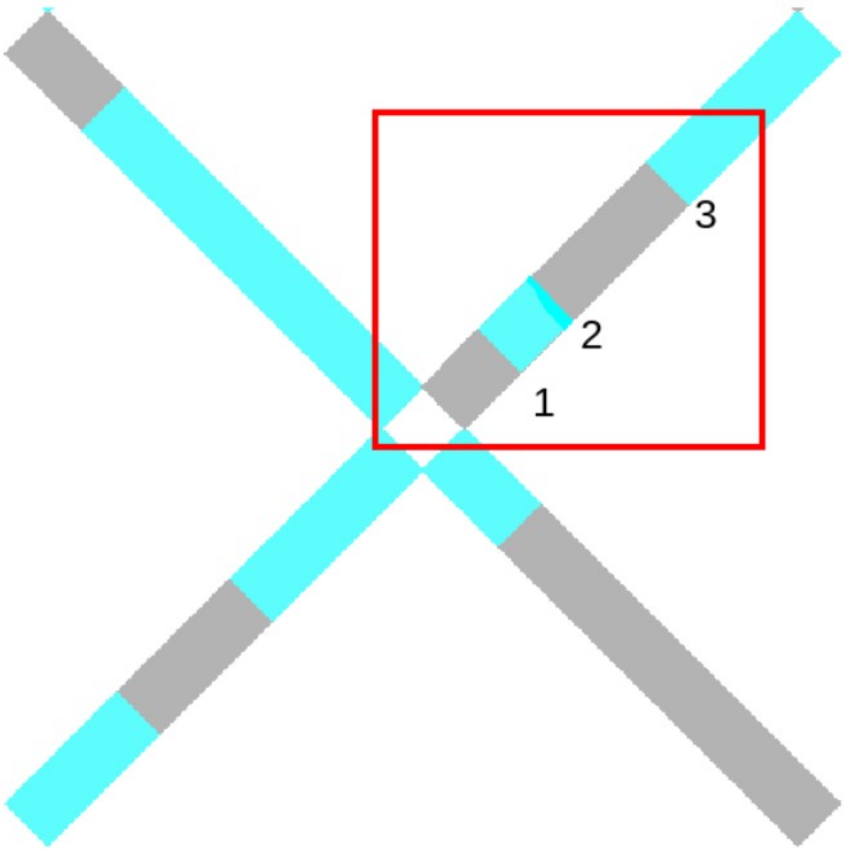


New Feature: Distribution of Phases at the nodes



New Feature: Distribution of Phases at the nodes





C++ Implementation Moving Meniscus Forward in Tube

```cpp
func::Integration::TFluid func::Integration::calculate_fluid_table
(
    const Tdouble& radius,
    const TMns& mnsc,
    const Tdouble& velocity,
    const Tdouble& volume,
    const dst::Diamension& diamension,
    const double time_step
)
{
    TFluid fluid_addition_table = diamension.empty_table_templated<Fluid>();

    for(int row = 0; row < diamension.node_rows(); ++ row)
    {
        for(int col = 0; col < diamension.node_cols(row); ++ col)
        {
            Tank tank;
            std::vector<Tube_FromNode> tubes_from_node_vec;

            const std::vector<dst::Tube> tubes_connected_vec =
                diamension.generate_tubes_connected_to_node(row, col);

            const int connections_size = tubes_connected_vec.size();
            for(int direction = 0; direction < connections_size; ++ direction)
            {
                const dst::Tube& connection = tubes_connected_vec[direction];
                if(connection.active)
                {
                    const double rad = radius[connection.row][connection.col];
                    const dst::Mns& mns = mnsc[connection.row][connection.col];
                    const double vel = velocity[connection.row][connection.col];

                    if(mns.is_the_flow_from_tube_into_node(direction, vel))
                    {
                        const std::vector<double> add_fluid =
                            mns.vol_fluid_into_nodes(
                                rad,
                                direction,
                                vel,
                                time_step,
                                declconst::TUBE_LENGTH
                            );
```

```cpp
                            );

                        tank.add_fluid(add_fluid);
                        continue;
                    }

                    Tube_FromNode tube
                    {
                        rad,
                        connection.row,
                        connection.col
                    };

                    tubes_from_node_vec.push_back(tube);
                }
            }


            std::sort(tubes_from_node_vec.begin(),
                tubes_from_node_vec.end(),
                compare_where_wetting_fluid_go_first);


            for(const Tube_FromNode& tpshin: tubes_from_node_vec)
            {
                fluid_addition_table[tpshin.row][tpshin.col]
                    = tank.pour_out_fluid(volume[tpshin.row][tpshin.col]);
            }

        }
    }

    return fluid_addition_table;
}
```

Recombination, centre of mass

Grey: $m_1 = l_1$ at $d_1 = \frac{l_1}{2}$,

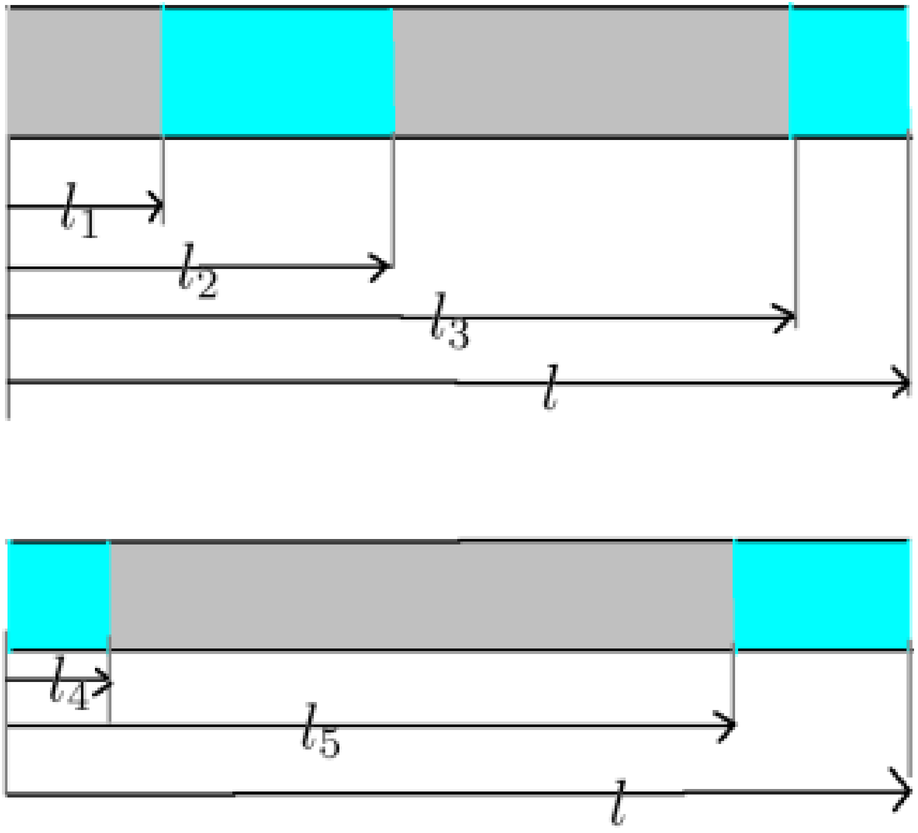
$m_2 = l_3 - l_2$ at $d_2 = \frac{(l_3+l_2)}{2}$

$$m = m_1 + m_2$$

$$c_{grey} = \frac{m_1 d_1 + m_2 d_2}{m}$$

$$l_4 = c_{grey} - \frac{m}{2}$$

$$l_5 = l_4 + m$$



Recombination, centre of mass

```
dst::Mns::PosNew_Type_Result dst::Mns::centre_of_mass_recombination
(
    const bool type_begin,
    const std::vector<double>& pos_new
) const
{
    if(pos_new.size() < 3)
    {
        return {type_begin, pos_new};
    }

    bool new_type_begin = type_begin;
    double l1 = -1;
    double l2 = -1;
    double l3 = -1;
    double l4 = -1;

    if(pos_new.size() == 3)
    {
        if(type_begin)
        {
            l1 = pos_new[0];
            l2 = pos_new[1];
            l3 = pos_new[2];
            l4 = 1;
        }
        else
        {
            l1 = 0;
            l2 = pos_new[0];
            l3 = pos_new[1];
            l4 = pos_new[2];
            new_type_begin = !type_begin;
        }
    }
    if(pos_new.size() == 4)
    {
        l1 = pos_new[0];
        l2 = pos_new[1];
        l3 = pos_new[2];
        l4 = pos_new[3];
    }

    return {new_type_begin, centre_of_mass_equation(l1, l2, l3, l4)};
}
```

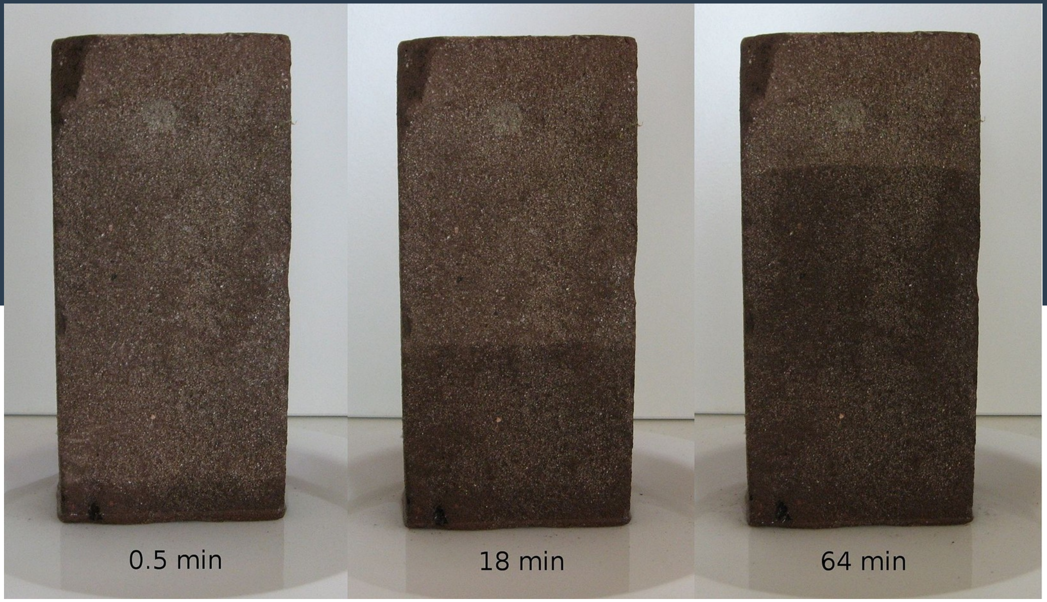
```
std::vector<double> dst::Mns::centre_of_mass_equation
(
    const double l1,
    const double l2,
    const double l3,
    const double l4
)
{

    const double d1 = l2 - l1;
    const double d2 = l4 - l3;
    const double d = d1 + d2;
    const double c1 = (l1 + l2) / 2;
    const double c2 = (l3 + l4) / 2;

    const double L = (c1 * d1 + c2 * d2) / d;
    const double L1 = L - d / 2;
    const double L2 = L1 + d;

    return std::vector<double> {L1, L2};
}
```

Test 1



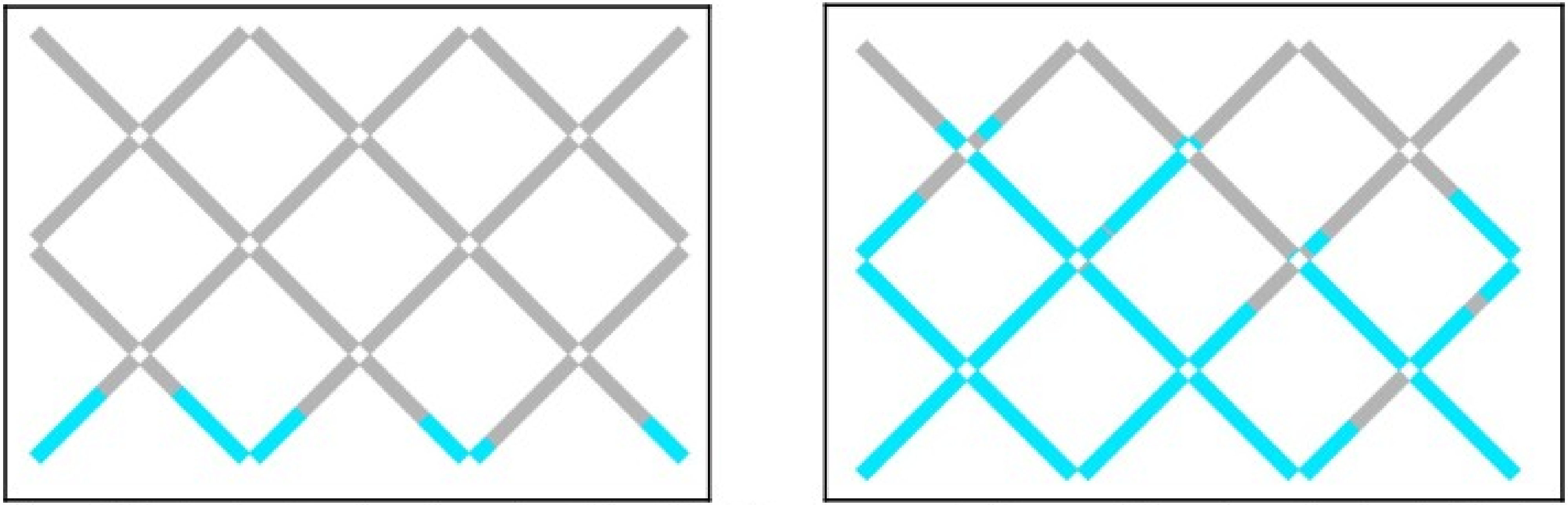
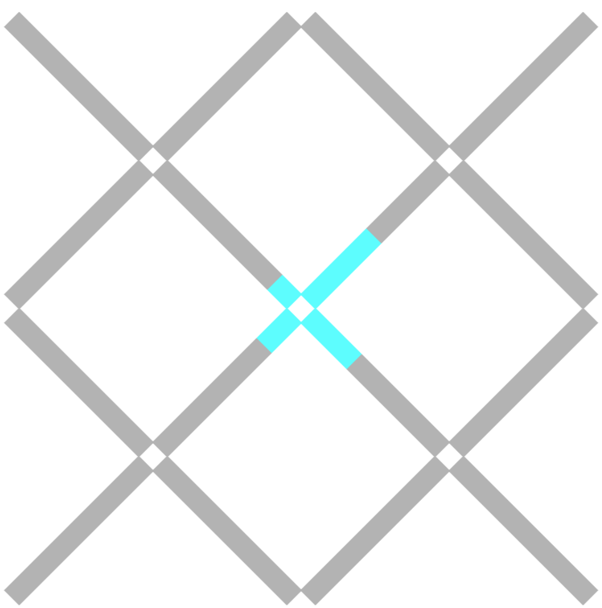


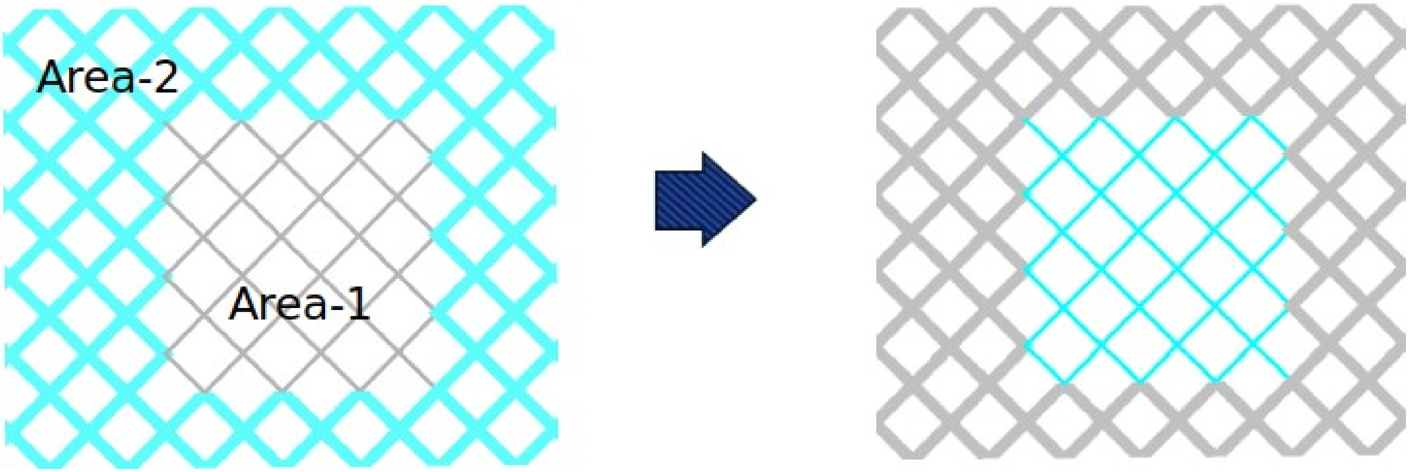
Figure 3 & 4: showing invasion of wetting fluid into a porous body consisting of randomly distributed tube radii for a highly simplified model (4 x 6), at saturation levels of 22% and 61% using our model.

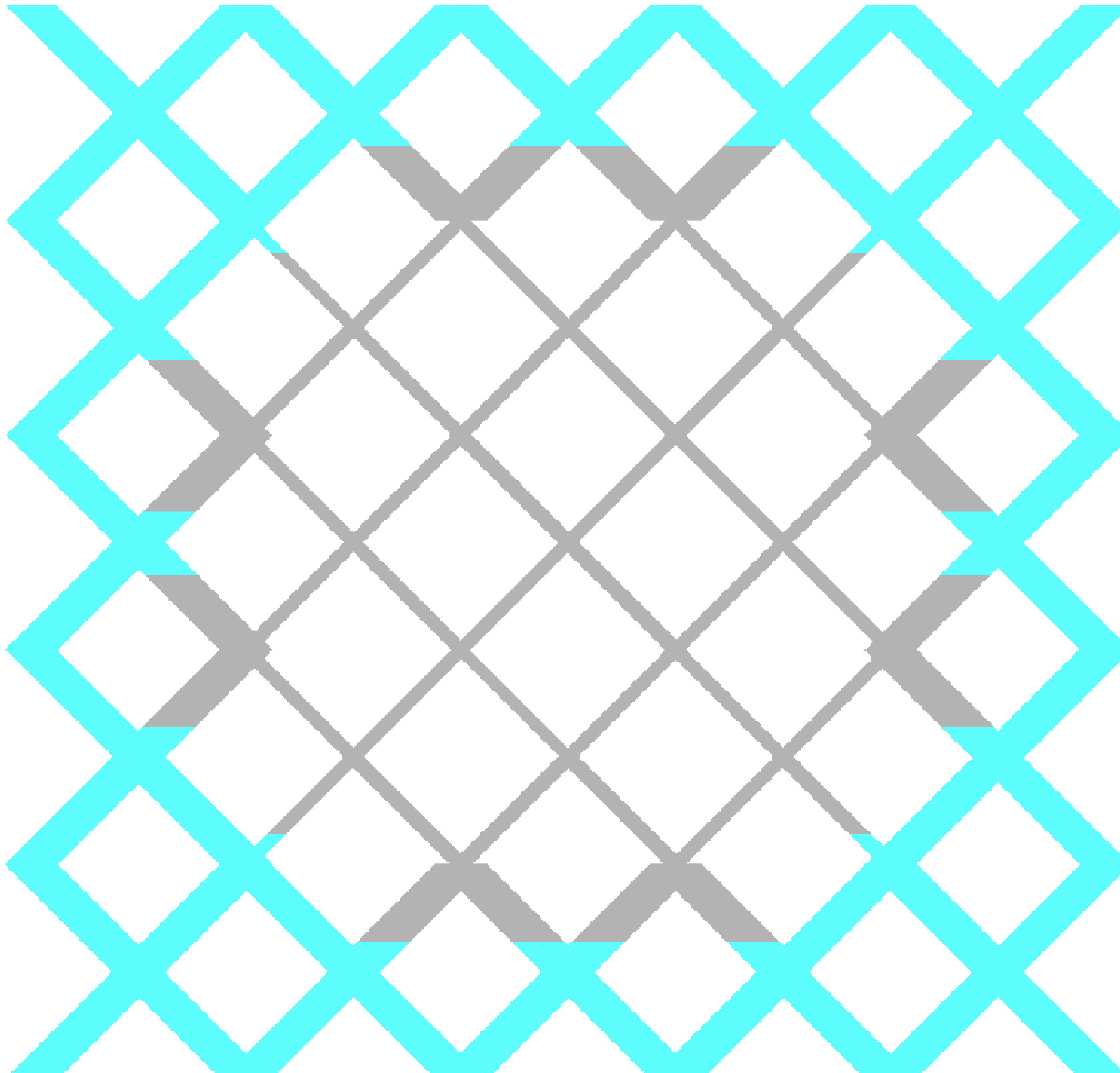
Test 2

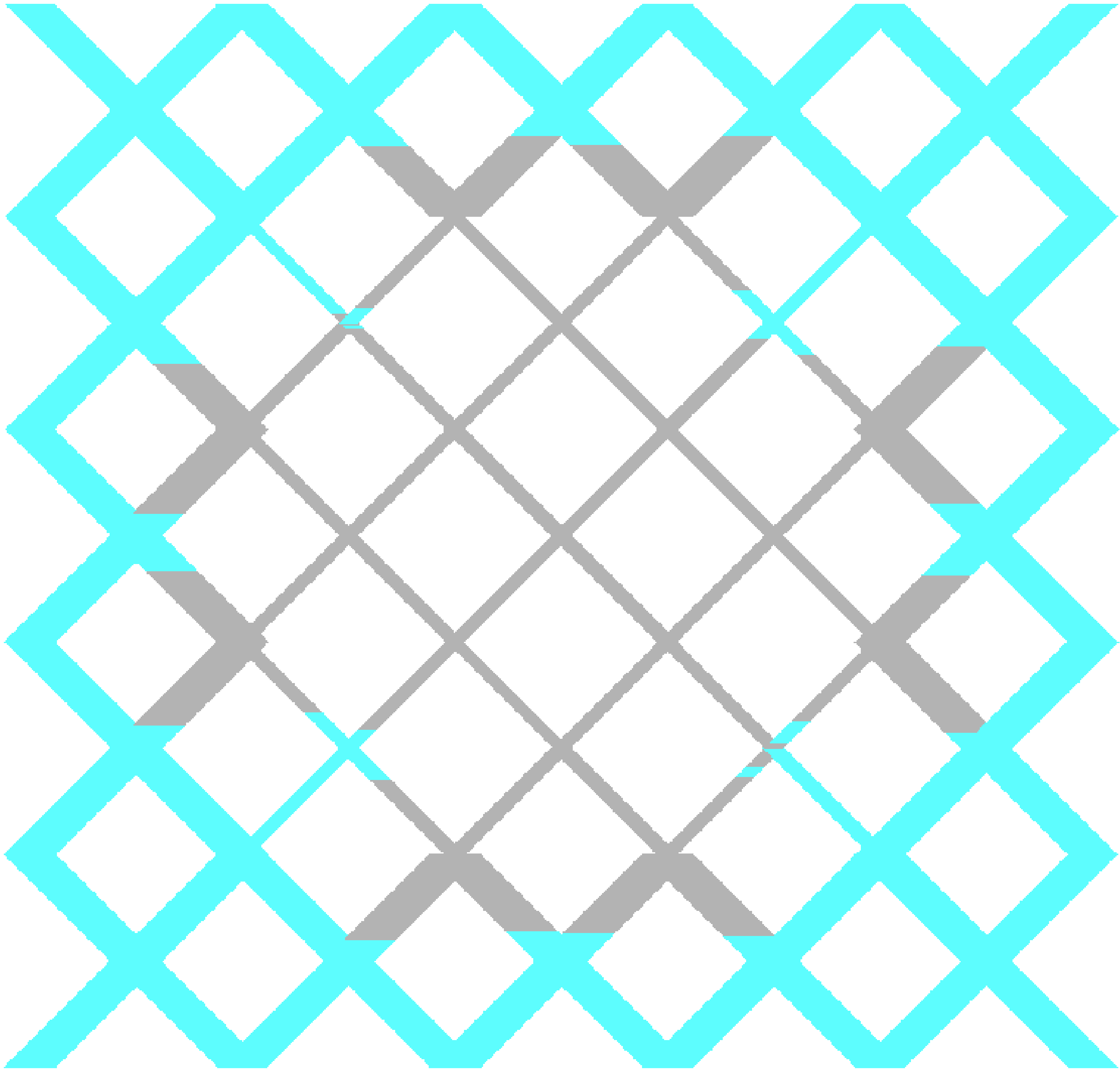


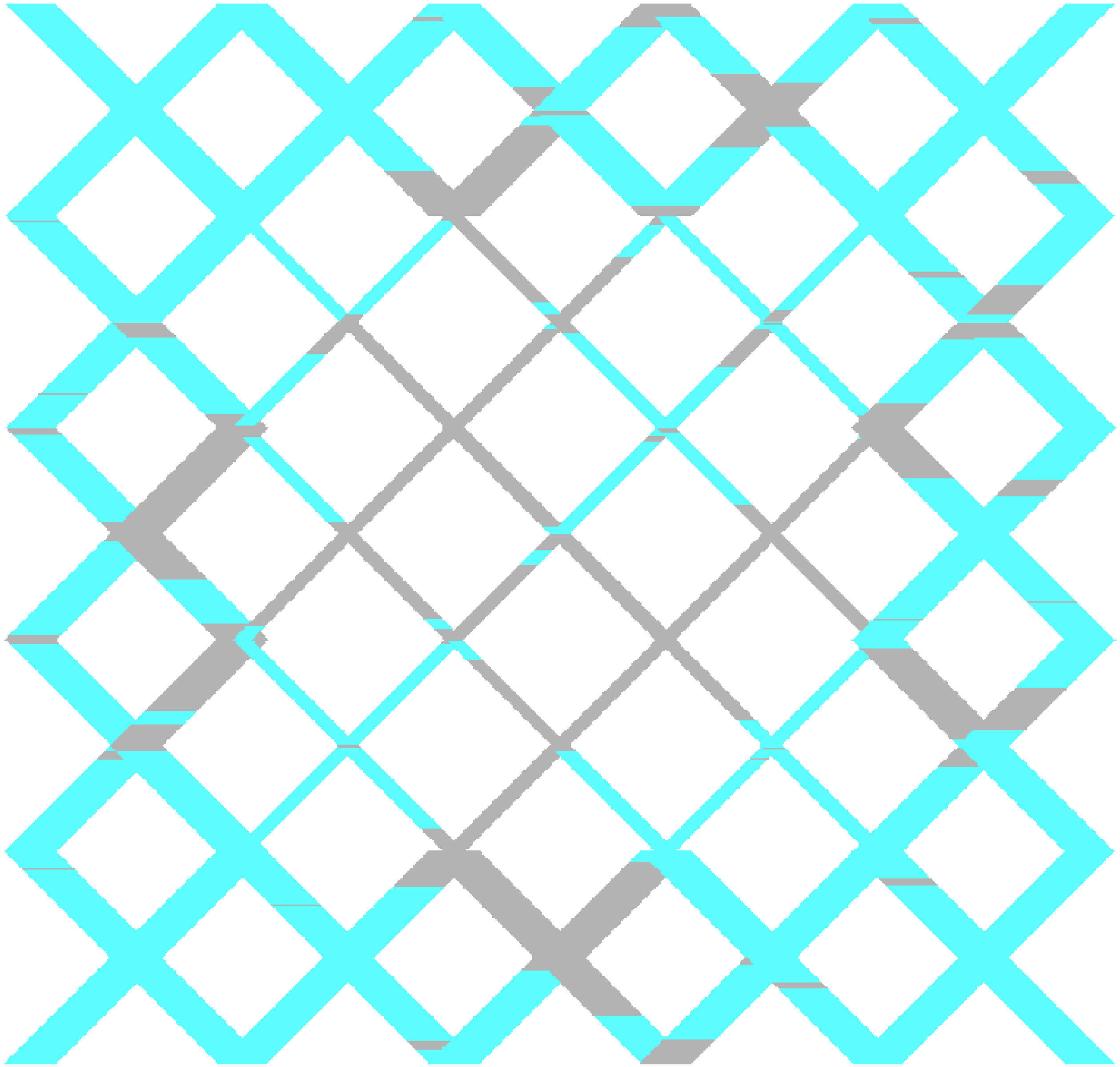
Main Problem

- **Closed Boundaries**
- **$V_w = V_{nw}$**
- **$R_{out} = 3\ R_{in}$**
- **Measure S(t)**
- **Expected:**

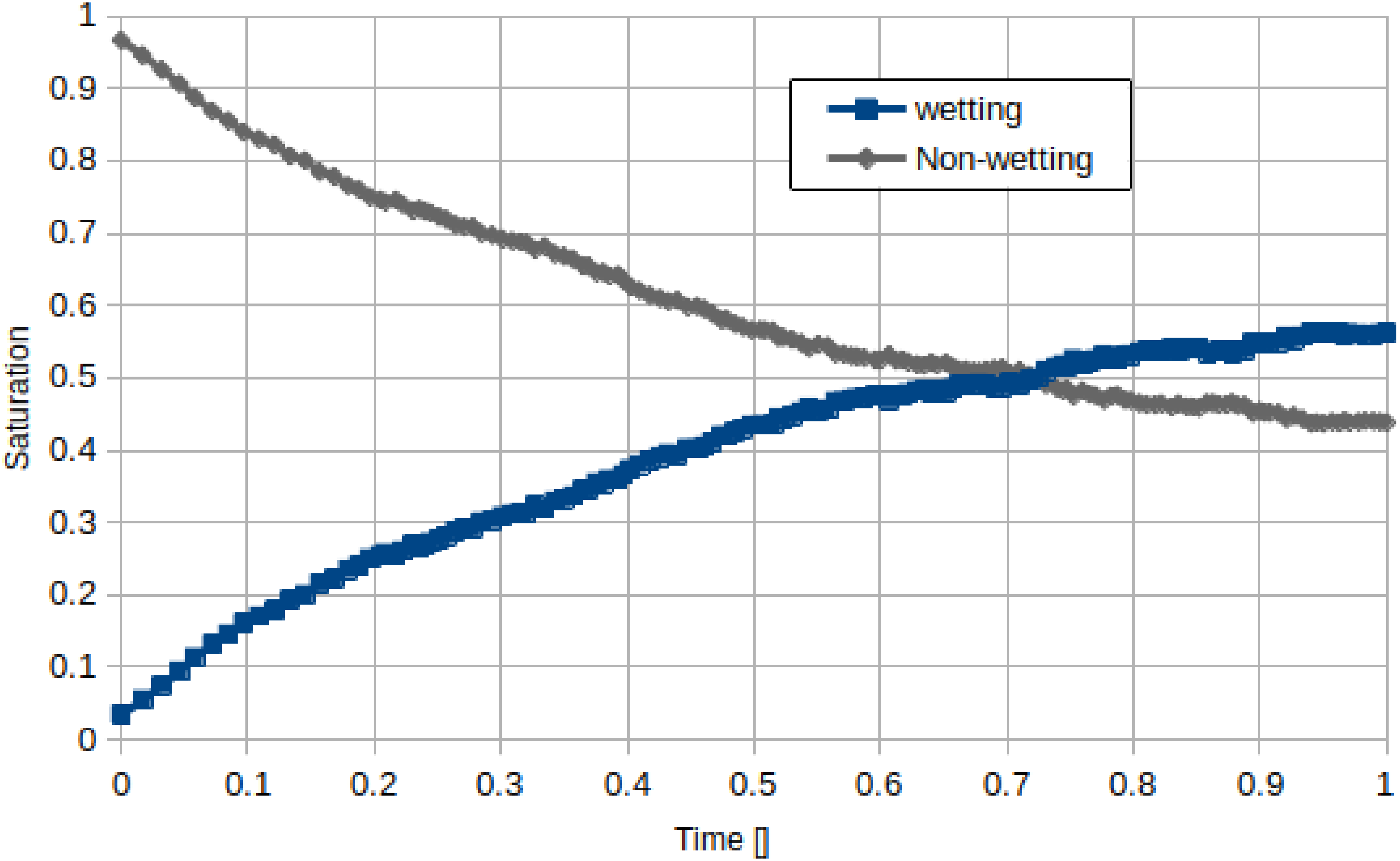




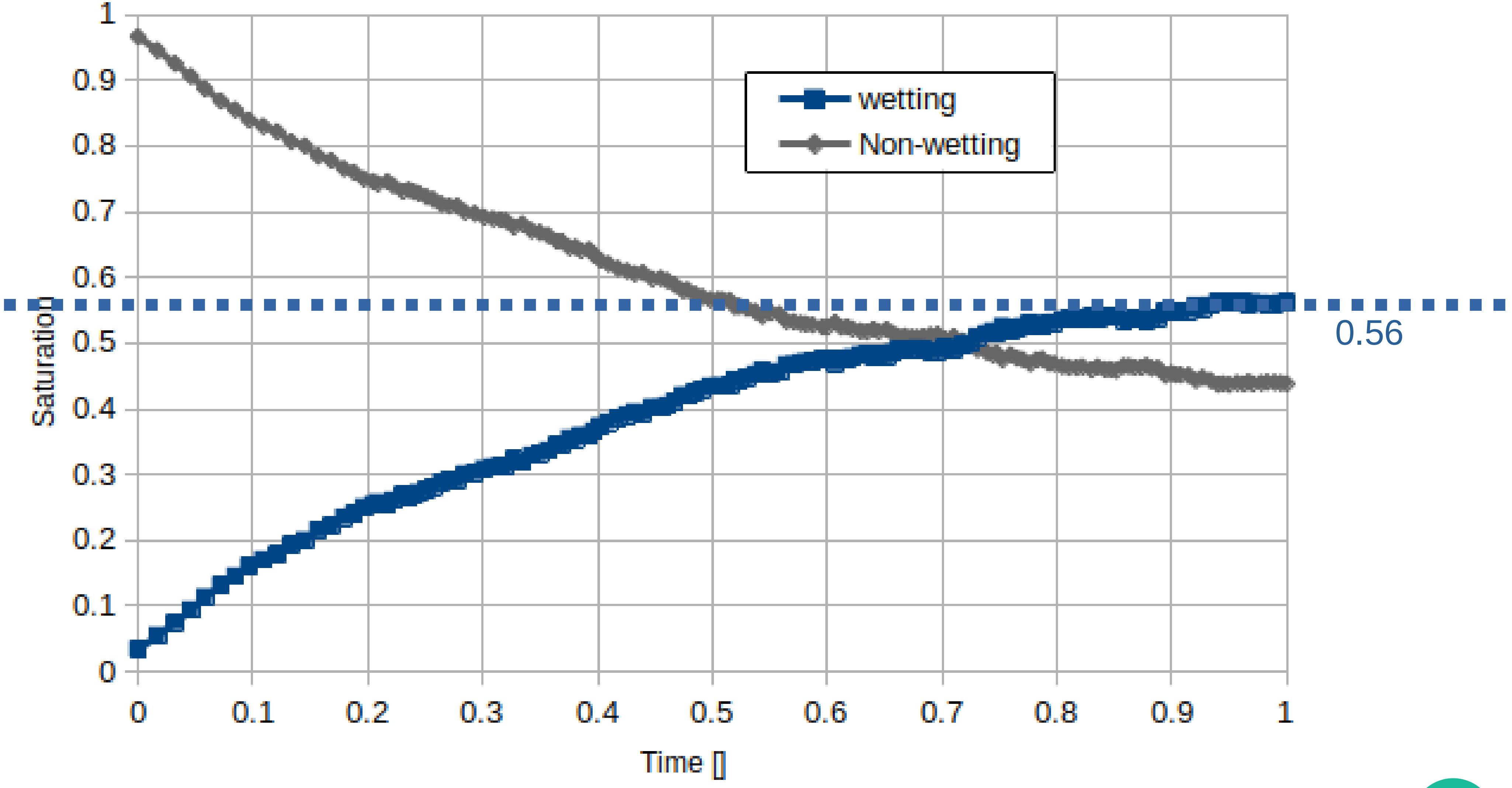


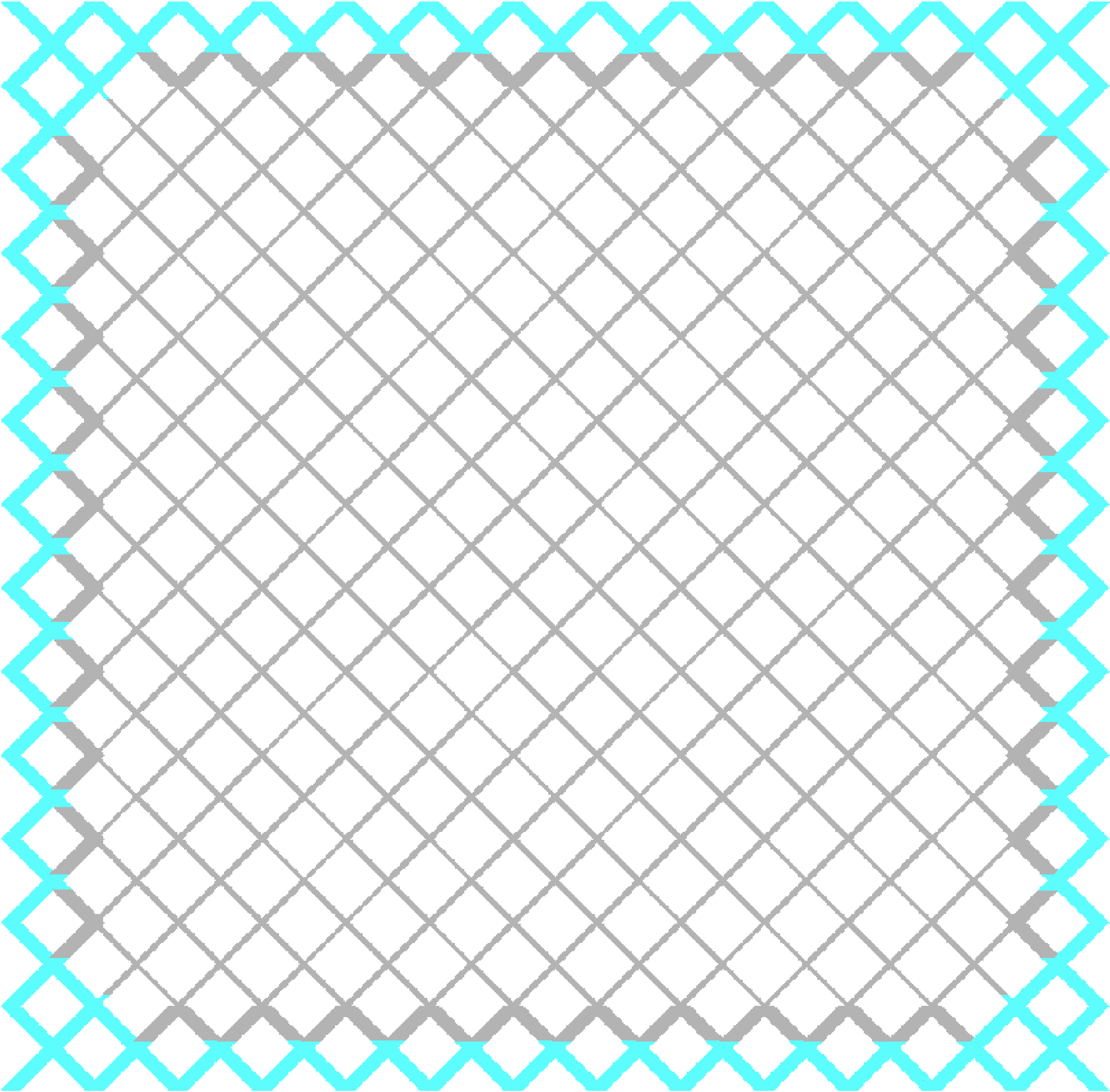


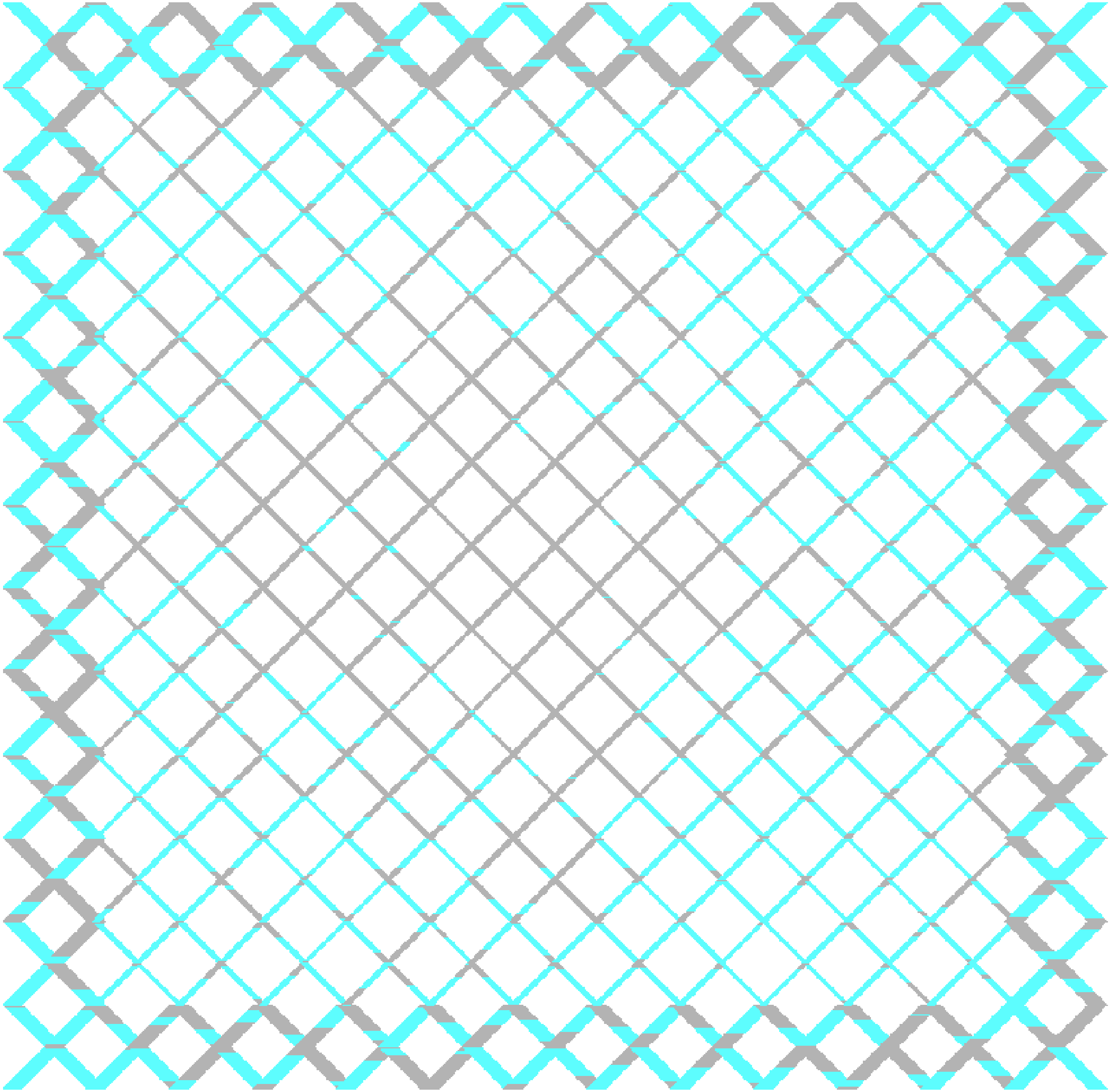
Result: Plot of Saturation 10x10



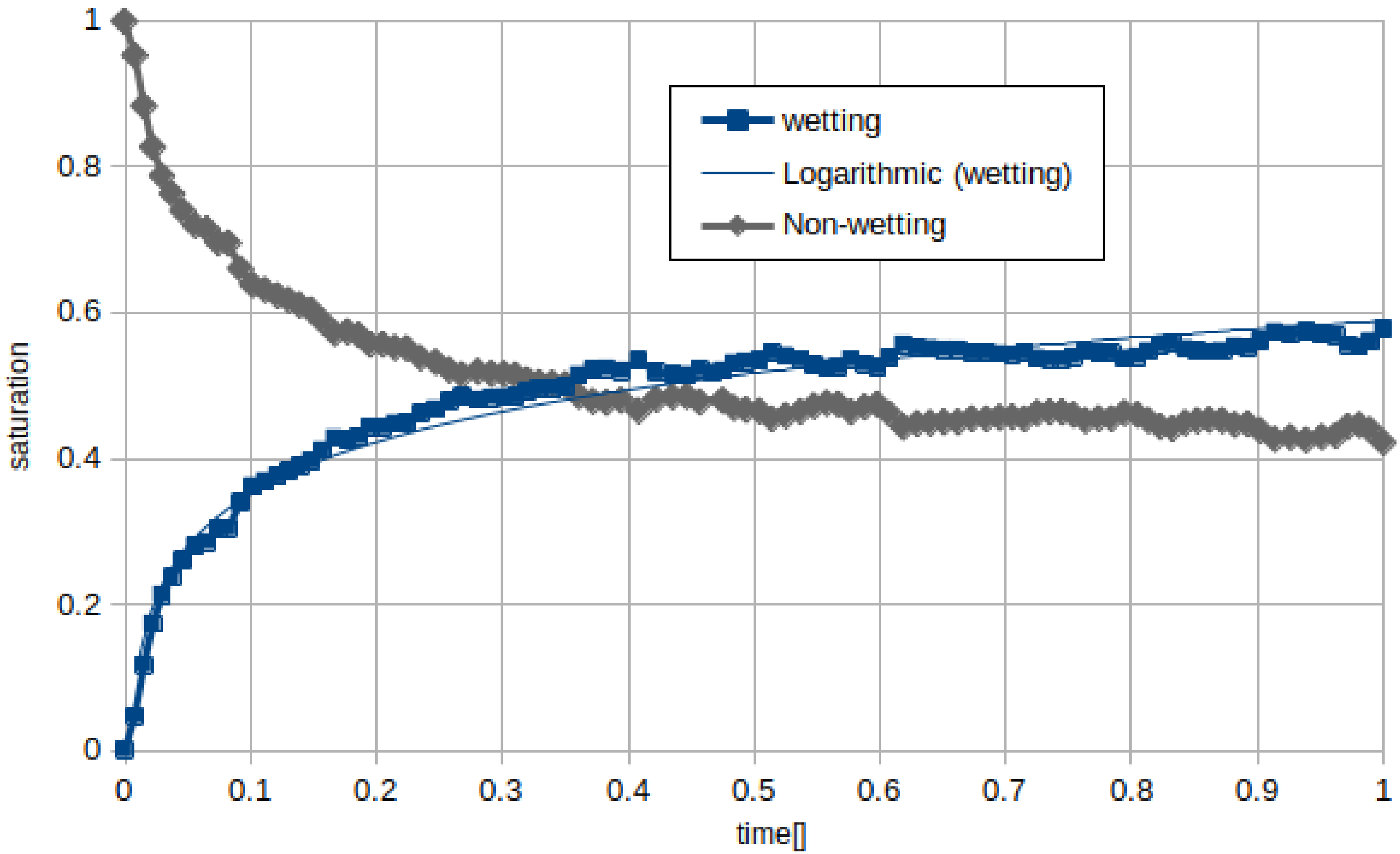
Result: Plot of Saturation 10x10



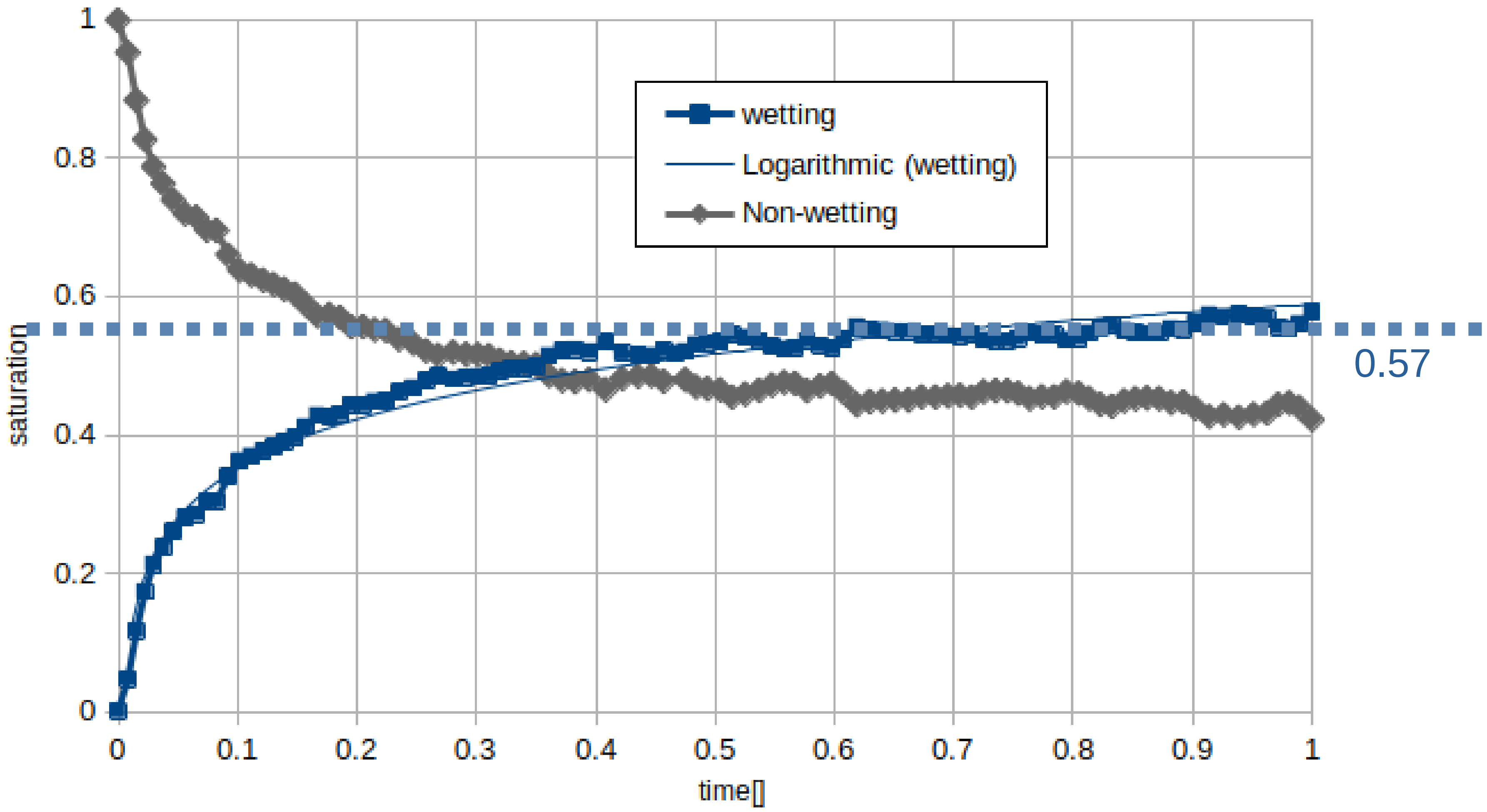




Result: Plot of Saturation 26x26

- 

Result: Plot of Saturation 26x26

- 

Conclusion

- The saturation -> equilibrium value.
- Explains relaxation phenomena in porous body.
- Imbibition observed.
- Method of distributing fluid such the wetting fluid first goes to the tube
- Modified Poiseuille equation
- Gaussian-elimination, more accurate than iterative methods.
- Conservation of volume for phases, high accuracy.
- To verify the Kondaurov model
- Determine the physical meaning of the non equilibrium parameter
- Scope of its applicability
- To be continued as a part of Master’s thesis.

Main References

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УДК 532.685

Simulation of Two-Phase Flow in Porous Media using a Two-Dimensional Network Model

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The algorithms and methods used to simulate two-phase flow in porous media has many practical applications in oil recovery, hydrology, electricity production where pressurized water is passed through heated pipes and is transformed into steam, etc. Our algorithm presented here is used to find the saturation of a phase with respect to time, model imbibition, the hysteresis curve when the pressure across the porous body is reversed, total capillary pressure as a function of saturation [1], and determination of permeability which appears in Darcy's law.

Our model is initially set up such that the wetting fluid is low in saturation and is confined to the bottom of our network. A higher pressure is fixed for all nodes at the bottom layer, while a low pressure is fixed for the top row. In all nodes, law of conservation of volume is applied, since mass is conserved and the phases are non-compressible. However for the bottom layer of nodes, the wetting fluid is injected as much required according to the sum of flow rates determined in the tubes connected to those nodes, while from the top layer of nodes a fluid is removed.

$$\sum Q_i = 0 \quad (1)$$

Where Q is the flow rate in [m^3/s] in a tube connected to a particular node. The flow rate formula used is

$$Q = \frac{\pi R^4}{8Ml}\left(\Delta P + \frac{2s\sigma}{R}\right) \quad (2)$$

s = {-1, 0, 1}, 0 when there are an even number of meniscus or no meniscus in a tube, +1 or -1 is due to the orientation. Here

$$M = \sum \mu_i \frac{l_i}{l} \quad (3)$$

Note that the case when no meniscus is present (s = 0) the flow rate formula is reduced to the well known Poiseuille's equation

$$Q = \frac{\pi R^4}{8\mu}\frac{\Delta P}{l} \quad (4)$$

From (1) we obtain an equation which relates five pressures. The number of equations obtained is equal to the number of nodes in our network. The equations are solved using Gauss-Jordan Elimination optimized for our case to determine the pressures in each of the nodes, then flow rates are calculated using (2). The time step is chosen according to the nearest meniscus reaching the node. At each of the nodes the flow is distributed to the outgoing tubes such that the tube with the smallest radius is filled first with the wetting fluid, this is due to the favor of energy. The tubes are inclined as suggested in [3], to ensure that a fluid flows equally well in tubes inclined right or left.

Thank You