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The NETWORK MODEL of POROUS MEDIA*
I.
Capillary Pressure Characteristics

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ABSTRACT

This paper proposes the network of tubes as a model more closely representing real porous media than does the bundle of tubes. Capillary pressure curves are derived from network models and pore size distributions are calculated from these curves. In this way is shown the difference between the true and calculated pore size distributions when the capillary pressure curve is used to obtain pore size distribution for porous media.

INTRODUCTION

Despite the technological importance of the laws governing flow through porous media, many of these laws have not yet been clearly formulated. This is especially true of the laws governing multiphase flow. The static properties, such as the capillary pressure curve, are also not at present interpretable correctly in terms of the pore size distribution and other structural properties of porous media. In the absence of any well founded theoretical description of fluid flow through porous media, many empirical descriptions have been proposed. In addition to the strictly empirical flow equations, some equations have been developed rigorously from simple geometrical models of the pore spaces. These equations are only as valid as is the model used in their development. The two models used in the past, the sphere pack and the bundle of tubes, have been too simple, and as a result, the equations derived from them have failed to predict the observed properties. Agreement between theory and observation has been achieved for these models by inserting parameters of doubtful physical significance.

The work to be reported here was undertaken in the belief that the study of a model more closely resembling most real porous media than does the sphere pack or bundle of tubes would yield useful information concerning the structure of real porous media and the relation between structure and flow properties. There was no intention of developing any calculational procedures, as for example calculating relative permeability from capillary pressure data. The objective of the study was instead to develop a better qualitative understanding of the relation between the various measurable properties of porous media.

The model chosen for study is the network of tubes in which each tube represents a pore space in the porous medium. Microscopic observations leave no doubt that sandstones and clastic carbonates are three-dimensional irregular networks of irregularly shaped pores. The irregularity of these structures makes any accurate theoretical study of their properties impossible. Results from an oversimplified model such as the bundle of tubes are worthless when applied to real systems. The sphere pack model, on the other hand, not

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Discussion of this paper is invited. Discussion in writing (3 copies) may be sent to the offices of the Journal of Petroleum Technology. Any discussion offered after Dec. 31, 1956, should be in the form of a new paper.

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only oversimplifies the structure, but retains enough complication to make the model too complicated for theoretical study.

The present study proposes a two-dimensional regular network of tubes of randomly distributed radii as a model which does not depart far enough from reality to distort the properties of real porous media significantly and yet is amenable to quantitative evaluation. The capillary pressure characteristics of the model can be evaluated by paper and pencil procedures. Evaluation of the flow properties requires the use of a simple analog computer formed by a network of electrical resistors.

The results obtained in this paper, first of a series of three, show clearly that the bundle of tubes is, in general, inadequate to interpret even the capillary pressure curve. The second and third papers of this series will show that while multiphase flow properties are a function of both network form and pore size distribution, it is the network structure which determines the overall behavior of real systems.

PREVIOUS MODELS USED IN THE STUDY OF POROUS MEDIA

The Sphere Pack

The earliest studies of fluid flow through porous media used the sphere pack as a model[1,2,3,4]. The complexity of the pore geometry in sphere packs prevented the derivation of any accurate descriptions of flow through this model. Kozeny[5] recognized the difficulties in treating the geometry of pore spaces. Using inductive methods, Kozeny arrived at an equation which related permeability to porosity and internal surface area of a sphere pack. An unspecified proportionality constant appears in this equation.

The Kozeny equation, as modified by Carman[6], has not proved to be of great value, aside from its use to estimate surface area of powders, because its basis on a physical model is not clear[7]. Rapoport and Leas[8] have attempted to obtain a relation between the capillary pressure curve and the wetting phase relative permeability curve by means of the Kozeny-Carman equation. However, Rapoport and Leas have made an assumption in their treatment of the problem that reduces the porous medium to a bundle of tubes.

Parallel to the development of the permeability equations from the sphere pack model has been the development of the electrical resistivity equations from the same model[9,10,11,12]. These equations relate the resistivity of a solution containing nonconducting solids to the resistivity of the same solution when it does not contain solids. This ratio is the "formation factor" used in electric logging. The theoretical equations for the formation factor derived from the sphere pack model do not fit observed data in the porosity range normally encountered in consolidated rock. In electric log interpretation, empirical equations must be used to relate formation factor to porosity[12,13].

The Bundle of Tubes Model

If a bundle of tubes is taken as the model of a porous medium, equations describing almost all of the flow properties of the medium can be derived by comparatively simple mathematical operations. The equations which relate the properties of porous media to the tube radius frequency distribution of the equivalent bundle of tubes have been given by Childs and Collis-George[14], Gates and Tempelaar Lietz[15], Fatt and Dykstra[16], Purcell[17], and Burdine, *et al*[18].

The advantages gained by being able to make rigorous derivations from the model are offset by the failure of the model to represent accurately real porous media. Several obvious characteristics of real porous media are not present in the bundle of tubes model. For example, real porous media are more or less isotropic with respect to fluid flow, whereas the bundle of tubes model is perfectly anisotropic. Many other differences between the bundle of tubes model and real porous media are uncovered when the flow properties of the model are compared to observed flow properties of porous media. Despite these weaknesses, the bundle of tubes model has been used with fair success to correlate certain properties of porous media[19,20,21]. The bundle of tubes model has also been used to calculate the formation factor and relative resisitivity (resistivity index) of porous media by applying Ohm's law to the flow of electricity through conducting fluid in the tubes of the model[22]. The equations for formation factor and relative resistivity derived from the model are not in agreement with equations obtained empirically from data on real porous media.

THE NETWORK MODEL

Previous considerations have indicated that the major weakness of the bundle of tubes model lies in the absence of cross-connections between the tubes. An examination of thin sections of sandstone indicates that the cross-connections between pores is a major structural feature of these porous media. The sphere pack model does have interconnected pores, but the shape of these pores is so complex that no analysis of flow through them is possible. If the two models are combined by substituting a uniform cylindrical tube for each pore space in the sphere pack model, a three-dimensional network of tubes is obtained on which, in principle, exact flow calculations can be made.

The network of tubes seems to be a valid model in many respects. In contrast to the bundle of tubes, the network of tubes is isotropic and thus meets the most obvious requirement of a model. The replacement of the cavernous pore spaces of the sphere pack model by cylindrical tubes does not seem to be a radical departure from reality for many well consolidated sandstones.

To simplify later network operations, it is desirable to replace the three-dimensional network by a two-dimensional network. In terms of the porous medium, the replacement is justified by assuming that a very thin slice of the porous medium with impermeable planes sealing the two large surfaces has the same properties as a cube of the same material. This is equivalent to assuming that the change produced by making the porous medium thin, and thereby eliminating a number of cross-connections in the third dimension, may be compensated by introducing additional channels within the two-dimensional network. It is assumed furthermore that changes in tube radius distribution, network form, and other network variables influence the properties in the same direction and in the same qualitative manner

[1]References given at end of paper.

in the two-dimensional network as in the three-dimensional network.

Having decided upon a two-dimensional network of cylindrical tubes as the model to be studied, there remains the network form to be chosen. Photomicrographs of sandstone thin sections show that the pores are connected in an irregular network. If the network form has an important influence on properties of porous media, it is desirable to study networks of different form. For this reason, four different regular networks were chosen for study. These networks are shown in Figs. 1, 2, 3, and 4. They are named the single hexagonal, square, double hexagonal, and triple hexagonal, respectively. This terminology is based on the fact that the double and triple hexagonal networks are obtained by superimposing two and three single hexagonal networks, respectively. The only parameter used to describe the network form is the number of tubes connected to each tube. This parameter is called the β factor and is given in Table 1 for the four networks and for two kinds of bundles of tubes. The explanation of the values of β for the bundles of tubes will be given later. The composite bundle of tubes is a bundle in which the tubes are made up of short tubes in series. The radii of these short tubes are distributed according to a tube radius distribution. The short tubes are randomly distributed along the length of each tube in the bundle. The simple bundle of tubes is a bundle in which the tube radius is constant along the tube length. These two kinds of bundles of tubes are included in the table to show that the limits of β are two and infinity.

TABLE 1—β FACTOR FOR FOUR NETWORKS AND TWO KINDS OF BUNDLES OF TUBES

Network	β Factor
Composite Bundle of Tubes	2
Single Hexagonal Network	4
Square Network	6
Double Hexagonal Network	7
Triple Hexagonal Network	10
Simple Bundle of Tubes	∞

There are several assumptions implicit in the use of a network of tubes as a model of a porous medium. All tubes in any network composed of tubes of different flow resistance conduct fluid when a potential gradient is applied. This implies that for single phase flow, there are no "blind" or "dead end" pores in the porous medium that is being modeled. There is very little experimental information on the existence of blind pores in real porous media. The best evidence to date suggests that there are very few blind pores in sandstones and sintered glass. This evidence is presented as incidental information by Russell, Morgan, and Muskat[23], Everett, Gooch, and Calhoun[24], and Mysels and Stigter[25]. These authors have shown that 97 per cent of a fluid in sandstone or sintered glass is displaced by flooding with a single pore volume of a miscible fluid. After one to 1½ pore volumes have passed through the porous medium, the displacement is essentially 100 per cent. This indicates that the blind pore volume in single phase flow in these porous media is probably not much greater than 1 per cent of the total pore volume. When there are two or more immiscible phases present in the porous medium as in multiphase fluid flow, the dead end pore volume is very much greater as will be shown later.

In deriving relative permeability and resistivity data from the network model, the assumption is made that a given pore contains either wetting phase or nonwetting phase, but not both at the same time. That is, the two immiscible phases do not flow concentrically in the same channel. Multiphase fluid flow in separate channels in porous media has been termed channel flow, while concentric flow in the same channel has been termed filament flow. The best observational support for the assumption of channel flow comes from the cinemicrophotographic studies of Chatenevar[26], and from the Stanolind group[27]. Leverett[28] has shown on theoretical grounds that the interfaces between two immiscible phases in porous media will distribute themselves so that the radii of curvature of the interfaces, r_1 and r_2, in the pore spaces will obey the LaPlace equation

$$P = \delta\left(\frac{1}{r_1} + \frac{1}{r_2}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The Chatenevar and Stanolind motion pictures show the distribution of the radii and the resulting channels filled either with wetting phase or nonwetting phase.

Operations on the network model to be described later will disclose that for reasons of convenience, the size of the network models is limited to from 200 to 400 tubes. In experimental measurements on porous media, the test sample usually contains about 40,000 pores. The assumption is that the statistics of the 400-tube model are not different from that of a 40,000-tube model. This is true only if the tubes in the model are randomly distributed. A table of random numbers is therefore used to locate the tubes of various radii in the network. Support for the validity of the relatively small networks will be given later when results from various size models are given. Extrapolation to infinite size shows that the results obtained on the 400-tube network are not seriously in error because of the small number of tubes.

When wetting and nonwetting phase are simultaneously in the network of tubes, the assumption is made

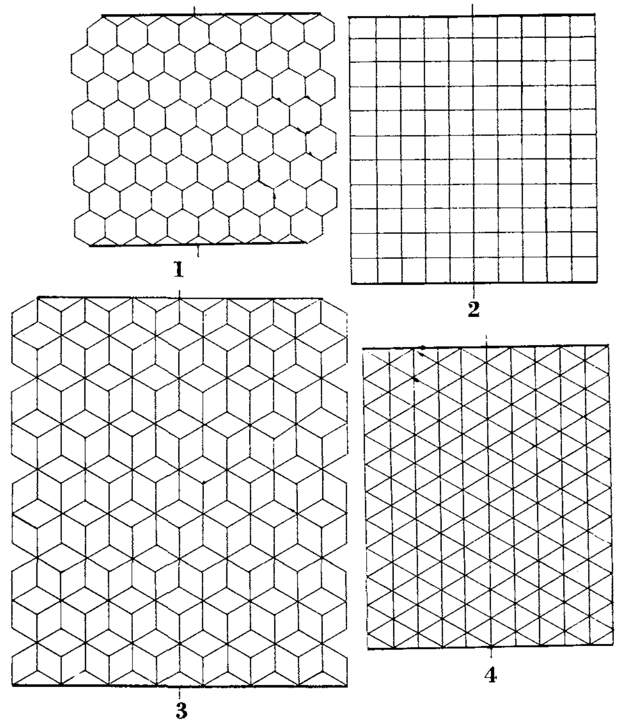

FIG. 1—SINGLE HEXAGONAL NETWORK.
FIG. 2—SQUARE NETWORK.
FIG. 3—DOUBLE HEXAGONAL NETWORK.
FIG. 4—TRIPLE HEXAGONAL NETWORK.

that the contact angle of the wetting phase on the inner surface of the tubes is zero. This condition leads to a continuous film of wetting phase throughout the network. In this way, no tube containing wetting phase is ever isolated from the continuous wetting phase.

There are two additional minor assumptions made in adopting the network of tubes model. One is that no reaction occurs between the fluid in the tubes and the tube material. This assumption eliminates the effects of hydration and swelling that are sometimes observed when water flows through porous materials. The other is that the tube wall is a nonconductor.

In addition to the above general assumptions which establish the network of tubes as a model for porous media, there are several special assumptions concerning different parameters of the model. These will be discussed as they appear in the development of the different tests of the network model.

THE CAPILLARY DESATURATION MECHANISM AND THE CAPILLARY PRESSURE CURVE

The network of tubes model, although developed primarily to study dynamic properties of porous media, can also be used to study static properties. The most commonly measured static properties are the porosity and the capillary pressure characteristics. The most useful of these properties is the capillary pressure vs saturation curve because this curve is characteristic of a particular porous medium. The network study gives an interpretation of the capillary pressure curve which is different from that obtained by the conventional bundle of tubes analysis.

The network model allows a detailed examination of the capillary desaturation mechanism which leads to a capillary pressure curve. Before this can be done, however, it is necessary to establish a network of tubes of different radius. If all tubes in a network are of the same radius, r, the wetting phase which initially saturates the network will be completely displaced by the nonwetting phase at a single pressure given by

$$P = \frac{2\,\delta \cos \Theta}{r} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

The assumption of zero contact angle gives

$$P = \frac{2\,\delta}{r} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

The capillary pressure curve for a network in which all tubes are of the same radius will be a straight horizontal line on the conventional capillary pressure plot.

If the tubes of the network have different radii, the desaturation proceeds stepwise as the pressure is raised. Nonwetting phase fluid first enters the network when the pressure between the phases is sufficient to force nonwetting phase into the largest tube if one or more of the largest tubes are at an outer edge of the network. The spatial arrangement of the tubes now becomes an important factor in the desaturation mechanism. Assuming that the pressure is P_1, the nonwetting phase will enter only those tubes of radius r_1 (P_1 and r_1 are related by Eq. 3) which are at the outer edge of the network or are connected to the outer edge by tubes of radius r_1 or larger. Tubes of radius r_1 surrounded by tubes of smaller radius will not be penetrated by nonwetting phase at pressure P_1. To study the capillary desaturation mechanism in a network of different size tubes, therefore, it is necessary to assume a tube radius

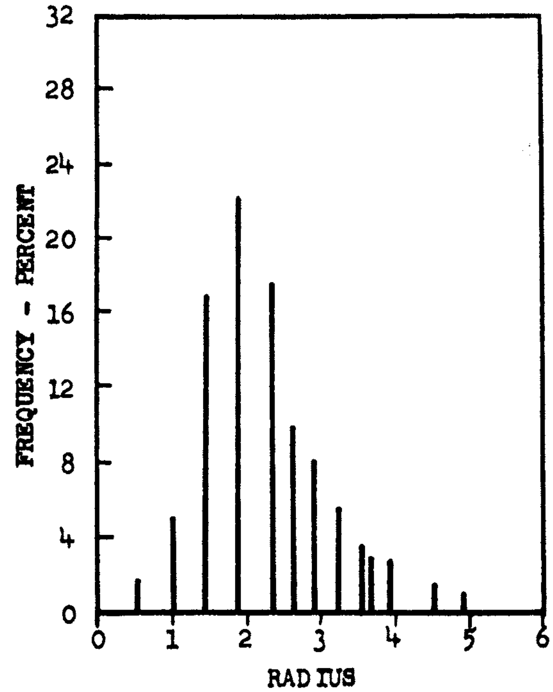

FIG. 5—TUBE RADIUS DISTRIBUTION.

frequency distribution and then randomly distribute the various size tubes in the network. The desaturation mechanism is best described by an example in which a capillary pressure curve is derived from a network.

EXAMPLE OF CAPILLARY DESATURATION OF A NETWORK OF TUBES

For this example, the triple hexagonal network of 331 tubes, shown in Fig. 4, is used. The tube radius frequency distribution is taken to be that of Fig. 5. The tube length is assumed to be inversely proportional to the tube radius. This seemingly arbitrary assumption will be justified in paper III of this series when the flow properties of the network of tubes are studied. It will then be shown that this particular relation between tube length and radius is more satisfactory than any other because it gives closer agreement between network flow properties and those observed on real porous media. This relation also gives network properties similar to those from networks in which tube radius and length are combined at random. The radius scale of Fig. 5 and all tube radius frequency distributions that follow is arbitrary because its only function is to determine the capillary pressure scale through the relation $P = 2\,\delta/r$ where δ is constant. The radius scale factor can be absorbed in this constant.

As mentioned above, the assumption is made that the pores of different radii in porous media which are being modeled are randomly distributed in space. The corresponding random spatial distribution of tubes in the model is obtained as follows: each tube position in the network is numbered, Fig. 6. The numbers are consecutive starting in the lower left-hand corner. Numbers from 1 to 331 are then tabulated in the sequence in which they appear in a random number table[20]. Available tube radii from the frequency distribution were listed in decreasing order alongside this list of random numbers. Each tube position is thus assigned a tube radius. In this way, the different tubes are randomly distributed in the network.

The network is assumed to be initially filled with wetting phase and surrounded by nonwetting phase. Since all the tubes are interconnected and the wetting phase is assumed to be always continuous, a single connection to one of the tubes at the edge of the network permits withdrawal of wetting phase. The model now closely resembles the experimental arrangement of Fatt and Dykstra[16] for obtaining capillary pressure and relative permeability data on sandstones.

The successive stages of penetration are shown in Fig 6. The resultant capillary pressure curve is shown in Fig. 7. The procedure used to obtain these data is described below.

The pressure difference between phases is assumed to be increased until the pressure of the nonwetting phase is equal to the entry pressure of the largest tube in the network, i.e., $r_1 = 4.96$. A diagram of the network in which tube radii are indicated at each position, Fig. 6, is then examined to determine if any tubes of the largest radius are on the edge of the network. In this example, the tube in position 15 is the only tube penetrated by nonwetting phase. There are two other of this radius in the network, at positions 198 and 291, but these are surrounded by tubes of smaller radius and will not be penetrated until later. The volume displaced at equilibrium at this capillary pressure is the volume of a single tube of radius 4.96. The wetting phase saturation in per cent is then given by

$$S_{w_1} = 100\left(1 - \frac{\pi r_1^2 l}{\Sigma n_i \pi r_i^2 l}\right). \quad . \quad . \quad . \quad . \quad (4)$$

Since tube length is assumed inversely proportional to radius, Eq. 4 can be written

$$S_{w_1} = 100\left(1 - \frac{r_1}{\Sigma n_i\, r_i}\right) \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Substituting into Eq. 5 the values of r_1 and the r_i's and n_i's from the distribution gives S_{w_1} equal to 99.4 per cent. The capillary pressure at this saturation is

$$P_1 = \frac{2\delta}{r_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

where 2δ is a scale factor as described previously.

Upon increasing the capillary pressure to that required to enter tubes of the second largest radius, $r_2 = 4.45$, examination of the network shows that nonwetting phase will enter tubes of radius r_2 in positions 11 and 328. The capillary pressure is now

$$P_2 = \frac{2\delta}{r_2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

and the equilibrium saturation is

$$S_{w_2} = 100\left(1 - \frac{r_1 + 2r_2}{\Sigma n_i\, r_i}\right) = 98.1 \quad . \quad . \quad (8)$$

An increase in capillary pressure to $2\delta/r_3$ causes nonwetting phase to enter tubes of radius r_3 in positions 9, 16, 147, 178, and 223. The wetting phase saturation becomes

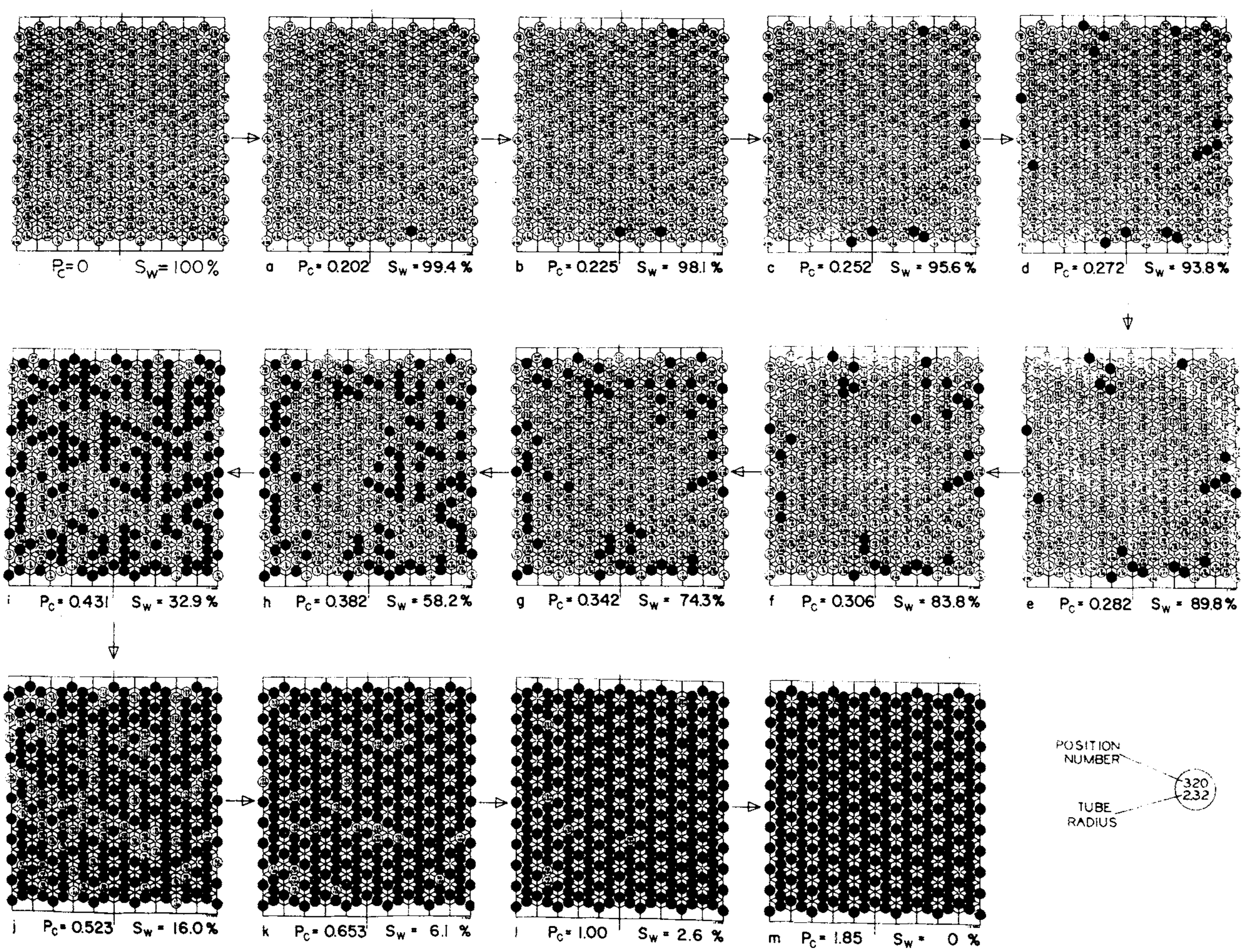

FIG. 6—CAPILLARY DESATURATION SEQUENCE. TUBE RADIUS DISTRIBUTION OF FIG. 5.

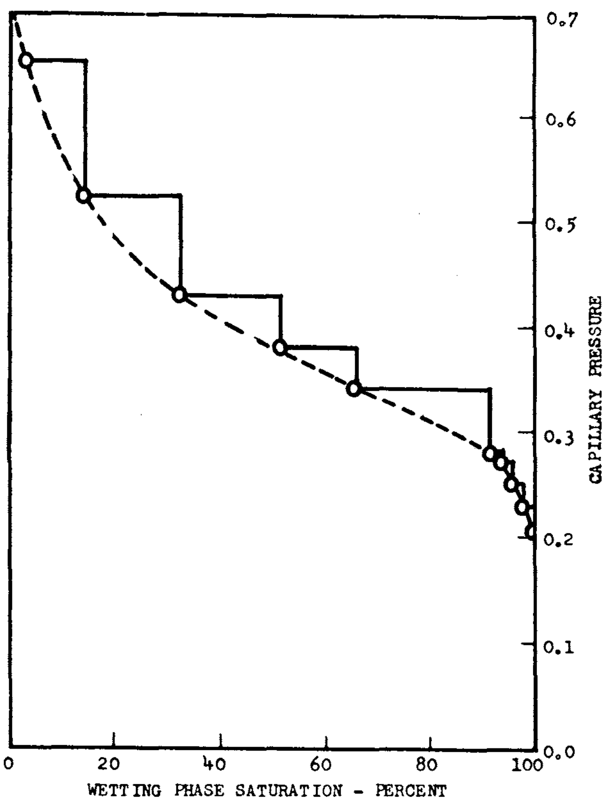

FIG. 7—CAPILLARY PRESSURE CHARACTERISTICS OF TRIPLE HEXAGONAL NETWORK USING TUBE RADIUS DISTRIBUTION OF FIG. 5.

$$S_{w_3} = 100\left(1 - \frac{r_1 + 2r_2 + 5r_3}{\Sigma n_i\, r_i}\right) = 95.6 \quad . \quad . \quad (9)$$

At capillary pressure, $2\delta/r_4$ tubes of radius r_4 in positions 129, 143, 144, 310, and 321 are filled with nonwetting phase. In addition, a tube of radius r_1 at position 291 fills with nonwetting phase because it is only now that this tube becomes connected to the nonwetting phase outside the network by the tube in position 310. The wetting phase saturation at pressure $2\delta/r_4$ is

$$S_{w_4} = 100\left(1 - \frac{2r_1 + 2r_2 + 5r_3 + 5r_4}{\Sigma n_i r_i}\right) = 93.8 \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (10)$$

The capillary pressure is increased in steps given by $P_5 = 2\delta/r_5$, $P_6 = 2\delta/r_6$, $P_7 = 2\delta/r_7$, etc. until the capillary pressure is sufficient to cause nonwetting phase to enter the smallest tube in the network. At each pressure, the network is examined to find the tubes that will fill with nonwetting phase at the given pressure. Fig. 6 shows the desaturation sequence for this example. The curve of capillary pressure vs wetting phase saturation derived from this example is shown in Fig. 7. The network data gives the stepped curve because the tube radius distribution is discontinuous. In real porous media with a continuous pore radius distribution, the capillary pressure curve is smooth. In Fig. 7, the dashed line shows the results of smoothing the network data. Future capillary pressure results in this study will report only the smoothed curve.

The curve obtained from the network model resembles a typical capillary pressure curve obtained from sandstone. There is one difference that should be noted. The curve derived from the network is asymptotic to zero wetting phase saturation, whereas curves from real porous media tend to be asymptotic to 20 to 30 per cent wetting phase saturation. There are probably several reasons for this difference. In the network model, the contact angle of the wetting phase on the solid was assumed to be zero, but for real porous media, the contact angle may be finite. A finite contact angle would result in a break of the wetting phase film at low wetting phase saturations, thereby trapping some of the wetting phase in the small tubes.

A second reason for the difference in asymptote between the network and real porous media may arise from the failure to reach equilibrium in capillary desaturation of real porous media. As the wetting phase saturation decreases, the mobility of the wetting phase in a porous medium is tremendously reduced. At each increment of capillary pressure at low saturations, a great deal of time must be allowed for the displaced phase to leave the porous test sample. In some laboratory determinations of capillary pressure characteristics, it is probable that insufficient time is allowed at low saturations. As a result, the reported capillary pressure curves show a high asymptotic wetting phase saturation that would not exist if the system were at equilibrium.

Wetting phase fluid trapped in the pores of a real porous medium may give a capillary pressure curve asymptotic to perhaps as much as 25 per cent wetting phase saturation. As will be pointed out presently, any trapping of fluid causes a discrepancy between a pore size distribution calculated from the capillary pressure curve and the real distribution. In the network model used here, no fluid was allowed to be trapped. A possible refinement of the model in future studies would be to permit trapping of wetting phase when a tube containing this phase is completely surrounded by nonwetting phase.

Before the network can be said to model satisfactorily the desaturation mechanism operating in porous media, two factors introduced by the model must be investigated. The first factor is the reduced size of the network, compared to the usual test sample of a porous material. As mentioned previously, the network of 331 tubes is two orders of magnitude smaller than the usual test sample. However, a comparison of capillary pressure curves obtained at the La Habra Laboratory of California Research Corp. by Purcell's mercury injection method on a ¼-in. cube of sandstone with curves obtained by the same method on 1-in. cubes of the same material indicated that a 64-fold reduction in the number of pores in a sample did not change the capillary pressure curve. The ¼-in. cube has only 10 times the number of pores that are in the network model.

Another test of the effect of network size on the capillary pressure curve was made by obtaining a capillary pressure curve from a network of only one-half the number of tubes that are in the example network, Fig. 6. The difference in the curves from the one-half size and full size networks was less than the scatter of points about each curve. This test, together with the study of capillary pressure curves from ¼-in. cubes of sandstone, indicates that the capillary pressure curves from sandstone and from the network are not sensitive functions of the size of the system in the range studied.

The effect of network size will again be studied when the network model is used to obtain relative permeability curves. Those studies will confirm the conclusion that the size of the network model adopted here for study of the capillary desaturation mechanism is sufficient to give data which are almost independent of size, and therefore, comparable to data from real porous media.

The second factor introduced into the network model is the random spatial distribution of the tubes. The capillary pressure curve obtained in the above example is for a particular random distribution. The question is then whether the same capillary pressure curve would be obtained from another random distribution, and in general with any random distribution. An exact and rigorous answer to this question could not be obtained. An empirical answer was obtained by comparing capillary pressure curves from four different random distributions of the same network and tube radius distribution. The difference in the curves from the four different random distributions was less than the scatter of points about each curve. The conclusion was then made that in a network of several hundred tubes, there was a very small probability of having random distribution of tubes which would give a different capillary pressure curve.

Effect of Variation of Network And Tube Radius Distribution on The Capillary Pressure Curve

Having established a valid capillary desaturation mechanism in the network of tubes model, it is now possible to examine the effect of variation of network and tube radius distribution on the capillary pressure curve. After describing the effect of these variables on the capillary pressure curve, the inverse problem will be considered; that is, the effect of network and tube radius distribution on the tube radius distribution calculated from a network capillary pressure curve.

Fig. 8 shows the capillary pressure curves for the tube radius distribution of Fig. 5 from the single, double, and triple hexagonal networks and from the bundle of tubes. The bundle of tubes referred to here is the simple bundle in which each tube is of constant radius along its length. The capillary pressure curve for the bundle of tubes is calculated directly from the tube radius distribution, the curves for the networks according to the capillary desaturation mechanism as described in the above example.

Several conclusions can be drawn from these curves. The first is that the networks studied and the bundle of tubes give capillary pressure curves which closely resemble the curves obtained from sandstones and sintered glass. This can be taken as evidence for the proper choice of the tube radius distribution because, as will be shown later, the tube radius distribution largely determines the shape of the capillary pressure curve.

The second conclusion is that as the β factor increases, that is, as the number of tubes joined to each tube increases, the network capillary pressure curve approaches the curve obtained from the bundle of tubes model. This results from the fact that as the number of paths to each tube increases, there is an increase in the probability of having a chain of larger tubes from the outer edge of the network to each tube. There are then fewer tubes that are surrounded by smaller tubes, and thereby isolated from the nonwetting phase during capillary desaturation. The network model then approaches the bundle of tubes model in which no tube is isolated from the nonwetting phase at any stage of the desaturation. The β factor is infinite for a bundle of tubes in which the tube radius is constant along the tube length.

The third observation is that the capillary pressure curve is not very sensitive to changes in network for

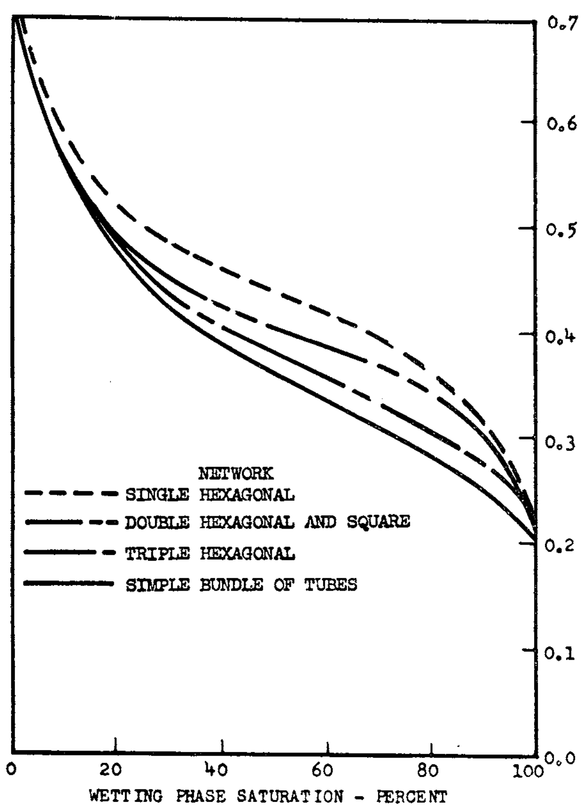

Fig. 8—Capillary Pressure Characteristics of All Four Networks and the Simple Bundle of Tubes for the Tube Radius Distribution of Fig. 5.

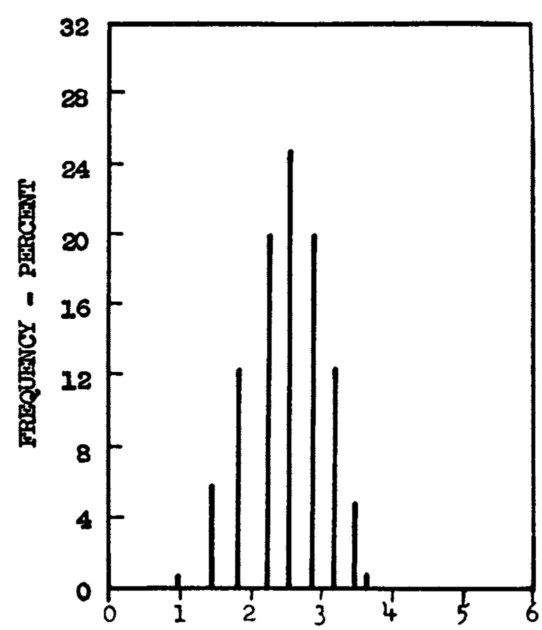

Fig. 9—Tube Radius Distribution.

the tube radius distribution of Fig. 5. The sensitivity of the capillary pressure curve to changes in network is shown later to be a function of the tube radius distribution.

To study the effect of changes in tube radius distribution on the capillary pressure curve, three typical distributions were used. Fig. 9 is a very narrow distribution, Fig. 5 is an intermediate distribution, and Fig. 10 is a very broad distribution. Capillary pressure curves were obtained for these three distributions in the four different networks in exactly the same way as for the example. The 12 capillary pressure curves so obtained are grouped according to network in Figs. 11 to 15 and according to distribution in Fig. 8 and Figs. 16 to 17.

Examination of Fig. 8 and Figs. 11 to 17 shows that although the capillary pressure curve for a given

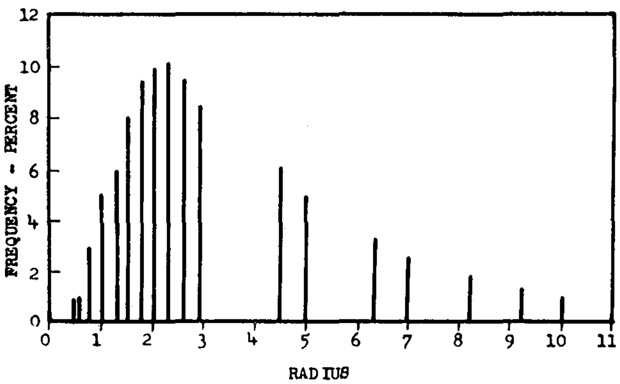

FIG. 10—TUBE RADIUS OR LENGTH DISTRIBUTION.

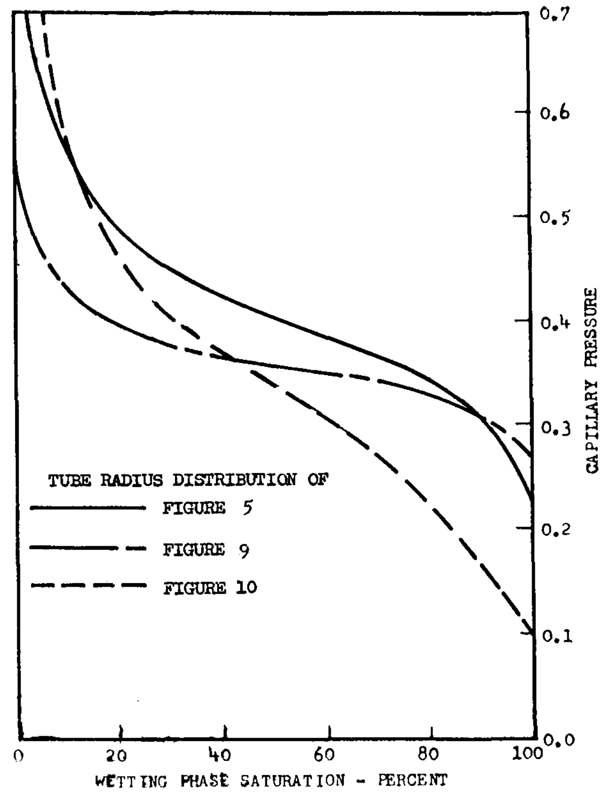

FIG. 12—CAPILLARY PRESSURE CHARACTERISTICS OF SQUARE NETWORKS FOR THREE DIFFERENT TUBE RADIUS DISTRIBUTIONS.

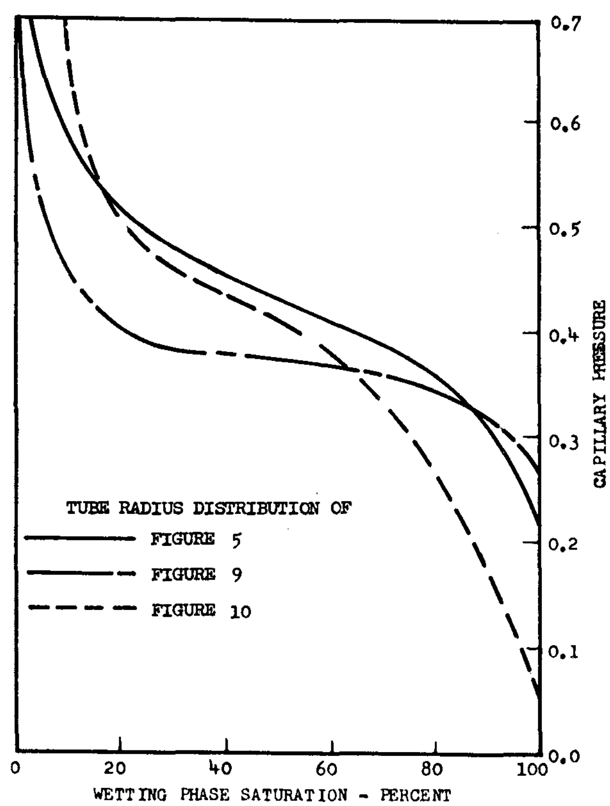

FIG. 11—CAPILLARY PRESSURE CHARACTERISTICS OF SINGLE HEXAGONAL NETWORKS FOR THREE DIFFERENT TUBE RADIUS DISTRIBUTIONS.

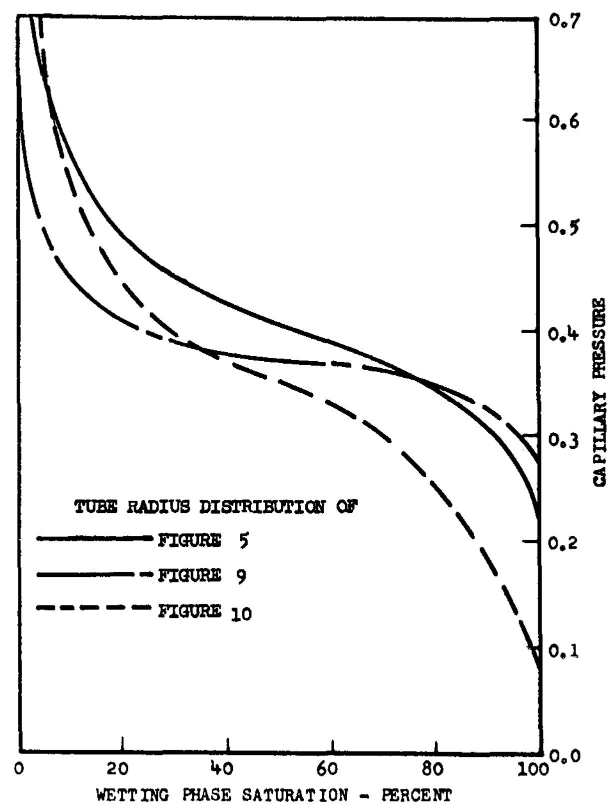

FIG. 13—CAPILLARY PRESSURE CHARACTERISTICS OF DOUBLE HEXAGONAL NETWORKS FOR THREE DIFFERENT TUBE RADIUS DISTRIBUTIONS.

tube radius distribution is somewhat different for each network, the greatest change in the capillary pressure curve is brought about by a change in tube radius distribution. This fact leads to the tentative conclusion that the capillary pressure characteristics of porous media can give information concerning the pore size distribution.

At present, the only method known whereby the pore size distribution can be calculated from the capillary pressure curve is by assuming the bundle of tubes model. The method of calculation is described in the Appendix. The variations in the capillary pressure curves obtained from the different networks and tube radius distributions show that two factors will cause error in the pore size distribution calculated from the capillary pressure curve by means of the bundle of tubes model. The first factor is the network. The bundle of tubes model becomes more applicable as the β factor increases.

The second factor is the range of radii in the pore size distribution. The capillary pressure curves from the three tube radius distributions used in Fig. 8 and in Figs. 16 to 17 show that as the range of radii is reduced, the capillary pressure curves from all three networks approach the curve from the bundle of tubes.

These observations can be used as a guide to the interpretation of the capillary pressure curve in terms of the pore size distribution. Any pore size distribution calculated from the capillary pressure curve of a real porous medium will be too narrow because the isolated pores at the large radius end of the distribution do not contribute to the saturation at low capillary pressures. The peak of the distribution will be too high, because these large pores are calculated as pores of intermediate size when the capillary pressure becomes high enough to cause entry of nonwetting phase into most of the pores. If the pore size distribution calculated from the capillary pressure curve is broad, the real distribution can be expected to be even broader. If the calculated distribution is narrow, then it may be not far from the real distribution. Meyer[30] has made a somewhat similar suggestion concerning the effect of the distribution only; he did not consider the effect of the network.

Quantitative Comparison of True and Calculated Pore Radius Distribution

The widespread use of the capillary pressure method to determine pore size distribution and the technological importance of the distribution makes desirable a quantitative method for comparing the real and calculated distribution. The guides to interpretation given in the preceding paragraph can be used to modify qualitatively a pore size distribution calculated from a capillary pressure curve so that the distribution becomes more like that existing in the porous medium. Engineering calculations, however, very often require quantitative data from the pore size distribution. It therefore seems worthwhile to make a detailed and quantitative examination of the effect of network and distribution parameters on the calculated pore size distribution.

The procedure by which a known tube radius distribution appearing in a network is compared with the distribution calculated from the network capillary pressure curve is shown schematically below.

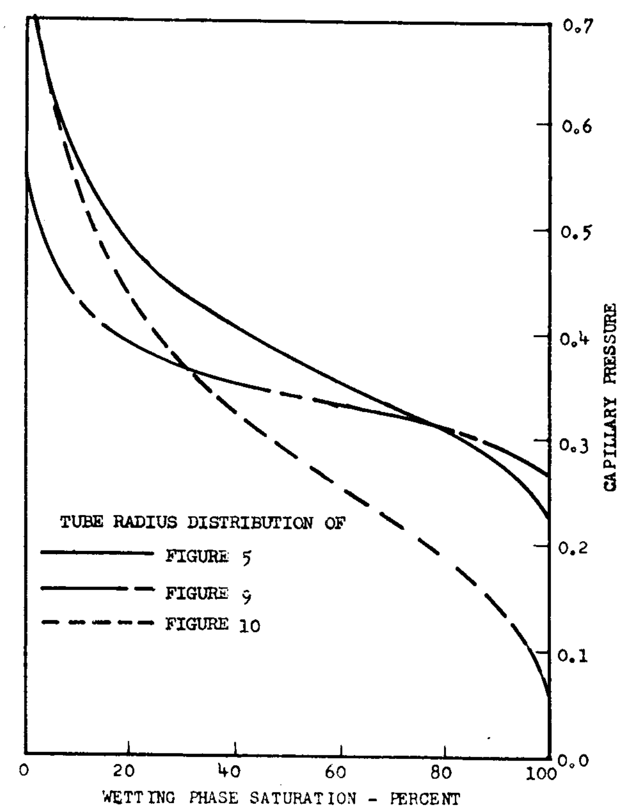

Fig. 14—Capillary Pressure Characteristics of Triple Hexagonal Networks for Three Different Tube Radius Distributions.

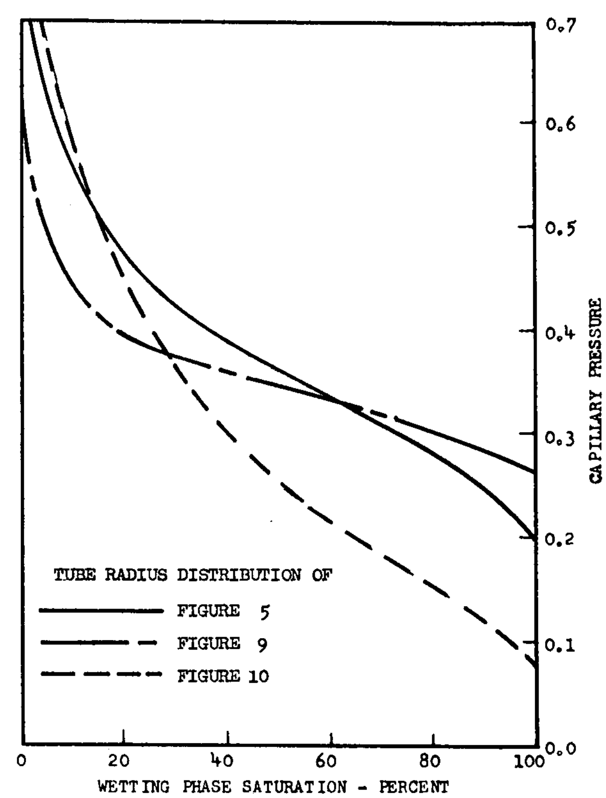

Fig. 15—Capillary Pressure Characteristics of a Simple Bundle of Tubes for Three Different Tube Radius Distributions.

true tube radius → true tube radius distribution distributed in the network

compared — capillary desaturation mechanism operates on network

tube radius distribution calculated by methods of Appendix from capillary pressure curve by assuming structure is bundle of tubes. — capillary pressure curve obtained from network

A quantitative comparison of tube radius distributions requires that the distributions be characterized by one or more parameters. For a symmetrical distribution, these parameters are the average tube radius and the dispersion. These parameters are the first and second moments of the distribution, and are defined as:

$$m_1 = \bar{r} = \frac{\sum_1^N n_i r_i}{N} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

$$m_2 = \overline{r^2} - \bar{r}^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

where

$$\overline{r^2} = \frac{\sum_1^N n_i r_i^2}{N} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

Since only ratios of average tube radius will be used in this analysis, a dimensionless radius will not be required, but a dimensionless dispersion is required. The dimensonless dispersion may be defined here as

$$\mu_2 = \overline{\left(\frac{r}{\bar{r}}\right)^2} - 1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

An asymmetrical distribution should be described by at least the first, second, and third moments. The third moment, m_3, is given by

$$m_3 = \overline{r^3} - 3\bar{r}\,\overline{r^2} + 2\,\bar{r}^3 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

where

$$\overline{r^3} = \frac{\sum_1^N n_i r_i^3}{N} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

The third moment can be made dimensionless by rearranging Eq. 15 to give

$$\mu_3 = \overline{\left(\frac{r}{\bar{r}}\right)^3} - 3\,\overline{\left(\frac{r}{\bar{r}}\right)^2} + 2 \quad . \quad . \quad . \quad . \quad (17)$$

A frequently used index of asymmetry (skewness) is the ratio

$$\frac{\mu_3^2}{\mu_2^3} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

This ratio is zero for the normal error distribution.

In addition to the parameters of the distribution, it is necessary to have a parameter which is characteristic of the network. The inverse of the β factor is a convenient parameter for this purpose. The range of $1/\beta$ for the four networks and the bundle of tubes is 0.00 to 0.50. The two types of the bundle of tubes form the extremes as shown by Table 1. As explained previously, the composite bundle of tubes is a bundle in which the tubes are made up of short tubes in series. The radii of these short tubes are distributed according to a tube radius distribution. The short tubes are randomly distributed along the length of each tube in the bundle. The simple bundle of tubes is a bundle

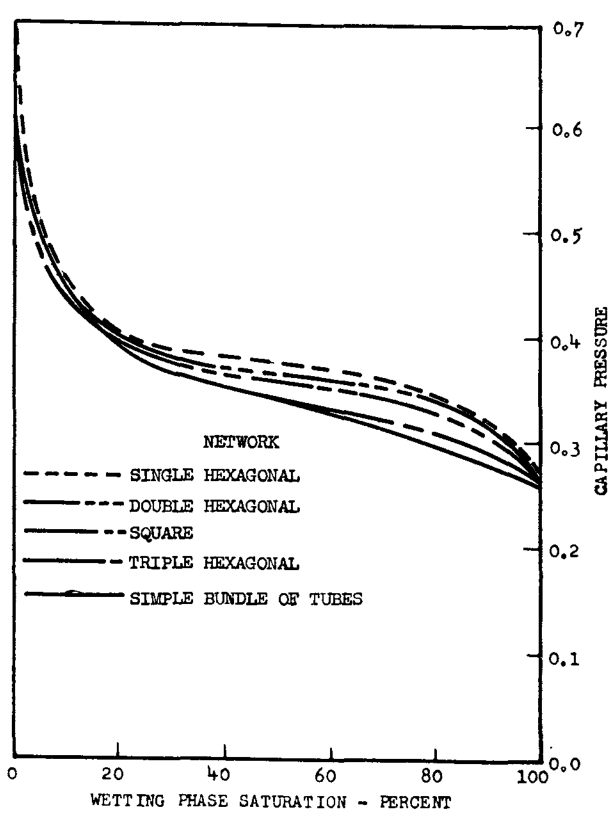

FIG. 16—CAPILLARY PRESSURE CHARACTERISTICS OF ALL FOUR NETWORKS AND THE SIMPLE BUNDLE OF TUBES FOR THE TUBE RADIUS DISTRIBUTION OF FIG. 9.

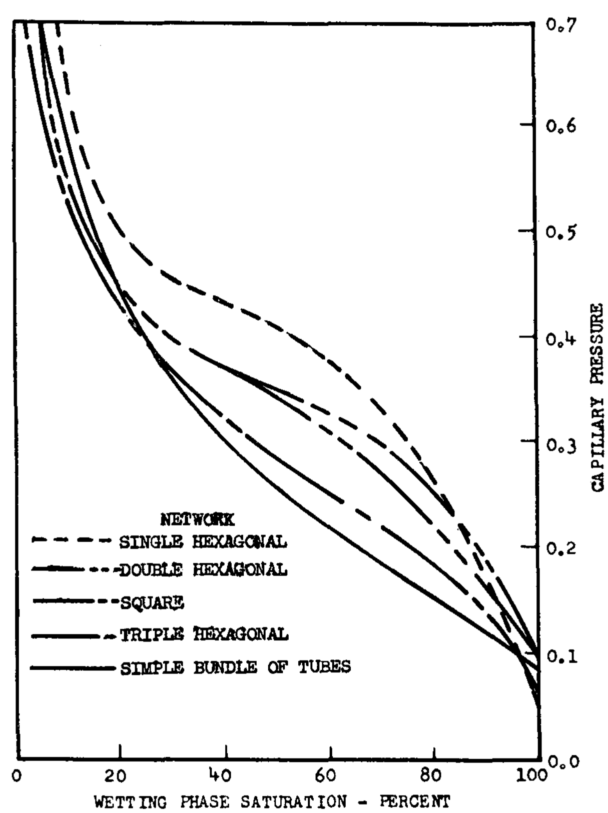

FIG. 17.—CAPILLARY PRESSURE CHARACTERISTICS OF ALL FOUR NETWORKS AND THE SIMPLE BUNDLE OF TUBES FOR THE TUBE RADIUS DISTRIBUTION OF FIG. 10.

in which the tube radius is constant along the tube length.

Tube radius distributions were calculated for each distribution and network from the capillary pressure curves in Figs. 11 to 15. The distribution was calculated by assuming the bundle of tubes model according to the procedure described in the Appendix.

The tube radius distribution of the simple bundle of tubes, for which $1/\beta$ is zero, is the original distribution that was set up in the network. The subscript $_n$ signifies a property calculated from a network capillary pressure curve; the subscript $_t$ signifies a property of the original tube radius distribution that was used to construct the network. Since the tube radius distribution is calculated from the capillary pressure curve by assuming that the curve came from the simple bundle of tubes, the properties of the original tube radius distribution are the same as for the simple bundle of tubes model.

The plot of the ratio $\frac{\bar{r}_n}{\bar{r}_t}$ against $1/\beta$ in Fig. 18 shows that the average pore radius calculated from a capillary pressure curve will be smaller than the true average pore radius. The difference between the calculated and true average pore radius is a function of both $1/\beta$ and the dispersion of the distribution. The straight lines obtained in Fig. 18 indicate that the average radius and $1/\beta$ are related by an equation of the form

$$\frac{\bar{r}_n}{\bar{r}_t} = 1 - \frac{d\left(\frac{\bar{r}_n}{\bar{r}_t}\right)}{d\,(1/\beta)}\,\frac{1}{\beta} \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

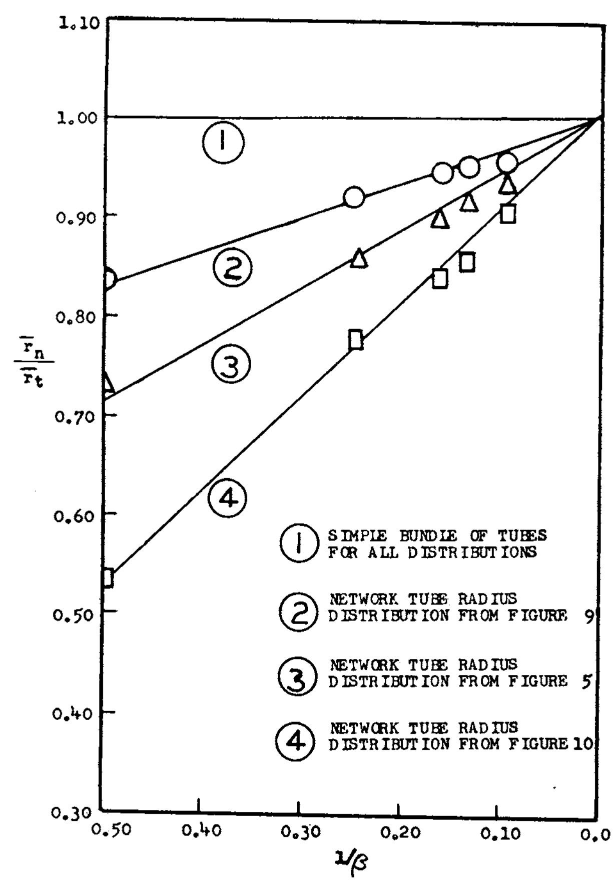

FIG. 18—EFFECT OF NETWORK β FACTOR ON CALCULATED AVERAGE TUBE RADIUS.

The magnitude of $\frac{d\left(\frac{\bar{r}_n}{\bar{r}_t}\right)}{d\,(1/\beta)}$ will be discussed later.

The maximum error in the average pore radius calculated from the capillary pressure curve of a real porous medium can be estimated from Fig. 18. The tube radius distribution of Fig. 10 probably has the maximum dispersion to be found in distributions in real porous media. On the other hand, there are at least three pores connected to each pore in a real porous medium, that is, the $1/\beta$ factor is not above 0.30. For the extreme conditions of a distribution, Fig. 10, and a network with $1/\beta$ equal to 0.30, the calculated average pore radius is about 70 per cent of the true average radius. When the experimental errors in measurement of the capillary pressure curve and the approximations and assumptions of the network treatment are taken into consideration, this error in the calculated average pore radius cannot be considered significant. This leads to the conclusion that the capillary pressure curve can be used in conjunction with the bundle of tubes model to obtain the average pore radius for most real porous media.

The dispersion of the calculated distribution is plotted against the dispersion of the true distribution in Fig. 19 to give a straight line with slope equal to 0.51. The relation between dispersion for all networks and distributions is then $(\mu_2)_n = 0.51\ (\mu_2)_t$ . . . . (20) The conclusion can thus be drawn that the dispersion of a pore radius distribution calculated from a capillary pressure curve by assuming the simple bundle of tubes model will be about one-half of the dispersion of the actual pore radius distribution in the porous medium.

In Fig. 20, the slopes of the $\frac{\bar{r}_n}{\bar{r}_t}$ vs $1/\beta$ lines are plotted against the calculated dispersion to give a smooth continuous curve. The equation of this curve in the range shown in Fig. 20 is

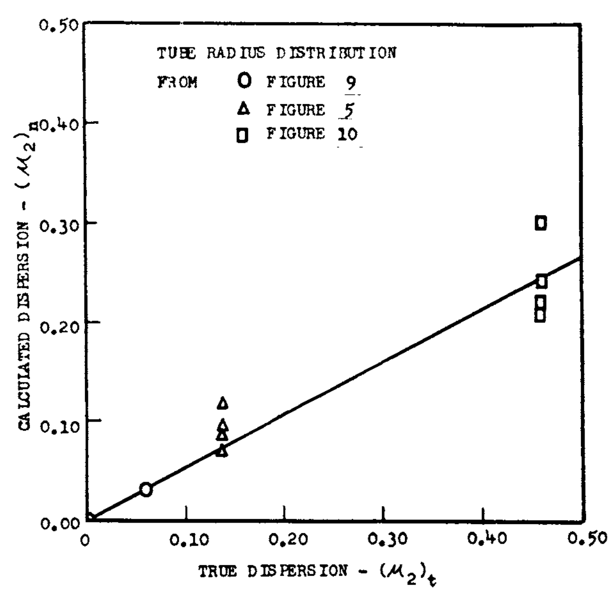

FIG. 19—RELATION BETWEEN CALCULATED AND TRUE DISPERSION FOR ALL FOUR NETWORKS.

$$\log\left[1-\left[\frac{d\left(\frac{\bar{r}_n}{\bar{r}_t}\right)}{d\,\frac{1}{\beta}}\right]^2\right] = 3.52 \log\left[10 - (\mu_2)_t\right] - 3.52 \qquad (21)$$

The relations between the average pore radius ratios, the dispersion ratios, and the network parameter $1/\beta$, permit the calculation of the true average pore radius and the true dispersion from the capillary pressure curve if the network parameter is known. Capillary pressure data alone cannot be used to evaluate $1/\beta$, but if in addition, certain flow measurements are made, $1/\beta$ can be calculated by methods to be described in the third paper of this series.

The relative indices of asymmetry, $\dfrac{\left(\frac{\mu_3{}^2}{\mu_2{}^3}\right)_n}{\left(\frac{\mu_3{}^2}{\mu_2{}^3}\right)_t}$ for the network derived distributions are plotted as a function of $1/\beta$ in Fig. 21. From these data, it is apparent that the asymmetry of the distribution calculated from a capillary pressure curve by means of the bundle of tubes model approaches the true asymmetry for porous media of high β factor. The relative index of asymmetry vs $1/\beta$ curve for the distribution of Fig. 5 falls below 1.0 at $1/\beta = 0.10$ because in the triple hexagonal network, the network derived distribution is more symmetrical than the starting distribution.

The question arises as to whether or not conclusions drawn above from the study of two-dimensional networks can be used to interpret capillary pressure curves from three-dimensional real porous media. There is a small difference in the possible paths and their length between two points separated by the same linear distance in a two-dimensional and three-dimensional network of the same type. This difference tends to give a larger β for a three-dimensional network than for a two-dimensional network. It is, however, reasonable to expect three-dimensional networks to behave in much the same manner as a two-dimensional network when a lower value of β is used for the three-dimensional network.

A possible procedure for calculating the true average pore radius and the true dispersion of real porous media is summarized below.

1. Calculate the pore size distribution from the capillary pressure curve using the simple bundle of tubes model and the procedure given in the Appendix.

2. Calculate the average pore radius, $\bar{r}_n$, and the dispersion, $(\mu_2)_n$, from the distribution obtained in Step 1.

3. Multiply the calculated dispersion by 1/0.50 to give the true dispersion.

4. From Fig. 20 or from Eq. 17, find the slope of the $\frac{\bar{r}_n}{\bar{r}_t}$ vs $1/\beta$ line for the value of the true dispersions calculated in Step 3.

5. If $1/\beta$ is known (from flow measurements to be described in paper III of this series), the true average pore radius can be calculated from

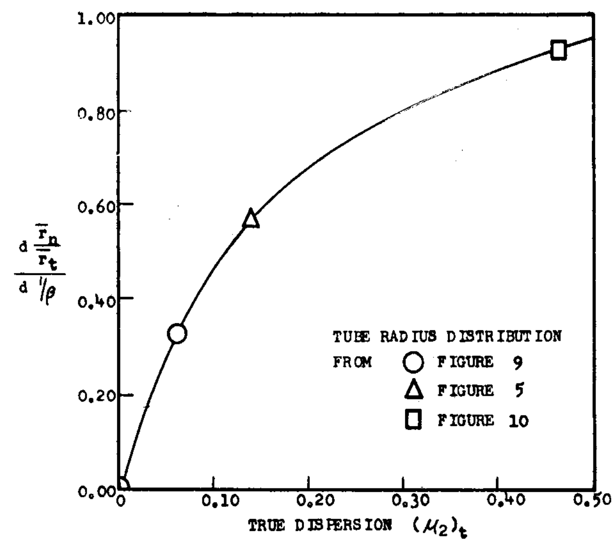

FIG. 20—SLOPE OF $\frac{\bar{r}_n}{\bar{r}_t}$ VS $1/\beta$ LINE AS A FUNCTION OF THE TRUE DISPERSION.

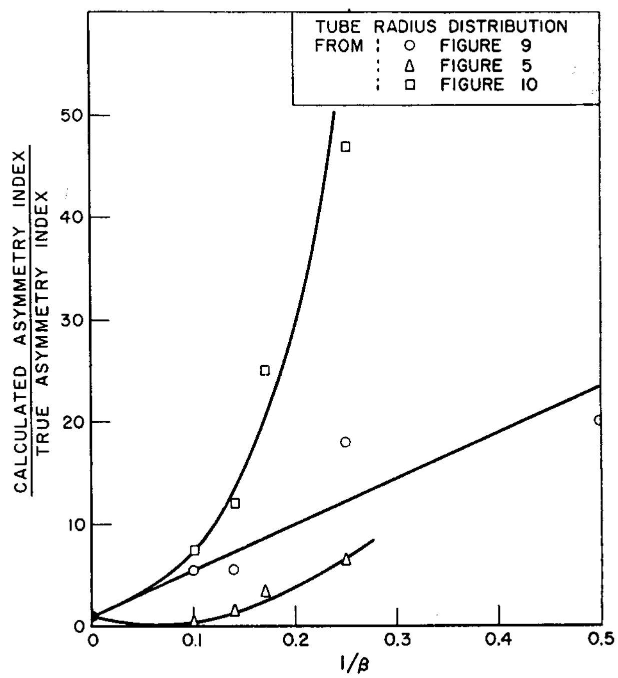

FIG. 21—EFFECT OF NETWORK AND RADIUS DISTRIBUTION ON CALCULATED ASYMMETRY INDEX.

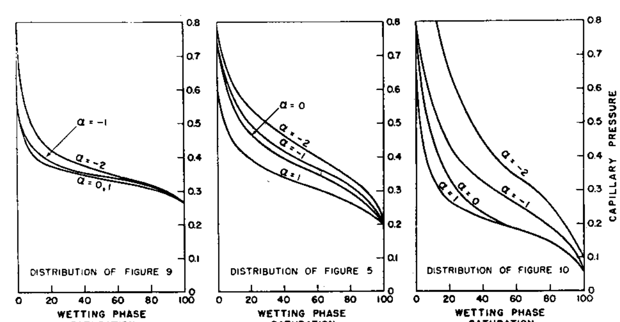

FIG. 22—EFFECT OF TUBE LENGTH TO RADIUS RELATION ON TRIPLE HEXAGONAL NETWORK CAPILLARY PRESSURE CURVE USING RADIUS DISTRIBUTION OF FIGS. 5, 9, AND 10.

$$\bar{r}_t = \frac{\bar{r}_n}{1 - \dfrac{d\left(\dfrac{\bar{r}_n}{\bar{r}_t}\right)}{d\,(1/\beta)}\,(1/\beta)} \quad . \quad . \quad . \quad . \quad . \quad . \quad (22)$$

Capillary Pressure Curves from Several Tube Length to Radius Relations

Although the inverse relation between tube length and radius can be shown[31] to be the most reasonable, it is of interest to determine the effect of the relation between tube length and radius on the capillary pressure characteristics of a network. There are two methods which can be used to relate tube length and radius. The tube length can be related to the radius by a fixed relation, for example of the form $l = Cr^{\alpha}$ where α and C are constants. The inverse relation used in the preceding network studies was a special case in which α is -1. The other method is to combine randomly tube radii and lengths so that there is no fixed relation between these quantities. Both methods have been used to obtain the network capillary pressure curves which are described below.

Capillary pressure curves for the distributions of Figs. 5, 9, and 10 were obtained for the four networks and the simple bundle of tubes in which α was 1.0, and -2, in addition to the case already described, in which α is -1. The curves for a given distribution and the triple hexagonal network for different α's are shown in Fig. 22. Curves for other networks are given in Reference 31.

Fig. 22 shows that the changes in α cause only a small change in the position and shape of the capillary pressure curve if the dispersion of the tube radius distribution is small. The difference in capillary pressure curves obtained from the distribution of Fig. 9 using different α's is less than the scatter of points about the individual curves. This is consistent with the requirement that for a network composed of a single size tube, dispersion equal to zero, the capillary pressure curves for all α must coincide and become a horizontal

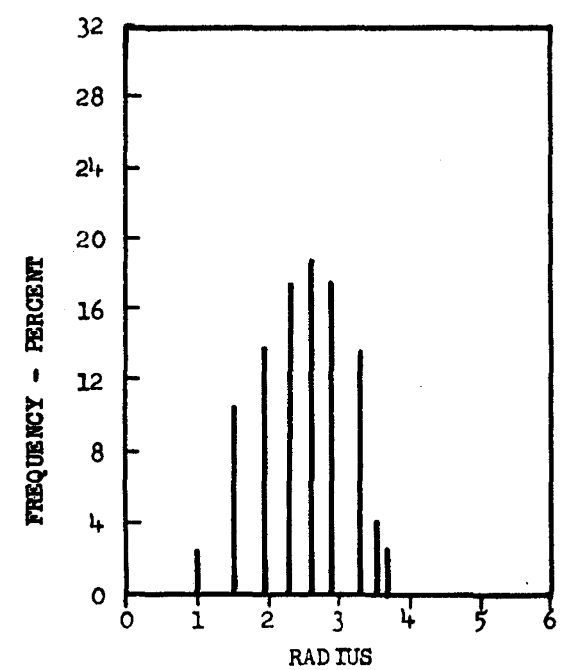

Fig. 23—Tube Radius or Length Distribution.

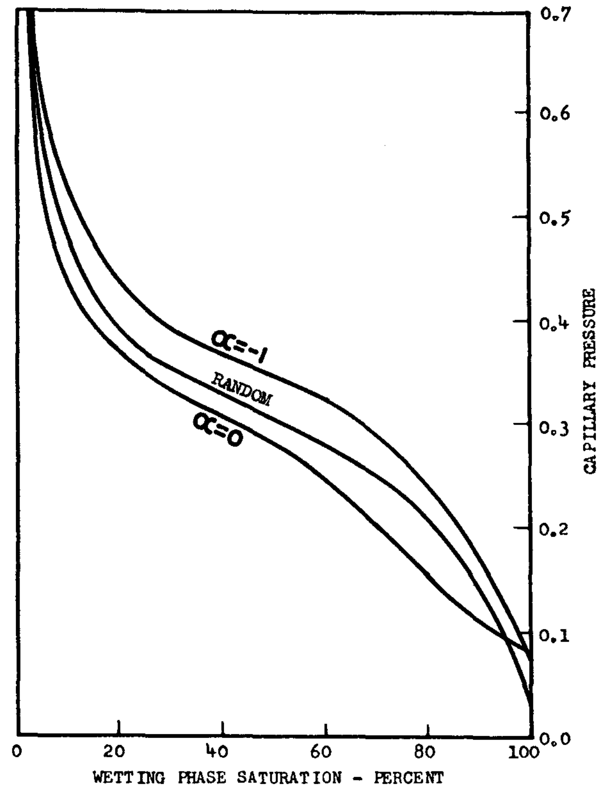

Fig. 24—Capillary Pressure Characteristics of Double Hexagonal Network with Randomly Combined Tube Radius and Length. Radius and Length Distribution from Fig. 10.

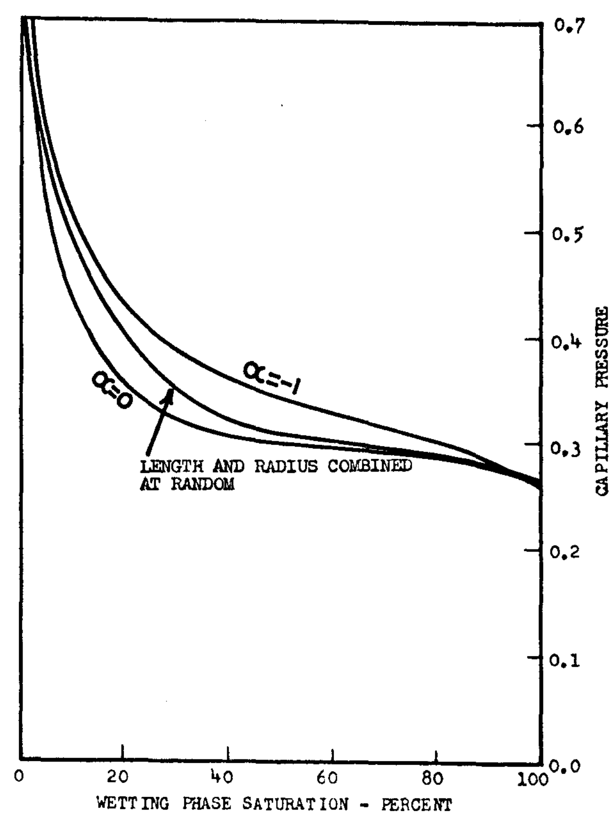

Fig. 25—Capillary Pressure Characteristics of Double Hexagonal Network with Randomly Combined Tube Radius and Length. Radius and Length Distribution from Fig. 23.

straight line. If the dispersion is large, as for the distribution of Fig. 10, the capillary pressure curve is sensitive to changes in α.

Capillary Pressure Curves from a Random Combination of Tube Length and Radius

The second method of relating tube length and radius is by randomly assigning different lengths to the tubes in a manner similar to that used to assign radii. At first thought, this may seem to be the preferred method and to give a network closely resembling the network in a real porous medium. However, examination of thin sections of well consolidated sandstone shows that there seems to be a fixed relation between pore length and radius. Pores of large radius seem to be short, whereas pores of small radius, usually in the form of crevices, are long.

There is an objection to the random combination of tube length and radius that is not in any way connected to the fundamental properties of networks. This objection arises from the fact that a great deal more time is required to obtain data on networks established by the random combination than on networks in which there is a fixed relation of tube length and radius.

Two networks in which tube radii and lengths were randomly combined were studied. The radius and length frequency distribution were taken to be that of Figs. 10 and 23 because these distributions represent the upper and lower limits respectively of the dispersion of pore size frequency distributions found in natural porous media. Radius and length were both randomly distributed in the network positions by using two different random number tables. The double hexagonal network was used. The capillary desaturation was carried out exactly as for the example given earlier. For each tube, the volume was calculated from $\pi r^2 l$ where both r and l depend upon the position in the network.

The resulting capillary pressure curves are shown in Figs. 24 and 25, together with the curves for the same distribution and network, but using the relations $l = Cr^{-1}$ and $l = Cr^0$. The capillary pressure curves from the random combination of length and radius have the characteristic shape observed for real porous media and are not much different from the curves for a network with a fixed tube length to radius relationship. When the flow properties of networks are discussed in paper III of this series, it will be shown that the random combination also leads to flow properties which are very much like those from a network with tube length inversely proportional to radius.

CONCLUSIONS

The similarity between capillary pressure curves from networks and from sandstones and sintered glass does not prove the validity of the network model. The simple bundle of tubes gives equally satisfactory capillary pressure curves, despite its lack of resemblance to real porous media. The test of the network model will be made when its flow properties are compared to those of real porous media.

The study of the capillary pressure characteristics of networks as presented in this section, however, does lead to certain conclusions concerning the relation between several network parameters and the capillary pressure curve. These conclusions are given below.

1. Network capillary pressure curves approach the simple bundle of tubes curve as the β factor increases, that is, as the number of tubes joined to each tube increases.

2. The sensitivity of network capillary pressure curves to changes in network is a function of the dispersion of the tube radius distribution. For low dispersion, all networks of a given distribution have almost identical capillary pressure curves. A distribution with high dispersion gave capillary pressure curves which were different for each network.

3. The capillary pressure curve is more sensitive to changes in tube radius distribution than to changes in network. For this reason, the capillary pressure curve can be used to estimate pore size distribution of real porous media.

4. A pore size distribution calculated from the capillary pressure curve of a real porous medium by means of the bundle of tubes model will have an average pore radius and dispersion which are smaller than the true average pore radius and dispersion.

5. The ratio between the average pore radius calculated from a capillary pressure curve and the true average pore radius is less than unity and is a function of the network. For networks of low β factor, the ratio is small; as β increases, the ratio increases, finally becoming unity when β is infinite.

6. The dispersion of the pore size distribution calculated from the capillary pressure curve of a real porous medium is about one-half of the true dispersion.

7. The sensitivity of the capillary pressure curve to changes in α in the relation $l = Cr^{\alpha}$ is a function of the dispersion of the tube radius distribution. For low dispersion, changes in α have little effect on the capillary pressure curve, whereas for high dispersions, changes in α cause significant changes in the curves.

8. The capillary pressure curve from networks in which tube length and radius are randomly combined, instead of by the relation $l = Cr^{\alpha}$, fall between the curves from networks in which α is zero and -1.

APPENDIX

CALCULATION OF THE PORE SIZE DISTRIBUTION FROM THE CAPILLARY PRESSURE CURVE BY MEANS OF THE BUNDLE OF TUBES MODEL

A derivation of the pore size distribution function for porous materials has been given by Ritter and Drake.* Their derivation gives a pore volume distribution. For fluid flow calculations, the pore number distribution derived below is more useful.

The pressure difference between the wetting and nonwetting phases required to cause nonwetting phase to displace wetting phase from a tube of radius r is given by

$$P = \frac{2\,\delta \cos \Theta}{r} \quad \ldots\ldots\ldots \quad (1\text{-I})$$

*Ritter, L. C. and Drake, R. L.: *Ind. Eng. Chem. Anal. Ed.* (1945), 17, 782.

where δ is the surface or interfacial tension and Θ is the contact angle of the wetting phase on the inner wall of the tube.

The number of tubes having radii between r and $r + dr$ is defined as

$$dN = N(r)\, dr \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2\text{-I})$$

where $N(r)$ is the radius distribution function. If δ and Θ are not functions of pressure or radius, Eq. 1-I can be differentiated to give

$$Pdr + rdP = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3\text{-I})$$

Eliminating dr from Eqs. 2-I and 3-I gives

$$N(r) = \frac{P}{r}\frac{dN}{dP} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4\text{-I})$$

Inasmuch as dN cannot be measured directly, it is necessary to relate dN to the increment of pore volume, dV, which is displaced by a change in pressure dP. An analytical expression for the pore volume can be obtained only if a simple pore geometry is assumed. The porous material is therefore assumed to be equivalent to a bundle of tubes of different radii but of constant radius along the length of a tube.

If the port space is a cylinder of length l and radius r, then

$$dV = \pi r^2\, l\, dN \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5\text{-I})$$

Substituting for dN in Eq. 4-I gives

$$N(r) = \frac{P}{\pi r^3 l}\frac{dV}{dP} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6\text{-I})$$

The saturation of a porous medium is defined as the volume of a given phase in the porous medium relative to the total pore volume, or

$$S = \frac{V}{V_p} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7\text{-I})$$

Then $dV = V_p dS$. Substituting this into Eq. 6-I gives

$$N(r) = \frac{P\, V_p}{\pi\, r^3 l}\frac{dS}{dP} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8\text{-I})$$

But

$$r = \frac{2\,\delta\cos\Theta}{P}$$

Therefore

$$N(r) = \frac{P^4\, V_p}{\pi\, l\,(2\,\delta\cos\Theta)^3}\frac{dS}{dP} \quad . \quad . \quad . \quad (9\text{-I})$$

In Eq. 9-I, all quantities except l are known or calculable from the capillary pressure curve. The l term can be treated in several ways. If l is assumed to be the same for all tubes, then Eq. 9-I may be rewritten

$$l\, N(r) = \frac{P^4 V_p}{\pi\,(2\delta\cos\Theta)^3}\frac{dS}{dP} \quad . \quad . \quad . \quad (10\text{-I})$$

The l term is now a frequency scale factor. In flow calculations, the term $lN(r)$ can be used directly so that l need not be evaluated separately.

An alternate method of treating l is to relate r and l by a relation of the form $l = C^\alpha$, where C is a proportionality constant and α is another constant. Using this relation, Eq. 9-I becomes

$$N(r) = \frac{P\, V_p}{\pi\, C r^{3+\alpha}}\frac{dS}{dP} \quad . \quad . \quad . \quad . \quad . \quad (11\text{-I})$$

or

$$CN(r) = \frac{P^{1+\alpha} + V_p}{(2\,\delta\cos\Theta)^{3+\alpha}}\frac{dS}{dP}. \quad . \quad . \quad . \quad (12\text{-I})$$

The C term is now the frequency scale factor in the pore size distribution. If α is taken as zero, then C is equal to l. The l term is usually taken as -1 as is described in the sections on the capillary pressure and relative permeability characteristics of networks of tubes. In any flow calculation, the quantity $CN(r)$ can be used directly if the same relation between r and l is used in the flow calculation as in the pore size distribution calculation.

Graphic integration from zero to infinity of a plot of $CN(r)$ vs r gives the total number of pores multiplied by the constant C. That is

$$CN_T = \int_o^\infty CN(r)\,dr \quad . \quad . \quad . \quad . \quad . \quad . \quad (13\text{-I})$$

The number of pores in the radius interval $r_2 - r_1$ multiplied by the constant C is

$$CN(r_2 - r_1) = \int_{r_1}^{r_2} CN(r)\,dr \quad . \quad . \quad . \quad (14\text{-I})$$

The frequency of occurrence of pores in the interval $r_2 - r_1$ is then

$$f(r_2 - r_1) = \frac{\int_{r_1}^{r_2} CN(r)\,dr}{\int_o^\infty CN(r)\,dr} \quad . \quad . \quad . \quad . \quad (15\text{-I})$$

Substituting for $CN(r)$ from Eq. 12-I gives

$$f(r_1 - r_2) = \frac{\int_{r_1}^{r_2} P^{1+\alpha}\left(\frac{dS}{dP}\right) dr}{\int_o^\infty P^{1+\alpha}\left(\frac{dS}{dP}\right) rd}. \quad . \quad . \quad (16\text{-I})$$

All other terms in Eq. 12-I cancel from numerator and denominator because they are not functions of S, P, or r.

From Eq. 1-I

$$dr = -\frac{2\delta}{P^2}\cos\Theta\, dP \quad . \quad . \quad . \quad . \quad . \quad . \quad (17\text{-I})$$

Substituting into Eq. 16-I gives

$$f(r_2 - r_1) = \frac{\int_{S_1}^{S_2} P^{2+\alpha}\, dS}{\int_o^1 P^{2+\alpha}\, dS}. \quad . \quad . \quad . \quad . \quad (18\text{-I})$$

The limits of the integrals of Eq. 18-I are now expressed in terms of saturation. In the denominator, the limits in saturation are from zero to unity, because at unit saturation, which is at the lowest capillary saturation, all pores, including the largest in the distribution, are filled. In the numerator, the lower limit is the saturation S_1 at which the capillary pressure causes entry of nonwetting phase into tubes of radius r_1, and similarly for the upper limit S_2.

To obtain the tube radius distribution at discreet radii as used in the network study, the radius limits of integration, r_1 and r_2, were chosen at the midpoint between the previously selected radii.

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The NETWORK MODEL of POROUS MEDIA*
II.
Dynamic Properties of a Single Size Tube Network

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ABSTRACT

Networks of resistors are used as analog computers to obtain relative permeability and resistivity index curves for networks of tubes. These curves have all of the characteristics of those obtained on porous media and, therefore demonstrate the validity of the network of tubes model.

INTRODUCTION

In paper I[1] of this series the network model was shown to give information on the structure of porous media that could not be obtained from the bundle of tubes model. The agreement between the capillary pressure characteristics of a network of tubes and those of typical porous media does not prove the validity of the network made. The bundle of tubes model, although it gives less information, also shows this agreement.

In this paper the flow properties of a network of tubes are shown to be in agreement with flow properties of porous media. The flow properties of the bundle of tubes model are not in adequate agreement with those of porous media.

The dynamic properties of porous media, such as fluid permeability and electrical conductivity, have been found experimentally to be more sensitive functions of structure and pore size distribution than are the static properties such as porosity and capillary pressure characteristics. The network model can, therefore, be best tested by comparing the dynamic properties of a network of tubes with the observed properties of porous media. All static and dynamic properties of a completely valid model must agree with the observed properties.

The most commonly measured sensitive dynamic properties are the relative fluid permeability and the relative electrical resistivity. Experimental measurements of these dynamic properties on sandstone, sintered glass, sintered alumina, and soil are reported in the petroleum and soil science literature. These properties were chosen, therefore, for the test of the network model.

In the network model, relative permeability is the permeability of the network when some of the tubes are not conducting fluid, relative to the permeability when all tubes are conducting fluid. Similarly, the relative resistivity is the electrical resistivity of the network of tubes when some of the tubes are filled with a non-conductor, relative to the resistivity when all tubes are filled with the electrical conducting fluid.

Single Size Tube Network

The simplest porous medium is one in which all pores are of the same size. Such media can be approached by packing beds or columns of uniform size particles of sand, glass, or metal. The model of such medium in this study is a network of single size tubes. The network form, the β factor, is usually not known for real porous media. If the validity of the network model can be established, then the study of the flow properties

*This work is taken from a dissertation submitted by the author to the Graduate School, University of Southern California, in partial fulfillment of the requirements for the PhD degree while on leave from California Research Corp., La Habra, Calif. Much of the detail omitted from this paper can be found in the dissertation which is available on microfilm from the Library of University of Southern California, Los Angeles 7, Calif.

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[1]References given at end of paper.

of networks may give information concerning the network form of real porous media.

In principle, network theory can be applied to the network of tubes to calculate the network flow resistance. The network of Figs. 1-I, 2-I, 3-I, and 4-I,* when composed of single size tubes, can be reduced to a series-parallel arrangement for which the flow resistance can be easily calculated. As the network is desatuarated, that is, as some of the tubes are made non-conducting, the network is no longer a series-parallel arrangement and network theory must be applied. In practice, the calculation of total network resistance of a network of several hundred tubes is impossibly laborious. Network theory leads to a determinant as the solution for the total network resistance. The network models used in this study lead to determinants of several hundred rows and columns; that is, determinants of order 100 or more. Such determinants cannot be evaluated by any reasonable amount of labor. Even modern high-speed computers cannot evaluate determinants of greater than 30th order unless the determinant has some symmetry condition which permits reduction to a lower order. The determinants derived from the network models have no symmetry, and therefore, cannot be reduced.

Instead of calculating flow properties of the network model from network theory, networks of electrical resistors were constructed and the total resistance was measured by a conventional Wheatstone bridge. The network of electrical resistors is, therefore, an analog computer which is used to solve network problems when these problems cannot be solved conveniently by analytical or numerical methods.

The choice of a sequence of desaturation is a problem that arises in the study of single size tube networks. As has already been pointed out in the study of the capillary pressure characteristics of the single size tube network[1], such networks desaturate completely when the capillary pressure is equal to the entry pressure. There are no intermediate equilibrium saturations. In real porous media, the pores are never exactly the same size, so the capillary desaturation mechanism operates stepwise to desaturate the smaller pores as the capillary pressure is raised.

The capillary desaturation mechanism can be made to operate on the single size tube network if the tubes are assumed to have a very narrow radius distribution. This distribution can be made as narrow as desired so that for purposees of calculating permeability, resistivity, or volume, the tubes can be considered of equal size. The smallest difference in tube radii, however, will allow the capillary desaturation mechanism to operate.

The capillary desaturation mechanism operating on the single size tube network is equivalent to a random desaturation with the requirement that the desaturation starts from the edge of the network and proceeds inward only through tubes that are already desaturated. The concept of a very small but finite distribution of tube radii causing the capillary desaturation mechanism to operate is closer to reality even for very unfirm porous media than is the concept of random desaturation. However, a random desaturation mechanism may be operating in a system where gas comes out of solution, thus randomly filling the pores without regard to size or position.

A tube radius distribution in which all radii were considered grouped very close to the average radius was used. The single, double, and triple hexagonal, and the square network were constructed of one-half watt, 4,700 ohm carbon resistors (Allied Radio Co., Chicago, Ill., Catalog No. 1-820-1953). These resistors were within ± 10 per cent of 4,700 ohms. The resistance of the network was measured with a precision of 1 per cent by a portable Wheatstone bridge.

The operations on the network of resistors and the calculation of the flow properties were essentially as follows. The complete network of resistors was constructed by inserting the wire leads of the resistors in Fahnstock clips mounted on a masonite sheet. The total resistance was measured between the bus bars shown on top and bottom of the networks in Figs. 1-I, 2-I, 3-I, and 4-I. Resistors were removed from the numbered positions according to the capillary desaturation mechanism. At each capillary pressure step the network resistance was measured. The ratio of network resistance at a partial saturation to the total network resistance was the wetting phase relative resistance and the reciprocal of this ratio was the wetting phase relative permeability. The saturation was simply the fraction of the total number of resistors remaining in the network.

As each resistor was removed from the network, it was inserted in the same position in another identical network. The second network represented the network of pores containing nonwetting phase. When sufficient resistors appeared in the nonwetting phase network to give a continuous path between the bus bars, the resistance of this network was measured at each capillary pressure step. The relative resistances so obtained gave the nonwetting phase relative resistivity and permeability.

Effect of Spatial Distribution and Network Size on the Dynamic Properties

Before discussing the results of the study of flow properties in a network of uniform size tubes, it is necessary to how that, as for the capillary pressure curves, the small size of the resistor network relative to the size of the network in real porous media does not invalidate the results. The effect of different random spatial arrangement of the tubes must also be investigated, because although the tube radius distribution was taken to be very narrow, the spatial distribution of the tubes determines the order in which they desaturate.

The effect of different random spatial distributions was investigated by comparing flow properties of networks obtained from four different random number tables. The difference between the smoothed relative permeability curves obtained from the four different random number tables was less than the scatter of points about each curve.

The effect of network size on the flow properties of the networks was determined by using networks of ¾, ½, and ¼ the size of the original networks as shown in Figs. 1-I, 2-I, 3-I, and 4-I. The reduction in size was obtained by two methods. In the first method, the reduced size network maintained the approximately square outer form of the original network. In the second method, the width of the original network was preserved, but the length was reduced.

Several conclusions were drawn from the study of the reduced size networks. It was noted that the two methods of reducing the size gave essentially the same results, except for the one-fourth size network. This network gave very scattered flow data from which no

*The roman numeral I after the figure number indicates that the figure appears in paper I of this series.

smooth relative permeability versus saturation curves could be drawn. The one-half size network containing from 115 to 195 tubes probably represent the smallest network that has the same statistical properties as the larger networks. The one-fourth size networks were not used in the study of the effect of network size.

The relative permeability curves from the other reduced size networks showed a small but continuous trend as a function of network size. At each saturation, the relative permeability was plotted as a function of the reciprocal of the network size relative to the size of the original network. The best straight line was drawn through these points and extrapolated to zero reciprocal size, which is equivalent to extrapolation to a network of infinite size. Table 1 shows the range in relative permeability at constant saturation when extrapolated from the network of Figs. 1-I, 3-I, and 4-I to infinite size networks.

Extrapolation to infinite size places all network data on the same size basis. The infinite size network must be considered a hypothetical system because of the rather long extrapolation required and because of the absence of any proof that the extrapolation should be linear. An alternate method of putting all network data on the same size basis is to extrapolate the data from all networks to a finite network of some standard size. For example, all networks relative permeabilities can be extrapolated to a network of 500 tubes by a very short extrapolation. Relative permeability characteristics of such networks are only very slightly different from those of the hypothetical infinite size network. For this reason, data from the hypothetical infinite size network is used in the following discussion.

The relative permeability curves for the four networks each containing tubes of a single size and extrapolated to a hypothetical infinite size network are shown in Fig. 1. The pair of straight lines which represents the relative permeability of a simple bundle of single size tubes is also shown on this graph for comparison.

For a network of single size tubes, the relative fluid resistance (the reciprocal of the relative permeability) is the same as the relative electrical resistance at a given fluid saturation. The relative resistivity is therefore the reciprocal of the relative permeability at each saturation. The wetting phase relative resistivity curves of the networks are plotted in Fig. 2. The nonwetting phase relative resistivity curves are shown in Fig. 3.

TABLE 1 — EFFECT OF NETWORK SIZE ON RELATIVE PERMEABILITY AT CONSTANT SATURATION

Wetting Phase Relative Permeability—Per Cent

Wetting Phase Saturation	Single Hexagonal		Double Hexagonal		Triple Hexagonal	
	Fig. 1-I	Infinite	Fig. 3-I	Infinite	Fig. 4-I	Infinite
90	72	69	78	76	82	82
70	31	29	43	40	50	50
50	11	9	18	17	25	25
30	2	1	3	3	8	8

Nonwetting Phase Relative Permeability

Nonwetting Phase Saturation	Single Hexagonal		Double Hexagonal		Triple Hexagonal	
	Fig. 1-I	Infinite	Fig. 3-I	Infinite	Fig. 4-I	Infinite
90	77	75	83	82	82	82
70	35	32	47	43	53	50
50	7	5	12	12	8	8

INTERPRETATION OF THE RESULTS FROM THE SINGLE SIZE TUBE NETWORK

Comparison of the relative permeability curves from the single size tube networks with typical relative permeability curves obtained from sandstone and sintered glass shows a remarkable similarity. The characteristic shape of the relative permeability curves of real porous media seems to be a direct consequence of the interconnection of the pores to form a network. The network structure alone is sufficient to give the characteristic shape, no pore size distribution is required.

The wetting phase relative resistivity curves are also found to be similar to the typical curves observed for porous media. The nonwetting phase relative resistivity curves cannot be compared, because very little information about this property has been reported for real porous media.

The relation between wetting phase relative resistivity and wetting phase saturation can be expressed as $R = S_w^{-n}$ where n is the slope of the straight lines in Fig. 2. The most recent experimental study of this relation for real porous media by Wyllie and Spangler[2] has shown that for sandstone, unconsolidated sand, sintered glass,

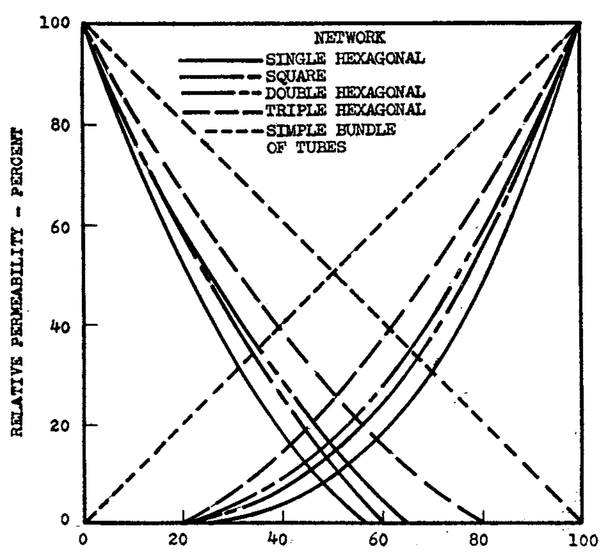

FIG. 1 — RELATIVE PERMEABILITY CHARACTERISTICS OF AN EXTRAPOLATED HYPOTHETICAL INFINITE SIZE NETWORK OF SINGLE SIZE TUBES.

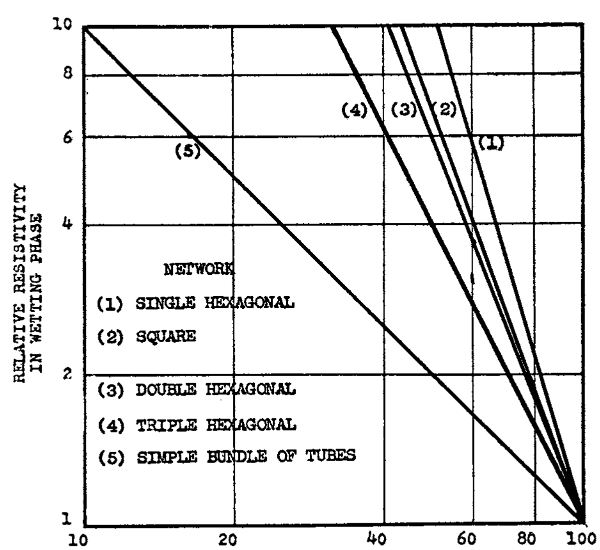

FIG. 2 — WETTING PHASE RELATIVE RESISTIVITY OF EXTRAPOLATED HYPOTHETICAL INFINITE SIZE NETWORKS OF SINGLE SIZE TUBES.

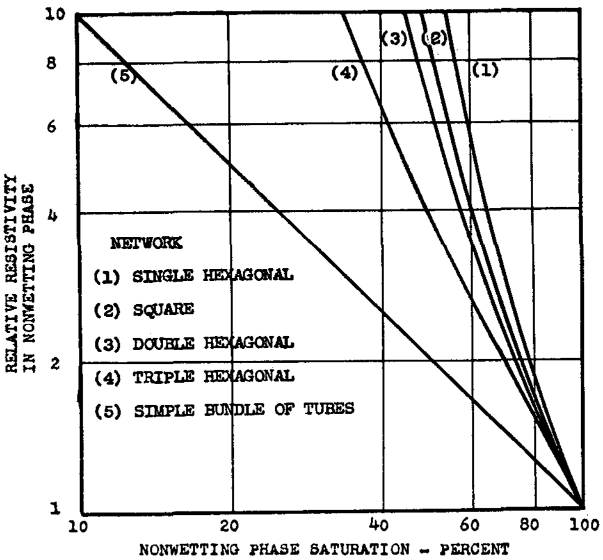

FIG. 3 — NONWETTING PHASE RELATIVE RESISTIVITY OF EXTRAPOLATED HYPOTHETICAL INFINITE SIZE NETWORKS OF SINGLE SIZE TUBES.

and sintered aluminum n varies from 1.42 to 2.55. The network relative resistivity data give values of n as shown in Table 2.

TABLE 2

Network	β	$1/\beta$	n
Composite Bundle of Tubes	2	0.50	—
Single Hexagonal	4	0.25	3.6
Square	6	0.167	2.8
Double Hexagonal	7	0.142	2.6
Triple Hexagonal	10	0.10	2.0
Simple Bundle of Tubes	∞	0.00	1.0

The β factor is the same network form parameter that was introduced in the study of the capillary pressure characteristics of a network.[1] The composite bundle of tubes has tubes with radius varying along the length. Relative resistivity data from the composite bundle of tubes did not obey the relation $R = S_w^{-n}$, and therefore, n could not be obtained for $1/\beta$ equal to 0.50.

Fig. 4 shows a plot of n vs $1/\beta$. A detailed comparison of these results with those from experimental studies on real porous media will be postponed until the effect of a tube radius distribution is described in paper III of this series. The results from the single size tube networks can be used at this point to make a few generalizations concerning the behavior of real porous media.

The most important observation is that the characteristic multiphase flow properties of porous media are

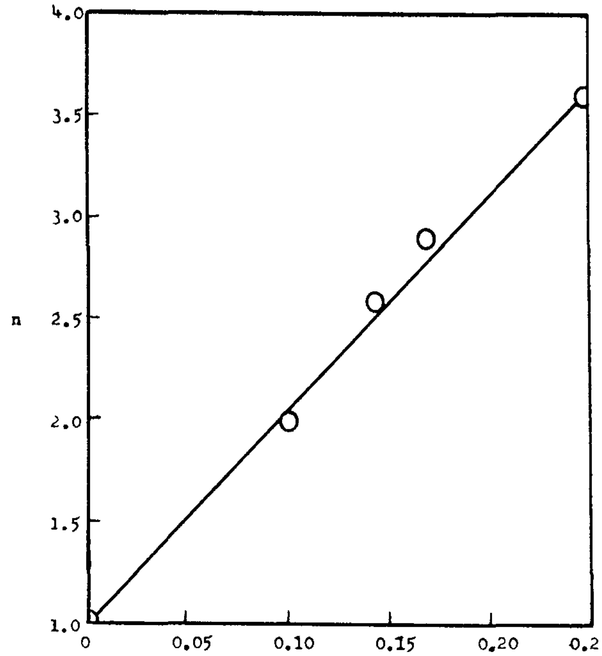

FIG. 4 — RELATION BETWEEN RECIPROCAL OF NETWORK $1/\beta$ FACTOR AND n IN THE RELATION $R = S^{-n}$ FOR NETWORKS OF SINGLE SIZE TUBES.

a direct consequence of the interconnection of the pore spaces to form a network.

A second and almost equally important observation is that the flow properties of networks are a function of the network for a system of uniform pores.

A third observation is that in a network containing uniform size pores, all relative flow properties which are functions of pore geometry only, such as relative permeability and relative resistivity, will be identical functions of the saturation.

The relation between n and $1/\beta$ as shown in Fig. 4 permits use of experimental flow data to determine the network form of porous media known to be of uniform pore size. As shown in paper I of this series, the shape of the capillary pressure curve can be used to estimate the degree to which a porous medium approaches a network of uniform size pores.

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The NETWORK MODEL of POROUS MEDIA*
III.
Dynamic Properties of Networks with Tube Radius Distribution

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ABSTRACT

Relative permeability and relative electrical resistivity curves are obtained for networks of tubes with a tube radius distribution by means of a network of resistors used as an analog model. These curves are similar to those obtained from real porous media. The effects of pore size distribution and network structure on relative permeability and resistivity are demonstrated.

INTRODUCTION

Study of the dynamic properties of networks of single size tubes in paper II[1] of this series has shown that the network model has flow properties similar to those observed on real porous media. The comparison of properties of single size tube networks with those of real porous media cannot be carried far because very few porous media have uniform size pores. The capillary pressure curve from most porous materials such as soil, sandstone, and sintered glass indicate that these materials have pore size distribution similar to those shown in Figs. 5-I, 9-I, and 10-I.† To properly test the network model, it is therefore necessary to compare the dynamic properties of real porous media with those of networks of tubes in which the tube radii are distributed according to the distributions in the aforementioned figures.

A network in which the tube sizes are distributed cannot be treated in the same manner as a network of single size tubes. In the single size tube network, the relation between tube geometry, flow resistance, and tube volume did not have to be known if the assumption was made that whatever relation did exist was the same for all tubes in a particular network. In a network of distributed tube size, the relation between tube geometry and flow resistance must be known or assumed in order to replace the tubes by equivalent electrical resistors.

RELATION OF TUBE GEOMETRY TO FLOW RESISTANCE

The interchangeability of tubes and electrical resistors in the network model is possible because of the analogy between Poiseuille's law and Ohm's law. Poiseuille's law for a cylinder is

$$q = \frac{\pi r^4}{8\mu l} \triangle P \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where q is the volumetric rate of flow, μ is the viscosity. $\triangle P$ is the pressure gradient, r is the tube radius, and l is the tube length.

Ohm's law is

$$I = \frac{1}{R} \triangle E \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where I is the flux and is equivalent to the volumetric rate in Poiseuille's law, R is the resistance, and $\triangle E$ is the voltage gradient. Both laws give the flux as a function of the potential drop and the resistance of the medium.

By analogy then,

$$\frac{1}{R} = \frac{\pi r^4}{8\mu l} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

*This work is taken from a dissertation submitted by the author to the Graduate School, University of Southern California, in partial fulfillment of the requirements for the PhD degree while on leave from California Research Corp., La Habra, Calif. Much of the detail omitted from this paper can be found in the dissertation which is available on microfilm from the Library of University of Southern California, Los Angeles 7, Calif.

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[1]References given at end of paper.

†The roman numerals following the figure number indicate the paper in this series in which the figure appears[1,2].

When only relative permeability data are to be obtained from a network of resistors, Eq. 3 can be written

$$\frac{1}{R} = \frac{r^4}{l} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

because the coefficient $\frac{\pi}{8\mu l}$ is common to all tubes in the saturated and partially saturated network.

When the network is used to study relative resistivity, Ohm's law for a tube filled with a conducting fluid gives the relation between tube geometry and electrical resistance. For the tube

$$I = \frac{\pi r^2}{\rho l} \triangle E \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

where ρ is the electrical resistivity of the conductor in the tube. For the resistor that is to replace the tube, Ohm's law is

$$I = \frac{1}{R} \triangle E \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

By analogy then

$$\frac{1}{R} = \frac{r^2}{l} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

Relative resistivity of the network will be independent of the coefficient $\frac{\pi}{\rho}$ so the equivalence of Eq. 7 can be written

$$\frac{1}{R} = \frac{r^2}{l} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

Eqs. 4 and 8 permit the network of tubes to be replaced by a network of resistors. The resistance of each resistor in the network will depend upon whether the network is to be used for obtaining relative permeability or relative resistivity data. For each tube in the network to be used for relative permeability data, the resistance of the equivalent resistor is given by

$$R_i = C \frac{l_i}{r_i^4} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

For each tube in the network to be used for relative resistivity data, the equivalent resistor is given by

$$R_i = C \frac{l_i}{r_i^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

The constant, C, can be the same in Eqs. 9 and 10. This constant is chosen to give convenient values to the electrical resistors. The constant does not influence the relative permeability or relative resistivity properties of the network.

The volume of each tube in the network is given by

$$V_i = \pi r_i^2 l_i \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

The total network volume is then

$$V_T = \sum_{i=1}^{j} \pi n_i r_i^2 l_i \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

where n_i is the number of tubes of radius r_i and length l_i. The percentage wetting phase saturation is

$$S = 100 \left[1 - \frac{\sum_{i=1}^{g} m_i r_i^2 l_i}{\sum_{i=1}^{j} n_i r_i^2 l_i} \right] \quad . \quad . \quad . \quad (13)$$

where m_i is the number of tubes of radius r_i that are empty. The terms m_i and n_i are not necessarily equal because, as was demonstrated in the discussion on the capillary desaturation mechanism in paper I of this series, not all tubes in a radius group are emptied at the same pressure.

The capillary desaturation mechanism operates to desaturate the network of tubes in a sequence that depends upon the random spatial distribution and the tube radius distribution. The volume and flow resistance of the tubes are functions of both tube radius and length. Therefore, in establishing the network of tubes in which tube size is distributed, both radius and length must be assigned to each tube. This problem was encountered in paper I when capillary pressure curves of networks with distributed tube size were desired. The capillary pressure curve calculations required tube radius and length in order to calculate tube volume. For relative permeability and resistivity data from a network of tubes, the tubes radius and length are required to fix both volume and resistance of each tube.

In the study of the flow properties of networks with distributed tube size the problem of specifying tube radius and length was treated in the same manner as for the study of capillary pressure properties. The two methods of specifying tube radius and length are: (1) by random combination of radius and length to arrive at the volume and resistance of each tube, or (2) by fixing the relation between radius and length thereby eliminating length as a variable. The disadvantage of the method of random combination was that the resultant networks were more time consuming to study and therefore fewer network properties could be studied within the time available for this work.

Only two networks in which tube radius and length are randomly combined were studied. The relative permeabilities of these networks were found to be similar to those of networks with a fixed relation between tube radius and length. The data for the randomly combined networks will be presented after the fixed relation networks are discussed.

The fixed relation between tube radius and length was assumed to be of the form

$$l = Cr^{\alpha} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

where C and α are constants. The constant C can have any value, provided that it has the same value for every tube in the network.

There now remains the problem of choosing a value for α and showing that the chosen value is reasonable and gives networks which have properties similar to those of real porous media. The main structural feature of porous media, namely that they are networks of pore spaces, has already been given substantial support from the study of networks of single size tubes. Using the network structure as a working hypothesis, it is possible to show that experimental data on porous media lead to a small range for the possible values of α[3].

Permeability and porosity data reported by five authors[4] for sand packs show that α lies in the range -0.24 to -1.25 with an average of -1.00. Resistivity and porosity data on the same kind of sand packs show α to be in the range 0.00 to -0.66 with an average of -0.26. The combined permeability and resistivity data give an over-all average α of -0.63. This average rounded off to -1.00, gives the final choice of α for the network of tubes model.

The α term derived from porous media is simply that α which gives a network of tubes with the same permeability-porosity and resistivity-porosity relationship as do real porous media.

Other values of α, namely 0.00, 1.00, and -2.00, were also used in a few cases to show the effect of variation of α on the network flow properties. Later in this paper, flow properties of real porous media will be compared with those of networks with different values of α. These comparisons will show that networks with α of -1.00 have flow properties most closely resembling those of real porous media.

Network Measurements

The adoption of a relation between tube radius and length, namely $l = Cr^{-1}$, permits substitution of electrical resistors for the tubes in the network of tubes model. Having adopted $l = Cr^{-1}$ as the relation between tube radius and length, the resistance of each tube can be given in terms of the radius alone. The resistance of the equivalent resistor is then

$$R \text{ fluid flow} = \frac{C}{r^5} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

and

$$R \text{ electrical cond.} = \frac{C}{r^3} \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

Using the relation $l = Cr^{-1}$ the volume of each tube becomes

$$V_i = C\pi r_i \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

and the total volume becomes

$$V_T = Cn_i\pi r_i \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

The radius distribution alone now determines the size distribution of resistors in the networks. The range of radii was taken to be from 0 to 10 in arbitrary units. The constant C in Eqs. 15 and 16 was chosen to be 10^6 to make the resistance values fall in the range of commercially available carbon resistors. The value chosen for C does not influence the relative permeability or relative resistivity properties of a network.

The resistance values for a given radius distribution are calculated from Eqs. 15 or 16, depending upon whether the network is to be used to study relative permeability or relative resistivity. As a consequence, the resistance values for a given radius distribution will depend upon the flow property which is being studied. The choice of tube radii was dictated by the resistance of commercially available resistors. Having chosen the radii from the set of resistors to be used for studying relative permeability, it was possible, by means of Eq. 16, to calculate the sizes of the resistors needed for the study of relative resistivity. Many of the resistance values so calculated were not commercially available, so the next nearest available size was chosen. For example, the tube radius of 2.92 was calculated from a resistor of 4,700 ohms, using Eq. 15. The equivalent relative resistivity resistor, calculated from Eq. 16, is 40,400 ohms, but the nearest available resistor is 39,000 ohms.

The resistors were one-half watt carbon resistors supplied by Allied Radio Corp., Chicago, Ill. Catalog No. 1-820 (1953). The supplier claimed the resistors to be within ± 10 per cent of the rated value. A study of 500 resistors rated at 8,200 ohms showed them to be randomly distributed about the rated value with a standard deviation of 5 per cent.

The procedure for obtaining relative permeability and relative resistivity curves from networks is best described by means of an example. This example uses the tube radius distribution of Fig. 5-I and the triple hexagonal network. The random spatial distribution of resistors in the network positions is obtained in the same way as for the capillary pressure curves for this distribution.

A random number table is used to construct the network of resistors, because for each tube radius, there is an equivalent resistor. The resulting network of resistors is equivalent to a network of tubes in which each tube is filled with wetting phase. The resistance of the network, measured from the top to the bottom bus bar, is equivalent to the reciprocal of the permeability or to the resistivity, depending upon whether the network is being used to model a fluid flow or an electrical conducting system.

The capillary desaturation mechanism dictates the sequence in which tubes containing wetting phase are removed from the network. Column 2 of Table 1 gives the sequence for this example. Resistors are removed in the sequence given in Column 2 of Table 1. At each capillary pressure step, the network resistance is measured. These measurements are given in Columns 4 and 8 of Table 1. The saturation at each capillary pressure step, calculated by the method previously described in the section on capillary pressure curves, is given in Column 3 of Table 1.

The wetting phase relative permeabilities at the various saturations are simply the network resistance, of a network established to study relative permeability, when all the resistors are in place divided by the network resistance when some of the resistors have been removed.

The wetting phase relative resistivities are the network resistance of the appropriate network at each saturation divided by the network resistance when all resistors are in place.

The removal of resistors from a network is equivalent to replacing tubes containing wetting phase by the same tube containing nonwetting phase. When the tubes containing nonwetting phase form a continuous path through the network, there can be conductance of fluid and electricity in the nonwetting phase.

The network of tubes containing nonwetting phase was formed by inserting the resistors removed from the wetting phase network into an identical nonwetting phase network. The sequence in which the nonwetting phase network is formed is given in Columns 1 and 2 of Table 1, together with the network resistance in Column 9 at each saturation.

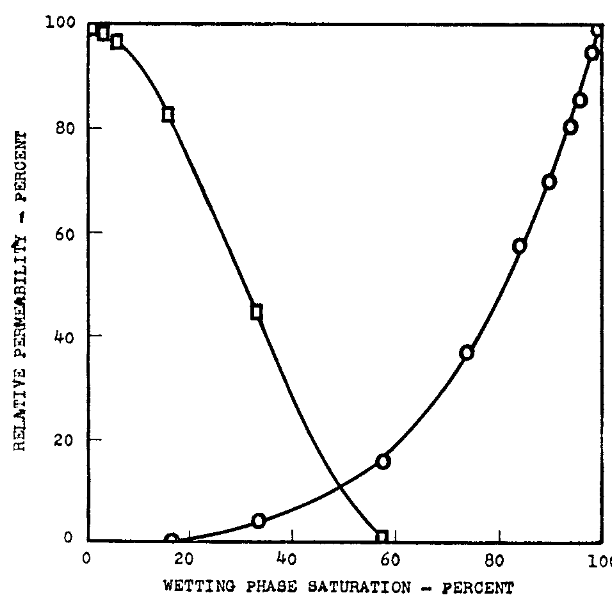

FIG. 1—Relative Permeability Characteristics of a Triple Hexagonal Network with Tube Radius Distribution of Fig. 5-I.

TABLE I—CAPILLARY DESATURATION SEQUENCE AND MEASURED RESISTANCE IN TRIPLE HEXAGONAL NETWORK WITH TUBE RADIUS DISTRIBUTION OF FIG. 5-1

(1) Tube Penetrated by Non-wetting Phase, Tube Radius	(2) Tube Position	(3) Network Wetting Phase Saturation, Per Cent	(4) Wetting Phase Resistance in Relative Permeability Network, Ohms	(5) Nonwetting Phase Resistance in Relative Permeability Network, Ohms	(6) Wetting Phase Relative Permeability	(7) Per Cent Nonwetting Phase Relative Permeability, Per Cent	(8) Wetting Phase Resistance in Relative Resistivity Network, Ohms	(9) Nonwetting Phase Resistance in Relative Resistivity Network, Ohms	(10) Wetting Phase Relative Resistivity	(11) Nonwetting Phase Relative Resistivity	(12) State of Desaturation in Fig. 6-1
4.96	15	99.4	20,000		99	0	6,050	∞	1.00	∞	a
4.45	328										
	11	98.1	20,800		95	0	6,410	∞	1.06	∞	b
3.96	16										
	178										
	147										
	9										
	223	95.6	23,000		86	0	6,710	∞	1.11	∞	c
3 67	129										
	310										
	321										
	144										
	143										
4.96	291	93.8	24,500		81	0	7,030	∞	1.16	∞	d
3.54	24										
	44										
	12										
	279										
	115	89.8	28,300		70	0	7,630	∞	1.26	∞	e
3.27	270										
	191										
	280										
	98										
	14										
	61										
	269										
	241										
3.96	268										
3.54	194										
4.45	298										
	297										
3.27	265										
	234	83.8	34,100		58	0	9,080	∞	1.50	∞	f
2.92	317 224										
	278 159										
	18 209										
	77 312										
	330 2										
	162 39										
	294 161										
	46 69										
	31 319										
	295 1										
3.67	288										
	59										
3.54	287										
3.27	134	74 3	53,500	∞	37	0	12,100	∞	2.00	∞	g
2.62	151 274 100										
	142 236 113										
	28 255 130										
	22 170 173										
	38 83 179										
	171 293 306										
	52 256 145										
	235 121 253										
	225 4 54										
	275 13 120										
	214										
3.67	34										
	140										
3.54	182										
	180										
	204										
3 27	65										
	112										
2.92	183										
	138										
	82										
	139										
	119										
	86	58.2	124,000	2,480,000	16	0.8	21,200	60,500	3.50	10	h
2.32	227 84 326 311										
	216 177 237 277										
	189 64 266 322										
	305 48 211 8										
	23 260 232 240										
	127 272 10 168										
	68 267 33 282										
	205 290 19 213										
	118 301 188 181										
	60 41 88 221										
	289 29 300 325										
	200 146 20 331										
	243 271 203 196										
	299 320 116										
	324 80 148										
4.96	198										
4.45	231										

TABLE 1—CONTINUED

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
3.96	199										
	166										
	74										
3.67	219										
3.54	247 105										
3.27	56										
	197										
	258										
	73										
	93										
	220										
2.92	230										
	165										
2.62	187 124										
	185	32.9	495,000	43,800	4.0	45	76,400	16,600	12.6	2.75	i
1.91	111 302 226 154 329 71										
	259 101 91 49 135 47										
	316 276 149 107 228 206										
	169 308 155 212 296 72										
	150 3 208 81 43 5										
	94 238 21 327 92 262										
	63 37 36 210 245 176										
	45 117 75 190 184 6										
	103 172 70 97 250 201										
	318 153 252 26 286 195										
	99 303 85 261 222 229										
	76 57 141 132 292 51										
	58	16.0	∞	23,900	0	83	∞	7,860	∞	1.30	i
1.53	96 79 251 329 55										
	323 242 90 53 102										
	163 25 309 273 133										
	264 193 136 158 257										
	248 307 104 125 215										
	87 202 110 106 167										
	157 263 62 217 7										
	233 95 17 313 27										
	207 218 254 42										
	175 284 285 160										
	78 30 137 131										
	174 89 152 314	6.1	∞	20,400	0	97	∞	6,410	∞	1.06	k
1.00	32 156 192 281										
	35 122 315 67										
	114 249 128										
	186 246 244										
	123 109 50	2.6	∞	20,000	0	99	∞	6,300	∞	1.04	l
0.54	304										
	164										
	108										
	126										
	283										
	40										
	66	0	∞	19,800	0	100	∞	6,050	∞	1.00	m

The relative permeability curves obtained from this example are shown in Fig. 1. The wetting phase and nonwetting phase relative resistivity curves are shown in Figs. 2 and 3 respectively.

Results of Network Study of Flow Properties for $\alpha = -1.00$

Relative permeability curves were obtained for each of the four networks using the tube radius distributions of Figs. 5-I, 9-I, 10-I, and 23-I, and Figs. 4 and 5. The tube radius to length relation was $l = Cr^{-1}$ for all networks. The wetting and nonwetting phase relative permeability curves so obtained are grouped according to network in Figs. 6 to 9 and according to tube radius distribution in Figs. 10 to 13.

Relative resistivity curves were obtained for each of

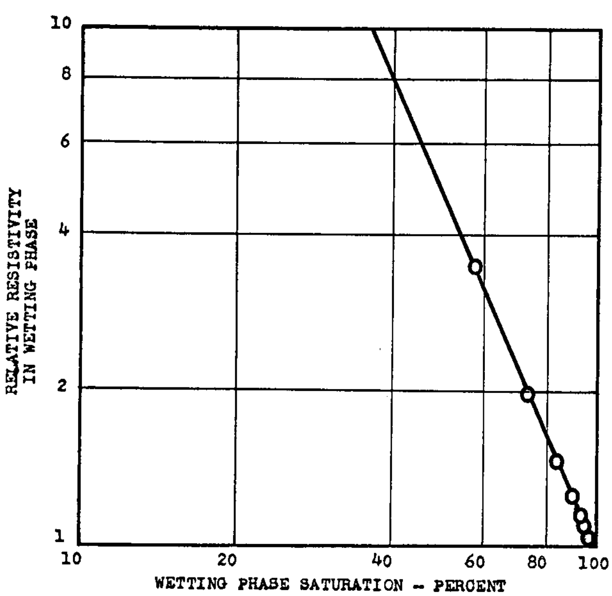

Fig. 2—Wetting Phase Relative Resistivity of Triple Hexagonal Network with Tube Radius Distribution of Fig. 5-I.

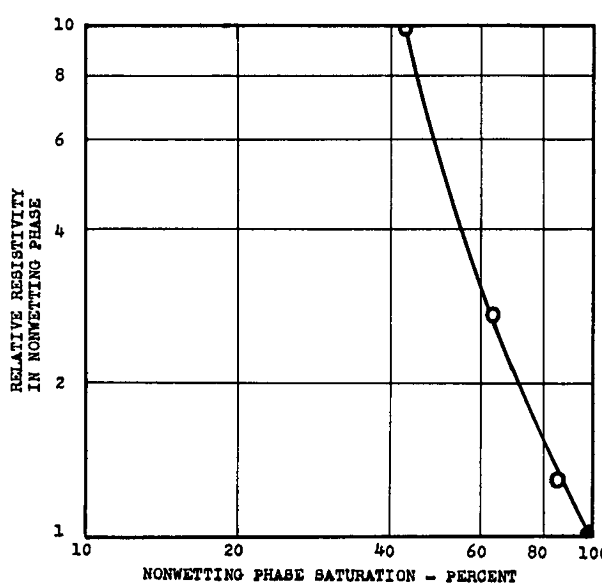

Fig. 3—Nonwetting Phase Relative Resistivity of a Triple Hexagonal Network with Tube Radius Distribution of Fig. 5-I.

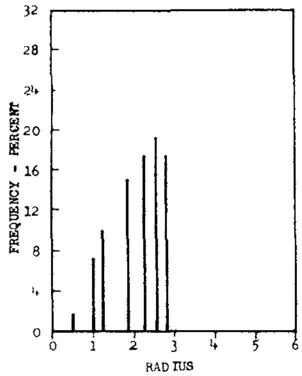

FIG. 4—TUBE RADIUS DISTRIBUTION.

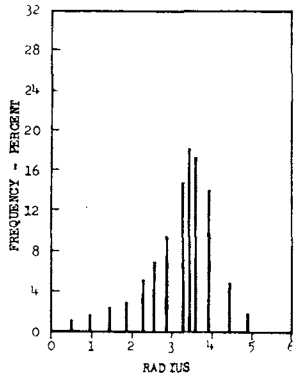

FIG. 5—TUBE RADIUS DISTRIBUTION.

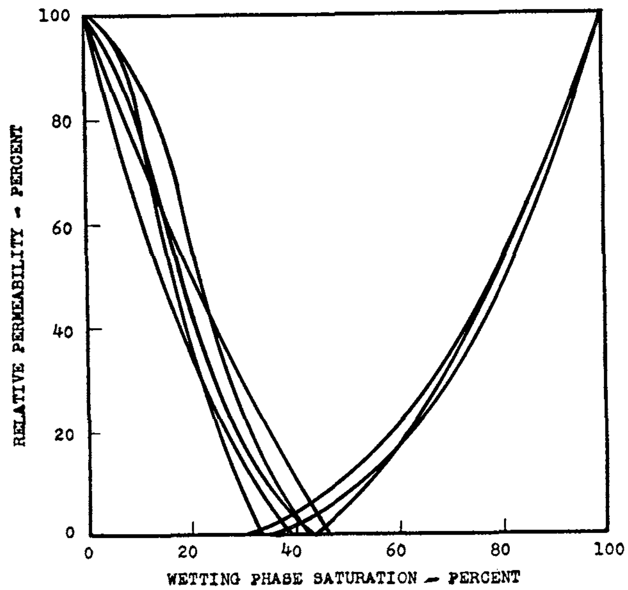

FIG. 6—RELATIVE PERMEABILITY CHARACTERISTICS OF TUBE RADIUS DISTRIBUTION OF FIGS. 5-I, 9-I, 10-I, 23-I, 4 AND 5 IN THE SINGLE HEXAGONAL NETWORK.

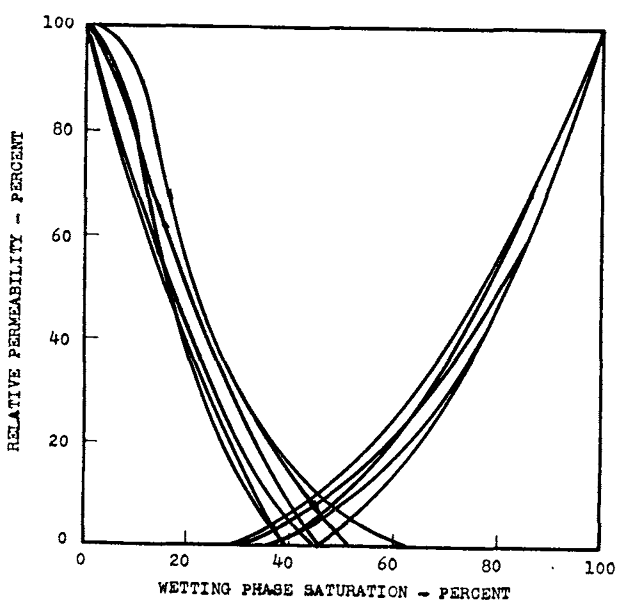

FIG. 7—RELATIVE PERMEABILITY CHARACTERISTICS OF TUBE RADIUS DISTRIBUTIONS OF FIGS. 5-I, 9-I, 10-I, 23-I, 4 AND 5 IN THE SQUARE NETWORK.

the four networks using the tube radius distributions of Figs. 5-I, 9-I, 10-I, and Fig. 5. The tube radius to length relation was $l = Cr^{-1}$. The wetting and nonwetting phase relative resistivity curves are grouped according to network in Figs. 14 to 21. These curves are not grouped according to tube radius distribution because, as the points in Figs. 14 to 21 show, relative resistivity is not sensitive to changes in tube radius distribution. Relative resistivities from different distributions and networks are scattered, but can be grouped according to network. This behavior of the relative resistivity is consistent with the requirement that the sensitivity of a flow property to changes in tube radius distribution is a function of the exponent of r_i in Eqs. 15 and 16. As the exponent approaches zero from either the positive or negative direction, the sensitivity of the flow property to changes in tube radius distribution is reduced. For an exponent of zero, the network has single size resistors for all tube radius distributions.

The effect of network size on relative permeability and relative resistivity was not determined on networks of distributed tube radius. If changes in size has the same effect on these networks as on the single size tube networks, then the correction to some standard finite size or to infinite size is small and does not change the general appearance or any of the curves given in Figs. 6 to 21.

EFFECT OF CHOICE OF α ON RELATIVE PERMEABILITY CHARACTERISTICS OF A NETWORK OF TUBES

Before proceeding to an interpretation of the results of the studies on networks with distributed tube radius, additional justification for use of the relation $l = Cr^{-1}$ will be given. The relative permeability characteristics of a network of tubes provide a useful test of the validity of this relation.

Relative permeability curves were obtained from triple hexagonal networks in which α was 1.0, 0.00, -1.00, and -2.00. The tube radius distributions of Figs. 5-I, 9-I, and 10-I were used.

The relative permeability curves for α of 1.00, 0.00, -1.00, and -2.00 are shown in Figs. 22, 23, and 24 for the various tube radius distributions. Only α of

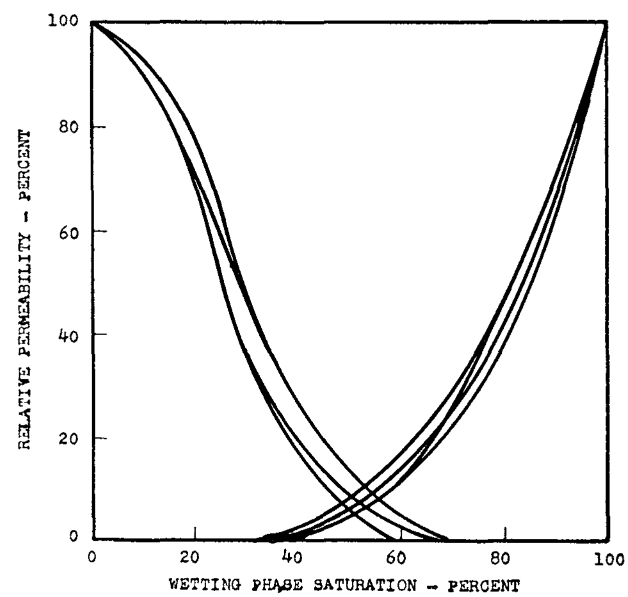

FIG. 8—RELATIVE PERMEABILITY CHARACTERISTICS OF TUBE RADIUS DISTRIBUTIONS OF FIGS. 5-I, 9-I, 10-I, 23-I, 4 AND 5 IN THE DOUBLE HEXAGONAL NETWORK.

-1.00 gives relative permeability curves for all tube radius distributions which are comparable to those observed on real porous media. The effect of change in α is a function of the dispersion of the tube radius distribution. Relative permeability curves from the tube radius distribution of Fig. 9-I with a dispersion of 0.0606 are least affected, while the curves from the distribution of Fig. 10-I with a dispersion of 0.4620 are most affected. This result is consistent with the requirement that for a single size tube network, dispersion of zero, α has no effect on the relative permeability curves.

It is also interesting to note here that, as shown in Fig. 24, relative permeability curves from a network in which tube radius and length were randomly combined fall between the curves for $\alpha = -1.00$ and $\alpha = 0$ for the same distribution.

INTERPRETATION OF RESULTS FROM NETWORKS WITH DISTRIBUTED TUBE RADIUS

Study of networks with distributed tube radius and using the tube radius to length relation $l = Cr^{-1}$ yields useful information concerning the properties of porous media. Previously, the capillary pressure curve was shown to be a sensitive function of the tube radius distribution and relatively insensitive to the network form. The small sensitivity to network form was further shown to be a function of the dispersion of the tube radius distribution. For distributions of small dispersion, the capillary pressure curve was almost insensitive to network form, whereas for distributions of large dispersion, the sensitivity was much greater. This was consistent with the requirement that for zero dispersion

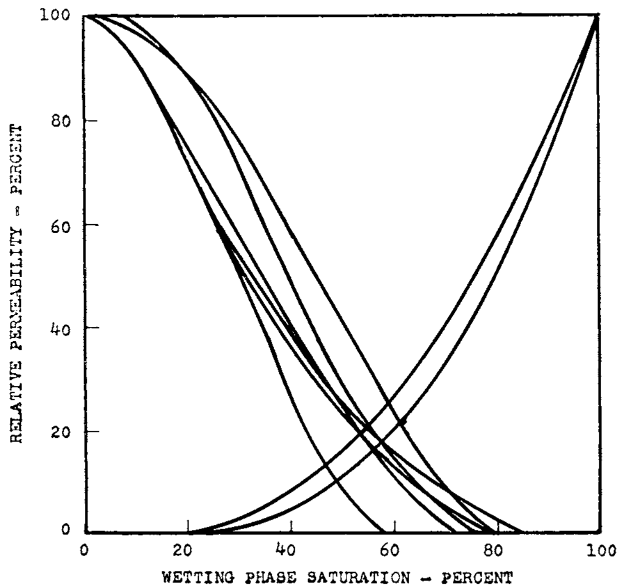

FIG. 9—RELATIVE PERMEABILITY CHARACTERISTICS OF TUBE RADIUS DISTRIBUTIONS OF FIGS. 5-I, 9-I, 10-I, 23-I, 4 AND 5 IN THE TRIPLE HEXAGONAL NETWORK.

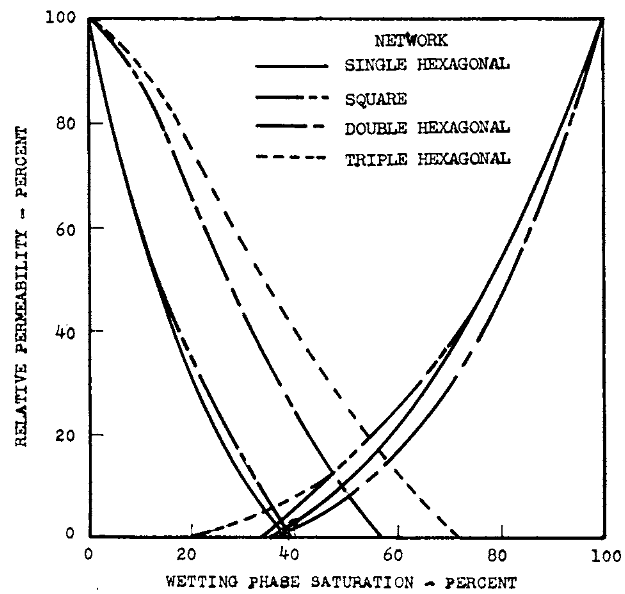

FIG. 11—RELATIVE PERMEABILITY CHARACTERISTICS OF NETWORKS WITH TUBE RADIUS DISTRIBUTION OF FIG. 9-I.

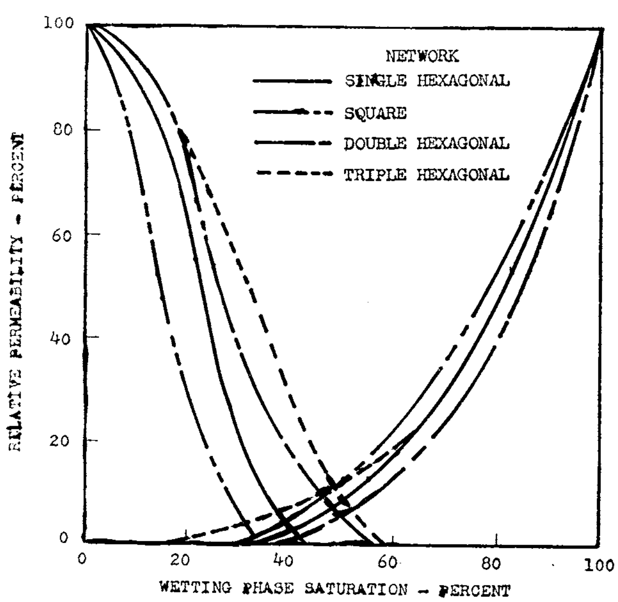

FIG. 10—RELATIVE PERMEABILITY CHARACTERISTICS OF NETWORKS WITH TUBE RADIUS DISTRIBUTION OF FIG. 5-I.

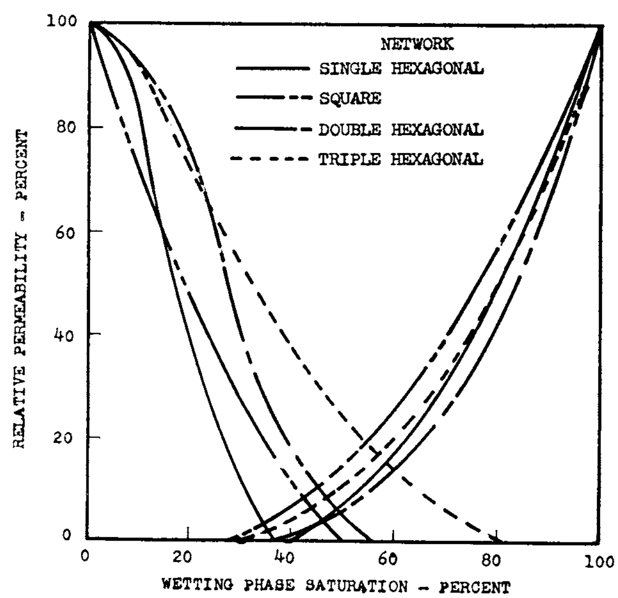

FIG. 12—RELATIVE PERMEABILITY CHARACTERISTICS OF NETWORKS WITH TUBE RADIUS DISTRIBUTION OF FIG. 10-I.

(single size tubes), all networks have a horizontal, straight line as their capillary pressure curve.

The behavior of the nonwetting phase relative permeability is opposite to that of the capillary pressure. As shown by Figs. 6 to 13, nonwetting phase relative permeability is largely a function of network form, and is much less sensitive to tube radius distribution. The wetting phase relative permeability is not very sensitive to either network form or tube radius distribution.

The relative permeability curves from all networks and tube radius distributions studied resemble curves which are obtained experimentally from real porous media. The insensitivity of the network wetting phase relative permeability curves to both network form and tube radius distribution may explain the observed similarity of this property measured on very different porous media.

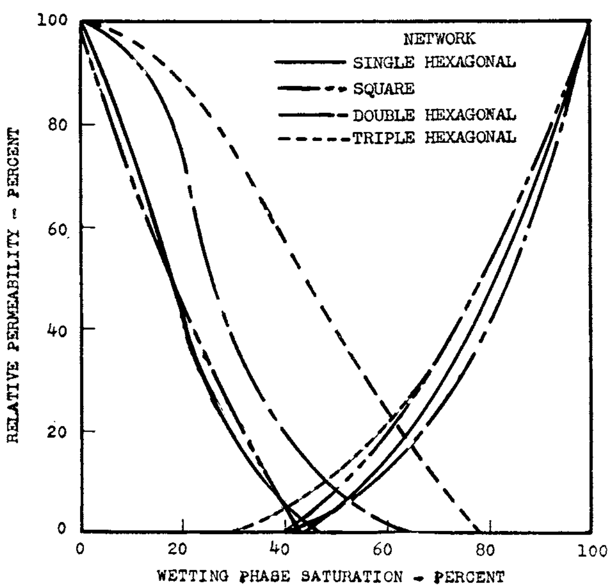

Fig. 13—Relative Permeability Characteristics of Networks with Tube Radius Distribution of Fig. 23-I.

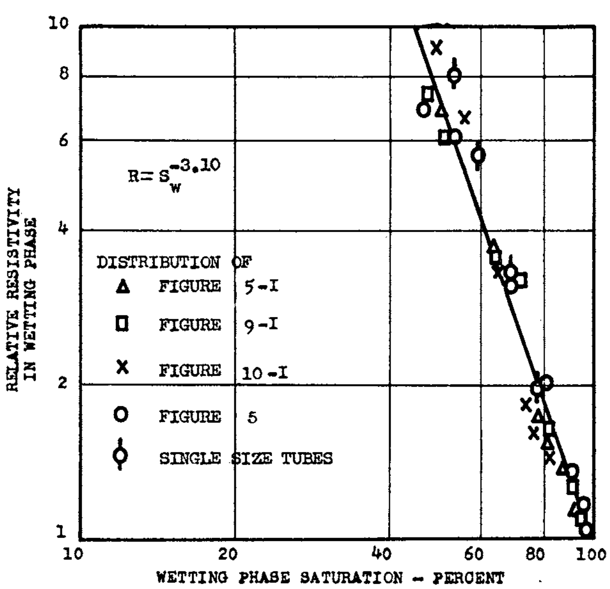

Fig. 14—Wetting Phase Relative Resistivity of Single Hexagonal Network for Tube Radius Distributions of Figs. 5-I, 9-I, 10-I, and 5, and for Single Size Tubes.

The network relative resistivity data is more sensitive to network form than to tube radius distribution. For this reason, relative resistivity data are grouped in Figs. 14 to 21 according to network only.

Although the wetting phase relative resistivity data are somewhat scattered, they do seem to follow Archie's law, that is, $R = S_w^{-n}$ (see Table 2).

TABLE 2 — EXPONENT n FOR THE DIFFERENT NETWORKS AND THE SIMPLE BUNDLE OF TUBES

Network	β	$1/\beta$	n
Single Hexagonal	4	0.25	3.10
Square	6	0.167	2.97
Double Hexagonal	7	0.142	2.67
Triple Hexagonal	10	0.10	2.22
Simple Bundle of Tubes	∞	0.00	1.00

Fig. 25 shows the above data graphically.

As noted previously, Wyllie and Spangler[5] have found that for sandstones, unconsolidated sand, sintered glass,

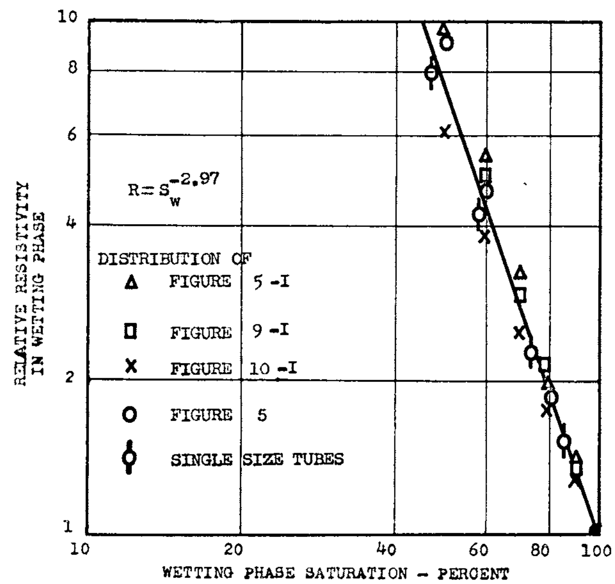

Fig. 15—Wetting Phase Relative Resistivity of Square Network for Tube Radius Distributions of Figs. 5-I, 9-I, 10-I, and 5 and for Single Size Tube.

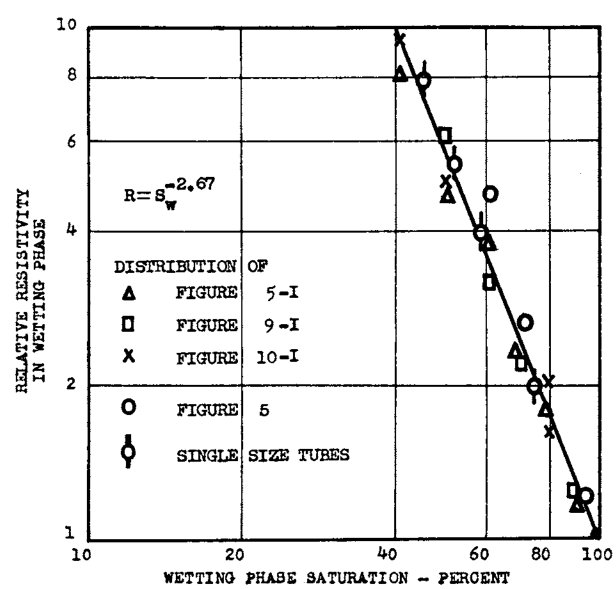

Fig. 16—Wetting Phase Relative Resistivity of Double Hexagonal Network for Tube Radius Distributions of Figs. 5-I, 9-I, 10-I, and 5 and for Single Size Tubes.

and sintered alumina, n varies from 1.42 to 2.55. From Fig. 25, this would indicate that $1/\beta$ for these materials ranges from 0.04 to 0.13. This means that in these materials from seven to 25 channels are connected to each channel, and that from four to 13 channels meet at each junction. A three-dimensional network with these properties seems quite reasonable.

In Fig. 25, n vs $1/\beta$ is plotted for both the single size and the distributed size networks. There seems to be little difference in the n vs $1/\beta$ relation between the two different kinds of networks except for $1/\beta$ near 0.25. Since the Wyllie and Spangler data indicate that real porous media do not have network structures with $1/\beta$ greater than 0.13, the linear relation between n and $1/\beta$ permits use of this relation to determine $1/\beta$ from relative resistivity data. These data thus become a means for obtaining information concerning the network structure of real porous media, irrespective of the pore size distribution.

Using $1/\beta$ calculated from relative resistivity data and Fig. 25, it is possible to use the methods outlined in the section on quantitative interpretation of the capillary pressure curve of paper I of this series to correct the average pore radius and the dispersion calculated from the capillary pressure curve. A combination of capillary pressure and relative resistivity data leads to more correct values for average pore radius and dispersion than can be obtained from the capillary pressure curve alone.

The nonwetting phase relative resistivity curves of Figs. 18 to 21 cannot be compared with experimental data because this property has not yet been measured. In all laboratory measurements made to date, the non-

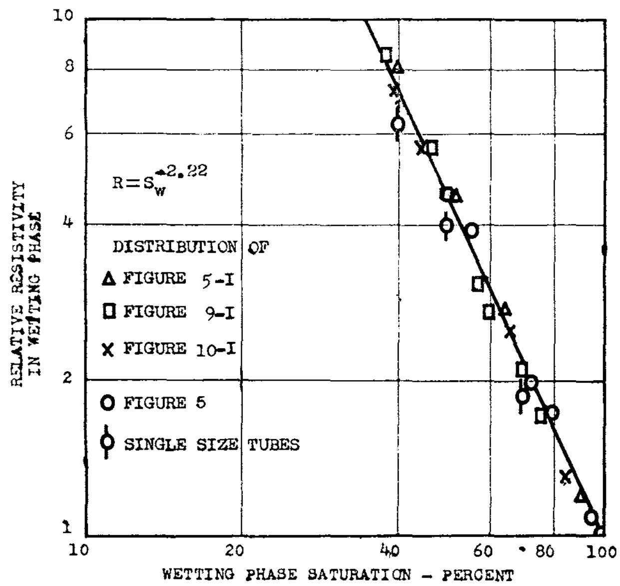

FIG. 17—WETTING PHASE RELATIVE RESISTIVITY OF TRIPLE HEXAGONAL NETWORK FOR TUBE RADIUS DISTRIBUTIONS OF FIGS. 5-I, 9-I, 10-I, AND 5, AND FOR SINGLE SIZE TUBES.

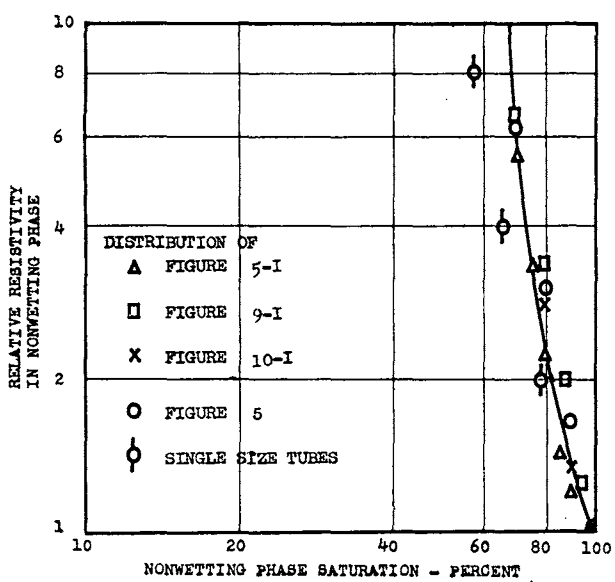

FIG. 18—NONWETTING PHASE RELATIVE RESISTIVITY OF SINGLE HEXAGONAL NETWORK FOR TUBE RADIUS DISTRIBUTIONS OF FIGS. 5-I, 9-I, 10-I, AND 5, AND FOR SINGLE SIZE TUBES.

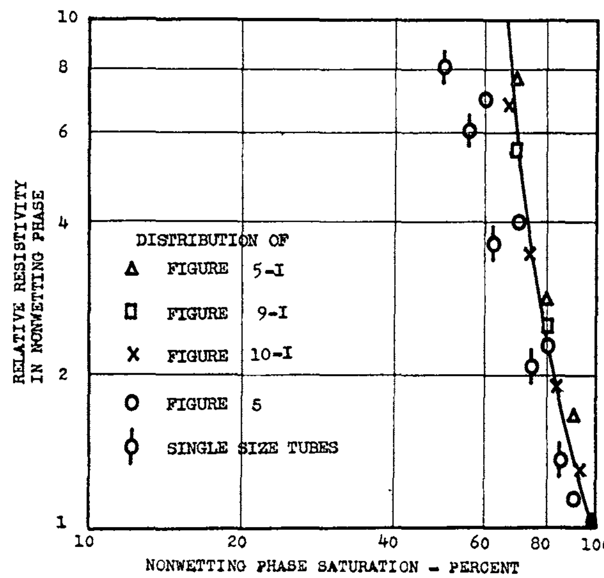

FIG. 19—NONWETTING PHASE RELATIVE RESISTIVITY OF SQUARE NETWORK FOR TUBE RADIUS DISTRIBUTIONS OF FIGS. 5-I, 9-I, 10-I, AND 5.

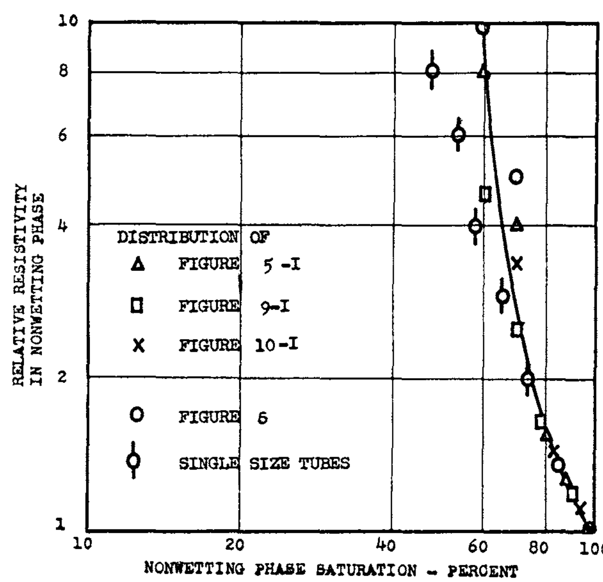

FIG. 20—NONWETTING PHASE RELATIVE RESISTIVITY OF DOUBLE HEXAGONAL NETWORK FOR TUBE RADIUS DISTRIBUTIONS OF FIGS. 5-I, 9-I, 10-I, AND 5, AND FOR SINGLE SIZE TUBES.

wetting phase is either a gas or a nonconducting oil. Experimental nonwetting phase relative resistivity data may be obtained by using porous materials which have been treated with a silicone fluid to render them oil-wet. Oil can then be used as the wetting phase and an electrolyte as the nonwetting phase. Reliable data on this system have not yet been reported.

One of the most interesting predictions to come from this network study is that relative flow properties which are functions of pore geometry only, such as relative permeability and relative resistivity, will be identical functions of the saturation for a network of uniform size pores. This implies that as the dispersion of the tube radius distribution is reduced, the relative permeability curve approaches the relative conductivity curve.

Experimental data on sand packs reported by Wycoff and Botset[6] and by Leverett[7] and data from networks with distributed tube size provide a check of the above prediction. The sand pack data are shown in Figs. 26 and 27. In both figures, it is evident that the wetting phase relative permeability curve approaches the wetting phase relative conductivity curve as the sand becomes more uniform. (It is assumed here that a more uniform grain size sand pack has a more uniform pore size.)

Fig. 28 shows the wetting phase relative permeability and relative conductivity curves for the distributions of minimum and maximum dispersion, Figs. 9-I and 10-I, in the triple hexagonal network. Here again the more uniform distribution, Fig. 9-I, gives a wetting

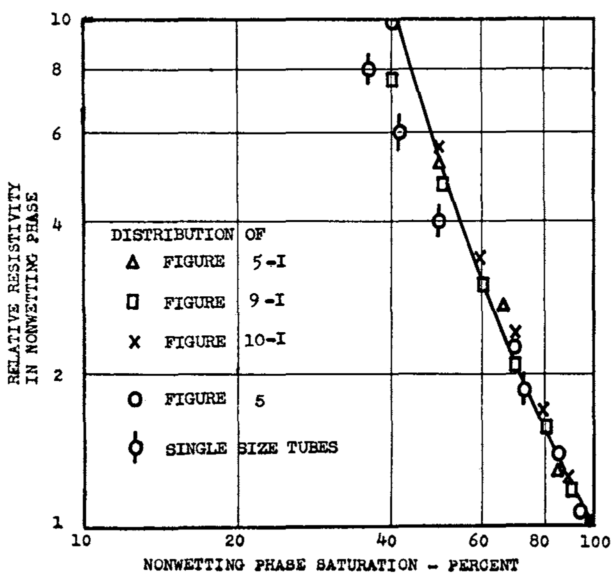

FIG. 21—NONWETTING PHASE RELATIVE RESISTIVITY OF TRIPLE HEXAGONAL NETWORK FOR TUBE RADIUS DISTRIBUTIONS OF FIGS. 5-I, 9-I, 10-I AND 5, AND FOR SINGLE SIZE TUBES.

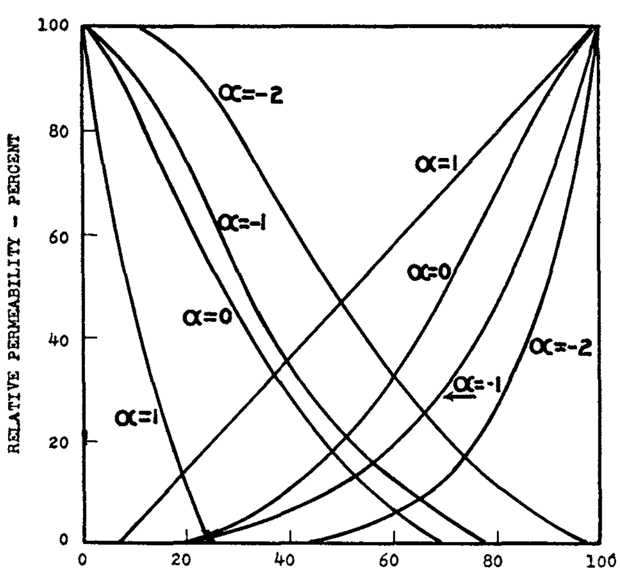

FIG. 22—RELATIVE PERMEABILITY CHARACTERISTICS OF TRIPLE HEXAGONAL NETWORK WITH THE TUBE RADIUS DISTRIBUTION OF FIG. 5-I FOR α = 1.00, 0.00, −1.00, AND −2.00.

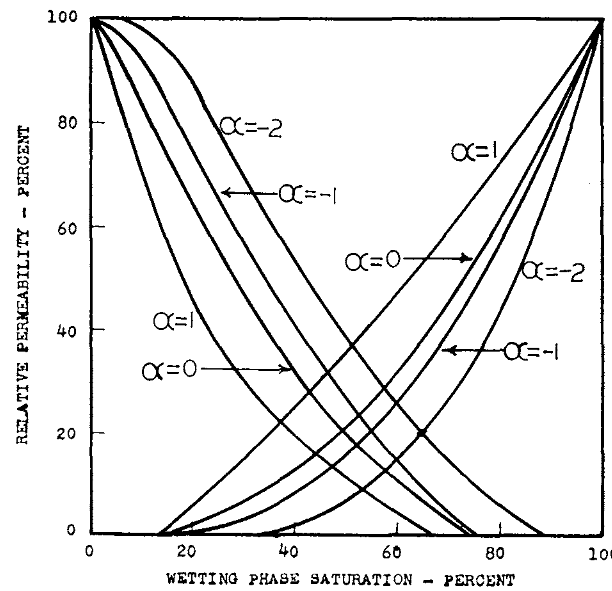

FIG. 23—RELATIVE PERMEABILITY CHARACTERISTICS OF TRIPLE HEXAGONAL NETWORK WITH TUBE RADIUS DISTRIBUTION OF FIG. 9-I FOR α = 1.00, 0.00, −1.00, AND −2.00.

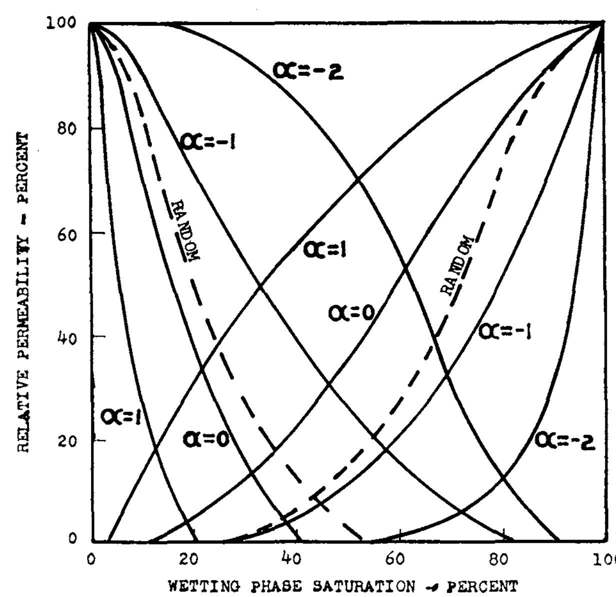

FIG. 24—RELATIVE PERMEABILITY CHARACTERISTICS OF TRIPLE HEXAGONAL NETWORK WITH TUBE RADIUS DISTRIBUTION OF FIG. 10-I FOR α = 1.00, 0.00, − 1.00, AND −2.00, AND FOR RANDOM COMBINATION OF TUBE RADIUS AND LENGTH.

phase relative permeability curve that is closer to the relative conductivity curve.

Fig. 29 shows an interesting phenomena that has has been noted in the network study. The nonwetting phase relative permeability is greater than the nonwetting phase relative conductivity at all saturations in contrast to the opposite behavior of the wetting phase relative permeability and relative conductivity as shown in Fig. 28. The relative position of the curves shown in Fig. 29 is typical of all networks and distributions studied. Network nonwetting phase relative permeability data tend to be more scattered than the wetting phase data, and therefore, a detailed comparison from the different distributions, as was done for the wetting phase in Fig. 28, is not possible.

The network relative permeability curves of Figs. 6 to 13 show that k_{nw}/k_w vs saturation where k_{nw} and k_w are nonwetting phase and wetting phase relative permeability respectively, should be largely a function of network form and much less sensitive to tube radius distribution. Data from the tube radius distributions of Figs. 5-I, 9-I, and 10-I and the four networks and simple bundle of tubes are plotted as k_{nw}/k_w vs nonwetting phase saturation in Figs. 30, 31, and 32. Data for networks and the simple bundle of tubes with single size tubes are shown in Fig. 33. It is immediately apparent from these curves that the network determines the average slope of the curve. The single hexagonal network gives a k_{nw}/k_w curve of greatest average slope, while the triple hexagonal network and the simple bundle of tubes give curves of much lower average

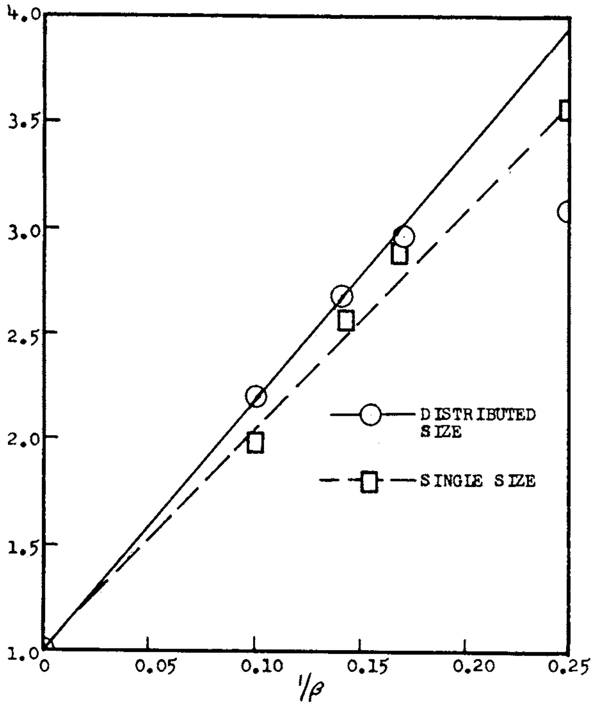

FIG. 25—RELATION BETWEEN RECIPROCAL OF NETWORK β FACTOR AND n IN THE RELATION $R = S_w^{-n}$ FOR NETWORKS OF SINGLE SIZE AND DISTRIBUTED SIZE TUBES.

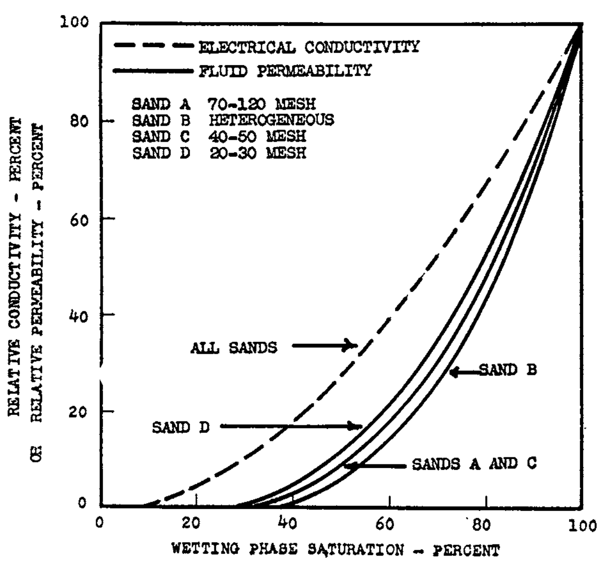

FIG. 26—COMPARISON OF WETTING PHASE RELATIVE PERMEABILITY AND RELATIVE CONDUCTIVITY FOR UNCONSOLIDATED SAND. DATA FROM WYCOFF AND BOTSET[6].

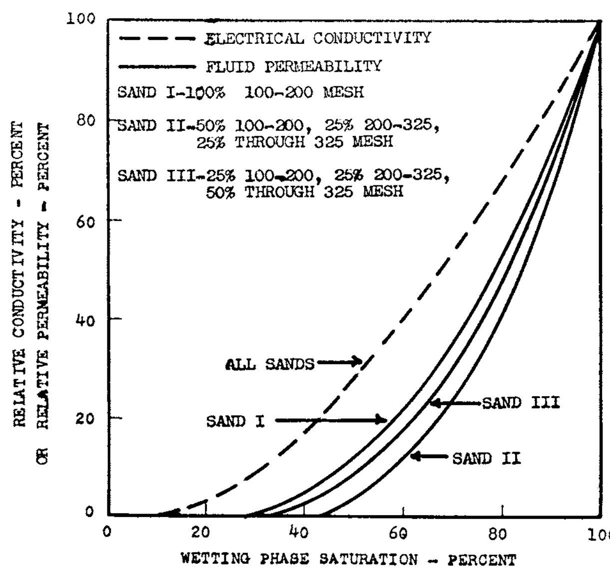

FIG. 27—COMPARISON OF WETTING PHASE RELATIVE PERMEABILITY AND RELATIVE CONDUCTIVITY FOR UNCONSOLIDATED SAND. DATA FROM LEVERETT[7].

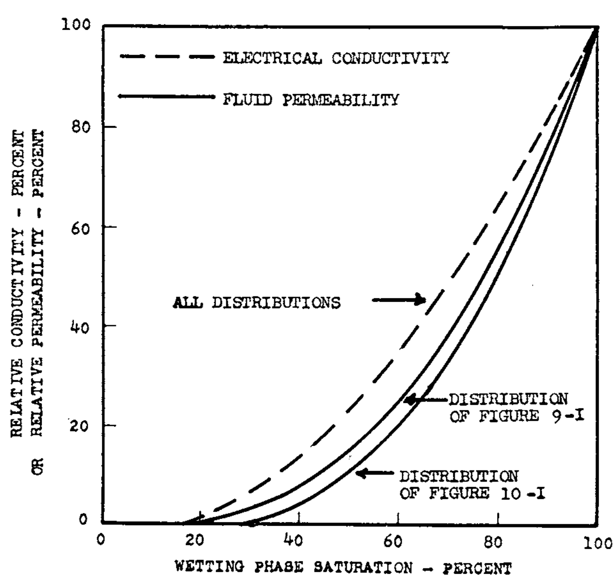

FIG. 28—COMPARISON OF WETTING PHASE RELATIVE PERMEABILITY AND CONDUCTIVITY FOR TUBE RADIUS DISTRIBUTIONS OF FIGS. 9-I AND 10-I IN THE TRIPLE HEXAGONAL NETWORK.

slope. These observations suggest a method of estimating the average slope of the k_g/k_o curve of porous media from relative resistivity data. The relation between the exponent in the relative resistivity relation $R = S_w^{-n}$ and the network structure, as shown in Fig. 25, suggests that the exponent should be related to the average slope of the k_g/k_o curve. In comparing a group of samples of porous material, those with the highest exponent in the relative resistivity relation should have a k_g/k_o curve with greatest slope, and conversely for samples with a low exponent.

The relation between n and $\left(\frac{d \log k_{nw}/k_w}{d S_w}\right)_{ave.}$ for the four networks and the bundle of tubes is shown in Fig. 34. There is some uncertainty in the value of n for single hexagonal networks as shown in Fig. 25. An extrapolation of the plot of n vs $1/\beta$ below $1/\beta = 0.167$ gives $n = 4.00$ for the single hexagonal network, whereas the measured n is 3.10. The horizontal arrows in Fig. 34 indicate the points for both $n = 4.00$ and $n = 3.10$.

Notwithstanding the uncertainty in n for single hexagonal networks, Fig. 34 clearly shows the trend of

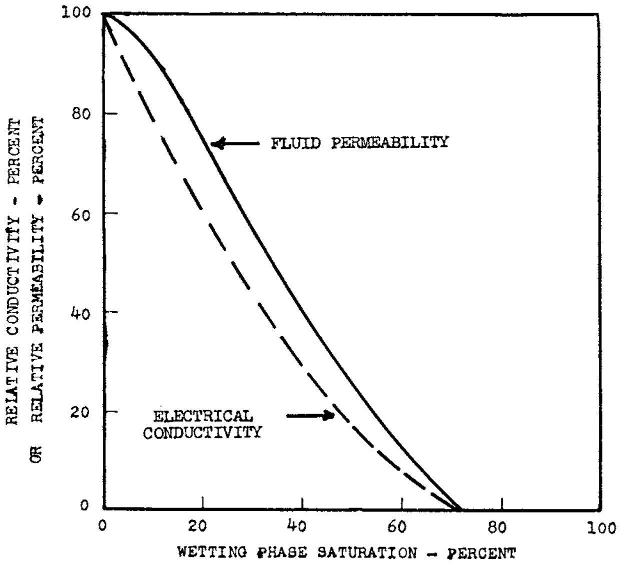

FIG. 29—COMPARISON OF NONWETTING PHASE RELATIVE PERMEABILITY AND RELATIVE CONDUCTIVITY OF NETWORKS.

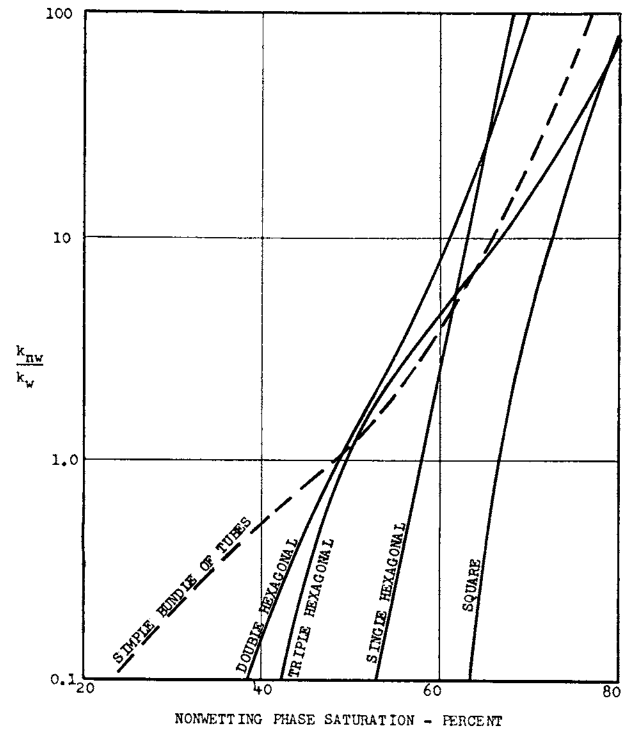

FIG. 31—k_{nw}/k_w RELATION FOR THE TUBE RADIUS DISTRIBUTION OF FIG. 5-I.

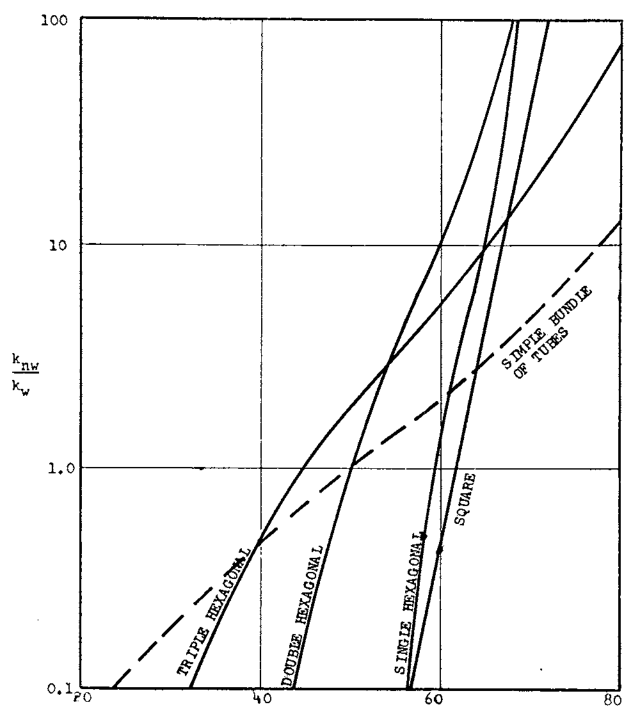

FIG. 30—k_{nw}/k_w RELATION FOR THE TUBE RADIUS DISTRIBUTION OF FIG. 9-I.

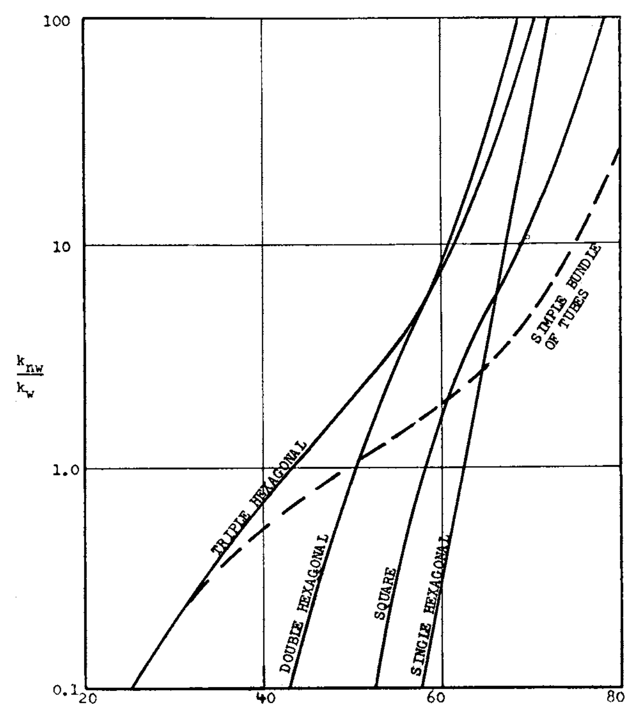

FIG. 32—k_{nw}/k_w RELATION FOR THE TUBE RADIUS DISTRIBUTION OF FIG. 10-I.

the average slope of the k_{nw}/k_w curve as a function of n.

There are almost no published data on porous media which can be used to test the relation shown in Fig. 34. Fluid flow and electrical resistivity data on the same sample are very rarely reported. The only data that are of some help are those of Wycoff and Botset[6] and Leverett[7]. The average slope of the unconsolidated sand curve is lower than that for consolidated sand. Observed values of n for unconsolidated sand are lower than those for consolidated sand. This relation between the average slope and n for real porous media is in the same direction as that predicted by Fig. 34 from network studies.

SUMMARY AND CONCLUSIONS

SUMMARY

In an attempt to correlate the matrix properties of porous media such as grain diameter and grain shape to flow behavior, two models have been proposed. The sphere pack model has given some useful generalizations concerning the relation between porosity and permeability and between porosity and electrical conductance. However, the complexity of the shape of the pore spaces in this model has precluded the calculation of any flow properties from the geometry of the model. The equations now in use for calculating flow properties of sand beds are based on the sphere pack model but have been developed by inductive reasoning or by empirical fitting of observed data.

The bundle of tubes is the second model which has been proposed. For this model, comparatively simple mathematical operations yield equations which describe almost all of the flow properties of porous media. However, the advantages gained by being able to make rigorous derivations from the model are offset by the failure of the model to represent accurately real porous media. As a result of this failure, most of the

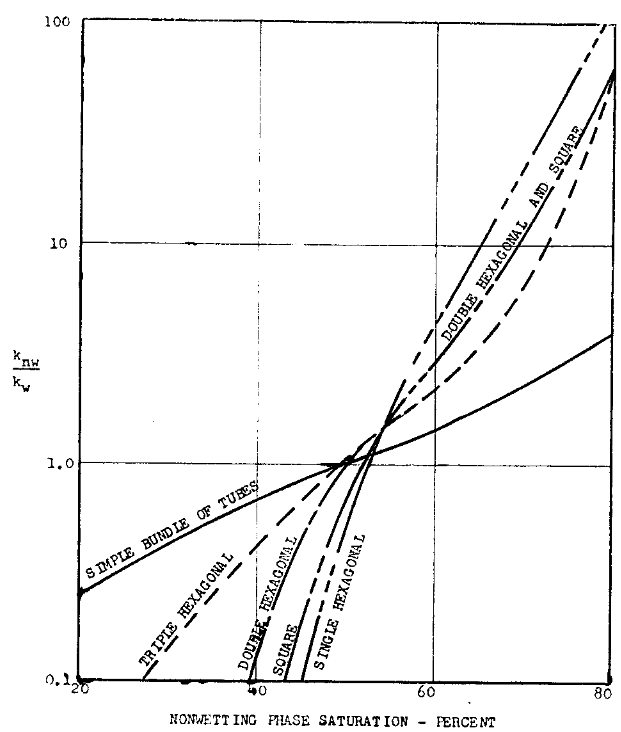

FIG. 33—k_{nw}/k_w RELATION FOR NETWORKS AND THE SIMPLE BUNDLE OF TUBES COMPOSED OF SINGLE SIZE TUBES.

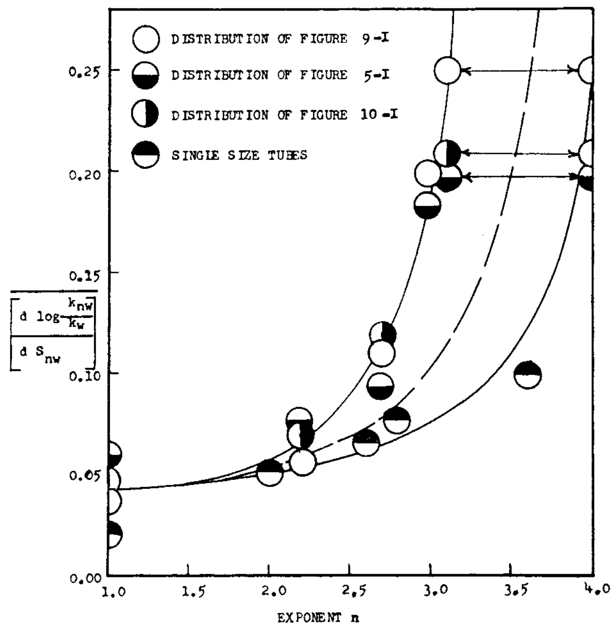

FIG. 34—AVERAGE SLOPE OF k_{nw}/k_w RELATIONS VS THE EXPONENT n IN THE RELATION $R = S_w^{-n}$.

equations derived from the bundle of tubes model do not describe the behavior of real porous media.

If the sphere pack and the bundle of tubes models are combined by substituting a uniform cylindrical tube for each pore space in the sphere pack model, a three-dimensional network of tubes is obtained. In principle, exact calculations can be made on this network. The network of tubes model is believed to be the most useful model considered to date.

For convenience in manipulation, the three-dimensional network is reduced to two dimensions. There is reason to believe that this does not reduce the validity of the model.

Real porous media are shown by photomicrographs of sandstone-thin sections to have pores connected in an irregular network. However, in order to show the effect of network structure on flow properties, four different regular networks were studied.

A network of different size tubes in which there is random spatial distribution of tubes was desaturated by the capillary desaturation mechanism to give a capillary pressure curve closely resembling such curves obtained from sintered glass or sandstone.

Three different tube radius distributions were studied in four different networks and in the simple bundle of tubes. From the resultant capillary pressure curves, the separate effects of the tube radius distribution and network were determined.

The flow properties of networks of tubes were studied by use of an electrical analog of the network. The equivalence of Poiseuille's law and Ohm's law was the basis for the substitution of an electrical resistor for each tube in the network. The electrical resistance of the network of resistors was then taken to be equivalent to the fluid resistance of a network of tubes.

Ohm's law for the electrical resistance of a tube filled with a conductor was used to construct networks of resistors which were equivalent to a network of tubes filled with an electrical conducting fluid. The electrical resistance of this network of resistors was taken to be equivalent to the electrical resistance of a network of tubes filled with an electrical conducting fluid.

The capillary desaturation mechanism applied to the networks of tubes and then to the equivalent network of resistors gave relative permeability and relative resistivity curves. These curves had the same shape as those obtained from real porous media. Networks with single size tubes and those with distributed size tubes both gave curves which closely resembled the curves obtained from porous media.

The effects of network structure and tube radius distribution on the flow properties of the network were studied. The results of these studies led to a correlation of the flow properties with network structure and showed the interrelation of the different flow properties of a given porous medium.

Conclusions

The over-all conclusion drawn from the study of the network of tubes is that the network is a valid model of porous media. This conclusion is supported by the resemblance of the network relative permeability and relative resistivity curves to those observed on porous media. Added support to this conclusion is given by the observation that the relation between these curves is in qualitative agreement with that observed for porous media.

From this general conclusion, certain specific conclusions can be drawn concerning the relations between the pore size distribution, network structure, capillary pressure characteristics, and flow properties.

Some of these specific conclusions are supported by experimental data on porous media, others are predictions concerning properties of porous media which have not yet been measured or compared.

Study of the network model leads to the following conclusions.

1. The relative permeability and relative resistivity characteristics of porous media are a direct consequence of the network structure of these media.

2. The exponent n in Archie's relation between wetting phase saturation and wetting phase relative resistivity, $R = S_w^{-n}$ is a fraction of the network structure of the porous medium and independent of the pore radius distribution. The exponent is about 4.0 for a network structure in which only four channels are joined to each channel and approaches 1.0 as the number of channels joined together approaches infinity.

3. In porous media of uniform size pores, all relative flow properties which are functions of pore geometry only, such as relative permeability and relative resistivity, will be identical functions of the saturation.

4. In porous media of non-uniform size pores, the wetting phase electrical conductivity will, at a given saturation, be greater than the wetting phase relative permeability. Conversely, the nonwetting phase relative electrical conductance will be, at a given saturation, less than the nonwetting phase relative permeability.

5. The average slope of the k_g/k_o curve, as usually plotted in petroleum reservoir engineering, can be correlated with the exponent in the relative resistivity relation for a given porous medium. The average slope of the k_g/k_o curve will be large for porous media of large exponent; conversely, the k_g/k_o curve will have a small average slope for media of low exponent.

ACKNOWLEDGMENT

The author is grateful to Karol J. Mysels for his guidance and encouragement during the course of the work report in this series of papers.

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DISCUSSION

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Science does not make uniform progress in any one field, but the advance may be likened to the pincer movements which we heard about in the last war in which there is a sudden encircling movement followed by consolidation of the gains. For example, the development of the whole field of antibiotics started with the discovery of penicillin and was thereafter followed by the discovery of a number of disease-controlling chemicals through a systematic search of earthen bacteria. I would classify the work in this paper as one of these pincer movements. It is a definite step forward in our understanding of the properties of porous media and will be followed by a great deal of comparative and correlating work.

Up until this time we have had to be satisfied with a model of the porous media as either a bundle of tubes upon which calculations could be made, or a pack of spheres upon which mathematical calculations were nearly impossible. Many of the theories for the flow of fluids through porous media have been developed using the bundle of tubes as a model; however, in using this model we have realized that there were several points of inconsistencies with natural porous media. For one thing, porous media are isotropic, that is the properties tend to be the same in all directions, while with the bundle of tubes, flow can take place in only one direction. One other point which bothered us was the fact that in a porous media the pores are cross-connected throughout their length while tubes are not.

The author of this paper has proposed a model which overcomes both of these difficulties and with which it is feasible to make either mathematical calculations or use an analog computer. The use of this model eliminates the "bugger factor," sometimes termed tortuosity, which is used to make the theoretical calculations on the bundle of tubes model fit experimental data.

In measuring the capillary pressure curves of cores, many of us have been disturbed by the fact that the non-wetting phase, such as mercury, first enters the large pores on the surface of the core but not those in the center of the core. Upon continued injection of mercury it may be forced through smaller pores before it encounters these large pores. The large pores will then be classified as the size of pore which is equivalent to the pressure being applied to the mercury at that point. The calculations in this paper indicate that the mercury capillary pressure curves are primarily a function of the pore size distribution and are not greatly affected by the degree of interconnection of the pores. From this we can conclude that the fact that mercury is being injected from the surface of the core does not invalidate the calculation of pore size distributions from these measurements. By using the corrections proposed in this paper, it is therefore possible to obtain a reasonably accurate estimate of pore size distributions from capillary pressure curves.

In measuring relative permeability in the laboratory by the capillary desaturation method, it is necessary to change the saturation within the core by injecting the non-wetting phase, such as gas, into the core from each end. Here again there is a question as to the uniformity of saturation. It is probable that the gas partially desaturates each end first and that this partially desaturated zone moves through the core until the whole core is partially desaturated. The measured relative permeabilities taken during this initial desaturation period are open to question since the saturations are non-uniform throughout the length of the core. The procedures developed in this paper would be ideal for comparing relative permeabilities measured in this manner to those wherein uniform desaturation occurs throughout the length of the core. One of the questions which has been raised regarding the capillary desaturation method for measuring relative permeability is the fact that in the field, desaturation is attained not by injecting gas into the formation but by formation of bubbles from solution gas. The desaturation mechanism proposed in this paper could be opened to the same criticism since it assumes a gas drive rather than a solution gas desaturation mechanism. Different operational procedures would be required if it was assumed that the gas was formed within the capillary tubes.

Some of us have been interested in the relationship between the relative permeability and the relative resistivity of cores. A simple relationship can be developed if a bundle of tubes is taken as a model; however, experimental data have shown that this simple relationship does not hold for actual field cores. Work in our laboratory has indicated that the relationship between the relative resistivity and the relative permeability is influenced by the rate of change of the pore size distribution. It is interesting to note that on the basis of the proposed model the author has found that there is a relationship between Archie's saturation exponent "n" and the k_g/k_o ratio determined from relative permeability measurements.

Relative resistivities have been measured in our laboratory on quite a number of California sandstones. These results have of course been influenced by the surface conductivities of the shaly constituents in the cores, but when their effect has been eliminated it has been found that the majority of the cores have an "n" of between 1.7 and 2.4 for the relationship proposed by Archie, i.e., the resistivity ratio is equal to the saturation to the minus "n" power. In the calculations in this paper surface conductivity was not considered. Therefore, we may conclude that for many California sandstones the degree of interconnection would be equivalent to a β of between eight and 17 or that their equivalent network model should have between eight and 17 tubes connecting to each tube.

Some of the important assumptions which the author has made in carrying out his calculations are that: (1) there is only one phase flowing in any one tube at any time; (2) flow follows Poiseuille's law for cylindrical flow; and (3) the two-dimensional model upon which calculations were carried out is equivalent to three-dimensional flow. Although all of these assumptions appear to be reasonable, experimental verification would seem desirable. The over-all conclusion arrived at by the author that these interconnected tube systems are valid models is based upon the fact that the results from these models appear to fit fairly close to experimental data on porous media. This is in itself a partial validation of the assumptions made.

In reviewing this paper two questions have come to my mind which I should like to put to the author.

1. Have you done any work on a system in which the number of pore connections are distributed in some random manner according to an assumed distribution curve? It seems probable that natural pores do not have regular networks but are a combination of many types of interconnections.

2. Have any actual cores been examined under a microscope so that the shape of the networks can be estimated and properties of these particular cores compared to those estimated theoretically by the procedure outlined in this paper?

DISCUSSION

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This paper makes a notable advance towards a complete understanding of the characterization of reservoir rock. The thorough manner in which the author has investigated all the variables in his network model is commendable. The inadequacy of the bundle of capillaries model has been recognized for a long time. The much more realistic model presented in this paper should enable us to be more precise in our thinking about flow behavior in porous media. Also, the accurate method presented for calculating pore size distribution from capillary pressure curves is a definite contribution.

The theoretically derived capillary pressure, relative permeability and resistivity curves closely resemble curves obtained from measurements on actual systems. The fact that the model desaturates to zero wetting-phase saturation does not appear to be a serious objection as the theoretical curves can be used to represent only the region above interstitial water saturation. A notable feature of the model is that the relative permeability curves do not add to one.

However, the relative permeability curves for the model do differ somewhat from experimental oil-gas curves. In the network model, the curves appear to be displaced too far to the left, in the direction of high non-wetting phase saturations. The non-wetting phase saturation which must be attained before flow starts varies from 15 to 70 per cent. In natural porous media, this critical non-wetting phase saturation is usually below 15 per cent. It is interesting that for all pore size distributions studied, the triple hexagonal curves for the non-wetting phase and the double hexagonal curve for the wetting phase lie farthest to the right. One can conclude that in reservoir porous media the β factor is high; that is, each pore is connected to a large number of other pores.

I should like to suggest three other ways in which the network model might be applied to the study of fluid flow behavior in porous media: (1) in miscible flood studies, (2) in waterflood studies assuming spontaneous imbibition of water, and (3) in waterflood studies assuming Darcy's law flow in each capillary.

Miscible floods might be studied using the network model in order to demonstrate the separate effects of pore size distribution and network type on displacement efficiency. An analog computer might be used which consists of a network of resistors and capacitors and the transient phenomenon measured when a voltage is applied across the network.

The author quotes three references on miscible floods and states that 97 per cent of a fluid in sandstone or sintered glass is displaced by flooding with a single pore volume of miscible fluid. However, I believe this is true only for porous media having a uniform pore size. We have found on reservoir cores that up to 14 pore volumes of fluid are required to displace 97 per cent of the original fluid.

Imbition water floods might be studied employing the network model. Rules would have to be set up governing the order in which the capillary tubes imbibed water. One assumption that might have validity under certain flow conditions would be that water would fill completely capillary tubes throughout the network as the tube radius was increased in increments. Relative permeability curves and the residual oil saturation could be obtained by the author's scheme of transferring resistors to another network.

Water floods, assuming no capillary effects such as imbibition, might conceivably be studied using network models in order to determine the factors influencing residual oil saturations. Here the assumption would be that oil and water flow in each capillary tube according to Poiseuille's law, and that when all the connections to the downstream end of the tube are filled with water, oil remaining in the tube will be unrecoverable. It is not apparent how some of the details of such an experiment would be worked out.

AUTHOR'S REPLY to J. I. GATES and to V. A. JOSENDAL

I wish to thank John I. Gates and V. A. Josendal for their comments. Their comments seem to be a result of a detailed and thoughtful study of a rather long paper, and for this I am grateful. Both commentators have proposed tests on the network model in addition to those reported in the paper. My purpose in writing the paper seems to be fulfilled, for I wanted only to point out the possibilities of gaining further insight into multiphase flow in porous media by means of the network model.

Gates points out the difference between desaturation by external drive and desaturation by a solution gas mechanism. I chose the external drive mechanism for developing the network model because published relative permeability and resistivity index data were obtained by this method. A comparison of network curves with published curves for real systems was obviously desirable. However, a slight modification in the network manipulations allows the internal gas drive mechanism to be used to desaturate the network. Capillary pressure, relative permeability, and resistivity index curves can then easily be obtained for internal gas drive.

I would like to point out in reply to Josendal's suggestion for waterflood studies in networks that tremendous difficulties arise if one tries to study the dynamics of frontal movement in the network. In my work I was interested only in the network permeability at a fixed saturation. I did not concern myself with the rate of change of the saturation or the change in shape of the immiscible fluid front. At present I do not know how to handle dynamic displacement problems in the network.

Nonsteady-state single phase flow can be studied in the network model by the addition of a capacitor to each resistor and by measuring electrical transients in the network. Such studies could show the effect of pore size distribution and network structure on transient flow behavior. For most porous media it is generally believed that nonsteady-state single phase behavior can be described by equations using steady-state permeability and porosity as normally measured. However, I believe that for fractured or vugular systems in which there is a large amount of porosity that does not contribute to permeability the steady-state permeability and the total porosity cannot be used to calculate nonsteady-state behavior. Network model studies could show how the porosity distribution influences nonsteady-state behavior.

The answer is no to both of the specific questions asked by Gates. Randomly connected networks may be studied in the future; networks of this kind were not examined during the course of the work reported in this paper. Many thin sections of reservoir sandstone have been examined. All had irregular networks with a random number of pores joined at each junction point. No attempt was made to compare network structure as seen in the thin sections with the flow properties of the sand.

DISCUSSION

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DONALD W. CARPENTER
PAUL A. WITHERSPOON
MEMBER AIME

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The writers agree with other reviewers that Fatt has developed an ingenious mathematical and analog model which can be used to investigate the microscopics of mixture flow in porous media such as petroleum reservoirs. Valuable and heretofore nonexistent information has already been obtained and reported by Fatt, and additional important postulations will come from others who study the properties of network models. For example, work is already underway in this laboratory to treat imbibition and hysteresis effects as characteristic of three-dimensional models, making use of special digital computer (Illiac) facilities.

At the moment, however, it seems important to question several features of Fatt's pioneer work. First, we note that Fatt's results appear to be essentially reproducible in that only minor discrepancies have been uncovered in the checks made to date. Since future work is being done on Illiac, it will be possible eventually to know if the results of capillary desaturation and relative permeability depend on the "path" followed, which is the implication of our failure so far to check Fatt's results in detail.

More to the point, we wish to question the sense, and indeed the necessity, of Fatt's neglect of the fact that the wetting phase must have an escape path (i.e. the wetting phase relative permeability must be finite) if capillary pressure desaturation is to continue. Conceivably, the postulation of zero contact angle and transfer via film flow is reasonable if, over geologic time, this is what actually occurs. However, if the laboratory capillary pressure experiment is the prototype, the model must have a different construction.

Fig. 1 shows our check of Fatt's Fig. 7 in Part I (Curve A), and our results (Curves B and C) obtained by imposing the condition that water must always have a path of continuity for exit in the same way that Fatt required a continuous path for oil entry. The boundary condition leading to Curve B was that oil could enter and water could exit at all four network edges, giving an "irreducible wetting phase saturation" of 13.6 per cent: the boundary condition leading to Curve C was that oil could enter at the top, bottom, and right edges, and that water could exit only at the left edge, giving a minimum water saturation of 23.3 per cent. The writers note that these values (Curves B and C) correspond more closely to laboratory experimental results than do Fatt's curves.

In these regards, Curve D of Fig. 1 will be of special interest. This depicts the transition zone shape and the residual oil saturation which would be left after first bringing oil into a water-filled system (i.e. Curve B), and then bringing gas into the oil-water system as a second non-wetting phase. Thus, a residual oil saturation of 18.3 per cent (i.e. 31.9 per cent from Curve D minus 13.6 per cent from Curve B) is obtained. This is to say that if Fatt's network model is a good representation of the reservoir rock prototype, there is still a possibility for trapping sizable volumes of oil even when the smallest pores are filled with irreducible (interstitial) water.

Our major point is to raise the question whether or not the implication should be given by Fatt and his other reviewers (e.g. see discussion by Gates above) that the tortuosity concept is useless and that the need for it can be circumvented by use of the network model approach. In fact, it is shown below that tortuosity is implicitly considered in the network model, and that it continues to be a useful and valid concept as does the underlying Kozeny-Carman theory.

Considering the data of Table I in Fatt's Paper III, it is found that a value for porosity and specific surface area can be assigned to the model being discussed. Also, the permeability is independently known from the resistance measurement on the analog computer, and hence all the parameters are available that are needed to calculate the tortuosity implicit in the model according to the Kozeny-Carman equation. As might be expected from the network form under consideration (i.e. the triple hexagonal pattern), tortuosity comes out to be nearly the square of the reciprocal of the cosine of 30 degrees! (N.B. The above calculation, while lengthy, is easy to check and will be presented by the writers in a later publication.)*

We think the tendency to regard the tortuosity factor as an invalid concept comes from the difficulty in defining the term exactly and in attaining an independent measurement of it. So far as Fatt's network models are concerned, however, we feel it is intuitively evident that when the pore spaces are completely filled with a given fluid, the square network will have unit tortuosity, the single and double hexagonal networks will have a tortuosity equal to 9/4ths, and (as noted above) the triple hexagonal network will have a tortuosity between unity and the factor $(\cos 30°)^{-2}$, depending on flow direction. Any of these networks will be characterized by higher values of tortuosity when there is more than one pore saturant, but it is clear that no simple method of calculation is available to actually determine what the wetting and non-wetting phase tortuosities will be. The most that can be said is that they are the statistical

*To assign numerical values for permeability, porosity, and specific surface area, reasonable conversion factors must be selected which relate length in Fatt's arbitrary units to length in centimeters, and which relate tube length to tube radius according to Fatt's Eq. 14-III. The former determines specific surface area, the latter determines porosity, and a complex function of the two factors determines permeability and its relationship to electrical conductivity of the network. On the other hand, it can be shown that tortuosity itself is independent of the numerical value of these factors, and is determined entirely by the network pattern form.

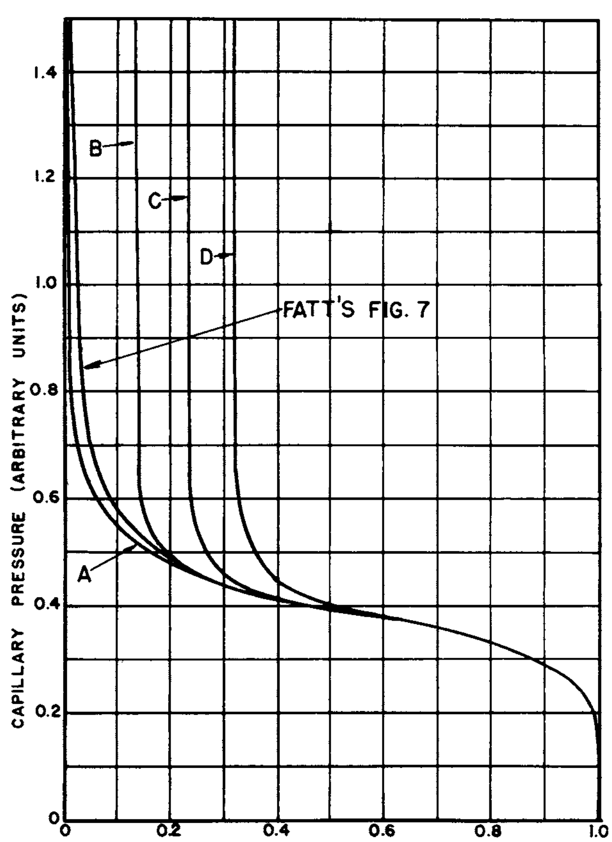

FIG. 1—CHECK OF FATT'S FIG. 7-I (CURVE A).

equivalents of ratios of path lengths to bed lengths; but in a more fundamental sense, tortuosities are a measure of the degree of interconnection between pores and the continuity between groups of pores. Indeed, Fatt's model is extremely useful in providing a visualization of the true significance of the tortuosity concept, for clearly tortuosity must bear some direct relationship to Fatt's "beta" factor. It will prove extremely interesting, therefore, to test the Kozeny-Carman derived relative permeability theories on the network model, since in this case all contributing factors can be independently evaluated.

AUTHOR'S REPLY to W. D. ROSE, D. W. CARPENTER, and P. A. WITHERSPOON

I am gratified that Rose, Carpenter, and Witherspoon have undertaken a study of the network model. My work must be considered only as a beginning. It was an attempt to point out directions in which there may be profitable study. The use of high speed computers, such as the Illiac, will make possible tests of the network model that I could not even consider while limited to my pencil-and-paper accounting procedure.

Rose, *et al*, have an excellent point in preferring a system in which there is a pore escape path for wetting phase over my system in which wetting phase escaped from the network as a film on the pore walls. My system was chosen because it was amenable to pencil-and-paper methods and yet was, as Rose admits, a conceivable mechanism for desaturation. There are, however, many alternate desaturation schemes.

I still see little to be gained from use of the tortuosity concept. Perhaps this stems from my unwillingness to accept the Kozeny-Carman relation as fundamentally sound or meaningful when applied to relative permeability. ★★★