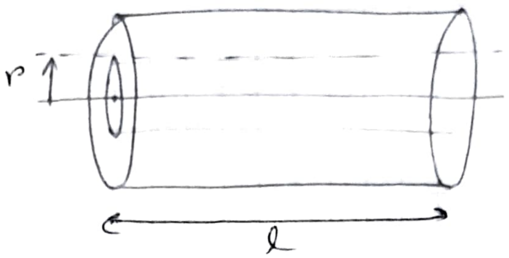
Draft of Thesis 30·01·2023

1. Volumetric Flow Rate through a cylinder




$$\frac{F}{A} = \mu \left| \frac{dv}{dr} \right|$$ [Definition of viscosity]

$$\frac{(\pi r^2) A}{2\pi r l} = -\mu \frac{dv}{dr}$$

$$\frac{1}{2} \frac{\Delta P}{l} \int r\, dr = -\mu \int v\, dv$$

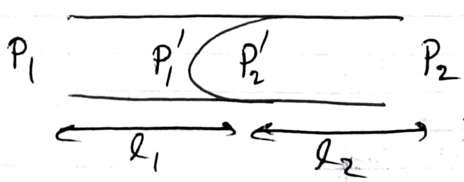
$$\frac{1}{4} \frac{\Delta P}{l} r^2 = C - \mu V$$

$$v = \frac{1}{4\mu} \frac{\Delta P}{l} (R^2 - r^2)$$ $[r = R,\ v = 0]$

$$Q = \int_0^R v\, 2\pi r\, dr$$

$$\boxed{Q = \frac{\pi}{8\mu} \frac{\Delta P}{l} R^4}$$ (Pousielle's equation)

2. Volumetric flow rate with meniscus present




$$Q = \frac{\pi}{8\mu_1} \frac{P_1 - P_1'}{l_1} R^4$$

$$Q = \frac{\pi}{8\mu_2} \frac{P_2' - P_1}{l_2} R^4$$

$$Q\mu_1 l_1 = \frac{\pi}{8}R^4 (P_1 - P_1')$$
$$Q\mu_2 l_2 = \frac{\pi}{8}R^4 (P_2' - P_2)$$

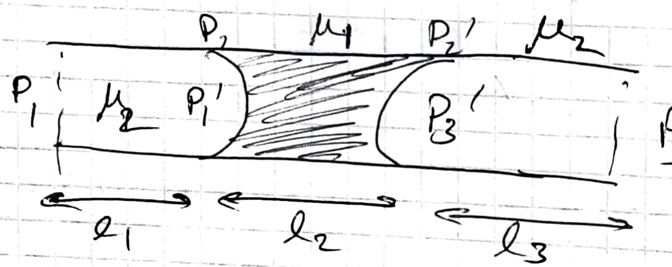
$$Q(\mu_1 l_1 + \mu_2 l_2) = \frac{\pi}{8}R^4 (P_1 - P_2 + P_2' - P_1')$$

$$Q = \frac{\pi R^4}{8(\mu_1 l_1 + \mu_2 l_2)}\left(\Delta P + \frac{2\sigma}{R}\right)$$

$$Q = \frac{\pi}{8(\mu_1 l_1 + \mu_2 l_2)}\left(\Delta P R^4 + 2\sigma R^3\right)$$

$$\boxed{Q = \frac{\pi R^3}{8(\mu_1 l_1 + \mu_2 l_2)}\left(R\Delta P + 2\sigma\right)}$$

3. Flow rate with two menisci




$$P_3' - P_2' = \frac{2\sigma}{R}$$
$$P_1' - P_2 = \frac{2\sigma}{R}$$

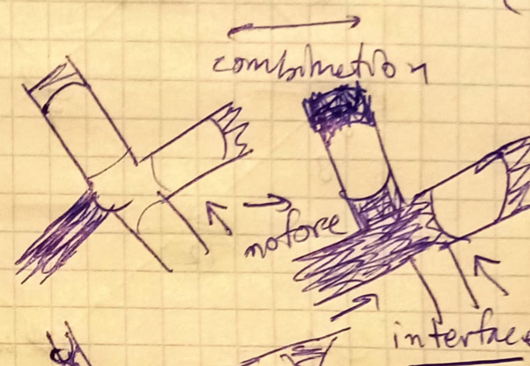
$$Q = \frac{\pi R^4}{8\mu_2 l_1} \quad P_1 - P_1'$$
$$- \frac{2\sigma}{R}$$
$$Q = \frac{\pi R^4}{8\mu_1 l_2} \quad P_2 - P_2'$$
$$= + \frac{2\sigma}{R}$$
$$Q = \frac{\pi R^4}{8\mu_2 l_3} \quad P_3' - P_3$$

$$Q(\mu_2 l_1 + \mu_1 l_2 + \mu_2 l_3) = \frac{\pi R^4}{8}(P_1 - P_3)$$

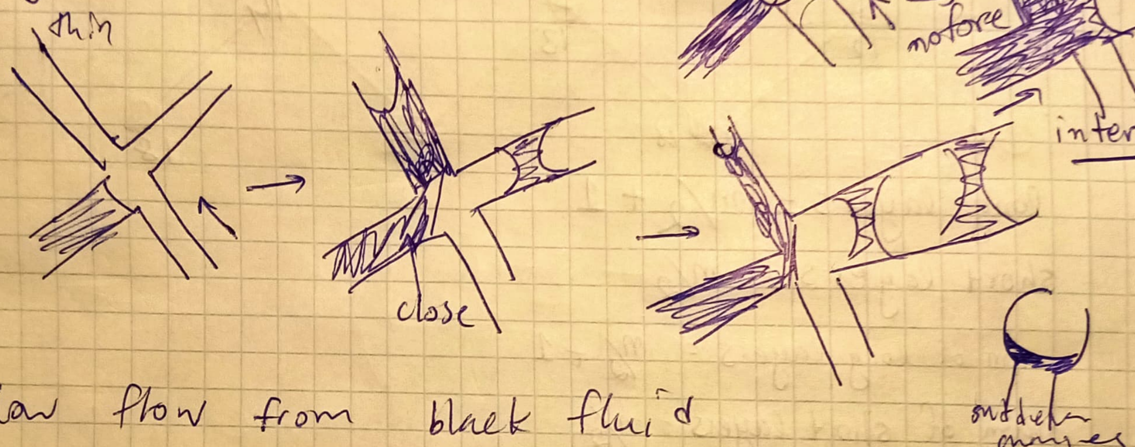
$$\boxed{Q = \frac{\pi R^4}{8(\mu_2 l_1 + \mu_1 l_2 + \mu_2 l_3)}\Delta P}$$

## 4. Cases of Distribution

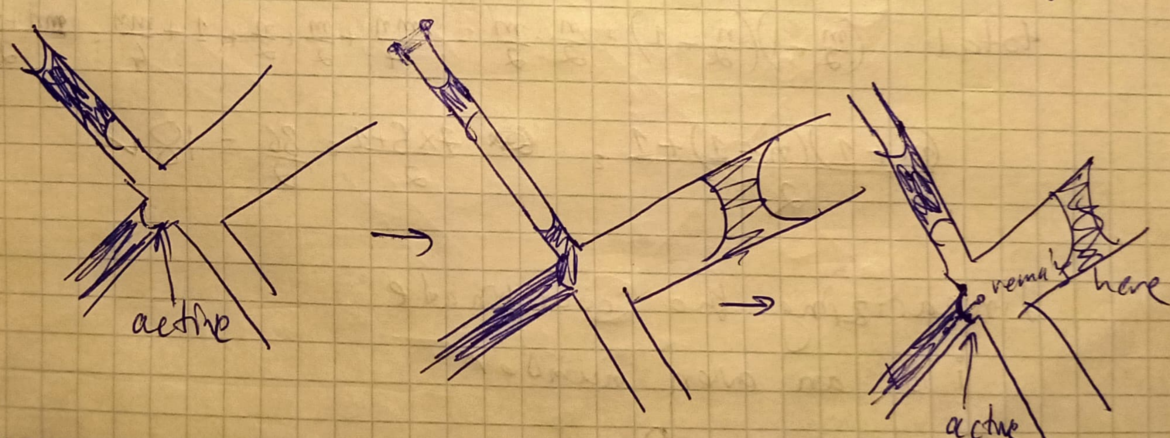
### 4.1 Equal radius exit

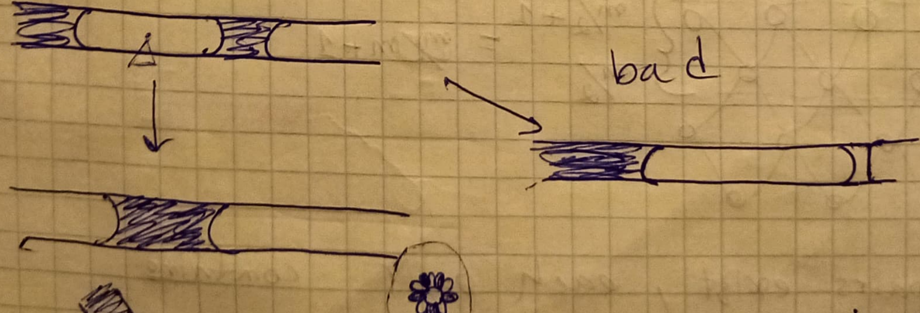



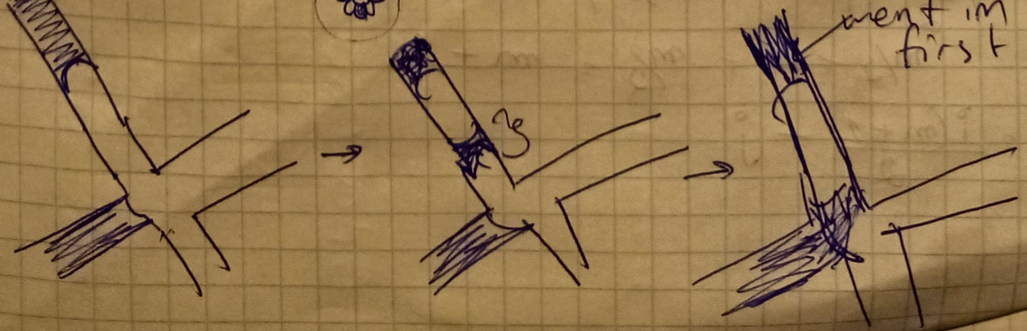

### 4.2 High flow from black fluid



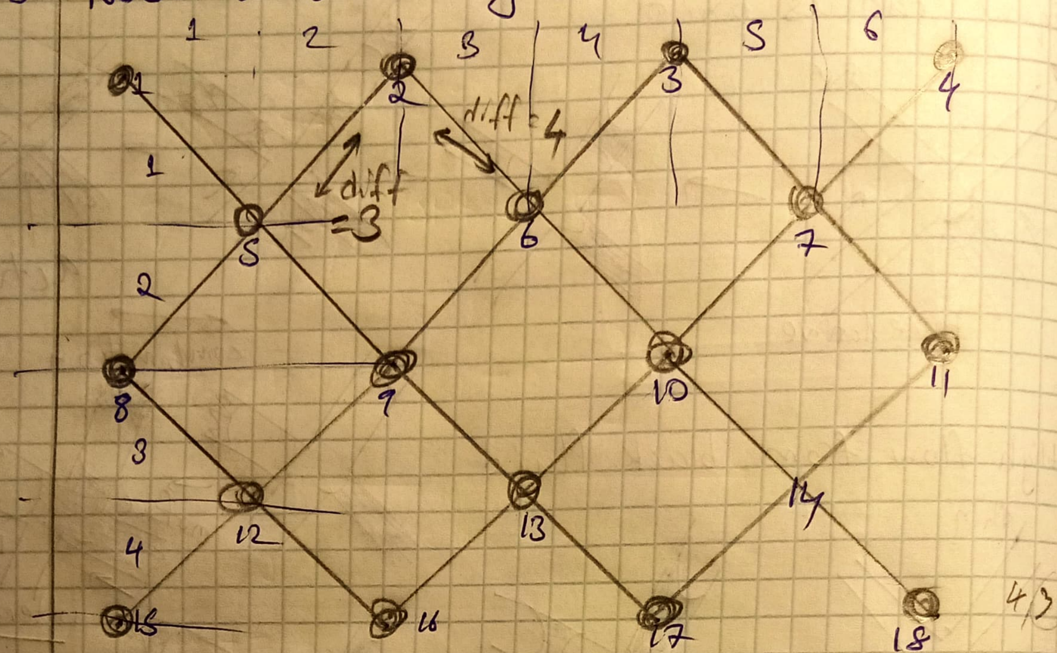

### 4.3 Low flow from black fluid



5. Node coordinat system




long layers = $m/2 + 1$

short layers = $m/2$

num of long layers = $n/2 + 1$

num of short layers = $n/2$

total = $\left(\frac{m}{2}+1\right)\left(\frac{n}{2}+1\right) + \frac{n}{2}\cdot\frac{m}{2} = \frac{mn}{4} + \frac{m}{2} + \frac{n}{2} + 1 + \frac{mn}{4} = \frac{mn+m+n}{2}$

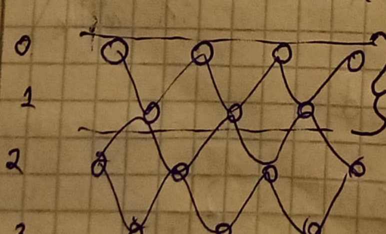
$\frac{(m+1)(n+1)+1}{2} = \frac{7\times 5+1}{2} = \frac{36}{2} = 18$ ✓

4/3

$\frac{4\times 7+}{2}$

Linearizing the coordinate

i is an even number




$\left.\begin{matrix} m/2+1 \\ m/2 \end{matrix}\right\} = m+1$

$\frac{i}{2}$ pairs exist, each pair contains

$m/2 + 1 + m/2 = m+1$

$= \frac{i(m+1)}{2} + j$

for the odd case

$\frac{i-1}{2}$ pairs

long layer

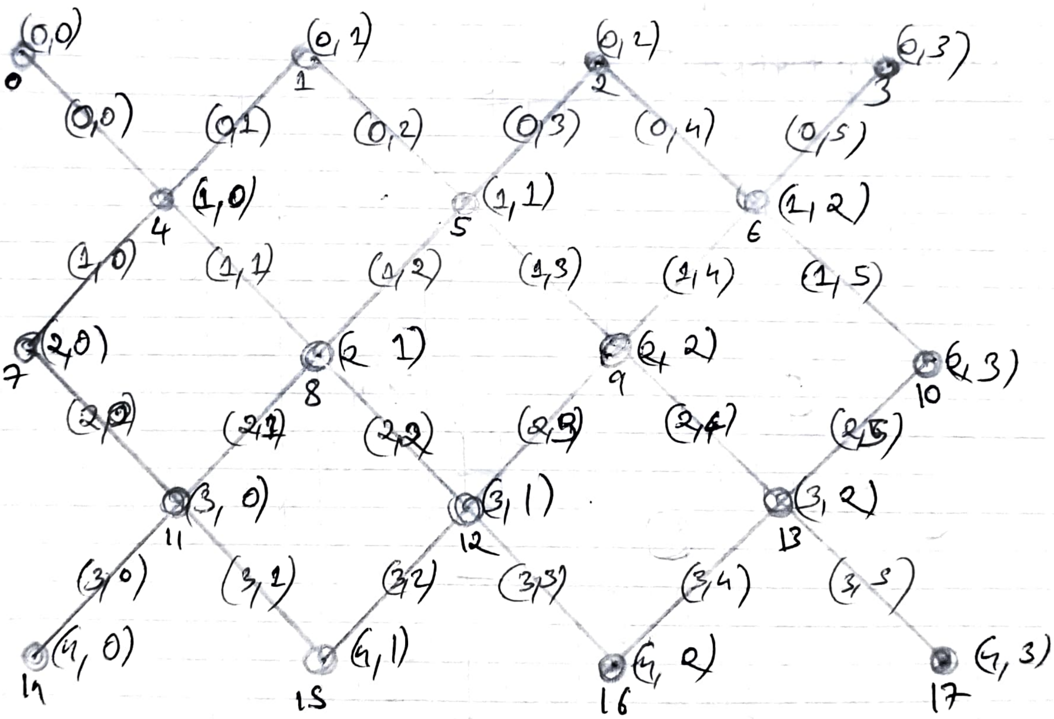
$\frac{i-1}{2}(m+1) + \left(\frac{m}{2}+1\right) + j$

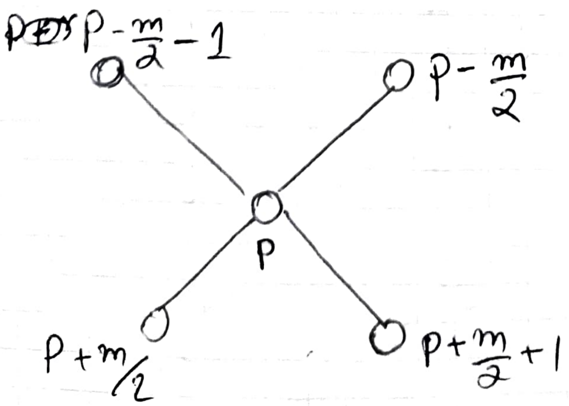
$\frac{(m+1)(i-1)}{2} + \frac{(m+1)+1}{2} + j$

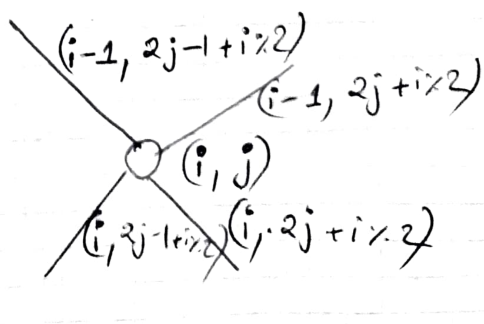
$= \frac{(m+1)i+1}{2} + j$

radius size 4

$$\boxed{linear(i,j,m) = \frac{i(m+1)+(i\%2)}{2} + j}$$



6. The Linear Equations

$$\mu = \sum_i \mu_i l_i$$

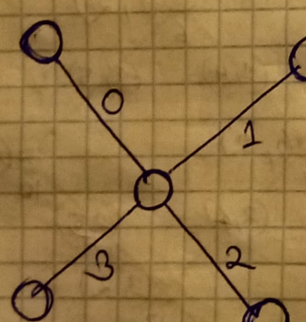
$$q = \frac{\pi}{8}\frac{R^3}{\mu}\left(\Delta P R + 2\sigma\right)$$

Note that all of these eq the actual l does not ma

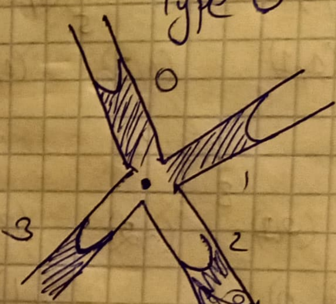
$$\sum_{j=1}^{4} \frac{R_{ij}^3}{\mu_{ij}}\left[(P_i - P_j)R_{ij} + 2\sigma\right]$$

$$K = \frac{R_{ij}^3}{\mu_{ij}}$$

$$\boxed{\begin{array}{l} P_i \mathrel{+}= R_{ij} K_{ij} \\ P_j \mathrel{-}= R_{ij} K_{ij} \\ \text{equation.back()} \mathrel{-}= 2\sigma K_{ij} \end{array}}$$




Type 0




sign of $2\sigma$

| Direction | No of meniscus | Type 0 | Type 1 |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 1 | +1 | -1 |
| 0 | 2 | 0 | 0 |
| 1 | 0 | 0 | 0 |
| 1 | 1 | +1 | -1 |
| 1 | 2 | 0 | 0 |
| 2 | 0 | 0 | 0 |
| 2 | 1 | -1 | +1 |
| 2 | 2 | 0 | 0 |
| 3 | 0 | 0 | 0 |
| 3 | 1 | -1 | +1 |
| 3 | 2 | 0 | 0 |

```
struct dst::FillProperty
{
    int n;
    bool type;
    std::vector<float> pos;
};

float type() const
{
    return (type ? 1:-1);
}
```

→

```
static float sig (bool c)
{
    return c ? -1: 1;
}

float scontb(int directn)
{
    return sig (d) * sig(type) *(n)
}
```

7. Volumetric flow rate

$$\overset{up}{\left(i, \ j/2 + 1 - (i \% 2)\right)} \quad \overset{down}{\left(i+1, \ j/2 + (i \% 2)\right)}$$

$$q = \frac{\pi}{8} \frac{R^3}{\mu L} \left[\Delta P R + 2\sigma\right]$$

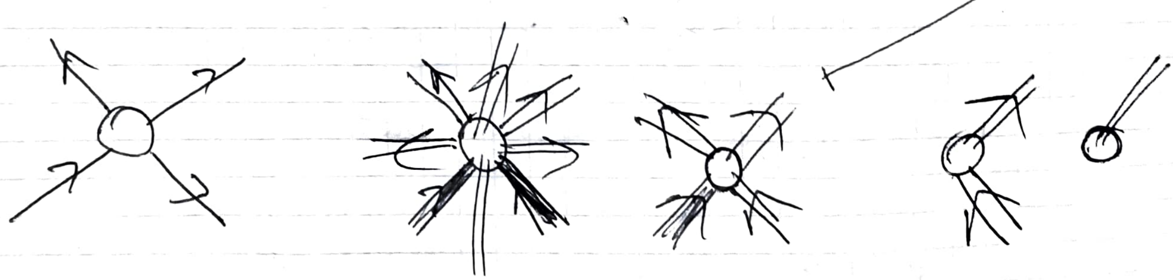
$$v = \frac{1}{8} \frac{R^2}{\mu L} \left(\Delta P R + 2\sigma\right)$$

Note:
in the calculation of $t$: the nearest time,

$$v = \frac{K}{L}$$

$$\delta t = \frac{QL}{Q \times K/L} = \frac{QL^2}{K}$$

8. Possible cases of fluid flow



sum_of_fluid_1 going in
sum_of_fluid_2 going in