

Simulation of Two-Phase Flow in Porous Media using a Two-Dimensional Network Model

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Moscow Institute of Physics and Technology
The International Summer Conference on
Theoretical Physics
Phystech.Digital 2.35, 17:15 – 17:35, 07.07.2023
Supported by PHΦ № 23-21-00175



About Presenter



- Bachelors Technical Physics' 23
- Masters Applied Mathematics and Physics' 25
- Chair of Applied Mechanics
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УДК 532.685

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The algorithms and methods used to simulate two-phase flow in porous media has many practical applications in oil recovery, hydrology, electricity production where pressurized water is passed through heated pipes and is transformed into steam, etc. Our algorithm presented here is used to find the saturation of a phase with respect to time, model imbibition, the hysteresis curve when the pressure across the porous body is reversed, total capillary pressure as a function of saturation [1], and determination of permeability which appears in Darcy's law.

Our model is initially set up such that the wetting fluid is low in saturation and is confined to the bottom of our network. A higher pressure is fixed for all nodes at the bottom layer, while a low pressure is fixed for the top row. In all nodes, law of conservation of volume is applied, since mass is conserved and the phases are non-compressible. However for the bottom layer of nodes, the wetting fluid is injected as much required according to the sum of flow rates determined in the tubes connected to those nodes, while from the top layer of nodes a fluid is removed.

$$\sum Q_i = 0 \quad (1)$$

Where Q is the flow rate in $[m^3/s]$ in a tube connected to a particular node. The flow rate formula used is

$$Q = \frac{\pi R^4}{8\mu l} \left(\Delta P + \frac{2s\sigma}{R} \right) \quad (2)$$

$s = \{-1, 0, 1\}$, 0 when there are an even number of meniscus or no meniscus in a tube, +1 or -1 is due to the orientation. Here

$$M = \sum \mu_i \frac{l_i}{l} \quad (3)$$

Note that the case when no meniscus is present ($s = 0$) the flow rate formula is reduced to the well known Poiseuille's equation

$$Q = \frac{\pi R^4}{8\mu} \frac{\Delta P}{l} \quad (4)$$

From (1) we obtain an equation which relates five pressures. The number of equations obtained is equal to the number of nodes in our network. The equations are solved using Gauss-Jordan Elimination optimized for our case to determine the pressures in each of the nodes, then flow rates are calculated using (2). The time step is chosen according to the nearest meniscus reaching the node. At each of the nodes the flow is distributed to the outgoing tubes such that the tube with the smallest radius is filled first with the wetting fluid, this is due to the favor of energy. The tubes are inclined as suggested in [3], to ensure that a fluid flows equally well in tubes inclined right or left.

- Model under development was presented at the 65th All Russia Scientific Conference – MIPT
- 08.04.2023
- Link to thesis p.196: <https://mipt.ru/upload/medialibrary/002/aerokosmicheskie-tekhnologii.pdf>



in order to more accurately represent the porous body.
This work is supported by grant PHΦ №23-21-00175

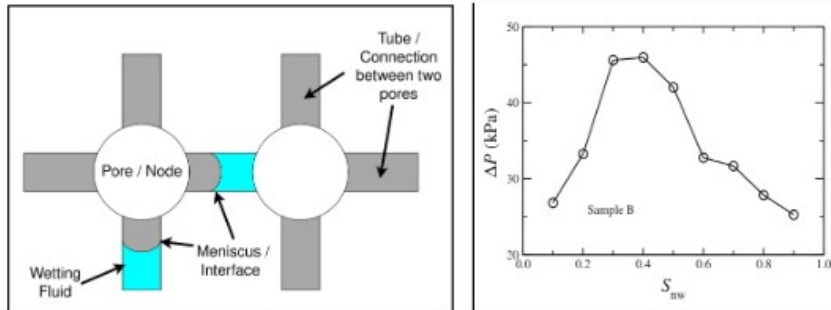


Figure 1: showing two nodes from the network where the size of the node is much larger than the radius such that the capillary force tends to zero when the meniscus enters a node. Figure 2: Showing the dependence of the average capillary pressure on the saturation of the wetting fluid, the average capillary pressure drop is maximum at the middle level of saturation, due to the maximum number of interfaces as also concluded by [2].

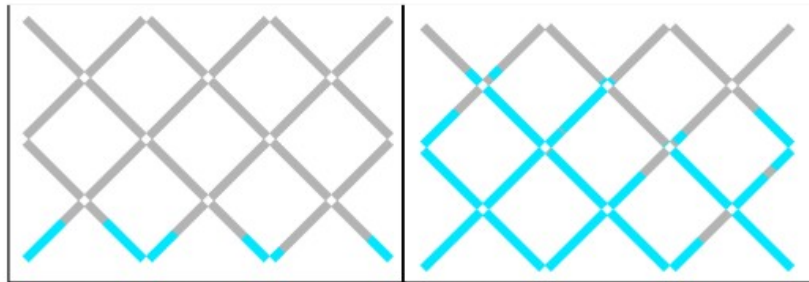


Figure 3 & 4: showing invasion of wetting fluid into a porous body consisting of randomly distributed tube radii for a highly simplified model (4 x 6), at saturation levels of 22% and 61% using our model.

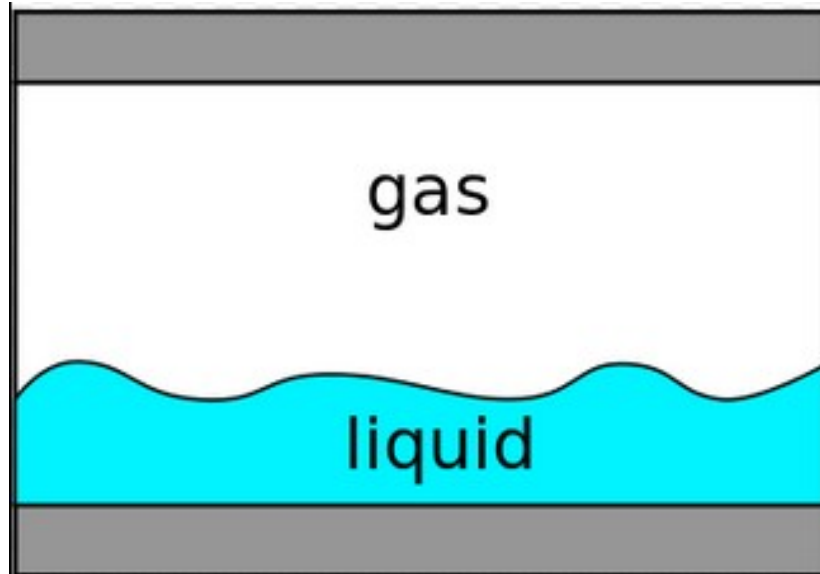
Reference

1. Fatt I. The network model of porous media 3, dynamic properties of networks with tube radius distribution // Petroleum Trans. AIME 1956. V. 207. P. 164
2. Sinha S. et al. Effective rheology of two-phase flow in three-dimensional porous media: experiment and simulation // Transp. Porous Med. 2017. V. 119. P. 77
3. Aker E., Måløy K.J., Hansen A., Batrouni G.G. A two-dimensional network simulator for two-phase flow in porous media // Transp. Porous Med. 1998 V. 32 P. 163
4. Raouf A., Hassanizadeh S. A new method for generating pore-network models of porous media // Transp. Porous Med. 2010. V. 81. P. 391

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Motivation - Two phase flow

- **Oil recovery**
- **Hydrology**
- **Electricity production**



Example of Porous Media



Classical Continuum models

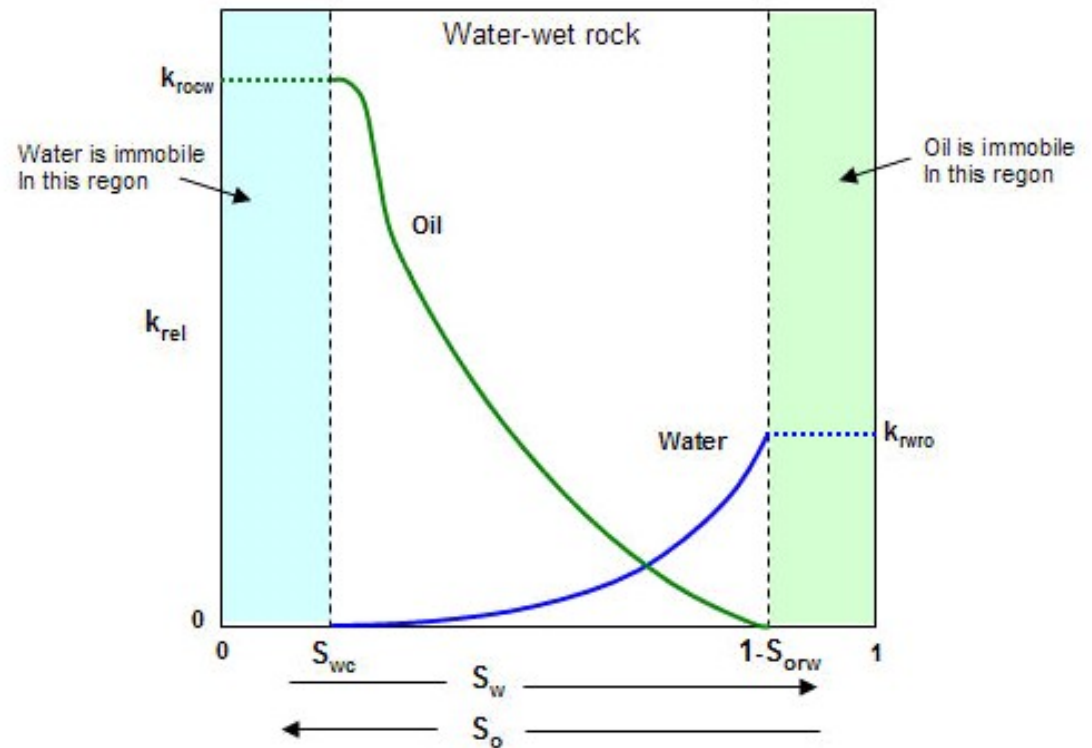
Darcy's Law

-

$$q = -\frac{k}{\mu} \nabla p$$

$$k = k(S)$$

$$S = \frac{dV_w}{dV_{void}}$$



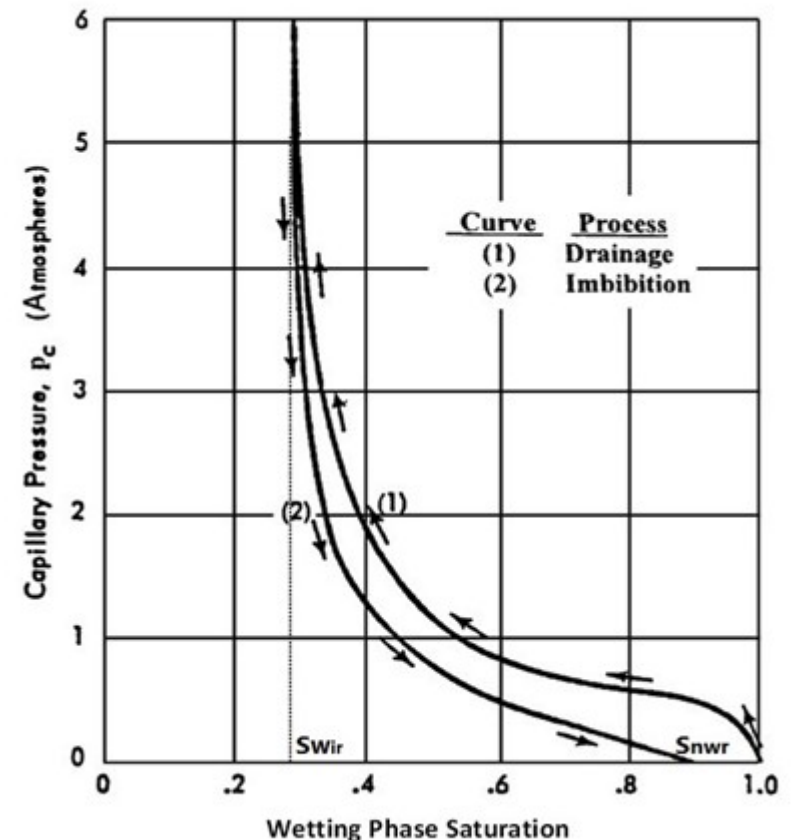
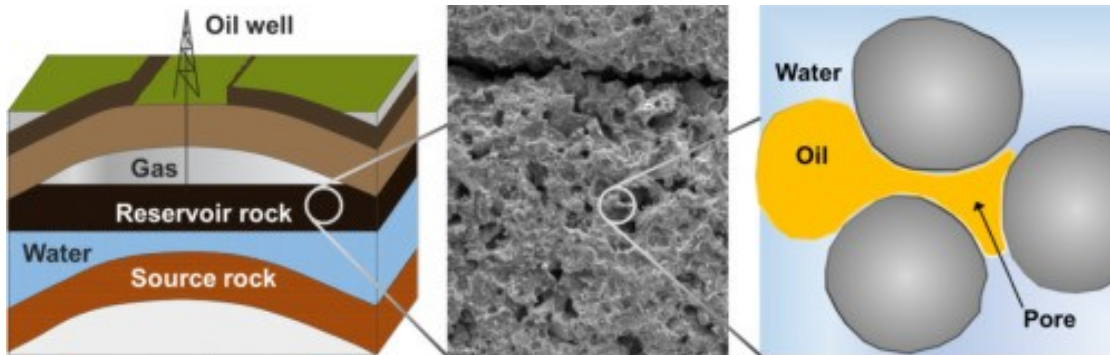
Limitation of Classical Models

$$k = k(S)$$

Only true when

Characteristic time of the processes
is much greater than characteristic
time of fluid redistribution in the
capillary space.

Fractured-porous medium with
blocks and cracks



Advanced Continuum models

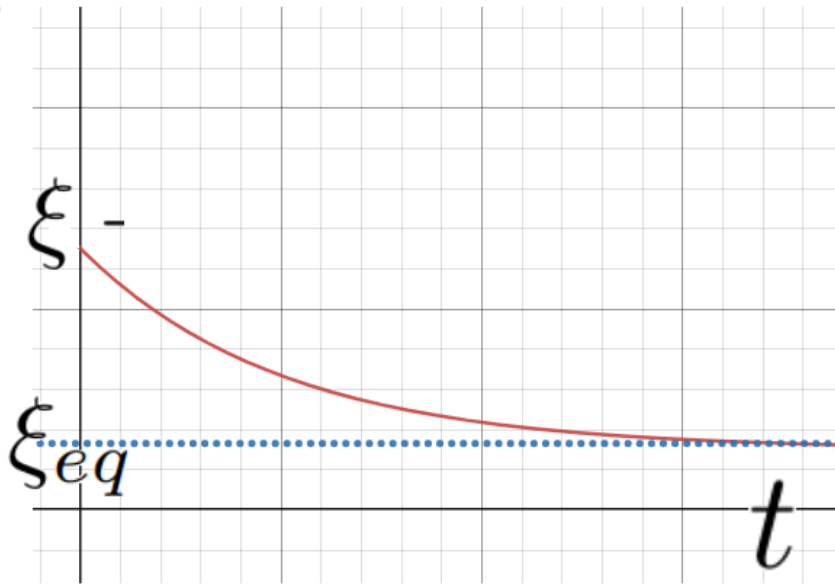
- **Hasanizadeh[6], Barenblatt[7]**
- **Kondaurov model[1]**

$$q = -\frac{k}{\mu} \nabla p$$

$$k = k\left(S, \frac{\partial S}{\partial t}\right)$$

$$k = k(S, \xi)$$

$$\frac{\partial \xi}{\partial t} = \Omega(S(t), \xi)$$



Pore Scale

- Direct Navier-Stokes Simulation
- Network Models

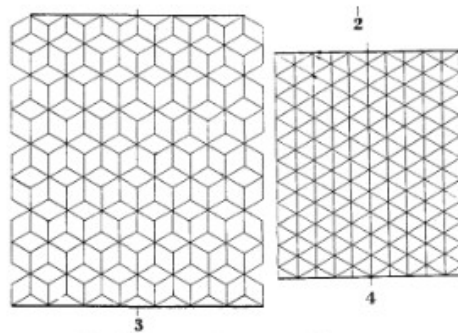


FIG. 1—SINGLE HEXAGONAL NETWORK.

FIG. 2—SQUARE NETWORK.

FIG. 3—DOUBLE HEXAGONAL NETWORK.

FIG. 4—TRIPLE HEXAGONAL NETWORK.

the same channel. Multiphase fluid flow in separate channels in porous media has been termed channel flow, while concentric flow in the same channel has been termed filament flow. The best observational support for the assumption of channel flow comes from the cinematographic studies of Chatenevar²¹, and from the Stanolind group²². Leverett²³ has shown on theoretical grounds that the interfaces between two immiscible phases in porous media will distribute themselves so that the radii of curvature of the interfaces, r_1 and r_2 , in the pore space will obey the LaPlace equation

$$P = \delta \left(\frac{1}{r_1} + \frac{1}{r_2} \right) \quad (1)$$

[1] Fatt I. The network model of porous media: Model using Resistors 1956

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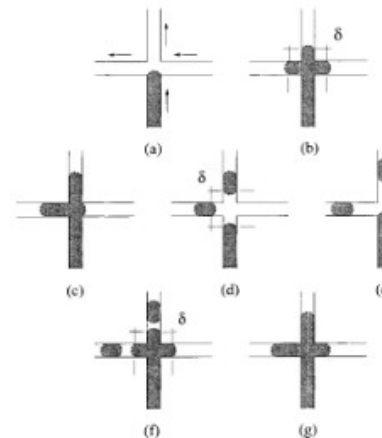


Figure 7. A 'mixture' of non-wetting (shaded) and wetting (white) which flow into the neighbor tubes. The different arrangements (a)–(g) are a result of applying the rules which are described earlier in this section. For all figures the fluids flow towards the node from the bottom and right tube while the fluids in the top and the left tube flow away from it (denoted by the arrows in (a)).

EYVIND AKER ET AL.

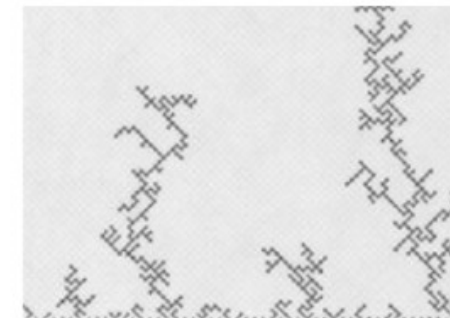
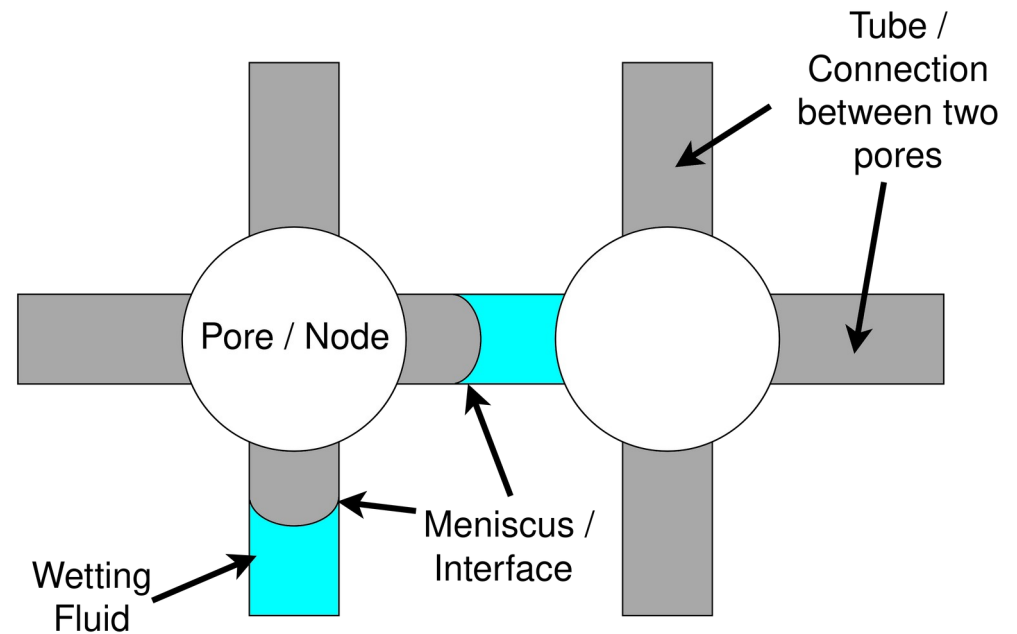
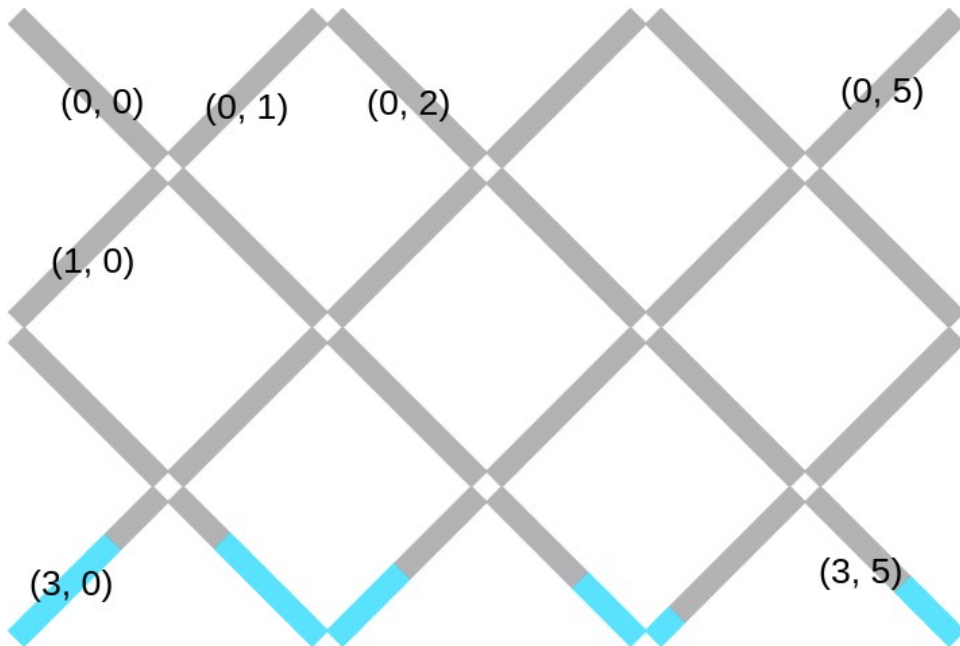


Figure 8. The pattern obtained of a simulation in the region of 60×80 nodes. $C_a = 4.6 \times 10^{-3}$ and $M = 1.0 \times 10^{-2}$ (black) displaces the defending wetting fluid (gray) from $7 \frac{1}{2}$ h on a Cray T90 vector machine.

Aker, E., Måløy, K.J., Hansen, A., Batrouni, G.G.
A two-dimensional network simulator for two-phase flow in porous media // Transp. Porous Med. 1998 V. 32 P. 163

Our Model

- Zero node volume
- Cylinder tubes
- Flow due to capillary pressure
- Maximum two meniscus



Algorithm implemented in C++

1) Read initial and radius

2) Linear Equations for Pressure

$$\sum Q_i = 0 \quad (1)$$

3) Solution using Gaussian-Elimination

$$Q = \frac{\pi R^4}{8 M l} \left(\Delta P + \frac{2 s \sigma}{R} \right) \quad (2)$$

4) Velocity

$$M = \sum \mu_i \frac{l_i}{l} \quad (3)$$

5) Time Step

6) Volume Distribution (New Method)

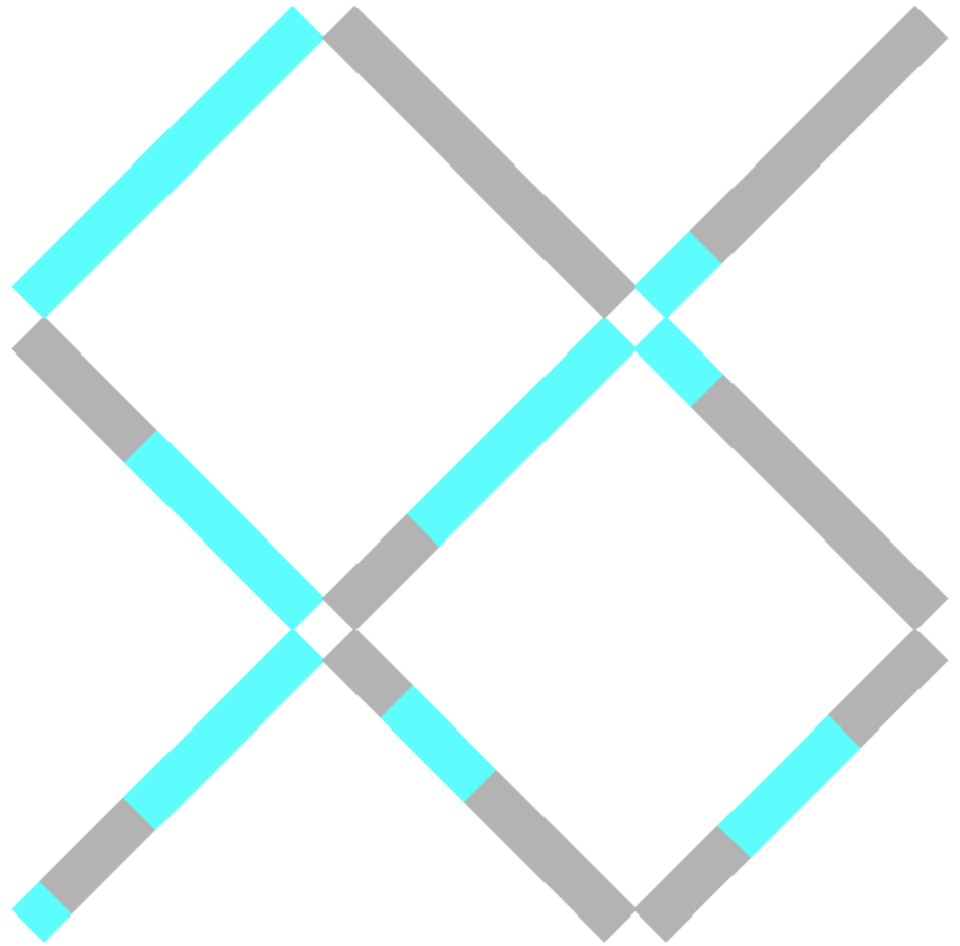
$$Q = \frac{\pi R^4}{8 \mu} \frac{\Delta P}{l} \quad (4)$$

7) Recombination



Meniscus Configuration Data

- [n, type, pos1, pos2]
- 0 0 0 0
- 0 1 0 0
- 1 0 0.2 0
-
- 1 0 0.6 0
- 1 1 0.3 0
- 1 1 0.8 0
-
- 2 0 0.1 0.4
- 2 1 0.5 0.8
- 2 1 0.3 0.7



Flow Rate of a Node

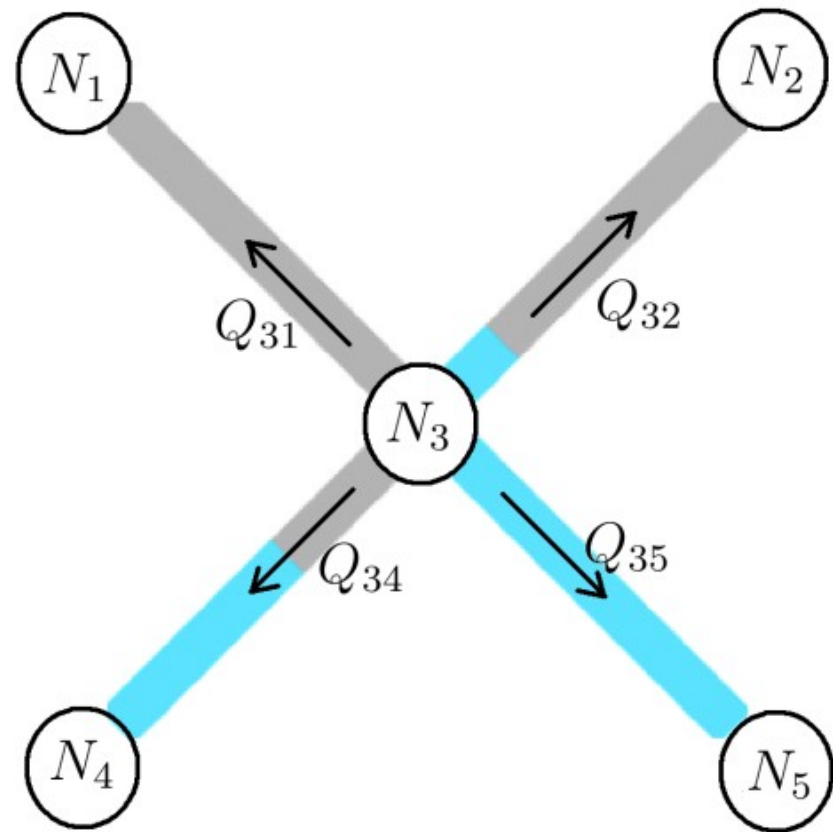
$$Q_{31} = \frac{\pi R_{31}^3}{8lM_{31}}(R_{31}\Delta P_{31} + 2s_{31}\sigma)$$

$$Q_{32} = \frac{\pi R_{32}^3}{8lM_{32}}(R_{32}\Delta P_{32} + 2s_{32}\sigma)$$

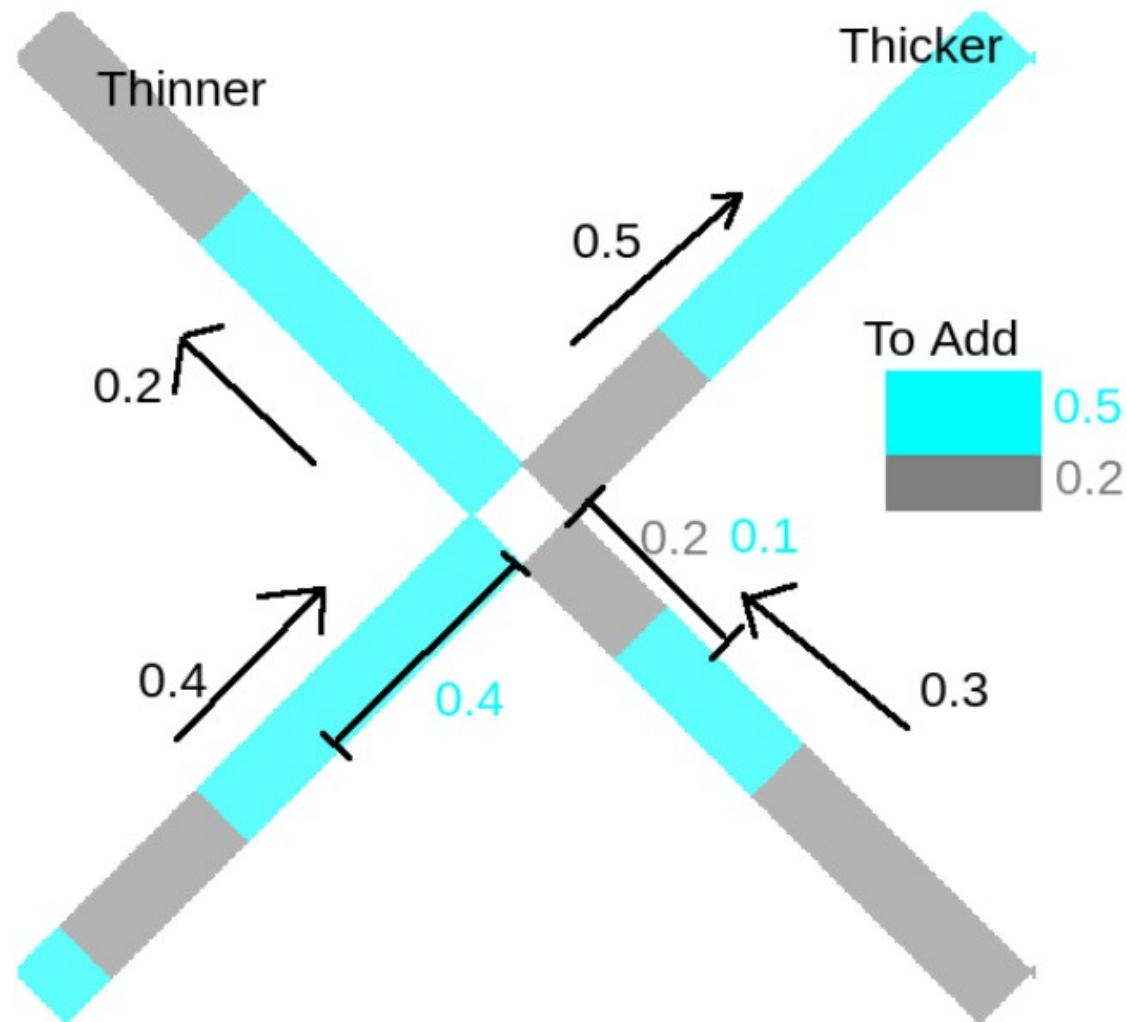
$$Q_{34} = \frac{\pi R_{34}^3}{8lM_{34}}(R_{34}\Delta P_{34} + 2s_{34}\sigma)$$

$$Q_{35} = \frac{\pi R_{35}^3}{8lM_{35}}(R_{35}\Delta P_{35} + 2s_{35}\sigma)$$

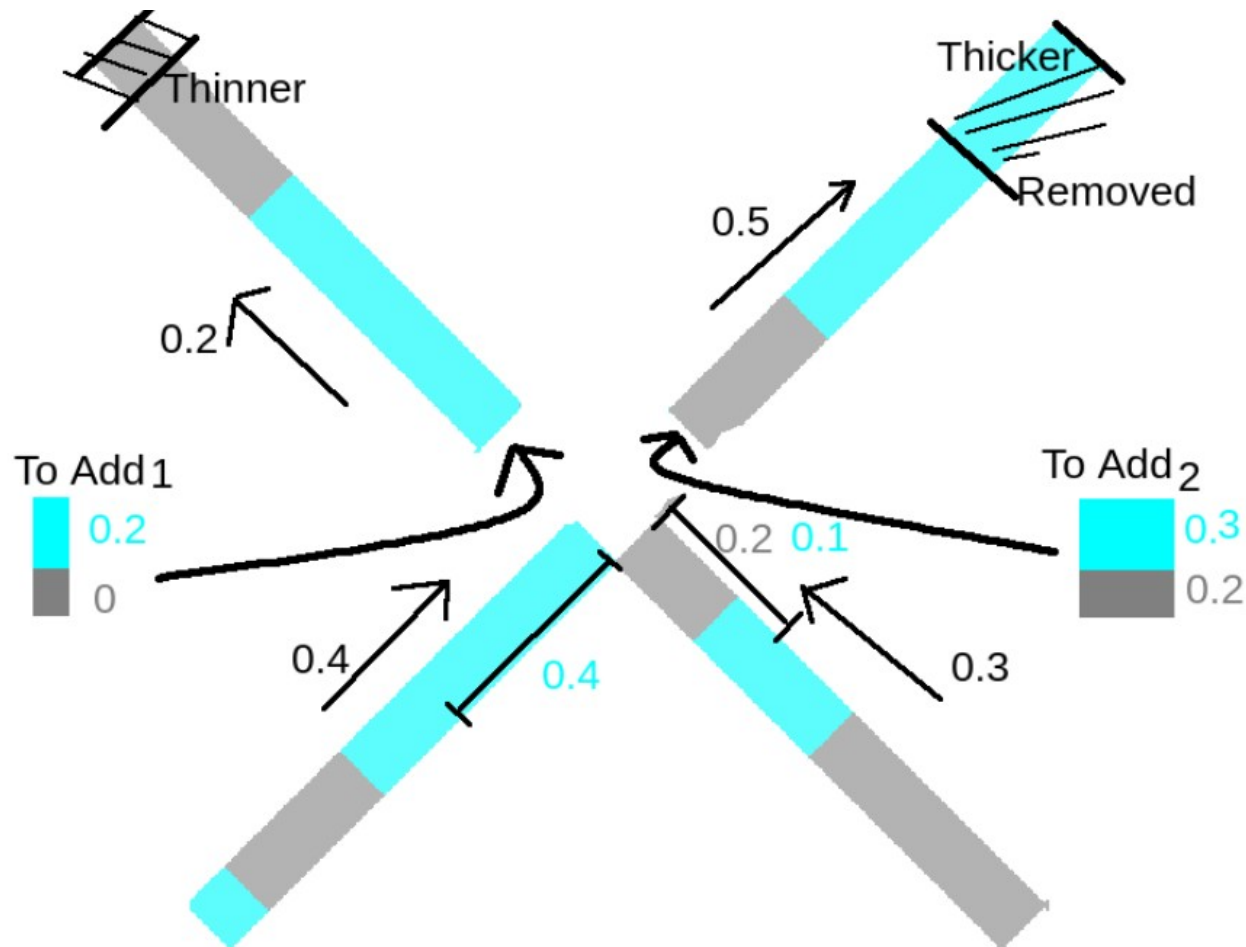
$$\sum_k Q_{3k} = 0$$



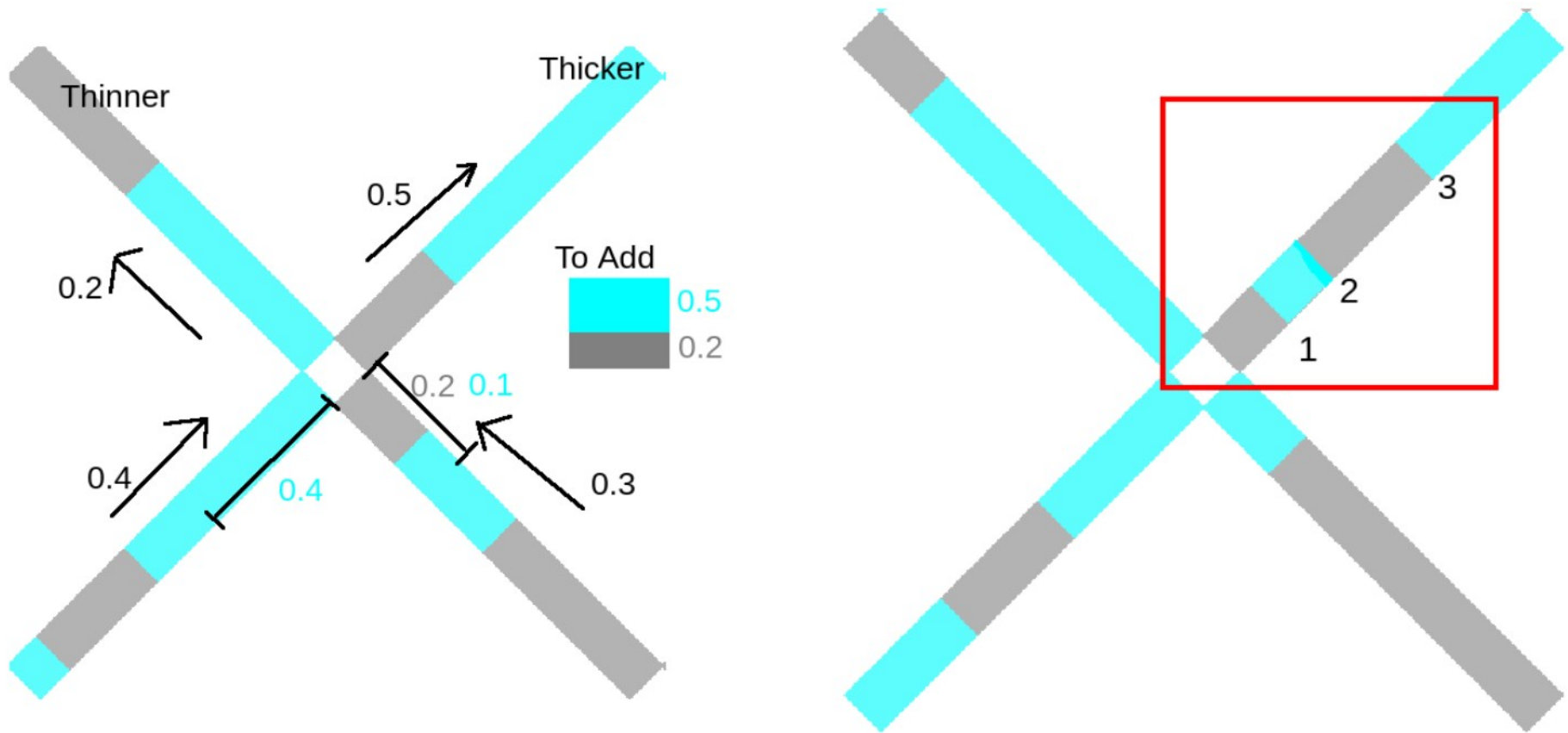
New Feature: Distribution of Phases at the nodes



New Feature: Distribution of Phases at the nodes



New Feature: Distribution of Phases at the nodes



C++ Implementation Moving Meniscus Forward in Tube

```
func::Integration::TFluid func::Integration::calculate_fluid_table
(
    const Tdouble& radius,
    const TMns& mns,
    const Tdouble& velocity,
    const Tdouble& volume,
    const dst::Dimension& dimension,
    const double time_step
)
{
    TFluid fluid_addition_table = dimension.empty_table_templated<Fluid>();

    for(int row = 0; row < dimension.node_rows(); ++ row)
    {
        for(int col = 0; col < dimension.node_cols(row); ++ col)
        {
            Tank tank;
            std::vector<Tube_FromNode> tubes_from_node_vec;

            const std::vector<dst::Tube> tubes_connected_vec =
                dimension.generate_tubes_connected_to_node(row, col);

            const int connections_size = tubes_connected_vec.size();
            for(int direction = 0; direction < connections_size; ++ direction)
            {
                const dst::Tube& connection = tubes_connected_vec[direction];
                if(connection.active)
                {
                    const double rad = radius[connection.row][connection.col];
                    const dst::Mns& mns = mns[connection.row][connection.col];
                    const double vel = velocity[connection.row][connection.col];

                    if(mns.is_the_flow_from_tube_into_node(direction, vel))
                    {
                        const std::vector<double> add_fluid =
                            mns.vol_fluid_into_nodes(
                                rad,
                                direction,
                                vel,
                                time_step,
                                declconst::TUBE_LENGTH
                            );

                        );
                        tank.add_fluid(add_fluid);
                        continue;
                    }

                    Tube_FromNode tube
                    {
                        rad,
                        connection.row,
                        connection.col
                    };

                    tubes_from_node_vec.push_back(tube);
                }
            }

            std::sort(tubes_from_node_vec.begin(),
                tubes_from_node_vec.end(),
                compare_where_wetting_fluid_go_first);

            for(const Tube_FromNode& tpshin: tubes_from_node_vec)
            {
                fluid_addition_table[tpshin.row][tpshin.col]
                    |= tank.pour_out_fluid(volume[tpshin.row][tpshin.col]);
            }
        }
    }

    return fluid_addition_table;
}
```



Recombination, centre of mass

Grey: $m_1 = l_1$ at $d_1 = \frac{l_1}{2}$,

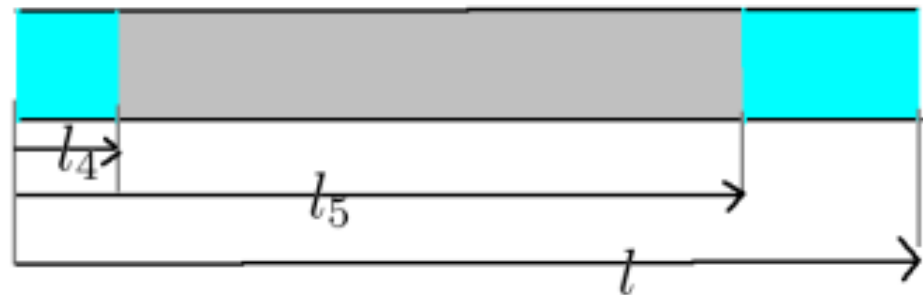
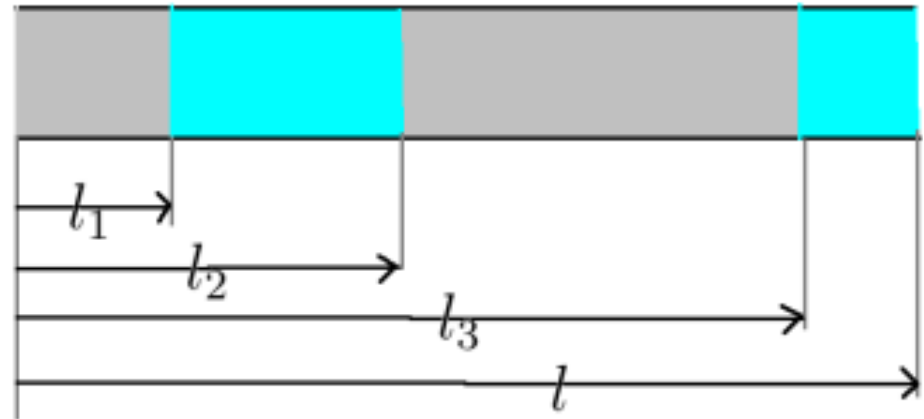
$m_2 = l_3 - l_2$ at $d_2 = \frac{(l_3 + l_2)}{2}$

$$m = m_1 + m_2$$

$$c_{grey} = \frac{m_1 d_1 + m_2 d_2}{m}$$

$$l_4 = c_{grey} - \frac{m}{2}$$

$$l_5 = l_4 + m$$



Recombination, centre of mass

```
396 dst::Mns::PosNew_Type_Result dst::Mns::centre_of_mass_recombination
397 {
398     const bool type_begin,
399     const std::vector<double>& pos_new
400 } const
401 {
402     if(pos_new.size() < 3)
403     {
404         return {type_begin, pos_new};
405     }
406
407     bool new_type_begin = type_begin;
408     double l1 = -1;
409     double l2 = -1;
410     double l3 = -1;
411     double l4 = -1;
412
413     if(pos_new.size() == 3)
414     {
415         if(type_begin)
416         {
417             l1 = pos_new[0];
418             l2 = pos_new[1];
419             l3 = pos_new[2];
420             l4 = 1;
421         }
422         else
423         {
424             l1 = 0;
425             l2 = pos_new[0];
426             l3 = pos_new[1];
427             l4 = pos_new[2];
428             new_type_begin = !type_begin;
429         }
430     }
431     if(pos_new.size() == 4)
432     {
433         l1 = pos_new[0];
434         l2 = pos_new[1];
435         l3 = pos_new[2];
436         l4 = pos_new[3];
437     }
438
439     return {new_type_begin, centre_of_mass_equation(l1, l2, l3, l4)};
440 }
```

```
442 std::vector<double> dst::Mns::centre_of_mass_equation
443 {
444     const double l1,
445     const double l2,
446     const double l3,
447     const double l4
448 }
449 {
450     const double d1 = l2 - l1;
451     const double d2 = l4 - l3;
452     const double d = d1 + d2;
453     const double c1 = (l1 + l2) / 2;
454     const double c2 = (l3 + l4) / 2;
455
456     const double L = (c1 * d1 + c2 * d2) / d;
457     const double L1 = L - d / 2;
458     const double L2 = L1 + d;
459
460     return std::vector<double> {L1, L2};
461 }
```



Test 1

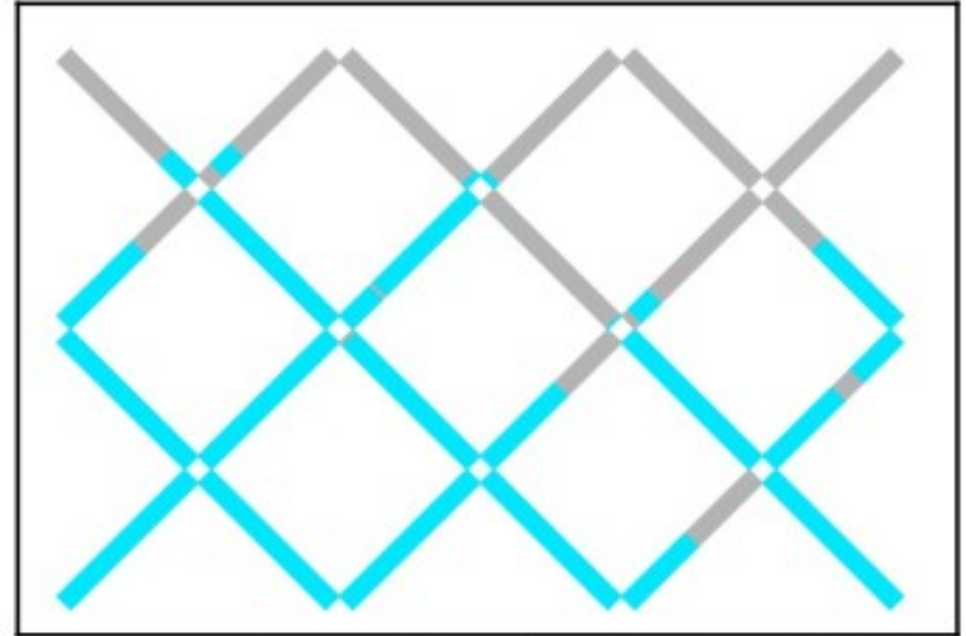
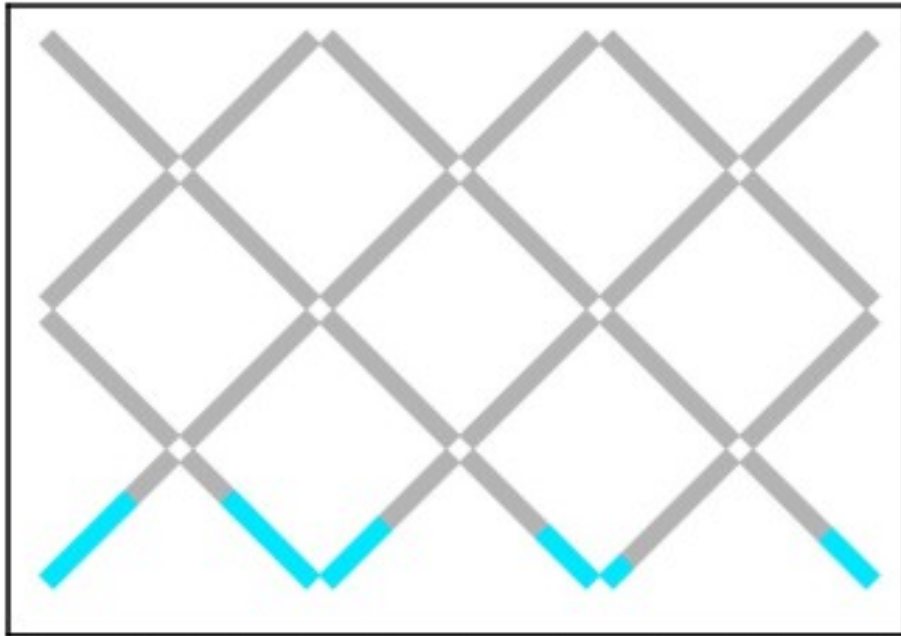
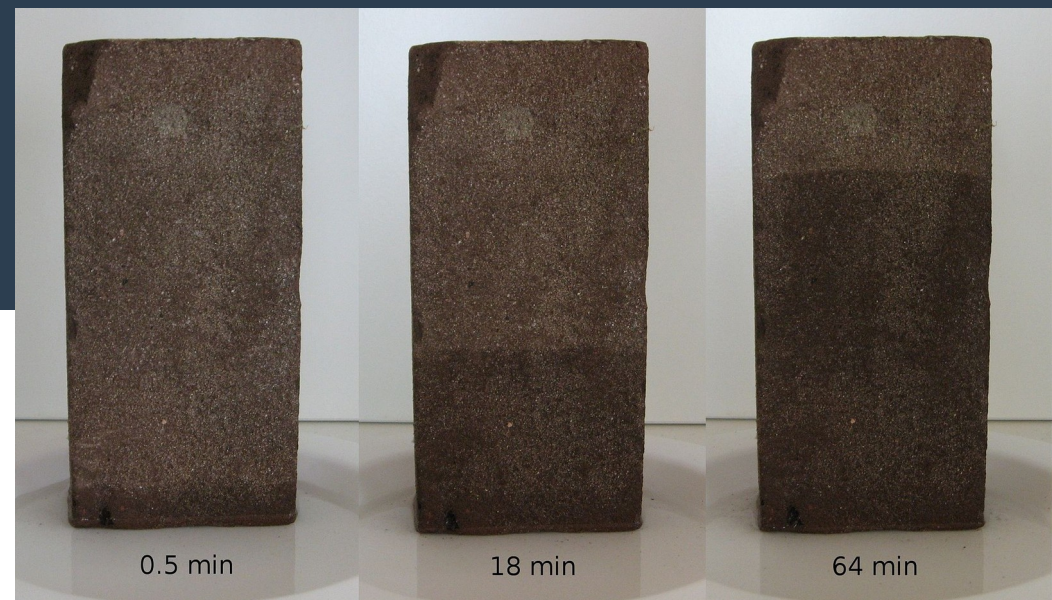
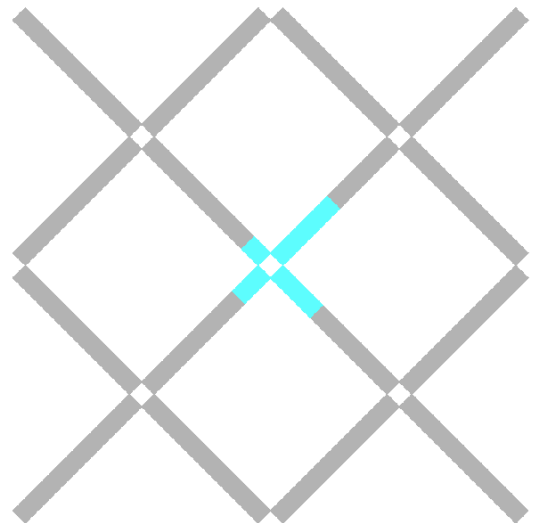
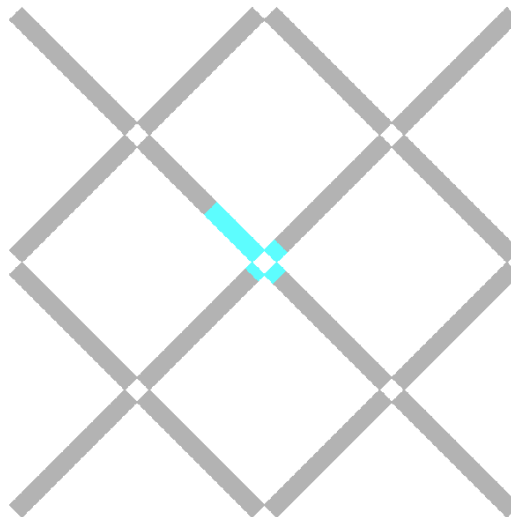
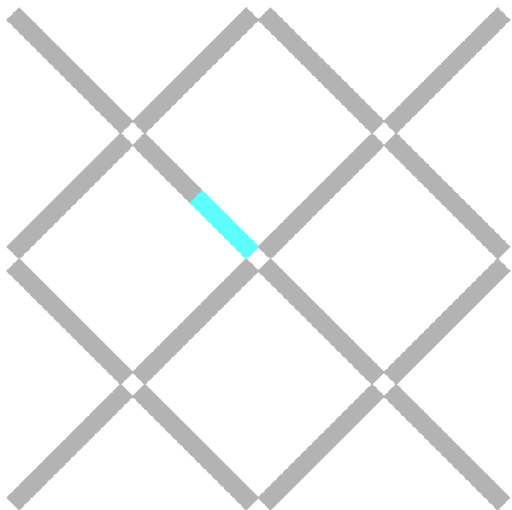
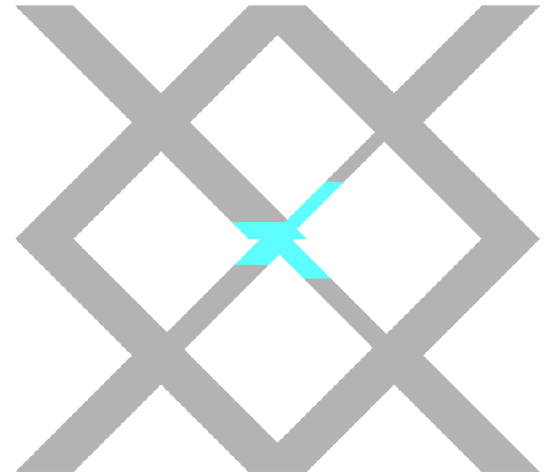
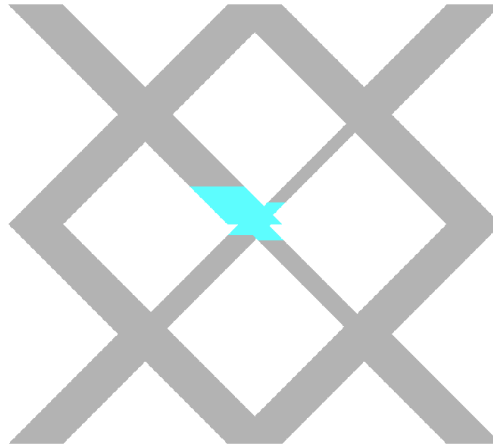
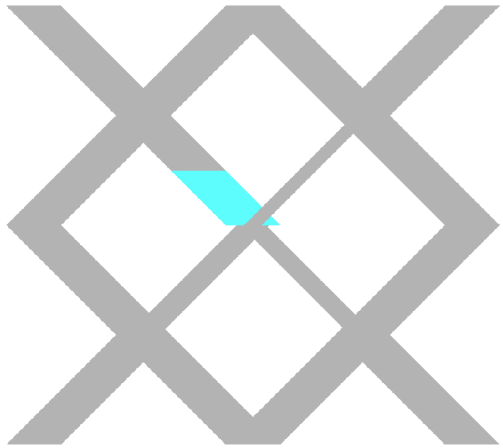


Figure 3 & 4: showing invasion of wetting fluid into a porous body consisting of randomly distributed tube radii for a highly simplified model (4 x 6), at saturation levels of 22% and 61% using our model.

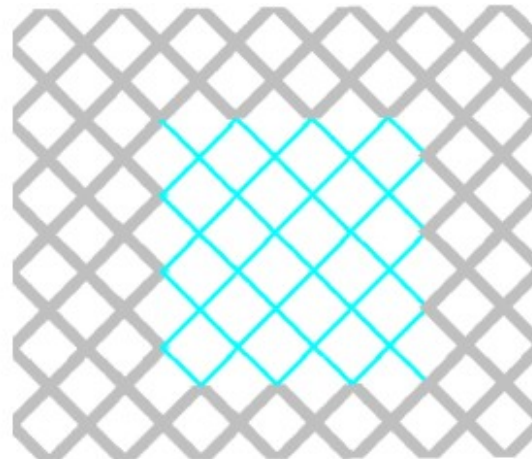


Test 2

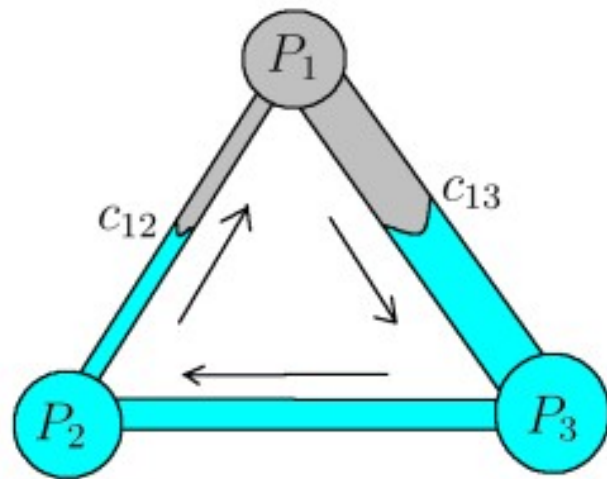


Main Problem: Model Imbibition

- **Closed Boundaries**
- $V_w = V_{nw}$
- $R_{out} = 3 R_{in}$
- **Measure $S(t)$**
- **Expected:**



Case of Infinitely many solutions



$$q_{ij} = k_{ij}\Delta P + c_{ij}$$

$$q_{12} = k_{12}(P_1 - P_2) + c_{12}$$

$$q_{13} = k_{13}(P_1 - P_3) + c_{13}$$

$$q_{12} + q_{13} = 0$$

$$(k_{12} + k_{13})P_1 - k_{12}P_2 - k_{13}P_3 = -c_{12}$$

$$\begin{pmatrix} (k_{12} + k_{13}) & -k_{12} & -k_{13} & -c_{12} - c_{13} \\ -k_{21} & (k_{21} + k_{23}) & -k_{23} & -c_{21} - c_{23} \\ -k_{31} & -k_{32} & (k_{31} + k_{32}) & -c_{31} - c_{32} \end{pmatrix}$$

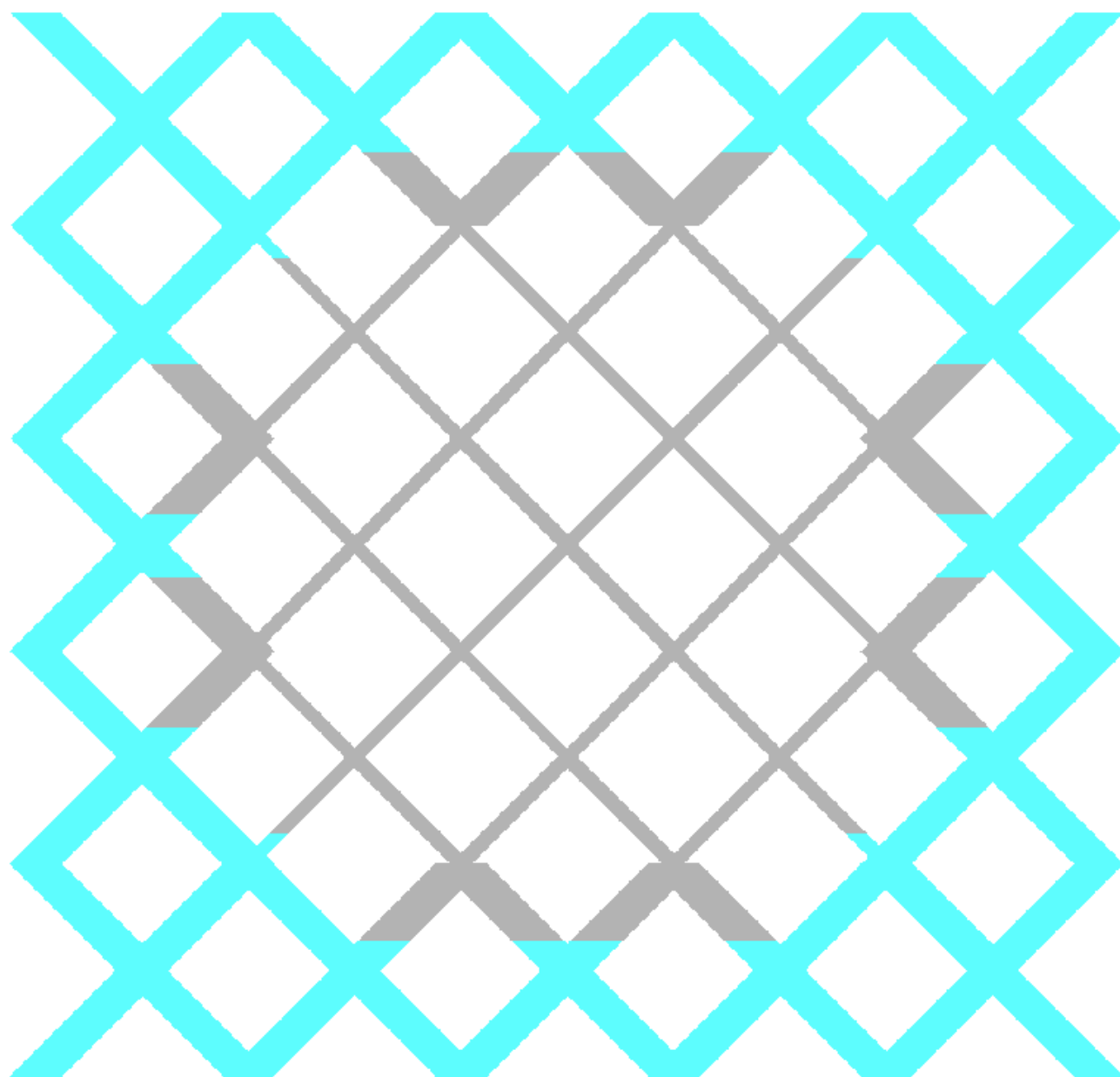
$$k_{ij} = k_{ji}$$

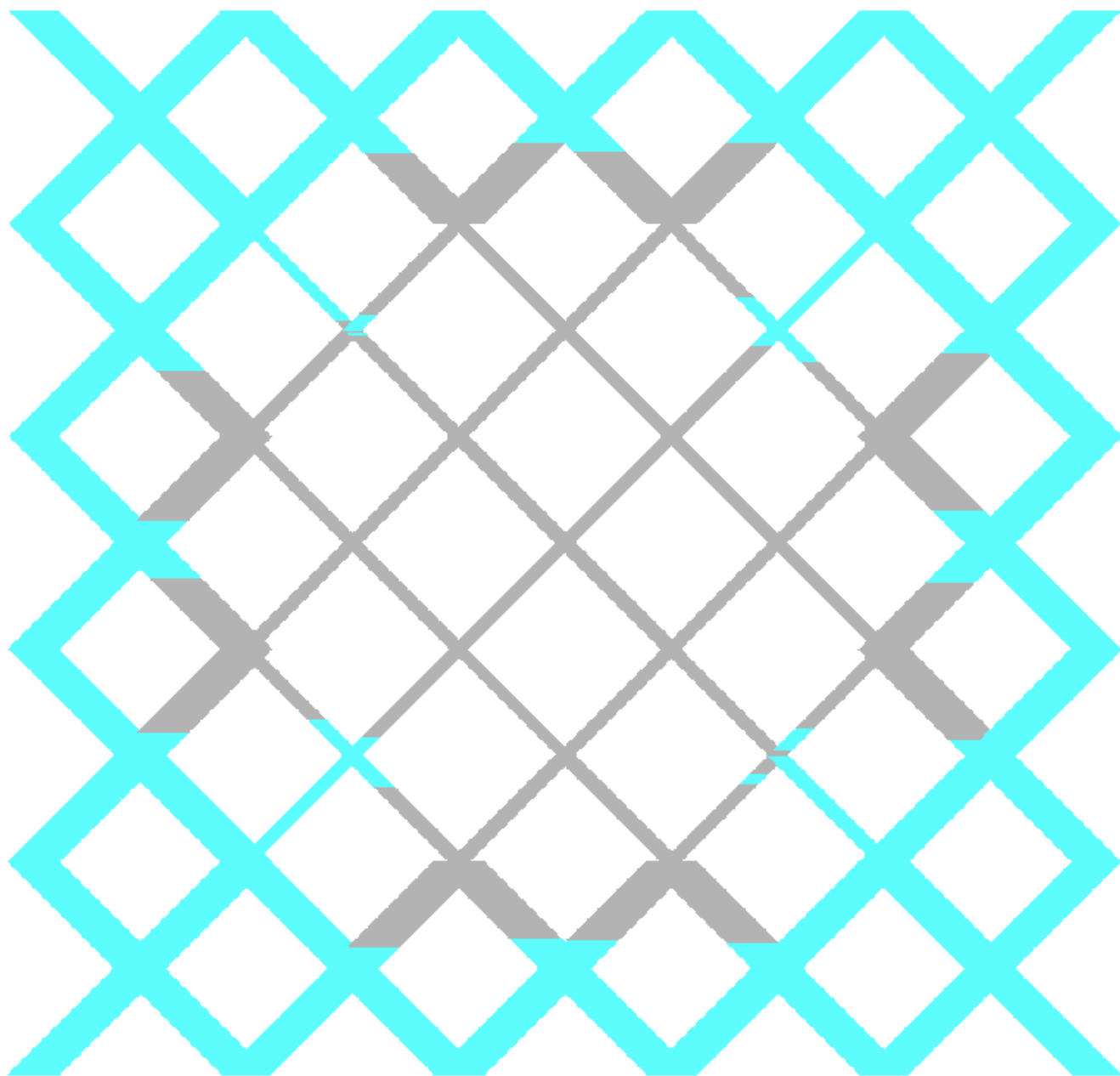
$$c_{ij} = -c_{ji}$$

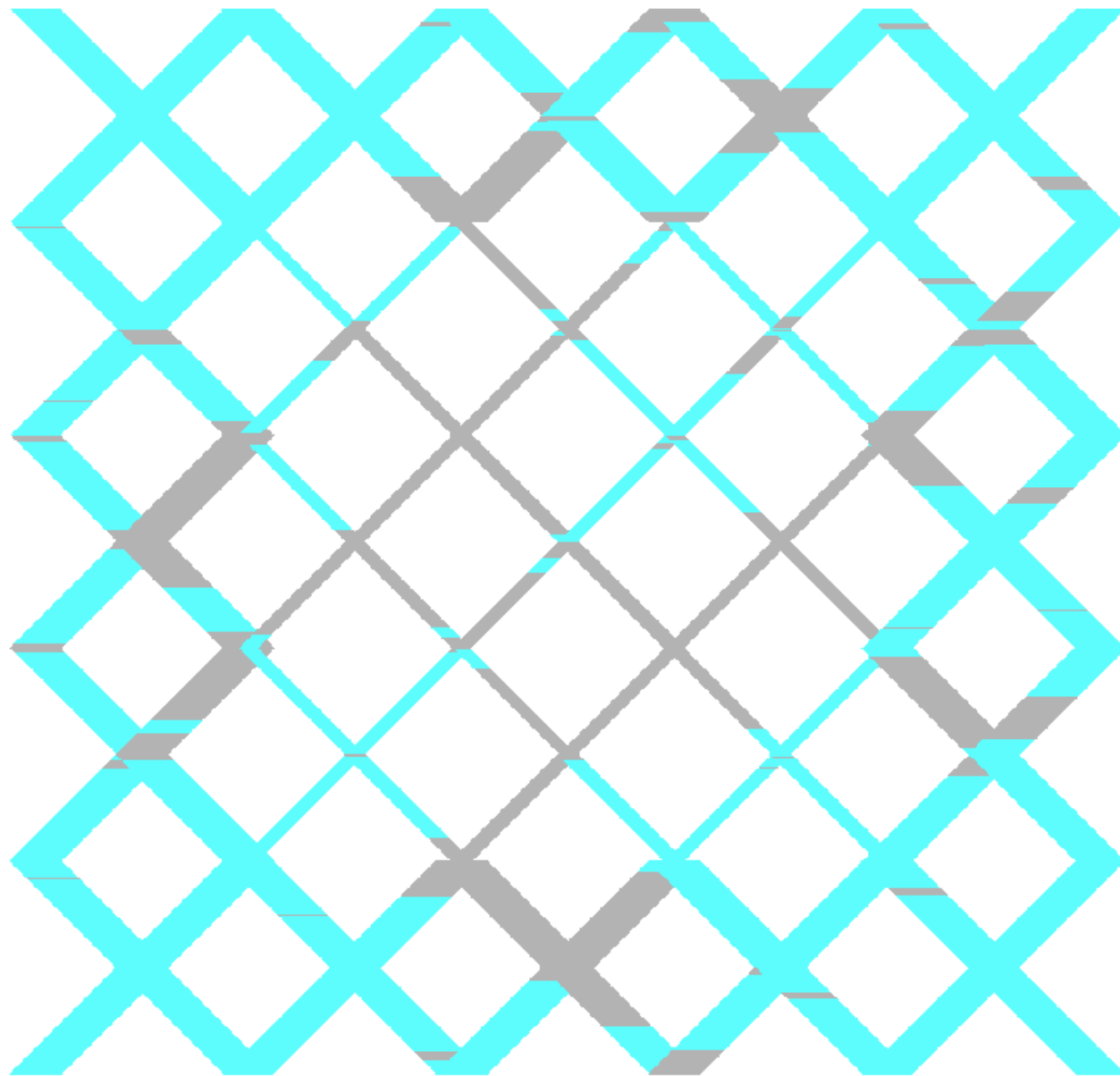
$$\begin{pmatrix} (k_{12} + k_{13}) & -k_{12} & -k_{13} & -c_{12} - c_{13} \\ -k_{21} & (k_{21} + k_{23}) & -k_{23} & -c_{21} - c_{23} \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} (k_{12} + k_{13}) & -k_{12} & -k_{13} + a & -c_{12} - c_{13} \\ -k_{21} & (k_{21} + k_{23}) & -k_{23} + a & -c_{21} - c_{23} \\ 0 & 0 & 3a & 0 \end{pmatrix}$$

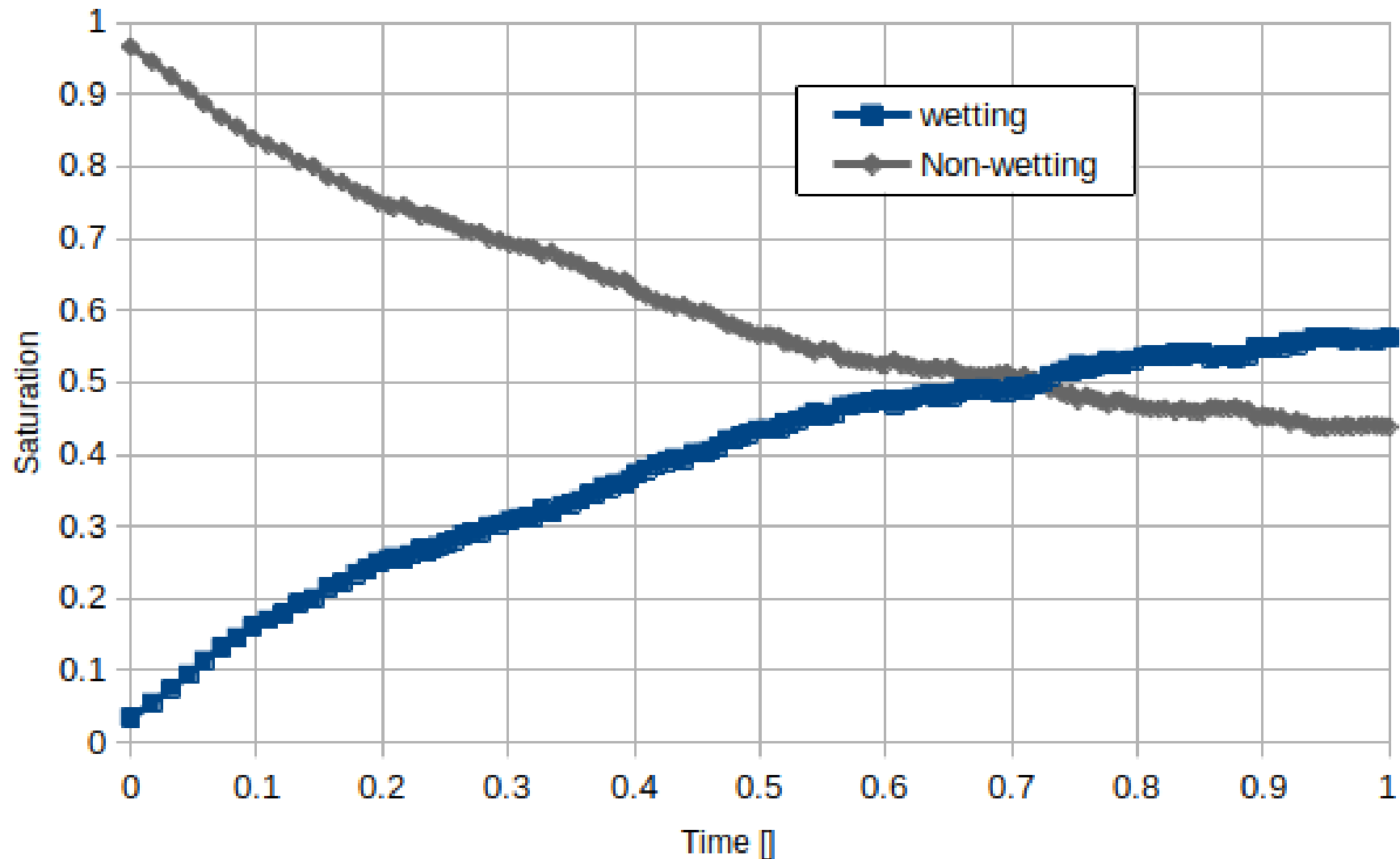
$$3aP_3 = 0$$



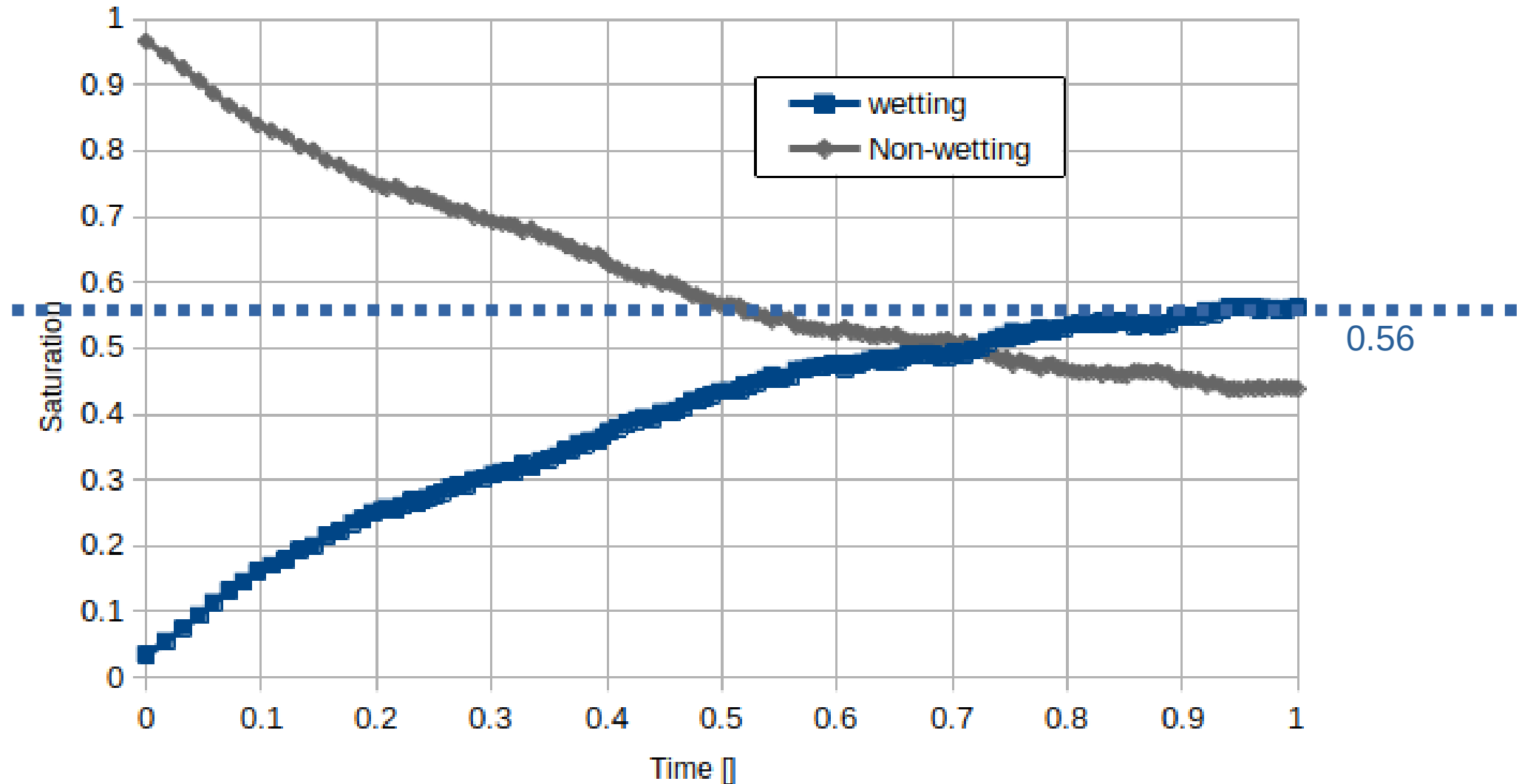


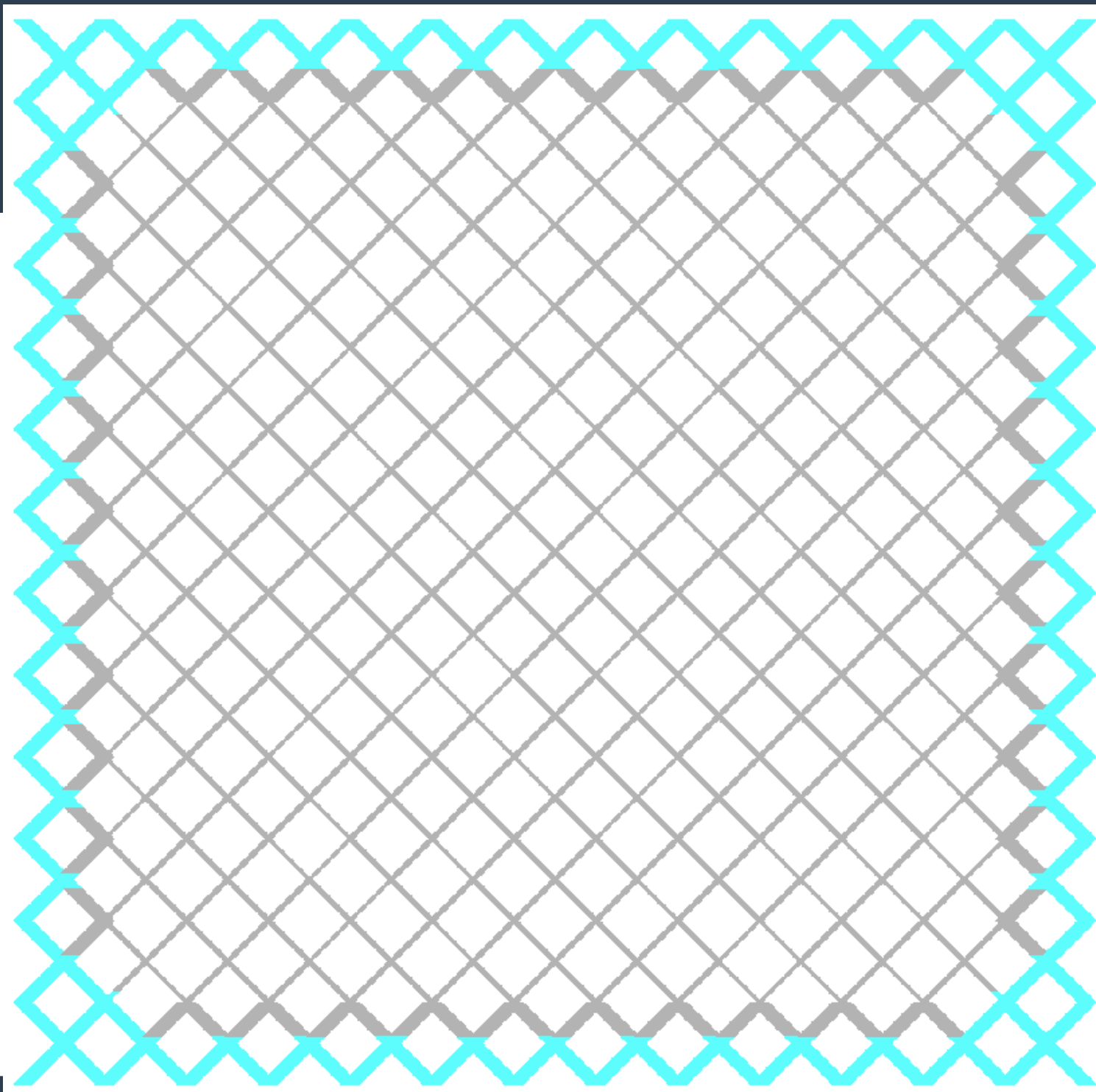


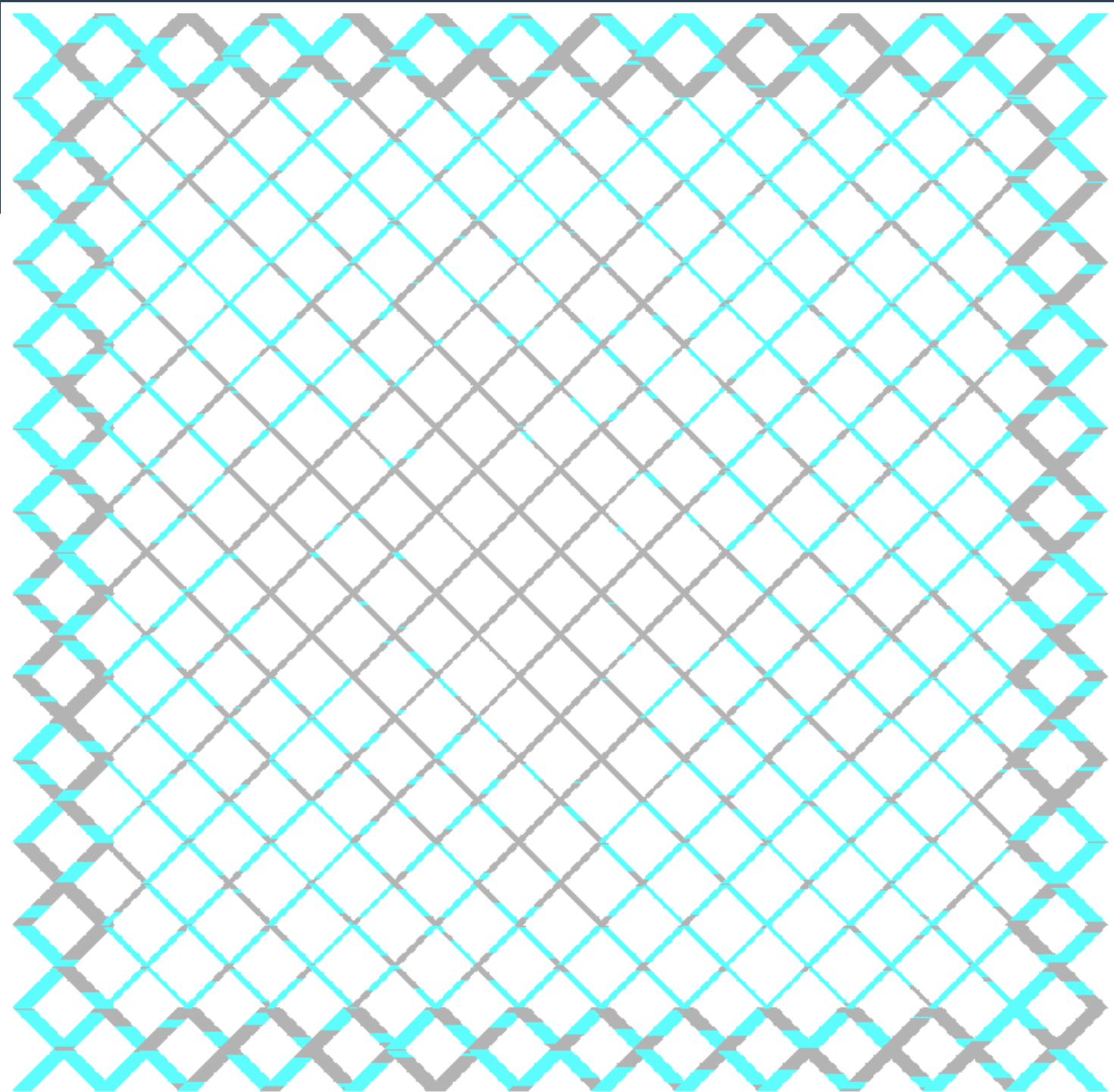
Result: Plot of Saturation 10x10



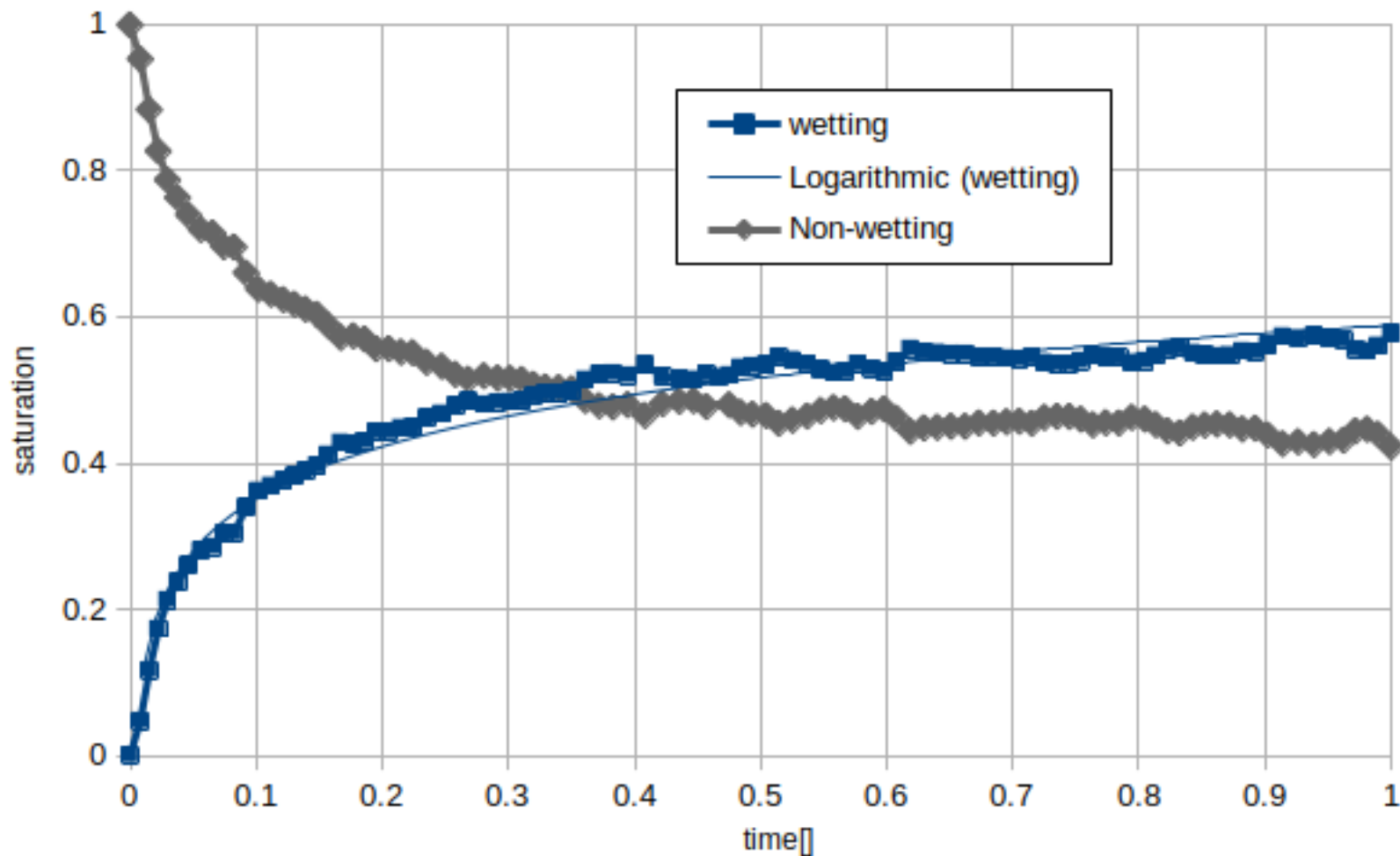
Result: Plot of Saturation 10x10



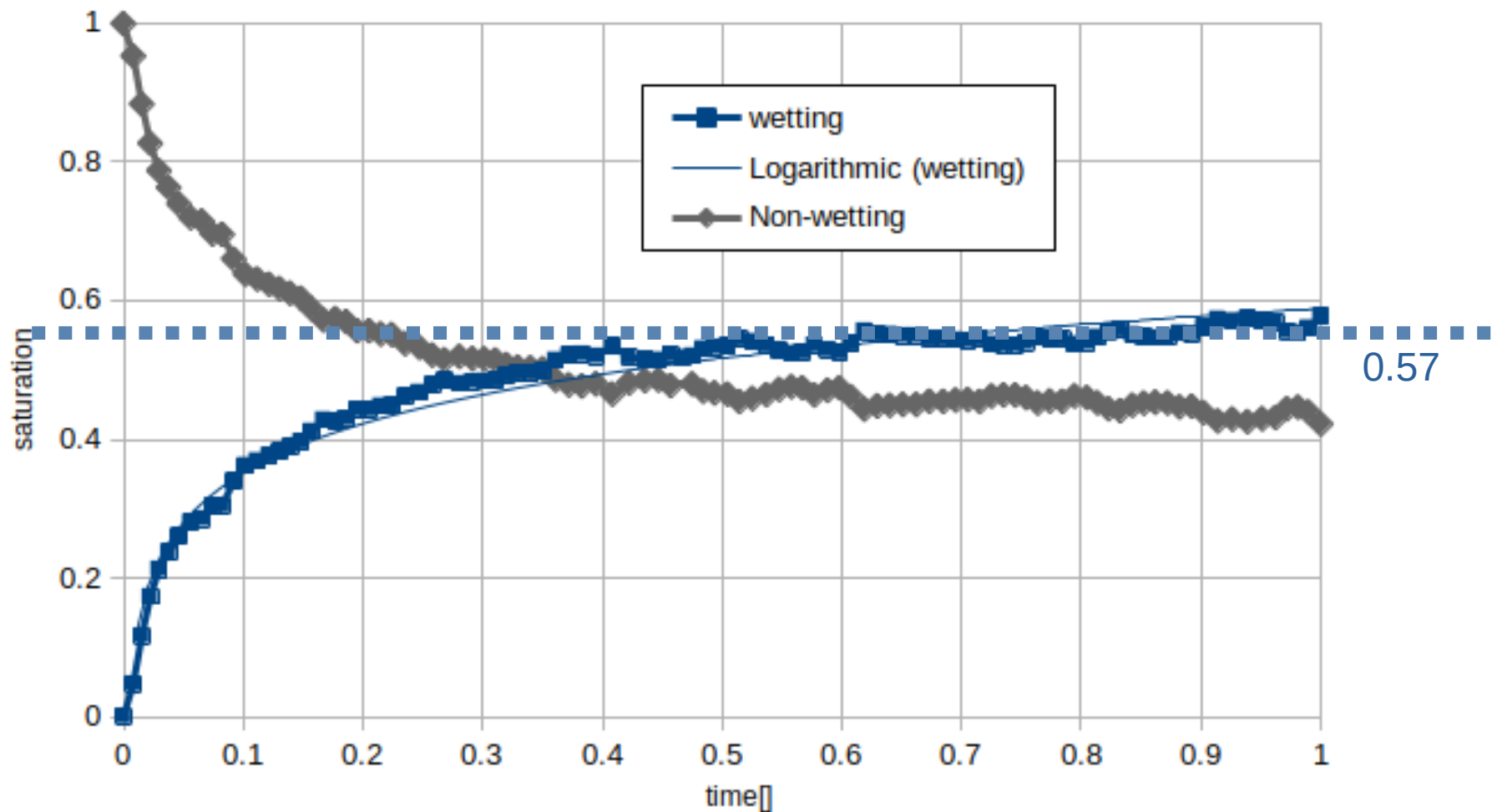




Result: Plot of Saturation 26x26



Result: Plot of Saturation 26x26



Conclusion

- The saturation \rightarrow equilibrium value.
- Explains relaxation phenomena in porous body.
- Imbibition observed.
- Method of distributing fluid such the wetting fluid first goes to the tube
- Modified Poiseuille equation
- Gaussian-elimination, more accurate than iterative methods.
- Conservation of volume for phases, high accuracy.
- To verify the Kondaurov model
- Determine the physical meaning of the non equilibrium parameter
- Scope of its applicability
- To be continued as a part of Master's thesis.



Main References

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Questions

