Mini Quiz (10 marks)

EE320

30 Aug. 2024

Name: ______________________ **Roll Number:** ______________________

1. Recall that the complex baseband signal of VSB modulated signal is given by $X_z(f) = 2M(f)G(f)$, where $M(f)$ is the spectrum of the real message signal (such as video) $m(t)$ and $G(f)$ is the transmit filter, whose spectrum is shown in Fig. 1. The receiver is also shown in Fig. 1. Plot the spectrum of the receive filter $H_r(f)$ such that the receiver works as intended for $f_c \gg 600$ kHz.

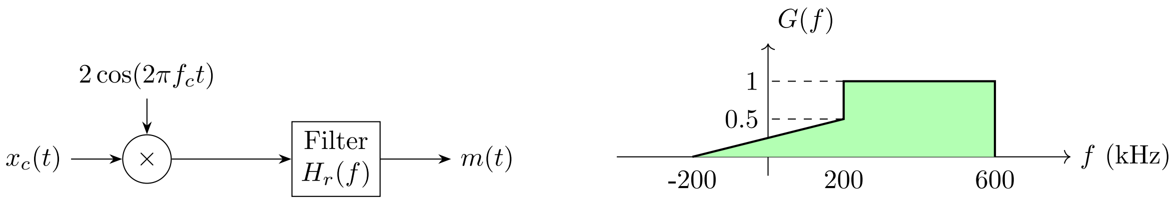
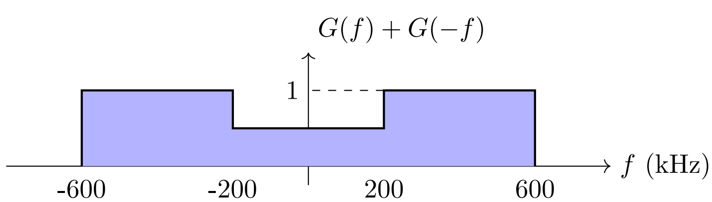


Figure 1: Receiver structure and spectrum of $G(f)$

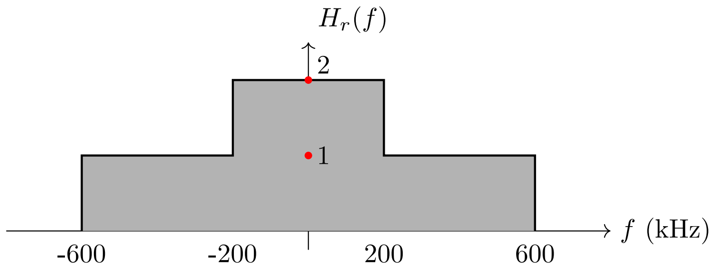
Solution: As seen from the lecture notes, the spectrum of the mixer output at the receiver is given by

$$2x_c(t)\cos(2\pi f_c t) \rightleftharpoons M(f-2f_c)G(f-2f_c) + M^\star(-f-2f_c)G(-f-2f_c) + M(f)\left[G(f)+G(-f)\right] \tag{1}$$

Firstly, $H_o(f)$ should reject all frequencies above 600 kHz, so that the first two terms can be dropped. Second observe that $G(f)+G(-f)$ as shown in the figure is one 1 in the required range of frequencies.

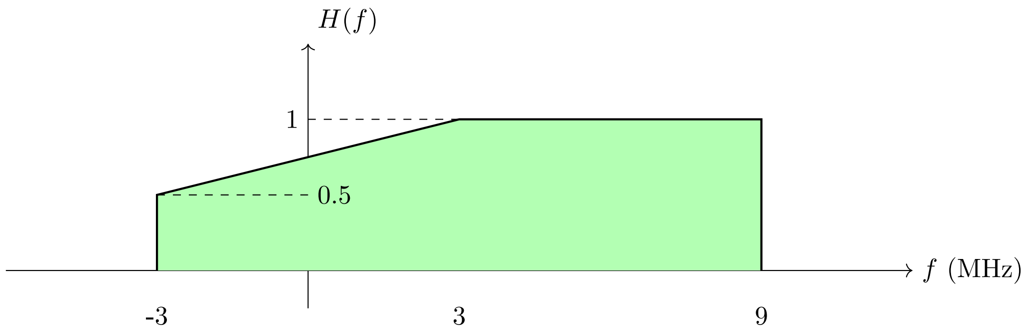


Therefore, to ensure that $m(t)$ is received without distortion at the receiver, we need to use the filter.



It can be seen that $M(f)[G(f)+G(-f)]H_r(f) = M(f)$ as required.

2. Consider that the complex baseband representation of a vestigial sideband modulated signal is defined as $X_z(f) = 2M(f)H(f)$, where $M(f)$ represents the spectrum of the real message signal $m(t)$ (e.g., video). Below, in Fig. 2, you will find the receiver setup and the spectral characteristics of the transmit filter $H(f)$. Your task is to sketch the spectrum of the receive filter $G(f)$ to ensure that the receiver functions correctly given that $f_c \gg 9$ MHz.



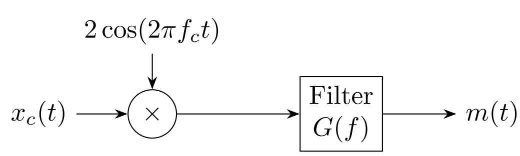
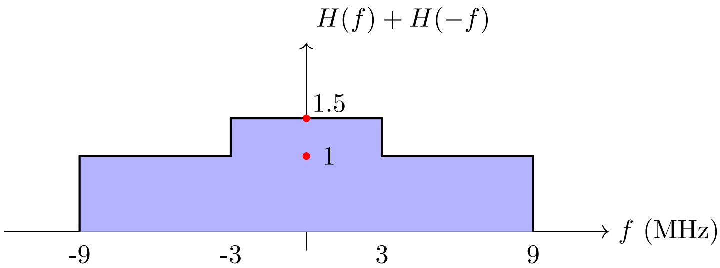


Figure 2: Frequency response $H(f)$ and receiver block diagram

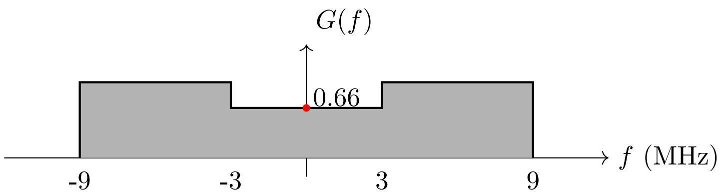
Solution: As seen from the lecture notes, the spectrum of the mixer output at the receiver is given by

$$2x_c(t)\cos(2\pi f_c t) \leftrightharpoons M(f-2f_c)H(f-2f_c) + M^{\star}(-f-2f_c)H(-f-2f_c) + M(f)\left[H(f)+H(-f)\right] \tag{2}$$

Firstly, $H_o(f)$ should reject all frequencies above 6 MHz, so that the first two terms can be dropped. Second observe that $H(f)+H(-f)$ as shown in the figure is not 1 in the required range of frequencies.



Therefore, to ensure that $m(t)$ is received without distortion at the receiver, we need to use the filter.



It can be seen that $M(f)[H(f) + H(-f)]G(f) = M(f)$ as required.

Note: Full marks if the y-axis is scaled by 2 or 1/2.

3. Recall that the complex baseband signal of VSB modulated signal is given by $X_z(f) = 2M(f)H(f)$, where $M(f)$ is the spectrum of the real message signal (such as video) $m(t)$ and $H(f)$ is the transmit filter with response

$$H(f) = \begin{cases} 0.5 & -20 \leq f \leq 0 \\ 1 & 0 \leq f \leq 40 \end{cases} \tag{3}$$

where f and the values are specified in MHz. The receiver is shown in Fig. 3. Plot the spectrum of the receive filter $V(f)$ such that the receiver works as intended for $f_c \gg 40$ MHz.

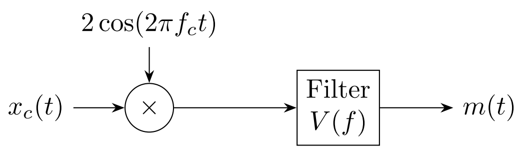
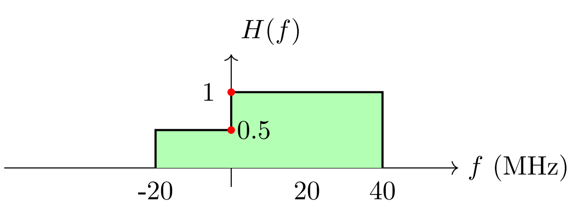


Figure 3: Receiver structure and spectrum of $H(f)$

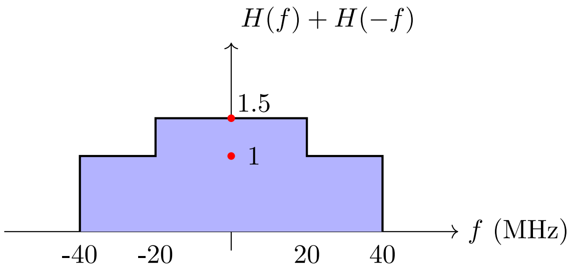
Solution: Let us plot the given $H(f)$



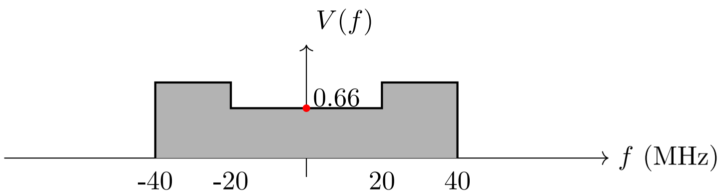
As seen from the lecture notes, the spectrum of the mixer output at the receiver is given by

$$2x_c(t)\cos(2\pi f_c t) \rightleftharpoons M(f-2f_c)H(f-2f_c) + M^\star(-f-2f_c)H(-f-2f_c) + M(f)\left[H(f)+H(-f)\right] \tag{4}$$

Firstly, $V(f)$ should reject all frequencies above 40 MHz, so that the first two terms can be dropped. Second observe that $H(f)+H(-f)$ as shown in the figure is not 1 in the required range of frequencies.



Therefore, to ensure that $m(t)$ is received without distortion at the receiver, we need to use the filter.



It can be seen that $M(f)[H(f) + H(-f)]V(f) = M(f)$ as required.

Note: Full marks if the y-axis is scaled by 2 or 1/2.

4. Recall that the complex baseband representation of VSB modulated signal is given by $X_z(f) = 2M(f)V(f)$, where $M(f)$ is the spectrum of the real message signal (such as video) $m(t)$ and $V(f)$ is the transmit filter, whose spectrum is shown in Fig. 4. At the receiver, the bandpass signal $x_c(t)$ is multiplied with $2\cos(2\pi f_c t)$ and passed through the filter with response $H_o(f)$. Plot the spectrum of the receive filter $H_o(f)$ such that the receiver works as intended for $f_c \gg 12$ MHz.

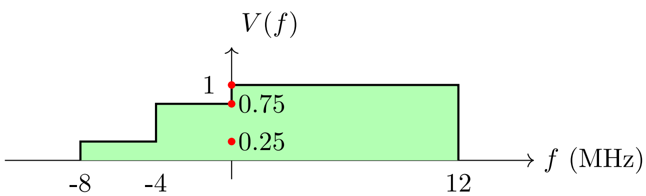
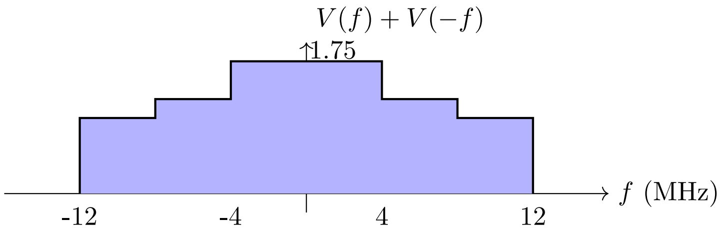


Figure 4: Spectrum of $V(f)$

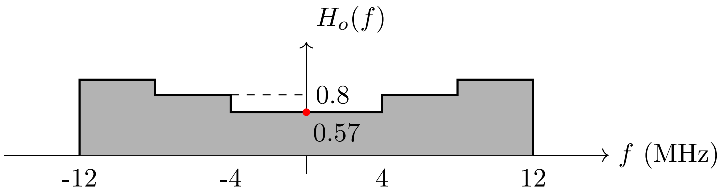
Solution: As seen from the lecture notes, the spectrum of the mixer output at the receiver is given by

$$2x_c(t)\cos(2\pi f_c t) \leftrightharpoons M(f-2f_c)V(f-2f_c) + M^\star(-f-2f_c)V(-f-2f_c) + M(f)\left[V(f)+V(-f)\right] \tag{5}$$

Firstly, $H_o(f)$ should reject all frequencies above 12 MHz, so that the first two terms can be dropped. Second observe that $V(f)+V(-f)$ as shown in the figure is not 1 in the required range of frequencies.



Therefore, to ensure that $m(t)$ is received without distortion at the receiver, we need to use the filter.



It can be seen that $M(f)[V(f)+V(-f)]H_o(f) = M(f)$ as required.

Note: Full marks if the y-axis is scaled by 2 or 1/2.

Mini Quiz (10 marks)

EE320

6 Sept. 2024

M	R (Mbps)	f_s (MHz)	α (%)	BW (MHz)	f_c (GHz)
8	3	1	25	1.25	2.4 (Bluetooth)
4	40	20	50	30	5.0 (Wi-Fi)
16	4	1	50	1.5	2.4 (Bluetooth)
64	120	20	25	25	5.0 (Wi-Fi)

Table 1: Table of M, R, f_s, α, and BW for communication system parameters.

1. How can we relate
   1. Data rate R measured in bits per second
   2. Sampling rate f_s measured in samples per second
   3. Modulation order M so that each symbol has $\log_2(M)$ bits per symbol
   4. Symbol duration T_s so that there are $\frac{1}{T_s}$ symbols per second
   5. Bandwidth BW at f_c and excess bandwidth α

Solution: There are following key points to consider:

1. Suppose we want to send R bits per second and modulation order is M. Then there are $\log_2(M)$ bits per symbol. Hence to achieve R bits per second, we need to send at the rate of $\frac{R}{\log_2(M)}$ symbols per second. Hence the symbol rate is $\frac{1}{T_s} = \frac{R}{\log_2(M)}$ and the symbol duration is $T_s = \frac{\log_2(M)}{R}$.
2. With a sampling rate of f_s we can only use it to sample or reconstruct a complex baseband signal with maximum frequency $f_s/2$. This is as per the Nyquist sampling theorem.
3. A complex signal with maximum frequency $f_s/2$, after application of modulation, would result in a real signal *around* f_c which would have a bandwidth of f_s. It would range from $f_c - f_s/2$ to $f_c + f_s/2$. This can be the case when we use reconstruction using sinc pulses.
4. When using sinc pulses spaced T_s apart, the reconstructed signal would be

$$\sum_n b[n]\text{sinc}(\frac{\pi}{T_s}(t - nT_s)) \leftrightharpoons T_s\Pi(fT_s)\sum_n b[n]e^{-j2\pi fnT_s} \tag{1}$$

   which occupies the range of frequencies $[-\frac{1}{2T_s}, \frac{1}{2T_s}]$. Since the maximum frequency is $\frac{1}{2T_s}$, the Nyquist sampling rate would be $f_s = \frac{1}{T_s}$. Hence by Nyquist criterion the sampling rate and symbol rate are the same. **Each sample carries 1 complex symbol.**
5. The Nyquist zero intersymbol interference criteria allows us to use a special types of pulses (such as raised cosine or trapezoidal) such that

- The signal would have an excess bandwidth of α and a total bandwidth of $(1+\alpha)f_s$ instead of f_s
- The symbol duration would still be same T_s and these pulses would satisfy Nyquist's zero ISI condition $p(mT_s) = 0$ for $m \neq 0$.
- The symbol rate would therefore be the same $\frac{1}{T_s} = f_s$ as in the case of sinc pulses.

6. Hence the total bandwidth of a signal using raised cosine or trapezoidal pulses with excess bandwidth of α would be $W = (1+\alpha)f_s$.

2. Bluetooth, a widely used wireless technology for short-range communication, operates in the 2.4 GHz ISM (Industrial, Scientific, and Medical) band. In Bluetooth communication, data transmission is designed to balance efficiency and reliability, utilizing a range of modulation schemes and bandwidth options. For this problem, consider a Bluetooth transmission operating at the center frequency of 2.4 GHz with a channel bandwidth of 1.25 MHz. The sampling rate is 1×10^6 samples per second and target data rate is 3 Mbps (i.e., 3×10^6 bits per second).

   (a) (2 points) What is the duration of a symbol in this transmission system? Express your answer in microseconds.

   Solution: Since the sampling rate is given to be $f_s = 1$ MHz, the symbol duration should be $Ts = \frac{1}{f_s} = 1$ microseconds.

   (b) (4 points) If a raised cosine pulse is used for pulse shaping to reduce inter-symbol interference, what would be the excess bandwidth required for this system?

   Solution: Using the formula BW $= (1+\alpha)f_s$, it follows that $\alpha = 25$ %.

   (c) (4 points) Referring to the available modulation constellations listed in Table 2, which constellation should be used? Justify your choice.

   Solution: Using the formula $R = f_s \log_2(M)$, we obtain $\log_2(M) = R/f_s = 3$. Therefore, we should use 8-PSK scheme.

M	Constellation Name
2	BPSK (Binary Phase Shift Keying)
4	QPSK (Quadrature Phase Shift Keying)
8	8-PSK (8-Phase Shift Keying)
16	16-QAM (16-Quadrature Amplitude Modulation)
64	64-QAM (64-Quadrature Amplitude Modulation)

Table 2: Constellation names corresponding to modulation order M.

3. Wi-Fi, operating in the 5 GHz band, is commonly used for high-speed wireless data communication. It utilizes various modulation schemes and channel bandwidths to achieve different data rates depending on the requirements of the application. For this problem, consider a Wi-Fi system operating in the 5 GHz band with a data rate of 40 Mbps (i.e., 40×10^6 bits per second) and channel bandwidth of 30 MHz. It is given that the system utilized trapezoidal pulses with excess bandwidth of 50%.

   (a) (5 points) Determine the sampling rate f_s of the system. Express your answer in samples per second.

Solution: Using the formula for bandwidth in a system with a raised cosine pulse:

$$BW = (1 + \alpha) f_s$$

Substituting $BW = 30$ MHz and $\alpha = 0.5$:

$$30\,\text{MHz} = (1 + 0.5) f_s$$

$$f_s = \frac{30}{1.5} = 20\text{Msps}$$

Therefore, the sampling rate is $f_s = 20 \times 10^6$ samples per second.

(b) (5 points) Referring to the available modulation constellations listed in Table 3, which constellation should be used? Justify your choice.

Solution: Using the formula for data rate R:

$$R = f_s \log_2(M)$$

Rearranging to solve for $\log_2(M)$:

$$\log_2(M) = \frac{R}{f_s} = \frac{40 \times 10^6}{20 \times 10^6} = 2$$

$$M = 2^1 = 4$$

Therefore, the modulation scheme used should be Quadrature Phase Shift Keying (QPSK), which corresponds to $M = 4$.

M	Constellation Name
2	BPSK (Binary Phase Shift Keying)
4	QPSK (Quadrature Phase Shift Keying)
8	8-PSK (8-Phase Shift Keying)
16	16-QAM (16-Quadrature Amplitude Modulation)
64	64-QAM (64-Quadrature Amplitude Modulation)

Table 3: Constellation names corresponding to modulation order M.

4. Bluetooth, a widely used wireless technology for short-range communication, operates in the 2.4 GHz ISM (Industrial, Scientific, and Medical) band. In Bluetooth communication, data transmission is designed to balance efficiency and reliability, utilizing a range of modulation schemes and bandwidth options. In this problem, we consider a Bluetooth transmission system using 16-QAM modulation ($M = 16$) and a sampling rate of 1×10^6 samples per second. The system uses raised cosine pulses with excess bandwidth of 50%.

(a) (2 points) What is the duration of a symbol in this transmission system? Express your answer in microseconds.

Solution: Since the sampling rate is given to be 1×10^6 samples per second, the symbol duration should be $Ts = \frac{1}{f_s} = 1$ microsecond.

(b) (4 points) Determine the bandwidth of the system. Express you answer in MHz.

Solution: Using the formula for bandwidth with a raised cosine pulse:

$$BW = (1 + \alpha) f_s$$

Substituting $\alpha = 0.5$ and $f_s = 1$ MHz:

$$BW = (1 + 0.5) \times 1\,\text{MHz} = 1.5\,\text{MHz}$$

Therefore, the bandwidth $BW = 1.5\,\text{MHz}$.

(c) (4 points) Determine the data rate of the system. Express your answer in Mbps ($= 10^6$ bps).

Solution: Using the formula for data rate R:

$$R = f_s \log_2(M)$$

Substituting $f_s = 1\,\text{MHz}$ and $M = 16$:

$$R = 1\,\text{MHz} \times \log_2(16) = 1\,\text{MHz} \times 4 = 4\,\text{Mbps}$$

Therefore, the data rate $R = 4\,\text{Mbps}$.

5. Wi-Fi systems in the 5 GHz band are known for their high data rates, suitable for modern wireless applications. It utilizes various modulation schemes and channel bandwidths to achieve different data rates depending on the requirements of the application. For this problem, consider a Wi-Fi system operating in the 5 GHz band with a data rate of 120 Mbps (i.e., 120×10^6 bits per second) and sampling rate of 20×10^6 samples per second. The system utilizes trapezoidal pulses with excess bandwidth of 25%.

(a) (2 points) What is the duration of a symbol in this transmission system? Express your answer in microseconds.

Solution: Since the sampling rate is given to be 1×10^6 samples per second, the symbol duration should be $Ts = \frac{1}{f_s} = 0.05$ microseconds.

(b) (4 points) Determine the bandwidth of the system. Express you answer in MHz.

Solution: Using the formula for bandwidth with a raised cosine pulse:

$$BW = (1 + \alpha) f_s$$

Substituting $\alpha = 0.25$ and $f_s = 20\,\text{MHz}$:

$$BW = (1 + 0.25) \times 20\,\text{MHz} = 25\,\text{MHz}$$

Therefore, the bandwidth $BW = 25\,\text{MHz}$.

(c) (4 points) Referring to the available modulation constellations listed in Table 4, which constellation should be used? Justify your choice.

Solution: Using the formula for data rate R:

$$R = f_s \log_2(M)$$

Rearranging to solve for $\log_2(M)$:

$$\log_2(M) = \frac{R}{f_s} = \frac{120 \times 10^6}{20 \times 10^6} = 6$$

$$M = 2^6 = 64$$

Therefore, the modulation scheme used should be 64-QAM (Quadrature Amplitude Modulation), which corresponds to $M = 64$.

M	Constellation Name
2	BPSK (Binary Phase Shift Keying)
4	QPSK (Quadrature Phase Shift Keying)
8	8-PSK (8-Phase Shift Keying)
16	16-QAM (16-Quadrature Amplitude Modulation)
64	64-QAM (64-Quadrature Amplitude Modulation)

Table 4: Constellation names corresponding to modulation order M.

EE320 Mini Quiz Set A
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1. Consider a communication channel
$$Y = X + N$$
where X takes values -2 and 1 for bit 0 and 1 respectively. N is a Gaussian random variable with mean 0. However, N's variance depends on X. In particular, N's variance is 4 and 9 for bit 0 and 1 respectively. Consider the following decoding
$$\begin{aligned}&\text{Decode bit } = 1 \text{ if } X > 1.5\\ &\text{else decode } 0\end{aligned}$$
What will be the probability of error? Assume that bit 1 and bit 0 are transmitted with equal probabilities. You can express the answer in terms of Q function.

Solution:

The probability of error for the symbol 1 is

$$P_{e|1} = \mathbb{P}[Y < 1.5|X = 1] = \mathbb{P}[X + N < 1.5|X = 1] = \mathbb{P}[N < 0.5|X = 1] = 1 - Q(0.5/\sqrt{9}) = 1 - Q(1/6)$$

The probability of error for the symbol -2 is

$$P_{e|0} = \mathbb{P}[Y > 1.5|X = -2] = \mathbb{P}[X + N > 1.5|X = -2] = \mathbb{P}[N > 3.5|X = -2] = Q(3.5/\sqrt{4}) = Q(7/4)$$

Total error is
$$P_e = \frac{1}{2}(P_{e|0} + P_{e|1})$$

EE320 Mini Quiz Set B
13 Sept 2024

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1. Consider a communication channel

$$Y = X + N$$

where X takes values -10 and 6 for bit 0 and 1 respectively. N is a Gaussian random variable with mean 0. However, N's variance depends on X. In particular, N's variance is 2 and 9 for bit 0 and 1 respectively. Consider the following decoding

$$\begin{aligned} &\text{Decode bit } = 1 \text{ if } X > 0 \\ &\text{else decode } 0 \end{aligned}$$

What will be the probability of error? Assume that bit 1 and bit 0 are transmitted with equal probabilities. You can express the answer in terms of Q function.

Solutions
The probability of error for the symbol 6 is

$$P_{e|1} = \mathbb{P}[Y < 0|X = 6] = \mathbb{P}[X + N < 0|X = 6] = \mathbb{P}[N < -6|X = 6] = Q(6/\sqrt{9}) = Q(2)$$

The probability of error for the symbol -10 is

$$P_{e|0} = \mathbb{P}[Y > 0|X = -10] = \mathbb{P}[X + N > 0|X = -10] = \mathbb{P}[N > 10|X = -10] = Q(10/\sqrt{2}) = Q(7.07)$$

Total error is

$$P_e = \frac{1}{2}(P_{e|0} + P_{e|1})$$

1. Consider a communication channel

$$Y = X + N$$

where X takes values 0 and 10 for bit 0 and 1 respectively. N is a Gaussian random variable with mean 0. However, N's variance depends on X. In particular, N's variance is 5 and 10 for bit 0 and 1 respectively. Consider the following decoding

$$\begin{aligned}&\text{Decode bit } = 1 \text{ if } X > 3\\&\text{else decode } 0\end{aligned}$$

What will be the probability of error? Assume that bit 1 and bit 0 are transmitted with equal probabilities. You can express the answer in terms of Q function.

Solutions

The probability of error for the symbol 10 is

$$P_{e|1} = \mathbb{P}[Y < 3|X = 10] = \mathbb{P}[X + N < 3|X = 10] = \mathbb{P}[N < -7|X = 10] = Q(7/\sqrt{10}) = Q(2.21)$$

The probability of error for the symbol 0 is

$$P_{e|0} = \mathbb{P}[Y > 3|X = 0] = \mathbb{P}[X + N > 3|X = 0] = \mathbb{P}[N > 3|X = 0] = Q(3/\sqrt{5}) = Q(1.34)$$

Total error is

$$P_e = \frac{1}{2}(P_{e|0} + P_{e|1})$$

EE320 Mini Quiz Set D
13 Sept 2024

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1. Consider a communication channel

$$Y = X + N$$

where X takes values -5 and 5 for bit 0 and 1 respectively. N is a Gaussian random variable with mean 0. However, N's variance depends on X. In particular, N's variance is 4 and 9 for bit 0 and 1 respectively. Consider the following decoding

$$\text{Decode bit } = 1 \text{ if } X > 1.5$$
$$\text{else decode } 0$$

What will be the probability of error? Assume that bit 1 and bit 0 are transmitted with equal probabilities. You can express the answer in terms of Q function.

Solutions
The probability of error for the symbol 5 is

$$P_{e|1} = \mathbb{P}[Y < 1.5|X = 5] = \mathbb{P}[X + N < 1.5|X = 5] = \mathbb{P}[N < -3.5|X = 5] = Q(3.5/\sqrt{9}) = Q(7/6)$$

The probability of error for the symbol -5 is

$$P_{e|0} = \mathbb{P}[Y > 1.5|X = -5] = \mathbb{P}[X + N > 1.5|X = -5] = \mathbb{P}[N > 6.5|X = -5] = Q(6.5/\sqrt{4}) = Q(13/4)$$

Total error is

$$P_e = \frac{1}{2}(P_{e|0} + P_{e|1})$$

EE320 Mini Quiz Set A
27 Sept 2024

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1. Consider a communication channel

$$Y = X + N$$

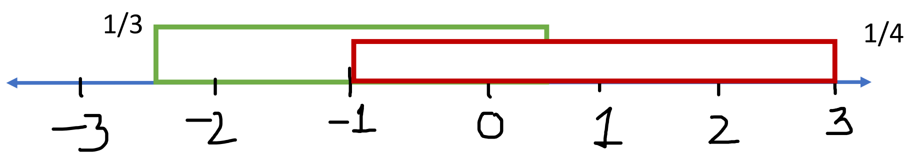
where X takes values -1 and 1 for bit 0 and 1 respectively. N is a uniform random variable. However, N's distribution depends on X. In particular, N is a uniform random variable between $(-2, 2)$ when X is 1. Further, N is a uniform random variable between $(-1.5, 1.5)$ when X is -1.

   (a) Compute and sketch the likelihood of Y for both values of X.

   (b) Derive the ML decoding rule.

Solution:

(a)



(b) Decoding rule

$$\begin{aligned} \text{Decode bit } &= 1 \text{ if } 3 > X > 0.5 \\ \text{Decode bit } &= 0 \text{ if } -2.5 < X < 0.5 \end{aligned}$$

1. Consider a communication channel

$$Y = X + N$$

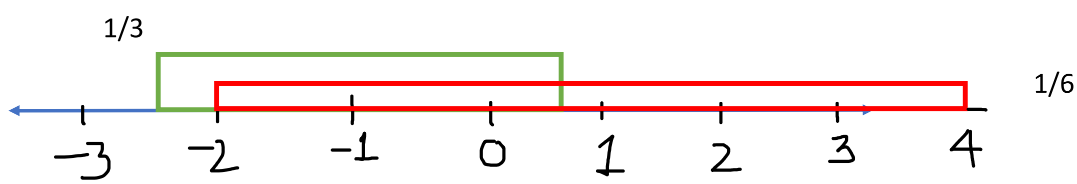
where X takes values -1 and 1 for bit 0 and 1 respectively. N is a uniform random variable. However, N's distribution depends on X. In particular, N is a uniform random variable between $(-3, 3)$ when X is 1. Further, N is a uniform random variable between $(-1.5, 1.5)$ when X is -1.

   (a) Compute and sketch the likelihood of Y for both values of X.

   (b) Derive the ML decoding rule.

Solution:

(a)



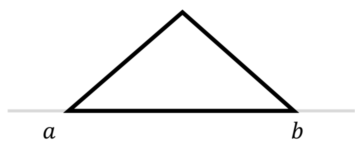
(b) Decoding rule Decoding rule

$$\begin{aligned} \text{Decode bit } &= 1 \text{ if } 0.5 < X < 4 \\ \text{Decode bit } &= 0 \text{ if } -2.5 < X < 0.5 \end{aligned}$$

1. Consider a communication channel

$$Y = X + N$$

where X takes values -1 and 1 for bit 0 and 1 respectively. N is a random variable with isosceles-triangular-shaped PDF spread between a and b as shown below.



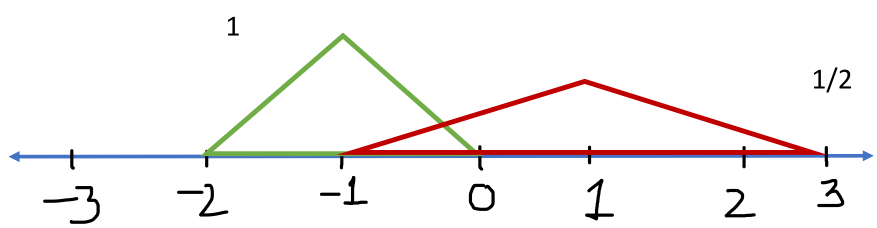
However, N's distribution depends on X. In particular, N spreads between $(a, b) = (-2, 2)$ when X is 1. Further, N spreads between $(a, b) = (-1, 1)$ when X is -1.

(a) Compute and sketch the likelihood of Y for both values of X.

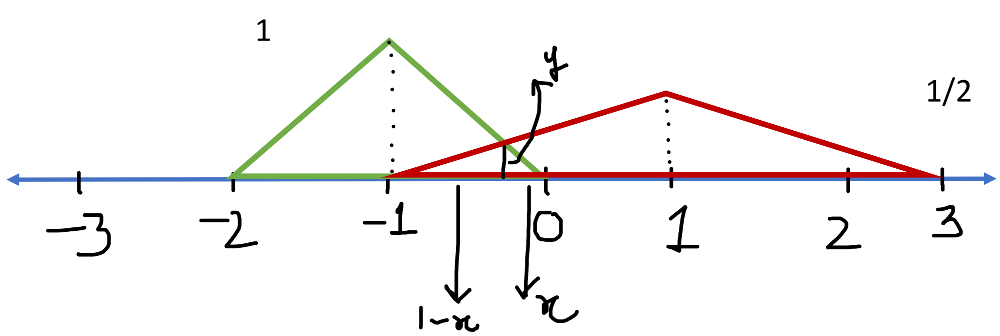
(b) Derive the ML decoding rule.

Solution:

(a)



(b) Decoding rule



to compute intersection point, we notice

$$\frac{y}{x} = \frac{1}{1} \implies y = x$$

$$\frac{y}{1-x} = \frac{1}{4} \implies 4y = 1 - x \implies y = \frac{1}{3}$$

$$\text{Decode bit } = 1 \text{ if } \frac{1}{3} < X < 3$$
$$\text{Decode bit } = 0 \text{ if } -2 < X < \frac{1}{3}$$

---

1. Consider a communication channel

$$Y = X + N$$

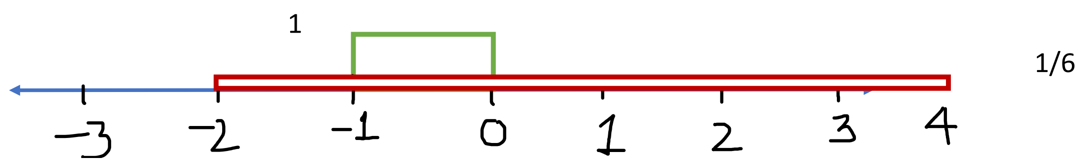
where X takes values -1 and 1 for bit 0 and 1 respectively. N is a uniform random variable. However, N's distribution depends on X. In particular, N is a uniform random variable between $(-3, 3)$ when X is 1. Further, N is a uniform random variable between $(0, 1)$ when X is -1.

   (a) Compute and sketch the likelihood of Y for both values of X.

   (b) Derive the ML decoding rule.

Solution:

(a)



(b) Decoding rule

$$\begin{aligned}\text{Decode bit } &= 1 \text{ if } 0 < X < 4 \text{ OR} - 2 < X < -1\\ \text{Decode bit } &= 0 \text{ if } -1 < X < 0\end{aligned}$$

EE320 Mini Quiz Set A
4 Oct 2024

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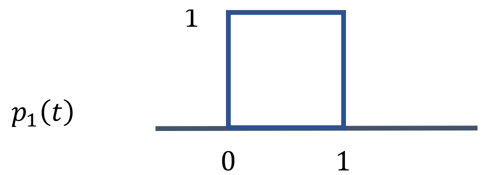
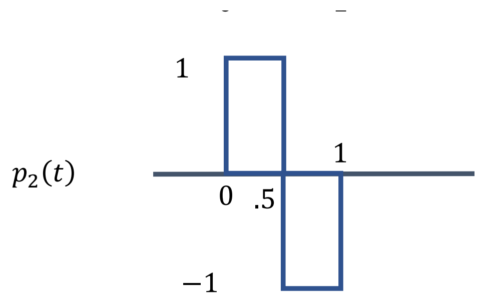
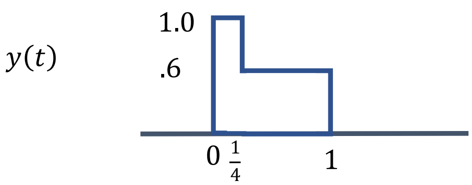
1. Let p_1 and p_2 are two pulses given below:





The receiver receives the signal y(t).



Compute the $y(t)$ component in the direction of $p_1(t)$ and $p_2(t)$.

Solution:

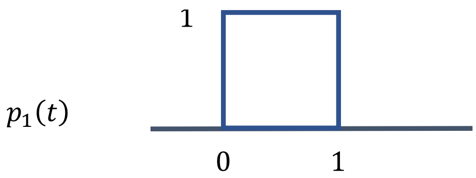
The $y(t)$ component in the direction of $p_1(t)$ is

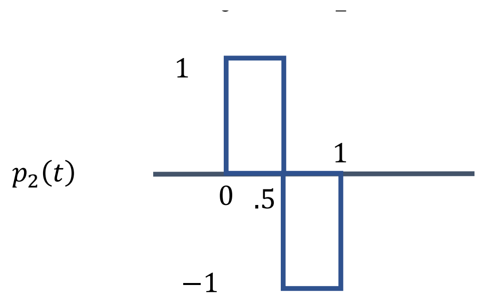
$$y_1 = \int y(t)p_1(t)dt = 1 \times \frac{1}{4} + 0.6 \times \frac{3}{4} = \frac{2.8}{4} = 0.7 \tag{1}$$

The $y(t)$ component in the direction of $p_2(t)$ is

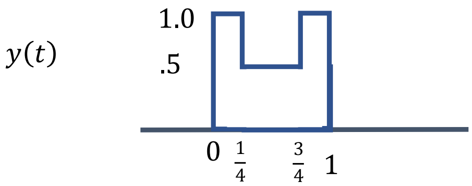
$$y_2 = \int y(t)p_2(t)dt = 1 \times \frac{1}{4} + 0.6 \times \frac{1}{4} - 0.6 \times \frac{1}{2} = \frac{.4}{4} = 0.1 \tag{2}$$

1. Let p_1 and p_2 are two pulses given below:





The receiver receives the signal y(t).



Compute the $y(t)$ component in the direction of $p_1(t)$ and $p_2(t)$.

Solution:

The $y(t)$ component in the direction of $p_1(t)$ is

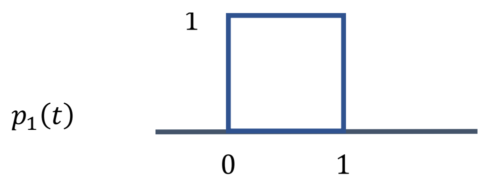
$$y_1 = \int y(t)p_1(t)dt = 1 \times \frac{1}{4} + .5 \times \frac{2}{4} + 1 \times \frac{1}{4} = \frac{3}{4} = 0.75 \tag{3}$$

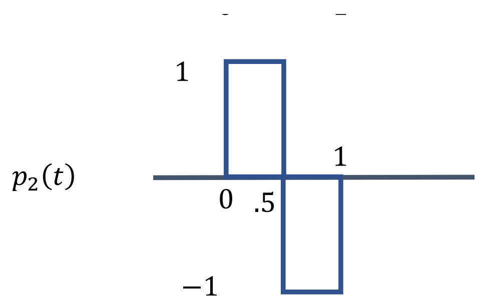
The $y(t)$ component in the direction of $p_2(t)$ is

$$y_2 = \int y(t)p_2(t)dt = 1 \times \frac{1}{4} + 0.5 \times \frac{1}{4} - 0.5 \times \frac{1}{4} - 1 \times \frac{1}{4} = 0 \tag{4}$$

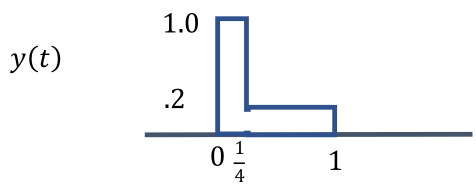
---

1. Let p_1 and p_2 are two pulses given below:





The receiver receives the signal y(t).



Compute the $y(t)$ component in the direction of $p_1(t)$ and $p_2(t)$.

Solution:

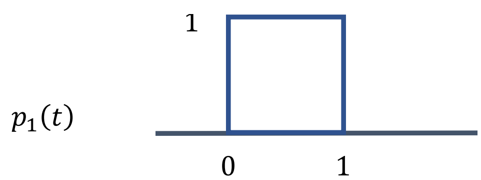
The $y(t)$ component in the direction of $p_1(t)$ is

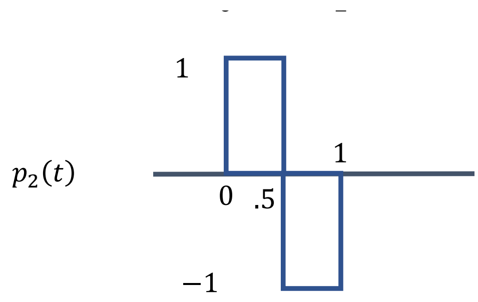
$$y_1 = \int y(t)p_1(t)dt = 1 \times \frac{1}{4} + .2 \times \frac{3}{4} = \frac{1.6}{4} = 0.4 \tag{5}$$

The $y(t)$ component in the direction of $p_2(t)$ is

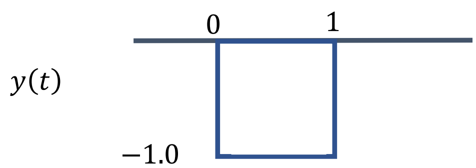
$$y_2 = \int y(t)p_2(t)dt = 1 \times \frac{1}{4} + 0.2 \times \frac{1}{4} - 0.2 \times \frac{2}{4} = \frac{0.8}{4} = 0.2 \tag{6}$$

1. Let p_1 and p_2 are two pulses given below:





The receiver receives the signal y(t).



Compute the $y(t)$ component in the direction of $p_1(t)$ and $p_2(t)$.

Solution:

The $y(t)$ component in the direction of $p_1(t)$ is

$$y_1 = \int y(t)p_1(t)dt = -1 \times 1 = -1 \tag{7}$$

The $y(t)$ component in the direction of $p_2(t)$ is

$$y_2 = \int y(t)p_2(t)dt = -1 \times \frac{1}{2} + 1 \times \frac{1}{2} = 0 \tag{8}$$