CMT107 Visual Computing

X.1 Camera Calibration

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Overview

- Cameras
- Pinhole Cameras
  - Vanishing points
- Real Camera
  - Aperture adjustment
  - Thin lens formula
  - Lens flaws
- Pinhole Camera Model
  - Camera parameters
    - Intrinsic parameters
    - Extrinsic parameters
- Camera Calibration
  - Linear method

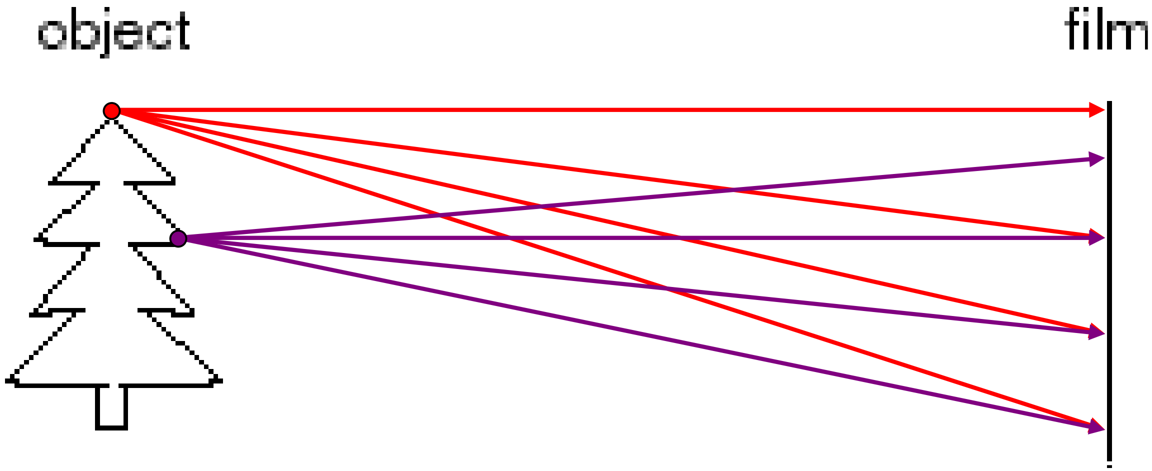
Acknowledgement

The majority of the slides in this section are from Svetlana Lazebnik at University of Illinois at Urbana-Champaign

Cameras

Source: D. Lowe

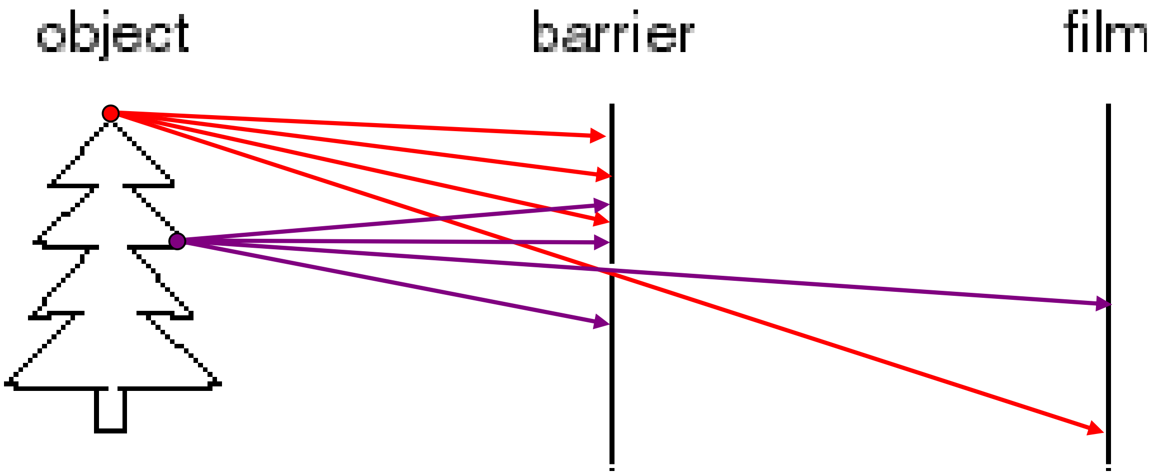
Let’s Design a Camera



- Idea 1: put a piece of film in front of an object
- Do we get a reasonable image?

Slide by Steve Seitz

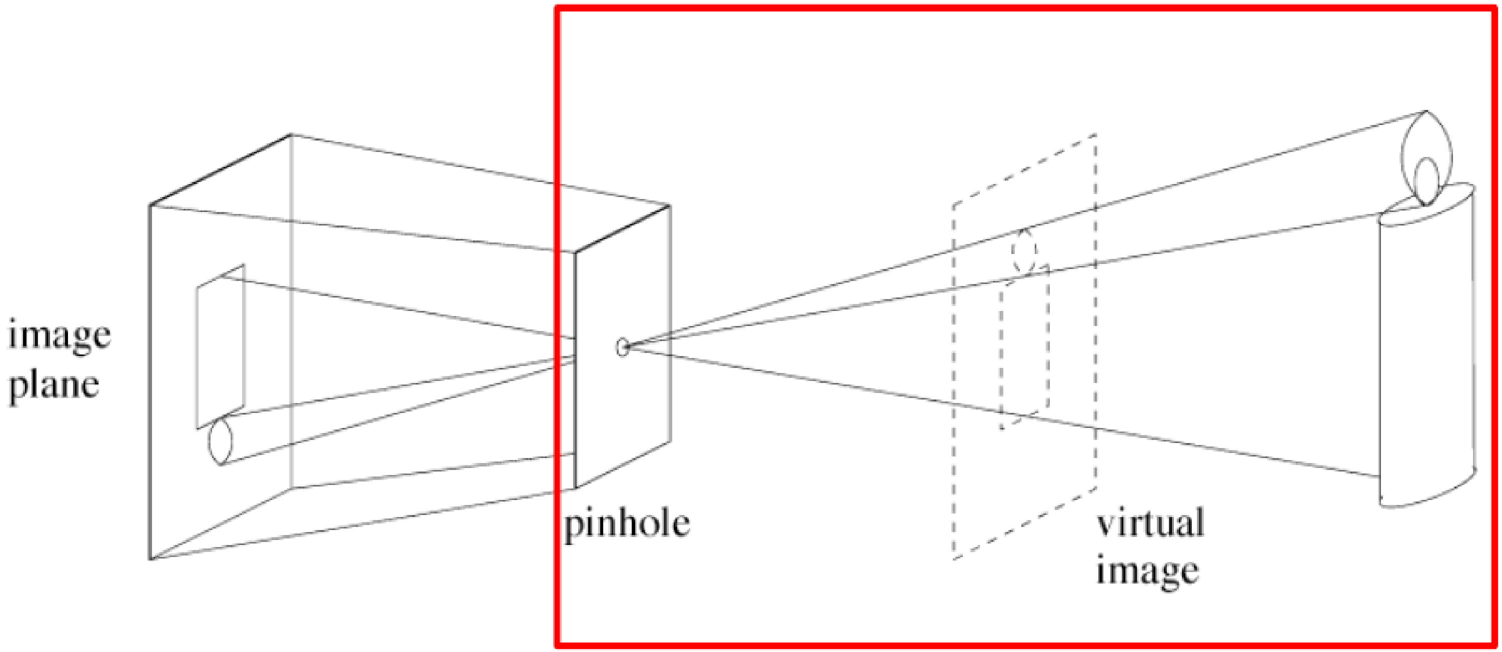
Pinhole Camera



- Add a barrier to block off most of the rays
  - This reduces blurring
  - The opening is known as the aperture

Slide by Steve Seitz

Pinhole Camera Model

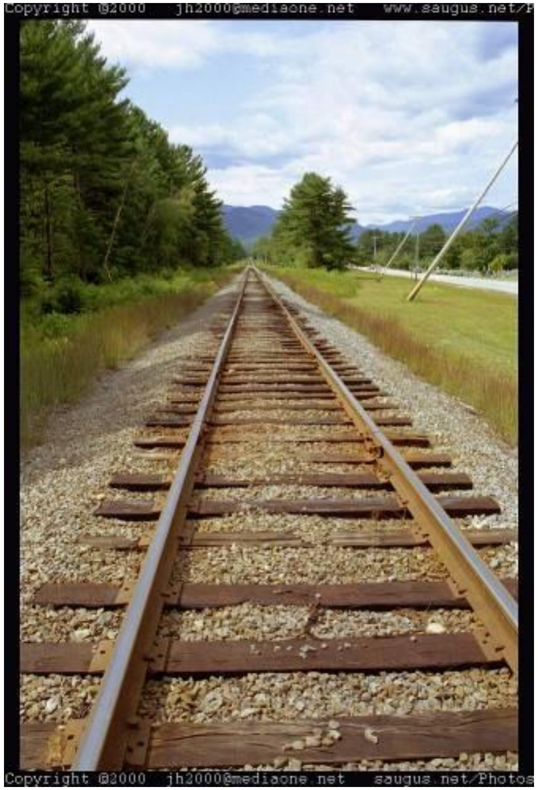


- Pinhole model:
  - Captures pencil of rays – all rays through a single point (Pinhole)
  - The point is called centre of Projection (focal point)
  - The image is formed on the Image Plane
  - A virtual image plane is used as mathematical description of the real image plane

Slide by Steve Seitz

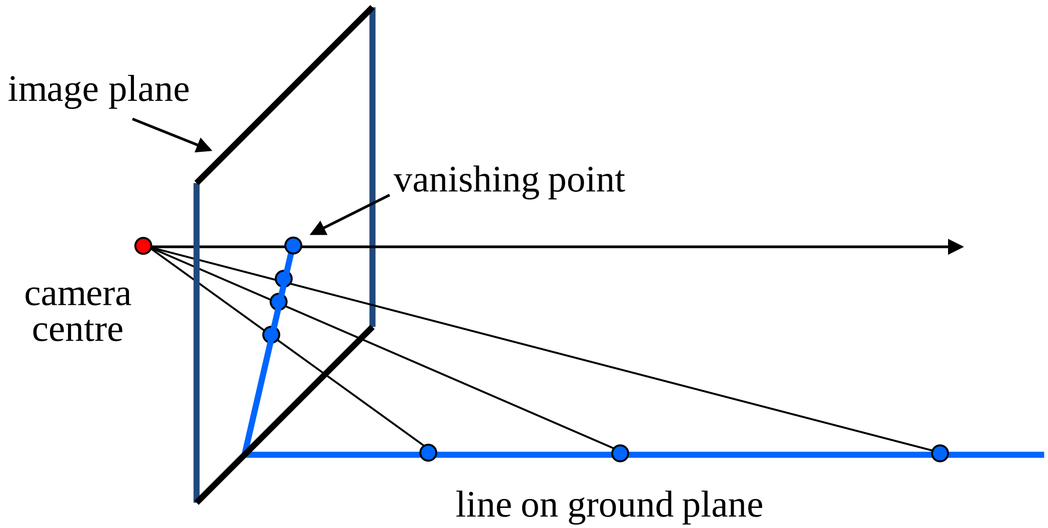
Vanishing Points

- Each direction in space has its own vanishing point
  - All lines going in that direction converge at that point
  - Exception: directions parallel to the image plane



Vanishing Points

- Each direction in space has its own vanishing point
  - All lines going in that direction converge at that point
  - Exception: directions parallel to the image plane
- How do we construct the vanishing point of a line?
  - What about the vanishing line of a plane?



Building a Real Camera

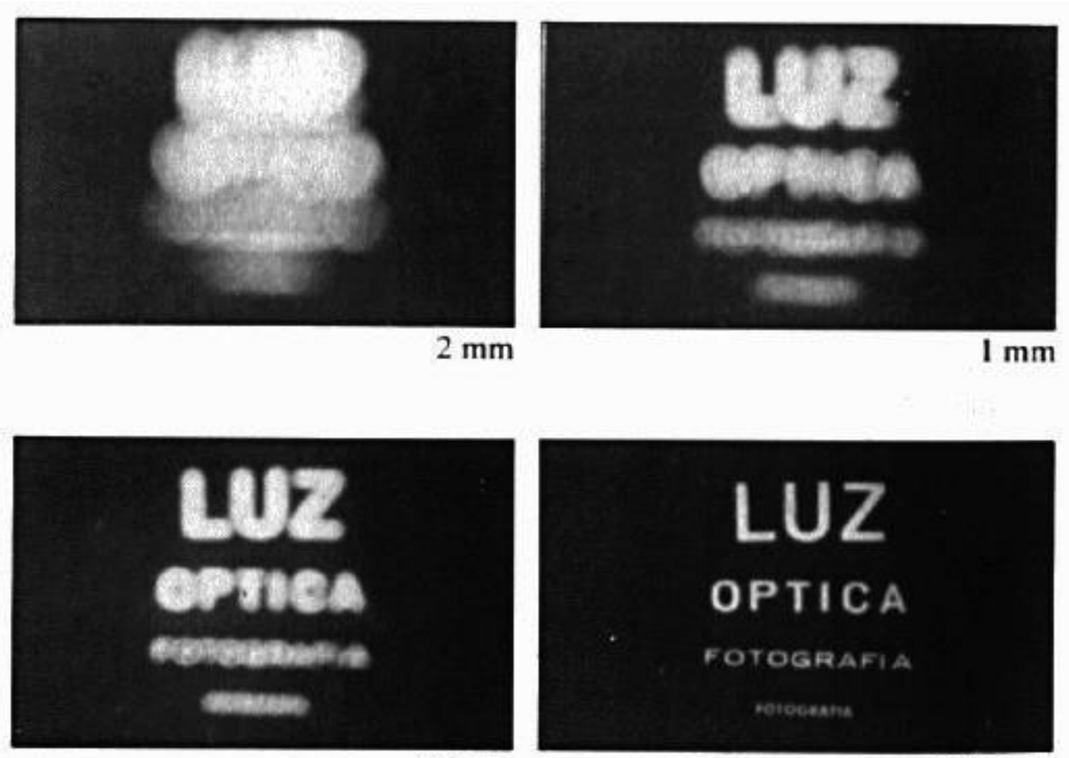
Home-made Pinhole Camera

Why so blurry?

http://www.debevec.org/Pinhole/

Slide by A. Efros

Shrinking the Aperture

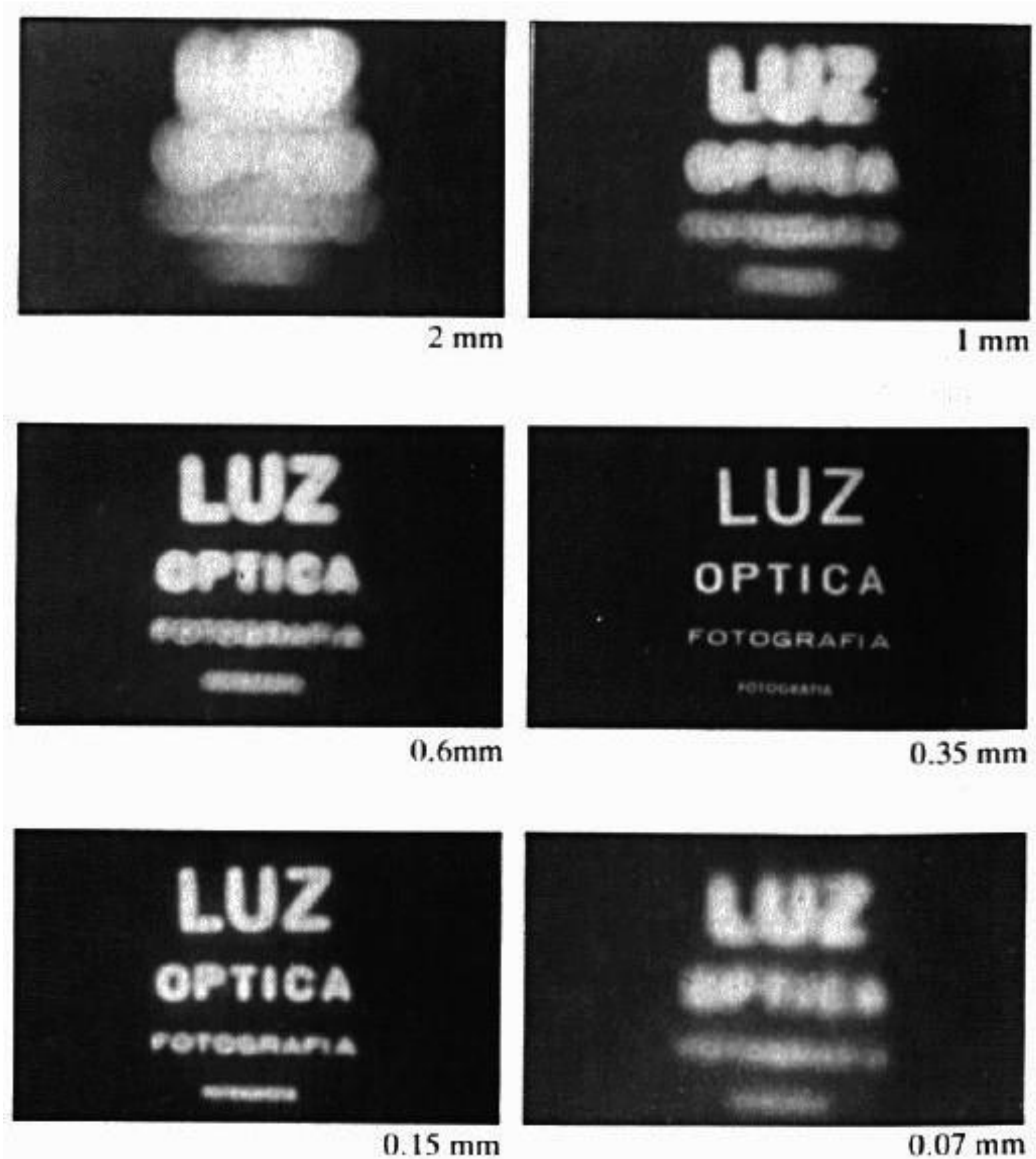


2 mm 1 mm 0.6mm 0.35 mm

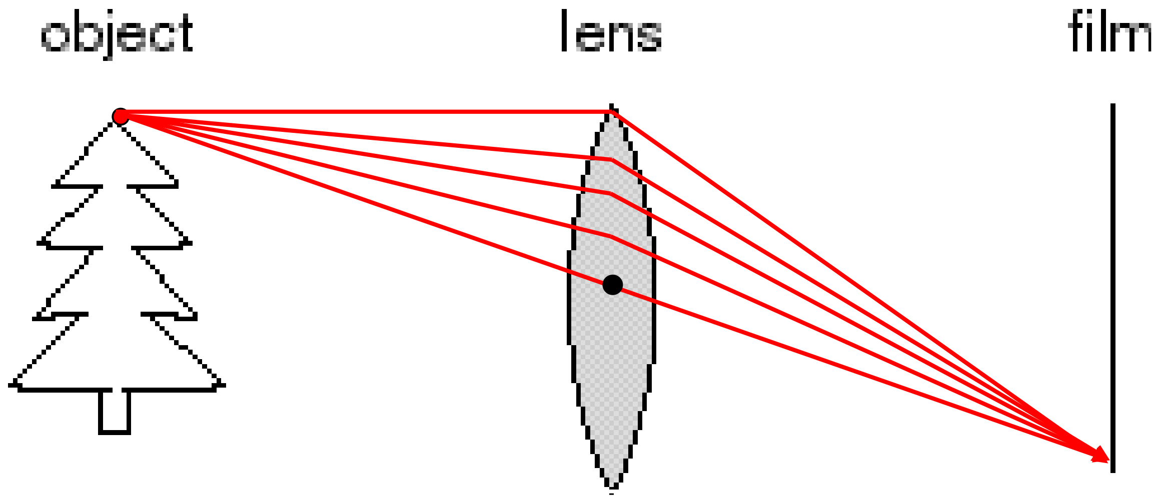
- Why not make the aperture as small as possible?
  - Less light gets through
  - Diffraction effects…

Slide by Steve Seitz

Shrinking the Aperture



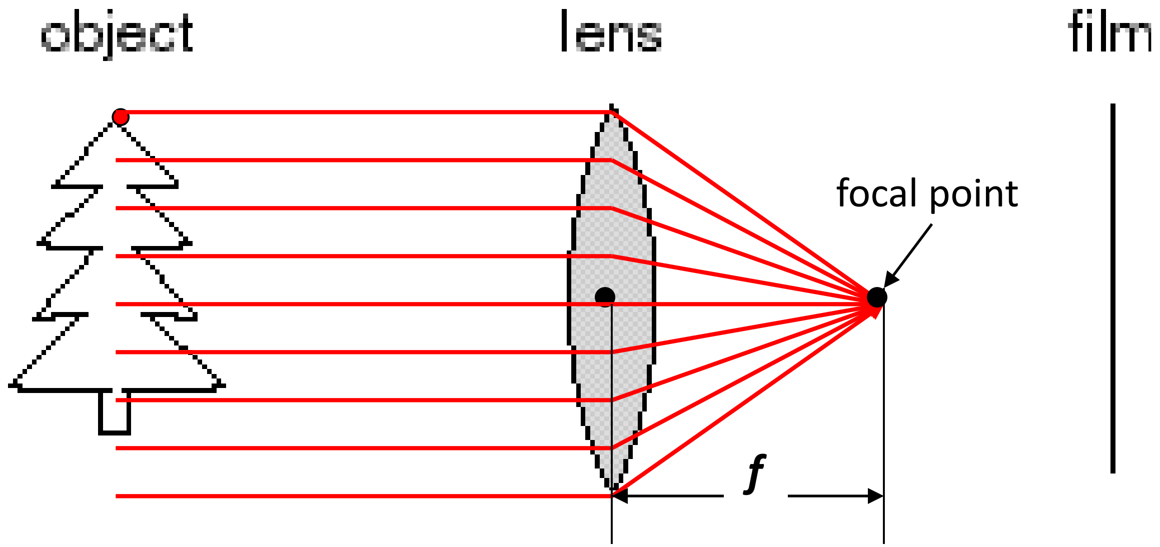
Adding a lens



- A lens focuses light onto the film
  - Thin lens model:
    - Rays passing through the centre are not deviated (pinhole projection model still holds)

Slide by Steve Seitz

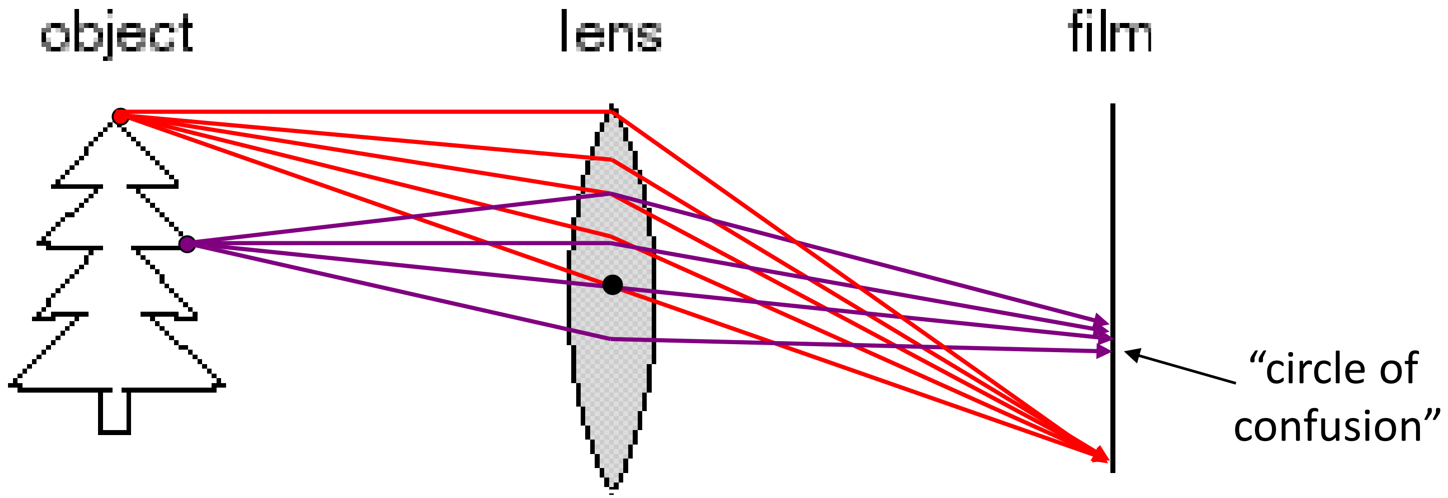
Adding a lens



- A lens focuses light onto the film
  - Thin lens model:
    - Rays passing through the centre are not deviated (pinhole projection model still holds)
    - All parallel rays converge to one point on a plane located at the *focal length f*

Slide by Steve Seitz

Adding a lens

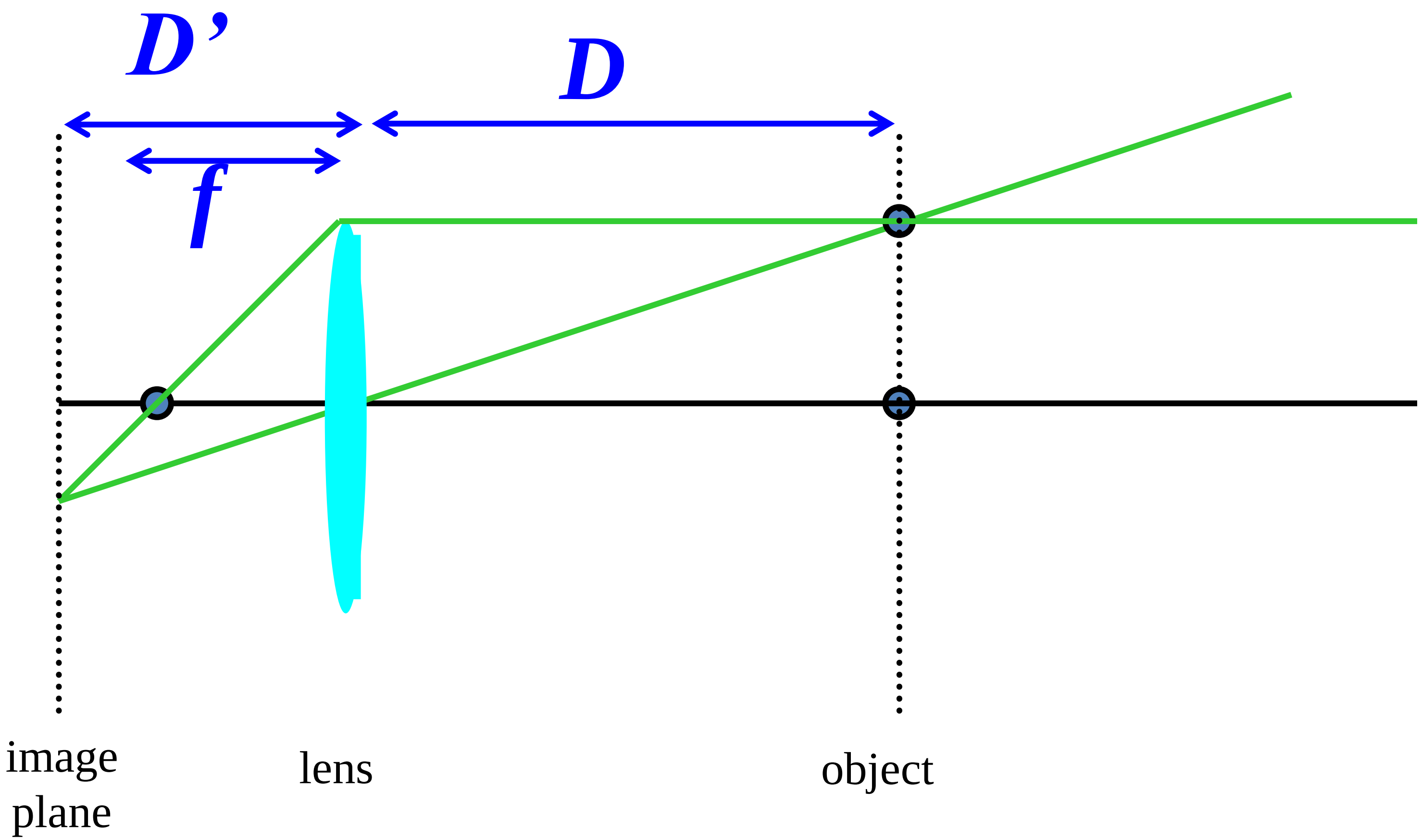


- A lens focuses light onto the film
  - There is a specific distance at which objects are “in focus”
    - other points project to a “circle of confusion” in the image

Slide by Steve Seitz

Thin Lens Formula

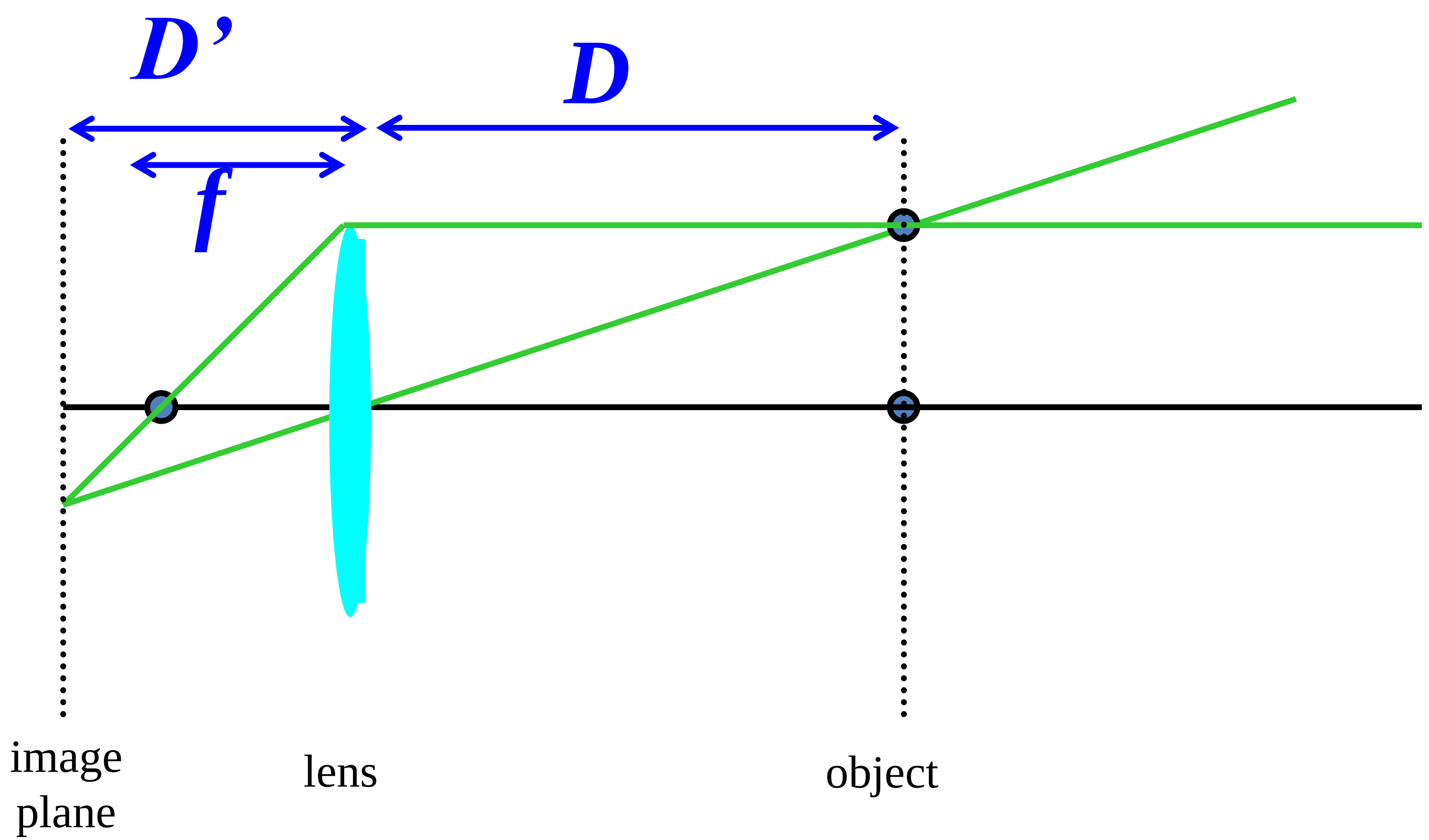
- What is the relation between the focal length (f), the distance of the object from the optical centre (D), and the distance at which the object will be in focus (D')?



Slide by Frédo Durand

Thin Lens Formula

- Similar triangles everywhere!

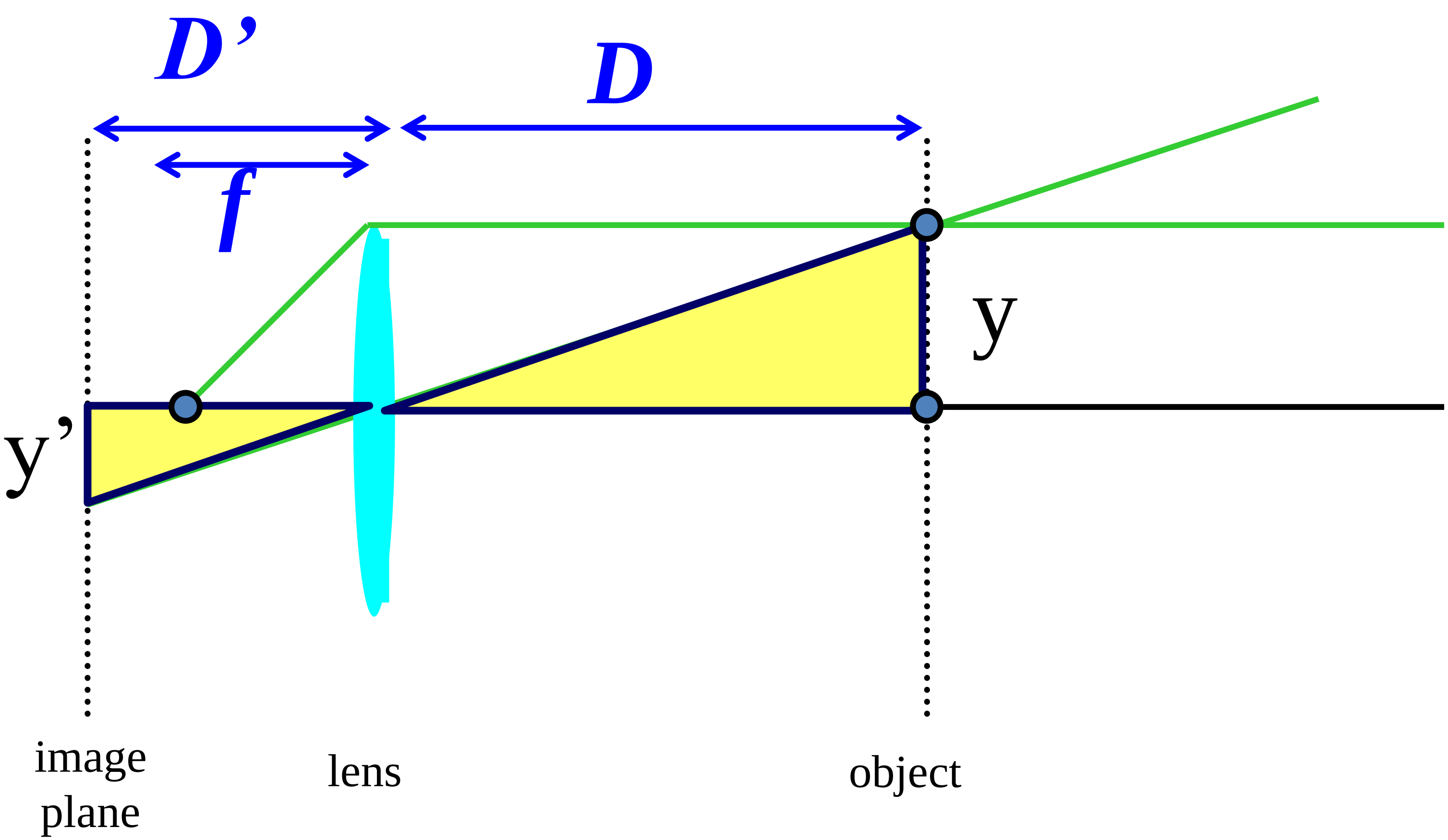


Slide by Frédo Durand

Thin Lens Formula

- Similar triangles everywhere!

$$y'/y = D'/D$$



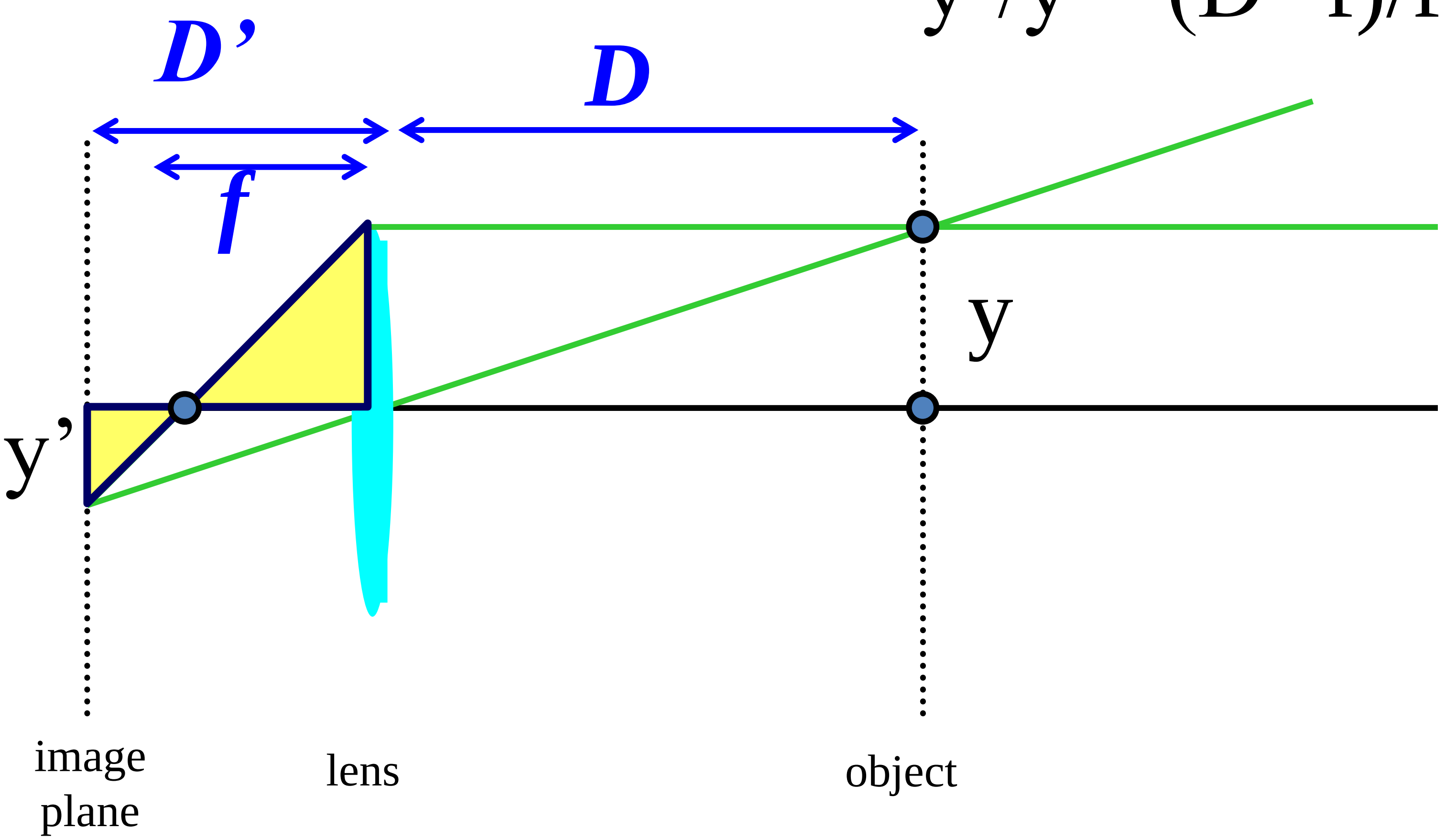
Slide by Frédo Durand

Thin Lens Formula

- Similar triangles everywhere!

$$y'/y = D'/D$$

$$y'/y = (D'-f)/f$$

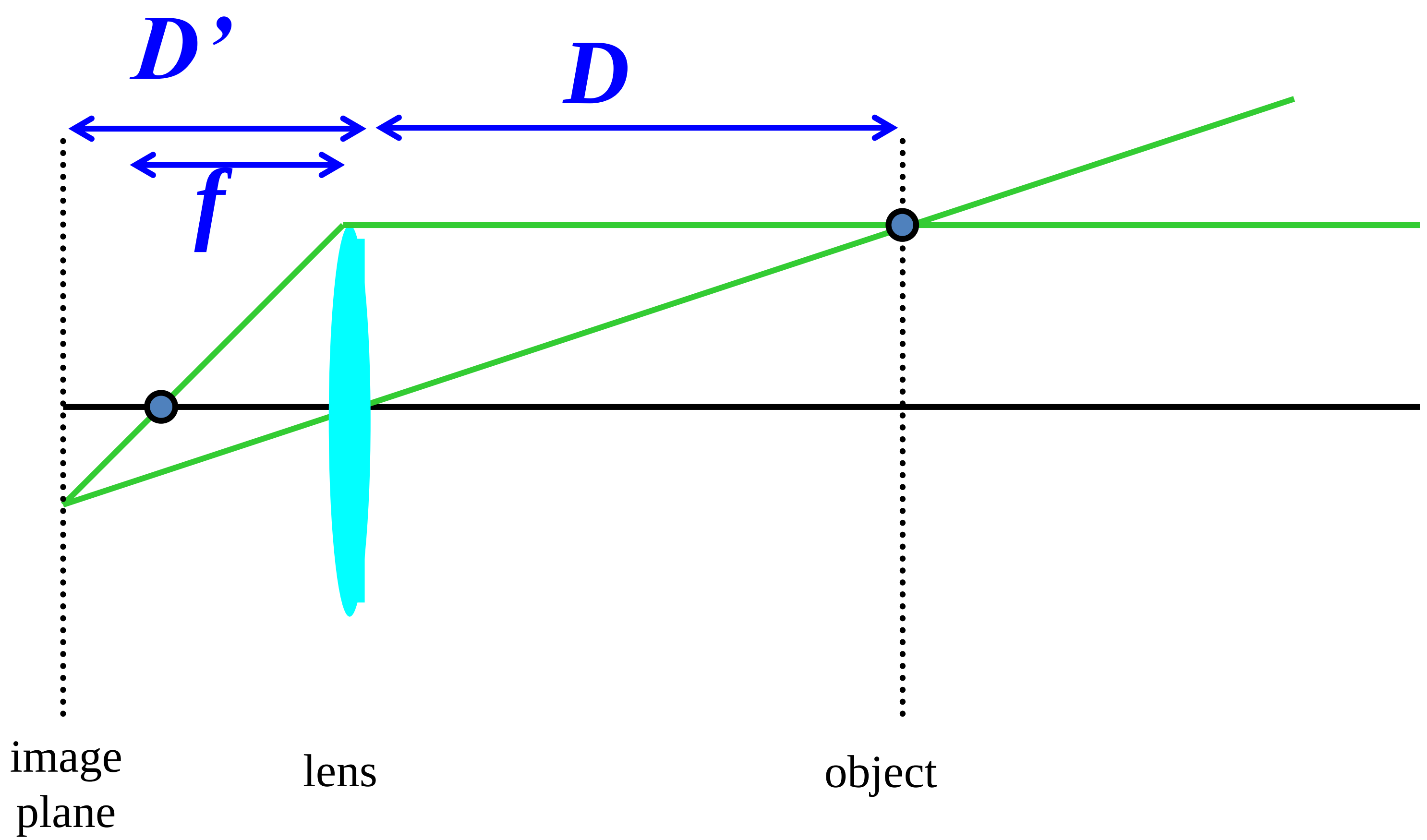


Slide by Frédo Durand

Thin Lens Formula

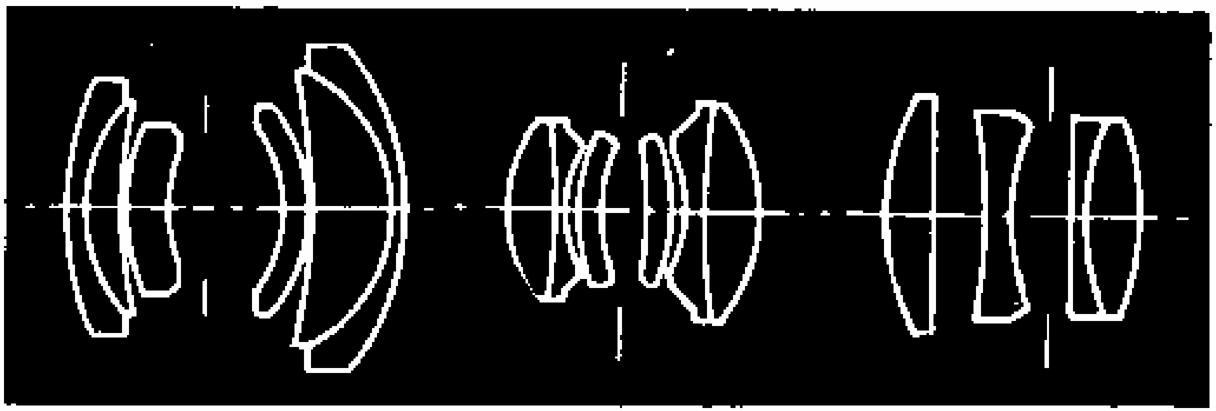
$$\boxed{\frac{1}{D'} + \frac{1}{D} = \frac{1}{f}}$$

Any point satisfying the thin lens equation is in focus.



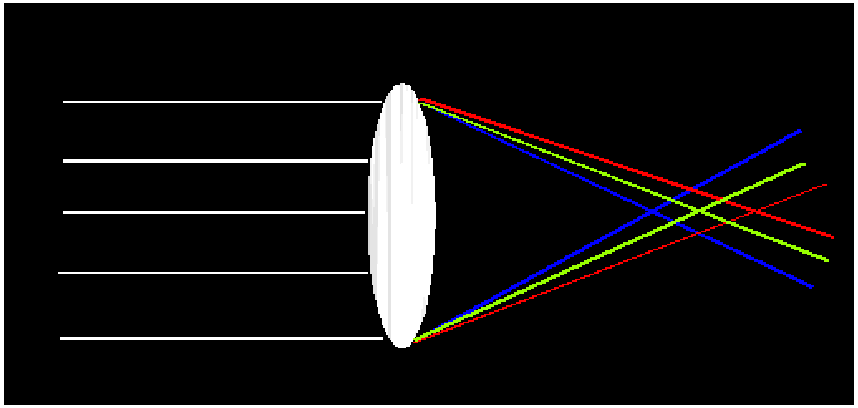
Slide by Frédo Durand

Real Lenses



Lens Flaws: Chromatic Aberration

- Lens has different refractive indices for different wavelengths: causes color fringing

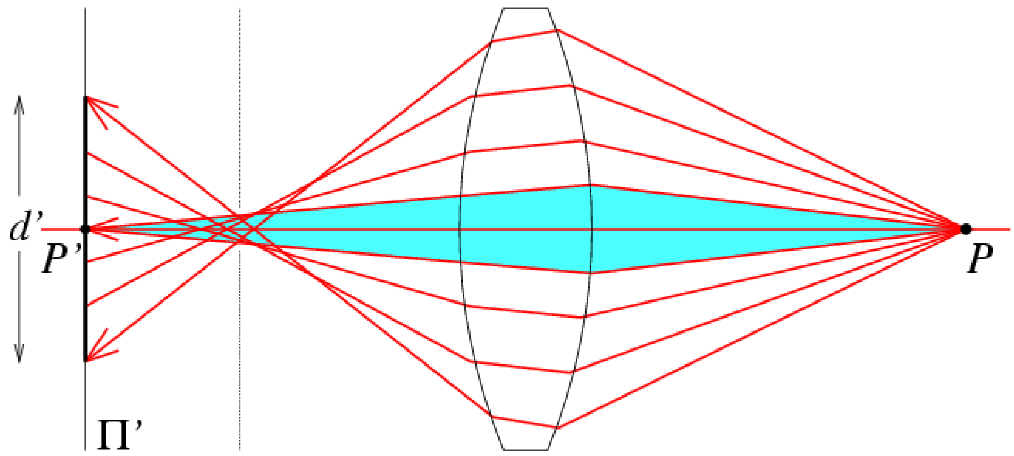


Near Lens centre

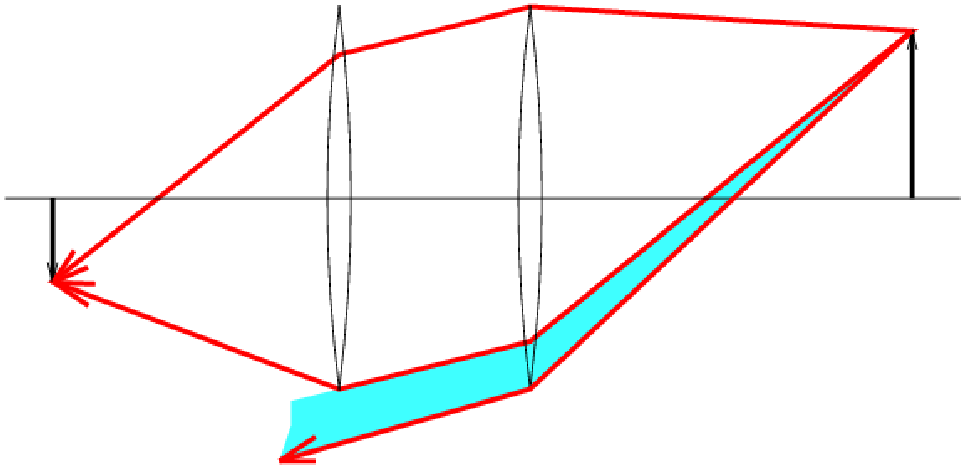
Near Lens Outer Edge

Lens Flaws: Spherical aberration

- Spherical lenses don't focus light perfectly
- Rays farther from the optical axis focus closer

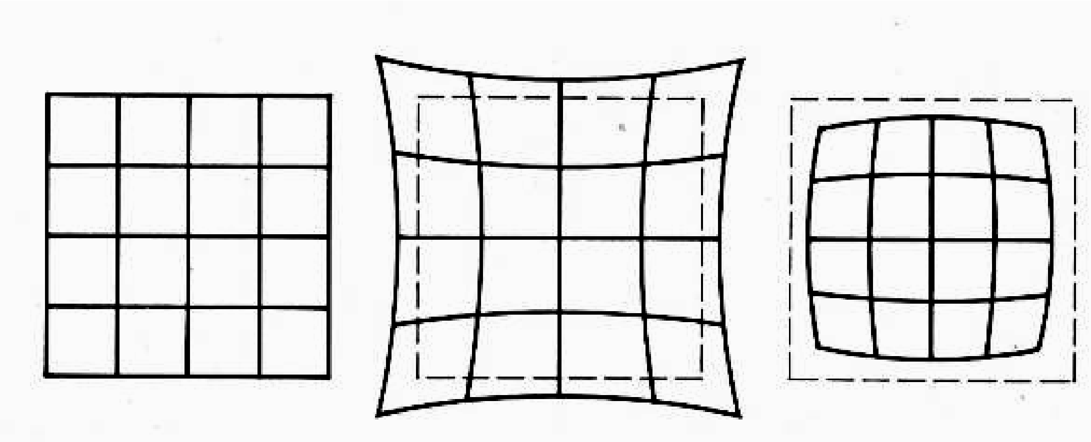


Lens Flaws: Vignetting



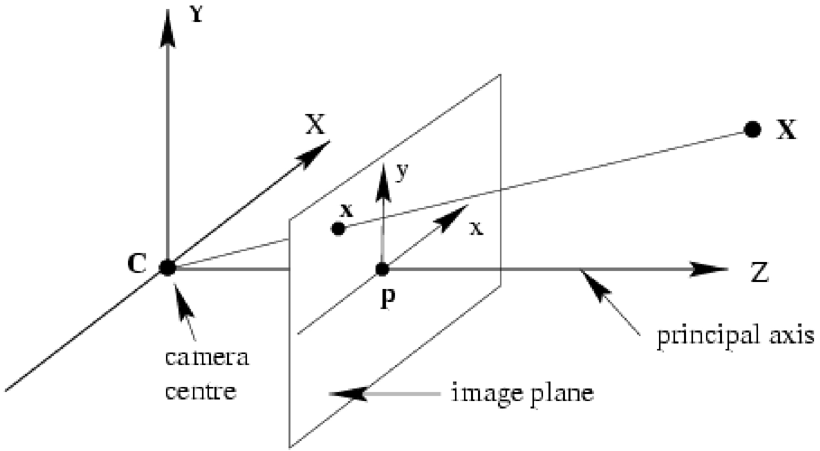
Radial Distortion

- Caused by imperfect lenses
- Deviations are most noticeable near the edge of the lens



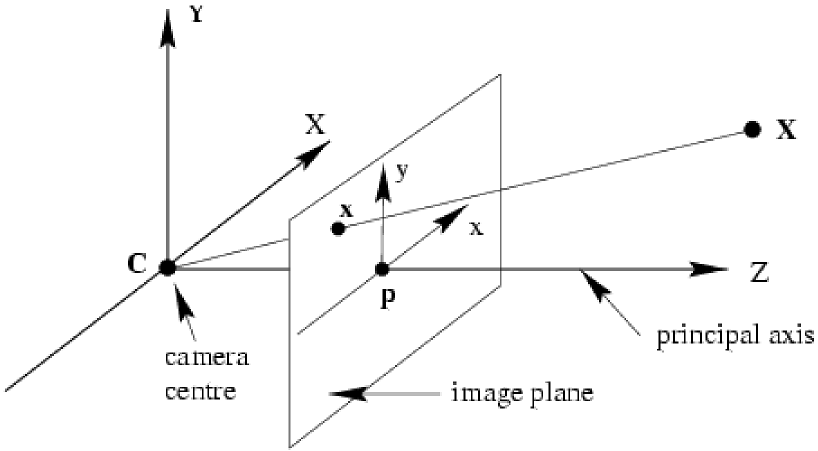
No distortion Pin cushion Barrel

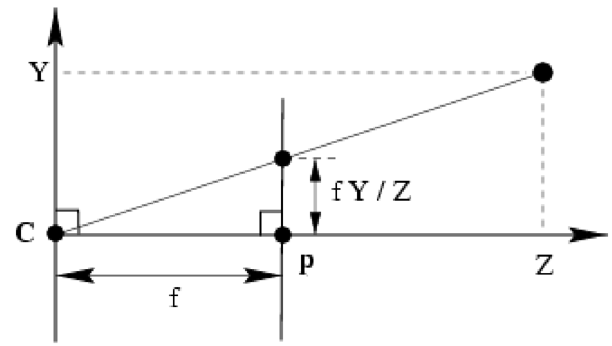
Pinhole Camera Model Revisit



- Principal axis: line from the camera centre perpendicular to the image plane
- Camera coordinate system: camera centre is at the origin and the principal axis is the z-axis

Pinhole Camera Model Revisit

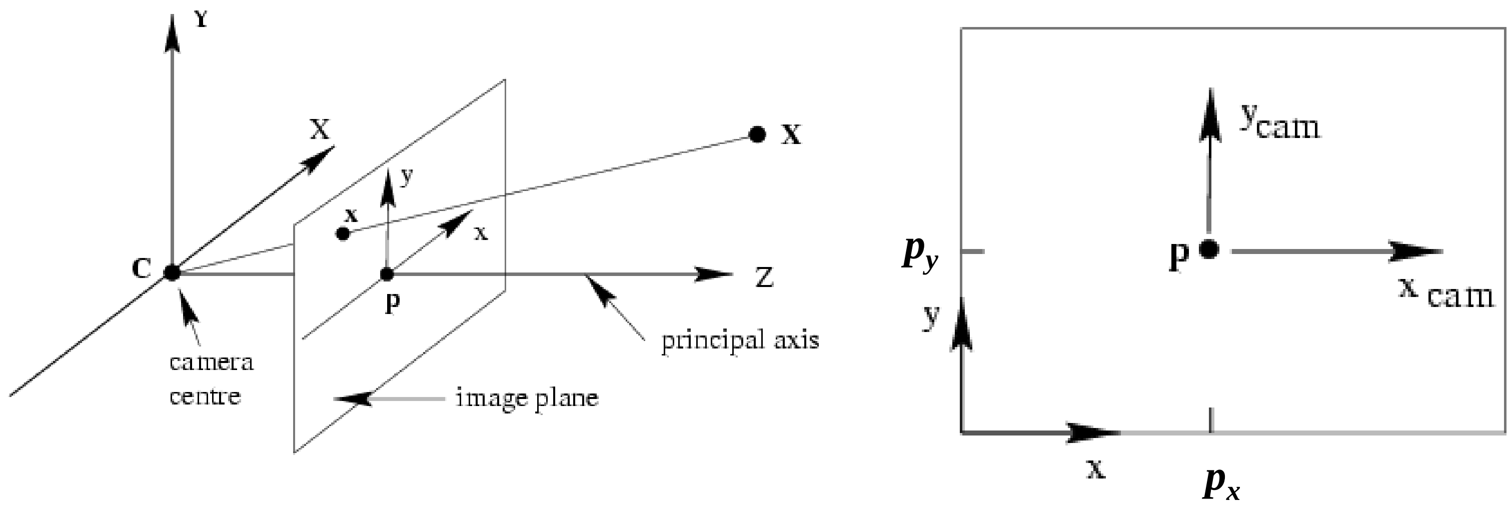




$$(X, Y, Z) \mapsto (f X / Z, f Y / Z)$$

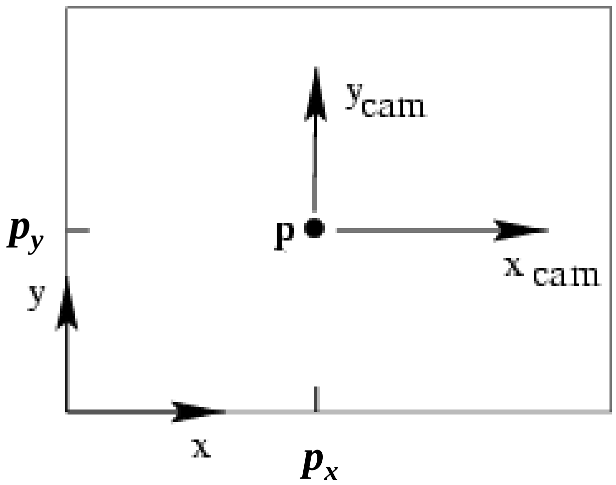
$$\begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix} \mapsto \begin{pmatrix} f X \\ f Y \\ Z \end{pmatrix} = \begin{bmatrix} f & & & 0 \\ & f & & 0 \\ & & 1 & 0 \end{bmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix} \qquad \mathrm{x} = \mathrm{P}\mathrm{X}$$

Principal Point



- Principal point (p): point where principal axis intersects the image plane (origin of normalized coordinate system)
- Normalized coordinate system: origin is at the principal point
- Image coordinate system: origin is in the corner
- How to go from normalized coordinate system to image coordinate system?

Principal Point Offset

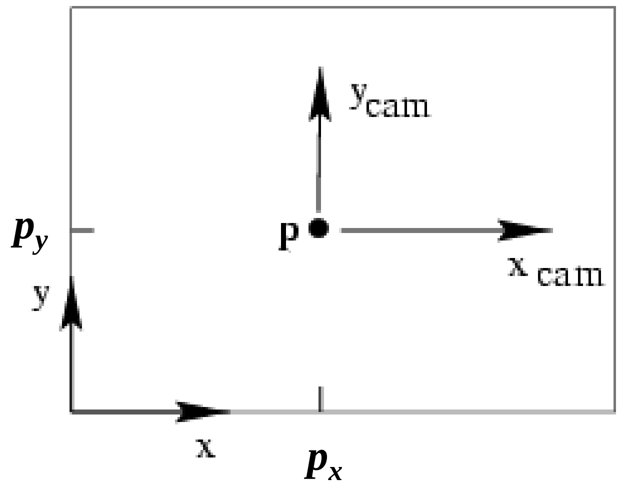


Principal Point: (p_x, p_y)

$$(X, Y, Z) \mapsto (f X / Z + p_x, f Y / Z + p_y)$$

$$\begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix} \mapsto \begin{pmatrix} f X + Z p_x \\ f Y + Z p_y \\ Z \end{pmatrix} = \begin{bmatrix} f & & p_x & 0 \\ & f & p_y & 0 \\ & & 1 & 0 \end{bmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

Principal Point Offset



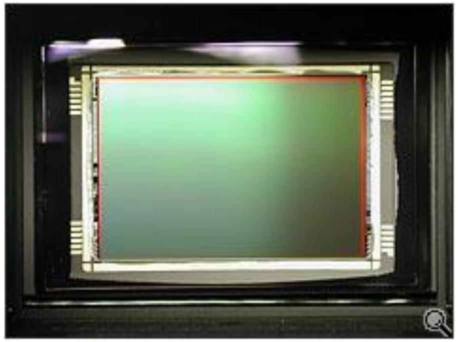
Principal Point: (p_x, p_y)

$$\begin{pmatrix} fX + Zp_x \\ fY + Zp_y \\ Z \end{pmatrix} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} 1 & & & 0 \\ & 1 & & 0 \\ & & 1 & 0 \end{bmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

$$K = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix}$$

Calibration Matrix $\mathrm{P} = \mathrm{K}[\mathrm{I} \mid 0]$

Pixel Coordinates



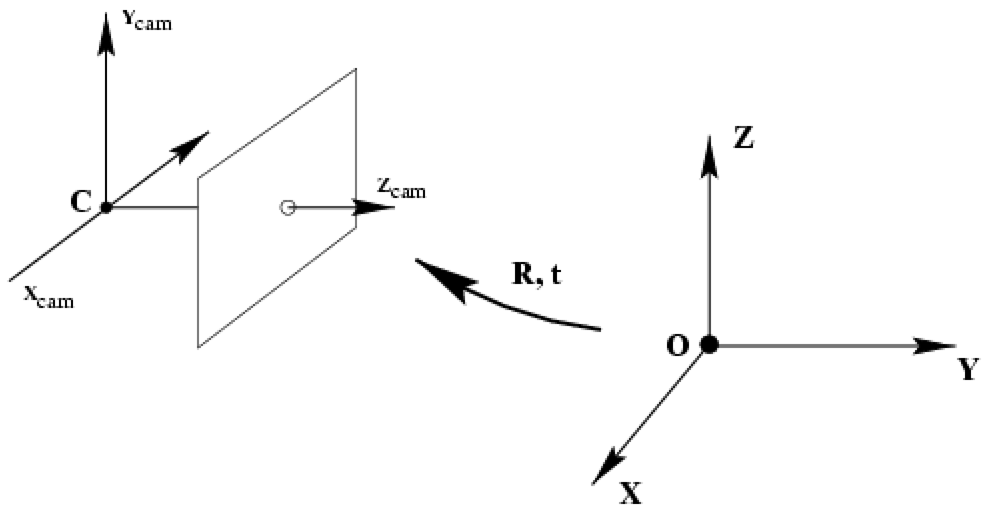
Pixel size: $\frac{1}{m_x} \times \frac{1}{m_y}$

- m_x pixels per metre in horizontal direction, m_y pixels per metre in vertical direction

$$K = \begin{bmatrix} m_x & & \\ & m_y & \\ & & 1 \end{bmatrix} \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} = \begin{bmatrix} \alpha_x & & \beta_x \\ & \alpha_y & \beta_y \\ & & 1 \end{bmatrix}$$

pixels/m   m   pixels

Camera Rotation and Translation



- In general, the camera coordinate frame will be related to the world coordinate frame by a rotation and a translation

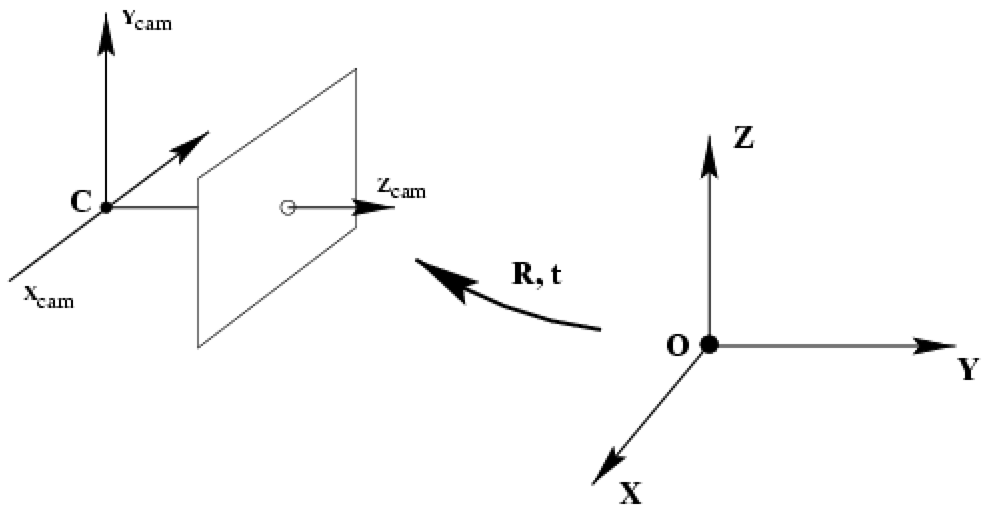
$$\tilde{X}_{cam} = R\left(\tilde{X} - \tilde{C}\right)$$

coords. of point in camera frame

coords. of a point in world frame (nonhomogeneous)

coords. of camera centre in world frame

Camera Rotation and Translation



In non-homogeneous coordinates:

$$\tilde{X}_{cam} = R\left(\tilde{X} - \tilde{C}\right)$$

$$X_{cam} = \begin{bmatrix} R & -R\tilde{C} \\ 0 & 1 \end{bmatrix} \begin{pmatrix} \tilde{X} \\ 1 \end{pmatrix} = \begin{bmatrix} R & -R\tilde{C} \\ 0 & 1 \end{bmatrix} X$$

$$x = K[I \mid 0]X_{cam} = K\left[R \mid -R\tilde{C}\right]X \qquad P = K[R \mid t], \qquad t = -R\tilde{C}$$

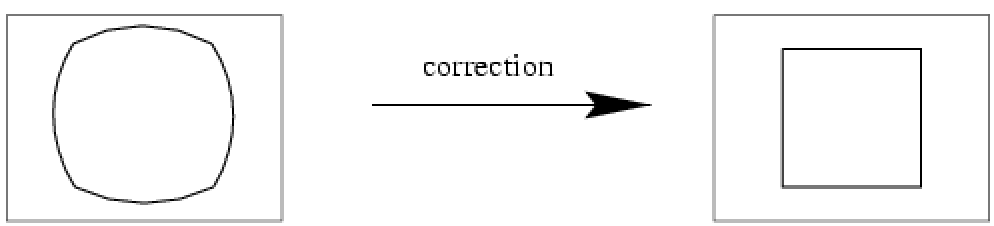
Camera Parameters

- Intrinsic parameters
  - Principal point coordinates
  - Focal length
  - Pixel magnification factors
  - *Skew (non-rectangular pixels)*
  - *Radial distortion*

$$K = \begin{bmatrix} m_x & & \\ & m_y & \\ & & 1 \end{bmatrix} \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} = \begin{bmatrix} \alpha_x & & \beta_x \\ & \alpha_y & \beta_y \\ & & 1 \end{bmatrix}$$

radial distortion

linear image



Camera Parameters

- Intrinsic parameters
  - Principal point coordinates
  - Focal length
  - Pixel magnification factors
  - *Skew (non-rectangular pixels)*
  - *Radial distortion*
- Extrinsic parameters
  - Rotation and translation relative to world coordinate system

Camera Calibration

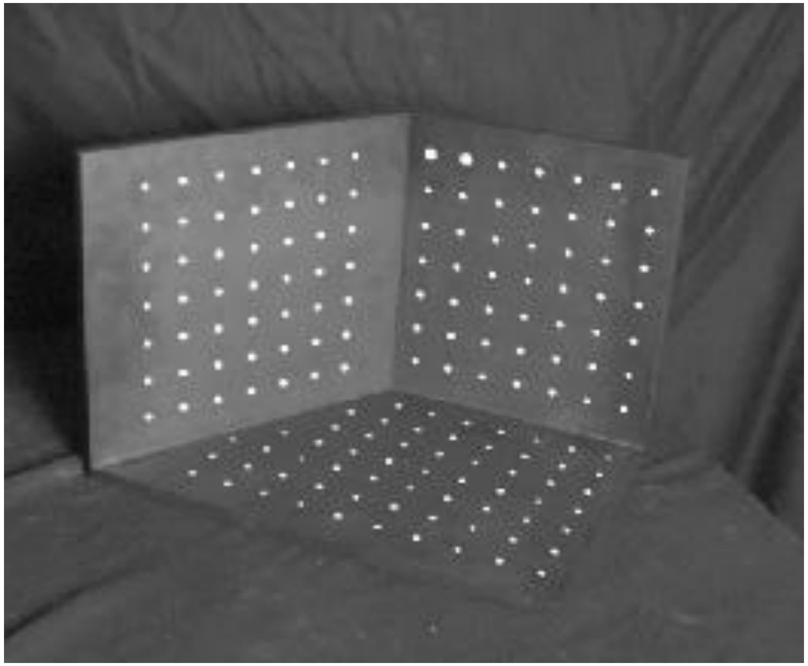
$$\mathbf{x} = \mathbf{P}\mathbf{X} = \mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix}\mathbf{X}$$

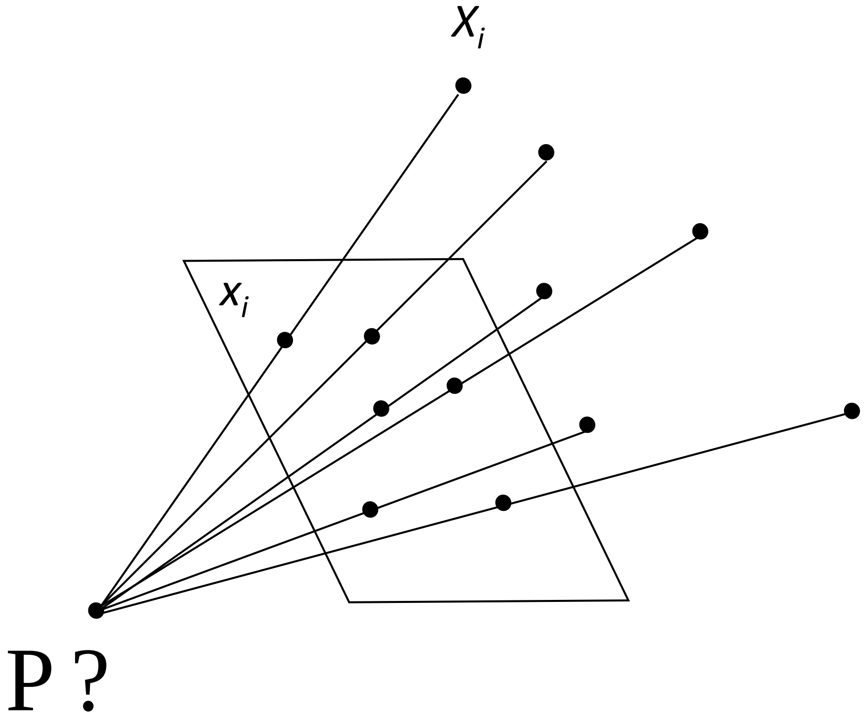
$$\begin{bmatrix} \lambda x \\ \lambda y \\ \lambda \end{bmatrix} = \begin{bmatrix} P_{11} & P_{12} & P_{13} & P_{14} \\ P_{21} & P_{22} & P_{23} & P_{24} \\ P_{31} & P_{32} & P_{33} & P_{34} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

Source: D. Hoiem

Camera Calibration

- Given n points with known 3D coordinates X_i and known image projections x_i, estimate the camera parameters





Camera Calibration: Linear Method

$$\lambda \mathrm{x}_i = \mathrm{P}\mathrm{X}_i \qquad \mathrm{x}_i \times \mathrm{P}\mathrm{X}_i = 0 \qquad \begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix} \times \begin{bmatrix} \mathrm{P}_1^T \mathrm{X}_i \\ \mathrm{P}_2^T \mathrm{X}_i \\ \mathrm{P}_3^T \mathrm{X}_i \end{bmatrix} = 0$$

$$\mathrm{P}_1^{\boldsymbol{T}} = \begin{bmatrix} \boldsymbol{P}_{11} & \boldsymbol{P}_{12} & \boldsymbol{P}_{13} & \boldsymbol{P}_{14} \end{bmatrix}$$

$$\mathrm{P}_2^{\boldsymbol{T}} = \begin{bmatrix} \boldsymbol{P}_{21} & \boldsymbol{P}_{22} & \boldsymbol{P}_{23} & \boldsymbol{P}_{24} \end{bmatrix}$$

$$\mathrm{P}_3^{\boldsymbol{T}} = \begin{bmatrix} \boldsymbol{P}_{31} & \boldsymbol{P}_{32} & \boldsymbol{P}_{33} & \boldsymbol{P}_{34} \end{bmatrix}$$

$$\begin{bmatrix} 0 & -\mathrm{X}_i^T & y_i \mathrm{X}_i^T \\ \mathrm{X}_i^T & 0 & -x_i \mathrm{X}_i^T \\ -y_i \mathrm{X}_i^T & x_i \mathrm{X}_i^T & 0 \end{bmatrix} \begin{pmatrix} \mathrm{P}_1 \\ \mathrm{P}_2 \\ \mathrm{P}_3 \end{pmatrix} = 0$$

Two linearly independent equations

Camera Calibration: Linear Method

$$\begin{bmatrix} 0^T & X_1^T & -y_1 X_1^T \\ X_1^T & 0^T & -x_1 X_1^T \\ \cdots & \cdots & \cdots \\ 0^T & X_n^T & -y_n X_n^T \\ X_n^T & 0^T & -x_n X_n^T \end{bmatrix} \begin{pmatrix} P_1 \\ P_2 \\ P_3 \end{pmatrix} = 0 \qquad Ap = 0$$

- P has 11 degrees of freedom (12 parameters, but scale is arbitrary)
- One 2D/3D correspondence gives us two linearly independent equations
- 6 correspondences needed for a minimal solution
- Homogeneous least squares
  - the eigenvector corresponding to the smallest eigenvalue of A^TA

Camera Calibration: Linear Method

$$\begin{bmatrix} 0^T & X_1^T & -y_1 X_1^T \\ X_1^T & 0^T & -x_1 X_1^T \\ \cdots & \cdots & \cdots \\ 0^T & X_n^T & -y_n X_n^T \\ X_n^T & 0^T & -x_n X_n^T \end{bmatrix} \begin{pmatrix} P_1 \\ P_2 \\ P_3 \end{pmatrix} = 0 \qquad \mathrm{Ap} = 0$$

- Note: for coplanar points that satisfy $\Pi^T X=0$, we will get degenerate solutions $(\Pi,0,0)$, $(0,\Pi,0)$, or $(0,0,\Pi)$

Camera Calibration: Linear Method

- Advantages:
  - easy to formulate and solve
- Disadvantages
  - Doesn't directly tell you camera parameters
  - Can't impose constraints, such as known focal length and orthogonality
  - Doesn't model radial distortion
  - Only an approximate solution

- Non-linear methods are preferred
  - Define error as difference between projected points and measured points
  - Minimise error using Newton's method or other non-linear optimisation

Source: D. Hoiem

Summary

- Describe pinhole model.
- What is vanishing point?
- What are intrinsic camera parameters, and what are extrinsic camera parameters?
- Describe the linear camera calibration method.
- What are the advantages and disadvantages of linear method for camera calibration?

CMT107 Visual Computing

X.2 Stereo Vision

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Overview

- Stereo Vision
  - Multi-view Geometry Problems
- Triangulation
- Epipolar Geometry
  - The epipolar constraint
  - Essential matrix and fundamental matrix
  - Eight-point Algorithm

Acknowledgement

The majority of the slides in this section are from Svetlana Lazebnik at University of Illinois at Urbana-Champaign

Stereo Vision

- Generic problem formulation: given several images of the same object or scene, compute a representation of its 3D shape

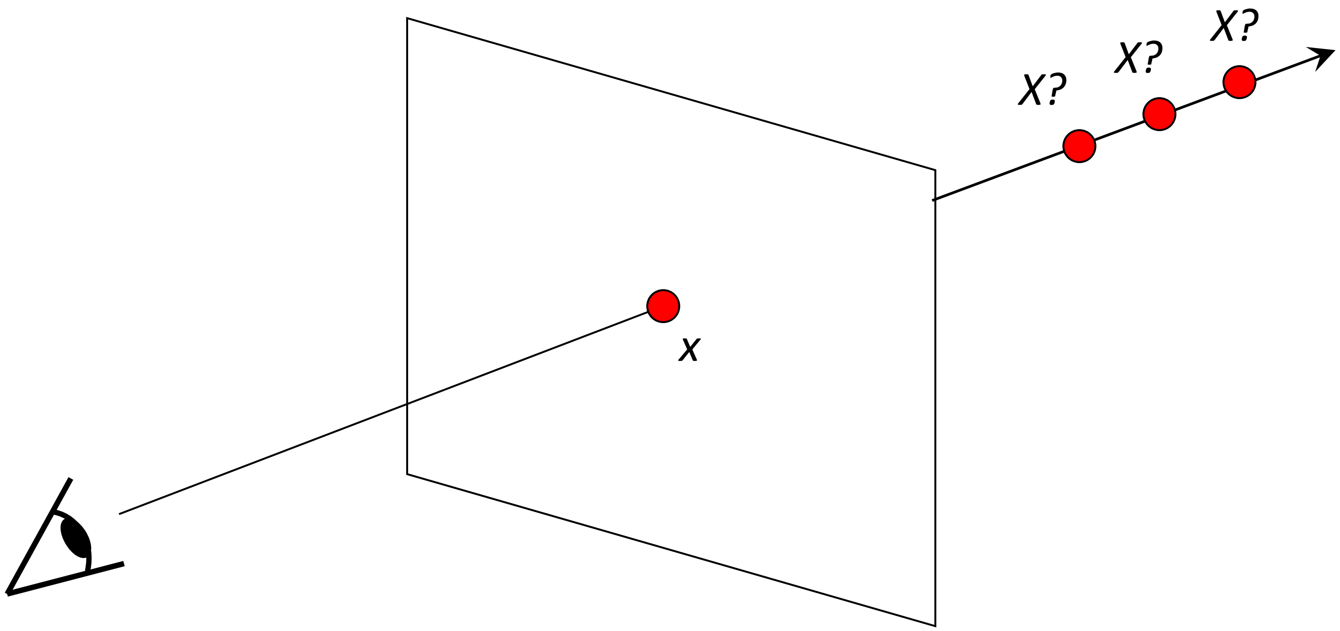


Stereo Vision

- Generic problem formulation: given several images of the same object or scene, compute a representation of its 3D shape
- "Images of the same object or scene"
  - Arbitrary number of images (from two to thousands)
  - Arbitrary camera positions (camera network or video sequence)
  - Calibration may be initially unknown
- "Representation of 3D shape"
  - Depth maps
  - Meshes
  - Point clouds
  - Patch clouds
  - Volumetric models
  - Layered models

Goal: Recovery of 3D Structure

- Recovery of structure from one image is inherently ambiguous



Goal: Recovery of 3D Structure

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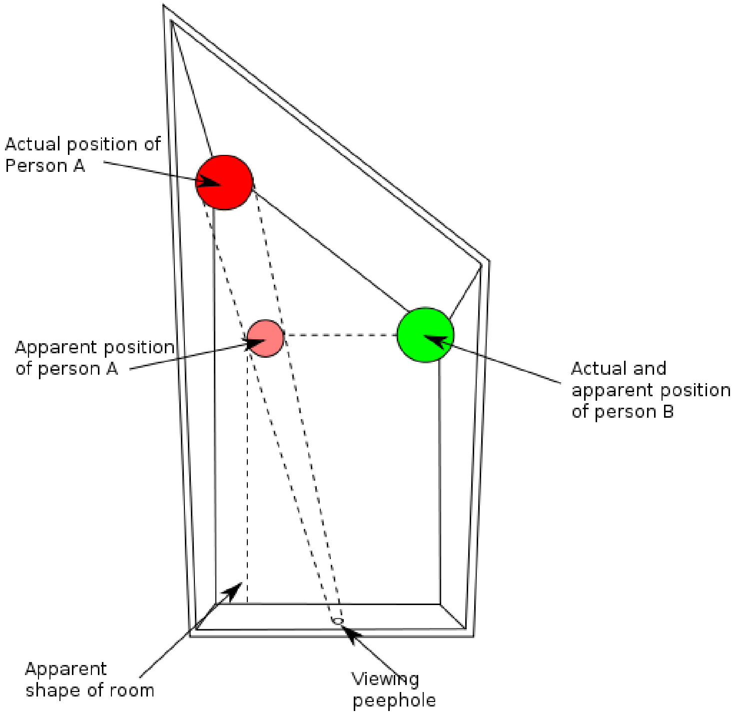


- Ames Room

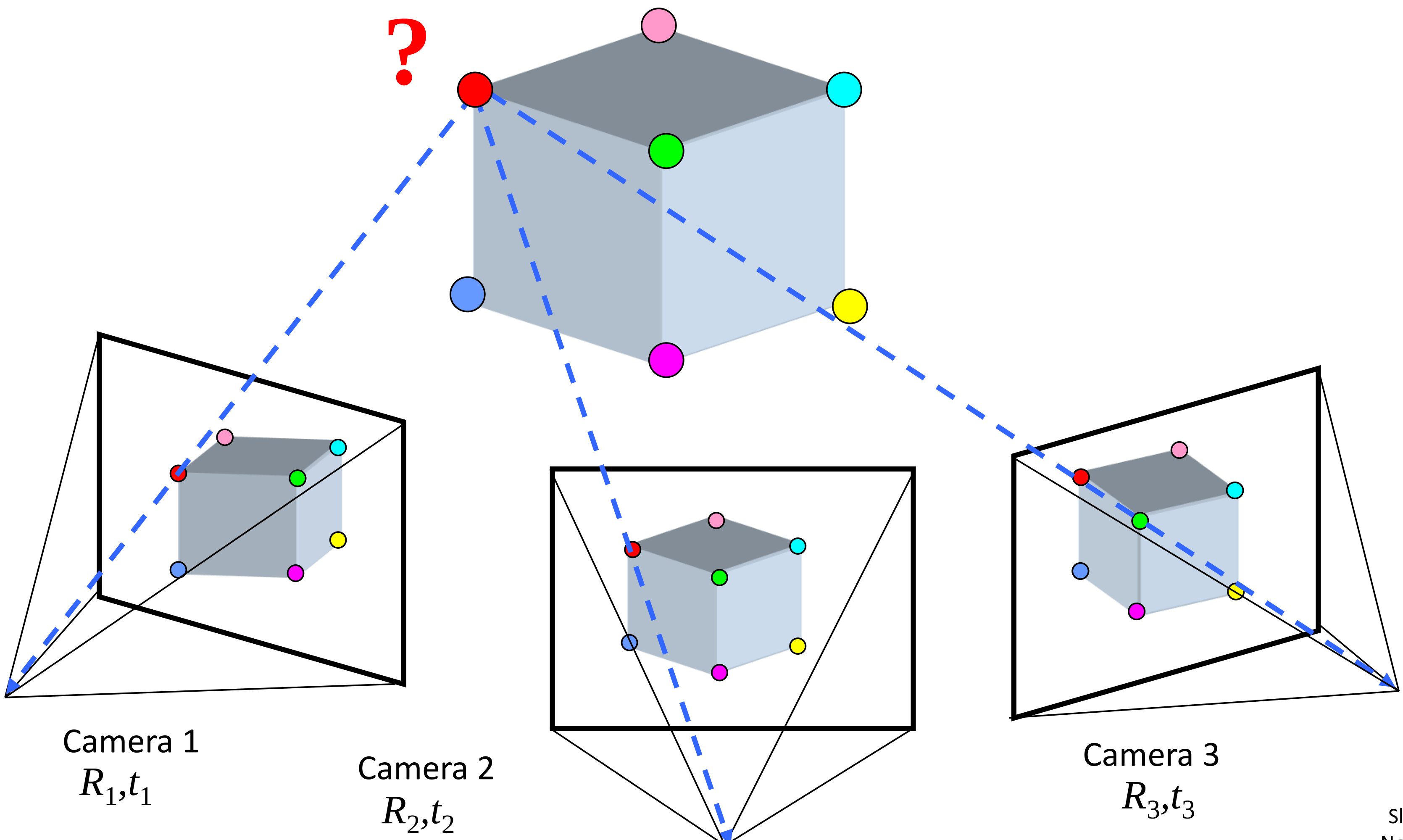
http://en.wikipedia.org/wiki/Ames_room

Goal: Recovery of 3D Structure

- We will need multi-view geometry

Multi-view Geometry Problems

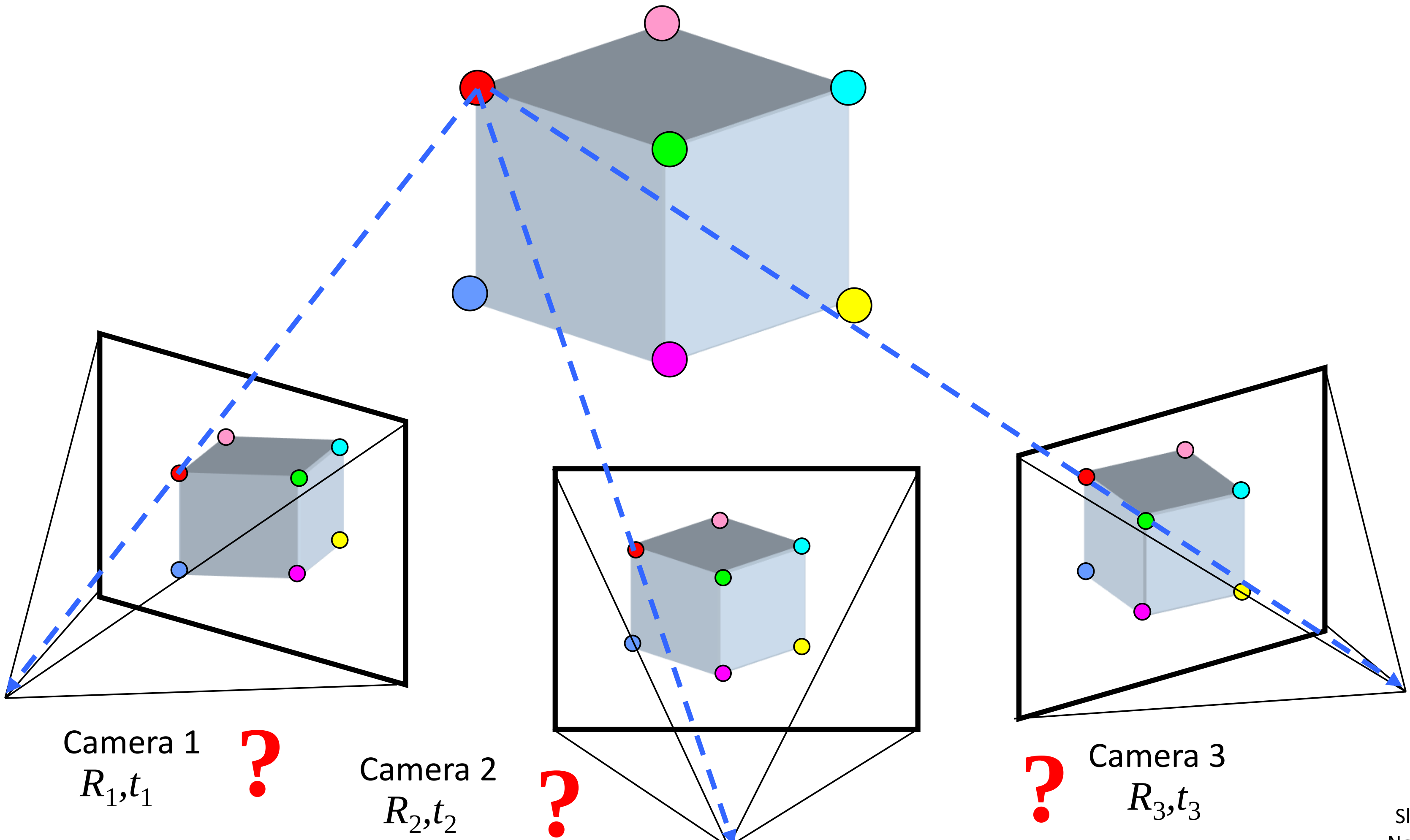
- **Structure:** Given projections of the same 3D point in two or more images, compute the 3D coordinates of that point



Slide credit: Noah Snavely

Multi-view Geometry Problems

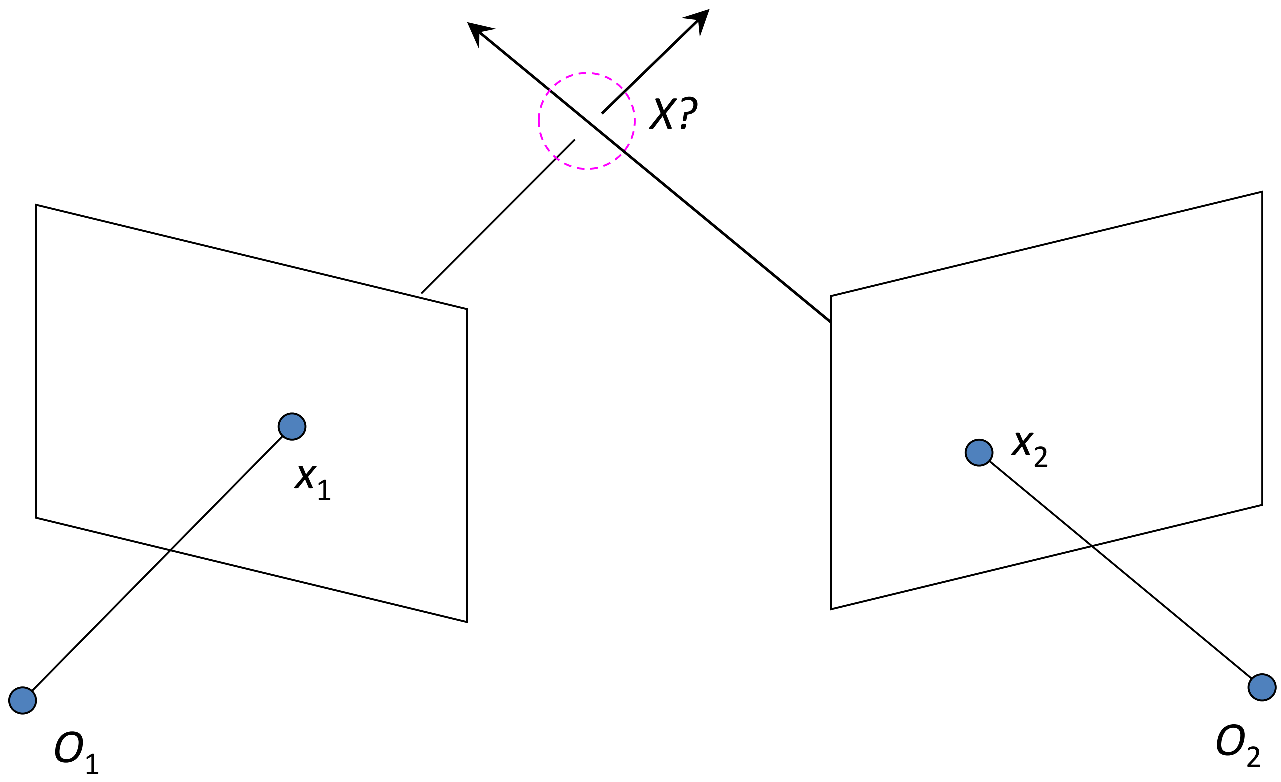
- **Motion:** Given a set of corresponding points in two or more images, compute the camera parameters



Slide credit: Noah Snavely

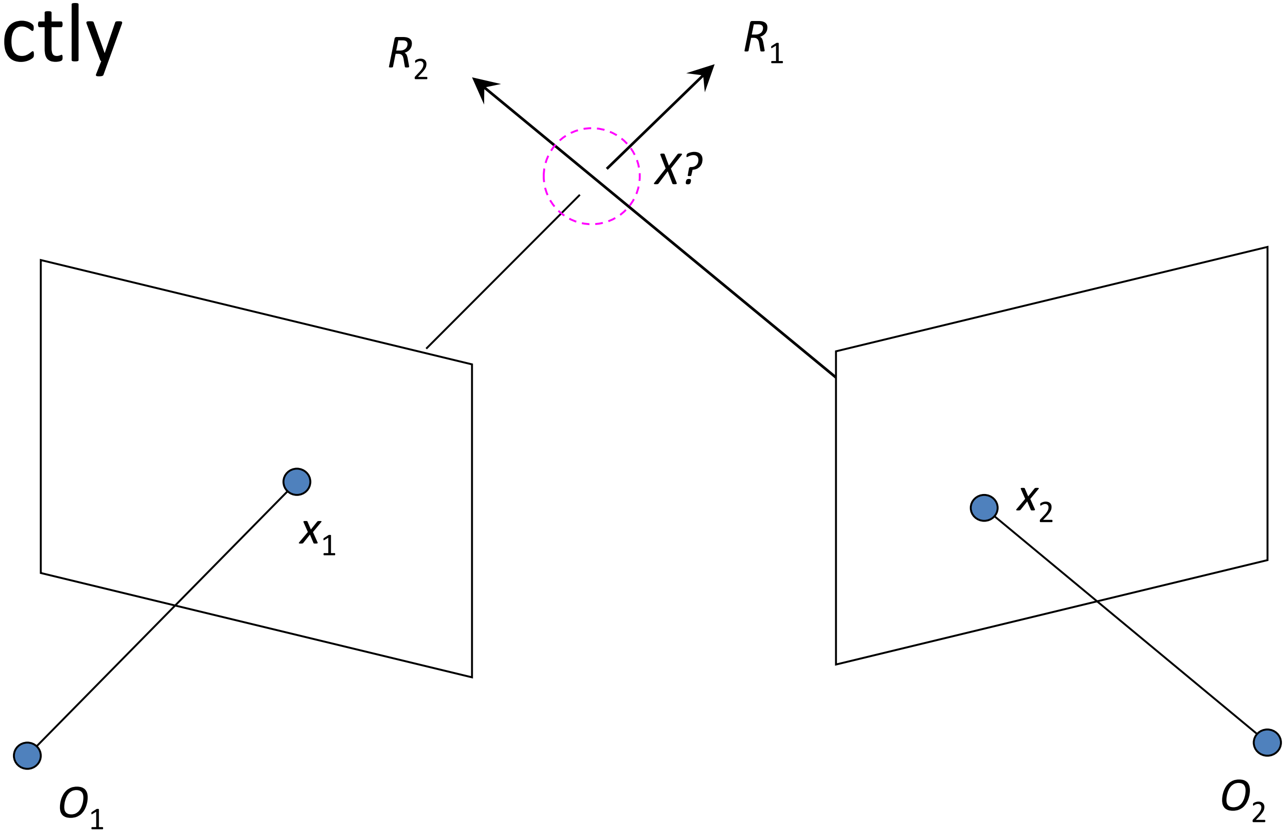
Triangulation

- Given projections of a 3D point in two or more images (with known camera matrices), find the coordinates of the point



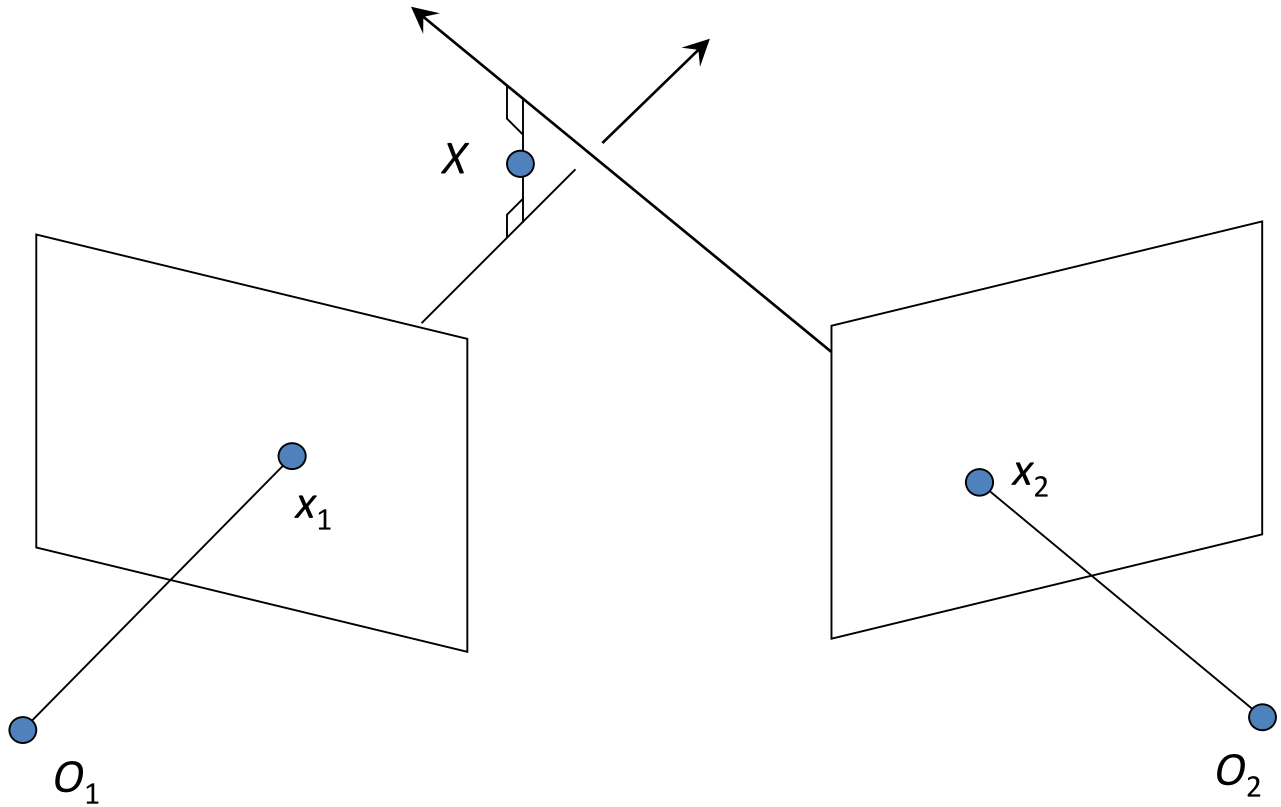
Triangulation

- We want to intersect the two visual rays corresponding to x1 and x2, but because of noise and numerical errors, they don't meet exactly



Triangulation: Geometric approach

- Find shortest segment connecting the two viewing rays and let X be the midpoint of that segment



Triangulation: Linear Approach

$$\lambda_1 \mathrm{x}_1 = \mathrm{P}_1 \mathrm{X} \qquad \mathrm{x}_1 \times \mathrm{P}_1 \mathrm{X} = 0 \qquad [\mathrm{x}_{1\times}]\mathrm{P}_1 \mathrm{X} = 0$$

$$\lambda_2 \mathrm{x}_2 = \mathrm{P}_2 \mathrm{X} \qquad \mathrm{x}_2 \times \mathrm{P}_2 \mathrm{X} = 0 \qquad [\mathrm{x}_{2\times}]\mathrm{P}_2 \mathrm{X} = 0$$

Cross product as matrix multiplication:

$$\mathbf{a} \times \mathbf{b} = \begin{bmatrix} 0 & -a_z & a_y \\ a_z & 0 & -a_x \\ -a_y & a_x & 0 \end{bmatrix} \begin{bmatrix} b_x \\ b_y \\ b_z \end{bmatrix} = [\mathbf{a}_\times]\mathbf{b}$$

Triangulation: Linear Approach

$$\lambda_1 \mathrm{x}_1 = \mathrm{P}_1 \mathrm{X} \qquad \mathrm{x}_1 \times \mathrm{P}_1 \mathrm{X} = 0 \qquad [\mathrm{x}_{1\times}]\mathrm{P}_1 \mathrm{X} = 0$$

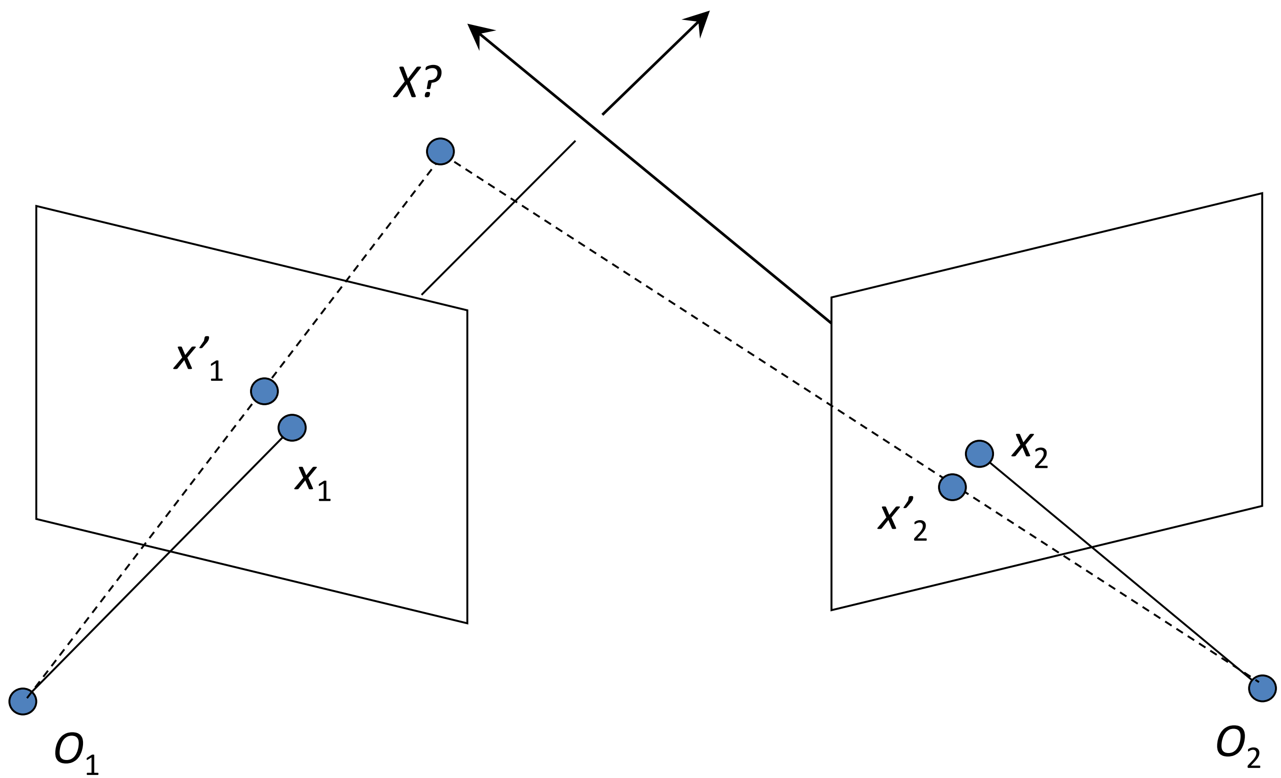
$$\lambda_2 \mathrm{x}_2 = \mathrm{P}_2 \mathrm{X} \qquad \mathrm{x}_2 \times \mathrm{P}_2 \mathrm{X} = 0 \qquad [\mathrm{x}_{2\times}]\mathrm{P}_2 \mathrm{X} = 0$$

Two independent equations each in terms of three unknown entries of X

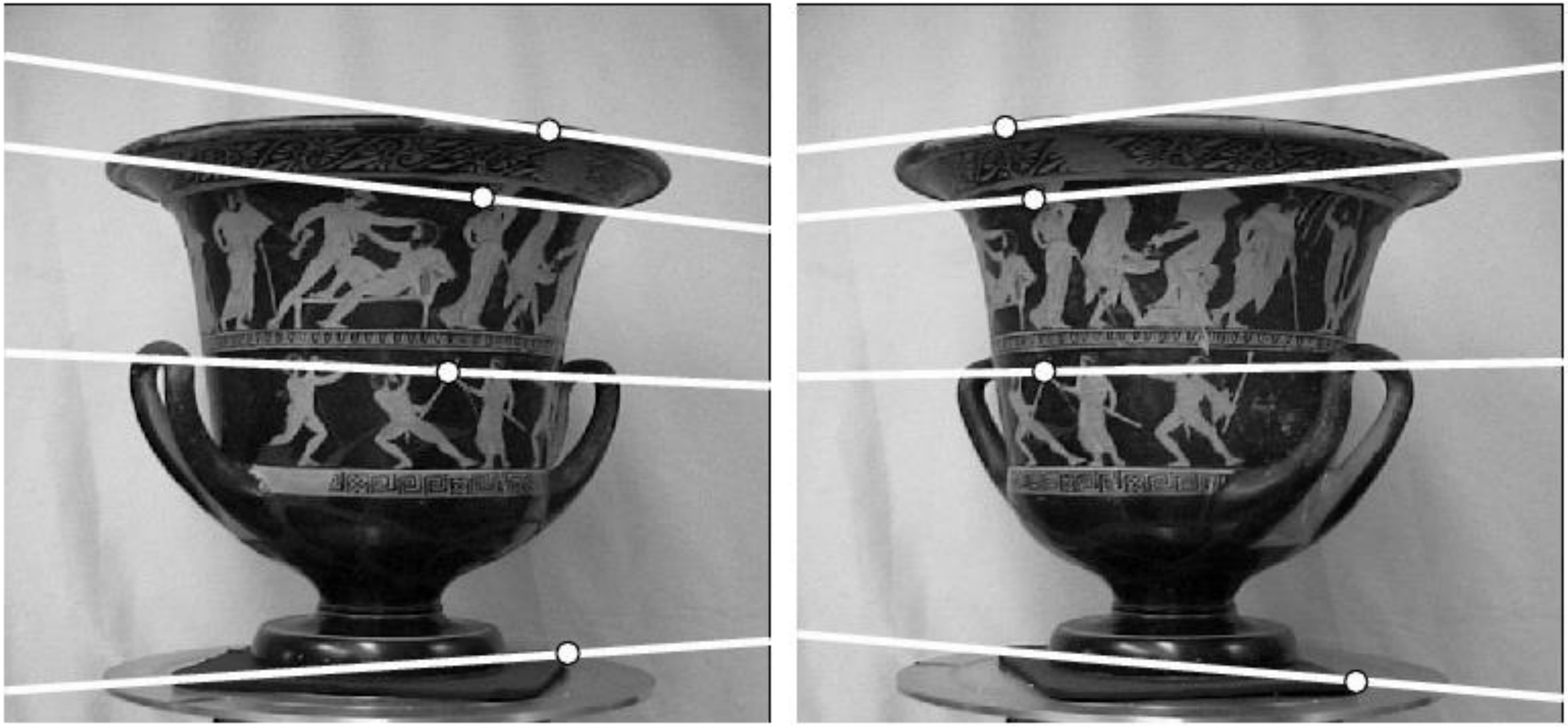
Triangulation: Nonlinear Approach

- Find X that minimizes

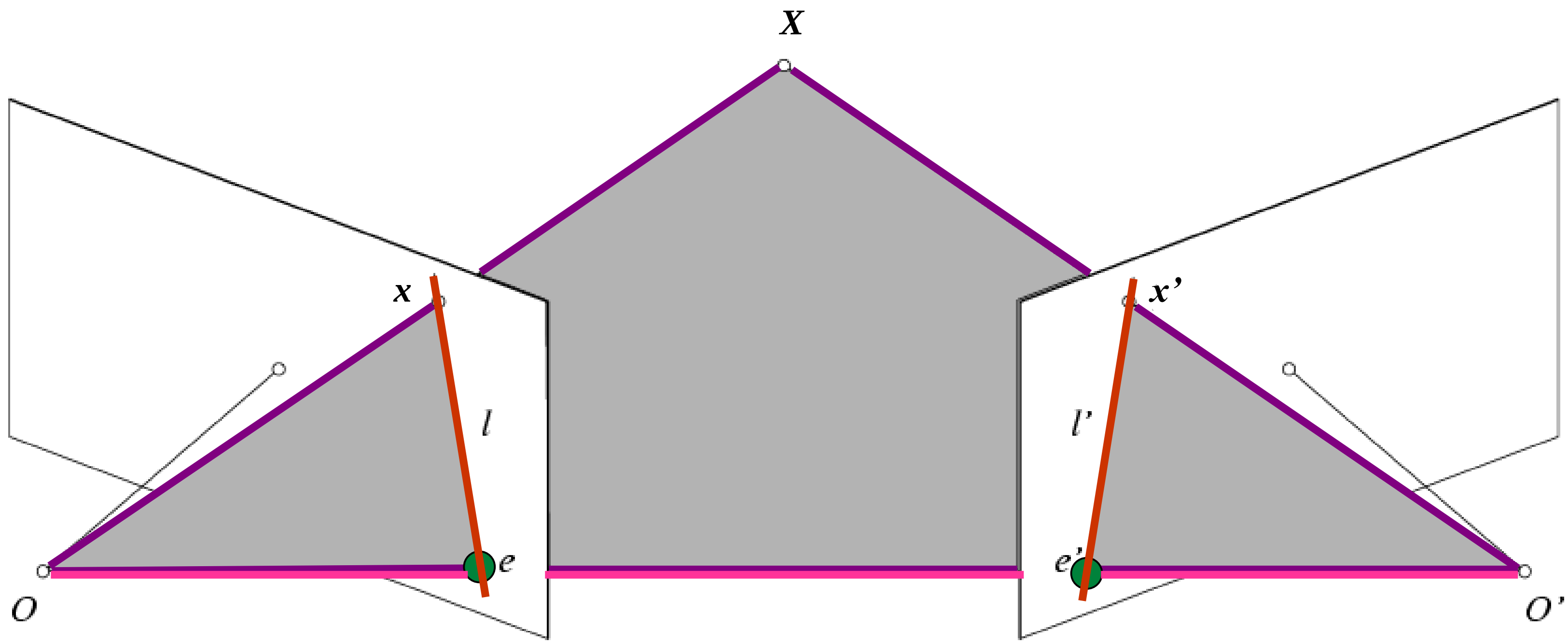
$$d^2(x_1, P_1 X) + d^2(x_2, P_2 X)$$



Two-view Geometry



Epipolar Geometry



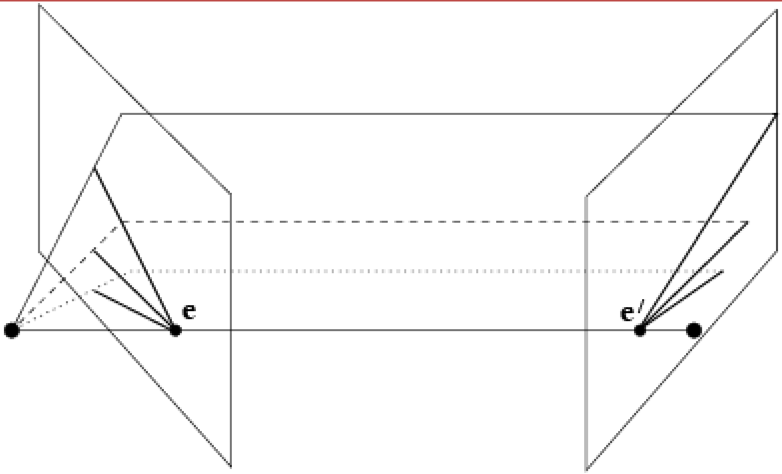
- **Baseline** – line connecting the two camera centres
- **Epipolar Plane** – plane containing baseline (1D family)
- **Epipoles**

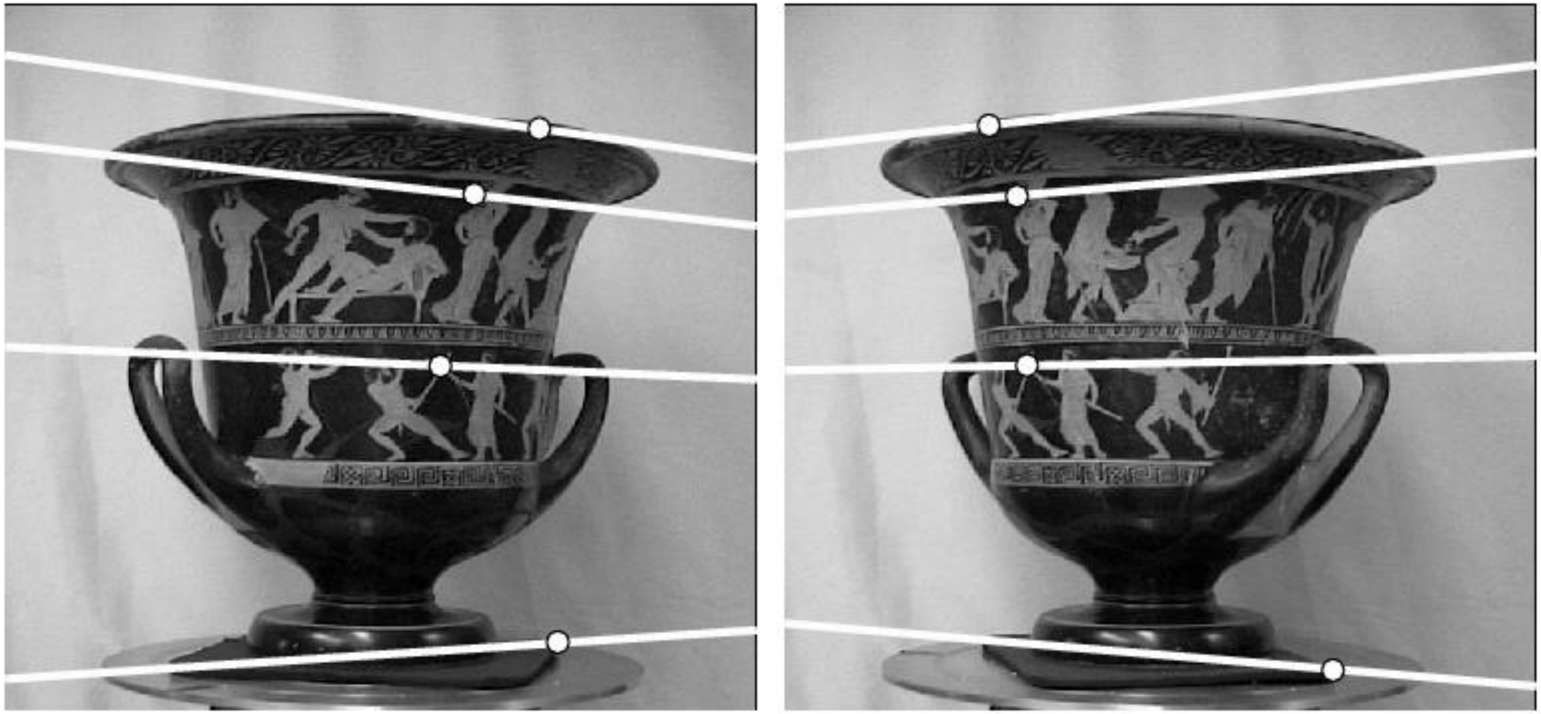
= intersections of baseline with image planes

= projections of the other camera centre

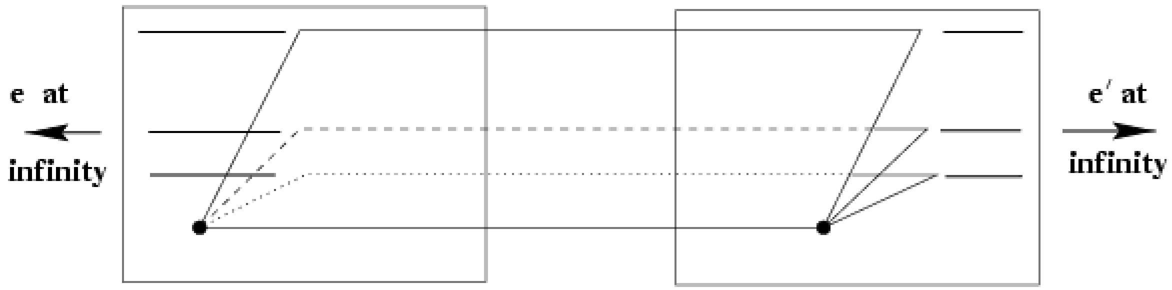
- **Epipolar Lines** - intersections of epipolar plane with image planes (always come in corresponding pairs)

Example: Converging Cameras





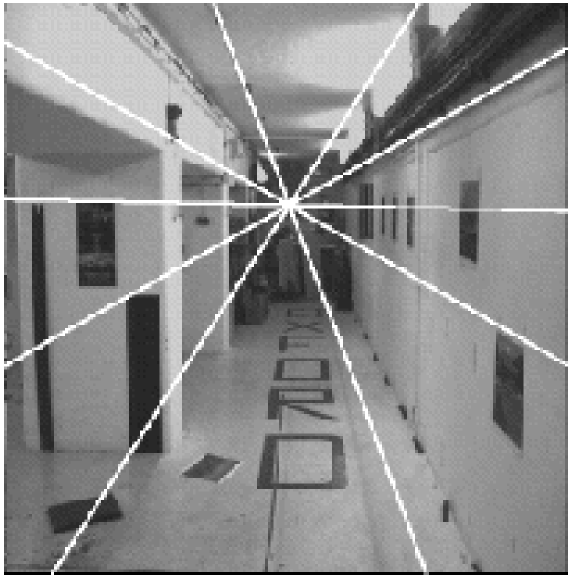
Example: Motion Parallel to Image Plane

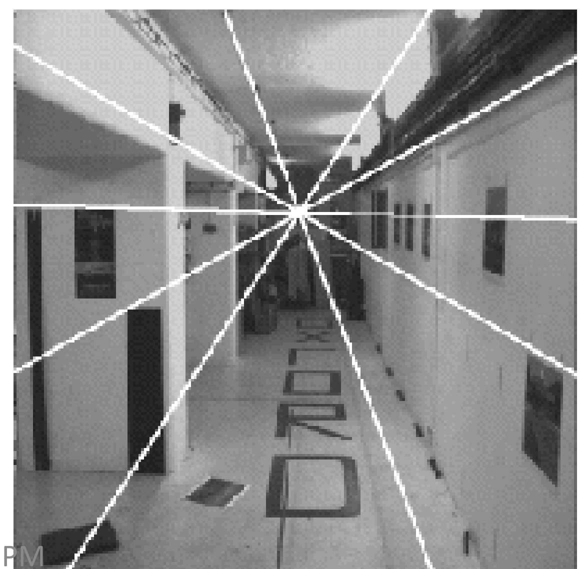


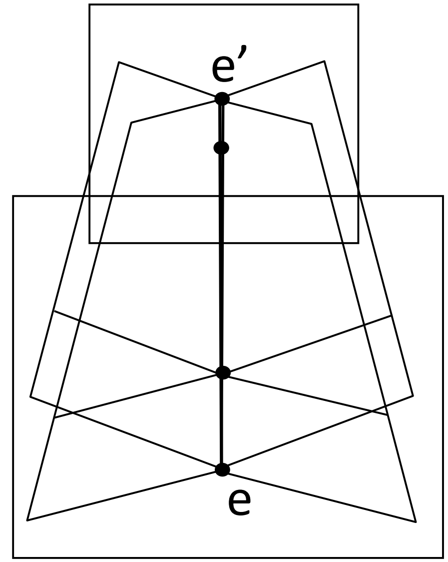
Example: Motion Perpendicular to Image Plane

Example: Motion Perpendicular to Image Plane

Example: Motion Perpendicular to Image Plane

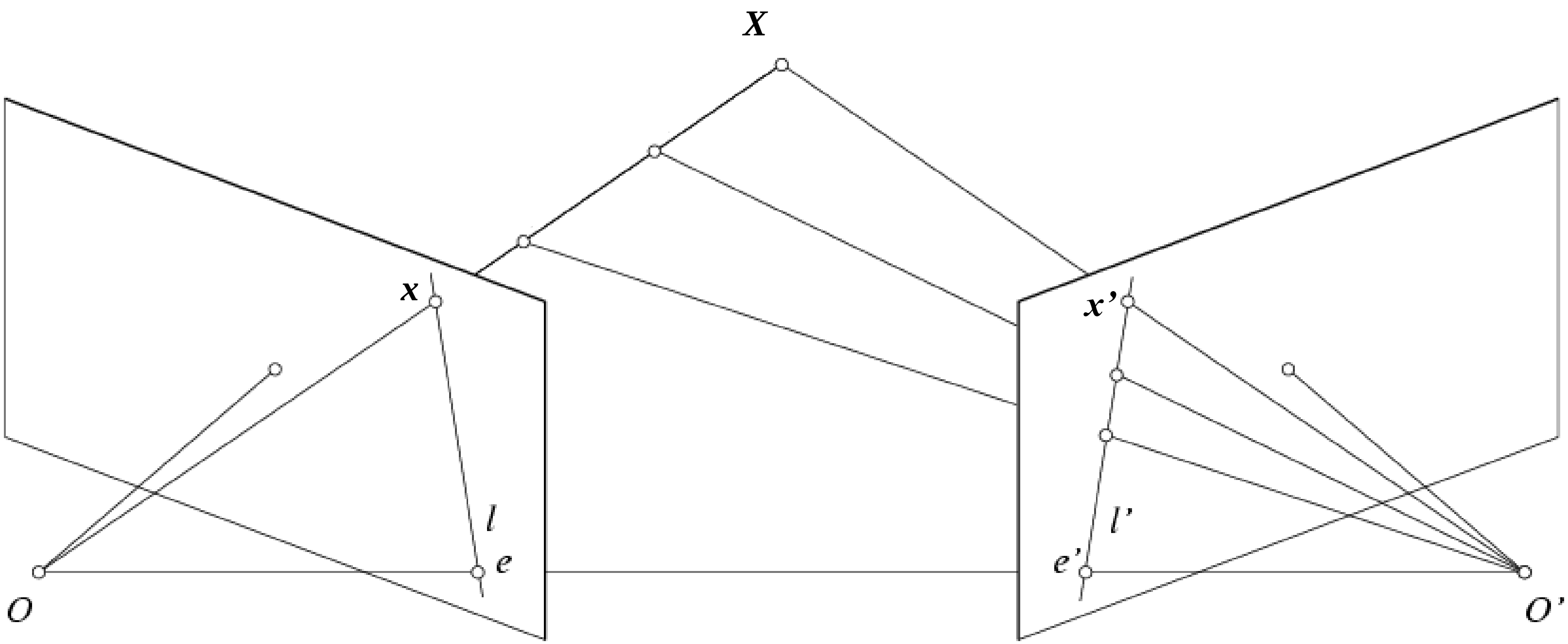






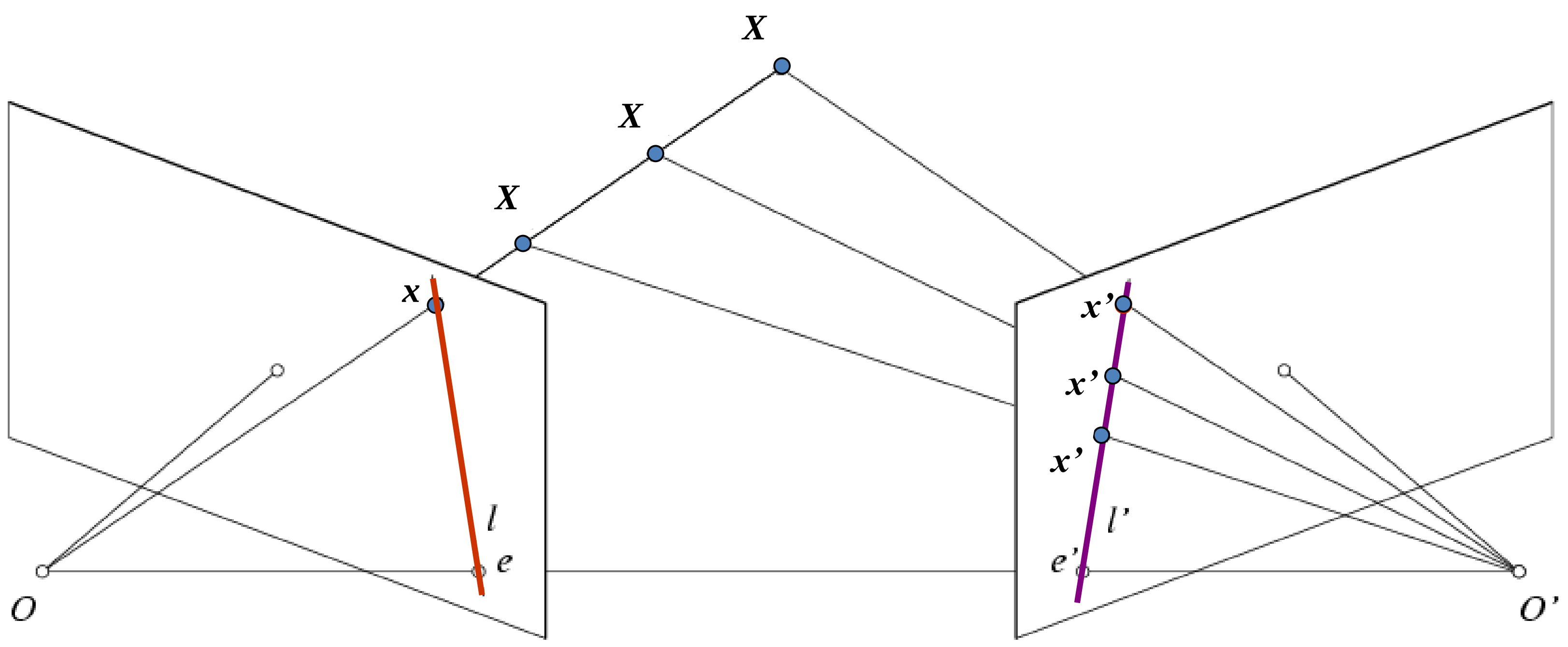
Epipoles have same coordinates in both images.
Points move along lines radiating from e: “Focus of expansion”

Epipolar Constraint



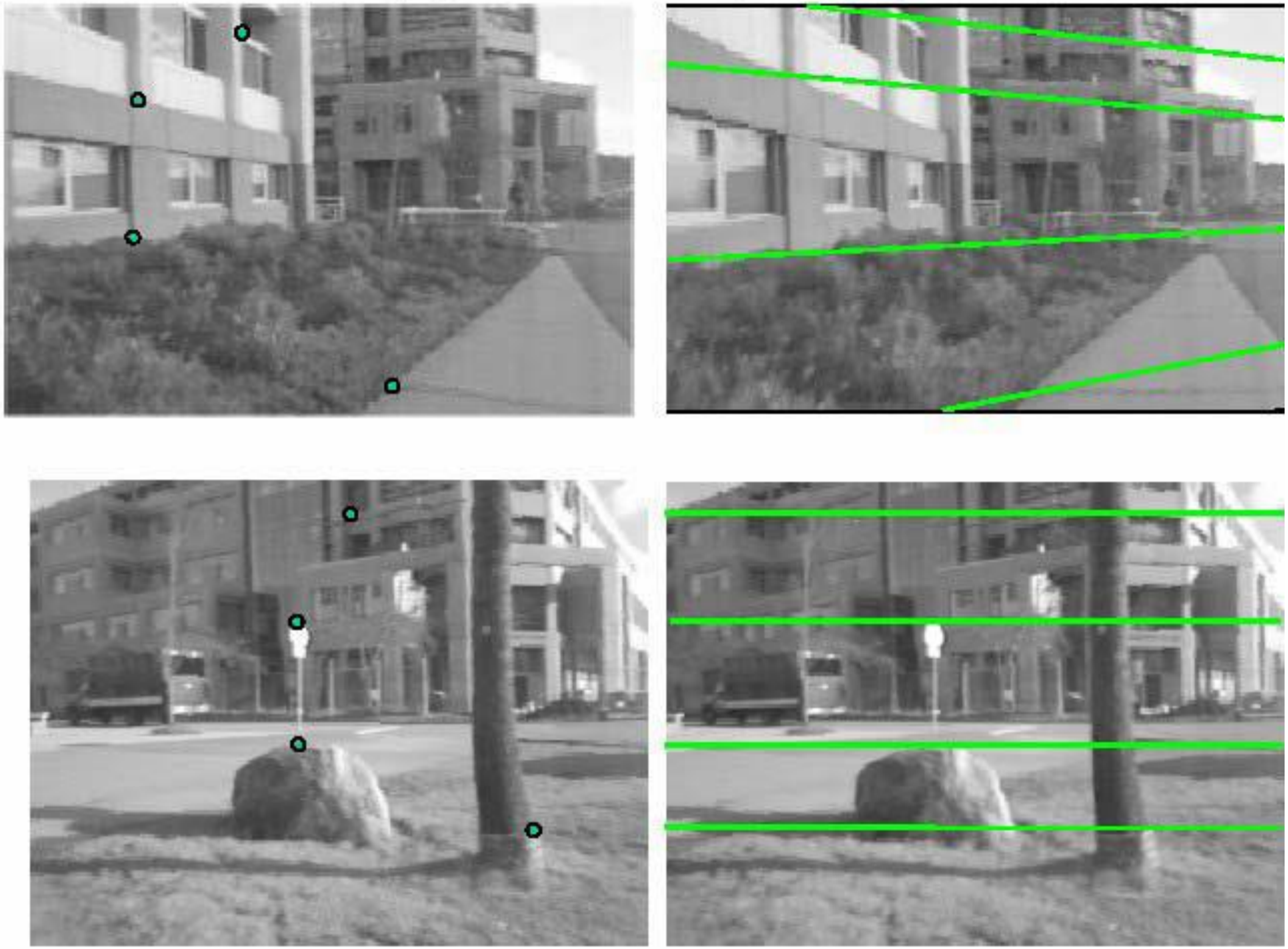
- If we observe a point x in one image, where can the corresponding point x' be in the other image?

Epipolar Constraint

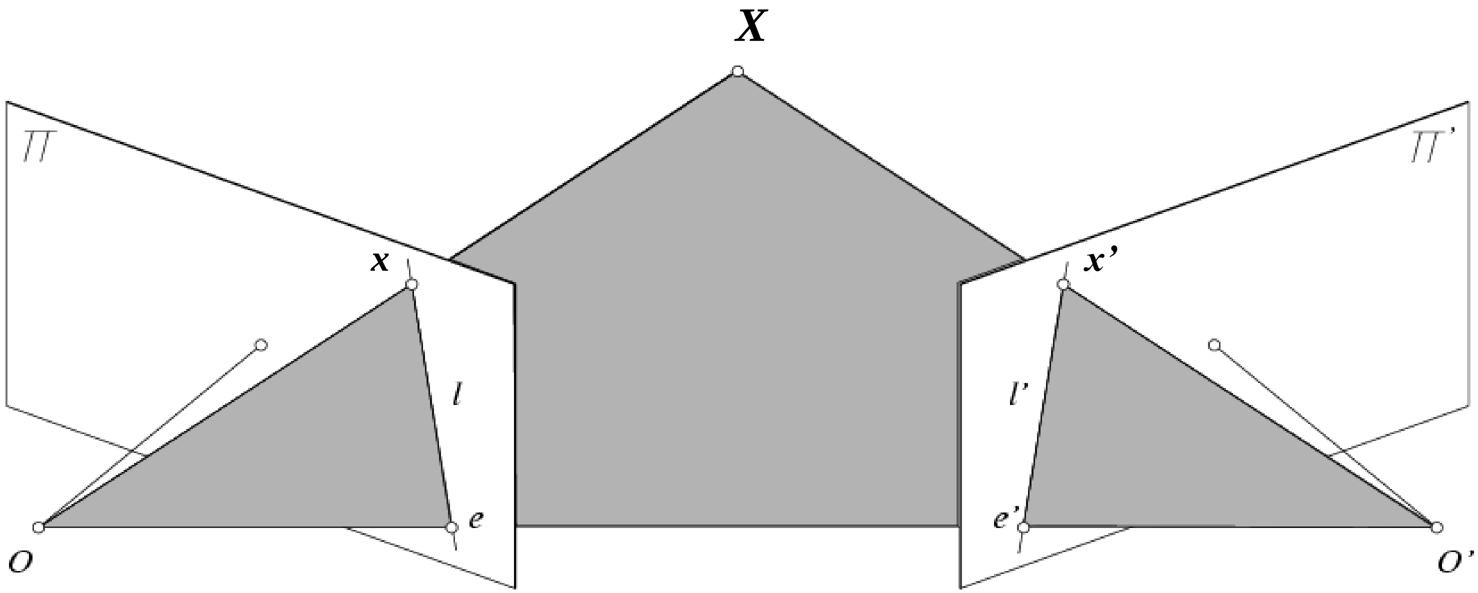


- Potential matches for *x* have to lie on the corresponding epipolar line *l'*.
- Potential matches for *x'* have to lie on the corresponding epipolar line *l*.

Epipolar Constraint Example

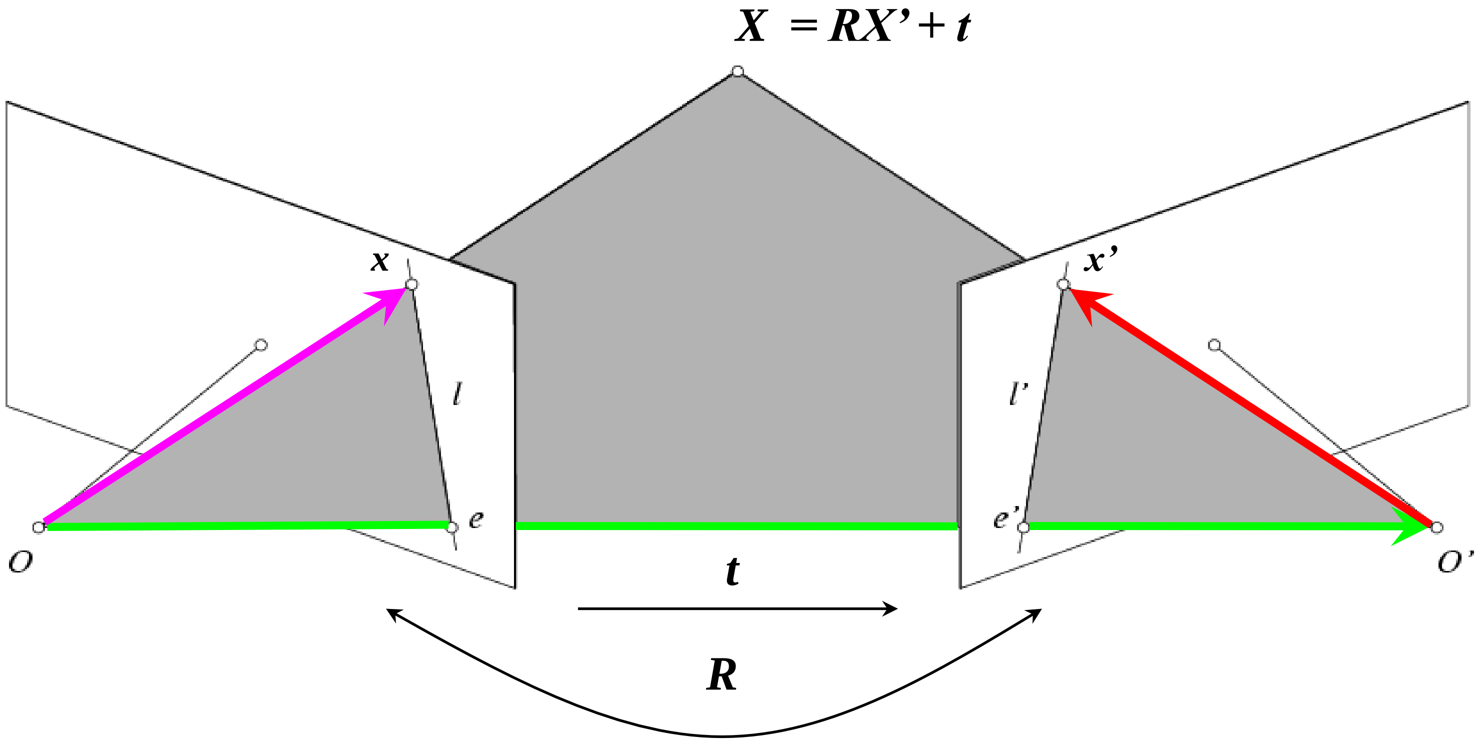


Epipolar Constraint: Calibrated Case



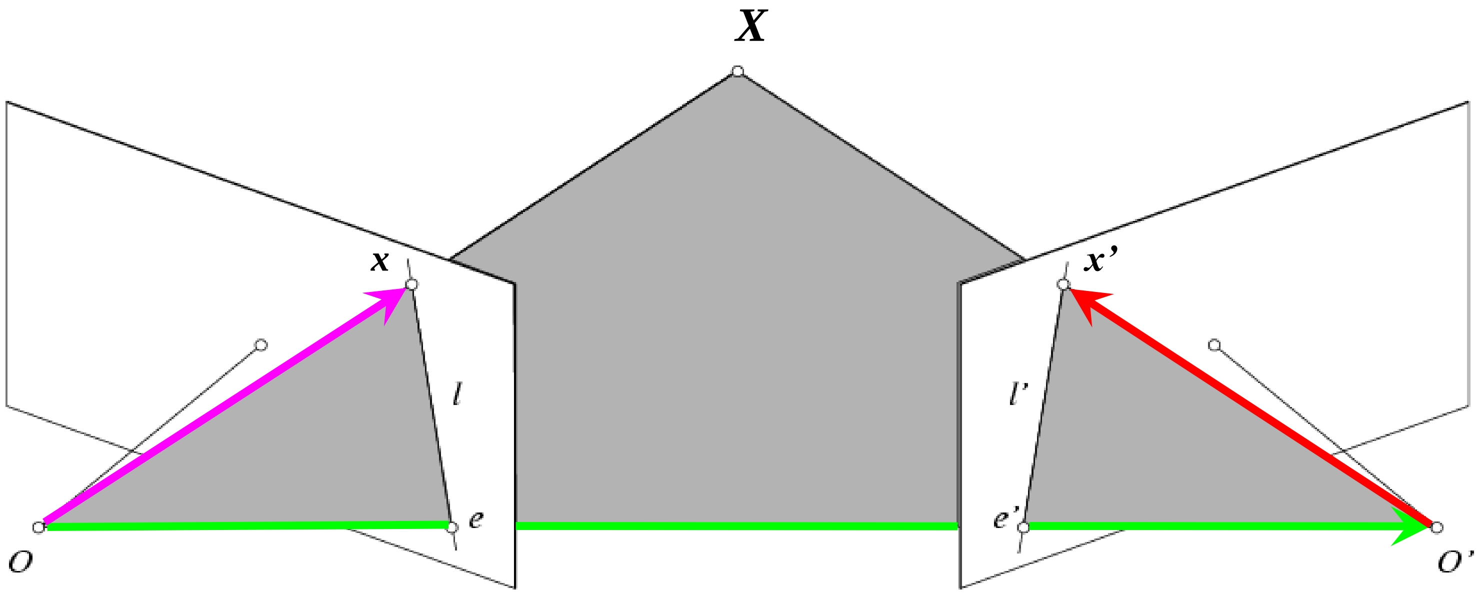
- Assume that the intrinsic and extrinsic parameters of the cameras are known
- We can multiply the projection matrix of each camera (and the image points) by the inverse of the calibration matrix to get *normalized* image coordinates
- We can also set the global coordinate system to the coordinate system of the first camera. Then the projection matrix of the first camera is $[\mathbf{I} \mid \mathbf{0}]$.

Epipolar Constraint: Calibrated Case



The vectors $\mathbf{x}$, $\mathbf{t}$, and $R\mathbf{x}'$ are coplanar

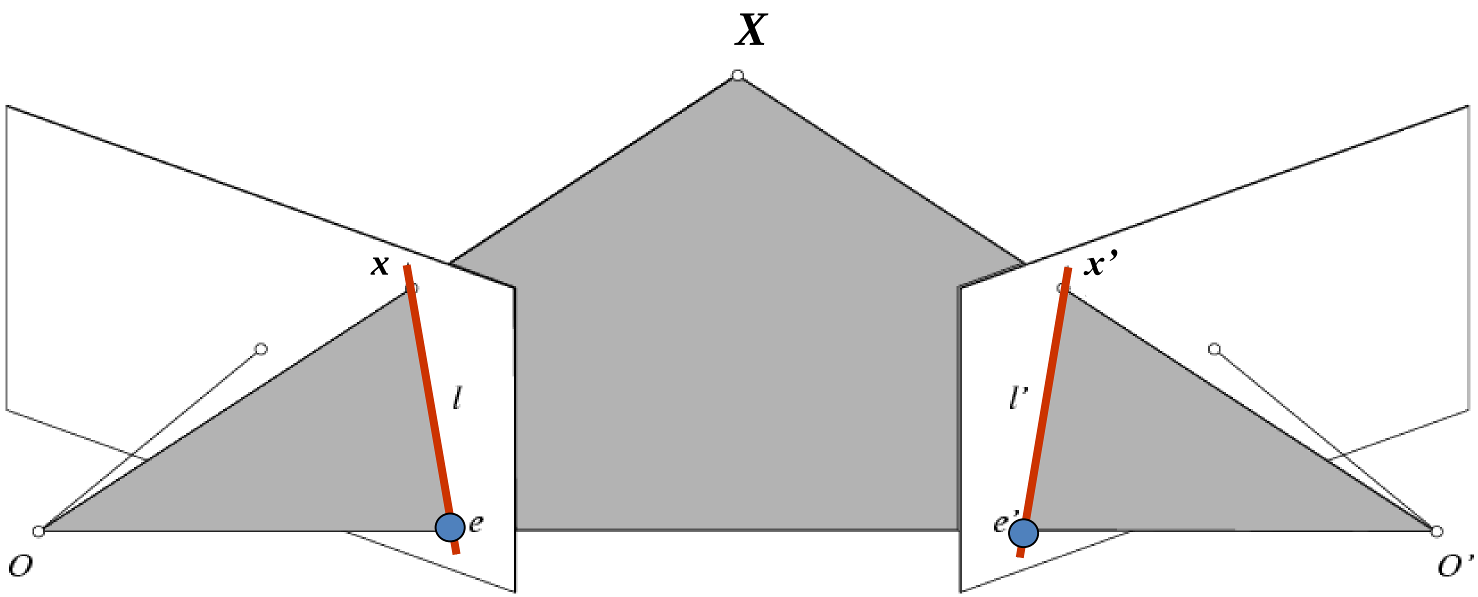
Epipolar Constraint: Calibrated Case



$$x \cdot [t \times (Rx')] = 0 \quad \Rightarrow \quad x^T E x' = 0 \quad \text{with} \quad E = [t_\times]R$$

Essential Matrix
(Longuet-Higgins, 1981)

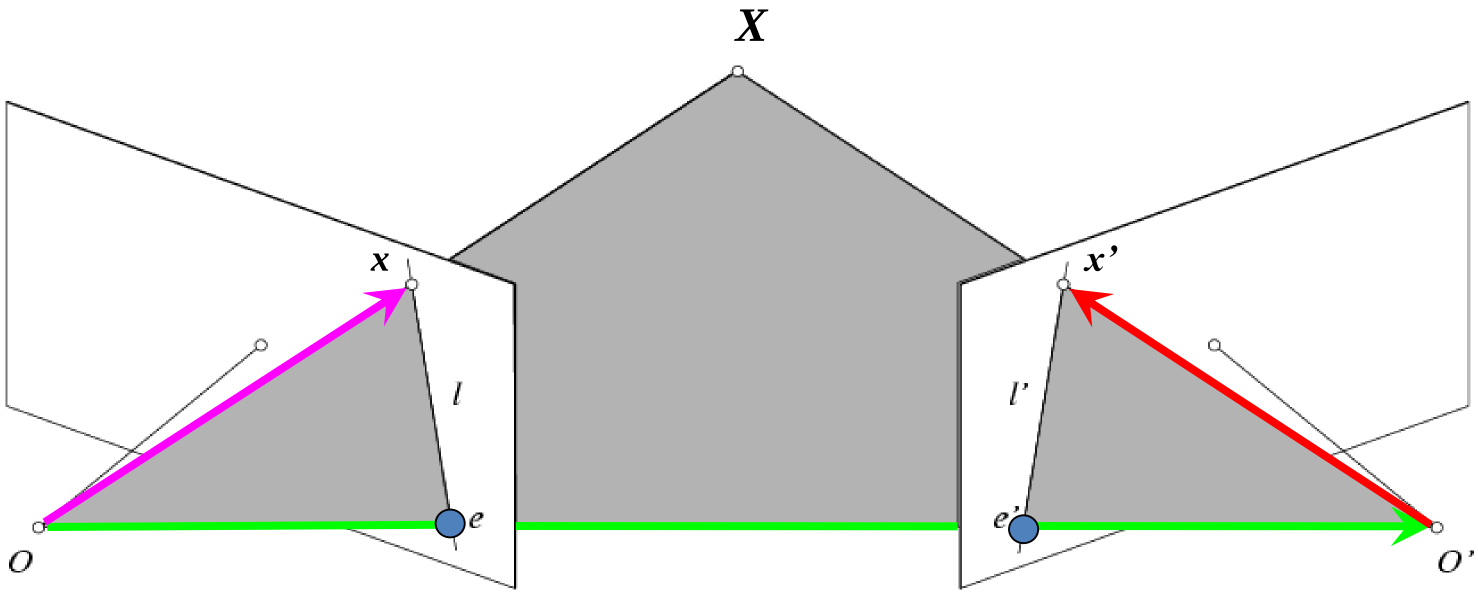
Epipolar Constraint: Calibrated Case



$$x \cdot [t \times (Rx')] = 0 \quad \Rightarrow \quad x^T E x' = 0 \quad \text{with} \quad E = [t_\times]R$$

- $E\,x'$ is the epipolar line associated with x' ($l = E\,x'$)
- $E^T x$ is the epipolar line associated with x ($l' = E^T x$)
- $E\,e' = 0$ and $E^T e = 0$
- E is singular (rank two)
- E has five degrees of freedom

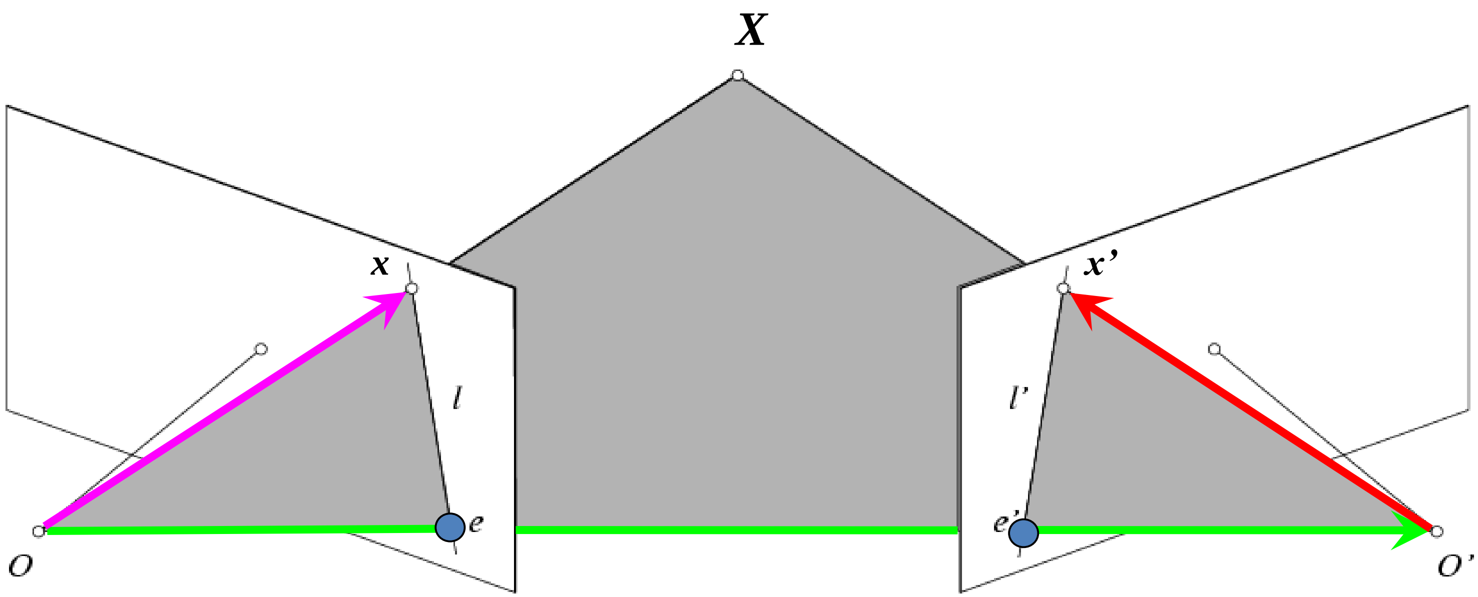
Epipolar Constraint: Uncalibrated Case



- The calibration matrices *K* and *K'* of the two cameras are unknown
- We can write the epipolar constraint in terms of *unknown* normalized coordinates:

$$\hat{x}^T E \hat{x}' = 0 \qquad\qquad x = K\hat{x}, \quad x' = K'\hat{x}'$$

Epipolar Constraint: Uncalibrated Case



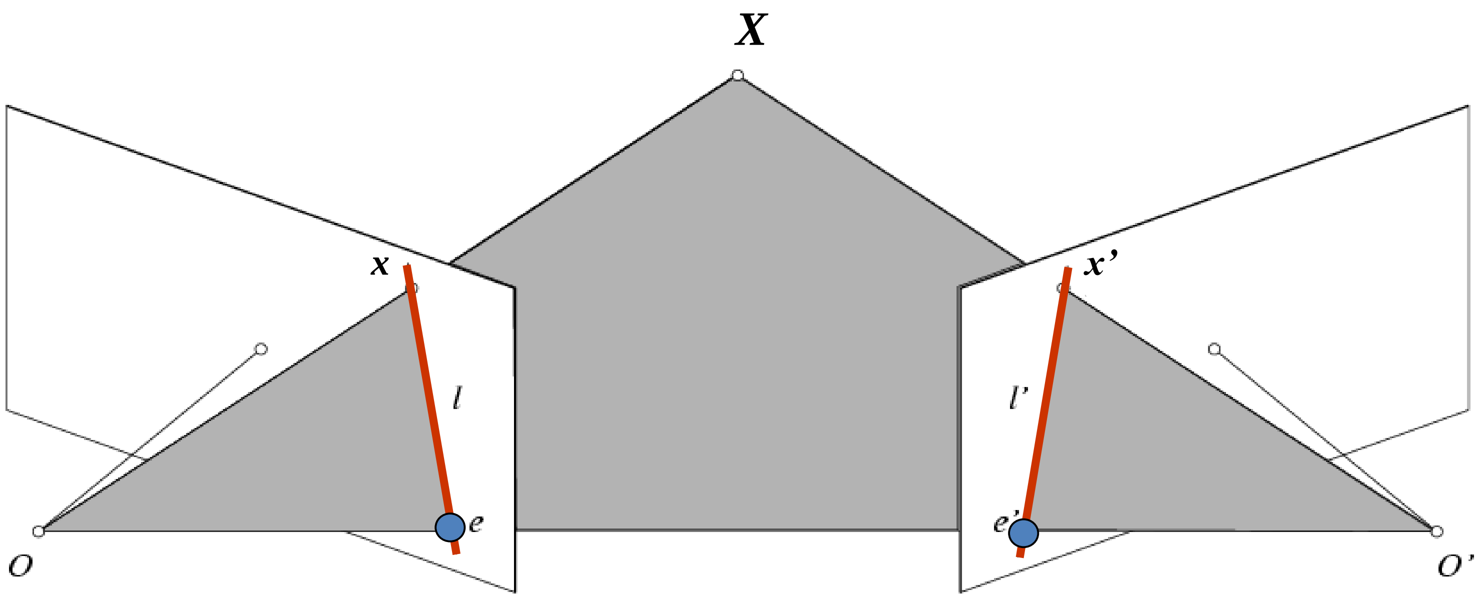
$$\hat{x}^T E \hat{x}' = 0 \quad \Rightarrow \quad x^T F x' = 0 \quad \text{with} \quad F = K^{-T} E K'^{-1}$$

$$\hat{x} = K^{-1} x$$

$$\hat{x}' = K'^{-1} x'$$

Fundamental Matrix
(Faugeras and Luong, 1992)

Epipolar Constraint: Uncalibrated Case



$$\hat{x}^T E \hat{x}' = 0 \quad \Rightarrow \quad x^T F x' = 0 \quad \text{with} \quad F = K^{-T} E K'^{-1}$$

- $F x'$ is the epipolar line associated with x' $(l = F x')$
- $F^T x$ is the epipolar line associated with x $(l' = F^T x)$
- $F e' = 0$ and $F^T e = 0$
- F is singular (rank two)
- F has seven degrees of freedom

The Eight-point Algorithm

$\boldsymbol{x} = (u, v, 1)^T, \quad \boldsymbol{x}' = (u', v', 1)^T$

$$(u, v, 1)\begin{pmatrix} F_{11} & F_{12} & F_{13} \\ F_{21} & F_{22} & F_{23} \\ F_{31} & F_{32} & F_{33} \end{pmatrix}\begin{pmatrix} u' \\ v' \\ 1 \end{pmatrix} = 0$$

$$(uu', uv', u, vu', vv', v, u', v', 1)\begin{pmatrix} F_{11} \\ F_{12} \\ F_{13} \\ F_{21} \\ F_{22} \\ F_{23} \\ F_{31} \\ F_{32} \\ F_{33} \end{pmatrix} = 0$$

$$\begin{pmatrix} u_1u_1' & u_1v_1' & u_1 & v_1u_1' & v_1v_1' & v_1 & u_1' & v_1' \\ u_2u_2' & u_2v_2' & u_2 & v_2u_2' & v_2v_2' & v_2 & u_2' & v_2' \\ u_3u_3' & u_3v_3' & u_3 & v_3u_3' & v_3v_3' & v_3 & u_3' & v_3' \\ u_4u_4' & u_4v_4' & u_4 & v_4u_4' & v_4v_4' & v_4 & u_4' & v_4' \\ u_5u_5' & u_5v_5' & u_5 & v_5u_5' & v_5v_5' & v_5 & u_5' & v_5' \\ u_6u_6' & u_6v_6' & u_6 & v_6u_6' & v_6v_6' & v_6 & u_6' & v_6' \\ u_7u_7' & u_7v_7' & u_7 & v_7u_7' & v_7v_7' & v_7 & u_7' & v_7' \\ u_8u_8' & u_8v_8' & u_8 & v_8u_8' & v_8v_8' & v_8 & u_8' & v_8' \end{pmatrix}\begin{pmatrix} F_{11} \\ F_{12} \\ F_{13} \\ F_{21} \\ F_{22} \\ F_{23} \\ F_{31} \\ F_{32} \end{pmatrix} = -\begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

Minimize:

$$\sum_{i=1}^{N} (x_i^T F x_i')^2$$

under the constraint

$F_{33} = 1$

The Eight-point Algorithm

- Meaning of error $\sum_{i=1}^{N}(x_i^T F x_i')^2$:

  sum of Euclidean distances between points x_i and epipolar lines Fx'_i (or points x'_i and epipolar lines $F^T x_i$) multiplied by a scale factor

- Nonlinear approach: minimize

$$\sum_{i=1}^{N}\left[\mathrm{d}^2(x_i, F x_i') + \mathrm{d}^2(x_i', F^T x_i)\right]$$

Problem with Eight-point Algorithm

$$
\begin{pmatrix}
u_1u'_1 & u_1v'_1 & u_1 & v_1u'_1 & v_1v'_1 & v_1 & u'_1 & v'_1 \\
u_2u'_2 & u_2v'_2 & u_2 & v_2u'_2 & v_2v'_2 & v_2 & u'_2 & v'_2 \\
u_3u'_3 & u_3v'_3 & u_3 & v_3u'_3 & v_3v'_3 & v_3 & u'_3 & v'_3 \\
u_4u'_4 & u_4v'_4 & u_4 & v_4u'_4 & v_4v'_4 & v_4 & u'_4 & v'_4 \\
u_5u'_5 & u_5v'_5 & u_5 & v_5u'_5 & v_5v'_5 & v_5 & u'_5 & v'_5 \\
u_6u'_6 & u_6v'_6 & u_6 & v_6u'_6 & v_6v'_6 & v_6 & u'_6 & v'_6 \\
u_7u'_7 & u_7v'_7 & u_7 & v_7u'_7 & v_7v'_7 & v_7 & u'_7 & v'_7 \\
u_8u'_8 & u_8v'_8 & u_8 & v_8u'_8 & v_8v'_8 & v_8 & u'_8 & v'_8
\end{pmatrix}
\begin{pmatrix} F_{11} \\ F_{12} \\ F_{13} \\ F_{21} \\ F_{22} \\ F_{23} \\ F_{31} \\ F_{32} \end{pmatrix}
= -
\begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}
$$

Problem with Eight-point Algorithm

250906.36	183269.57	921.81	200931.10	146766.13	738.21	272.19	198.81
2692.28	131633.03	176.27	6196.73	302975.59	405.71	15.27	746.79
416374.23	871684.30	935.47	408110.89	854384.92	916.90	445.10	931.81
191183.60	171759.40	410.27	416435.62	374125.90	893.65	465.99	418.65
48988.86	30401.76	57.89	298604.57	185309.58	352.87	846.22	525.15
164786.04	546559.67	813.17	1998.37	6628.15	9.86	202.65	672.14
116407.01	2727.75	138.89	169941.27	3982.21	202.77	838.12	19.64
135384.58	75411.13	198.72	411350.03	229127.78	603.79	681.28	379.48

$$\begin{pmatrix} F_{11} \\ F_{12} \\ F_{13} \\ F_{21} \\ F_{22} \\ F_{23} \\ F_{31} \\ F_{32} \end{pmatrix} = - \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

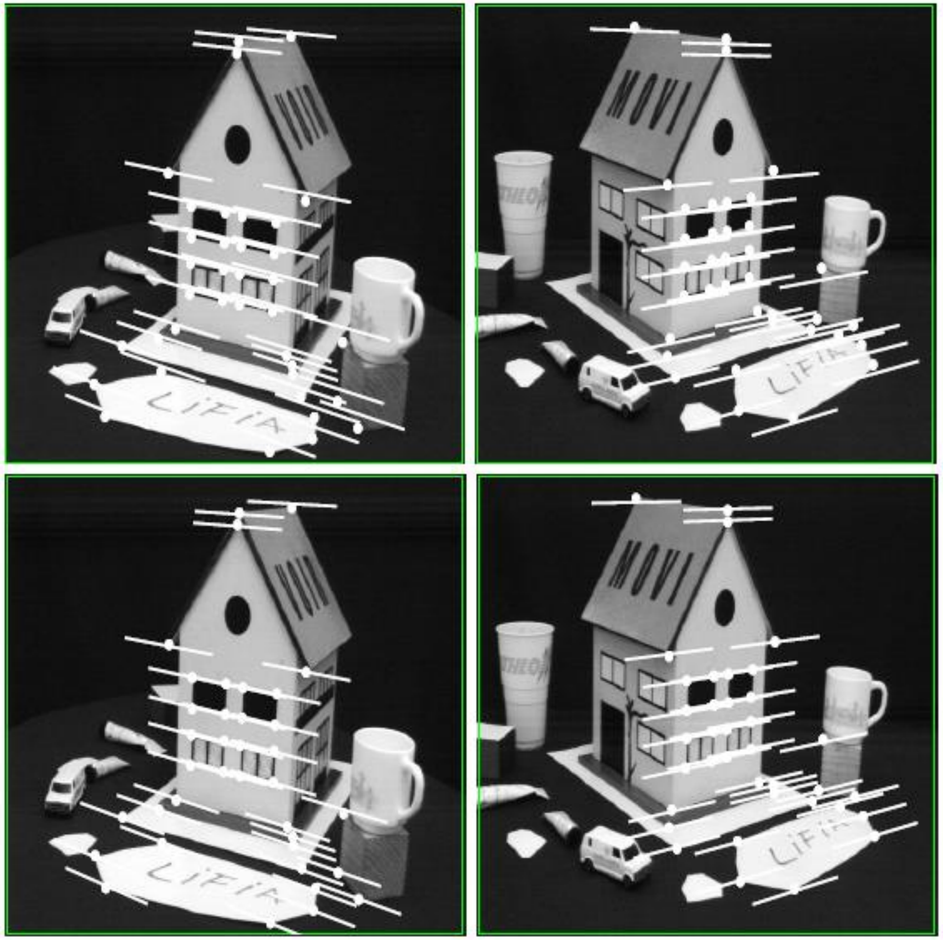
- Poor numerical conditioning
- Can be fixed by rescaling the data

The Normalized Eight-point Algorithm

(Hartley, 1995)

- Centre the image data at the origin, and scale it so the mean squared distance between the origin and the data points is 2 pixels
- Use the eight-point algorithm to compute *F* from the normalized points
- Enforce the rank-2 constraint (for example, take SVD of *F* and throw out the smallest singular value)
- Transform fundamental matrix back to original units: if T and T' are the normalizing transformations in the two images, than the fundamental matrix in original coordinates is $T^T F T'$

Comparison of Estimation Algorithms



	8-point	Normalized 8-point	Nonlinear least squares
Av. Dist. 1	2.33 pixels	0.92 pixel	0.86 pixel
Av. Dist. 2	2.18 pixels	0.85 pixel	0.80 pixel

From Epipolar Geometry to Camera Calibration

- Estimating the fundamental matrix is known as "weak calibration"
- If we know the calibration matrices of the two cameras, we can estimate the essential matrix: $E = K^T F K'$
- The essential matrix gives us the relative rotation and translation between the cameras, or their extrinsic parameters

Summary

- What is the problem of stereo vision?
- What is baseline? What are epipole, epipolar line, and epipolar plane? How to determine epipolar lines
- What is essential matrix? What is fundamental matrix?
- Describe eight-point algorithm.