Functional Programming

5. Analog Computing

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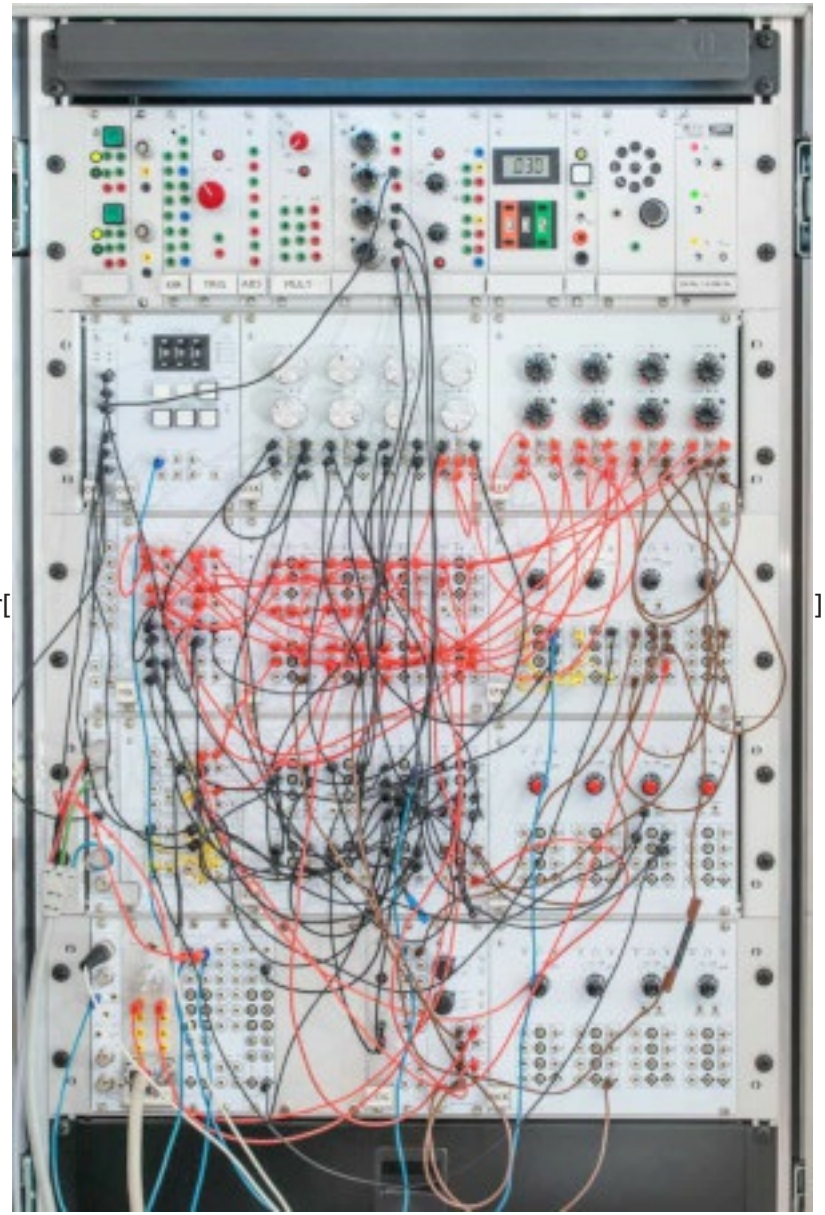
Analog Computers

- Network of interconnected components, very similar to functions, and more!
    - Combinatorial FPGA elements!
- Energy efficient? Additional capabilities?

.center[]

Operational Amplifier - Basic Building Block

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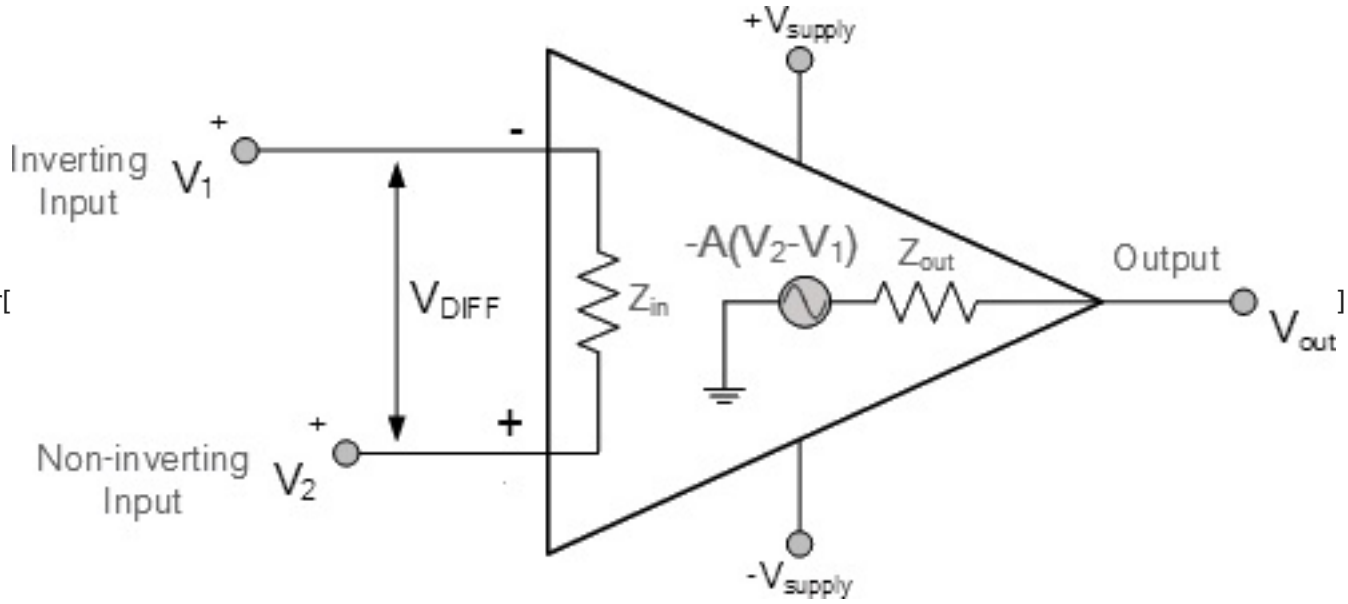

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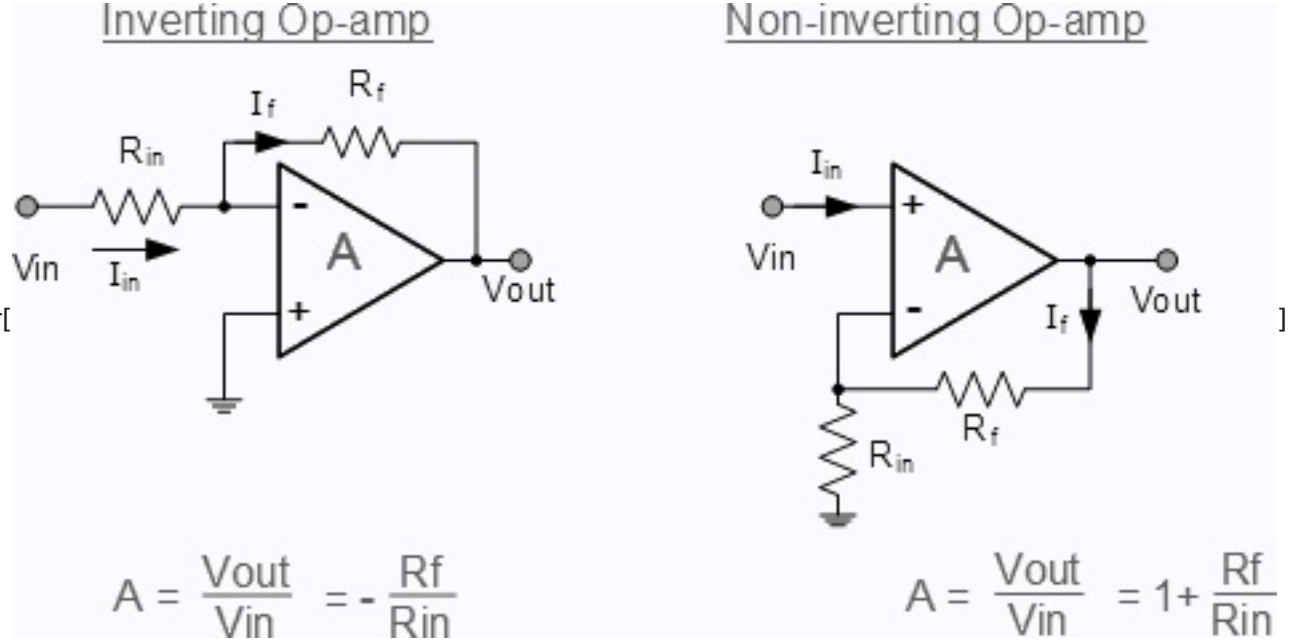

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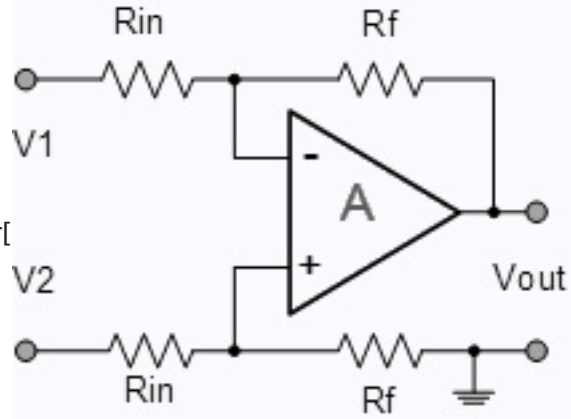
Operational Amplifier Circuits

Differential Op-amp

.center[

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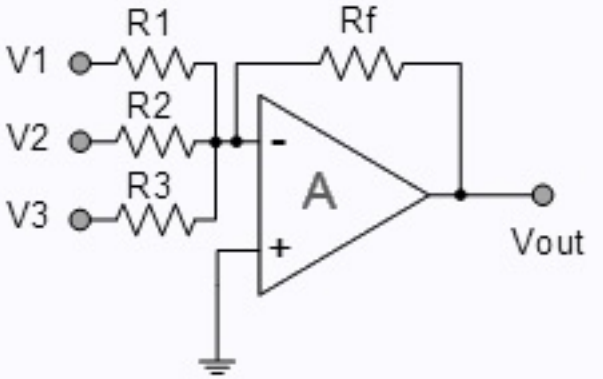
$$Vout = \frac{Rf}{Rin}(V2 - V1)$$

Summing Op-amp

.center[

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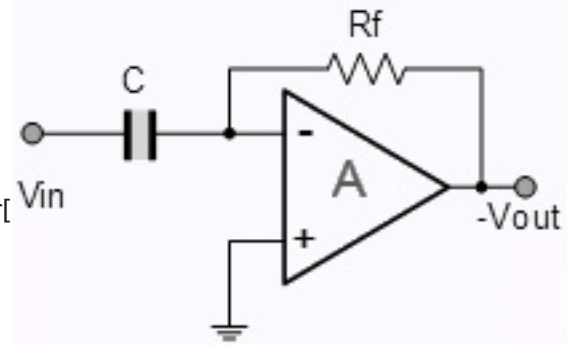
$$Vout = -\left(\frac{Rf}{R1}V1 + \frac{Rf}{R2}V2 + \frac{Rf}{R3}V3\right)$$

Differentiator Op-amp

.center[

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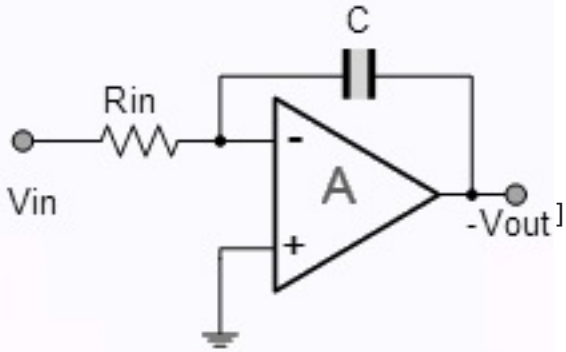
$$Vout = -Rf\ C\ \frac{dVin}{dt}$$

Integrator Op-amp

.center[

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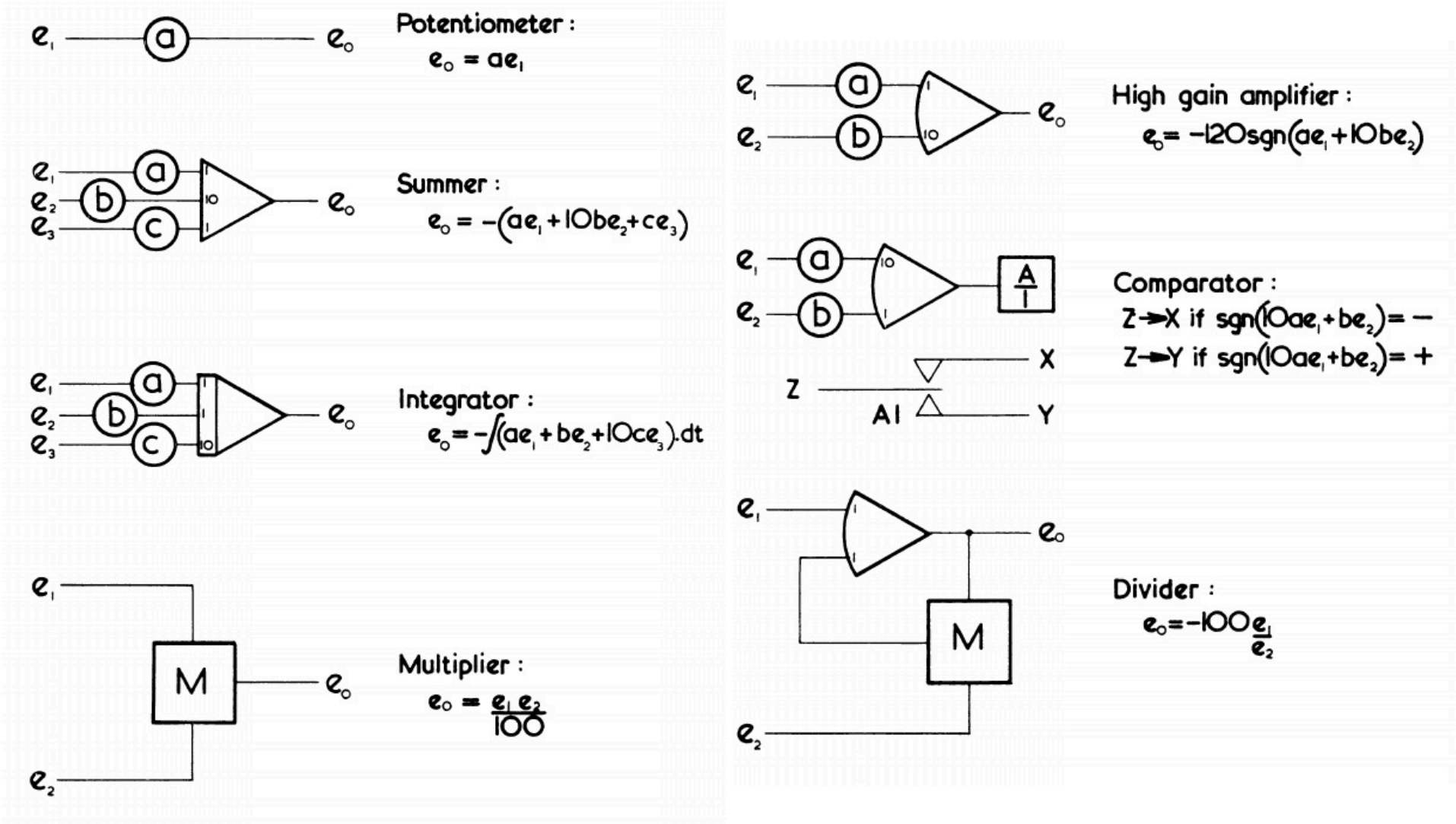
$$Vout = -\frac{1}{j\omega RinC}Vin$$

Basic Functions in Analog Circuits



- Based on feedback loops in operational amplifiers

- Evaluates continuous values!

Solving Differential Equations

- Predator-prey model (AJ Lotka; V Volterra)
    - Closed eco-system of two species: foxes and rabbits
    - Unlimited food supply for the rabbits
    - Foxes eat rabbits
    - Foxes can die of starvation
- Two coupled differential equations with `r` and `f` denoting the number of rabbits and foxes resp:

  ```
  dr/dt = alpha_1 r - alpha_2 r f
  ```

  ```
  df/dt = -beta_1 f + beta_2 r f
  ```

    - `alpha_1`: rabbit fertility rate
    - `alpha_2`: rate of rabbits killed by foxes
    - `beta_1`: fox fertility rate (negative sign, as foxes would die out without rabbits; note, rabbits without foxes would grow)
    - `beta_2`: rate of fox population increase due to rabbits eaten

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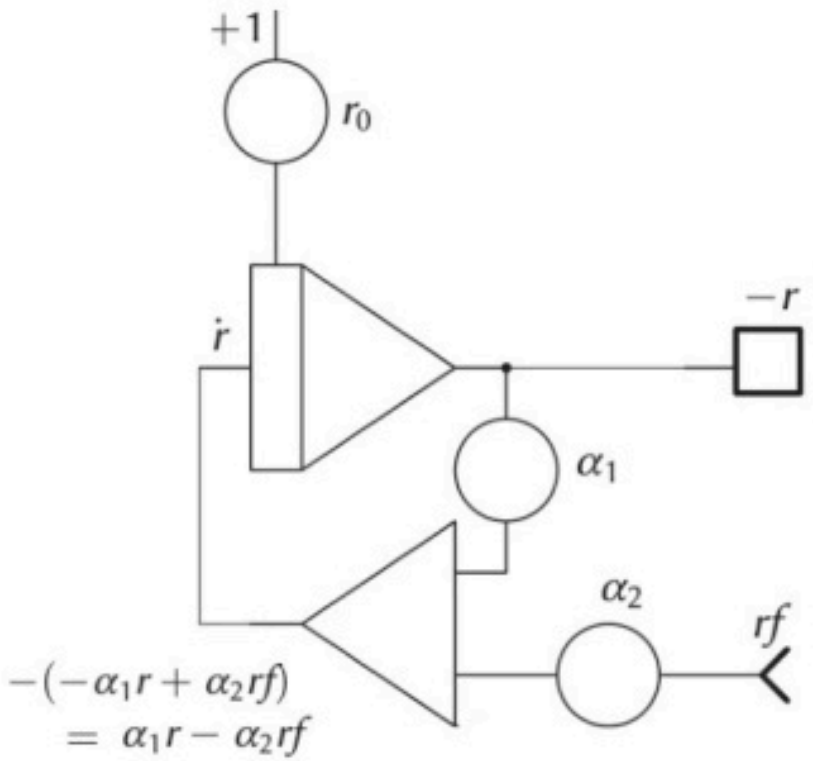
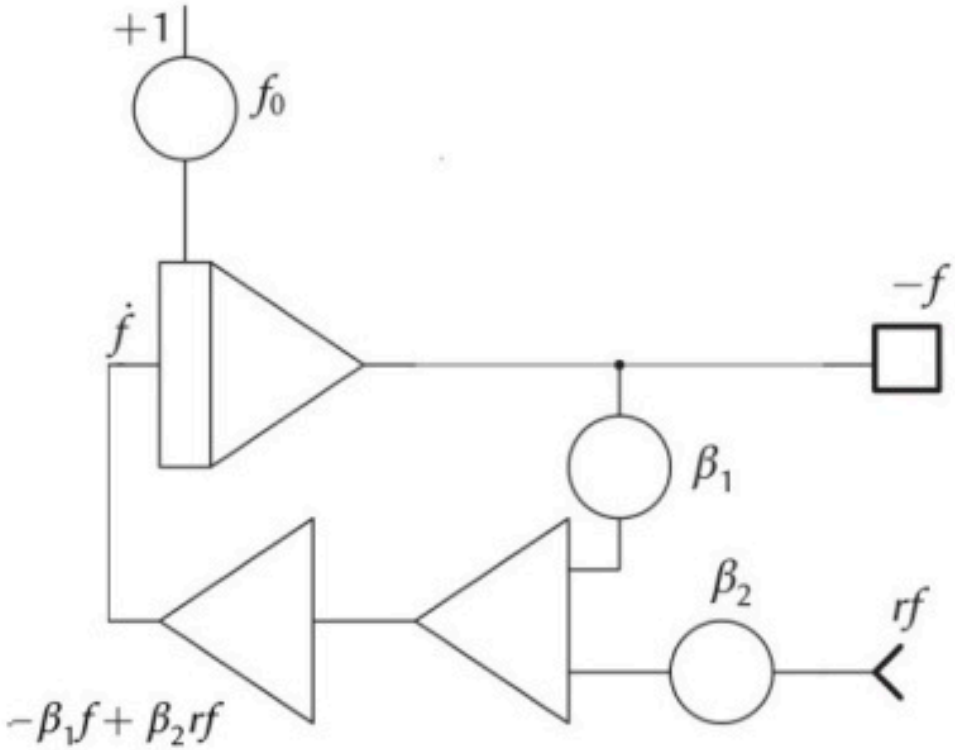
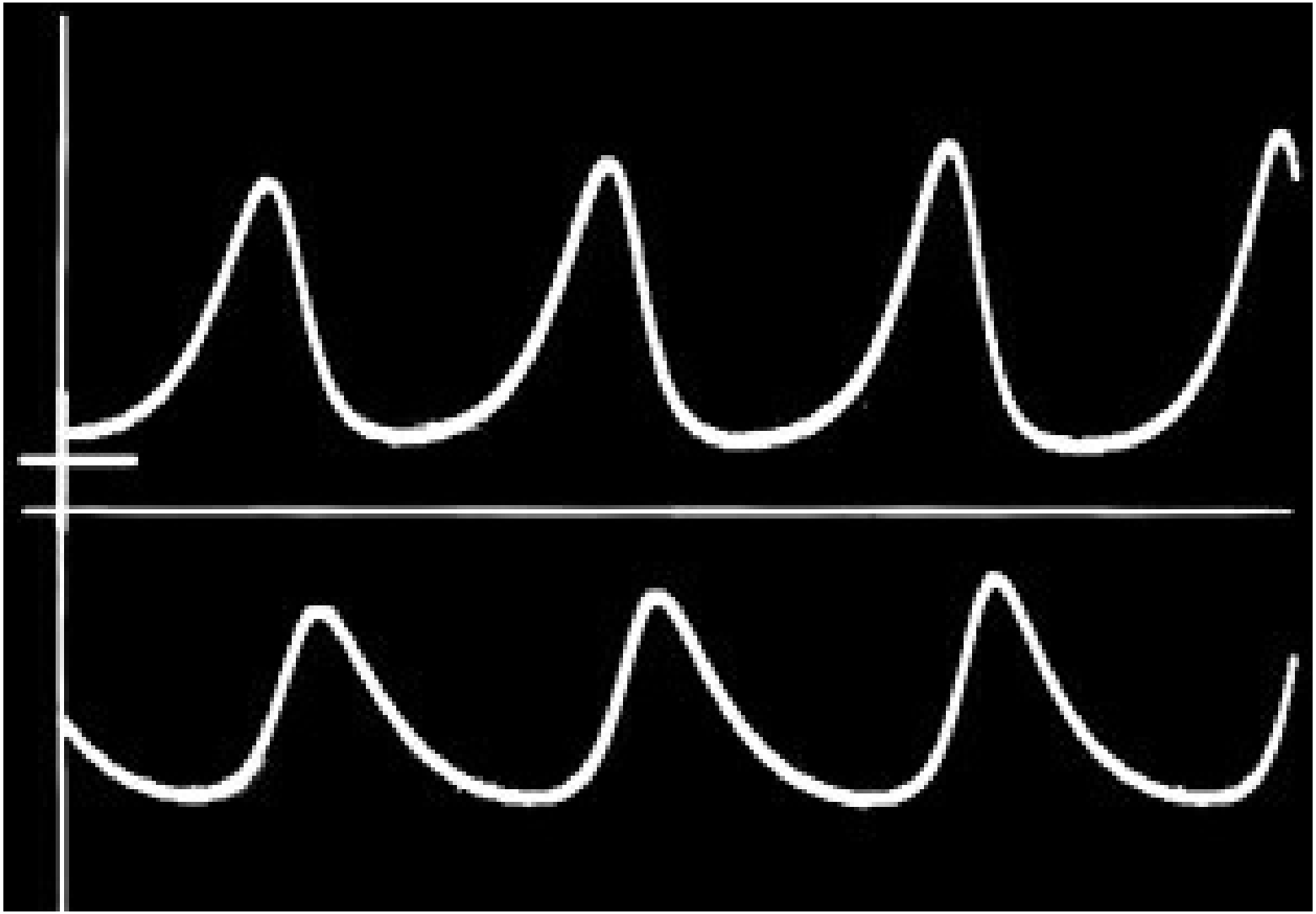
Analog Predator-Prey Simulator





- Left: Output `-r` with two inputs `r(0)` and `f(t)`
- Right: Output `-f` with input `f(0)`
- Still need a multiplier to get `rf` from `-r`, `-f`

Results



- Easy to change parameters