ASSIGNMENT2 - QUANTUM CIRCUIT ANALYSIS REPORT

CMT304 -Programming Paradigms

1. Quantum circuit analysis

1.1 circuit operators

There are 2 common single qubit gates in this circuit.

1. Pauli X Gate. This operator could half turn (π) around X, Y, Z axis which transform$|0\rangle$ to $|1\rangle$ and $|1\rangle$ to $|0\rangle$.

   $X = \begin{matrix} 0 & 1 \\ 1 & 0 \end{matrix}$

2. Control. This operator could control the gates in the same column, these gate only can work if this control meets the state $|1\rangle$.

There are 3 multi-qubit gates in this circuit.

1. Swap Gate. This operator could swap the values of two qubits, which transforms $|01\rangle$ to $|10\rangle$ and $|10\rangle$ to $|01\rangle$.

$$\mathrm{SWAP} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

2. Controlled-not (CNOT) gate. This operator contains a control and operates some qubits, and the x gates performs NOT operation only when the control is 1, otherwise it remains unchanged.

$$\mathrm{CNOT} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

3. Fredkin gate. This operator is a controlled swap gate that do the swap if control is 1, otherwise it remains unchanged.

$$\text{Fredkin gate} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

1.2 <u>mapping</u>

According to Input * (A|00b00a⟩ + B|00b01a⟩ + C|00b10a⟩ + D|00b11a⟩+ E|01b00a⟩+ F|01b01a⟩+ G|01b10a⟩+ H|01b11a⟩+ I|10b00a⟩+ J|10b01a⟩+ K|10b10a⟩+ L|10b11a⟩+ M|11b00a⟩+ N|11b01a⟩+ O|11b10a⟩+ P|11b11a⟩) = Output and after some experiments by Quirk we got the input-output mapping for |A⟩|B⟩ to |A’⟩|B’⟩:

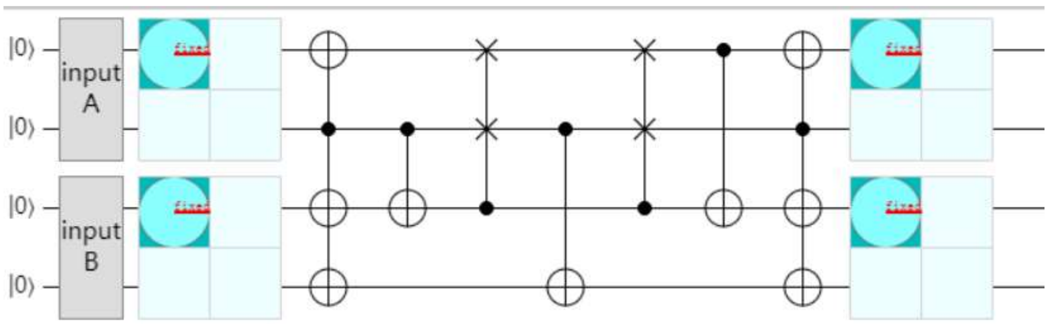

Figure 1 – quantum circuit for |A⟩|B⟩ to |A’⟩|B’⟩

	Input |B⟩	Input |A⟩	Output |B’⟩	Output |A’⟩
A	|00⟩	|00⟩	|00⟩	|00⟩
B	|00⟩	|01⟩	|01⟩	|01⟩
C	|00⟩	|10⟩	|10⟩	|10⟩
D	|00⟩	|11⟩	|11⟩	|11⟩
E	|01⟩	|00⟩	|01⟩	|00⟩
F	|01⟩	|01⟩	|10⟩	|01⟩
G	|01⟩	|10⟩	|11⟩	|10⟩
H	|01⟩	|11⟩	|00⟩	|11⟩
I	|10⟩	|00⟩	|10⟩	|00⟩
J	|10⟩	|01⟩	|11⟩	|01⟩
K	|10⟩	|10⟩	|00⟩	|10⟩
L	|10⟩	|11⟩	|01⟩	|11⟩
M	|11⟩	|00⟩	|11⟩	|00⟩
N	|11⟩	|01⟩	|00⟩	|01⟩
O	|11⟩	|10⟩	|01⟩	|10⟩
P	|11⟩	|11⟩	|10⟩	|11⟩

Table1 - input-output mapping for |A⟩|B⟩ to |A’⟩|B’⟩

According to the mapping data, finally we got the transformation matrix as below:

```
[[1. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.]
 [0. 0. 0. 0. 0. 1. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.]
 [0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 1. 0. 0. 0. 0. 0.]
 [0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 1.]
 [0. 0. 0. 0. 1. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.]
 [0. 0. 0. 0. 0. 0. 0. 0. 0. 1. 0. 0. 0. 0. 0. 0.]
 [0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 1. 0.]
 [0. 0. 0. 1. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.]
 [0. 0. 0. 0. 0. 0. 0. 0. 1. 0. 0. 0. 0. 0. 0. 0.]
 [0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 1. 0. 0.]
 [0. 0. 1. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.]
 [0. 0. 0. 0. 0. 0. 0. 1. 0. 0. 0. 0. 0. 0. 0. 0.]
 [0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 1. 0. 0. 0.]
 [0. 1. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.]
 [0. 0. 0. 0. 0. 0. 1. 0. 0. 0. 0. 0. 0. 0. 0. 0.]
 [0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 1. 0. 0. 0. 0.]]
```

1.3 the operation does the circuit implement

According to the input-output mapping above we can easily get that:

The input |A⟩ and output |A'⟩ is totally same, do nothing. This is because we can easily see that the two swaps are offset each other, and the two X Gates are also offset each other.

The input |B⟩ and output |B'⟩ will do the add operation, value of output |B'⟩ equals to the sum of value of |A⟩ and value of |B⟩. And the upper limit is 11 and the data will overflow if the upper limit is exceeded

2. Inverse quantum circuit

2.1 the inverse circuit

By reverse the original circuit we simply get the inverse circuit:

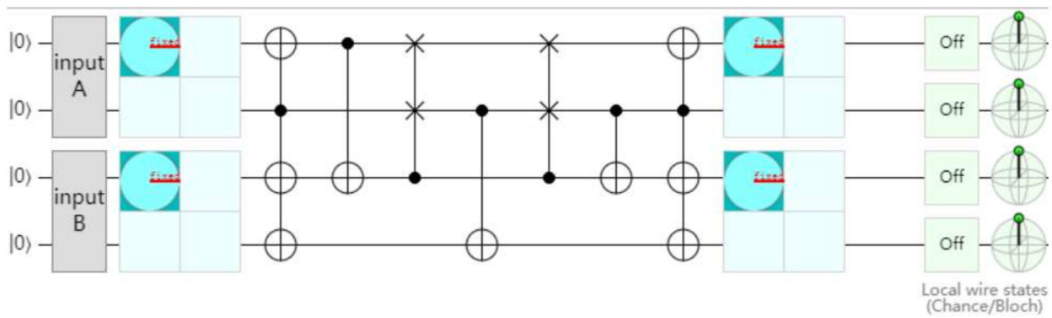

Figure 2 – quantum circuit of |A’⟩|B’⟩ to |A⟩|B⟩

Regarding the whole process as a unitary transformation, and according to the formula:

$$UU^{\dagger} = U^{\dagger}U = I$$

Then we got inverse transformation matrix:

$U^{\dagger}$ =

```
[[1. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.]
 [0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 1. 0. 0.]
 [0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 1. 0. 0. 0. 0. 0.]
 [0. 0. 0. 0. 0. 0. 0. 1. 0. 0. 0. 0. 0. 0. 0. 0.]
 [0. 0. 0. 0. 1. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.]
 [0. 1. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.]
 [0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 1. 0.]
 [0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 1. 0. 0. 0. 0.]
 [0. 0. 0. 0. 0. 0. 0. 0. 1. 0. 0. 0. 0. 0. 0. 0.]
 [0. 0. 0. 0. 0. 1. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.]
 [0. 0. 1. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.]
 [0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 1.]
 [0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 1. 0. 0. 0.]
 [0. 0. 0. 0. 0. 0. 0. 0. 0. 1. 0. 0. 0. 0. 0. 0.]
 [0. 0. 0. 0. 0. 0. 1. 0. 0. 0. 0. 0. 0. 0. 0. 0.]
 [0. 0. 0. 1. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.]]
```

2.2 correctness

And after some operations by Quirk we got the input-output mapping for |A'⟩|B'⟩ to |A⟩|B⟩ :

	Input |B'⟩	Input |A'⟩	Output |B⟩	Output |A⟩
A	|00⟩	|00⟩	|00⟩	|00⟩
B	|00⟩	|01⟩	|11⟩	|01⟩
C	|00⟩	|10⟩	|10⟩	|10⟩
D	|00⟩	|11⟩	|01⟩	|11⟩
E	|01⟩	|00⟩	|01⟩	|00⟩
F	|01⟩	|01⟩	|00⟩	|01⟩
G	|01⟩	|10⟩	|11⟩	|10⟩
H	|01⟩	|11⟩	|10⟩	|11⟩
I	|10⟩	|00⟩	|10⟩	|00⟩
J	|10⟩	|01⟩	|01⟩	|01⟩
K	|10⟩	|10⟩	|00⟩	|10⟩
L	|10⟩	|11⟩	|11⟩	|11⟩
M	|11⟩	|00⟩	|11⟩	|00⟩
N	|11⟩	|01⟩	|10⟩	|01⟩
O	|11⟩	|10⟩	|01⟩	|10⟩
P	|11⟩	|11⟩	|00⟩	|11⟩

Table2 - input-output mapping for |A'⟩|B'⟩ to |A⟩|B⟩

After careful comparison values of table 1(mapping for |A⟩|B⟩ to |A'⟩|B'⟩) and table 2(mapping for |A'⟩|B'⟩ to |A⟩|B⟩), the output value of inverse circuit is totally same as the input value of the original circuit ,therefore we can prove the correctness of this inverse circuit.