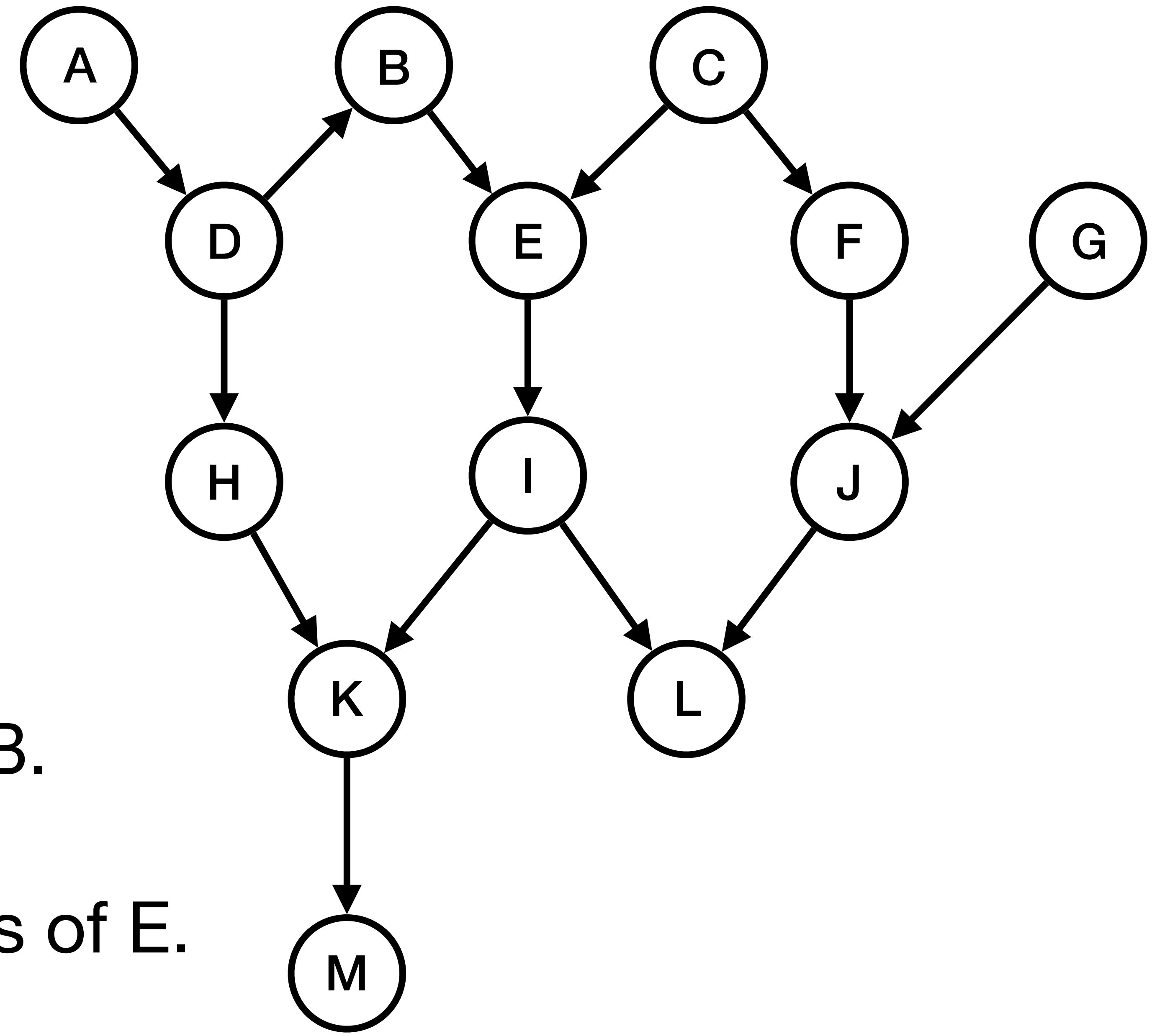
CMT311: Bayesian Networks

These exercises use material from
Jensen & Nielsen: Bayesian Networks and Decision Graphs

Exercise 1



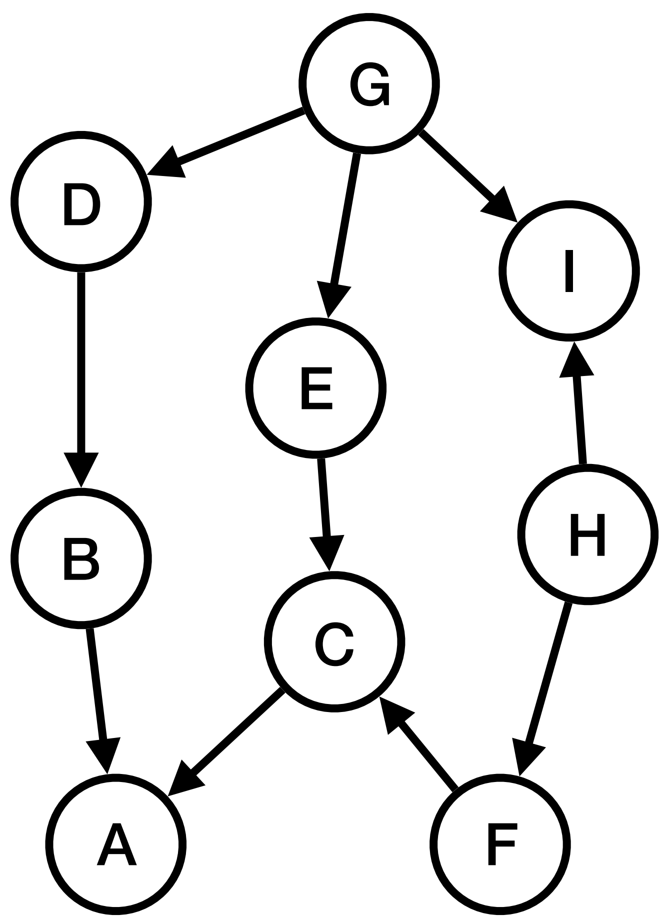
- List all parents of D.
- List all children of D.
- List all ancestors of L.
- List all descendants of B.
- List all non-descendants of E.
- List all nodes in the Markov blanket of H.
- List all nodes in the Markov blanket of E.

Exercise 2

Whether a car starts or not depends on whether there is fuel in the tank and whether the battery is dead or not. The battery is more likely to be dead in winter than at other times of the year. The car has a fuel meter indicating empty, half full or full. The fuel meter may be broken, and the tank may have a leak. If fuel is leaking from the tank, we may smell this. There have been recent reports about a fuel thief in the area. The car owner always fills up the tank on Saturday.

- Provide the graph of a Bayesian network modelling this domain.
- Which conditional probability tables (CPTs) need to be specified to turn your graph into a BN?
- How many parameters are needed to specify the CPTs of your Bayesian network?
- How many parameters would be needed to directly specify the full joint distribution over the random variables you used?

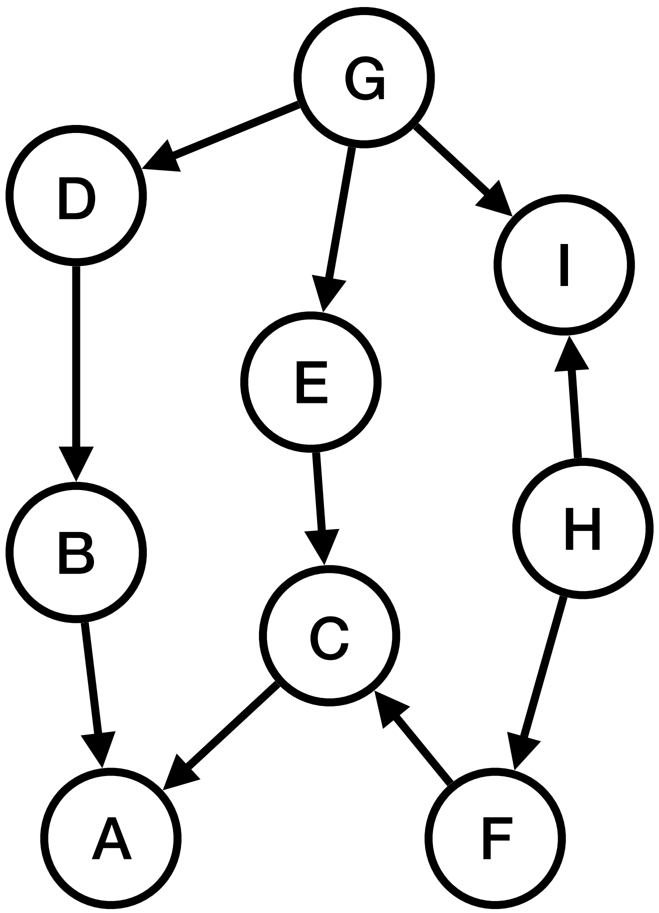
Exercise 3



- On the path D - G - I - H, which nodes are colliders?
- On the path D - G - E - C - F - H, which nodes are colliders?
- On the path D - B - A - C - F - H, which nodes are colliders?
- Which set(s) of evidence variables $\mathscr{Z}$ block the path D - G - I - H?
- Which set(s) of evidence variables $\mathscr{Z}$ block the path D - G - E - C - F - H?
- Which set(s) of evidence variables $\mathscr{Z}$ block the path D - B - A - C - F - H?

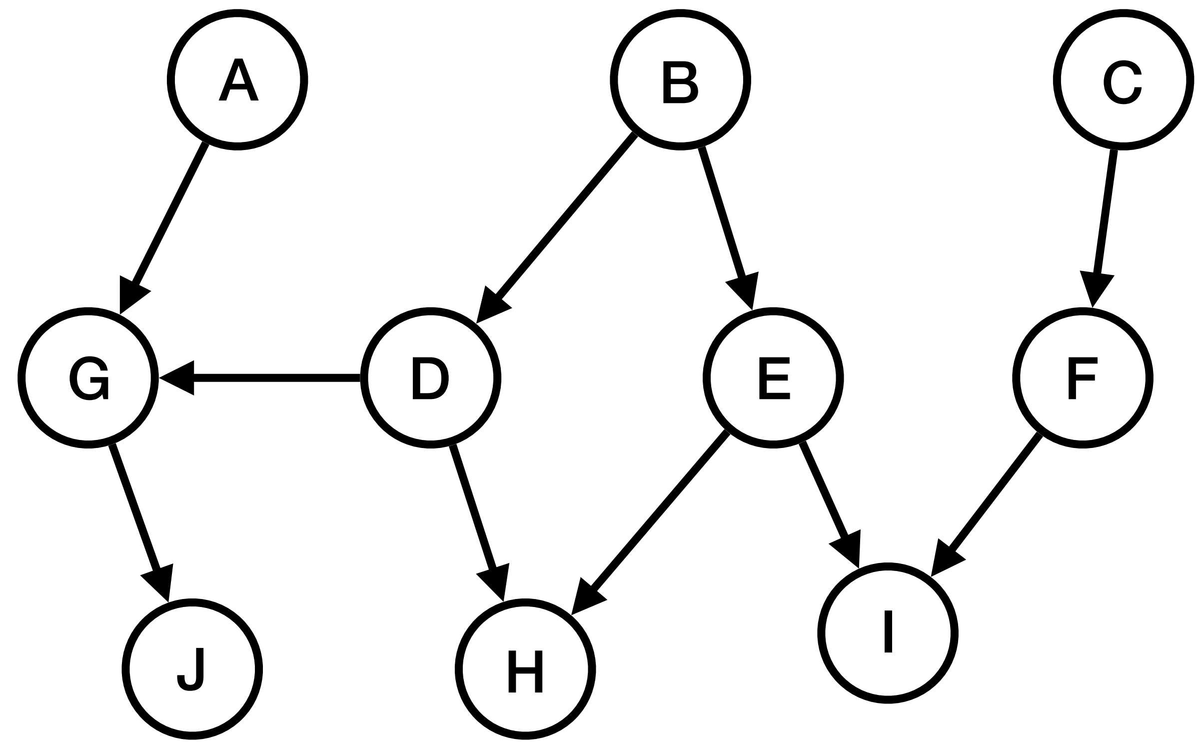
Exercise 4

a)



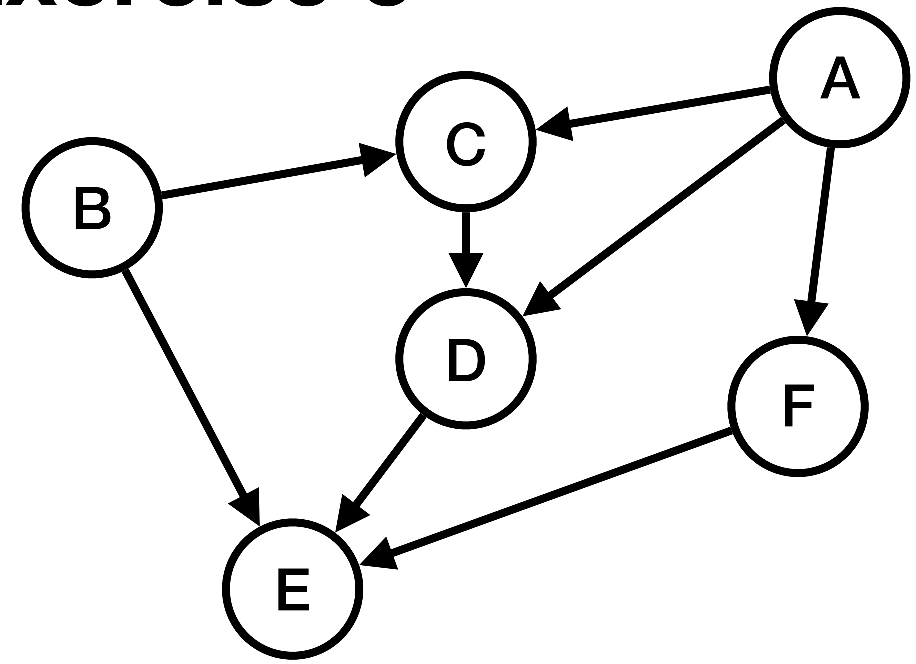
Assume B is the only observed node. Which nodes are d-separated from A?

b)



Assume J is the only observed node. Which nodes are d-separated from A?

Exercise 5



- What are the minimal set(s) of variables required to d-separate C and E (that is, sets of variables for which no proper subset d-separates C and E)?

- What are the minimal set(s) of variables required to d-separate A and B?

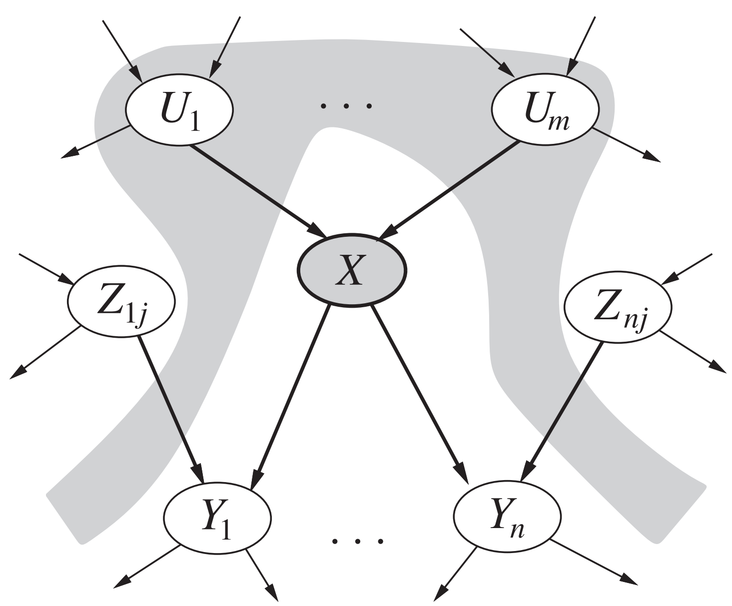
- What are the maximal set(s) of variables required to d-separate C and E (that is, sets of variables for which no proper superset d-separates C and E)?

- What are the maximal set(s) of variables required to d-separate A and B?

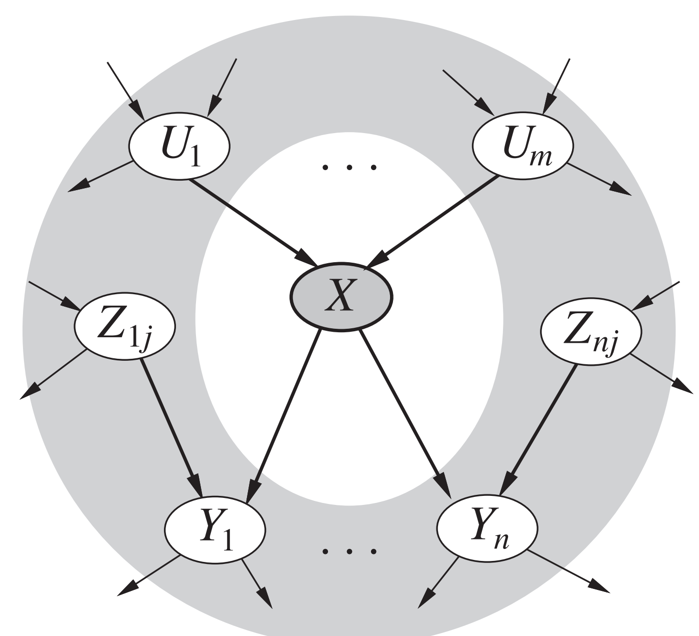
Exercise 6

Show that each of the two characterisations below follows from the more general notion of d-separation.

Each node is conditionally independent of its **non-descendants given** its **parents**.

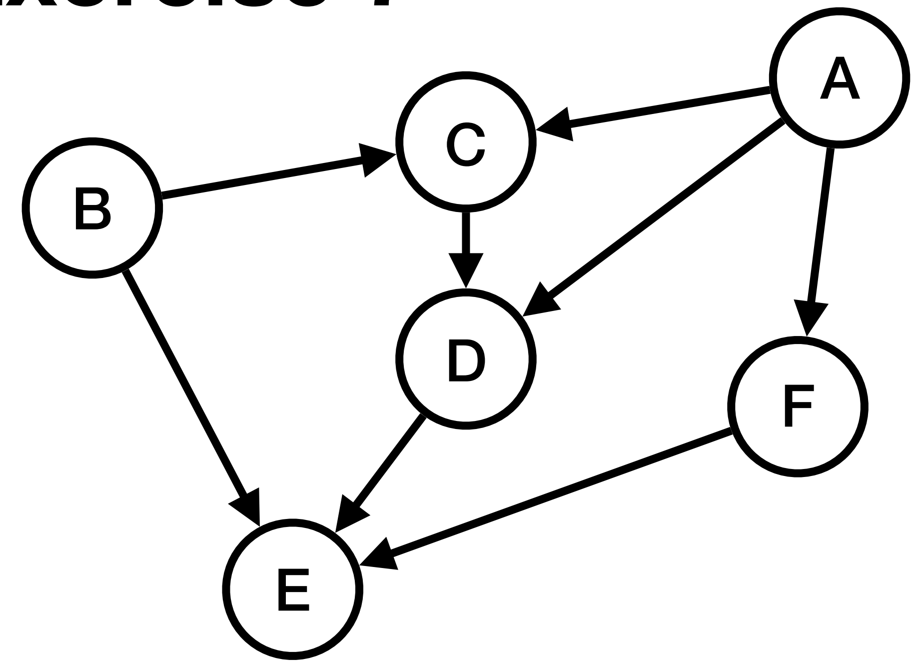


Each node is conditionally independent of **all other nodes** in the network **given** its **Markov blanket**.



[figures: Russell& Norvig]

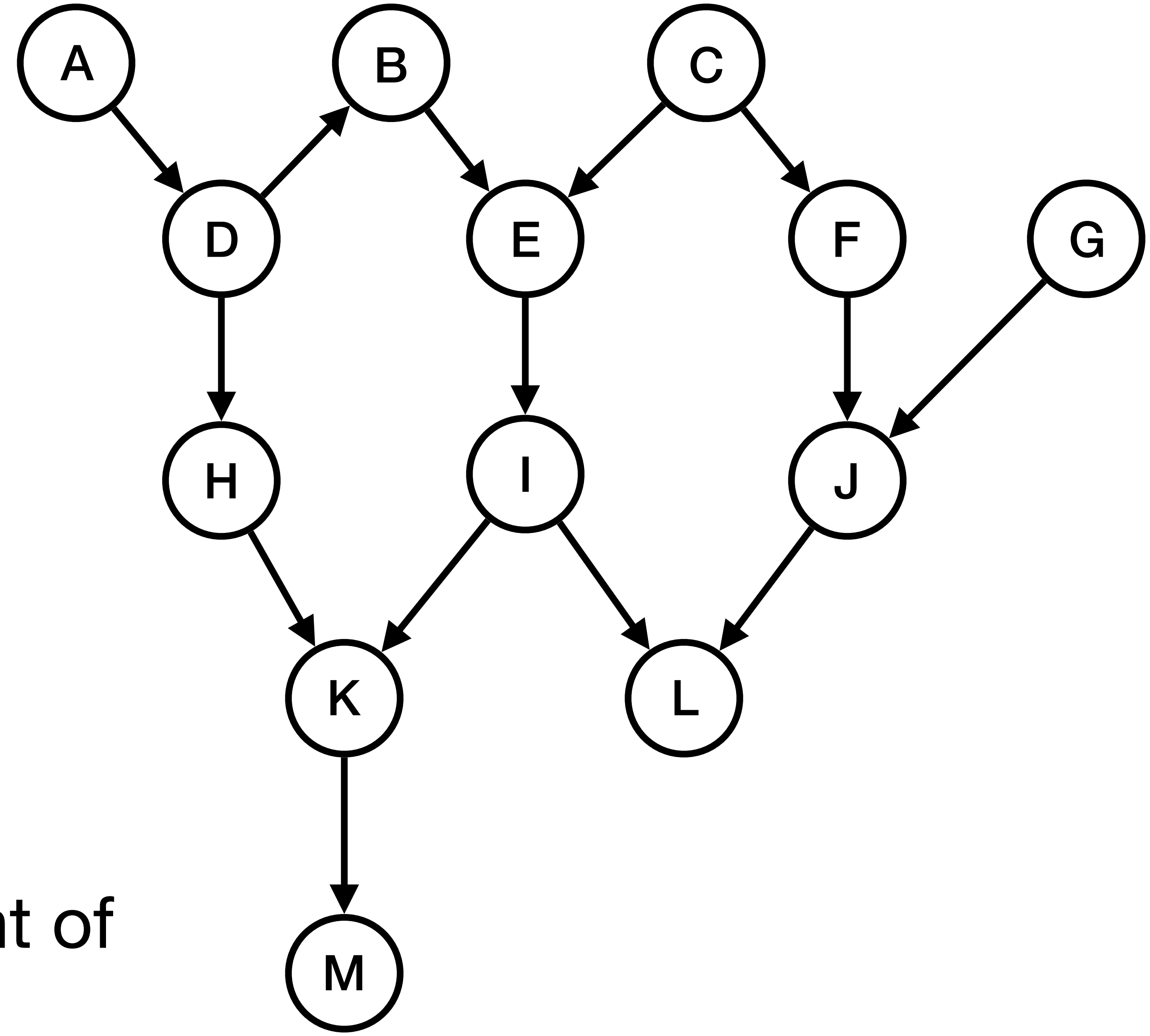
Exercise 7



- For each node X in this DAG, write a statement of the form $X \perp\!\!\!\perp \{\ldots\} | \{\ldots\}$ based on “A node X is conditionally independent of its **non-descendants given** its **parents**.”

- For each node X in this DAG, write a statement of the form $X \perp\!\!\!\perp \{\ldots\} | \{\ldots\}$ based on “A node X is conditionally independent of **all other nodes** in the network **given** its **Markov blanket**.”

Exercise 8



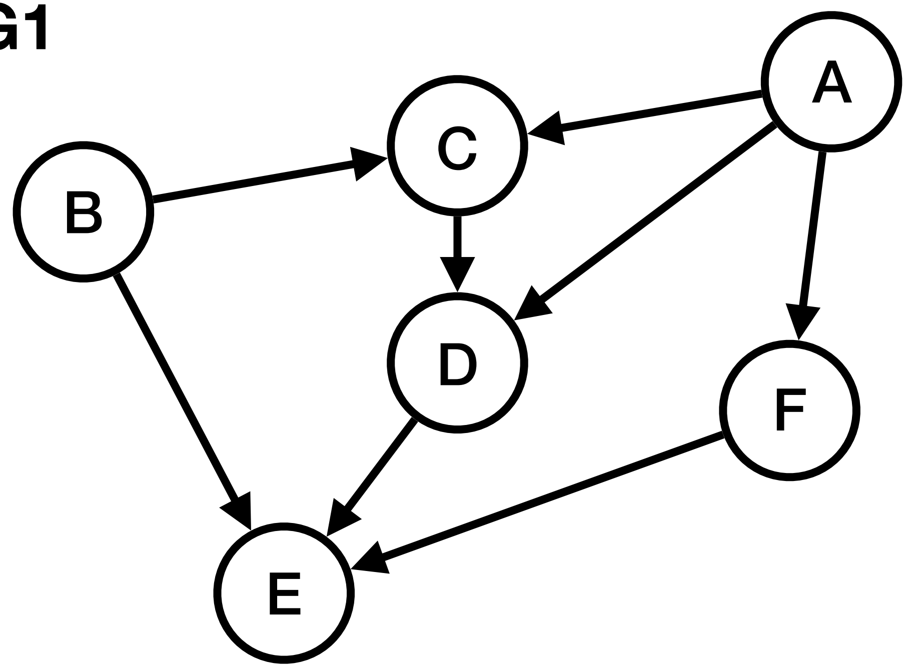
- List all nodes that are conditionally independent of A given B and M.

- List all nodes that are conditionally independent of A given B.

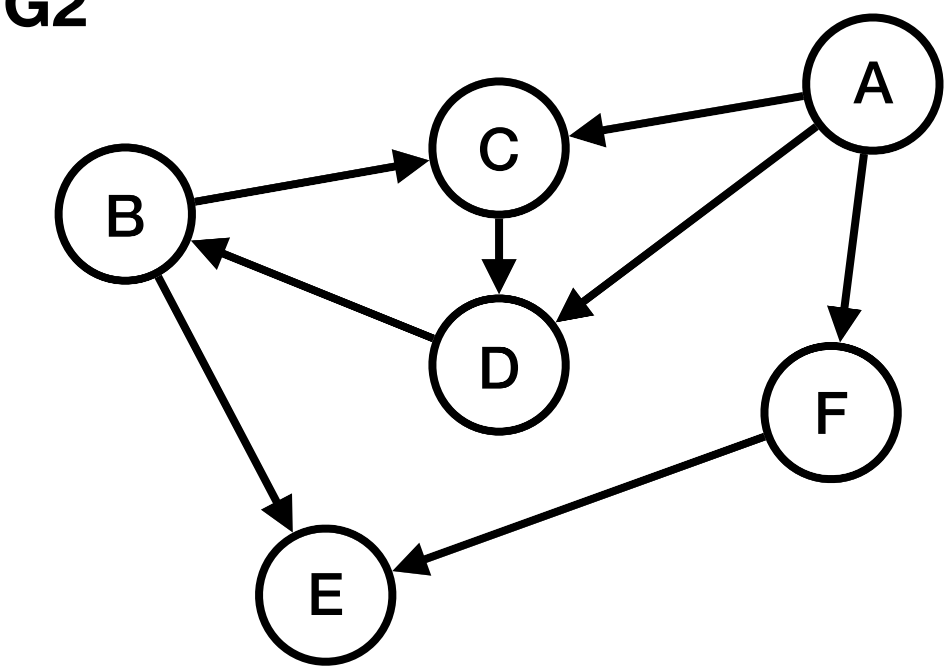
Exercise 9

List all immoralities in G1.
Which of these four graphs are Markov equivalent?

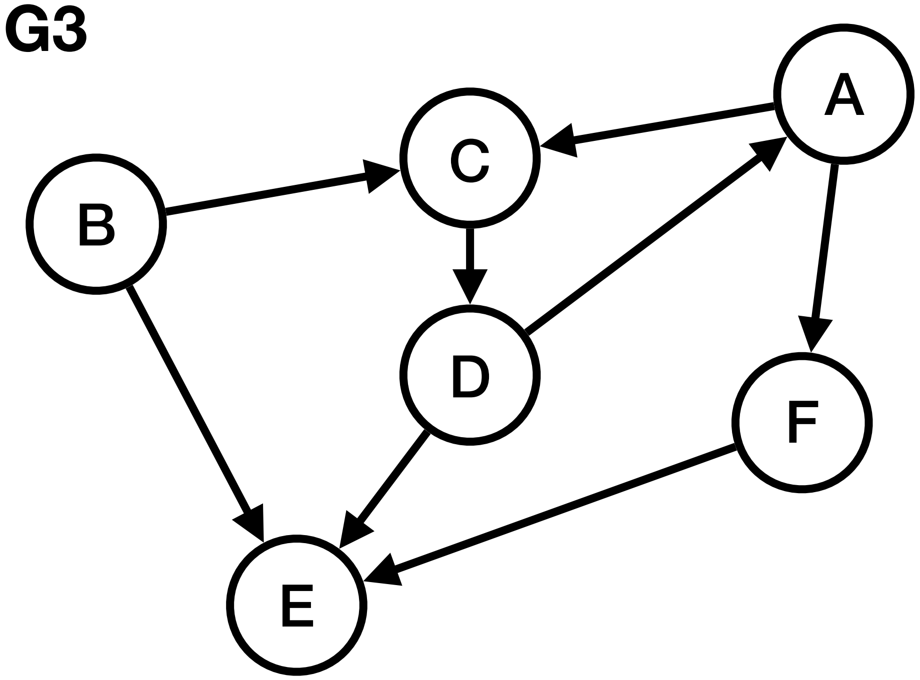
G1



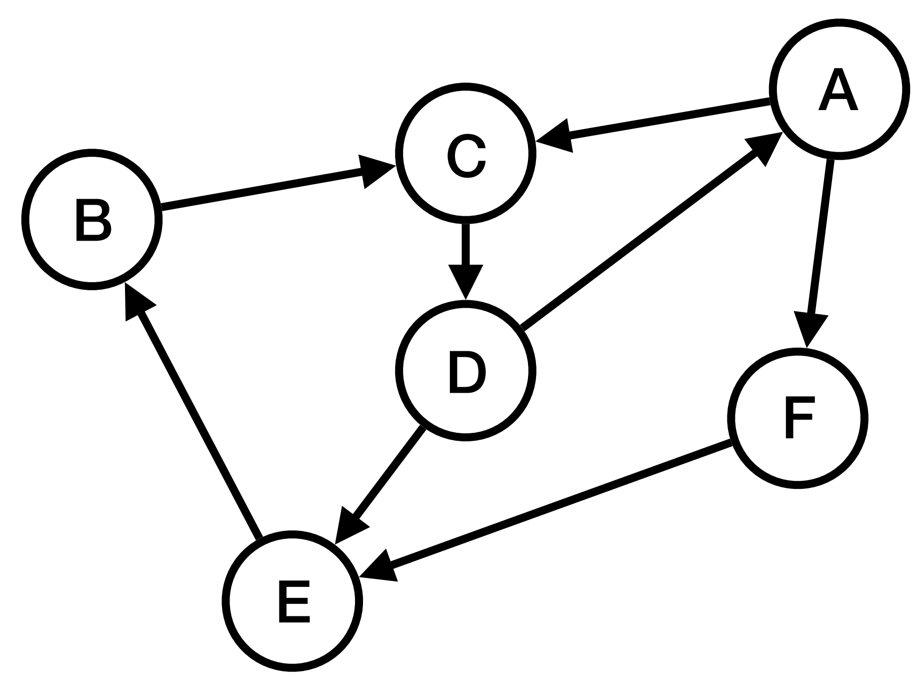
G2



G3



G4

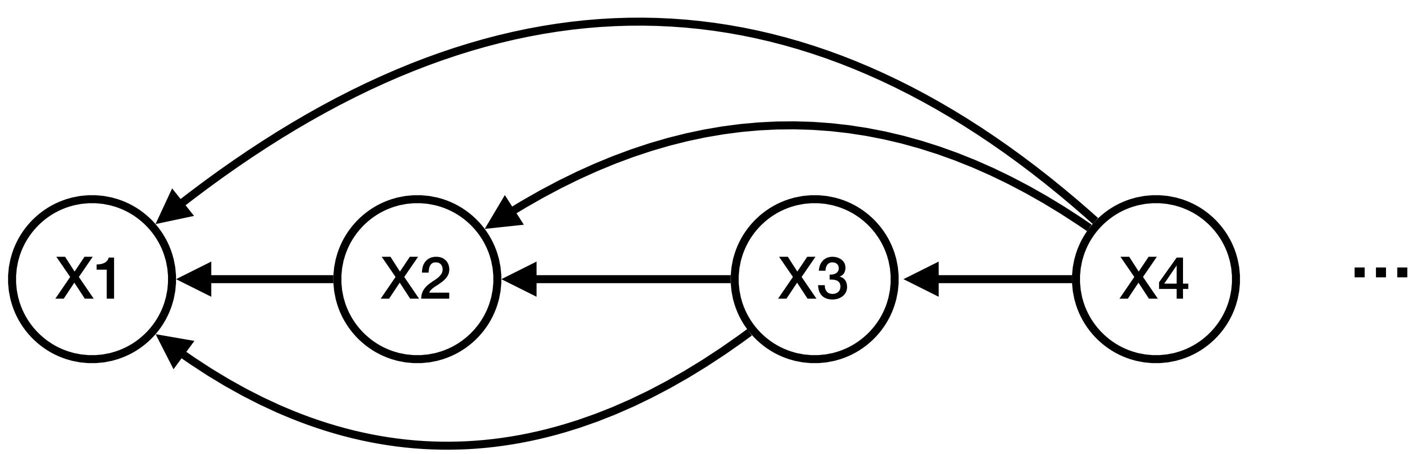


Exercise 10

Consider the factorisation of the joint distribution using the chain rule, where we fix an arbitrary order of the variables and use all variables after X_i in that order as X_i's parents,

$$P(X_1, \ldots, X_n) = P(X_n) \cdot \prod_{i=1}^{n-1} P(X_i \mid X_{i+1}, \ldots, X_n)$$

and the corresponding DAG structure



Show that the DAGs obtained in this way for different orders of nodes are all Markov equivalent.