CMT311: Probability Exercises

Question 1

Consider the following joint distribution over random variables A, B, C and D, each with domain {y,n}.

		A=y		A=n	
	P(A,B,C,D)	B=y	B=n	B=y	B=n
D=y	C=y	0.201	0.002	0.010	0.010
	C=n	0.038	0.007	0.105	0.109
D=n	C=y	0.034	0.300	0.005	0.151
	C=n	0.009	0.004	0.012	0.003

Compute the following probabilities:

P(A=y,B=n,C=y)

P("all four RVs have the same value")

P(A=y v B=y v C=y v D=y)

P("two variables have value y and two variables have value n")

P(A=y|B=n)

P(C=n,D=n | A=y, B=y)

P("C and D have same value" | B=y)

P(C=y | A=y v B=y v D=y)

Question 2

Consider the following joint distribution over random variables A, B and C, each with domain {y,n}. Show that A and B are independent.

	A=y		A=n	
P(A,B,C)	B=y	B=n	B=y	B=n
C=y	0.021	0.126	0.015	0.135
C=n	0.049	0.504	0.015	0.135

Question 3

Consider the following joint distribution over random variables A, B and C, each with domain {y,n}.

	A=y		A=n	
P(A,B,C)	**B=y**	**B=n**	**B=y**	**B=n**
C=y	0.0675	0.6075	0.03	0.07
C=n	0.0075	0.0675	0.045	0.105

Show that
a) B and C are not independent.
b) B and C are conditionally independent given A.

Question 4

A, B and C are random variables with domain {y,n}, and L is a random variable with domain {0,1}. The joint distribution of these random variables is unknown, but you are given the following sample of ten i.i.d. examples from that unknown distribution:

	A	B	C	L
1	y	y	n	0
2	y	n	y	0
3	n	y	n	1
4	n	y	y	0
5	n	n	y	1
6	n	n	y	1
7	y	n	n	1
8	n	y	n	0
9	n	n	y	1
10	y	y	n	0

a) Provide the ML parameters of the full joint distribution based on the sample.
b) Assume that the four variables are all pairwise independent. Provide the ML parameters based on the sample under this assumption.

c) Assume that A, B and C are pairwise independent given L. Provide the ML parameters based on the sample under this assumption.
d) For each of the models from parts a)-c), compute the likelihood of the following test data set. How would you judge the quality of these models?

	A	B	C	L
11	y	n	n	0
12	n	y	y	1
13	y	y	n	0
14	y	y	y	0
15	y	y	n	1
16	n	n	n	1
17	y	y	n	0
18	y	y	n	1
19	y	y	y	1
20	n	y	y	1