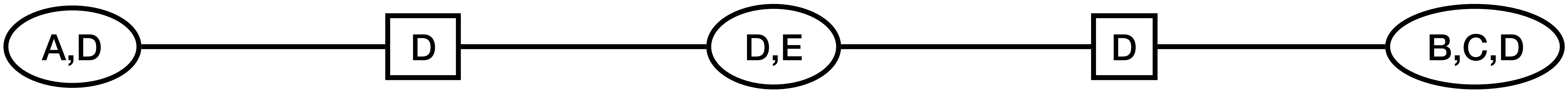


A	D	$f_{11}(A,D)$
0	0	85
0	1	42
1	0	17
1	1	21

D	$f_{10}(D)$
0	102
1	63

D	E	$f_9(D,E)$
0	0	102
0	1	0
1	0	0
1	1	63

D	$f_8(D)$
0	102
1	63

B	C	D	$f_7(B,C,D)$
0	0	0	60
0	0	1	3
0	1	0	6
0	1	1	15
1	0	0	6
1	0	1	15
1	1	0	30
1	1	1	30

these are unnormalised marginals: to compute the normalisation constant Z, pick one of the tables and sum out all variables

$$Z = \sum_D f_{10}(D) = 102 + 63 = 165$$

to compute the marginal P(A,D), normalise $f_{11}(A,D)$:

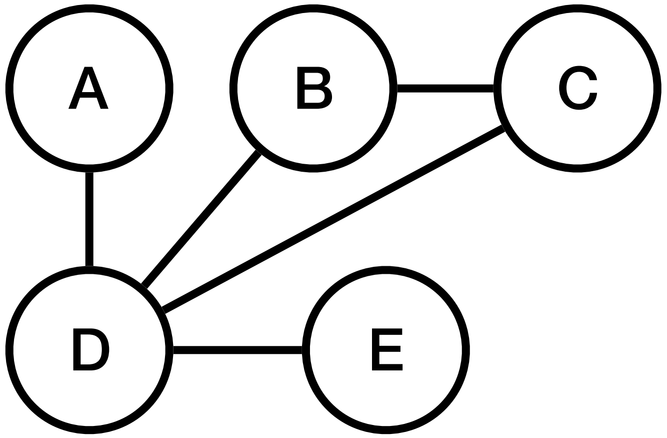
A	D	P(A,D)
0	0	85/165=0.515
0	1	42/165=0.255
1	0	17/165=0.103
1	1	21/165=0.127

to compute the marginal P(A), sum out D:

A	P(A)
0	85/165+42/165=0.7697
1	17/165+21/165=0.2303

similar for marginals of other variables

Homework



A	D	$f_1(A,D)$
0	0	5
0	1	2
1	0	1
1	1	1

D	E	$f_3(D,E)$
0	0	1
0	1	0
1	0	0
1	1	1

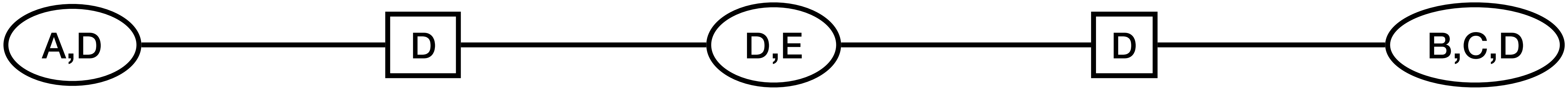
B	C	D	$f_2(B,C,D)$
0	0	0	10
0	0	1	1
0	1	0	1
0	1	1	5
1	0	0	1
1	0	1	5
1	1	0	5
1	1	1	10

- From the information on the previous slide, compute the marginals of the other variables, P(B), P(C), P(D) and P(E), as well.

- Compute the marginal of each variable using the original Markov network (repeated above) and the definition of the marginals, i.e.,
  $P(A) = \frac{1}{Z} \sum_{B,C,D,E} f_1(A,D) f_2(B,C,D) f_3(D,E)$ etc, where
  $$Z = \sum_{A,B,C,D,E} f_1(A,D) f_2(B,C,D) f_3(D,E)$$

- Construct the factor graph for the Markov network and use the sum-product algorithm (seen last week) to compute the marginal of each variable.

remaining marginals from JT



A	D	f_{11}(A,D)
0	0	85
0	1	42
1	0	17
1	1	21

D	f_{10}(D)
0	102
1	63

D	E	f_9(D,E)
0	0	102
0	1	0
1	0	0
1	1	63

D	f_8(D)
0	102
1	63

B	C	D	f_7(B,C,D)
0	0	0	60
0	0	1	3
0	1	0	6
0	1	1	15
1	0	0	6
1	0	1	15
1	1	0	30
1	1	1	30

$$Z = \sum_{D} f_{10}(D) = 102 + 63 = 165$$

compute P(B) by summing out C and D from f7 & normalising

B	P(B)
0	(60+3+6+15)/165=0.5091
1	(6+15+30+30)/165=0.4909

compute P(C) by summing out B and D from f7 & normalising

C	P(C)
0	(60+3+6+15)/165=0.5091
1	(6+15+30+30)/165=0.4909

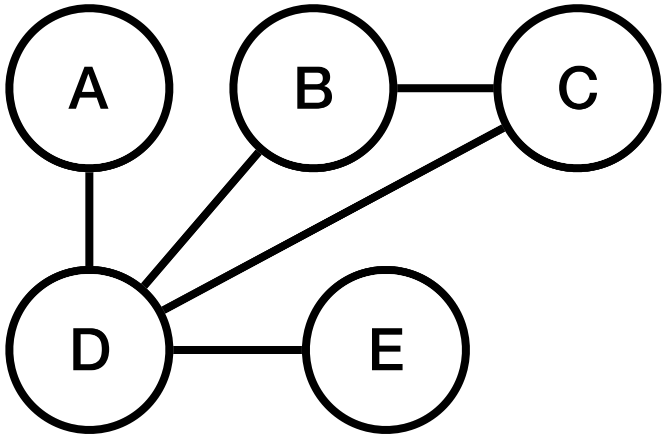
normalise f8 to get P(D)

D	P(D)
0	102/165=0.6182
1	63/165=0.3818

compute P(E) by summing out D from f9 & normalising

E	P(E)
0	(102+0)/165=0.6182
1	(0+63)/165=0.3818

marginals from definition



A	D	$f_1(A,D)$
0	0	5
0	1	2
1	0	1
1	1	1

D	E	$f_3(D,E)$
0	0	1
0	1	0
1	0	0
1	1	1

B	C	D	$f_2(B,C,D)$
0	0	0	10
0	0	1	1
0	1	0	1
0	1	1	5
1	0	0	1
1	0	1	5
1	1	0	5
1	1	1	10

A	B	C	D	E	$f_1*f_2*f_3$
0	0	0	0	0	50
0	0	0	0	1	0
0	0	0	1	0	0
0	0	0	1	1	2
0	0	1	0	0	5
0	0	1	0	1	0
0	0	1	1	0	0
0	0	1	1	1	10
0	1	0	0	0	5
0	1	0	0	1	0
0	1	0	1	0	0
0	1	0	1	1	10
0	1	1	0	0	25
0	1	1	0	1	0
0	1	1	1	0	0
0	1	1	1	1	20

A	B	C	D	E	$f_1*f_2*f_3$
1	0	0	0	0	10
1	0	0	0	1	0
1	0	0	1	0	0
1	0	0	1	1	1
1	0	1	0	0	1
1	0	1	0	1	0
1	0	1	1	0	0
1	0	1	1	1	5
1	1	0	0	0	1
1	1	0	0	1	0
1	1	0	1	0	0
1	1	0	1	1	5
1	1	1	0	0	5
1	1	1	0	1	0
1	1	1	1	0	0
1	1	1	1	1	10

$$Z = \sum_{A,B,C,D,E} f_1(A,D) f_2(B,C,D) f_3(D,E)$$

$$= 50 + 0 + 0 + 2 + \ldots + 5 + 0 + 0 + 10 = 165$$

A	B	C	D	E	$f_1*f_2*f_3$
0	0	0	0	0	50
0	0	0	0	1	0
0	0	0	1	0	0
0	0	0	1	1	2
0	0	1	0	0	5
0	0	1	0	1	0
0	0	1	1	0	0
0	0	1	1	1	10
0	1	0	0	0	5
0	1	0	0	1	0
0	1	0	1	0	0
0	1	0	1	1	10
0	1	1	0	0	25
0	1	1	0	1	0
0	1	1	1	0	0
0	1	1	1	1	20

A	B	C	D	E	$f_1*f_2*f_3$
1	0	0	0	0	10
1	0	0	0	1	0
1	0	0	1	0	0
1	0	0	1	1	1
1	0	1	0	0	1
1	0	1	0	1	0
1	0	1	1	0	0
1	0	1	1	1	5
1	1	0	0	0	1
1	1	0	0	1	0
1	1	0	1	0	0
1	1	0	1	1	5
1	1	1	0	0	5
1	1	1	0	1	0
1	1	1	1	0	0
1	1	1	1	1	10

$$Z = 165$$

$$P(A) = \frac{1}{Z} \sum_{B,C,D,E} f_1(A, D) f_2(B, C, D) f_3(D, E)$$

A	P(A)
0	(50+0+0+2+5+0+0+10+5+ 0+0+10+25+0+0+20)/165 = 0.7697
1	(10+0+0+1+1+0+0+5+1+0 +0+5+5+0+0+10)/165 = 0.2303

same principle gives:

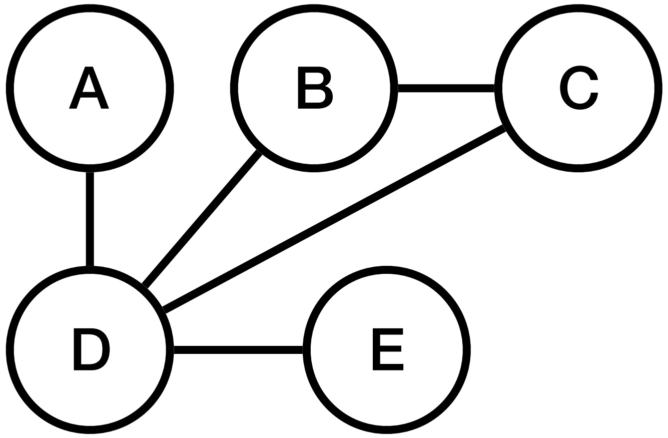
B	P(B)
0	0.5091
1	0.4909

C	P(C)
0	0.5091
1	0.4909

D	P(D)
0	0.6182
1	0.3818

E	P(E)
0	0.6182
1	0.3818

marginals from sum-product algorithm

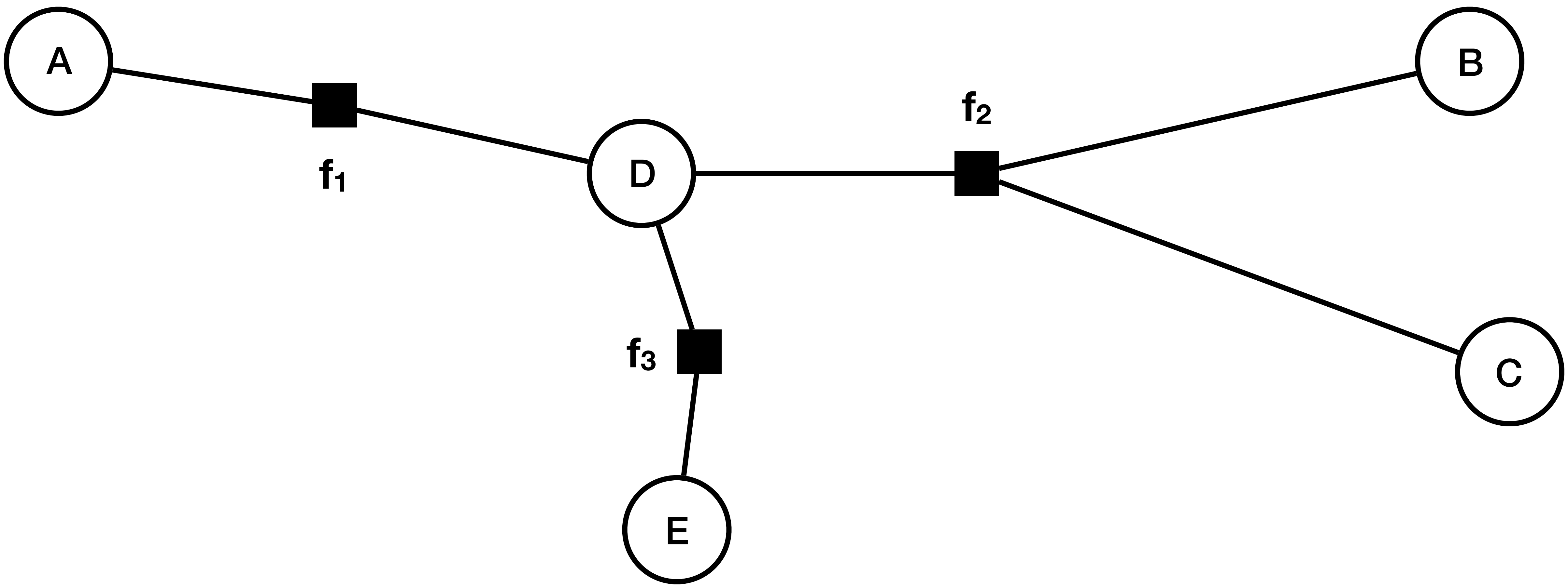


A	D	$f_1(A,D)$
0	0	5
0	1	2
1	0	1
1	1	1

D	E	$f_3(D,E)$
0	0	1
0	1	0
1	0	0
1	1	1

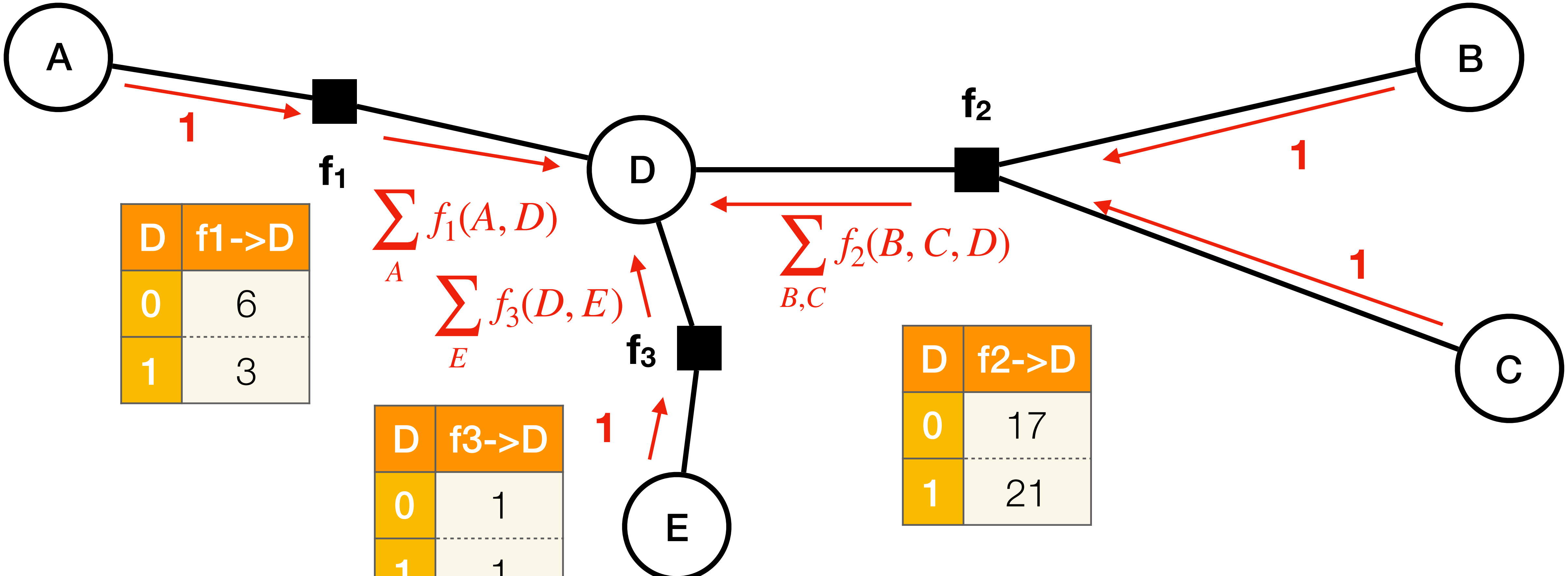
B	C	D	$f_2(B,C,D)$
0	0	0	10
0	0	1	1
0	1	0	1
0	1	1	5
1	0	0	1
1	0	1	5
1	1	0	5
1	1	1	10

Homework: use the sum-product algorithm (seen last week) on the factor graph



pick D as the root

propagate messages from leaves to root



D	f1->D
0	6
1	3

D	f3->D
0	1
1	1

D	f2->D
0	17
1	21

A	D	f_1(A,D)
0	0	5
0	1	2
1	0	1
1	1	1

D	E	f_3(D,E)
0	0	1
0	1	0
1	0	0
1	1	1

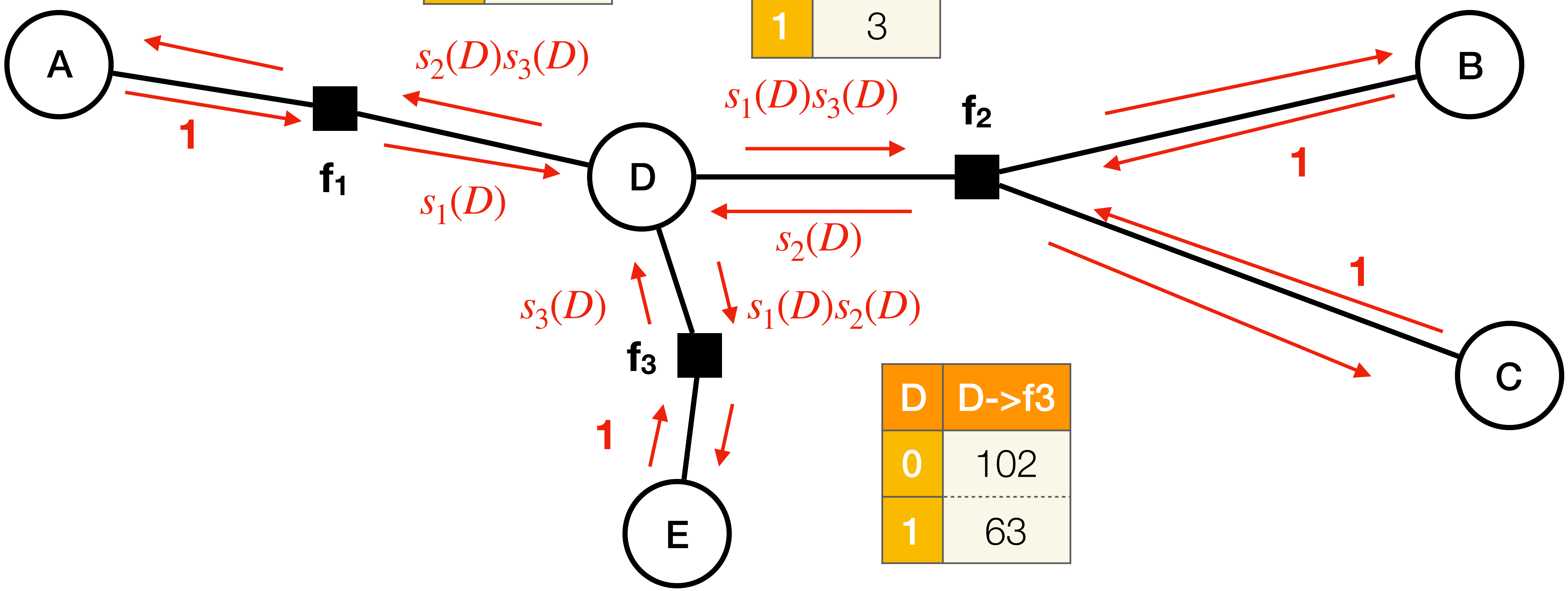
B	C	D	f_2(B,C,D)
0	0	0	10
0	0	1	1
0	1	0	1
0	1	1	5
1	0	0	1
1	0	1	5
1	1	0	5
1	1	1	10

propagate messages from root to leaves

we’re re-naming earlier messages for easier reference here

D	D->f1
0	17
1	21

D	D->f2
0	6
1	3



D	D->f3
0	102
1	63

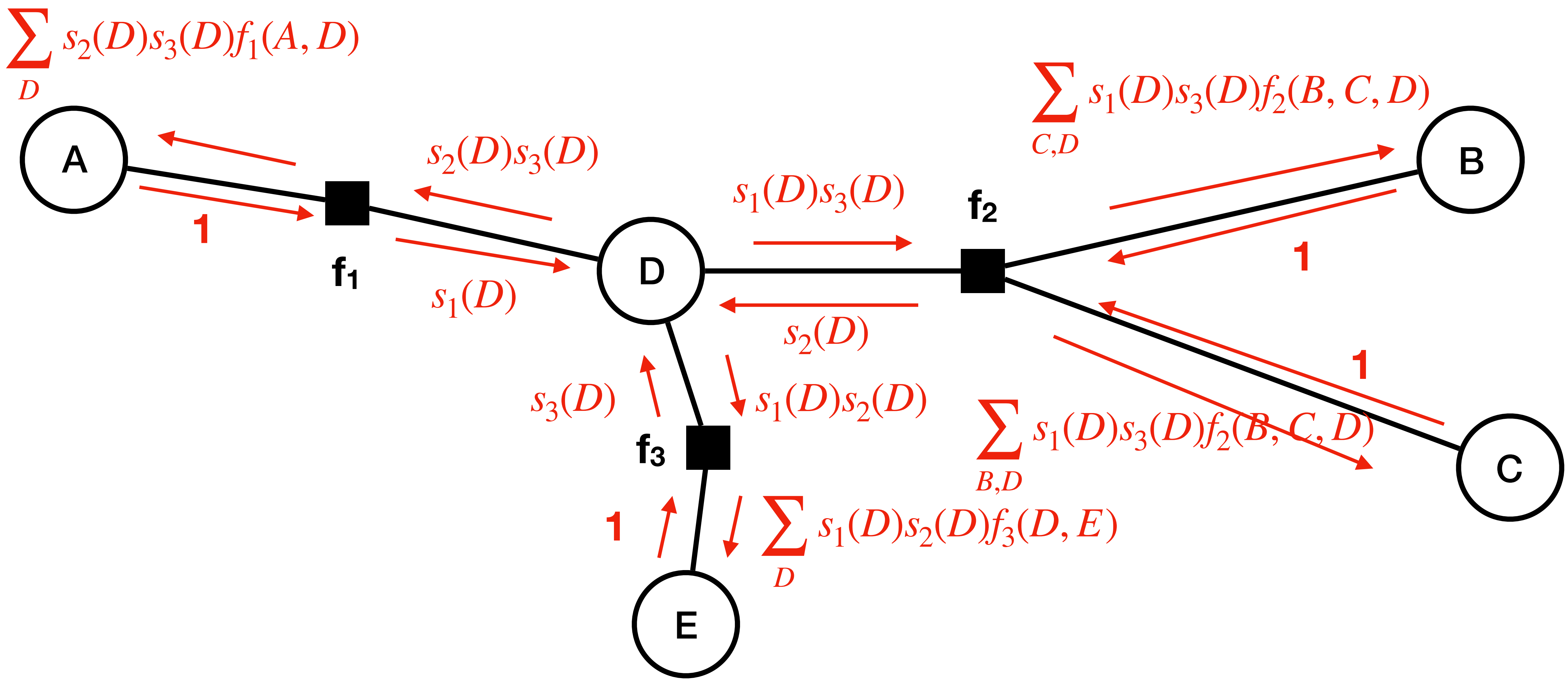
D	s1(D)
0	6
1	3

D	s3(D)
0	1
1	1

D	s2(D)
0	17
1	21

propagate messages from root to leaves

we're re-naming earlier messages for easier reference here



D	s1(D)
0	6
1	3

D	s3(D)
0	1
1	1

D	s2(D)
0	17
1	21

D	s1(D)
0	6
1	3

D	s3(D)
0	1
1	1

D	s2(D)
0	17
1	21

A	D	f_1(A,D)
0	0	5
0	1	2
1	0	1
1	1	1

D	E	f_3(D,E)
0	0	1
0	1	0
1	0	0
1	1	1

B	C	D	f_2(B,C,D)
0	0	0	10
0	0	1	1
0	1	0	1
0	1	1	5
1	0	0	1
1	0	1	5
1	1	0	5
1	1	1	10

$s_2(D)s_3(D)f_1(A, D)$

A	D	s2s3f1
0	0	85
0	1	42
1	0	17
1	1	21

sum out D:

A	e1(A)
0	127
1	38

$$\sum_D s_2(D)s_3(D)f_1(A, D)$$

$s_1(D)s_2(D)f_3(D, E)$

D	E	s1s2f3
0	0	102
0	1	0
1	0	0
1	1	63

sum out D:

E	e2(E)
0	102
1	63

$$\sum_D s_1(D)s_2(D)f_3(D, E)$$

$s_1(D)s_3(D)f_2(B, C, D)$

B	C	D	s1s3f2
0	0	0	60
0	0	1	3
0	1	0	6
0	1	1	15
1	0	0	6
1	0	1	15
1	1	0	30
1	1	1	30

sum out B and D:

C	e3(C)
0	84
1	81

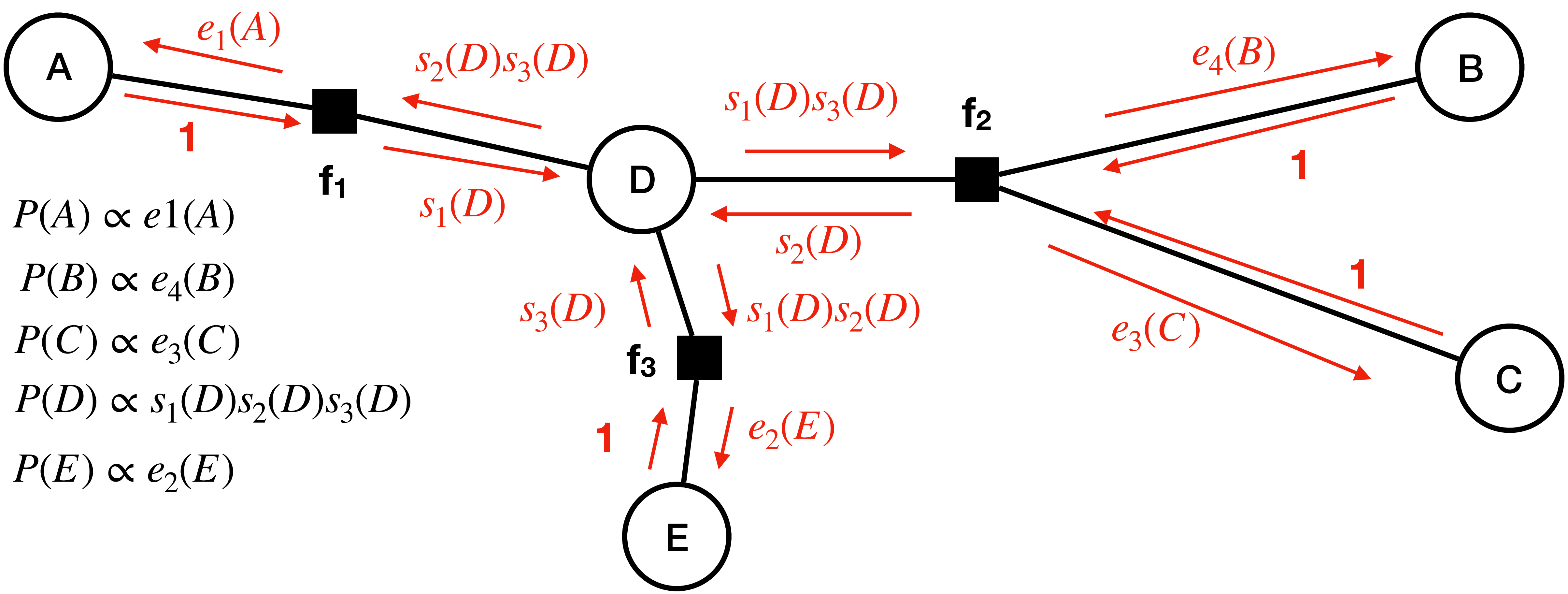
$$\sum_{B,D} s_1(D)s_3(D)f_2(B, C, D)$$

sum out C and D:

B	e4(B)
0	84
1	81

$$\sum_{C,D} s_1(D)s_3(D)f_2(B, C, D)$$

final state of graph



$$P(A) \propto e1(A)$$

$$P(B) \propto e_4(B)$$

$$P(C) \propto e_3(C)$$

$$P(D) \propto s_1(D)s_2(D)s_3(D)$$

$$P(E) \propto e_2(E)$$

D	P(D)
0	6*1*17/ (6*1*17+3*1*21)=0.6182
1	3*1*21/ (6*1*17+3*1*21)=0.3818

A	P(A)
0	127/(127+38) =0.7697
1	38/(127+38) = 0.2303

E	P(E)
0	0.6182
1	0.3818

C	P(C)
0	0.5091
1	0.4909

B	P(B)
0	0.5091
1	0.4909

D	s1(D)
0	6
1	3

D	s3(D)
0	1
1	1

D	s2(D)
0	17
1	21

A	e1(A)
0	127
1	38

E	e2(E)
0	102
1	63

C	e3(C)
0	84
1	81

B	e4(B)
0	84
1	81