Matrix Determinant

Matrix determinant in general cases: Matrix A of $n \times n$

$$A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{bmatrix}$$

Denote M_{ij} a sub-matrix, received by deleting row i and column j from A

M_{ij} is $(n-1)\times(n-1)$ matrix

So:

$$det(A) = (-1)^{1+1} det(M_{11}) + (-1)^{1+2} det(M_{12}) + \dots + (-1)^{1+j} det(M_{1j}) \dots + (-1)^{1+n} det(M_{1n})$$

Example (re-explain the example in lecture 02 ’s slide):

$$det \begin{bmatrix} 2 & 5 \\ 3 & 1 \end{bmatrix} = 2\times1 - 3\times5 = 2 - 15 = -13$$

Geometric Interpretation

This equation

$$\underbrace{\begin{bmatrix} 2 & 1 & 1 \\ 4 & -6 & 0 \\ -2 & 7 & 2 \end{bmatrix}}_{A}\underbrace{\begin{bmatrix} u \\ v \\ w \end{bmatrix}}_{x}=\underbrace{\begin{bmatrix} 5 \\ -2 \\ 9 \end{bmatrix}}_{b}$$

Ax = b

The above vector form equation is equivalent to a system of three linear equations (with three unknown variables u,v,w):
Let's compute the product between matrix A and vector x to receive:

(Inner product between first row of A and x) 2u + v + w = 5

(Inner product between second row of A and x) 4u – 6v + 0 = -2

(Inner product between third row of A and x) -2u + 7v + 2w = 9

Solution: $x = A^{-1}b$ (if you can find the inverse of A (let's use some linear algebra to find it))

Geometric Interpretation

This equation Ax = b

Solution: $x = A^{-1} b$

Example 1: A = [5] (a special matrix of 1 row and 1 column); b=[2] (a vector of 1 dimension)

Solution: $x = A^{-1} b = 5^{-1} \times 2 = \frac{2}{5}$

Example 2: $A = \begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix}$ $b = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

Solution: $x = A^{-1} b = \begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \underbrace{\frac{1}{4 \times 3 - 2 \times 1} \begin{bmatrix} 3 & -1 \\ -2 & 4 \end{bmatrix}} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \frac{1}{10} \begin{bmatrix} 1 \\ 6 \end{bmatrix} = \begin{bmatrix} \frac{1}{10} \\ \frac{3}{5} \end{bmatrix}$

See the definition for a 2x2 matrix inverse in the slide for matrix inversion