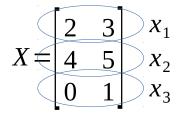


CSC4007 Advanced Machine Learning

Working example for RBF features

Example: RBF features

Assume there are three data points as



Assume there are a set of two centers

$$c_{1} = [1,0]$$
 Receives 2 features
$$c_{2} = [1,1]$$
 $\phi(x) = [1,\phi_{1}(x),\phi_{2}(x)]$ $\phi_{2}(x) = e^{\frac{-\|x-c_{1}\|}{2\sigma^{2}}}$ $\phi_{2}(x) = e^{\frac{-\|x-c_{2}\|}{2\sigma^{2}}}$

- So the data matrix used in computing optimum parameters is
- The **optimal** β **is the same**

$$\hat{\beta}^{\text{ls}} = (X^{\top}X)^{-1}X^{\top}Y \quad \text{but with} \quad X = \begin{pmatrix} \phi(x_1)^{\top} \\ \vdots \\ \phi(x_n)^{\top} \end{pmatrix} \in \mathbb{R}^{n \times k} \qquad \frac{-\|x_1 - c_1\|}{2\sigma^2} \qquad \frac{-\|x_1 - c_2\|}{2\sigma^2} \qquad \phi(x_1)^{\top} \\ X = \begin{pmatrix} \frac{-\|x_1 - c_1\|}{2\sigma^2} & \frac{-\|x_2 - c_2\|}{2\sigma^2} \\ \frac{-\|x_2 - c_1\|}{2\sigma^2} & \frac{-\|x_2 - c_2\|}{2\sigma^2} \\ \frac{-\|x_3 - c_1\|}{2\sigma^2} & \frac{-\|x_3 - c_2\|}{2\sigma^2} \end{pmatrix} \phi(x_3)^{\top}$$

Assignment: RBF Features

 Task 2.1: "Take all training instances to assign as centers" means create a set of centers

$$c_i = x_i$$
 For all input instance x_i

Create a matrix of all centers as

$$C = \begin{bmatrix} c_1^T \\ c_2^T \\ \vdots \\ c_n^T \end{bmatrix} = \begin{bmatrix} x_1^T \\ x_2^T \\ \vdots \\ x_n^T \end{bmatrix}$$
 n is the number of training instances

So we will receive n features as $\phi_i(x) = e^{\frac{-\|x-x_i\|}{2\sigma^2}}$

$$\phi(x) = [1, \phi_1(x), \phi_2(x), \dots, \phi_n(x)]$$

Assignment: RBF Features

• Snippet code to compute RBF features for given training input data X, and a matrix of centers C.