01

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1 Homework 01

Name: Gary Pham - gp492

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[109]:
```
using Plots
using Printf
using LaTeXStrings
```

[110]:
```
default(lw=2, markersize=6,
    xtickfont=font(12), ytickfont=font(12),
    guidefont=font(14), legendfont=font(12), titlefont=font(12))
```

[111]:
```
function bisection(f, a, b, n_max, tol; verbose=true)

    converged = false
    p = 0
    for i in 1:n_max

        p = 0.5 * (a + b) # compute the midpoint

        # print current iterate information to screen
        if verbose
            @printf("%3d: a = %.4f, b = %.4f, p = %.4f, f(a)f(p) = %7.4f,␣
 ↪|f(p)| = %.4g\n",
                i, a, b, p, f(a) * f(p), abs(f(p)))
        end

        # determine if the root is in the left or right interval
        if (f(a) * f(p) <= 0)
            b = p # root is interval [a,p]
        else
            a = p # root is in interval [p,b]
        end
        if (abs(f(p)) == 0)
            converged = true
```

```
            break
        end

        # test for convergence
        if 0.5 * (b - a) < tol
            converged = true
            break
        end
    end

    if !converged
        @printf("ERROR: Did not converge after %d iterations\n", n_max)
    end

    return p # return midpoint guess

end
```

```
bisection (generic function with 1 method)
```

[112]:
```
function fixed_point_iteration(g, p, n_max, rel_tol; verbose=true)
    p_old = p
    for i in 1:n_max
        p = g(p)
        @printf("%3d: p = %.12f\n", i, p)
        if (i > 1)
            if abs(p_old - p) / abs(p) < rel_tol
                break
            end
        end
        p_old = p
    end

    return p

end
```

```
fixed_point_iteration (generic function with 1 method)
```

1.1 Section 2.1

1.1.1 Ex 3.c

Use the Bisection method to find solutions accurate to within $f(x) = x^3 - 7x^2 + 14x - 6$ on $[3.2, 4]$.

[113]:
```
# code
f(x) = x^3 - 7x^2 + 14x - 6
a = 3.2
```

```
b = 4.0
n_max = 100
tol = 1e-2
bisection(f, a, b, n_max, tol)
```

```
 1: a = 3.2000, b = 4.0000, p = 3.6000, f(a)f(p) = -0.0376, |f(p)| = 0.336
 2: a = 3.2000, b = 3.6000, p = 3.4000, f(a)f(p) =  0.0018, |f(p)| = 0.016
 3: a = 3.4000, b = 3.6000, p = 3.5000, f(a)f(p) = -0.0020, |f(p)| = 0.125
 4: a = 3.4000, b = 3.5000, p = 3.4500, f(a)f(p) = -0.0007, |f(p)| = 0.04613
 5: a = 3.4000, b = 3.4500, p = 3.4250, f(a)f(p) = -0.0002, |f(p)| = 0.01302
 6: a = 3.4000, b = 3.4250, p = 3.4125, f(a)f(p) =  0.0000, |f(p)| = 0.001998
```

```
3.4125000000000005
```

1.1.2 Ex 14

Find an approximation to $\sqrt{3}$ correct to within 10^{-4} using the Bisection Algorithm.

[Hint: Consider $f(x) = x^2 - 3$].

[114]:
```
# code
f(x) = x^2 - 3
a = 1
b = 2
tol = 1e-4
n_max = 100
bisection(f, a, b, n_max, tol)
```

```
 1: a = 1.0000, b = 2.0000, p = 1.5000, f(a)f(p) =  1.5000, |f(p)| = 0.75
 2: a = 1.5000, b = 2.0000, p = 1.7500, f(a)f(p) = -0.0469, |f(p)| = 0.0625
 3: a = 1.5000, b = 1.7500, p = 1.6250, f(a)f(p) =  0.2695, |f(p)| = 0.3594
 4: a = 1.6250, b = 1.7500, p = 1.6875, f(a)f(p) =  0.0547, |f(p)| = 0.1523
 5: a = 1.6875, b = 1.7500, p = 1.7188, f(a)f(p) =  0.0070, |f(p)| = 0.0459
 6: a = 1.7188, b = 1.7500, p = 1.7344, f(a)f(p) = -0.0004, |f(p)| = 0.008057
 7: a = 1.7188, b = 1.7344, p = 1.7266, f(a)f(p) =  0.0009, |f(p)| = 0.01898
 8: a = 1.7266, b = 1.7344, p = 1.7305, f(a)f(p) =  0.0001, |f(p)| = 0.005478
 9: a = 1.7305, b = 1.7344, p = 1.7324, f(a)f(p) = -0.0000, |f(p)| = 0.001286
10: a = 1.7305, b = 1.7324, p = 1.7314, f(a)f(p) =  0.0000, |f(p)| = 0.002097
11: a = 1.7314, b = 1.7324, p = 1.7319, f(a)f(p) =  0.0000, |f(p)| = 0.000406
12: a = 1.7319, b = 1.7324, p = 1.7322, f(a)f(p) = -0.0000, |f(p)| = 0.0004397
13: a = 1.7319, b = 1.7322, p = 1.7321, f(a)f(p) = -0.0000, |f(p)| = 1.682e-05
```

```
1.7320556640625
```

1.2 Section 2.2

1.2.1 Ex 1

Use algebraic manipulation to show that each of the following functions has a fixed point at p precisely when $f(p) = 0$, where $f(x) = x^4 + 2x^2 - x - 3$.

- $g_1(x) = (3 + x - 2x^2)^{\frac{1}{4}}$
- $g_2(x) = \left(\frac{x+3-x^4}{2}\right)^{\frac{1}{2}}$
- $g_3(x) = \left(\frac{x+3}{x^2+2}\right)^{\frac{1}{2}}$
- $g_4(x) = \frac{3x^4+2x^2+3}{4x^3+4x-1}$

1.2.2 Ex 8

Use a fixed-point iteration method to determine a solution accurate to within 10^{-2} for $x^3-x-1=0$ on $[1,2]$. Use $p_0 = 1$

[115]:
```
# code
g(x) = x^3 - x - 1
p0 = 1
tol = 1e-2
n_max = 100
p = fixed_point_iteration(g, p0, n_max, tol)
```

```
 1: p = -1.000000000000
 2: p = -1.000000000000
```

```
-1
```