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TESTING THE CONVERGENCE HYPOTHESIS

Frank R. Lichtenberg*

Abstract—We show that, contrary to the beliefs of some previous analysts of international economic growth, the hypotheses of convergence and of mean-reversion are not equivalent. Under some assumptions, the rate of convergence is independent of the degree of mean-reversion; under other assumptions, mean-reversion is a necessary, but not a sufficient, condition for convergence. We show the relationship between the convergence test and the mean-reversion test, and provide an empirical example in which the null hypothesis of no mean reversion is rejected, but the null hypothesis of no convergence is not rejected.

The convergence hypothesis is the hypothesis that

$$\frac{d[\text{var}(\ln Y_t)]}{dt} < 0 \tag{1}$$

where Y_t is (labor or total-factor) productivity at time t and var( ) denotes the variance across countries. When there are only two time periods, indexed by 0 and 1, the hypothesis may be expressed as

$$[\text{var}(\ln Y_0)]/[\text{var}(\ln Y_1)] > 1. \tag{2}$$

A number of papers have examined evidence related to the convergence hypothesis, but few, if any, of these have formally tested it. Baumol and Wolff (1988) and Dowrick and Nguyen (1989)[1] calculated var(ln Y_t) (or a similar measure, the coefficient of variation of $Y_t, CV(Y_t) \equiv [\text{var}(Y_t)]^{1/2}/[\text{mean}(Y_t)]$ for various t, but they did not formally test the first inequality by regressing var(ln Y_t) or $cv(Y_t)$ on t or the second inequality by performing an F-test on the variance ratio.

Much of the empirical work concerning convergence has been based on the regression equation

$$y_1 - y_0 = \beta y_0 + u \tag{3}$$

where $y_1 \equiv \ln Y_1$, $y_0 \equiv \ln Y_0$, and the intercept is suppressed for simplicity.[2] This may also be expressed in the form

$$\begin{aligned} y_1 &= (1+\beta)y_0 + u \\ &= \pi y_0 + u \end{aligned} \tag{4}$$

where $\pi \equiv (1+\beta)$. It is assumed that $-1 \leq \beta \leq 0$ and that $0 \leq \pi \leq 1$. Analysts have estimated equation (3) or (4) in order to test the hypothesis that $\beta < 0$ or that $\pi < 1$. We refer to this as the "mean-reversion hypothesis," the hypothesis that countries with the lowest initial productivity level tended to have the highest subsequent productivity growth.

It appears that at least some writers considered the mean-reversion hypothesis to be *equivalent* to the convergence hypothesis, in the sense that rejection of the null hypothesis of no mean reversion (H_0: $\beta = 0$) is tantamount to rejection of the null hypothesis of no convergence (H_0: [var y_0]/[var y_1] = 1). Mankiw, Romer, and Weil (1992), for example, conclude from their finding that b (the OLS estimate of β) is significantly less than zero that "there is a significant tendency towards convergence in the OECD sample."

Barro and Sala-i-Martin (1991, p. 112) recognized that the mean-reversion and convergence hypotheses are not equivalent, and that a negative "β coefficient need not imply that the cross-sectional dispersion of per capita output... diminishes over time." They did not, however, show precisely how, and under what conditions, mean-reversion and convergence are related to one another. That is the purpose of this note.

We show that, under certain assumptions, the degree of convergence does not depend at all on the degree of mean-reversion. Under other assumptions (presumably those implicitly maintained by previous authors), mean-reversion is a necessary condition for convergence, but not a sufficient condition; convergency may fail to hold even when $\beta < 0$. We show that in this case the degree of convergence depends not only on β (or π) but also on the R^2 of equation (4), i.e., on the relative importance of random disturbances in determining productivity. We also provide a formal test of the convergence hypothesis for OECD countries during the period 1960–85.

I. Convergence When y_0 and y_1 Are Generated by the Same Process

Equation (4) may be considered a (one-period) special case of an equation of the form

$$y_t = \pi y_{t-1} + u_t. \tag{5}$$

If equation (5) applies to all t ($t = \ldots, -2, -1, 0, 1, 2, \ldots$) then y_t may be expressed as

$$y_t = u_t + \pi u_{t-1} + \pi^2 u_{t-2} + \pi^3 u_{t-3} + \cdots. \tag{6}$$

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[1] See also Barro (1991), De Long (1988), and De Long and Summers (1991).

[2] Actually, the equations estimated by most authors include other regressors (x) in addition to y_0; they are of the form $y_1 - y_0 = \beta y_0 + \theta x + u$. We show in section IV that our argument also applies to the multivariate case.

If we assume that the u's are i.i.d. with mean zero and variance σ_u^2, then one can easily show that the variance of y_t is

$$\text{var}\, y_t = \sigma_u^2/(1-\pi^2)$$

which does not depend on t. Even if the y series is characterized by mean reversion ($\pi < 1$), the distribution of y will fail to converge: $[(\text{var}\, y_t)/(\text{var}\, y_{t-k})] = 1$ for all t and k.

Suppose we permit y to converge by allowing the variance of the shocks to decline or depreciate over time. Does the rate of convergence depend upon the degree of mean reversion? Suppose that $\text{var}\, u_{t-k} = \sigma_u^2(1+(\delta/2))^{2k}$ where $0 < \delta < 1$ is the annual rate at which the variance of the shocks declines. Then

$$\text{var}\, y_t = \left[(1-\delta/2)^{2t}\sigma_u^2\right] \Big/ \left[(1-\pi^2(1-\delta/2)^{-2}\right].$$

The variance of y_t is decreasing in t:

$$[d\ln(\text{var}\, y_t)]/dt \approx -\delta.$$

The lower is π (the greater the degree of mean reversion), the lower the variance of y at any given t, but the derivative of $\ln(\text{var}\, y_t)$ with respect to t—the (percentage) rate of convergence—does not depend on π.

The above discussion indicates that if we regard y_0 as being determined by an autoregressive process similar to the process determining y_1, then either convergence cannot occur at all or if it can, its rate is independent of the degree of mean reversion.

II. Convergence When y_0 and y_1 Are Generated by Different Processes

For the degree of mean reversion to affect the rate of convergence, it seems that we must drop the assumption that y_1 and y_0 are generated by the same autoregressive process. Let us assume instead that y_1 is determined according to equation (4), and that the y_0 are simply random draws from some arbitrary distribution. In this case the convergence and mean-reversion hypotheses are related, but not equivalent.

From equation (4), the variance of y_1, σ_1^2, may be expressed as

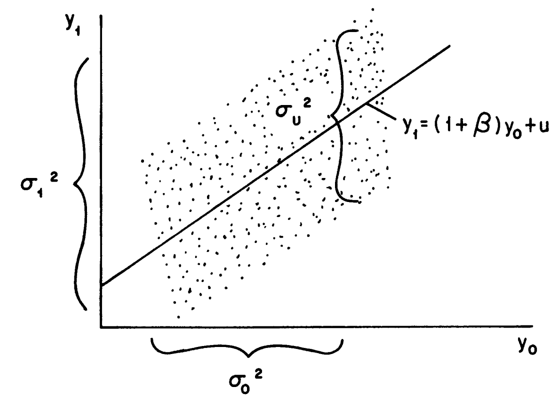
$$\sigma_1^2 = (1+\beta)^2\sigma_0^2 + \sigma_u^2 \tag{7}$$

where $\sigma_0^2 \equiv \text{var}\, y_0$. Hence

$$\begin{aligned}\sigma_1^2/\sigma_0^2 &= (1+\beta)^2 + \sigma_u^2/\sigma_0^2 \\ &= \pi^2 + \sigma_u^2/\sigma_0^2.\end{aligned} \tag{8}$$

Equation (8) reveals that the degree of convergence depends not only on the degree of mean-reversion π, but also on the variance of the disturbance relative to the variance of initial productivity. Mean reversion is a necessary, but not a sufficient, condition for productivity convergence: if there is enough "noise" in the process determining y_1, σ_1^2/σ_0^2 will exceed 1 even when $\pi < 1 (\beta < 0)$. The scatterplot of y_1 against y_0 in figure 1 provides a graphical illustration. Given the variance of y_0, the variance of y_1 depends positively on both the slope of the regression line (π) and the dispersion of observations around the regression line (σ_u^2). The goodness of fit, as well as the slope, of the relationship between y_1 and y_0 determines whether or not there is convergence.

FIGURE 1.—$\sigma_1^2 = (1+\beta)^2\sigma_0^2 + \sigma_u^2$



This implies that the usual t-test on the significance of β is not a proper test of the convergence hypothesis. However there is an appropriate test of the convergence hypothesis that involves β—and another regression statistic, R^2—that is equivalent to the familiar F-test of the equality of variances σ_1^2 and σ_0^2. By definition,

$$1 - R^2 \equiv \sigma_u^2/\sigma_1^2.$$

Hence,

$$\sigma_u^2 = (1-R^2)\sigma_1^2.$$

From (7),

$$\sigma_1^2 = (1+\beta)^2\sigma_0^2 + (1-R^2)\sigma_1^2.$$

Rearranging,

$$\sigma_0^2/\sigma_1^2 = R^2/(1+\beta)^2 = R^2/\pi^2. \tag{9}$$

The test of the mean reversion hypothesis $\beta < 0$ (based on the t distribution with $n-2$ degrees of freedom) is equivalent to a test of $1/(1+\beta)^2 > 1$ (based on the F-distribution with $1, n-2$ degrees of freedom). Equation (9) reveals, however, that this statistic will generally overstate the degree of convergence; we should multiply it by R^2; this "adjusts" for the amount of noise in the regression. The test statistic indicated by equation (9) has an F distribution with $n-2$, $n-2$ degrees of freedom.

Another way of expressing the test statistic of equation (9) is

$$R^2/\pi^2 = \left(\pi^2 + \left(\sigma_u^2/\sigma_0^2\right)\right)^{-1}. \tag{10}$$

The convergence test statistic is inversely related to π: the lower (closer to 0) is π, i.e., the greater the degree of mean-reversion, the higher is the test statistic, and the greater is the probability of rejecting the null hypothesis of no convergence. But for given π, the higher the relative noise level (σ_u^2/σ_0^2), the lower the probability of rejection.

III. An Empirical Example

To illustrate our argument, consider the following regression estimates based on 1960 and 1985 per capita output data for 22 OECD countries:

$$\ln GDP85 = \text{constant} + \underset{(0.079)}{0.715} \ln GDP60 + e$$

$$R^2 = 0.802$$

(standard error in parentheses). π, the coefficient on $\ln GDP60$, is highly significantly less than 1, so we can reject the null hypothesis of no mean reversion at the 0.0001 level. To test the null hypothesis of no convergence, we compute the statistic

$$R^2/\pi^2 = 0.802/(0.715)^2 = 1.57$$

which is equal, of course, to the ratio

$$\mathrm{var}(\ln GDP60)/\mathrm{var}(\ln GDP85).$$

Although this ratio is greater than one, it is far from significant: the probability value is 0.31. In a sample of 22 countries, the variance of log productivity has to decline about 53% (66%) to reject the null hypothesis of no convergence at the 0.05 (0.01) level; the observed variance declined by only 36%.[3]

IV. Generalization to the Multivariate Case

As noted earlier, the models analyzed by most investigators include other regressors (or covariates) in addition to initial productivity. We show here that our argument about the interpretation of the coefficient on y_0 still applies in the presence of other right-hand-side variables.

Consider the Solow model analyzed by Mankiw, Romer, and Weil (1991); that model consists of three basic equations:

$$y^* = \alpha x \tag{11}$$

$$y_0 = y^* + \epsilon \tag{12}$$

$$y_1 - y_0 = \mu(y^* - y_0) + v \tag{13}$$

where y^* denotes steady-state productivity. Equation (11) says that y^* (which is not directly observable) depends on a set of exogenous variables such as rates of investment in physical and human capital and the population growth rate. The disturbance ϵ in equation (12) is assumed to have the properties $E(\epsilon) = E(\epsilon y^*) = 0$: deviations of countries' initial productivity from their steady-state productivity are assumed to be random. Equation (13) is an adjustment equation implied by the Solow model: the growth rate of productivity is proportional to the deviation of initial productivity from y^*, where $0 < \mu < 1$. (μ is the "speed of adjustment" and depends on α.) Substituting (11) into (13) and rearranging terms, we obtain a reduced-form equation of the kind estimated in previous studies:

$$y_1 = (1 - \mu)y_0 + \mu\alpha x + u. \tag{14}$$

y_1 depends on x as well as y_0. Of course, y_0 also depends on x; substituting (11) into (12),

$$y_0 = \alpha x + \epsilon. \tag{15}$$

We can express y_1 as a function of x alone by substituting (15) into (14):

$$y_1 = \alpha x + (1 - \mu)\epsilon + u. \tag{16}$$

We now define $y_0' \equiv y_0 - y^*$ and $y_1' \equiv y_1 - y^*$ as deviations of productivity from its steady-state level at times 0 and 1, respectively. From (15) and (16),

$$y_0' = y_0 - \alpha x = \epsilon \tag{17}$$

$$y_1' = (1 - \mu)\epsilon + u. \tag{18}$$

Hence $\mathrm{var}(y_0') = \sigma_\epsilon^2$, $\mathrm{var}(y_1') = (1 - \mu)^2\sigma_\epsilon^2 + \sigma_u^2$, and

$$\frac{\mathrm{var}(y_1')}{\mathrm{var}(y_0')} = (1 - \mu)^2 + \frac{\sigma_u^2}{\sigma_\epsilon^2} \tag{19}$$

where $\sigma_\epsilon^2 \equiv \mathrm{var}(\epsilon)$ and $\sigma_u^2 \equiv \mathrm{var}(u)$.

Equation (19) is analogous to equation (8): the relative variances of time − 1 and time − 0 *deviations from steady-state productivity* are related to the *partial* regression coefficient of y_0 in equation (14) in the same way that the relative variances of "unstandardized" productivity are related to the *simple* regression coefficient in equation (4). A faster speed of adjustment to the steady state (μ) will lower the variance of deviations of current income (y_1) from the steady state, by reducing the impact of "old" shocks (ϵ). But whether or not these deviations tend to converge also depends on the ratio of the variance of the new shocks (u) to that of the old shocks.

[3] When a similar regression is run on a much larger sample of countries (including less developed countries), the slope is insignificantly different from unity: we fail to reject not only the null hypothesis of no convergence, but also the (weaker) hypothesis of no mean reversion.

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THE MACROECONOMIC CONSEQUENCES OF THE SAVINGS AND LOAN DEBACLE

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Abstract—This paper used a general equilibrium framework to examine the macroeconomic consequences of the recent failures and subsequent bailout in the savings and loan industry. We distinguish between the losses in the capital stock, the economic effects of alternative methods of funding those real losses, and the intertemporal transfer of real resources implicit in backing the financial assets used. We then embed the analysis in a general equilibrium, multi-country model with intertemporal budget constraints that allows for the interaction of intertemporal adjustment and expectation revisions. The more complete model is used to explore the consequences of the S&L debacle on the evolution of the U.S. economy during the 1980's and 1990s.

I. Introduction[1]

The present value of the cost to U.S. taxpayers of cleaning up the savings and loan debacle was estimated in 1991 by the Congressional Budget Office (1991) to be more than $215 billion, but this represents only the direct monetary costs to the government of upholding the deposit insurance system.[2] Not included in this estimate is the value of production and consumption that has been, and will be, foregone as a result of poor investments made by thrift institutions during the 1980s and into the early years of the 1990s. In addition, this estimate ignores losses in physical capital accumulation caused by the overconsumption of households who underestimated the extent of the wealth loss during the 1980s.

Three aspects of the S&L failures and subsequent government bailout help to identify the macroeconomic effects of the S&L crisis. The first is the losses in productive capital due to the misallocation of resources within the savings and loan industry, and the second is the implicit guarantee that the government issued to make depositors whole. The third piece emerged when the government announced explicit funding for the deposit reimbursements.

Some authors have argued that resolving the S&L problems has no real consequences because the resolution is merely replacing an implicit guarantee by an

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[1] This paper describes work that underlies a recent Congressional Budget Office Study (1992).

[2] The estimate has been revised down to about $165 billion as of August 1993 because thrifts have recovered faster than anticipated. All macroeconomic effects would be reduced by about 23%. Other, much higher figures—on the order of $500 billion—are sometimes cited, but such estimates are misleadingly high because they include the undiscounted value of future interest costs.